2002 SCHEME

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MAT41

Fourth Semester B.E. Degree Examination, June 2012
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, choosing at least one question from each part.***
2. Use of statistical tables is permitted.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. If f(z) is analytic, then prove that, $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$ **(06 Marks)**

b. Find the analytic function whose imaginary part is $e^x(x\sin y + y\cos y)$. **(07 Marks)**

c. Prove that a bilinear transformation preserves cross-ratio of four points. **(07 Marks)**

2 a. State and prove Cauchy's theorem. **(06 Marks)**

b. Expand $f(z) = \frac{z}{(z+1)(z+2)}$ in a Laurent's series about z = –2. **(07 Marks)**

c. Find the residue of $f(z) = \frac{z^3}{(z-1)(z-2)(z-3)}$ at each of its poles. **(07 Marks)**

PART – B

3 a. Prove that, $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$. **(06 Marks)**

b. Prove that, $e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. **(07 Marks)**

c. Prove that, $\int_0^1 x\, J_n(\alpha x) J_n(\beta x)\,dx = 0$. **(07 Marks)**

4 a. Prove that, $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}$. **(06 Marks)**

b. Show that, $\int_{-1}^{1} xP_n(x)P_{n-1}(x)\,dx = \frac{2n}{4n^2-1}$ **(07 Marks)**

c. Prove that, $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$ **(07 Marks)**

PART – C

5 a. Fit a straight line to the following data: **(06 Marks)**

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

b. Calculate the coefficient of correlation between X and Y series from the following data:

	Series	
	X	Y
No. of pairs of observations	15	15
Arithmetic mean	25	18
Standard deviation	3.01	3.03
Sum of squares of deviations from mean	136	138

Summation of product deviations of X and Y series from their respective arithmetic means = 122. **(07 Marks)**

c. State and prove Baye's theorem. **(07 Marks)**

6 a. The probability distribution of a finite random variable X is given by the following table:

x_i	-2	-1	0	1	2	3
$P(x_i)$	0.1	K	0.2	2k	0.3	K

i) Find the value of k and calculate the mean and variance.
ii) Evaluate $P(X < 1)$. **(06 Marks)**

b. The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that
i) exactly two pens will be defective. ii) at most two pens will be defective. iii) none will be defective. **(07 Marks)**

c. It is known from past experience that in a certain plant there are on the average 4 industrial accidents per month. Find the probability that in a given year there will be less than 4 accidents. Assume Poisson distribution ($e^{-4} = 0.0183$). **(07 Marks)**

PART – D

7 a. The weights of workers in a large factory are normally distributed with mean 68 kgs and standard deviation 3 kgs. If 80 samples consisting of 35 workers each are chosen, how many of the 80 samples will have the mean between 67 and 68.25 kgs? Given $P[0 \le z \le 2] = 0.4772$ and $P[0 \le z \le 0.5] = 0.1915$. **(06 Marks)**

b. Explain the following terms:
i) Type I and Type II errors ii) Null hypothesis. **(07 Marks)**

c. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. **(07 Marks)**

8 a. Let X and Y be two random variables each taking three values –1, 0 and 1 and having the joint probability distribution, as shown below:

y \ x	-1	0	1
-1	0	0.1	0.1
0	0.2	0.2	0.2
1	0	0.1	0.1

Obtain the marginal probability distribution of X and Y and hence their expected values. **(06 Marks)**

b. The initial probability matrix of a Markov chain is given by $P = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$ and the initial probability distribution is $P^{(0)} = (\frac{1}{2}, \frac{1}{2}, 0)$. Find the $P_{13}^{(2)}$, $P_{23}^{(2)}$ and $P_1^{(2)}$. **(07 Marks)**

c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? **(07 Marks)**

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