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First Semester B.E. Degree Examination, June 2012
Engineering Mathematics – I

Time: 3 hrs.

Max. Marks:100

- Note: 1. Answer any FIVE full questions, choosing at least two from each part.**
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

- 1 a.** Choose your answers for the following :
- i) The n^{th} derivative of $\cos^2 x$ is
 A) $2^{n-1} \cos\left(2x + \frac{n\pi}{2}\right)$ B) $2^n \cos\left(2x + \frac{n\pi}{2}\right)$ C) $2^{n-1} \cos(2x + n\pi)$ D) $2^{n-1} \cos\left(\frac{n\pi}{2}\right)$
- ii) The value of C of the Cauchy mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in $[4, 5]$ is
 A) $\frac{5}{2}$ B) $\frac{3}{2}$ C) $\frac{9}{2}$ D) $\frac{1}{2}$
- iii) Find the n^{th} derivative of $y = x^{n-1} \log x$ is
 A) $y_n = \frac{(n+1)!}{x}$ B) $y_n = \frac{n!}{x}$ C) $y_n = \frac{(n-1)!}{x}$ D) $y_n = \frac{n!}{x^2}$
- iv) Maclaurin's series expansion of $\log(1+x)$ is
 A) $x + \frac{x^2}{2} + \frac{x^3}{5} + \dots$ B) $x - \frac{x^2}{3} + \frac{x^4}{5} - \dots$
 C) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{5} + \dots$ D) $x + \frac{x^2}{3} + \frac{x^3}{16} + \dots$ **(04 Marks)**
- b.** By informing in two different ways the n^{th} derivative of x^{2n} , prove that

$$1 + \frac{n^2}{1^2} + \frac{n^2(n-1)^2}{1^2 \times 2^2} + \frac{n^2(n-1)^2(n-2)^2}{1^2 \times 2^2 \times 3^2} + \dots = \frac{(2n)!}{(n!)^2}$$
 (06 Marks)
- c.** Verify Rolle's theorem for the function $f(x) = \frac{\sin 2x}{e^{2x}}$ in $\left[0, \frac{\pi}{2}\right]$. **(04 Marks)**
- d.** Using Maclaurin's series, expand $\log \sec x$ upto the term containing x^6 . **(06 Marks)**
- 2 a.** Choose your answers for the following :
- i) The value of $\lim_{x \rightarrow \infty} \frac{\log x}{x}$ is
 A) 0 B) 1 C) 2 D) -2
- ii) If s is the arc length of the curve $x = f(y)$ then $\frac{ds}{dy}$ is
 A) $\sqrt{1+y_1^2}$ B) $\sqrt{1+y_1}$ C) $\sqrt{1+\left(\frac{dx}{dy}\right)^2}$ D) $\sqrt{\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)^2}$
- iii) Pedal equation to the curve $\frac{2a}{r} = 1 - \cos \theta$ is
 A) $P = ar^2$ B) $P^2 = a^2 r$ C) $P^2 = a^2 r^2$ D) $P^2 = ar$
- iv) The angle between two curves $r = ae^\theta$ and $re^\theta = b$ is
 A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) 0 D) π **(04 Marks)**

- 2 b. For the curve $y = \frac{ax}{a+x}$, if ρ is the radius of curvature at any point (x, y) , show that:

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2 \quad (06 \text{ Marks})$$

- c. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^{1/x^2}$. (04 Marks)

- d. Find the angle between the curves $r = \frac{a}{1+\cos\theta}$; $r = \frac{b}{1-\cos\theta}$. (06 Marks)

- 3 a. Choose your answers for the following :

- i) When $u = y^2 \log\left(\frac{x}{y}\right)$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is

A) u B) u^2 C) $2u$ D) $3u$

- ii) The Taylor's series of $f(x, y) = xy$ at $(1, 1)$ is

A) $1 + [(x-1) + (y-1)]$ B) $1 + [(x-1) + (y-1)] + [(x-1)(y-1)]$
C) $[(x-1)(y-1)]$ D) None of these

- iii) The Jacobian of transformation from the Cartesian to polar coordinates system is

A) r^3 B) r C) r^2 D) $r \sin \theta$

- iv) The rectangular solid of maximum volume which can be inscribed in a sphere is

A) parallelogram B) rectangle C) cube D) triangle. (04 Marks)

- b. Examine the function $\sin x + \sin y + \sin(x+y)$ for extreme values. (06 Marks)

- c. Find the possible error in percent in computing the parallel resistance 'r' of two resistances

r_1 and r_2 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ are both in error by 2%. (04 Marks)

- d. If $z(x+y) = x^2 + y^2$ show that $\left[\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]^2 = 4\left[1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]$. (06 Marks)

- 4 a. Choose your answers for the following :

- i) A gradient of the scalar point function ϕ that is $\nabla\phi$ is

A) vector function B) scalar function C) zero D) ϕ

- ii) The directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the $(1, -2, -1)$ in the direction PQ where $P = (1, 2, -1)$, $Q = (-1, 2, 3)$ is

A) $\frac{28}{\sqrt{5}}$ B) $\frac{30}{\sqrt{4}}$ C) $\frac{-28}{\sqrt{5}}$ D) $\frac{20}{\sqrt{6}}$

- iii) If \vec{R} is the position vector of any point $P(x, y, z)$ then $\nabla \cdot \vec{R}$ is

A) 3 B) -3 C) 2 D) 0

- iv) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $\text{Curl } \vec{r} = \dots\dots\dots$

A) 0 B) 1 C) -1 D) ∞ (04 Marks)

- b. Find the constants a and b such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational and find scalar potential function ϕ such that $F = \nabla\phi$. (06 Marks)

- c. Prove that $\nabla_x \left[\frac{ax\vec{r}}{r^n} \right] = \frac{-a}{r^3} + \frac{3(a\cdot\vec{r})\vec{r}}{r^5}$. (04 Marks)

- d. Prove that the cylindrical coordinates system is orthogonal. (06 Marks)

PART – B

5 a. Choose your answers for the following :

i) The value of $\int_0^1 x^2(1-x^2)^{\frac{3}{2}} dx$ is

- A) $\frac{\pi}{23}$ B) $\frac{1}{32}$ C) $\frac{\pi}{32}$ D) $\frac{\pi}{16}$

ii) The tangent to the curve $y^2 = 4ax$ at origin is

- A) y-axis B) x-axis
C) both x-axis and y-axis D) does not exist

iii) The value of $\int_0^{\pi} \sin^4\left(\frac{x}{2}\right) dx$ is

- A) $\frac{3\pi}{18}$ B) $\frac{3\pi}{8}$ C) $\frac{3\pi}{16}$ D) $\frac{3\pi^2}{8}$

iv) The surface area of the sphere of radius 'a' is

- A) $4\pi a^2$ B) $4\pi^2 a$ C) $4\pi a$ D) $2\pi a^2$ (04 Marks)

b. Obtain the reduction formula for $\int \sin^m x \cos^n x dx$. (06 Marks)

c. Evaluate $\int_0^{\pi} \frac{\tan^{-1}(ax)}{x(1+x^2)} dx$ using the method of differentiation under integral sign. (04 Marks)

d. Find the area of the loop of the curve $ay^2 = x^2(a-x)$. (06 Marks)

6 a. Choose your answers for the following :

i) The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

- A) $e^x + e^{-y} = c$ B) $e^{-x} + e^{-y} = c$ C) $e^x + e^y = c$ D) $e^{x+y} = c$

ii) If the homogeneous differential equation $\frac{dy}{dx} = \frac{f_1(x, y)}{f_2(x, y)}$ the degree of the homogeneous functions $f_1(x, y)$ and $f_2(x, y)$ are

- A) different B) same
C) relatively prime D) degree of $f_1(x, y) >$ degree of $f_2(x, y)$

iii) The integrating factor of the differential equation $(1+x^2)\frac{dy}{dx} + xy = \sin^{-1}x$ is

- A) $\frac{1}{\sqrt{1+x^2}}$ B) $\sqrt{1-x^2}$ C) $\sqrt{1+x^2}$ D) $\frac{x}{\sqrt{1+x^2}}$

iv) If replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in the differential equation $f\left(x, y, \frac{dy}{dx}\right) = 0$ we get the differential equation of

- A) polar trajectory B) orthogonal trajectory
C) parametric trajectory D) parallel trajectory (04 Marks)

b. Solve $(1+xy^2)\frac{dy}{dx} = 1$. (06 Marks)

c. Solve $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y \cos y}$. (04 Marks)

d. Find the orthogonal trajectory of $r^n = a^n \sin n\theta$ (06 Marks)

7 a. Choose your answers for the following :

- i) In a system of linear equations if the rank of the co-efficient matrix = rank of the augmented matrix = n number of unknowns then the system has
 A) no solutions B) unique solutions
 C) infinite number of solutions D) trivial solutions

- ii) The rank of matrix $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$ is

A) 3 B) 4 C) 2 D) 5

- iii) A square matrix in which $a_{ij} = a_{ji}$ for all i and j then it is called a

A) unique matrix B) symmetric matrix C) skew symmetric D) triangular matrix

- iv) The inverse of the square matrix A is

A) $|A|$ B) $\frac{\text{adj } A}{|A|}$ C) $\text{adj } A$ D) $\frac{|A|}{\text{adj } A}$ (04 Marks)

- b. Investigate for what value of λ and μ the simultaneous equation $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have

i) no solutions ii) unique solutions iii) infinite number of solutions. (06 Marks)

- c. Apply Gauss-elimination method to solve the following equations:

$$2x - y + 3z = 1, \quad -3x + 4y - 5z = 0, \quad x + 3y - 6z = 0 \quad (04 \text{ Marks})$$

- d. Find the rank of $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$. (06 Marks)

8 a. Choose your answers for the following :

- i) The eigen values of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ are

A) 2, 3, 8 B) 2, 2, 8 C) 8, 4, 3 D) 2, -2, 8

- ii) A homogeneous expression of the second degree in any number of variables is called a

A) quadratic form B) diagonal form C) symmetric form D) spectral form

- iii) A square matrix A of order 3 has 3 linearly independent eigen vectors then a matrix P can be found such that $P^{-1}AP$ is a

A) diagonal matrix B) symmetric matrix C) unit matrix D) singular matrix

- iv) If the eigen vector is (1, 1, 1) then its normalized form is

A) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ B) $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$
 C) $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ D) $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (04 Marks)

- b. Reduce $6x^2 + 3y^2 - 4xy - 2yz + 4zx$ into canonical form. (06 Marks)

- c. Find all the eigen values for the matrix, $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$. (04 Marks)

- d. Reduce the matrix, $A = \begin{bmatrix} 11 & -4 & 7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ into a diagonal matrix. (06 Marks)

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