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On Status Indices of Some Graphs

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Abstract: Ramane, Yalnaik recently defined another molecular structural descriptor on the lines of Weiner index, Zagreb Index, etc. Here we construct new graphs of fixed diameter and compute the status indices as well as harmonic status indices of those graphs.

Key Words: Status of vertex, first status connectivity index, second status connectivity index, harmonic status index.

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§1. Introduction

There are several molecular structural graph descriptors such as Weiner Index, Zagreb Index, Hosoya Index etc which strongly correlate studies in graph theory with chemistry. Most of these indices are based on the distance between vertices in a graph.Motivated by harmonic mean we have harmonic index of a graph defined by Fajtlowicz [5]. For more work one can refer [6]. Further motivated by the same, Ramane and Yalnaik introduced the harmonic status index of graphs [4].

Definition 1.1([1]) *The status of a vertex $u \in V(G)$ is defined as the sum of its distance from every other vertex in $V(G)$ and is denoted by $\sigma(u)$. That is*

$$\sigma(u) = \sum_{u \in V(G)} d(u, v).$$

Definition 1.2 *The first status connectivity index $S_1(G)$and second status connectivity index $S_2(G)$ of a connected graph G are defined respectively as*

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)] \text{ and } S_2(G) = \sum_{uv \in E(G)} [\sigma(u)\sigma(v)].$$

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Similarly the first and second status connectivity coindices of a connected graph G are defined as

$$\overline{\mathrm{S}_1}(G) = \sum_{uv \notin E(G)} [\sigma(u) + \sigma(v)] \text{ and } \overline{\mathrm{S}_2}(G) = \sum_{uv \notin E(G)} [\sigma(u)\sigma(v)].$$

Definition 1.3([5]) *The Harmonic index of a graph G is defined as*

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d(u) + d(v)}.$$

The harmonic status index of a connected graph G as ([4])

$$HS(G) = \sum_{uv \in E(G)} \frac{2}{\sigma(u) + \sigma(v)}.$$

Similarly the harmonic status coindex of a connected graph G is defined as

$$\overline{\mathrm{HS}}(G) = \sum_{uv \notin E(G)} \frac{2}{\sigma(u) + \sigma(v)}.$$

§2. Status Connectivity Indices and Coindices of Some Graphs

In what follows, we consider a class of graphs constructed by first joining a path of length $l(\geq 1)$ to each vertex of G and then attaching k pendent vertices to each end vertex of the path attached. Such a graph can be called l level thorn graph denoted by $G^{l(+k)}$. The usual thorny graph G^{+k} can be regarded as 0 level thorn graph. If $l = 1$ we get first level thorn graph $G^{1(+k)}$.

Example 2.1 A graph G and it's first level thorn graph $G^{\wedge 1(+3)}$ are as shown below.

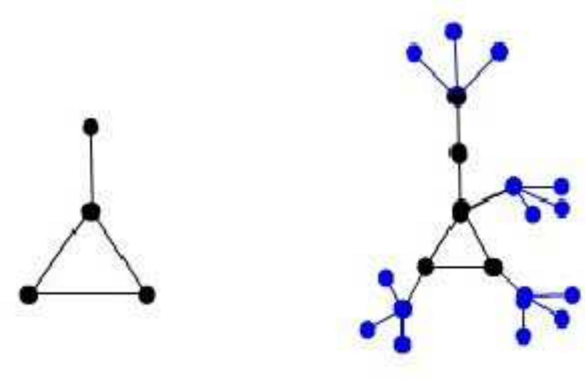

Figure 1

First we evaluate the status connectivity index and coindex of 0 level thorn graphs denoted by G^{+k}.To obtain the harmonic status connectivity index and coindex of this graph we need to calculate status of each vertex and number of pairs of adjacent vertices and pairs of non adjacent vertices in G^{+k}.If G is a r regular graph then,with respect to degree there are two types of vertices in G^{+k},nk pendent vertices (external),n vertices of degree $'r+nk'$ we call them as internal.

Theorem 2.1 *The first and second status connectivity index of thorn graph K_n^{+k} are given by*

$$\begin{aligned} S_1(K_n^{+k}) &= n(n-1)(2nk+n-k-1)+nk(5nk+3n-2k-4) \\ S_2(K_n^{+k}) &= (nk)C_2\times(3nk+2n-k-3)^2+nC_2\times(3nk+2n-k-3)(2nk+n-k-1) \end{aligned}$$

Proof The graph K_n^{+k} is of diameter3and there are two types of vertices in it. A set of $'nk'$ pendent vertices and n vertices of degree $'n+1'$. Let $u_i,\ i=1,2,\cdots,nk$ denote the pendent vertices and v_i,$i=1,2,\cdots,n$ denote the vertices of degree $'n+1'$. Then the status of pendent vertex is

$$\sigma(u_i)=1+2(k-1)+2(n-1)+3k(n-1)=3nk+2nk-3$$

and the status of the internal vertex v_i is

$$\sigma(v_i)=1(n-1)+k+2k(n-1)=(2nk+n-k-1).$$

Now in K_n^{+k} there are $\frac{n(n-1)}{2}$ adjacent pairs internal vertices and nk pairs of vertices forming edges formed by one internal and one external vertex. Hence by definition the status connectivity index of K_n^{+k} is

$$\begin{aligned} S_1(K_n^{+k}) &= \frac{n(n-1)2(2nk+n-k-1)}{2}+nk(3nk+2n-k-3+2nk+n-k-1) \\ &= n(n-1)(2nk+n-k-1)+nk(5nk+3n-2k-4). \end{aligned}$$

Also in K_n^{+k} there are $(nk)C_2$ pairs of nonadjacent pendent vertices and $nk(n-1)$ pairs of nonadjacent pairs of vertices formed by one pendant and one internal vertex. So that status connectivity coindex of K_n^{+k} is

$$S_2(K_n^{+k})=(nk)C_2\times(3nk+2n-k-3)^2+nC_2\times(3nk+2n-k-3)(2nk+n-k-1). \quad \Box$$

Theorem 2.2 *The harmonic status index and coindex of thorn graph K_n^{+k} are given by*

$$\begin{aligned} HS(K_n^{+k}) &= \frac{n(n-1)}{2}\frac{1}{(2nk+n-k-1)}+nk\frac{2}{(5nk+3n-2k-4)}, \\ \overline{HS}(K_n^{+k}) &= (nk)C_2\frac{1}{(3nk+2n-2k-4)}+nk(n-1)\frac{2}{(5nk+3n-k-3)}. \end{aligned}$$

Proof The graph K_n^{+k} is of diameter 4 and there are two types of vertices in it. A set of $'nk'$ pendent vertices and n vertices of degree $'n+1'$. Let $u_i, i=1,2,\cdots,nk$ denote the pendent vertices and $v_i, i = 1, 2, \cdots, n$ denote the vertices of degree $'n+1'$. Then the status of pendent vertex is

$$\sigma(u_i) = 1 + 2(k-1) + 2(n-1) + 3k(n-1) = 3nk + 2n - k - 3$$

and the status of the internal vertex v_i is

$$\sigma(u_i) = 1(n-1) + k + 2k(n-1) = 2nk + n - k - 1.$$

Now in K_n^{+k} there are $\frac{n(n-1)}{2}$ adjacent pairs internal vertices and nk pairs of vertices forming edges formed by one internal and one external vertex. Hence by definition the harmonic status index of K_n^{+k} is

$$\begin{aligned} HS(K_n^{+k}) &= \frac{n(n-1)}{2}\frac{2}{2(2nk+n-k-1)} + nk\frac{2}{(3nk+2n-k-3+2nk+n-k-1)} \\ &= \frac{n(n-1)}{2}\frac{1}{(2nk+n-k-1)} + nk\frac{2}{(5nk+3n-2k-4)}. \end{aligned}$$

Also in K_n^{+k} there are $(nk)C_2$ pairs of nonadjacent pendent vertices and $nk(n-1)$ pairs of nonadjacent pairs of vertices formed by one pendant and one internal vertex. So that harmonic status coindex of K_n^{+k} is

$$\begin{aligned} \overline{HS}(K_n^{+k}) &= (nk)C_2\frac{2}{2(3nk+2n-2k-4)} + nk(n-1)\frac{2}{(3nk+2n-k-3+2nk+n-k-1)} \\ &= nkC_2\frac{1}{(3nk+2n-2k-4)} + nk(n-1)\frac{2}{(5nk+3n-k-3)}. \end{aligned}$$ □

Now, we discuss the status connectivity indices and the coindices of regular graphs with diameter 2.

Theorem 2.3 *If G is 'r' regular graph of diameter 2 then the first and second status connectivity index of G^{+k} are given by*

$$\begin{aligned} S_1(G^{+k}) &= nr(2n+2kr+k-r-2) + nk(5n+5kr+3k-3r-6), \\ S_2(G^{+k}) &= \frac{nr}{2}(2n+2kr+k-r-2)^2 + nk(3n+3kr+2k-r-2). \end{aligned}$$

Proof The proof follows by direct counting. □

Theorem 2.4 *If G is 'r' regular graph of diameter 2 then the first and second status connectivity co index of G^{+k} are given by*

$$\begin{aligned} \overline{\mathrm{S}_1}(G^{+k}) &= \frac{nk(nk-1)}{2}2(3n+3kr+2k-r-4) + nk(n-1)(5n+5kr+3k-3r-6) \\ &\quad +(nC_2 - \frac{nr}{2})(4n+4kr+2k-2r-4) \end{aligned}$$

$$\begin{aligned}
&= nk(nk-1)(3n+3kr+2k-r-4)+nk(n-1)(5n+5kr+3k-3r-6) \\
&\quad +(nC_2-\frac{nr}{2})(4n+4kr+2k-2r-4), \\
\overline{\mathrm{S}_2}(G^{+k}) &= \frac{nk(nk-1)}{2}(3n+3kr+2k-r-4)^2 \\
&\quad +nk(n-1)(3n+3kr+2k-2r-4)(2n+2kr+k-r-2) \\
&\quad +(nC_2-\frac{nr}{2})(2n+2kr+k-r-2)^2.
\end{aligned}$$

Proof The proof follows by direct counting. □

Theorem 2.5 *If G is 'r' regular graph of diameter 2 then the harmonic status index of G^{+k} is*

$$HS(G^{+k}) = \frac{nr}{2}\frac{1}{(2n+2kr+k-r-2)} + nk\frac{2}{(5n+5kr+3k-2r-6)}.$$

Proof First, we observe that if G has diameter 2 then G^{+k} has diameter 4. Hence from the structure we have the status of each internal vertex v_i as

$$\sigma(v_i) = 1.(k+r) + 2kr + 2.(n-1-r) = 2n + 2kr + k - r - 2.$$

Also the status of each pendant vertex u_i as

$$\sigma(u_i) = 1 + 2.r + 2(k-1) + 3(n-1-r) = 3n + 3rk + 2k - r - 4.$$

There are $\frac{nr}{2}$ internal edges giving harmonic status contribution

$$\frac{nr}{2}\frac{2}{2(2n+2kr+k-r-2)} = \frac{nr}{2}\frac{1}{(2n+2kr+k-r-2)}.$$

Similarly the pendent $'nk'$ vertices adjacent to 'n' internal vertices contribute,

$$nk\frac{2}{(5n+5kr+3k-2r-6)}.$$

Hence the harmonic status index of G^{+k} is

$$HS(G^{+k}) = \frac{nr}{2}\frac{1}{(2n+2kr+k-r-2)} + nk\frac{2}{(5n+5kr+3k-2r-6)}.$$ □

Theorem 2.6 *If G is 'r' regular graph of diameter 2 then the harmonic status coindex of G^{+k} is*

$$\overline{\mathrm{HS}}(G^{+k}) = (nkC_2)\frac{1}{(3n+3kr+2k-r-4)} + nk(n-1)\frac{2}{(5n+5rk+3k-2r-6)} + (nC_2 - \frac{nr}{2}).$$

Proof We note that there are $n(k+1)C_2 - (\frac{nr}{2} + nk)$ non adjacent pairs of vertices in

G^{+k}. There are $(nk)C_2$ pendent nonadjacent pendent vertices, $nk(n-1)$ pairs of nonadjacent vertices combining one pendant and one internal vertex and finally $(nC_2 - \frac{nr}{2})$ nonadjacent internal vertices. Taking contribution from each of them we have status connectivity coindex of G^{+k} as

$$\begin{aligned}\overline{\mathrm{HS}}(G^{+k}) &= (nk)C_2\frac{2}{6n+6kr+4k-4r-8} + nk(n-1)\frac{2}{5n+5rk+3k-2r-6}\\ &\quad + \left(nC_2 - \frac{nr}{2}\right)\frac{2}{2(2n+2kr+k-r-2)}\\ &= (nk)C_2\frac{1}{(3n+3kr+2k-2r-4)} + nk(n-1)\frac{2}{(5n+5rk+3k-2r-6)}\\ &\quad +(nC_2 - \frac{nr}{2})\frac{1}{(2n+2kr+k-r-2)}. \qquad \square\end{aligned}$$

§3. Status Connectivity Indices and Coindices of First Level Thorn Graphs

Now we discuss the harmonic status index and coindex of first level thorn graphs.We need to calculate status of each vertex and number of pairs of adjacent vertices and pairs of non adjacent vertices in $G^{\wedge 1(+k)}$.With respect to degree there are three types of vertices in $G^{\wedge 1(+k)}$.nk pendent vertices,n vertices of degree $'k+1'$ we call them as internal and lastly $'n'$ vertices having degree sequence added by 1.We call them external, in particular if G is 'r'' regular their degrees will become '$r+1'$'.

Theorem 3.1 *The first and second status connectivity index and coindex of first level thorn graph of a 'r'' regular graph of order 'n'' and diameter* 2 *are given by*

$$\begin{aligned}S_1(G^{1(+k)}) &= \frac{nr}{2}\times(5n+4nk-rk-2r-2k-4) + n(12n+9nk-2rk-4r-4k-10)\\ &\quad +2n^2\times k(7n+5nk-rk-2r-3k-6),\\ S_2(G^{1(+k)}) &= \frac{nr}{2}\times(5n+4nk-rk-2r-2k-4)^2\\ &\quad +n(5n+4nk-rk-2r-2k-4)(7n+5nk-2r-4k-rk-6),\\ \overline{\mathrm{S_1(G^{1(+k)})}} &= (nk)C_2\times 2(9n+6nk-rk-4k-2r-8)\\ &\quad +(nC_2-\frac{nr}{2})\times 2(5n+4nk-rk-2r-2)\\ &\quad +2(nC_2)(7n+5nk-rk-2r-3k-6) + n^2k(7n+5nk-rk-2r-3k-6)\\ &\quad +2n(n-1)(12n+9nk-2rk-4r-6k-10)\\ \overline{\mathrm{S_2(G^{1(+k)})}} &= (nk)C_2\times(9n+6nk-rk-4k-2r-8)^2\\ &\quad +(nC_2-\frac{nr}{2})(5n+4nk-rk-2r-2)^2\\ &\quad +nC_2(7n+5nk-rk-2r-3k-6)^2 + n^2k(7n+5nk-rk-2r-3k-6)^2\\ &\quad +n(n-1)(12n+9nk-2rk-4r-6k-10)^2.\end{aligned}$$

Proof The proof follows by direct counting. $\square$

Theorem 3.2 *If G is 'r' regular graph of diameter 2 then the harmonic status connectivity index of $G^{\wedge 1(+k)}$ is*

$$\begin{aligned}
HS(G^{\wedge 1(+k)}) &= \frac{nr}{2}\frac{1}{(5n+4nk-rk-2r-2k-4)} \\
&+n\frac{2}{(5n+4nk-rk-2r-2k-4+7n+5nk-2r-4k-rk-6)} \\
&+n^2k\frac{2}{(5n+4nk-rk-2r-2k-4+9n+6nk-rk-4k-2r-8)}.
\end{aligned}$$

Proof First, we observe that if G has diameter2 then $G^{\wedge 1(+k)}$ has diameter 6. Hence from the structure we have the status of each internal vertex v_i as

$$\begin{aligned}
\sigma(v_i) &= 1(r+1)+2(r+k)+2.(n-1-r)+3rk+3(n-1-r)+4k(n-1-r) \\
&= 5n+4nk-rk-2r-2k-4.
\end{aligned}$$

Also the status of each external vertex u_i as

$$\begin{aligned}
\sigma(u_i) &= 1(k+1)+2r+3r+3(n-1-r)+4(n-1-r)+4rk+5k(n-1-r) \\
&= 7n+5nk-2r-4k-rk-6.
\end{aligned}$$

Finally the pendent vertices being the only vertices on the diametrical path have the status

$$\begin{aligned}
\sigma(w_i) &= 1+2\times 1+2(k-1)+3r+4(n-1-r)+4r+5kr+5(n-1-r)+6k(n-1-r) \\
&= 9n+6nk-rk-4k-2r-8.
\end{aligned}$$

In $G^{\wedge 1(+k)}$ there are $\frac{nr}{2}$ pairs of internal adjacent vertices, n pair of adjacent vertices formed of one internal and one external vertex and finally n^2k pairs of adjacent vertices formed of one internal and one pendant vertex.Hence the harmonic status index of $G^{\wedge 1(+k)}$ is given by

$$\begin{aligned}
HS(G^{\wedge 1(+k)}) &= \frac{nr}{2}\frac{1}{(5n+4nk-rk-2r-2k-4)} \\
&+n\frac{2}{(5n+4nk-rk-2r-2k-4+7n+5nk-2r-4k-rk-6)} \\
&+n^2k\frac{2}{(5n+4nk-rk-2r-2k-4+9n+6nk-rk-4k-2r-8)} \\
&= \frac{nr}{2}\frac{1}{(5n+4nk-rk-2r-2k-4)} \\
&+n\frac{2}{(12n+9nk-2rk-4r-4k-10)}+n^2k\frac{2}{(14n+10nk-2rk-4r-6k-12)} \\
&= \frac{nr}{2}\frac{1}{(5n+4nk-rk-2r-2k-4)}+n\frac{2}{(12n+9nk-2rk-4r-4k-10)} \\
&+n^2k\frac{1}{(7n+5nk-rk-2r-3k-6)}.
\end{aligned}$$

□

Theorem 3.3 *The harmonic status coindex of $G^{\wedge 1(+k)}$ is given by*

$$\begin{aligned}\overline{\mathrm{HS}}(G^{\wedge 1(+k)}) &= (nk)C_2\frac{1}{(9n+6nk-rk-4k-2r-8)}\\ &+(nC_2-\frac{nr}{2})\frac{1}{(5n+4nk-rk-2r-2k-4)}\\ &+nC_2\frac{1}{(7n+5nk-rk-2r-3k-6)}+n^2k\frac{1}{(7n+5nk-rk-2r-3k-6)}\\ &+n(n-1)\frac{2}{(12n+9nk-2rk-4r-6k-10)}.\end{aligned}$$

Proof In $G^{\wedge 1(+k)}$ there are $(nk)C_2$ pairs of nonadjacent pendent vertices, $(nC_2-\frac{nr}{2})$ pairs of nonadjacent vertices formed by internal vertices, nC_2 pairs of nonadjacent vertices formed by external vertices, n^2k nonadjacent pair of vertices formed by one pendant and one internal vertex and finally $n(n-1)$ pairs of nonadjacent vertices formed by one internal and one external vertex.Hence the harmonic status connectivity coindex is given by

$$\begin{aligned}\overline{\mathrm{HS}}(G^{\wedge 1(+k)}) &= (nk)C_2\frac{2}{2(9n+6nk-rk-4k-2r-8)}\\ &+(nC_2-\frac{nr}{2})\frac{2}{2(5n+4nk-rk-2r-2k-4)}\\ &+nC_2\frac{2}{2(7n+5nk-rk-2r-3k-6)}\\ &+n^2k\frac{2}{(5n+4nk-rk-2r-2k-4+9n+6nk-rk-4k-2r-8)}\\ &+n(n-1)\frac{2}{(5n+4nk-rk-2r-2k-4+7n+5nk-2r-4k-rk-6)}\\ &= (nk)C_2\frac{1}{(9n+6nk-rk-4k-2r-8)}\\ &+(nC_2-\frac{nr}{2})\frac{1}{(5n+4nk-rk-2r-2k-4)}\\ &+nC_2\frac{1}{(7n+5nk-rk-2r-3k-6)}+n^2k\frac{1}{(7n+5nk-rk-2r-3k-6)}\\ &+n(n-1)\frac{2}{(12n+9nk-2rk-4r-6k-10)}. \qquad \square\end{aligned}$$

§4. Conclusion

We considered general l level thorn graphs and obtained in particular, status connectivity indices and coindices as well as Harmonic status indices and coindices of 0 level and first level thorn graphs for some class of graphs.

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