What is tribology?

Tribology is the science and technology of friction, lubrication, and wear, derived from the Greek tribos meaning "rubbing". Formally, it is defined as the science and technology of interacting surfaces in relative motion and all practices related thereto. Webster's dictionary defines tribology as "a study that deals with the design, friction, wear, and lubrication of interacting surfaces in relative motion (as in bearings or gears)."

The study of tribology is perhaps most commonly associated with bearing design. The term became more widely used following a British study in 1966 (The Jost Report) in which huge sums of money were reported to have been lost in the UK annually due to the consequences of friction and wear. Since then the term has diffused into the international engineering field and a number of specialists now claim to be tribologists.

What is friction?

Friction is the force that resists relative motion between two bodies in contact.

Static friction occurs when the two objects are not moving relative to each other (like a desk on the ground). The coefficient of static friction is typically denoted as μ_s. The initial force required to get an object moving is often dominated by static friction.

Rolling friction occurs when two objects move relative to each other and one "rolls" on the other (like a car's wheels on the ground). This is classified under static friction because the patch of the tire in contact with the ground, at any point while the tire spins, is stationary relative to the ground.

Kinetic friction occurs when two objects are moving relative to each other and rub together (like a sled on the ground). The coefficient of kinetic friction is typically denoted as μ_k, and is usually less than the coefficient of static friction.

Sliding friction occurs when two objects are rubbing against each other. Putting a book flat on a desk and moving it around is an example of sliding friction.

Fluid friction is the friction between a solid object as it moves through a liquid or gas medium. The drag of air on an airplane, or that of water on a swimmer, are two examples of fluid friction.

Friction is important to keep us walking, driving, etc. Friction may also “bad” since it can cause damage to devices, and is a reason for wear. It is saif that “half of all energy produced is spent to overcome some type of friction.”

What is *wear*?

Wear is defined as the action of causing deterioration through use. In materials science, wear is considered to be the errosion of material from a solid surface by the action of another solid.

Adhesive wear – also known as scoring, galling, or seizing – occurs when two solid surfaces slide over one another under pressure. Surface projections, or asperities, are plastically deformed and eventually welded toghether by the high local pressure. As sliding continues, these bonds are broken, producing cavities on the surface, projections on the second surface, and, frequently, tiny, abrasive particles, all of which contribute to future wear of the surfaces.

What is lubrication?

Lubrication refers to the action of rendering something smooth or slippery.
Lubrication occurs when opposing surfaces are completely separated by a lubricant film. The applied load is carried by pressure generated within the fluid, and frictional resistance to motion arises entirley from the shearing of the viscous fluid.

TYPES OF LUBRICATION

Three distinct forms of lubrication may be identified:

1 Hydrodynamic

2 Hydrostatic

3 Elastohydrodynamic

Hydrodynamic lubrication means that the load-carrying surfaces of the bearing are separated by a relatively thick film of lubricant, so as to prevent metal-to-metal contact and that the stability thus obtained can be explained by the laws of fluid mechanics. Hydrodynamic lubrication does not depend upon the introduction of the lubricant under pressure, though that may occur; but it does require the existence of an adequate supply at all times. The film pressure is created by the moving surface itself pulling the lubricant into a wedge-shaped zone at a velocity sufficiently high to create the pressure necessary to separate the surfaces against the load on the bearing. Hydrodynamic lubrication is also called *full-film,* or *fluid, lubrication*.

Hydrostatic lubrication is obtained by introducing the lubricant, which is sometimes air or water, into the load-bearing area at a pressure high enough to separate the surfaces with a relatively thick film of lubricant. So, unlike hydrodynamic lubrication, this kind of lubrication does not require motion of one surface relative to another. We shall not deal with hydrostatic lubrication in this book, but the subject should be considered in designing bearings where the velocities are small or zero and where the frictional resistance is to be an absolute minimum.

Elastohydrodynamic lubrication is the phenomenon that occurs when a lubricant is introduced between surfaces that are in rolling contact, such as mating gears or rolling bearings. The mathematical explanation requires the Hertzian theory of contact stress and fluid mechanics. Insufficient surface area, a drop in the velocity of the moving surface, a lessening in the quantity of lubricant delivered to a bearing, an increase in the bearing load, or an increase in lubricant temperature resulting in a decrease in viscosity any one of these may prevent the buildup of a film thick enough for full-film lubrication. When this happens, the highest asperities may be separated by lubricant films only several molecular dimensions in thickness. This is called *boundary lubrication*.

Viscosity

In Figure.1 let a plate A is moving with a velocity U on a film of lubricant of thickness h. We imagine the film as composed of a series of horizontal layers and the force F causing these layers to deform or slide on one another just like a deck of cards. The layers in contact with the moving plate are assumed to have a velocity U; those in contact with the stationary surface are assumed to have a zero velocity. Intermediate layers have velocities that depend upon their distances y from the stationary surface. ***Newton's viscous effect states that the shear stress in the fluid is proportional to the rate of change of velocity with respect to y. Thus***

$$\tau = \frac{F}{A} = \mu \frac{du}{dy}$$

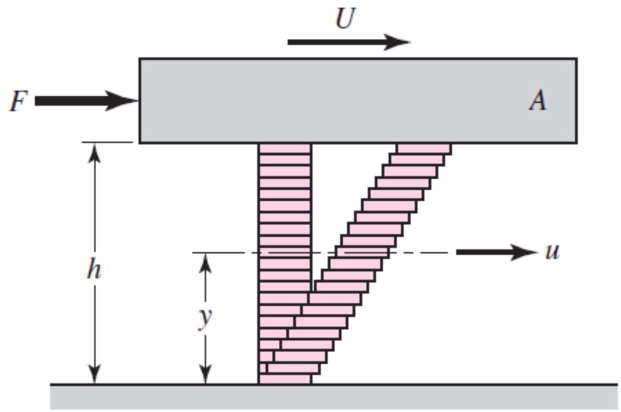


Figure.1. Moving plate

Where μ is the constant of proportionality and defines *absolute viscosity,* also called *dynamic viscosity*. The derivative $\frac{du}{dy}$ is the rate of change of velocity with distance and may be called the rate of shear, or the velocity gradient. The viscosity μ is thus a measure of the internal frictional resistance of the fluid. For most lubricating fluids, the rate of shear is constant, and $\frac{du}{dy}=\frac{U}{h}$

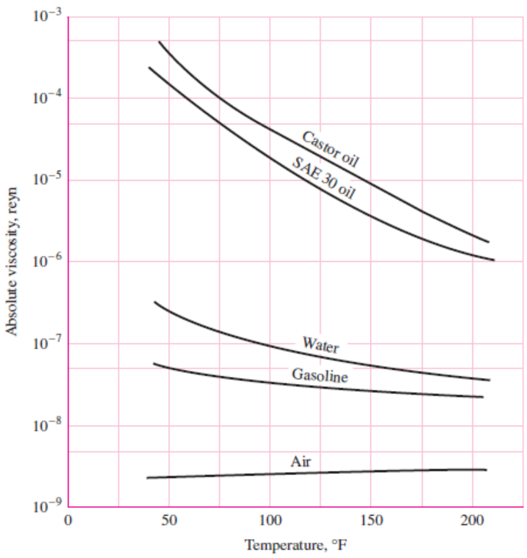
$$\tau=\frac{F}{A}=\mu\frac{u}{h}$$

$$\boxed{\mu=\frac{hF}{uA}}$$

Fluids exhibiting this characteristic are said to be *Newtonian fluids.* The unit of viscosity in the IPS system is seen to be the pound-force-second per square inch; this is the same as stress or pressure multiplied by time. The IPS unit is called the *reyn,* in honor of Sir Osborne Reynolds.

The absolute viscosity is measured by the Pascal-second (Pa · s) in SI; this is same as a Newton-second per square meter.

Effect of temperature on the viscosity of the liquid lubricants

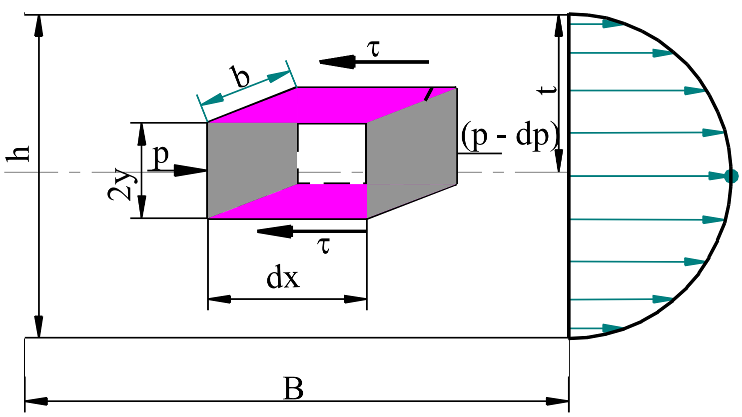


FLOW BETWEEN TWO PARALLEL STATIONARY PLATES

Assumptions:

- Flow is steady and laminar
- The fluid is incompressible and completely fills
- Pressure across the clearance is constant

Consider a rate of flow through the clearance between two parallel stationary plates. The width ‘B’ of the surface is assumed to be so large in comparison with the clearance ‘h’. The pressure at the left end of the clearance is higher than at the right end and is dropping gradually from left to right



The equation of equilibrium of forces acting on an element of a fluid having the thickness '2y' and length 'dx' and width 'b' is

Pressure force acting at the left end = $p2yb$

Pressure force acting at the right end = $(p-dp)2yb$

Shear force on element = $2\tau b\,dx$

If the body is under equilibrium sum of all the forces must be equal to zero

i. e. $\sum F=0$

-ve ←

+ ve →

Sign convention for the forces

$$p2yb-(p-dp)2yb-2\tau b\,dx=0$$

$$p2yb-2ybp+dp2yb-2\tau b\,dx=0$$

$$dp2yb=2\tau b\,dx$$

$$dp=\frac{\tau}{y}dx$$

$$\frac{dp}{dx}=\frac{\tau}{y}$$

According to Newton's law

$$\tau=\mu\frac{du}{dy} \quad \text{or} \quad \tau=-\mu\frac{du}{dy}$$

In this case velocity 'U' decreases gradually as 'y' increases

$$dp=\frac{-\mu}{y}\frac{du}{dy}dx$$

$$du=\frac{-dp\,.\,y}{dx\,.\,\mu}dy$$

Where $\frac{dp}{dx}$ is pressure gradient it is constant

Integrating the above equation w.r.t

$$u=\frac{-1}{\mu}\frac{dp}{dx}\int y\,dy \quad ; \qquad u=\frac{-1}{\mu}\frac{dp}{dx}\frac{y^2}{2}+c_1$$

Applying boundary conditions if $u=0$; $y=\frac{h}{2}=t$

$$0=\frac{-t^2}{2\mu}.\frac{dp}{dx}+c_1 \quad , \qquad c_1=\frac{t^2}{2\mu}.\frac{dp}{dx}$$

$$u=\frac{-1}{\mu}\frac{dp}{dx}\frac{y^2}{2}+\frac{t^2}{2\mu}.\frac{dp}{dx}$$

$$\boxed{u=\frac{1}{2\mu}\frac{dp}{dx}\left(t^2-y^2\right)}$$

This expression shows that velocity, distribution across the plates is parabolic

Maximum velocity

Maximum velocity occurs when $y=0$

$$u=\frac{1}{2\mu}\frac{dp}{dx}\left(t^2\right) \quad , \qquad \text{We know that} \quad t=\frac{h}{2}$$

$$\boxed{u_{max}=\frac{h^2}{8\mu}.\frac{dp}{dx}}$$

Rate of flow

We know that $Q=U\,dA$

$$dA=2b\,dy$$

$$dQ=u\,2\,dy\,b$$

$$dQ=\frac{1}{2\mu}\frac{dp}{dx}\left(t^2-y^2\right)2\,dy\,b$$

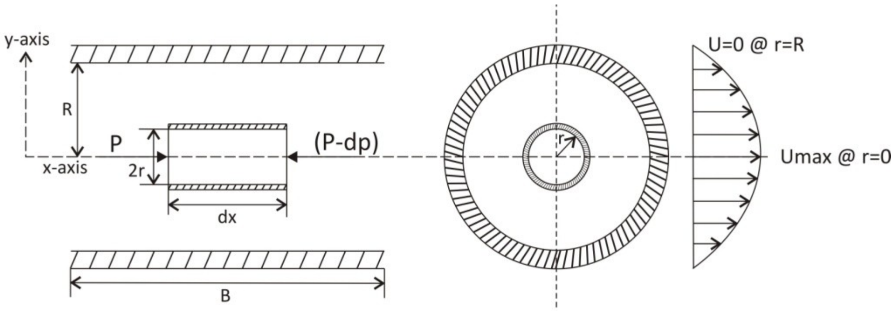
$$dQ=\frac{b}{\mu}\frac{dp}{dx}\left(t^2-y^2\right)dy$$

Integrating the above equation between $0-t$

$$Q=\frac{b}{\mu}\frac{dp}{dx}\int_0^t t^2\,dy-\int_0^t y^2\,dy$$

$$\boxed{Q=\frac{2b}{3\mu}\frac{t^3\,dp}{dx}}$$

FLOW THROUGH CAPILLARY TUBE (HAGEN POISEUILLE LAW)

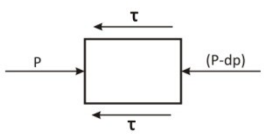


Assumptions:

1. Flow is steady and laminar.
2. The fluid is incompressible and completely fills.
3. Pressure across the clearance is constant.

Consider equilibrium of forces acting on a circular element of fluid having radius 'r' and length 'dx',

The equilibrium of the element is given by,



Pressure force on left end $= P\pi r^2$

Pressure force on right end $= (P-dp)\pi r^2$

Shear stress on the circumference of the tube $= 2\pi r\,dx\,\tau$

$\therefore$ For the equilibrium, $\Sigma F=0$

Sign convention for the forces, $\xrightarrow{+ve}$ = $\xleftarrow{-ve}$

$$P\pi r^2 - (P-dp)\pi r^2 - 2\pi r\, dx\,\tau = 0$$

$$dp\,\pi r^2 = 2\pi r\, dx\,\tau$$

$$\frac{dp}{dx} = \frac{2\tau}{r}$$

From the Newton's law,

$$\tau = \mu\frac{du}{dr} \qquad \therefore \tau = -\mu\frac{du}{dr}$$

-ve sign indicates that

Shear stress decreases with increase in 'y' or 'R'.

$$\therefore \frac{dp}{dx} = -2\mu\frac{du}{dr}\cdot\frac{1}{r}$$

$$du = -\frac{1}{2\mu}r\cdot dr\left(\frac{dp}{dx}\right)$$

$$\therefore du = -\frac{1}{2\mu}\cdot\frac{dp}{dx}\cdot r\cdot dr$$

Integrate above equation,

$$u = -\frac{1}{2\mu}\cdot\frac{dp}{dx}\int r\,dr$$

$$u = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{2} + C_1$$

Constant of integration can be obtained by applying boundary conditions,

$$\therefore r = R;$$

u=0;

$$0 = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\cdot\frac{R^2}{2} + C_1$$

$$C_1 = \frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)$$

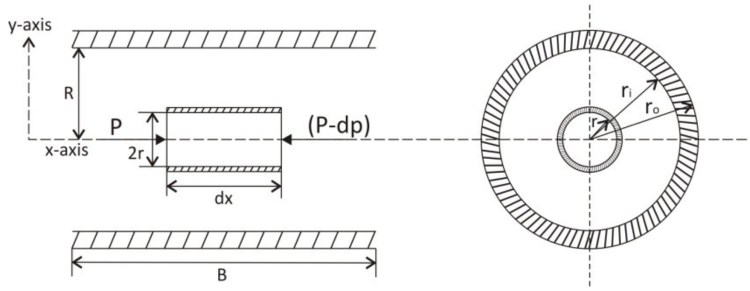
$$\therefore u = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\cdot\frac{r^2}{2} + \frac{R^2}{4\mu}\cdot\left(\frac{dp}{dx}\right)$$

$$u = \frac{1}{4\mu}\left(\frac{dp}{dx}\right)\left(R^2 - r^2\right)$$

$$\therefore u = u_{max} @ r = 0;$$

$$u_{max} = \frac{R^2}{4\mu}\left(\frac{dp}{dx}\right)$$

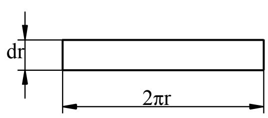
<u>Rate of flow through capillary tube</u>:-



Velocity assumed to be uniform throughout the ring shape element.

From the above equation,

$$u = \frac{1}{4\mu}\left(\frac{dp}{dx}\right)\left(R^2 - r^2\right)$$



dQ=U.dA

dA=2πr.dr

dQ=U. 2πr.dr

$$dQ = \frac{2\pi}{4\mu}\left(\frac{dp}{dx}\right)\left(R^2 - r^2\right).r.dr$$

Integrate above equation from 0-R,

$$Q=\frac{2\pi}{4\mu}\left(\frac{dp}{dx}\right)\left(R^2.r-r^3\right).dr$$

$$Q=\frac{2\pi}{4\mu}\left(\frac{dp}{dx}\right)\left[\int_0^R R^2 r dr-\int_0^R r^3 dr\right]$$

$$Q=\frac{2\pi}{4\mu}\left(\frac{dp}{dx}\right)\left[\left(R^2\left\{\frac{r^2}{2}\right\}_0^R\right)-\left(\frac{r^4}{4}\right)_0^R\right]$$

$$Q=\frac{\pi}{2\mu}\cdot\left(\frac{dp}{dx}\right)\cdot\left(\frac{R^4}{4}\right)$$

$$\therefore Q=\frac{\pi R^4}{8\mu}\cdot\frac{dp}{dx}$$ **Hagen-Poiseuille law**

Viscosity Index

The measure of variation of viscosity with temperature is the *viscosity index* (VI).
For Pennsylvania crude oils, VI = 100, which undergoes the least change of viscosity with temperature.
For Gulf coast oils, VI = 0, which undergoes the greatest change with temperature
Other oils were rated intermediately. VI of Multigrade oils such as SAE 10W-40 is more than that of single grade designation (as SAE 40 or SAE 10W).

$$VI=\frac{L-U}{L-H}x\,100 \quad (6)$$

Where L- viscosity of a standard 0% VI oil at 100oF
H- Viscosity of standard 100% VI oil at 100oF
U - Viscosity of oil with unknown VI oil at 100oF

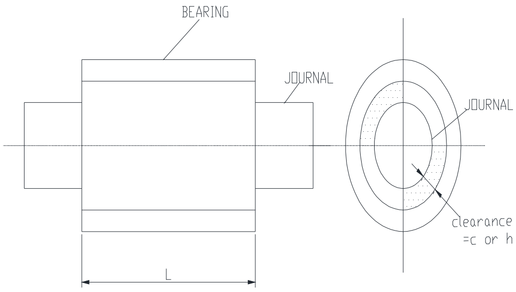
LUBRICANT PROPERTIES

Properties of a good lubricant are:

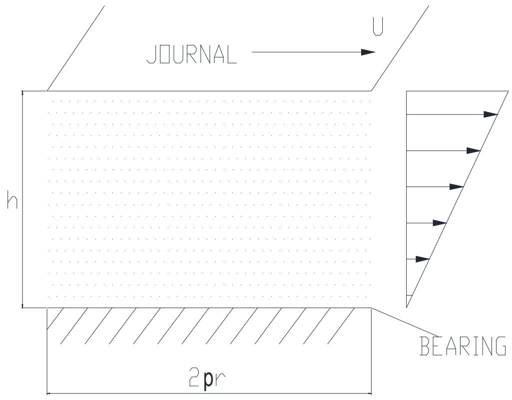
1. It should give rise to low friction.
2. It should adhere to the surface and reduce the wear.
3. It should protect the system from corrosion.
4. It should have good cleaning effect on the surface.
5. It should carry away as much heat from the surface as possible.
6. It should have thermal and oxidative stability.
7. It should have good thermal durability.
8. It should have antifoaming ability.
9. It should be compatible with seal materials.
10. It should be cheap and available in plenty.

2. Hydrodynamic lubrication

Petroff's equation for lightly loaded Journal bearing



Consider a full journal bearing with a journal running concentrically with the bearing this happens only when the radial load acting on the journal is equal to zero, viscosity of the lubricant is equal to infinity and speed of the journal is infinity. None of these conditions are practically possible, however if the load is light enough, the journal has sufficiently high speed and viscosity is also sufficiently high. The eccentricity of the journal relative to the bearing may also be so small then oil film around the journal may be considered to be practically of uniform thickness.



The oil film in the journal bearing can be considered as unwrapped straight body having thickness equal to radial clearance a length equal to " 2πr " length of bearing is equal to " l " (in width direction). Surface " B " represent bearing surface, Surface" A " represent journal and it is moving the frictional force resisting the relative motion of the surface is given by F=τ A where "A "is area of the journal surface.

τ = μ.v/h $\pi = \mu . R$

F = τ.A A= 2πrl

F = μ. $\frac{v}{h}$. A τ= Shear stress in the fluid film on the journal

= μ. $\frac{2\pi rn}{60h}$ .2πrl’ v = $\frac{2\pi rn}{60}$ m/sec

F = $\frac{4\pi^2 rNl}{60h}$. μ linear speed of journal

F = $\frac{4\pi^2 \mu r^2 Nl}{60h}$

Torque = F.r

T = $\frac{4\pi^2 \mu r^2 Nl}{60h}$ .r

$$\boxed{T = \frac{4\pi^2 \mu r^3 Nl}{60h} \text{ Nm}}$$

The above equation is known as **Petroff’s equation**.

A lightly loaded bearing is one which the eccentricity of a journal running concentric to the bearing give a good approximation for the frictional formula.

The co-efficient of friction is defined as the ratio of frictional force to the radial load on the journal and is given by

f = $\frac{F}{w}$

f = $\frac{4\pi^2 \mu r^3 Nl}{60h.w}$

unit load "p" is given by

$$P = w/\text{projected area} = \frac{w}{2\pi r l}$$

❷

$$\boxed{w = 2prl}$$

therefore $f = \dfrac{4\pi^2 \mu r^2 N l}{60h.2pr^4}$

$$f = 2\pi^2\left(\frac{r}{h}\right).\frac{\mu N}{60p} \qquad N' = \frac{N}{60} \text{ rps}$$

$$\boxed{f = 2\pi^2\left(\frac{r}{h}\right).\frac{\mu N}{p}}$$

Problems :-

1. A lightly loaded journal bearing as the following specifications,

Bearing diameter, d=25mm

Length of bearing, L=57mm

Clearance, h=c=5.08x10^{-2}mm

Journal speed, N=25000rpm

Load, W=910N

Viscosity of the Lubricant, μ=2.4x10^{-3} $\frac{Ns}{m^2}$

Determine, (1). Powerloss in the bearing and

(2). Co-efficient of friction.

Solution:-

Power loss, H=F.U

Frictional force, F= $\frac{4\pi^2 r^2 \mu NL}{60h}$

F= $\frac{4\pi^2 \times (12.5)^2 \times (2.4\times10^{-9}) \times 25000 \times 57}{60\times(5.08\times10^{-2})}$

F=6.9213 N

U= $\frac{2\pi\pi r}{60000}$

U= $\frac{2\pi\times12.5\times25000}{60000}$

U= 32.72 mm/sec

H=F.U

H=6.9213x32.72 $N\times\frac{mm}{sec}$

H=226.499 watts= 0.226 kw

f= $\frac{F}{w}$ = $\frac{6.9213}{910}$ $\frac{N}{N}$

f= 7.6x10^{-3}

Cross check, $f=2\pi^2 \left(\frac{r}{h}\right) \cdot \frac{\mu N^1}{P}$

wkt, $P=\frac{w}{2rL}=\frac{910}{2\times12.5\times57}$ = 0.638 N/mm^2

$N^l=\frac{N}{60}=\frac{25000}{60}$ = 416.66 rps

$$\therefore f = 2\pi^2 \times\left(\frac{12.5}{5.08\times10^{-2}}\right)\times\frac{\left(2.4\times10^{-3}\right)\times416.66}{0.638}$$

f = 7.6×10^{-3}

2. Two reservoirs 'A' & 'B' are connected by a capillary of bore 1.25mm,

Length of the capillary 1250mm,

Pressure in the reservoir 'A' is 40x10^4 N/m^2,

Pressure in the reservoir 'B' is 20x10^4 N/m^2,

Calculate the rate of flow between two reservoirs if they are filled with a oil of viscosity 180cp.

Solution:-

1cp = $\frac{1}{100}$ poise = 0.01 poise

1poise = 0.1 $\frac{N.s}{m^2}$

$$\therefore 180cp = 1.8poise = 0.18\frac{N.s}{m^2}$$

Q = $\frac{\pi R^4}{8\mu}\cdot\left(\frac{dp}{dx}\right)$ $\quad \frac{m^4}{\frac{N.s}{m^2}}\times\frac{N}{m^3}$

wkt, R = $\frac{d}{2}$ = $\frac{1.25\times10^{-3}}{2}$ = 0.625 $\times10^{-3}$ m

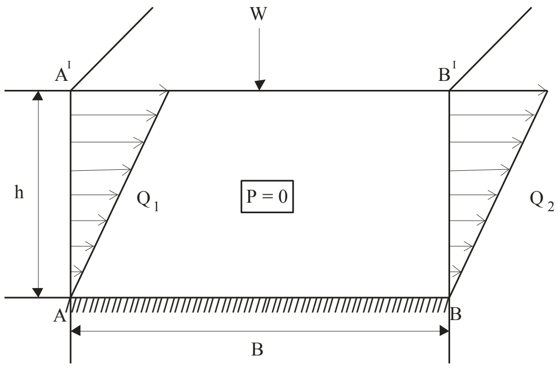
$$\frac{dp}{dx}=\frac{\left(40\times10^4\right)-\left(20\times10^4\right)}{1.25}$$ = 16 $\times10^4\ \frac{N}{m^3}$

$$\therefore Q=\frac{\pi\times\left(0.625\times10^{-3}\right)^4}{8\times0.18}\times\left(1.6\times10^4\right)$$

Q = 5.3263x10^{-8} $\frac{m^3}{sec}$

MECHANISM OF PRESSURE DEVELOPMENT IN OIL FILM

Case 1



$Q_1 = Q_2$

Let upper plate A'B' is moving with constant velocity 'u' while AB is stationary the velocity varies uniformly from 0 at surface A&B and U at the surface A'B' & rate of shear is constant throughout the oil film.

Assuming that this surfaces are wide in the direction perpendicular to the motion so that the flow of the lubricant in this direction is negligibly small

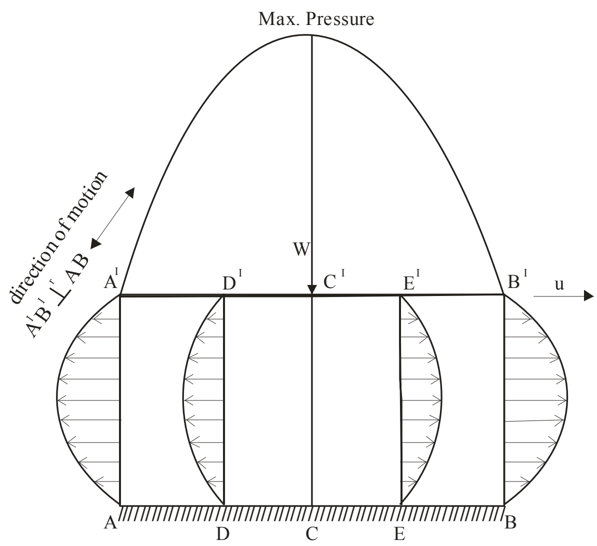
under these conditions. Volume of the fluid flowing across AA' is equal to that of flow across the BB'.

Therefore there is no pressure built up in the oil film and pressure at all the points in the oil film is zero.

The ability of the film to support the load depends upon the pressure build up in the film.

A bearing with parallel surfaces are not able to support the load by fluid film and if load is applied to the surface A'B' the lubricant will be squeezed out and bearing will operate under extreme boundary condition.

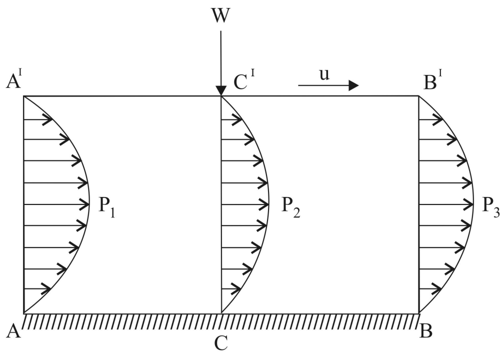
Case 2



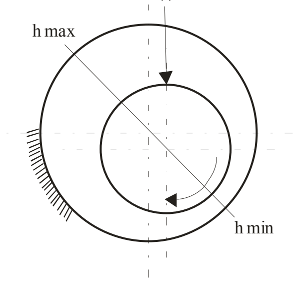
Consider 2 parallel surfaces AB & A'B' in which AB is stationary and A'B' is in motion in the direction perpendicular to A&B. There is no relative motion

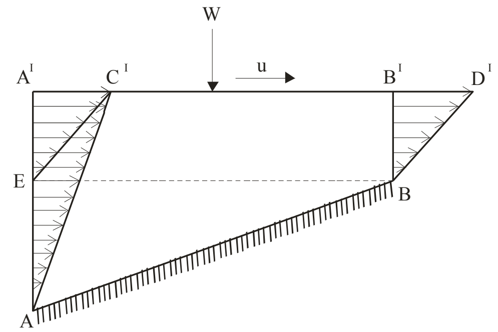
of the surface in the horizontal direction. The oil is squeezed so that it is going both to the right and left of the section CC', the rate of flow increases gradually as the distance from the centre of cross section increases so that maximum rate of flow across at the section AA' & BB' in each section the velocity of the oil film will be maximum in middle layer and zero at the surfaces this type of flow is possible if there is a pressure gradient along the surface with the maximum pressure at the middle section and falling gradually to zero at the end section. The pressure developed at the different section of film depends on viscosity, rate of flow, velocity and instantaneous clearance.

Case 3



The flow of the lubricant caused by difference is pressure in different cross section is called pressure induced flow when the lubricant flows between 2 parallel stationary plates with a pressure at AA' is P_1 which is greater than pressure P_2 & P_3 at sections CC' and BB'. The important factor is that pressure in the oil film is always present supports the applied load.Case (4) Converging Film

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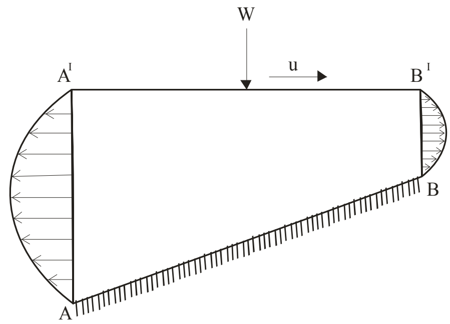
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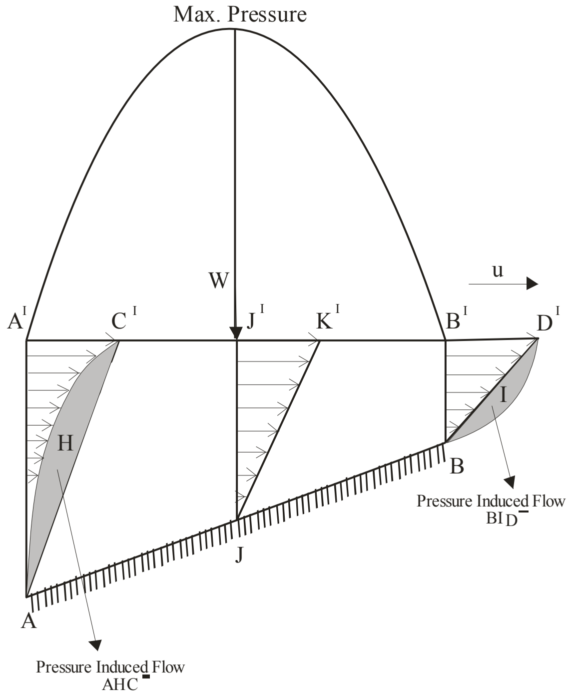


Area AHC'A' is equal to area BID'B'

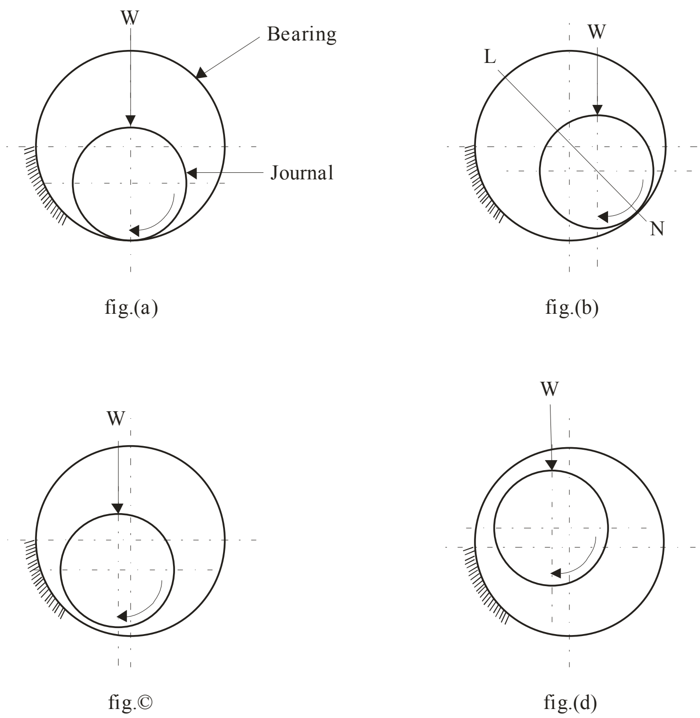
Area of JK'J' is equal to area of AHC'A + area of BID'B

The form of the velocity distribution curve obtained in this way must satisfy the condition that the rate of flow through AA' is equal to rate of flow at BB'. Because of the pressure developed in the oil film the plane A'&D' is able to support the vertical load.

Therefore the positive pressure in the oil film is developed in the direction of the motion of the surface A'B' is such that the volume of the lubricant which surface tends to drag in to the space between the surface is greater than the volume it tends to discharge from the space, such film is known as converging film.

If the motion of the upper surface is reversed with the inclination of lower surface unchanged the volume of the lubricant that the moving surface tends to drag in to the space becomes less than the volume it tends to discharge. Therefore pressure developed will be negative bearing is not to with stand the load such a film is known as diverging film.

Formation of continuous film



In fig (a) the journal is loaded by a load W is at rest. The metal to metal contact exist at point N. The journal rotate slowly and tends to rotate the bearing surface the point of contact will move towards N.

In fig (b) in this position continuous oil film consists of 2 parts converging part above the line LN and diverging film below line NL.

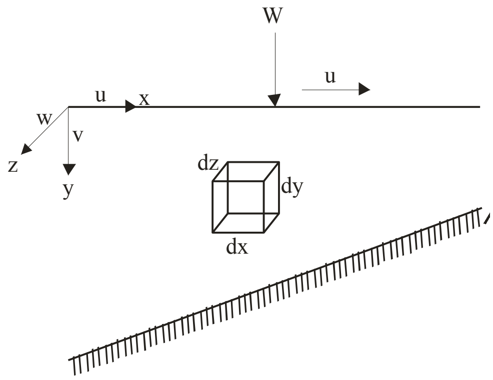
The positive pressure developed in converging portion tend to move the journal to the diverging film as shown in fig (c).

As the journal speed increases a minimum film thickness is maintained on the entire bearing surface as shown in fig (d).

REYNOLD'S EQUATION IN 2 DIMENSIONS

Assumptions

1) There is no flow in the direction perpendicular to the motion.
2) The flow of the fluid is laminar.
3) Fluid is incompressible.
4) There is no slip occurs between lubricant and bearing surface.
5) Viscosity of the lubricant is constant through out the film.
6) Shear stress is directly proportional to the rate of shear in the fluid. (It will be of Newtonian fluids).
7) The clearance between the bearing surface is so small that change in the pressure across the clearance can be neglected.
8) The inertia forces developed are so small that their influenced on the pressure developed in the film may be neglected.

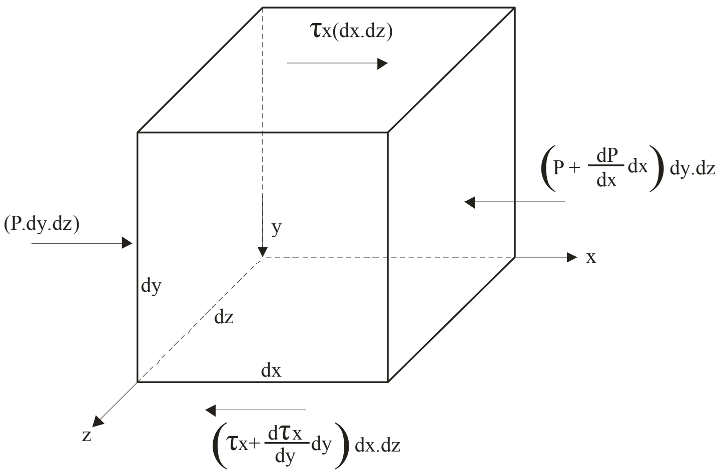


Consider the converging oil film as shown in fig. surface AB fixed while the surface A'B' is in motion with constant velocity ‘u’ in the x-direction.

The surface A'B is loaded with a vertical force ‘W’ because there is no flow in the Z-direction, the pressure along the Z-axis is constant and the pressure force acting on the surface of the element perpendicular to Z-axis are in Equilibrium.

Consider a infinite decimal element of lubricant between the surface. The positive direction of velocity components u,v,w of the element is as shown in fig.

Consider the equilibrium of infinite decimal element subjected to the forces as shown in fig.



$$\sum F_x = 0$$

Sign convention

$$\underset{\rightarrow}{+ve} = \underset{\leftarrow}{-ve}$$

$$\text{Pdydz} + \tau\text{xdxdz} - \left(P + \left(\frac{dp}{dx}\right)dx\right) \text{ dy dz} - \left(\tau_x + \frac{d}{dy}\tau\, xdy\right) \text{ dxdz} = 0$$

$$\text{Pdydz} + \tau\text{xdxdz} - \text{pdydz} - \left(\frac{dp}{dx}\right) \text{ dxdydz} - \tau\text{xdxdz} - \frac{d}{dy}\tau \text{ xdxdydz} = 0$$

$$-\left(\frac{dp}{dx}\right) dxdydz = \frac{d}{dy}\tau\, x\, dxdydz$$

$$\left(\frac{dp}{dx}\right) = \frac{-d}{dy}\tau\, x$$

$$\tau = \frac{-\mu\, du}{dy}$$

$$\frac{dp}{dx} = \frac{\mu\, d^2u}{dy^2}$$

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)$$

Integrate the above equation

$$\int \frac{d^2u}{dy^2} \quad = \quad \frac{1}{\mu}\left(\frac{dp}{dx}\right)\int dy$$

$$\frac{du}{dy} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)y + c_1$$

$$u = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{y^2}{2} + c_1 y + c_2 \qquad \ldots\ldots. (1)$$

Applying boundary conditions

(i) y = 0 u = U (max velocity)

(ii) y – h (film thickness at any point in the film) u = 0

Apply (i) condition

y = 0 u = U

U = $\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{0}{2}+C_1(0)+C_2$

.... (2)

Applying (ii) condition

y = h u = 0

$$O=\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{h^2}{2}+c_1 h+U$$

$$c_1=\frac{\frac{-1}{\mu}\left(\frac{dp}{dx}\right)\frac{h^2}{2}-U}{h}$$

$$c_1=\frac{-1}{\mu}\left(\frac{dp}{dx}\right) \text{ - } \frac{h}{2}-\frac{U}{h}$$

c_1 $2\mu\left(dx \right) h$ ....(3)

Sub (2) & (3) in (1)

$$u=\frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{y^2}{2}-\left(\frac{h}{2\mu}\left(\frac{dp}{dx}\right)+\frac{U}{h}\right)y+U$$

Parabolic Velocity Distribution

Linear Velocity Distribution

In the c/s jj, the pr. in the film is max the pr. Gradient $\frac{dp}{dx}=0$

The above equation contains two terms the second term gives linear velocity distribution which is caused by the velocity of moving surface relative to stationary one it has the form $u2=u\frac{(h-y)}{h}$. The first term gives a parabolic distribution due to Pr. Included flow in the film it has the form $u1=\frac{1}{2\mu}\frac{dp}{dh}(y^2+ny)$

Pr. Gradient $\frac{dp}{dx}=\text{is}+\text{ve}$ in the region AA' & JJ' also (y^2-ny) is always -ve

$\therefore u_1 = -ve$

$\frac{dp}{dx}$ in the region between JJ_1 and BB1 is -ve

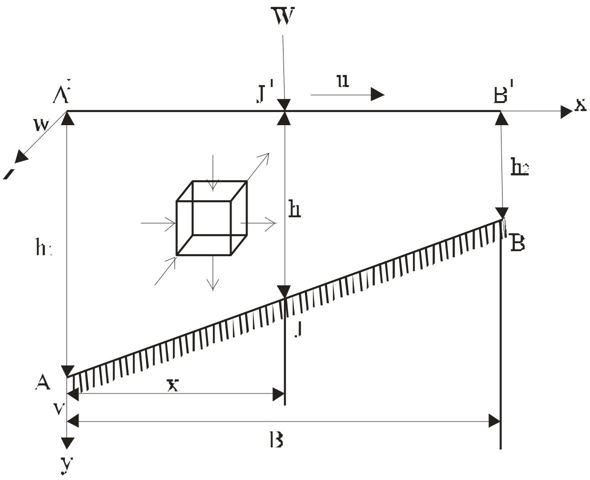
$\therefore u_1 = +ve$

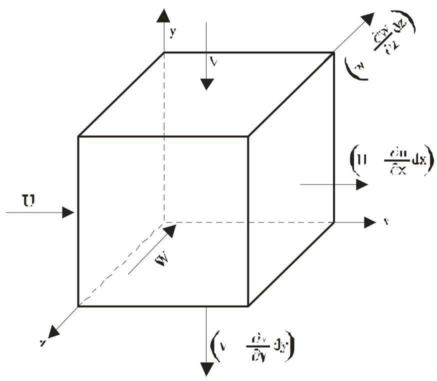
In fig (a) the velocity curve in any cross section between AA' and JJ' is shown 'U' is the velocity of the flow at any in the layer in this region $u_1 = -ve \quad \therefore u = u_2 - u_1$

In fig (b) The velocity curve in any c/s b/n JJ' and BB' is shown here

$u1 = +ve \quad \therefore u = u_2 + u_1$

Since the fluid has been assumed to be incompressible the Volume of lubricant flowing out of the elementary cube is equal to quantity of the lubricant flowing out of the same cube,





U.dy.dz + v.dx.dz + w.du.dx = $(u+\frac{\partial u}{\partial x}.dx)$ dy.dz + $(v+\frac{\partial y}{\partial y}.dy)$ dx.dz + $(w+\frac{\partial w}{\partial z}.dz).dy.dx$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=u$$

Since there is no flow in z-direction, $\therefore \frac{\partial w}{\partial x}=0$

$$\therefore \frac{\partial u}{\partial z}+\frac{\partial v}{\partial y}=0$$

$$\therefore \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$$

$$\partial v = -\frac{\partial u}{\partial x}\partial y$$

Since velocity is zero when y=0 and y=h,

$$\therefore \int_0^h dv = 0$$

But,

$$\partial v = -\frac{\partial u}{\partial x}\partial y$$

$$\therefore \int_0^h -\left(\frac{\partial u}{\partial x}\right)dy = 0$$

Integrating from 0 to h,

$$-\int_0^h \left(\frac{\partial u}{\partial x}\right)dy = -\int_0^h \left(\frac{\partial}{\partial x}\left(\left(\frac{y^2 - hy}{2\mu}\right)\frac{dp}{dx} + U\left(\frac{h-y}{h}\right)\right)\right)dy = 0$$

In this integral 'h' is a function of 'x' hence as per the following relation,

It is of the form,

$$\int_{\phi(x)}^{\psi(x)} \frac{\partial}{\partial x} f(x,\ y)dy = \frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(x,\ y)dy - \psi'(x)f(x,\ \psi(x)) + \phi'(x)f(x,\ \varphi(x)) = 0 \quad \rightarrow \text{By Green's Theorem}$$

Where,

$$\phi(x) = 0, \phi'(x) = 0, f(x,\ y) = u$$

$$\psi(x) = h, \psi'(x) = \frac{dh}{dx}$$

$$f(x, \psi(x)) = f(x, h) = 0$$

$$f(x, \phi(x)) = f(x, 0) = 0$$

$$\therefore -\int_0^h \frac{du}{dx}dy = -\frac{d}{dx}\int_0^h U.dy = 0$$

$$-\frac{d}{dx}\int_0^h U.dy = 0$$

$$-\int_0^h\left(\left(\frac{y^2-hy}{2\mu}\right)\frac{dp}{dx}+U\left(\frac{h-y}{h}\right)\right)dy = 0$$

$$-\frac{d}{dx}\left[\int_0^h \frac{y^2}{2\mu}\left(\frac{dp}{dx}\right)dy-\int_0^h \frac{hy}{2\mu}\left(\frac{dp}{dx}\right)dy+\int_0^h \frac{u}{h}h.dy-\int_0^h \frac{uy}{h}dy\right]=0$$

After Integrating above equation we get,

$$-\frac{d}{dx}\left[\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\left[\frac{y^3}{3}\right]_0^h-\frac{h}{2\mu}\left(\frac{dp}{dx}\right)\left[\frac{y^2}{2}\right]_0^h+U[y]_0^h-\frac{U}{h}\left[\frac{y^2}{2}\right]_0^h\right]=0$$

$$-\frac{d}{dx}\left[\frac{1}{2\mu}\left(\frac{dp}{dx}\right)\frac{h^3}{3}-\frac{h}{2\mu}\left(\frac{dp}{dx}\right)\frac{h^2}{2}+Uh-\frac{U}{h}\frac{h^2}{2}\right]=0$$

$$-\frac{d}{dx}\left[\frac{h^3}{6\mu}\left(\frac{dp}{dx}\right)-\frac{h^3}{4\mu}\left(\frac{dp}{dx}\right)+\frac{Uh}{2}\right]=0$$

$$-\frac{d}{dx}\frac{h^3}{6\mu}\left(\frac{dp}{dx}\right)+\frac{d}{dx}\frac{h^3}{4\mu}\left(\frac{dp}{dx}\right)-\frac{d}{dx}\frac{Uh}{2}=0$$

$$\frac{d}{dx}\left[h^3\left(\frac{dp}{dx}\right)\left(-\frac{1}{6\mu}+\frac{1}{4\mu}\right)\right]=\frac{U}{2}\frac{d}{dx}(h)$$

$$\frac{d}{dx}\left[\frac{h^3}{12\mu}\left(\frac{dp}{dx}\right)\right]=\frac{U}{2}\frac{dh}{dx}$$

$$\frac{d}{dx}\left[h^3\left(\frac{dp}{dx}\right)\right]=6\mu U\frac{dh}{dx}$$

This is the Reynolds differential equation in two dimensions for Pressure Gradient in a converging oil film with no end leakage.

IDEALISED FULL JOURNAL BEARING OR HYDRODYNAMIC LUBRICATION

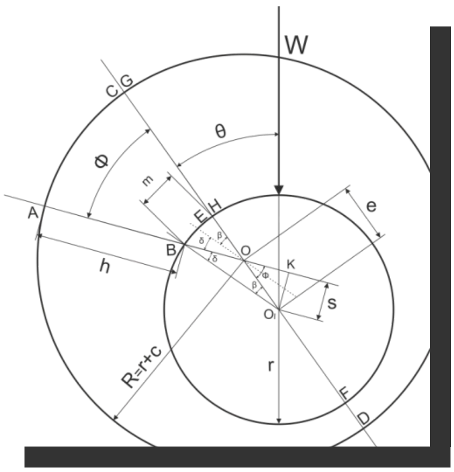
In full journal bearing usually journal rotates while the bearing head is fixed. However, a system in which a journal is a fixed member and a member is connected to a machine member revolving around the journal is rather than typical case of application of a full journal bearing.

The main difference in the analysis of full journal bearing when compared with analysis with plane slider bearing is due to the difference in the shape of oil film.

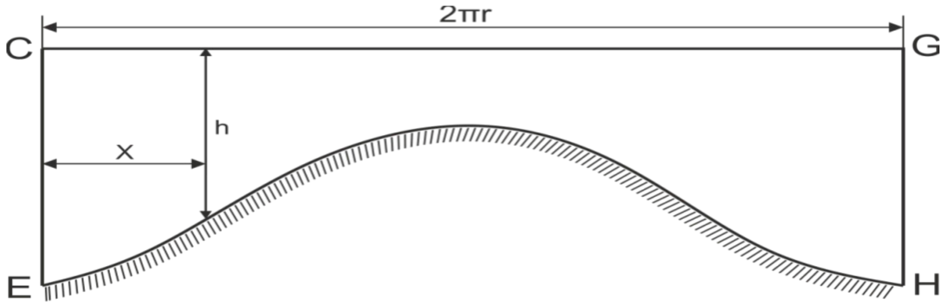
Assumptions:

In analysising full journal bearing, the assumptions of Reynolds's equation are made.

1. There is no flow in the direction perpendicular to the motion.
2. The flow of the fluid is laminar.
3. The fluid is incompressible.
4. There is no slip occurs between lubricant & bearing surface.
5. Viscosity of the lubricant is constant throughout the film.
6. Shear stress is directly proportional to the rate of shear in the fluid.
7. The clearance between bearing surface is the so small that change in pressure across the clearance can be neglected.
8. The inertia forces developed are so small that they are influenced on the pressure developed in the film may be neglected.



Unit 3 Idealised Journal Bearing



A schematic representation of full journal bearing with an exhausted clearance ‘c’ is as shown in fig. The radius of journal is ‘r’, the radius of the bearing ‘R’= r+c, where ‘c’ is radial clearance of bearing. If the journal is loaded with a vertical load ‘W’ and rotates in counter clockwise direction, if the operating conditions such that fluid film lubrication is possible.

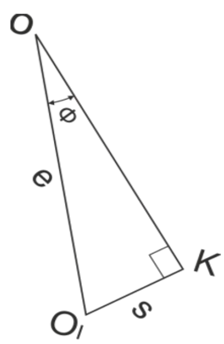
The journal surface moves into an eccentricity position relative to bearing. The journal surface is unwrapped so that its becomes straight line having length 2 π r, ‘h’ is the film thickness at any distance is given by ‘x’ from the origin can be expressed as the function of ' $\emptyset$ '. The film thickness ‘h’ is given by

$$h = AB$$

$$h = OA - OB$$

$$\mathbf{h = [r + c] - OB} \quad \ldots\ldots\ldots\ldots (1)$$

Consider the triangle O_1OK

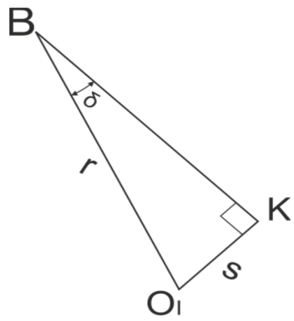


$$\sin\emptyset = \frac{opp.}{hyp.} = \frac{O_1K}{O_1O}$$

$$\sin\emptyset = \frac{s}{e}$$

$$s = e\sin\emptyset \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Consider the triangle O_1BK



$$\sin\delta = \frac{opp.}{hyp.} = \frac{O_1K}{O_1B}$$

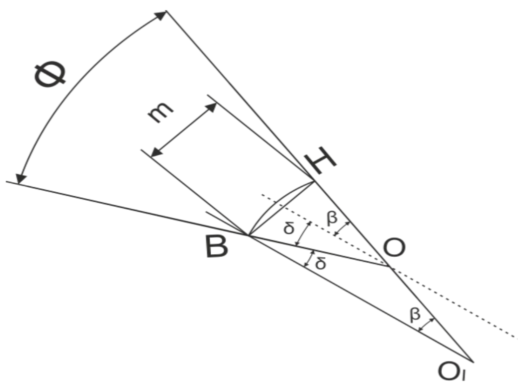
$$\sin\delta = \frac{s}{r}$$

$$s = r\sin\delta \qquad \ldots\ldots. (3)$$

Comparing equation 2 & 3,

$$e\sin\varnothing = r\sin\delta \qquad \ldots\ldots\ldots (4)$$

Consider the triangle O_1BH



$$\varnothing = \beta + \delta$$

$$\beta = \varnothing - \delta \qquad \ldots\ldots\ldots (5)$$

From the triangle O_1BH

$$\sin\delta = \frac{opp.}{hyp.} = \frac{BH}{O_1B} = \frac{m}{r}$$

$$m = r\sin\beta$$

Consider the triangle BOH,

$$\sin\varnothing = \frac{BH}{OB}$$

$$OB = \frac{m}{\sin\varnothing} = \frac{r\sin\beta}{\sin\varnothing}$$

$$OB = \frac{r.\sin(\varnothing - \delta)}{\sin\varnothing}$$

WKT, sin A – B = sin A cos B – cos A sin B

$$\cos^2\delta + \sin^2\delta = 1$$

$$\cos\delta = \sqrt{1+\sin^2\delta}$$

$$OB = \frac{r(\sin\varnothing\cos\delta - \cos\varnothing\sin\delta)}{\sin\varnothing}$$

$$OB = \frac{r}{\sin\varnothing}\left[\sin\varnothing\left(\sqrt{1-\frac{e^2}{r^2}\sin^2\varnothing}\right)\right] - \cos\varnothing\frac{e}{r}\sin\varnothing$$

$$OB = \frac{r}{\sin\varnothing}\frac{\sin\varnothing}{r}\left[\left(\sqrt{r^2-e^2\sin^2\varnothing}\right) - e(\cos\varnothing)\right]$$

$$OB = \left(\sqrt{r^2-e^2\sin^2\varnothing}\right) - e\cos\varnothing$$

Since the eccentricity “e” is very small, compare with radius of journal

$$e^2\sin^2\varnothing = 0$$

OB = r – e $\cos\varnothing$

Substitute “OB” in equation (1)

$$h = (r+c) - (r - e\cos\varnothing)$$

$$h = r + c - r + e\cos\varnothing$$

$$\boxed{\mathbf{h = c + e}\cos\varnothing}$$

$$n = \frac{e}{c}$$

Where, n = eccentricity ratio

e = n. c

$$h = c + nc\cos\varnothing$$

$$\boxed{\mathbf{h} = c(1 + n\cos\varnothing)}$$

Consider the Reynolds’s equation,

$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right) = 6\mu U\frac{dh}{dx}$$

From the triangle 3,

x = r β

x = r [$\varnothing - \delta$] $\therefore$ [δ " is very small]

x = r $\varnothing$

dx = r d$\varnothing$

$$\frac{d}{r\,d\varnothing}\left(h^3\frac{dp}{r\,d\varnothing}\right)=6\mu U\frac{dh}{r\,d\varnothing}$$

$$\frac{d}{d\varnothing}\left(h^3\frac{dp}{r\,d\varnothing}\right)=6\mu U\frac{dh}{d\varnothing}$$

Integrate w.r.t "$\varnothing$"

$$h^3\frac{dp}{r\,d\varnothing}=6\mu Uh+C_1$$

$$h^3\frac{dp}{d\varnothing}=6\mu Uhr+C_1$$

Assuming that r. C_1 = -K

$$\frac{dp}{d\varnothing}=\frac{(6\mu Urh-K)}{h^3}$$

$$\frac{dp}{d\varnothing}=6\mu Ur\left(\frac{1}{h^2}-\frac{K}{h^3}\right)$$

$$dp=6\mu Ur\left(\frac{1}{h^2}-\frac{K}{h^3}\right)d\varnothing$$

W.K.T,

h = C [1+ n $\cos\varnothing$]

$$dp=6\mu Ur\left(\frac{1}{C^2(1+n\cos\varnothing)^2}-\frac{K}{C^3(1+n\cos\varnothing)^3}\right)d\varnothing$$

$$dp=\frac{6\mu Ur}{C^2}\left(\frac{1}{(1+n\cos\varnothing)^2}-\frac{K}{(1+n\cos\varnothing)^3}\right)d\varnothing$$

Integrate the above equation from 0 to 2 π at that point pressure is zero.

P = 0 at $\varnothing=0\wedge\varnothing=2\pi$

$$p=0\,\frac{6\mu Ur}{C^2}\left[\int_0^{2\pi}\frac{1}{(1+n\cos\varnothing)^2}-\frac{K}{C}\int_0^{2\pi}\frac{1}{(1+n\cos\varnothing)^3}\right]d\varnothing$$

W.K.T, attitude or eccentricity ratio is the reciprocal of $\frac{1}{n}$ [pg. no. 4 from DHB]

$$n = \frac{e}{c} = \frac{1}{\alpha}$$

$$\alpha = \frac{1}{n}$$

Where the journal located concentric to bearing, the eccentricity[e] = 0.
If the eccentricity, e =0 then the " α lies between 1 to ∞.

$$\frac{6\mu Ur}{C^2}\left[\int_0^{2\pi} \frac{d\varnothing}{(1+n\cos\varnothing)^2} = \frac{6\mu Ur}{C^2}\frac{K}{C}\int_0^{2\pi} \frac{d\varnothing}{(1+n\cos\varnothing)^3}\right]$$

$$\frac{K}{C} = \frac{\int_0^{2\pi} \frac{d\varnothing}{(1+n\cos\varnothing)^2}}{\int_0^{2\pi} \frac{d\varnothing}{(1+n\cos\varnothing)^3}}$$

$$\frac{K}{C} = \frac{\frac{1}{n^2}\int_0^{2\pi} \frac{d\varnothing}{\left(\frac{1}{n}+\cos\varnothing\right)^2}}{\frac{1}{n^2}\int_0^{2\pi} \frac{d\varnothing}{\left(\frac{1}{n}+\cos\varnothing\right)^3}}$$

Therefore, from the integration tables [pg. 8]

$\alpha + \cos\varnothing ¿^2$
¿
¿
$\frac{d\varnothing}{¿}$
$J_2 = \int_0^{2\pi} ¿$

$\alpha + \cos\varnothing ¿^3$
¿
¿
$\frac{d\varnothing}{¿}$
$3 = ¿\int_0^{2\pi} ¿$
$J_{¿}$

$$\frac{K}{C}=\frac{\frac{\alpha^2 * 2\pi\alpha}{\left(\sqrt{\alpha^2-1}\right)^3}}{\frac{\alpha^3 * \pi\left(2\alpha^2+1\right)}{\left(\sqrt{\alpha^3-1}\right)^5}}$$

$$K=\frac{2C\left(\alpha^2-1\right)}{\left(2\alpha^2+1\right)}$$

WKT, $\alpha=\frac{1}{n}$

$$K=\frac{2C\left(1-n^2\right)}{\left(2+n^2\right)}$$

The physical meaning of ‘K’.

We know that,

$$\frac{dp}{d\varnothing}=6\mu Ur\left[\frac{1}{h^2}-\frac{k}{h^3}\right]$$

If the pressure gradient $\frac{dp}{d\varnothing}=0$,

$$0=6\mu Ur\left[\frac{1}{h^2}-\frac{k}{h^3}\right]$$

$$0=\left[\frac{1}{h^2}-\frac{k}{h^3}\right]$$

$$\frac{h-k}{h^3}=0$$

$$h-k=0$$

$$k=h$$

$$\therefore h = k = \frac{2C\left(1-n^2\right)}{\left(2+n^2\right)}$$

Where, ‘k’ is the thickness of the oil film in the section where the pressure gradient

$\frac{dp}{d\varnothing}=0,$ or the pressure is maximum value or minimum value.

We know that,

$$1+n\cos\varnothing ¿^2$$
$$¿$$
$$\frac{-K}{C}$$
$$¿$$
$$1+n\cos\varnothing ¿^3$$
$$¿$$
$$¿ d\varnothing$$
$$¿$$
$$\frac{1}{¿}$$
$$\int_0^{\varnothing} ¿$$
$$P=\frac{6\mu Ur}{C^2} ¿$$

Let us consider $'P_0'$, The pressure of the oil film at the point where $\varnothing=0$,

$$1+n\cos\varnothing ¿^2$$
$$¿$$
$$\frac{-K}{C}$$
$$¿$$
$$1+n\cos\varnothing ¿^3$$
$$¿$$
$$¿+P_0$$
$$¿$$
$$\frac{d\varnothing}{¿}$$
$$\int_0^{\varnothing} ¿$$
$$\therefore P_{\varnothing}=\frac{6\mu Ur}{C^2} ¿$$

Let P_0 is the initial pressure developed in the oil film, Where $n=\frac{1}{\alpha}$

$\alpha+\cos\varnothing ¿^2$

$¿$

$\frac{-K}{C}\alpha^3$

$¿$

$\alpha+\cos\varnothing ¿^3$

$¿$

$¿$

$¿$

$\frac{d\varnothing}{¿}$

$\alpha^2\int_0^{\varnothing} ¿$

$\therefore P_{\varnothing}-P_0=\frac{6\mu Ur}{C^2}¿$

From the standard integration table, pg.no.8

$\alpha+\cos\varnothing ¿^2$

$¿$

$¿$

$¿$

$\alpha^2-1 ¿^{\frac{3}{2}}$

$¿$

α

$\varnothing$

$\alpha+\cos ¿$

$¿$

$¿$

$\frac{(¿¿2-1)*\sin\varnothing}{¿}$

$¿$

$\frac{d\varnothing}{¿}$

$J_2=\int_0^{\varnothing} ¿$

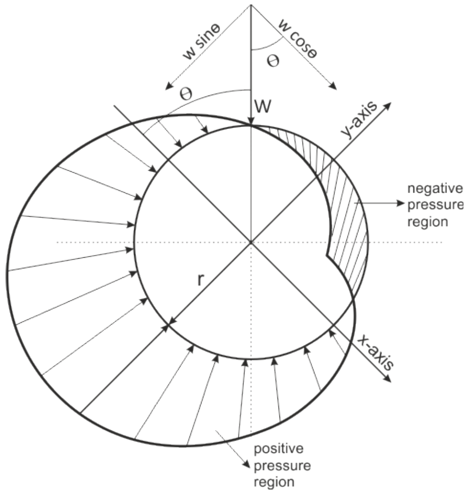
$$\alpha+\cos\varnothing ¿^3$$
$$¿$$
$$¿$$
$$¿$$
$$\alpha^2-1 ¿^{\frac{5}{2}}$$
$$¿$$
$$\alpha^2-1 ¿^2$$
$$¿$$
$$\varnothing$$
$$\alpha+\cos ¿$$
$$¿$$
$$\alpha+\cos\varnothing ¿^2$$
$$2(\alpha^2-1) ¿$$
$$\frac{\sin\varnothing}{¿}$$
$$2 ¿$$
$$\frac{3\alpha}{¿}$$
$$\left[\sqrt{\frac{\alpha-1}{\alpha+1}*\tan\frac{\varnothing}{2}}\right]-¿$$
$$¿$$
$$\frac{d\varnothing}{¿}$$
$$J_3=\int_0^{\varnothing} ¿$$

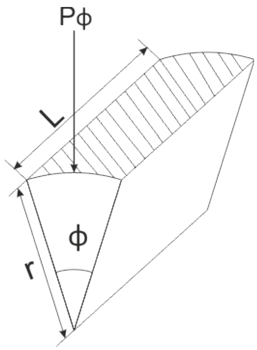
Substituting J_2 , J_3 and K in the above equation,

$$\varnothing$$
$$2+n\cos ¿\sin\varnothing$$
$$¿$$
$$1+n\cos\varnothing ¿^2$$
$$(2+n^2) ¿$$
$$n ¿$$
$$¿$$
$$P_{\varnothing}-P_0=\frac{6\mu Ur}{C^2} ¿$$

Load carrying capacity & Summer field Number:

The load carrying capacity may be defined as the [load corresponds to minimum film thickness which is at the point of nearest approach between the bearing surplus].





The minimum film thickness of full journal bearing depends upon the cleaner attribute for the equilibrium of the bearing resolve the load ‘w’ along x & y directions.

Resolving the component of the forces only on the journal in x direction

$$\sum fx = 0$$

$$\underbrace{\int_0^{2\pi} P\phi \cos\phi (Lr)\, d\phi}_{A} + w.\cos\theta$$

$$= Lr \int_o^{2\pi} (P\phi(\cos\phi).\ d\phi$$

$$= Lr \int_o^{2\pi} \frac{6\mu ur}{c^2}\left[\frac{n(2 + n\cos\phi.)\sin\phi}{(2+n)^2\ (1+n\cos\phi)} + Po\right] \cos\phi.\ d\phi$$

$$= \frac{6\mu ur^2 l\ n}{c^2\,(2+n^2)} = K$$

$$A = Lr\,K \int_o^{2\pi}\left(\frac{(2+n\cos\phi)\ \sin\phi}{(1+n\cos\phi)^2}\ Po\right)\cos\phi\ d\phi$$

$$A = \int_o^{2\pi} Po\, Lr.\cos\phi\, d\phi + k\underbrace{\int_o^{2\pi}\frac{2\sin\phi\ \cos\phi\ d\phi}{(1+n\cos\phi)^2}}_{A_1} + k\underbrace{\int_o^{2\pi}\frac{n\cos^2\phi \sin\phi.\ d\phi}{(1+n\cos\phi)^2}}_{A_2}$$

$$A_1 = K \int_o^{2\pi} \frac{2\sin\phi.\cos\phi \; d\phi}{(1+n\cos\phi)^2}$$

$$A_1 = A_1^1 + A_1^{11}$$

$$K = \underbrace{\int_\varphi^{A} \frac{2\sin\phi \; \cos\phi}{(1+n\cos\phi)^2}.d\phi}_{A_1^1} + k \underbrace{\int_\pi^{2\pi} \frac{2\sin\phi \quad \cos\phi}{(1+n\cos\phi^2}.d\phi}_{A_1^{11}}$$

$$A_1^1 = k \int_o^{\pi} \frac{2\sin\phi.\cos\phi}{(1+n\cos\phi)^2}.d\phi$$

Let – $1+n\cos\phi = t$

$-n\sin\phi.\, d\phi = dt$

$$d\phi = \frac{-dt}{n.\sin\phi}$$

$1+n\cos\phi = t$

$n\cos\phi = t-1$

$\cos\phi = \left(\frac{t-1}{n}\right)$

$\phi = 0$

$1+n\cos(0) = t$

$(1+n) = t$

$1+n\cos\phi = t$

$1+n\cos(180) = t$

$1-n = t$

$$A_1^1 = k \int_{1+n}^{1-n} \frac{2\sin\phi \; \cos\phi}{t^2}.\frac{.dt}{n\sin\phi} = -k \int_{1+n}^{1-n} \frac{2\sin\phi\,(t-1)/n}{t^2}\frac{.dt}{n\sin\phi}$$

Unit 3 Idealised Journal Bearing

$$A_1^1 = \frac{-2k}{n^2} \int_{1+n}^{1-n} \left(\frac{t-1}{t^2} \right) .dt$$

$$A_1^{11} = k \int_{\pi}^{2\pi} \frac{2\sin\phi . \cos\phi}{(1+n\cos\phi)^2} .d\phi$$

Let $(1+n\cos\phi) = t$

$-n\sin\phi . d\phi = dt$

$d\phi = \frac{-dt}{n\sin\phi}$

$(1+n\cos\phi) = t$

$n\cos\phi = t-1$

$\cos\phi = \left(\frac{t-1}{n}\right)$

where $\phi = 0$

$1+n\cos(\pi) = t$

$(1-n) = t$

When $\phi = 2\pi$

$1+n\cos 2\pi = t$

$(1+n) = t$

$$= -K \int_{(1-n)}^{(1+n)} \frac{2-\sin\phi \left(\frac{t-1}{n}\right)}{t^2} \frac{.dt}{n\sin\phi}$$

$$A_1^{11} = \frac{-2k}{n^2} \int_{(1-n)}^{(1+n)} \frac{t-1}{t^2} .dt$$

$$A_1^1 = \frac{-2k}{n^2}\int_{1+n}^{1-n}\left(\frac{t-1}{t^2}\right).dt$$

$$A_1 = A_1^1 + A_1^{11}$$

$$A_1^{11} = \frac{2k}{n}\int_{1+n}^{(1-n)}\frac{(t-1)}{t^2}dt \qquad \int_o^A (x) = -\int_A^o (x)$$

$$A_1 = \frac{-2k}{n^2}\int_{1+n}^{1-n}\left(\frac{t-1}{n^2}\right).dt \frac{+2k}{n^2}\int_{1+n}^{1-n}\left(\frac{t-1}{t^2}\right).dt$$

$$A_1 = o$$

2[nd] term equation is

$$A_2 = K\int_o^{2\pi}\frac{n\cos^2\phi\sin\phi}{(1+n\cos\phi)^2}.d\phi$$

$$A_2 = A_2{}^1 + A_2{}^{11}$$

$$A_2{}^1 = K\int_O^A \frac{n\cos^2\phi\sin\phi}{(1+n\cos\phi)^2}.d\phi$$

$$A_2{}^{11} = K\int_\pi^{2\pi}\frac{n\cos^2\phi.\sin\phi}{(1+n\cos\phi)^2}.d\phi$$

$$A_2 = A_2{}^1 + A_2{}^{11}$$

$$A2 = 0$$

Substituting equation 1

$$\int_0^{2\pi}(P\phi\cos\phi)\,L.r.d\,\phi + w.\cos\theta = 0$$

$$w\cos\theta = 0;$$

$$w \neq 0 \quad = 0$$

$$\theta = 90$$

This result indicates that as an identified full journal bearing the line passing through center journal and the bearing is always perpendicular to the direction of internal load on the journal.

Resending the forces along y-axis:-

$$\sum y = 0$$

$$\int_0^{2\pi} \frac{P\phi \sin\phi}{A}.Lr.d\phi - w\sin\theta = 0$$

$$\int_0^{2\pi} \left[Po + \frac{6\mu\, ur}{C^2}\left(\frac{n(2 + n\cos\phi)\sin\varphi\phi}{(2+n^2)(1+n\cos\phi)^2} \right)\right] Lr.\sin\phi.d\phi$$

$$\int_0^{2\pi} (PoLrSin\ \phi)\,d\phi + \int_0^{2\pi} \frac{6\mu\, r}{C^2}\left(\frac{n(2 + n\cos\phi)Sin\phi}{(2+n^2)(1+n\cos\phi)} \right) L.r\ Sin\phi.d\phi$$

$$\frac{6\mu\, ur}{C^2} x \frac{Lr.n}{(2+n^2)} \int_0^{2\pi} \frac{(2 + nCos\phi)Sin^2\phi}{(1+nCos^2\phi)^2}.d\phi$$

$$\int_0^{2\pi} \frac{(2 + n\cos\phi)Sin^2\phi}{(1+n\cos\phi)^2} \frac{2\pi}{\sqrt{1-n^2}}$$

$$\frac{6\mu\, ur^2 Ln}{C^2(2+n^2)} x \frac{2\pi}{\sqrt{1-n^2}}$$

$$\frac{12\pi\mu\, ur^2 n.L}{C^2(2+n^2)\sqrt{1-n^2}}$$

Equation 1 becomes:

$$\frac{12\pi\,\mu\, ur^2 n.L}{C^2(2+n^2)\sqrt{1-n^2}} - w.Sin\ \theta = 0;$$

$$w = \frac{12\pi\,\mu\, ur^2 nL}{C^2(2+n^2)\sqrt{1-n^2}}$$

W.K.T

$$P = \frac{load}{area} = \frac{w}{A} = \frac{w}{2rL}$$

Pressure,

$$U = \frac{2\pi\ Nr}{60}$$

W=2PrL

U= $2\pi rN^1$

$$2PrL = \frac{12\pi\mu r^2 n.L.2\pi rN^1}{C^2(2+n^2)(\sqrt{1-n^2})}$$

$$= \left(\frac{r}{C}\right)^2\left(\frac{\mu N^1}{P}\right) = \frac{(2+n^2)\sqrt{1-n^2}}{12\pi^2}$$

$$S = \frac{(2+n^2)\sqrt{1-n^2}}{12\pi^2 n}$$

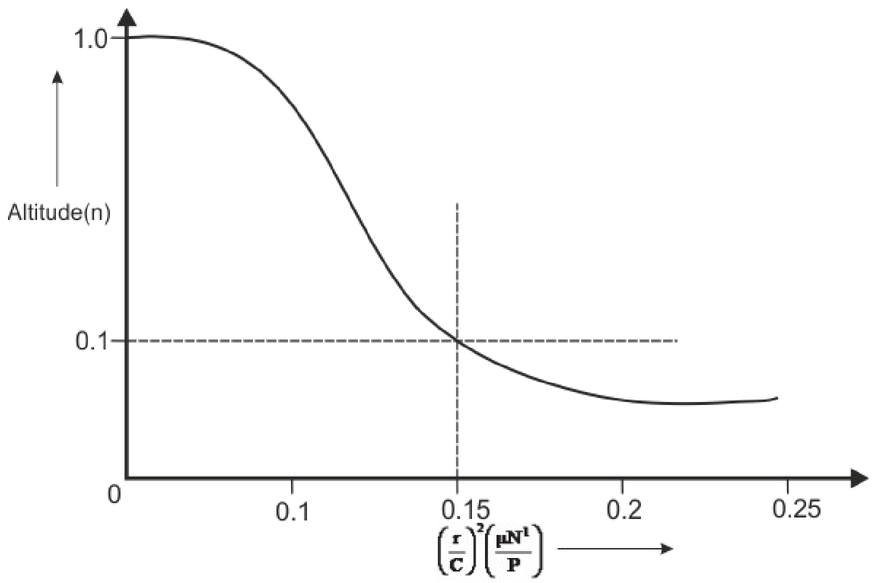
The quantities is known somewhere feld no.

Where $\alpha = \frac{C}{e} = \frac{1}{n}$

$$n = \frac{e}{c}$$

Significance of Somerfield Number :

Case1: Relationship between Somerfield Number $\left(\frac{\mu N'}{P}\right)*\left(\frac{r}{c}\right)$ and altitude 'n'.



The curved line in the figure shows the relationship between attitude and the Somerfield number for idealized full journal bearing the dashed line in the figure separates the region of lightly loaded moderately and heavily loaded bearing the exact meaning of the term lightly loaded bearing will be understand well when the friction in the full journal bearing is considered.

For the equilibrium condition frictional force and the applied load are equal to make the system under equilibrium.

$$F=\frac{8\pi^2\mu L r^2 N^I}{C}\times\frac{(1+2n^2)}{(2+n^2)\sqrt{1-n^2}}$$

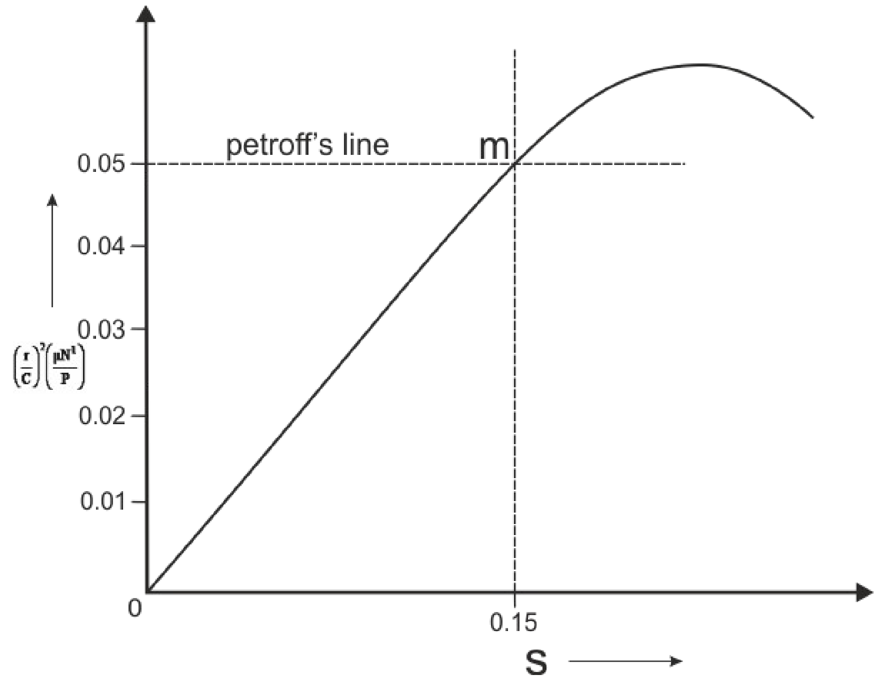
Wkt, $F=w, but, P=\frac{w}{2rL}, w=2PrL$

$$2PrL=\frac{8\pi^2\mu L r^2 N^I}{C}\times\frac{(1+2n^2)}{(2+n^2)\sqrt{1-n^2}}$$

$$\therefore\frac{(2+n^2)\sqrt{1-n^2}}{4\pi^2(1+2n^2)}=\frac{\mu N^I}{P}\left(\frac{r}{C}\right)\rightarrow(1)$$

LHS of the equation (1) is the function of attitude 'n' and 'n' is the faction of Somerfield number therefore the value of $\left(\frac{\mu N'}{P}\right)*\left(\frac{r}{c}\right)$ is also a function of Somerfield number.

Case2: Relationship between $\left(\frac{\mu N'}{P}\right)*\left(\frac{r}{c}\right)$ with Somerfield number is as shown in figure.



If pet raff's assumption of a concentric shaft is used the eccentricity and attitude n = 0,

The equation (1) becomes,

$$\frac{(2+n^2)\sqrt{1-n^2}}{4\pi^2(1+2n^2)}=\frac{\mu N^I}{P}\left(\frac{r}{C}\right)$$

$$\frac{2}{4\pi^2}=\frac{\mu N^I}{P}\left(\frac{r}{C}\right)$$

$$\frac{\mu N^I}{P}\left(\frac{r}{C}\right)=0.05$$

The value of the quantity $\left(\frac{\mu N'}{P}\right)*\left(\frac{r}{c}\right)$ is shown by a dotted line. The point 'm' indicates the value of Somerfield number that corresponds to the operating condition of full journal bearing. When the quantity $\left(\frac{\mu N'}{P}\right)*\left(\frac{r}{c}\right)$ closely approaches the value that can be determined by assuming that shaft eccentricity is zero.

Case3: The coefficient of friction in an idealized full journal bearing,

$$wkt, f=\frac{F_o}{w}$$

$$f=\frac{\dfrac{8\pi^2\mu L r^2 N^I(1+2n^2)}{C(2+n^2)\sqrt{1-n^2}}}{\dfrac{12\pi\mu U L r^2 n}{C^2(2+n^2)\sqrt{1-n^2}}}$$

$$f=\frac{2C\pi N^I(1+2n^2)}{3Un}$$

$$But, U=2\pi N^I r$$

$$f=\frac{2C\pi N^I(1+2n^2)}{6\pi N^I rn}$$

$$f=\frac{C}{r}\left(\frac{1+2n^2}{3n}\right)$$

It is convenient to use the Somerfield number as a variable quantity.

Multiplied both sides of the equation $\left(\frac{r}{c}\right)$

$$f\times\frac{r}{c}=\frac{C}{r}\times\frac{r}{c}\left(\frac{1+2n^2}{3n}\right)$$

$$f\left(\frac{r}{c}\right)=\left(\frac{1+2n^2}{3n}\right)$$

The quantity 'f $\left(\frac{r}{c}\right)$ ' is the function of attitude only and therefore it is the function of Somerfield number. Since the quantity of 'f $\left(\frac{r}{c}\right)$ ' is a function of Somerfield number it can be plotted as shown in figure (3) and indicated by a curve 'b'.

The coefficient of friction for a lightly loaded bearing is given by,

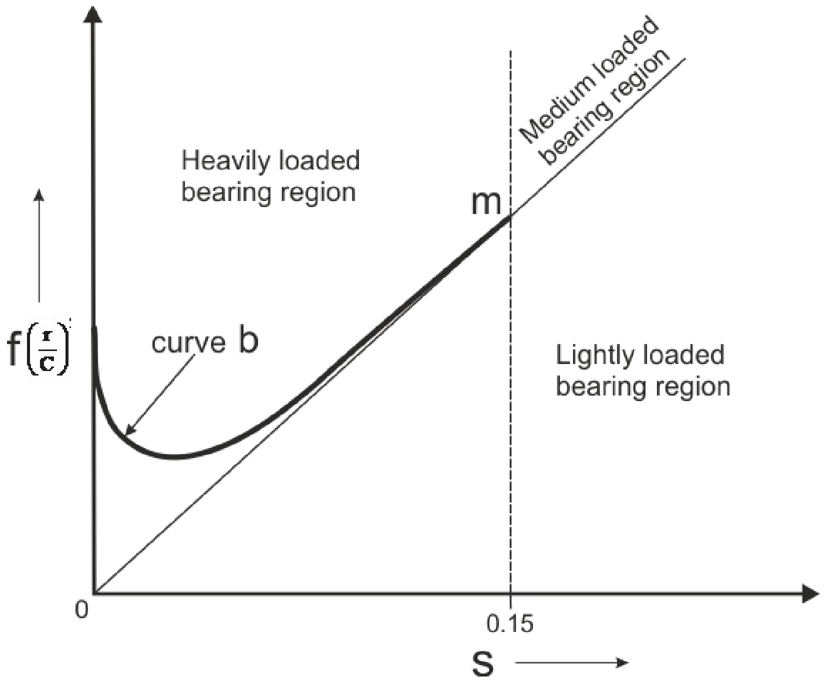
Multiplied both sides by $\left(\frac{r}{c}\right)$

Unit 3 Idealised Journal Bearing

$$f = 2\pi^2 \frac{\mu N^I}{P}\left(\frac{r}{C}\right)\left(\frac{r}{C}\right)$$

$$f = 2\pi^2 \frac{\mu N^I}{P}\left(\frac{r}{C}\right)^2$$

The right side of the equation is proportional to the Somerfield number while the left side of the equation (2) is same as in the equation (1).



The quantity ‘f $\left(\frac{r}{c}\right)$ ’ and the Somerfield number from the equation number (2) as shown in figure by a dashed line.

The point ‘m’ in the figure corresponds to the Somerfield number at which the curve ‘b’ and line ‘c’ coincides for all practical purpose.

Problems

1. A full journal beraing has the following specifications,
   Journal diameter:d = 50mm
   Bearing length:L = 63mm
   Journal speed:N = 1200rpm
   Radial clearence: c = 0.025mm
   Mean viscousity of the lubricant:μ = 12cps
   Attitude:n = 0.8
   Neglecting the effects of end leakage determine,
   I Load on the bearing could support

II Frictional froce

III Co-efficient of friction

IV Powerloss in the bearing.

Solution:

$$U=\frac{2\pi rN}{60}\,m/s$$

$$U=\frac{2\pi\times 25\times 10^{-3}\times 1200}{60}$$

$$U=3.14\,m/s$$

Load on the bearing,

$$W=\mu UL\left(\frac{r}{c}\right)^2\left[\frac{12\pi n}{(2+n^2)\sqrt{1-n^2}}\right]N$$

$$W=0.012\times 3.14\times 63\times 10^{-3}\left(\frac{25}{0.025}\right)^2\left[\frac{12\pi\,0.8}{(2+0.8^2)\sqrt{1-0.8^2}}\right]$$

$$W=45197.8\,N$$

Frictional froce,

$$F=\left[\frac{8\pi^2\mu L r^2 N^I(1+2n^2)}{c(2+n^2)\sqrt{1-n^2}}\right]N$$

$$F=\left[\frac{8\pi^2 0.012\times 63\times 10^{-3}(25\times 10^{-3})^2\times 1200(1+2(0.8)^2)}{60\times 0.025\times 10^{-3}(2+0.8^2)\sqrt{1-0.8^2}}\right]$$

$$F=42.93\,N$$

Co-efficient of friction,

$$f = \frac{F}{W}$$

$$f = \frac{42.93}{45197.8}$$

$$f = 9.5 \times 10^{-4}$$

Powerloss in the bearing,

$$P = F \times U \; Watts$$

$$P = 42.93 \times 3.14$$

$$P = 134.8 \quad \text{Watts}$$

Unit 3 Idealised Journal Bearing

2. A fulljournal bearing as the following specifications

Bearing dia= 64mm

Bearing length = 60mm

Radial dia to radious ratio = 0.05mm

Journal speed = 25000rpm

Radial load = 900N

Viscosity of the oil at effective temparature = 2.416cp

Determine

1. The friction torqe
2. The power loss
3. The unit pressure
4. The coefficent of friction.

Ans:

Given:

D= 64mm

L = 60mm

c= 0.05

N = 25000rpm

W = 900N

μ = 2.416cp

$$U = \frac{2\pi r N}{60} = \frac{2 \times \pi \times 0.032 \times 25000}{60} = 83.77\frac{m}{sec}$$

1. <u>The friction torqe,</u>

P = F $\times U$

$$P = \frac{4\pi r^2 \mu N L}{60h} \times U$$

$$\frac{4 \times \pi^2 \times 0.23^2 \times 2800 \times 0.066}{60 \times 5 \times 10^{-5}} \times 83.77$$

= 1.56N-M

2. The power loss,

$$P = F \times U$$

$$= \frac{4\pi^2 r^2 \mu N L}{60h} \times U$$

$$¿ \frac{4 \times \pi^2 \times 0.032^2 \times 0.0024 \times 25000 \times 0.06}{60 \times 5 \times 10^{-5}} \times 83.77$$

P = 4090.86 $\frac{Nm}{sec}$

3. The unit pressure,

$$P = \frac{W}{2rL} = \frac{900}{2 \times 0.32 \times 0.02} = \mathbf{234375}\ \frac{N}{m^2}$$

4. The coefficent of friction,

$$f = \frac{F}{W}$$

$$f = 2\pi^2 \left(\frac{\mu N}{60P}\right)\left(\frac{r}{c}\right)$$

$$f = 2\pi^2 \left(\frac{0.0024 \times 25000}{60 \times 234375}\right)\left(\frac{0.032}{5 \times 10^{-5}}\right)$$

$$f = 0.054$$

3.Aidealized full journal beraing has the following specifications,

Journal diameterD = 50mm
Bearing length:L = 64mm
Journal speed:N = 1200rpm
Radial clearence: c = 0.025mm
Mean viscousity of the lubricant:μ = 11cps
Attitude:n = 0.8
Determine,

V Load carrying capacity

VI Frictional force

VII Co-efficient of friction

VIII Power loss in the bearing.

IX Summerfeld number

X Minimum film thickness

Solution:

D = 50mm
L = 64mm
N = 1200rpm
c = 0.025mm
μ = 11cps
n = 0.8

Load carrying capacity

$$W=\mu UL\left(\frac{r}{c}\right)^2\left[\frac{12\pi n}{\left(2+n^2\right)\sqrt{1-n^2}}\right]$$

$$W=0.0011\times 3.14\times 64\times 10^{-3}\left(\frac{0.025}{0.0025}\right)^2\left[\frac{12\pi 0.8}{\left(2+0.8^2\right)\sqrt{1-0.8^2}}\right]$$

$$W=42.07\,kN$$

$$P=\frac{W}{2rL}$$

$$P=\frac{42078.30}{2\times 0.025\times 64\times 10^{-3}}=131419469.25\,W$$

Summerfeld number

$$S=\frac{(2+n^2)\sqrt{1-n^2}}{12\pi^2 n}$$

$$S=\frac{(2+0.8^2)\sqrt{1-0.8^2}}{12\times\pi^2\times0.8}$$

$$S=0.0167$$

Minimum film thickness

h_{min}= c(1-n)

= $0.025\text{x}10^{-3}(1-0.8)$

h_{min} = $5\text{x}10^{-6}$ m

$$U=\frac{2\pi rN}{60}$$

$$U=\frac{2\pi\times0.025\times10^{-3}\times1200}{60}$$

$$U=3.14\,m/s$$

Frictional force

$$F=\left[\frac{8\pi^2\mu L r^2 N^I(1+2n^2)}{c(2+n^2)\sqrt{1-n^2}}\right]$$

$$F=\left[\frac{8\pi^2 0.0011\times64\times10^{-3}(0.025\times10^{-3})^2\times20(1+2(0.8)^2)}{0.025\times10^{-3}(2+0.8^2)\sqrt{1-0.8^2}}\right]$$

$$F=39.89\,N$$

Co-efficient of friction

$$f=\frac{F}{W}$$

$$f=\frac{39.89}{42.07\times 10^{3}}$$

$$f=9.48\times 10^{-4}$$

Power loss in the bearing.

$$P=F\times U$$

$$P=39.89\times 3.14$$

$$P=125.25 \quad \textbf{Watts}$$

4.A full journal beraing has the following specifications,

Journal diameter:d = 45.2mm
Bearing length:L = 60.5mm
Journal speed:N = 1200rpm
Radial clearence: c = 0.0254mm
Mean viscousity of the lubricant:μ = 10cps
Attitude:n = 0.8
Determine,

XI Load carrying capacity

XII Frictional force

XIII Co-efficient of friction

XIV Powerloss in the bearing.

Solution:

$$S=\frac{\left(2+n^{2}\right)\sqrt{1-n^{2}}}{12\pi^{2}n}$$

$$S=\frac{\left(2+0.8^{2}\right)\sqrt{1-0.8^{2}}}{12\times \pi^{2}\times 0.8}$$

$$S = 0.0167$$

h_{min}= c(1-n)

= 0.0254x10^{-3}(1-0.8)

h_{min} = 0.00508x10^{-3} m

$$U = \frac{2\pi rN}{60}$$

$$U = \frac{2\pi \times 22.6 \times 10^{-3} \times 1200}{60}$$

$$U = 2.839\, m/s$$

$$W = \mu\, UL\left(\frac{r}{c}\right)^2 \left[\frac{12\pi n}{\left(2+n^2\right)\sqrt{1-n^2}}\right]$$

$$W = 0.01 \times 2.839 \times 60.5 \times 10^{-3}\left(\frac{22.6}{0.0254}\right)^2 \left[\frac{12\pi\, 0.8}{\left(2+0.8^2\right)\sqrt{1-0.8^2}}\right]$$

$$W = 25.89\, kN$$

$$F = \left[\frac{8\pi^2 \mu\, L r^2 N^I \left(1+2n^2\right)}{c\left(2+n^2\right)\sqrt{1-n^2}}\right]$$

$$F = \left[\frac{8\pi^2 0.01 \times 60.5 \times 10^{-3}\left(22.6 \times 10^{-3}\right)^2 \times 20\left(1+2\left(0.8\right)^2\right)}{0.0254 \times 10^{-3}\left(2+0.8^2\right)\sqrt{1-0.8^2}}\right]$$

$$F = 27.65\, N$$

$$f = \frac{F}{W}$$

$$f = \frac{27.65}{25.89 \times 10^3}$$

$$f = 1.06 \times 10^{-3}$$

$$P = F \times U$$

$$P = 27.65 \times 2.839$$

$P = 78.49$ Watts

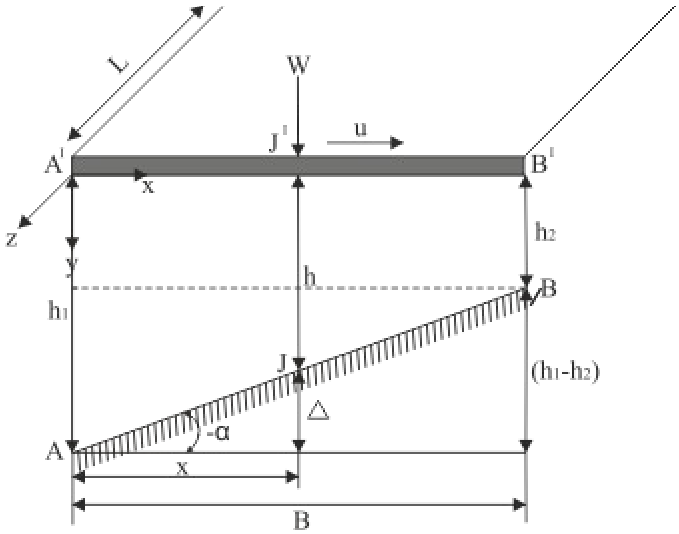
Unit 4 Slider/Pad Bearing with a fixed and pivoted Shoe

PRESSURE DISTRIBUTION IN IDEALISED PLAN SLIDERS BEARING WITH FIXED SHOE :

A hydrodynamic plane slider bearing consists two non parallel surfaces while other moves with uniform velocity. The direction of the motion and inclination of the plans are such that a converging oil film is found between the surfaces.

The positive pressure that is developed in the oil film is capable of supporting load.

The stationary plan may be fixed rigidly or oil may be pivoted so that it can assume any inclination relative to moving surface



From the figure thickness of the oil film at any point along the shoe is,

h = h_1 - Δ

From similar triangles AJH and ABC,

$$\frac{JH}{AH} = \frac{BC}{AC}$$

$$\frac{\Delta}{X} = \frac{X(h_1 - h_2)}{B}$$

$$\Delta = \frac{X(h_1 - h_2)}{B}$$

h = h_1 $-$ $\frac{X(h_1 - h_2)}{B}$ or

h = $h_1 \quad \frac{+X(h_2 - h_1)}{B}$ (1)

From the triangles ABC,

$\alpha = ¿ \frac{BC}{AC}$ = - $\left(\frac{(h_1 - h_2)}{B} \right)$

tan ¿

α = - $\left(\frac{(h_1 - h_2)}{B} \right)$ or α = $\left(\frac{(h_2 - h_1)}{B} \right)$

Let us designate $\frac{h_2}{B}$ = a, X= B X_1 or X_1 = $\frac{X}{B}$ and h_2 = B a, $h \, (¿¿2 - h_1) \, ¿$ =

αB

The equation (1) can be re written as,

h = h_1 $+\alpha$ X

B X_1

h_1 can be written as,

h_1 = h_2 + $h \, (¿¿1 - h_2) \, ¿$ or h_1 = $h \, (¿¿2 - h_1) \, h_2 - ¿$

h_1 = B a – αB

$\alpha ¿$ (2)

$\therefore h$ = B (a – $\alpha ¿$ + $\alpha B X_1$

h $\alpha ¿¿$ (3)

For a converging film as shown in figure the value of $h \, (¿¿2 - h_1) \, ¿$ is negative making the quantity $'\alpha'$ also negative.

Apply Reynolds equation,

$$\frac{d}{dx}\left(h^3 \frac{dp}{dx} \right) = 6 \, \mu U \frac{dh}{dx}$$

Integrating above equation,

$$h^3\left(\frac{dp}{dx}\right) + C_1 = 6 \mu U h + C_2$$

$$h^3\left(\frac{dp}{dx}\right) = 6 \mu U h + C_2 - C_1$$ Let, - $C_2 = C_2 - C_1$

$$h^3\left(\frac{dp}{dx}\right) = 6 \mu U h - C_2$$

$$\left(\frac{dp}{dx}\right) = \left(\frac{6 \mu U h}{h^3} - \frac{C_2}{h^3}\right)$$

$$\left(\frac{dp}{dx}\right) = \left(\frac{6 \mu U}{h^2} - \frac{C_2}{h^3}\right)$$

$$\frac{dp}{dx} = 6 \mu U\left[\frac{1}{h^2} - \frac{C_2}{6 \mu U h^3}\right]$$

$$\frac{dp}{dx} = 6 \mu U\left[\frac{1}{h^2} - \frac{C}{h^3}\right]$$ Let, $\frac{C_2}{6 \mu U} = c$

Substitute the value of ‘ h’ in the above equation,

$$\left(\frac{dp}{dx}\right) = 6 \mu U \quad \begin{matrix} \alpha X_1 + (a - \alpha) ¿^2 \\ ¿ \\ \alpha X_1 + (a - \alpha) ¿^3 \\ B^3 ¿ \\ B^2 ¿ \\ \frac{1}{¿} \\ ¿ \end{matrix}$$

Wkt, X= B X_1, $dx = B\, dx_1$

$$\frac{dp}{B\, dx_1} = \frac{6 \mu U}{B^2} \quad \begin{matrix} \alpha X_1 + (a - \alpha) ¿^2 \\ ¿ \\ \alpha X_1 + (a - \alpha) ¿^3 \\ B ¿ \\ ¿ \\ \frac{1}{¿} \\ ¿ \end{matrix}$$

$$dp = \frac{6\mu U}{B} \quad \begin{array}{c} \alpha X_1 + (a-\alpha)¿^2 \\ ¿ \\ \alpha X_1 + (a-\alpha)¿^3 \\ B¿ \\ ¿ \\ \dfrac{dx_1}{¿} \\ ¿ \end{array}$$

Integrating the above equation to obtained pressure distribution in the bearing,

$$P = \frac{6\mu U}{B} \quad \begin{array}{c} \alpha X_1 + (a-\alpha)¿^2 \\ ¿ \\ \dfrac{-C}{B} \\ ¿ \\ \alpha X_1 + (a-\alpha)¿^3 \\ ¿ \\ ¿ \\ ¿ \\ \dfrac{dx_1}{¿} \\ \int ¿ \\ ¿ \end{array}$$

From the standard integral table,

$$\begin{array}{c} \alpha X_1 + (a-\alpha)¿^2 \\ ¿ \\ ¿ \\ \dfrac{dx_1}{¿} \\ \int ¿ \end{array} \quad + \quad C'$$

$$\begin{array}{c} \alpha X_1 + (a-\alpha)¿^3 \\ ¿ \\ \alpha X_1 + (a-\alpha)¿^2 \\ ¿ \\ 2\alpha ¿ \\ ¿ \\ \dfrac{dx_1}{¿} \\ \int ¿ \end{array} \quad + \quad C''$$

$$P = \begin{array}{c} \alpha X_1 + (a-\alpha)¿^2 \\ ¿ \\ 2B\alpha ¿ \\ \dfrac{-1}{\alpha(\alpha X_1 + (a-\alpha))} + C' + \dfrac{C}{¿} \\ \dfrac{6\mu U}{B} ¿ \end{array}$$

$Let, C' \quad + \quad C'' \quad = D$

$$\alpha X_1 + (a-\alpha)¿^2$$
$$2B\alpha ¿$$
$$P = \frac{-1}{\alpha(\alpha X_1 + (a-\alpha))} + \frac{C}{¿} + D \qquad (4)$$
$$\frac{6\mu U}{B} ¿$$

(1). Apply first boundary condition

$$X = 0, P = 0 \wedge X = X_1 B, X_1 = \frac{X}{B} = 0$$

$$0 = \frac{6\mu U}{B}\left[\frac{-1}{\alpha(a-\alpha)} + \frac{C}{2B\alpha(a-\alpha)^2}\right] + D$$

$$0 = \frac{-6\mu U}{B\alpha(a-\alpha)} + \frac{6\mu UC}{2\alpha B^2(a-\alpha)^2} + D$$

$$\frac{6\mu U}{B\alpha(a-\alpha)} = ¿ \quad \frac{6\mu UC + 2\alpha B^2 (a-\alpha)^2 D}{2\alpha B^2 (a-\alpha)^2}$$

$$12 \quad \mu UB \quad (a-\alpha) = ¿ \quad 6\mu UC + 2\alpha B^2 (a-\alpha)^2 D$$

(5)

(2). Apply second boundary condition

X = B, P = 0 and X = $X_1 B$, B = $X_1 B$ or $X_1 = \frac{B}{B}$ = 1,

$$0 = \frac{6\mu U}{B}\left[\frac{-1}{\alpha(\alpha + a - \alpha)} + \frac{C}{2\alpha B(\alpha + a - \alpha)^2}\right] + D$$

$$0 = \frac{6\mu U}{B}\left[\frac{-1}{\alpha a} + \frac{C}{2a^2 B\alpha}\right] + D$$

$$0 = \frac{-6\mu U}{\alpha B a} + \frac{6\mu UC}{2a^2 B^2 \alpha} + D$$

$$\frac{6\mu U}{\alpha B a}=\frac{6\mu UC+2a^2B^2\alpha D}{2a^2B^2\alpha}$$

$$12\mu UaB=6\mu UC+2a^2B^2\alpha D$$

(6)

Subtract equation (6) from equation (5)

$$6\mu UB(a-\alpha)=3\mu UC+\alpha B^2(a-\alpha)^2D$$

$$6\mu UaB=3\mu UC+a^2B^2\alpha D$$

$$6\mu UB((a-\alpha)-a)=aB^2D((a-\alpha)^2-a^2)$$

$$-6\mu UB\alpha=\alpha B^2D(a-\alpha)^2-a^2 ¿$$

$$-6\mu UB\alpha=\alpha B^2D(\alpha^2-2a\alpha)$$

$$\frac{-6\mu U}{B(\alpha^2-2a\alpha)}=D$$

$$\therefore D=\frac{\quad}{B\alpha(2a-\alpha)} \qquad (7)$$

Substitute equation (7) in equation (5),

$$6\mu UB(a-\alpha)=3\mu UC+\alpha B^2(a-\alpha)^2\frac{6\mu U}{B\alpha(2a-\alpha)}$$

$$6\mu UB(a-\alpha)=3\mu UC+\frac{6\mu UB(a-\alpha)^2}{(2a-\alpha)}$$

$$6\mu UB(a-\alpha)-\frac{6\mu UB(a-\alpha)^2}{(2a-\alpha)}=3\mu UC$$

$$6\mu UB(a-\alpha)\left[1-\frac{(a-\alpha)}{(2a-\alpha)}\right]=3\mu UC$$

$$2B(a-\alpha)\left[\frac{(2a-\alpha)-(a-\alpha)}{(2a-\alpha)}\right]=C$$

$$2B(a-\alpha)\left[\frac{2a-\alpha-a+\alpha}{(2a-\alpha)}\right]=C$$

$$C = \left[\frac{2aB(a-\alpha)}{(2a-\alpha)} \right] \longrightarrow \qquad (8)$$

Substitute equation (7) and (8) in equation (4)

After simplification,

Where, $\alpha = \dfrac{h_2 - h_1}{B}$ and a = $\dfrac{h_2}{B}$, $X_1 = \dfrac{X}{B}$

Pressure at any point X in the oil film is given by ,

$$(\alpha - 2a) ¿$$

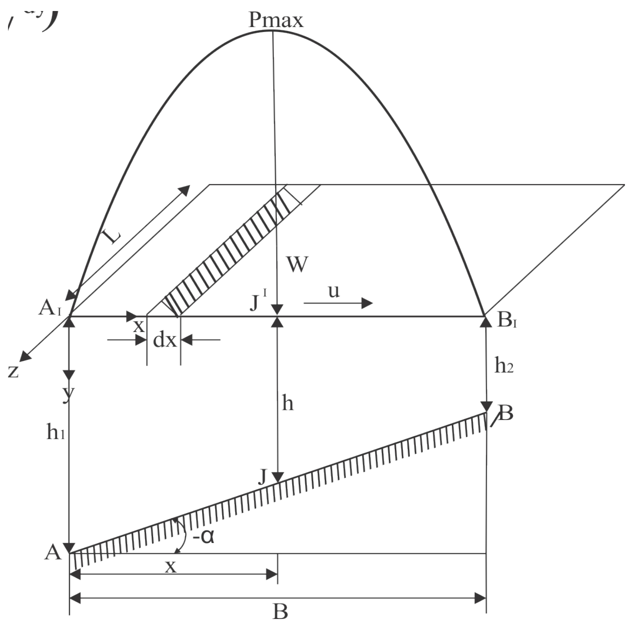
$$\frac{6\alpha X_1 (1 - X_1)}{¿}$$

$$P = \frac{\mu U}{B} ¿$$

Pressure at any point ‘x’ in the oil film is given by the above equation.

Load carrying capacity of idealized plane slider bearing with fixed shoe

The area under the curve multiplied by width of bearing is equal to external load acting on bearing it is the capacity of the bearing for a given condition. It is for the given minimum film thickness h2 relative indication of the surface ,the velocity of moving surface and viscosity of lubricant.



Elemental area =da=(dx.L)

Load on the elemental area $=w_E$ =Pressure on elemental area X elemental area

W_E= (P dx L)

Total load caring capacity w = $\int_0^B P\,dx\,L$

We know that

$X=Bx_1$ $\quad$ $x=0$; $x_1=0$

$dx=Bdx$ $\quad$ $x=B$; $x_1=1$

$$w=\int_0^1 BPLdx_1$$

$$w=BL\int_0^1 Pdx_1$$

W.K.T

$$P=\frac{\mu U}{b}\frac{6\alpha x_1(1-x_1)}{(\alpha-2a)(\alpha x_1+(a-\alpha))^2}$$

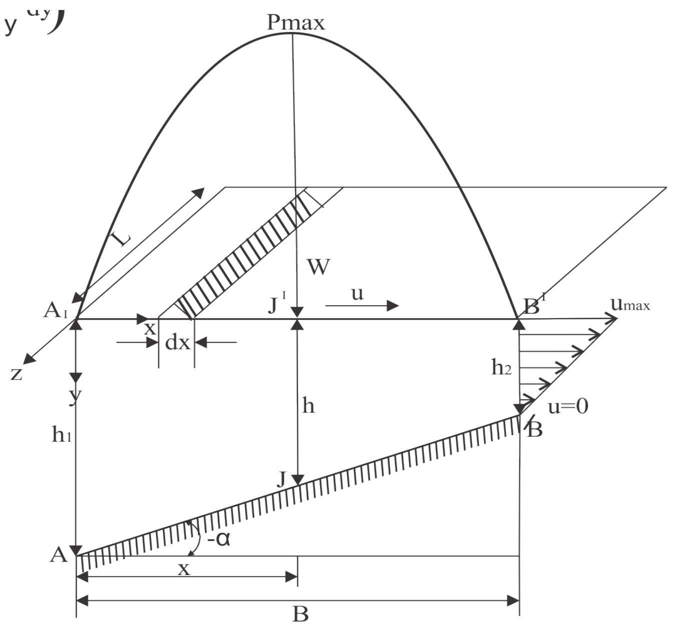
$$w=\frac{BL\mu U}{B}\frac{6\alpha}{(\alpha-2a)}\int_0^1\frac{x_1(1-x_1)}{(\alpha x_1+(a-\alpha))^2}dx_1\text{+--------}$$

$$W = \frac{6\mu UL}{\alpha^2}\left[\ln\left(\frac{a-\alpha}{a}\right)+\frac{2\alpha}{(2a-\alpha)}\right]$$

Friction in a idealized plane slider bearing with fixed shoe

The shear stress in the fluid film between two surfaces in the slider bearing tend to detain the mating member of the bearing unit and tend to drag (darw) the stationary member in to the motion.

Shear stress at any point in the fluid film from the figure as the value of Y increases velocity decreases.



Shear stress at any point in the fluid film

$$\tau = -\mu\frac{du}{dy}$$

------------------- (1)

From the Reynolds equation we know that

$$u = \frac{y^2 - hy}{2\mu}\left(\frac{dp}{dx}\right) + U\left(\frac{h-y}{h}\right)$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y^2-hy}{2\mu}\right)\frac{dp}{dx} + U\left(\frac{h-y}{h}\right)$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left[\frac{y^2}{2\mu}\left(\frac{dp}{dx}\right) - \frac{hy}{2\mu}\left(\frac{dp}{dx}\right) + U - \frac{U}{h}y\right]$$

$$\frac{\partial u}{\partial y} = \frac{2y}{2\mu}\left(\frac{dp}{dx}\right) - \frac{h}{2\mu}\left(\frac{dp}{dx}\right) + U - \frac{U}{h}$$

$$- \frac{U}{h} \qquad \text{--------------- (2)}$$

Substitute (2) in (1)

$$\tau = -\mu\left[\frac{1}{\mu}\left(\frac{dp}{dx}\right)\left(\frac{2y-h}{2}\right) + \frac{U}{h}\right]$$

$$\text{---------------- (3)}$$

W.K.T

$$\alpha x_1 + (a - \alpha)$$

¿

$$\frac{dp}{dx} = 6\mu u\left[\frac{1}{B^2\left((\alpha x_1 + (a-\alpha))\right)^2} - \frac{c}{B^3(¿¿3)}\right]$$

$$X = B x_1 \qquad\qquad c = \frac{2aB(a-\alpha)}{(2a-\alpha)}$$

$$dx = B d x_1$$

$$\alpha x_1 + (a-\alpha)$$
¿
$$¿2$$
¿
$$\alpha x_1 + (a-\alpha)$$
¿
$$\frac{dp}{d x_1} = 2(2a-\alpha)¿$$
¿
¿
$$\frac{1}{¿}$$
$$\frac{6\mu UB}{B^2}¿$$

$$\alpha x_{1+(a-\alpha)-2a(a-\alpha)}$$
¿
¿
$$(2a-\alpha)¿$$
¿
$$\frac{dp}{d x_1} = \frac{6\mu U}{B}¿$$

$$\alpha x_1 + (a-\alpha)$$
¿
$$(2a-\alpha)¿$$
¿
$$\frac{2a\alpha x_1 + 2a^2 - 2a\alpha - \alpha^2 x_1 - \alpha a + \alpha^2 - 2a^2 + 2a\alpha}{¿}$$
$$\frac{dp}{d x_1} = \frac{6\mu U}{B}¿$$

$$\frac{dp}{d x_1} = \frac{6\mu U}{B}\left[\frac{-\propto(a-\propto+\propto x_1 - 2a x_1)}{(\alpha-2a)(\alpha x_1 + (a-\alpha))^3}\right]$$

$$1$$
$$(a-\alpha)+\alpha x_1 - 2a x_{¿}$$
¿
$$6\alpha¿$$
¿
$$\frac{dp}{d x_1} = \frac{\mu U}{B}¿$$

W. K .T

$$X = B x_1 \qquad dx = Bd x_1$$

$$\frac{dp}{dx} = \frac{dp}{Bd x_1}$$

w.k.t

$$\tau=\left(\frac{h-2y}{2}\right)\frac{dp}{dx} \quad + \quad \frac{\mu u}{h}$$

$$\tau=\frac{1}{B}\frac{(h-2y)}{2}\left(\frac{dp}{d x_1}\right) \quad + \quad \frac{\mu u}{h}$$

$$1$$
$$a-\alpha+\alpha x_1-2ax_{¿}$$
$$¿$$
$$\alpha x_1+(a-\alpha)^3$$
$$(\alpha-2a)¿$$
$$6\alpha ¿ \quad + \quad \frac{\mu U}{h}$$
$$\frac{\mu u}{B}¿$$
$$\tau=\frac{1}{B}\left(\frac{h-2y}{2}\right)¿$$

W.K.T

h=B $(\alpha x_1+(a-\alpha))$

$$\alpha$$
$$a-¿$$
$$¿$$
$$\alpha x_1+(¿3¿)$$
$$\alpha x_1+(a-\alpha)$$
$$B¿$$
$$(\alpha.2a)(¿]+\frac{\mu U}{¿}$$
$$\frac{\mu U}{B}\frac{6\alpha(a-\alpha+\alpha x_1-2ax_1)}{¿}$$
$$\tau=\left(\frac{B(\alpha x_1+(a-\alpha))-2y}{2}\right)¿$$

$$\tau=\frac{\mu U}{B}\frac{3\alpha(a-\alpha+\alpha x_1-2ax_1)}{(\alpha-2a)((a-\alpha+\alpha x_1)^3)}\left(\frac{B(\alpha x_1+a-\alpha)-2y}{B}\right)+\left(\frac{1}{(\alpha x_1+a-\alpha)}\right)$$

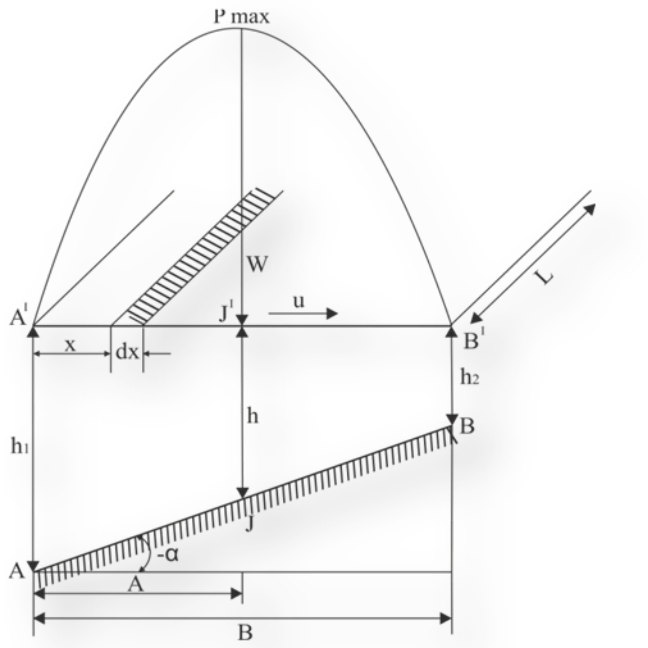
On the surface of the moving member y=0 shear stress on the moving member after simplification

$$\tau = \frac{\mu U}{B} \frac{3\alpha\left(a-\alpha+\alpha x_1 - 2a x_1\right)\left(a-\alpha+\alpha x_1\right)}{\left(\alpha - 2a\right)\left(\left(a-\alpha+\alpha x_1\right)^3\right)} + \left(\frac{1}{\left(\alpha x_1 + a - \alpha\right)}\right)$$

$$a-\alpha+\alpha x_1$$
$$¿$$
$$\tau_0 = \frac{\mu U}{B}\left[\frac{4}{\left(\alpha x_1 + a - \alpha\right)} - \frac{6a(a-\alpha)}{(2a-\alpha)(¿¿2)}\right]$$

Location of center of pressure

The distance 'A' that determines the location of center of pressure may be found by equating the sum of the movement of infinite decimal pressure forces about the horizon of the coordinates to the movement of external load 'W' about the same point.



Movement = Force × Perpendicular Distance or Load.

Movement due to External load = W × A.

ME ¿ W × A

Pressure force on the element ¿ pressure × Area.

¿ $p(dxL)$

Movement due to pressure force ¿ $\frac{p(dx\,l)x}{Force}$

Total movement due to pressure force ¿ $\int_0^B (p\,dx\,lx)$

Mp ¿ $\int_0^B (p\,dx\,l)x$

ME ¿ Mp

A 0

We know that,

X=B x_1 dx= Bd x_1

x=0 x_1 =0

x=B x_1 =1

WA= $\int_0^1 p\,B\,d\,x_1$ L B x_1

WA= B^2 L $\int_0^1 p\,x_1\,d\,x_1$

We know that,

$$p= \frac{\mu U}{B}\left[\frac{6\propto x_1(1-x_1)}{(\propto-2a)(\propto x_1+(a-\propto))^2}\right]$$

$$= B^2 L\left[\frac{6\propto x_1(1-x_1)}{(\propto-2a)(\propto x_1+(a-\propto))^2}\right]x_1\,d\,x_1$$

$$\propto x_1 + (a-\propto) ¿^2$$

$$¿$$

$$¿$$

$$= B^2 \;\; \mathrm{L} \;\; \frac{x_1^{\,2}(1-x_1)}{¿} \;\; \mathrm{d} \;\; x_1$$

$$\frac{\mu U}{B}\frac{6\propto}{(\propto-2a)}\int_0^1 ¿$$

WA =

We know that,

'W' is the load carrying capacity of idealized plain slider bearing with a fixed shoe.

$$\mathrm{W} = \begin{array}{c} \left(\frac{a-\propto}{a}\right) + ¿ \frac{2\propto}{2a-\propto} \\ \ln ¿ \\ \frac{6\mu UL}{\propto^2} ¿ \end{array}$$

$$\mathrm{WA} = \begin{array}{c} \left(\frac{a-\propto}{a}\right) + ¿ \left(\frac{2\propto}{2a-\propto}\right) \\ \ln ¿ \\ \frac{6\mu UL}{\propto^2} [¿] \\ A ¿ \end{array}$$

$$\mathrm{WA} = \frac{6\mu ULB\propto}{(\propto-2a)}\left[\frac{(a-\propto)(3a-\propto)\ln\left(\frac{a-\propto}{a}\right) - 2.5\propto^2 + 3a\propto}{\propto^4}\right]$$

$$\begin{array}{c} \left(\frac{a-\propto}{a}\right) + ¿ \frac{2\propto}{2a-\propto} \\ \ln ¿ \\ ¿ \\ \frac{6\mu UL}{\propto^2} ¿ \end{array}$$

$$\frac{A}{B} = \frac{\frac{6\mu UL\propto}{\propto-2a}\left[\frac{(a-\propto)(3a-\propto)\ln\left(\frac{a-\propto}{a}\right) - 2.5\propto^2 + 3a\propto}{\propto^4}\right]}{¿}$$

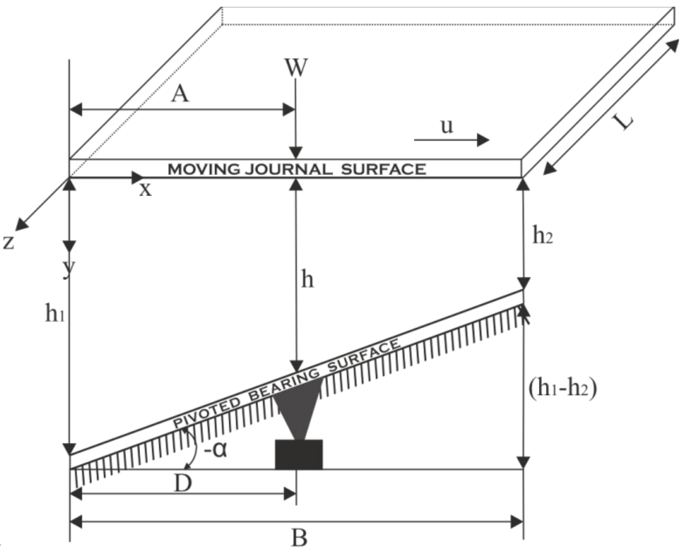
$\frac{A}{B}$

It is seen from the equation that ratio of $\frac{A}{B}$ indicating the location of center of pressure from the horizon of the coordinates dose not depends on the value of external load W, velocity μ. But depends upon the angle of inclination (-α) quantity a which is the function of the minimum thickness h_2 .

Idealized plain slider bearing with pivoted shoe.

The load carrying capacity of an idealized plane slider bearing with a pivoted for a given velocity, viscosity at minimum film thickness there is a definite value of angle of inclination that corresponds to maximum load there is also a definite value of angle of inclination. That will give minimum friction in the area consequently a bearing having fixed angle of inclination will show satisfactory performance in relatively narrow range of operating performance.

A slider bearing in which the shoe is supported at a single point so that the angle of inclination becomes variable instead of being supported along its length has much stability under baring condition of operation such a bearing with a pivoted shoe is has shown in the figure.



Load carrying capacity of idealized plane slider bearing with a fixed shoe.

$$W = \frac{\ln ¿ \left(\frac{a-\propto}{a}\right) + ¿ \frac{2\propto}{2a-\propto}}{\frac{6\mu UL}{\propto^2} ¿}$$

Let the dimension less variables are,

$$m = \frac{h_1}{h_2} - 1$$

Where,

$h_1 \rightarrow$ Maximum film thickness.

$h_2 \rightarrow$ Minimum film thickness.

$$m = \frac{h_1 - h_2}{h_2}$$

$$\propto = -ma$$

From the D.H.B page NO: 4.

$$\propto = -ma$$

$$-\frac{\propto}{a} = m \qquad \approx \frac{\propto}{a} = -m$$

$$- \propto = ma$$

$$\propto = -ma \, a = \frac{h_2}{B}$$

$$\propto = -m\frac{h_2}{B}$$

$$\propto^2 = m^2 a^2$$

$$\frac{a-\propto}{a} = 1 - \frac{\propto}{a} = 1 - (-m) = 1 + m$$

$$\frac{a-\propto}{a} = 1 + m$$

$$\frac{2\propto}{2a-\propto} = \frac{2(-ma)}{2a-(\propto)}$$

$$\text{¿}\frac{-2ma}{2a+ma}$$

$$\text{¿}\frac{-2ma}{a(2+m)}$$

$$\text{¿}\frac{-2m}{2+m}$$

$$\frac{2\propto}{2a-\propto}=\frac{-2m}{2+m}$$

Substitute the values in ‘W’

$$W = \frac{6\mu UL}{m^2a^2}\left[\ln(1+m)-\left(\frac{2m}{2+m}\right)\right]$$

We know that,

$$a=\frac{h_2}{B} \quad ; \quad \propto^2=\frac{m^2h_2^2}{B^2}$$

$$W = \frac{6\mu UL}{\frac{m^2h_2^2}{B^2}}\left[\ln(1+m)-\left(\frac{2m}{2+m}\right)\right]$$

$$W = \frac{6\mu ULB^2}{m^2h_2^2}\left[\frac{1}{m^2}\ln(1+m)-\frac{2m}{(2+m)m^2}\right]$$

$$= \frac{6\mu ULB^2}{h_2^2}\left[\frac{1}{m^2}\ln(1+m)-\frac{2m}{(2+m)m}\right]$$

$$K$$
$$(\text{¿¿}w)$$
$$W = \frac{6\mu ULB^2}{h_2^2}\text{¿}$$

Where, K_w is a dimension less quantity which is a function of $'m'$.

The value of K_w can be fined from the graph as shown in the figure 5.2 page no: 14 in the D.H.B.

Frictional resistance in a slider bearing with pivoted shoe

Substitute the quantities,

$$\propto = -ma$$

$$a = \frac{h_2}{B}$$

$$a = \frac{-\propto}{m}$$

$$\frac{h_2}{B} = \frac{-\propto}{m}$$

$$-\propto = \frac{m h_2}{B}$$

$$\propto = \frac{-m h_2}{B}$$

$$\left(\frac{a-\propto}{a}\right) = (1+m)$$

We know that,

$$\frac{2\propto}{2a-\propto} = \frac{-2m}{2+m}$$

$$(2a-\propto) = \frac{-2\propto(2+m)}{(2m)}$$

$$(2a-\propto) = \frac{-\propto(2+m)}{m}$$

$$\frac{m h_2}{Bm}(2+m)$$

$$(2a-\propto)=\frac{h_2}{B}(2+m)$$

Fixed bearing.

Pivoted Bearing.

Substitute the values of $\propto,\left(\frac{a-\propto}{a}\right)\wedge(2a-\propto)$ in F_0

$$F_0=-\mu UL\left[\frac{4}{\frac{-mh_2}{B}}\ln(1+m)+\frac{6}{\frac{h_2}{B}(2+m)}\right]$$

$$F_0=-\mu UL\frac{B}{h_2}\left[\frac{4}{m}\ln(1+m)+\frac{6}{(2+m)}\right]$$

$$F_0=-\mu UL\frac{B}{h_2}(K_F)$$

Where,

K_F The dimension less quantity is depends upon the quantity $'m'$.

The resistance in the slider Bearing is proportional to $\mu,U,L\wedge B$ and inversely proportional to minimum thickness of lubricating film between the surfaces.

The value of K_F can be optimized by using the table & figure no: 5.3.

Coefficient of friction,

$$f=\frac{F_0}{w}=\frac{\frac{\mu ULB}{h_2}(K_F)}{\frac{6\mu ULB^2}{h_2^2}(K_w)}$$

$$f=\frac{\mu ULB}{h_2}(K_F)\cdot\frac{h_2^2}{6\mu ULB^2}(K_w)$$

$$f = \frac{h_2}{B} \frac{K_F}{6K_w}$$

Where,

K_f Is the fraction of quantity $'m'$ only it can be determined from the

Figure no: 5.4 & the table.

Problems.

1) A pivoted shoe of a slider bearing has a square shape. The load acting on the bearing $w = 13500\ N \approx 13.5\ KN$. The velocity of the moving member is $U = 5\ m/sec$.

The lubricating oil is SAE40. The excepted mean temperature of the oil film is 190 $^\circ F$. The permissible minimum oil film thickness $h_2 = 1.905 \times 10^{-5} m$.

Find a)Required dimensions of the shoe.

b) Coefficient of the friction in the bearing under given operating condition.

c) Power loss.

Assume inclination of the surface that corresponding to maximum load carrying capacity. Neglect the effect of end flow from the bearing.

<u>Given:</u>

$w = 13500\ N \approx 13.5\ KN$.

$U = 5\ m/sec$.

$h_2 = 1.905 \times 10^{-5} m$.

Oil is SAE40, $T = 190\ ^\circ F$, from table 2.13

$\mu = 16\ cp$.

$$\mu = \frac{16}{1000} = 0.016\ Ns/m^2$$

¿table no: 5.2

For m ¿1.2

$K_f = 4.70034$.

$K_F = 0.753191$.

$K_w = 0.026707$.

a) Required dimensions of the shoe.

$$w = \frac{6\mu U L^3}{h_2^2}(K_w)$$

$$13500 = \frac{6 \times 0.016 \times 5 \times L^3}{(1.905 \times 10^{-5})^2}(0.026707)$$

$L = 0.0725\, m.$

$$w = \frac{6\mu UL B^2}{h_2^2}(K_w)$$

$$13500 = \frac{6 \times 0.016 \times 5 \times 0.0725 \times B^2}{(1.905 \times 10^{-5})^2}(0.026707)$$

$B = 0.355\, m.$

b) Coefficient of the friction in the bearing under given operating condition.

$$f = \frac{h_2}{B} K_f$$

$$f = \frac{1.905 \times 10^{-5}}{0.355} 4.70034$$

$f = 2.522 \times 10^{-4}$

c) Power loss.

$P = F_0 \times U$

$$F_0 = \frac{\mu ULB}{h_2}(K_F)$$

$$F_0 = \frac{0.016 \times 5 \times 0.0725 \times 0.355}{1.905 \times 10^{-5}}(0.753191)$$

$F_0 = 3.4022\, N$

$P = 3.4022 \times 5$

$P = 17.011$

2) A slider bearing with rectangular pivoted shoe ¿ $0.1143\, m.$

Width of Bearing $B = 0.0726\, m.$

Velocity $U = 2.032 \frac{m}{sec}.$

$h_2 = 2.286 \times 10^{-5}\, m.$

Expected mean temperature of the oil film175 °F oil SAE50.

1) Determine the load carrying capacity of the bearing.
2) Power loss of the Bearing.
3) Coefficient of friction in the Bearing.

<u>Given:</u>

$L = 0.1143\, m.$

$B = 0.0726\, m.$

$U = 2.032 \frac{m}{sec}.$

$h_2 = 2.286 \times 10^{-5}\, m.$

Temperature of the oil film175 °F oil SAE50.

From table no: 2.13.

$\mu = 28\, cp$.

$$\mu = \frac{28}{1000} = 0.028\, Ns/m^2$$

Assume maximum load carrying capacity

¿ $table\, no:$ 5.2

For m ¿1.2

$K_f = 4.70034.$

$K_F = 0.753191$.

$K_w = 0.026707$.

a) Load carrying capacity of the Bearing.

$$w = \frac{6 \mu U L B^2}{h_2^2} (K_w)$$

$$w = \frac{6 \times 0.028 \times 2.032 \times 0.1143 \times (0.0762)^2}{(2.286 \times 10^{-5})^2} (0.026707)$$

$w = 11578.75\, N$

$w = 11.57\, KN.$

b) Power loss.

$$P = F_0 \times U$$

$$F_0 = \frac{\mu ULB}{h_2}(K_F)$$

$$F_0 = \frac{0.028 \times 2.032 \times 0.1143 \times 0.0762}{2.286 \times 10^{-5}}(0.753191)$$

$$F_0 = 16.32\,N$$

$$P = 16.32 \times 2.032$$

$$P = 33.17\ Nm/sec.$$

b) Coefficient of the friction in the bearing under given operating condition.

$$f = \frac{h_2}{B}K_f$$

$$f = \frac{2.286 \times 10^{-5}}{0.0762}4.70034$$

$$f = 1.41 \times 10^{-3}\ .$$

3) The runner of the thrust Bearing is as shown in figure it is it is supported by 6 pivoted shoes.
It has the following specifications.
Diameter $D = 0.5588\,m.$

$$\frac{D}{d} = 2.5$$

Viscosity of the lubricating oil $0.04481\,Ns/m^2.$

$h_2 = 4.57 \times 10^{-5}\,m.$

1) Find what load can be supported by this bearing.
2) Coefficient of friction.
3) Power loss in the Bearing.

Assume dimension less variable $m=1.$ & speed $N = 350\,rpm.$ the load is equally distributed among the individualshoes.

<u>Given:</u>

$D = 0.5588\,m.$

$$\frac{D}{d} = 2.5$$

$\mu = 0.04481\, Ns/m^2.$

$h_2 = 4.57 \times 10^{-5}\, m.$

$N = 350\, rpm.$

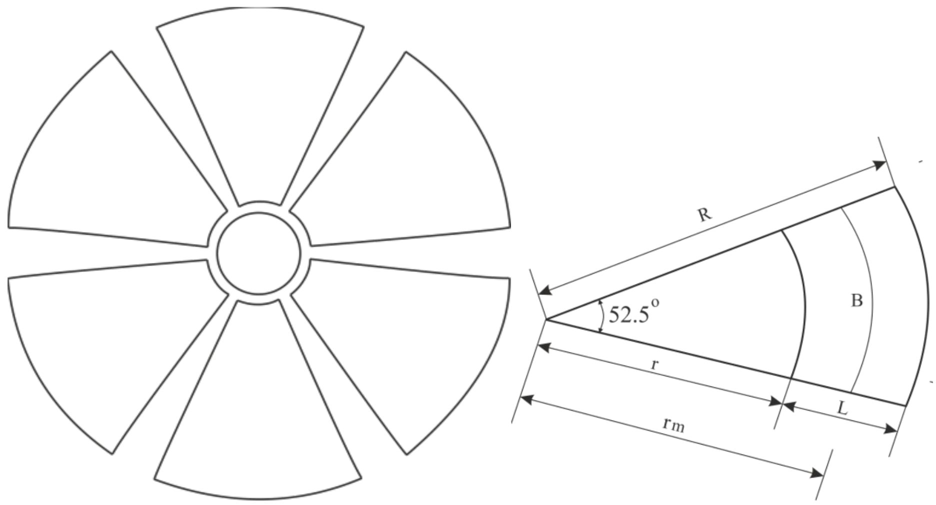
¿ $table\, no:$ 5.2.

For m ¿ 1

$K_f = 4.862537.$

$K_F = 0.772589$.

$K_w = 0.026481$.



$$R = \frac{D}{2} = \frac{0.5588}{2} = 0.2794\, m.$$

$$2.5 = \frac{D}{d}$$

$$d = \frac{0.5588}{2.5} = 0.2235\, m.$$

$r = 0.11176\, m.$

$$L = R - r$$

$$¿\,0.2794 - 0.11176$$

$$L = 0.16764\,m.$$

$$B = r_m \theta$$

$$r_m = \frac{R+r}{2} = \frac{0.2794+0.11176}{2}$$

$$r_m = 0.19558\,m.$$

$$B = r_m \theta$$

$$¿\,0.19558 \times \frac{\pi}{180} \times 52.5$$

$$B = 0.1742\,m.$$

We know that,

$$U = \frac{2\pi r_m n}{60}$$

$$¿\,\frac{2 \times \pi \times 0.19558 \times 350}{60}$$

$$U = 7.168\,m/sec.$$

1) Required dimensions of the shoe.

$$w = \frac{6\mu U L B^2}{h_2^2}\left(K_w\right)$$

0.1792
¿
¿
¿2
$6 \times 0.04481 \times 7.168 \times 0.16764 \times ¿$
$w = ¿$

$$w = 131547.15\,N/shoe.$$

$$w = 131547.15 \times 6 = 789282.9\,N.$$

$$w = 789.2\,KN.$$

2) Coefficient of the friction in the bearing under given operating condition.

$$f = \frac{h_2}{B} K_f$$

$$f = \frac{4.57 \times 10^{-5}}{0.1792} 4.862537$$

$$f = 1.24 \times 10^{-3}$$

3) Power loss.

$$P = F_0 \times U$$

$$F_0 = \frac{\mu ULB}{h_2} (K_F)$$

$$F_0 = \frac{0.0.14481 \times 7.168 \times 00.16764 \times 0.1792}{4.57 \times 10^{-5}} (0.772589)$$

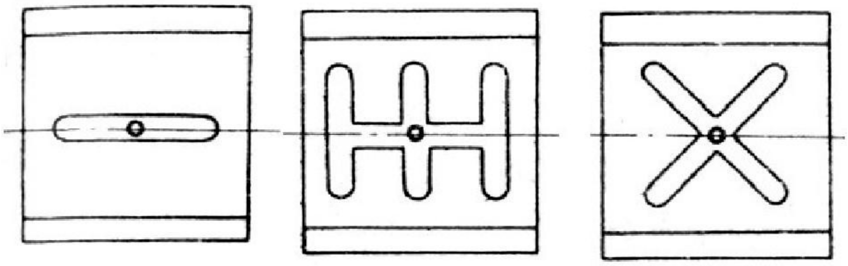
$$F_0 = 163.125 \, N/shoe.$$

$$F_0 = 163.125 \times 6 = 978.75 \, N.$$

$$P = 978.75 \times 7.168$$

$$P = 7015.68 \, Kw$$

Typical oil groove patterns



Some typical groove patterns are shown in the above figure. In general, the lubricant may be brought in from the end of the bushing, through the shaft, or through the bushing. The flow may be intermittent or continuous. The preferred practice is to bring the oil in at the center of the bushing so that it will flow out both ends, thus increasing the flow and cooling action.

Thermal aspects of bearing design

Heat is generated in the bearing due to the viscosity of the oil. The frictional heat is converted into heat, which increases the temperature of the lubricant. Some of the lubricant that enters the bearing emerges as a side flow, which carries away some of the heat. The balance of the lubricant flows through the load-bearing zone and carries away the balance of the heat generated. In determining the viscosity to be used we shall employ a temperature that is the average of the inlet and outlet temperatures, or

T_{av} T_1 $T/2$

where $= T +$ $_1$ is the inlet temperature and T is the temperature rise of the lubricant from inlet to outlet. The viscosity used in the analysis must correspond to T_{av}.

Self contained bearings:

These bearings are called ***selfcontained*** bearings because the lubricant sump is within the bearing housing and the lubricant is cooled within the housing. These bearings are described as *pillow-block* or *pedestal* bearings. They find use on fans, blowers, pumps, and motors, for example. Integral to design considerations for these bearings is dissipating heat from the bearing housing to the surroundings at the same rate that enthalpy is being generated within the fluid film.

Heat dissipated based on the projected area of the bearing:

Heat dissipated from the bearing, J/S $\mathbf{H_D = K_2\, ld\, (t_B - t_A) = C''ld}$

Where $C'' = K_2\,(t_B - t_A)$ a coefficient from fig15.16 or table 15.10 (From data hand book)

Another formula to determine the heat dissipated from the bearing

$\mathbf{H_D = ld\, (T+18)^2 / K_3}$

Where $K_3 = 0.2674 \times 10^6$ for bearings of heavy construction and well ventilated $= 0.4743 \times 10^6$ for bearings of light construction in still air air

T= t_B - t_A

Where,

t_B = Bearing surface temperature t_A = Ambient temperature

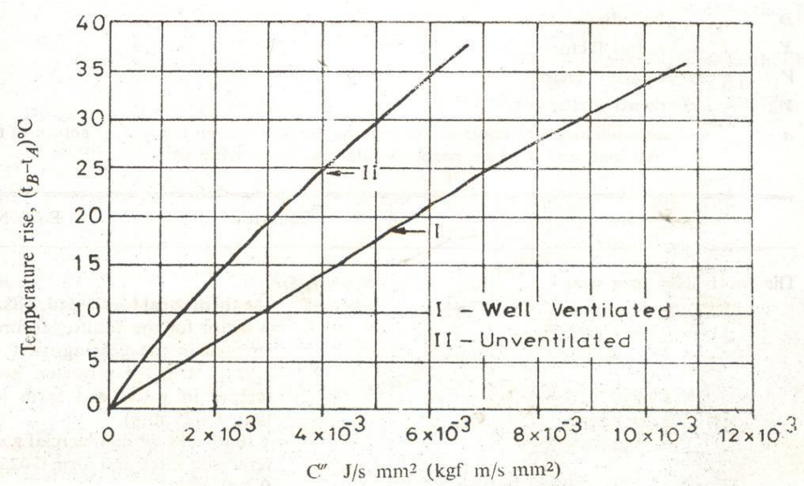


Fig. 15.16 Coefficient, C″

For good performance the following factors should be considered.

- **Surface finish of the shaft (journal):** This should be a fine ground finish and preferably lapped.
- **Surface hardness of the shaft:** It is recommended that the shaft be made of steel containing at least 0.35-0.45% carbon. For heavy duty applications shaft should be hardened.
- **Grade of the lubricant:** In general, the higher the viscosity of the lubricant the longer the life. However the higher the viscosity the greater the friction, so high viscosity lubricants should only be used with high loads. In high load applications, bearing life may be extended by cutting a grease groove into the bearing so grease can be pumped in to the groove.
- **Heat dissipation:** Friction generates heat and causes rise in temperature of the bearing and lubricant. In turn, this causes a reduction in the viscosity of the lubricating oil and could result in higher wear. Therefore the housing should be designed with heat dissipation in mind. For example, a bearing mounted in a Bakelite housing will not dissipate heat as readily as one mounted in an aluminum housing.
- **Shock loads:** Because of their oil-cushioned operation, sliding bearings are capable of operating successfully under conditions of moderate radial shock loads. However excessive prolonged radial shock loads are likely to increase metal to metal contact and reduce bearing life. Large out of balance forces in rotating members will also reduce bearing life.
- **Clearance:** The bearings are usually a light press fit in the housing. A shouldered tool is usually used in arbour press. There should be a running clearance between the journal and the bush. A general rule of thumb is to use a clearance of 1/1000 of the diameter of the journal.
- **Length to diameter ratio(l/d ratio):** A good rule of thumb is that the ratio should lie in the range 0.5-1.5. If the ratio is too small, the bearing pressure will be too high and it will be difficult to retain lubricant and to prevent side leakage. If the ratio is too

high, the friction will be high and the assembly misalignment could cause metal to metal contact.

Examples on journal bearing design

Example EI:

Following data are given for a 360º hydrodynamic bearing:

Radial load=3.2 kN
Journal speed= 1490 r.p.m.
Journal diameter=50 mm
Bearing length=50mm
Radial clearance=0.05 mm
Viscosity of the lubricant= 25 cP

Assuming that the total heat generated in the bearing is carried by the total oil flow in the bearing, calculate:

- Power lost in friction;
- The coefficient of friction;
- Minimum oil film thickness
- Flow requirement in 1/min; and
- Temperature rise.

Solution:

P= W/Ld = 3.2x1000/ (50x50) =1.28 MPa.= 1.28×10^6 Pa

Sommerfeld number = $S = (ZN'/p)(r/c)^2$

r/c =25/0.0.05 = 500

Z= 25 cP = 25×10^{-3} Pa.sec

= 1490/60= 24.833 r/sec. Substituting the above values, we get

S=0.121

For S= 0.121 & L/d=1,

Friction variable from the graph= (r/c) f= 3.22

Minimum film thickness variable= $h_o/c = 0.4$

Flow variable= $Q/rcN'L$= 4.33

f = 3.22x0.05/25= 0.0064

Frictional torque= T= fWr = 0.0064x3200x 0.025 = 0.512 N-m

Power loss in the Bearing= $2\pi N' T/1000$ kW = 0.080 kW

h_o = 0.4x 0.05= 0.02 mm

$Q/r\,c\,N^1 L$= 4.33 from which we get,

$Q = 6720.5\ mm^3 / sec.$

Determination of dimensionless variables is shown in the following figures. Assume that all the heat generated due to friction is carried away by the lubricating oil.

Heat generated = 80 watt = mC_p T

where:

m= mass flow rate of lubricating oil= ρQ in kg/sec

Cp= Specific heat of the oil= 1760 J/kg ºC

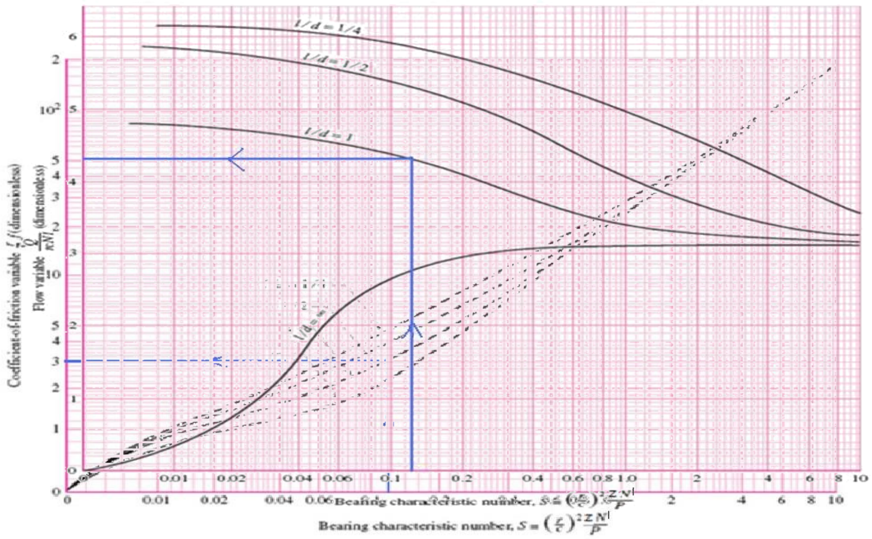
T= temperature rise of the oil

ρ= 860×10^{-9} kg/mm^3

Substituting the above values,

T= 7.9 ºC

The Average temperature of the oil= T_i + T/2 = 27+ ‹7.9/2›=30.85 ºC



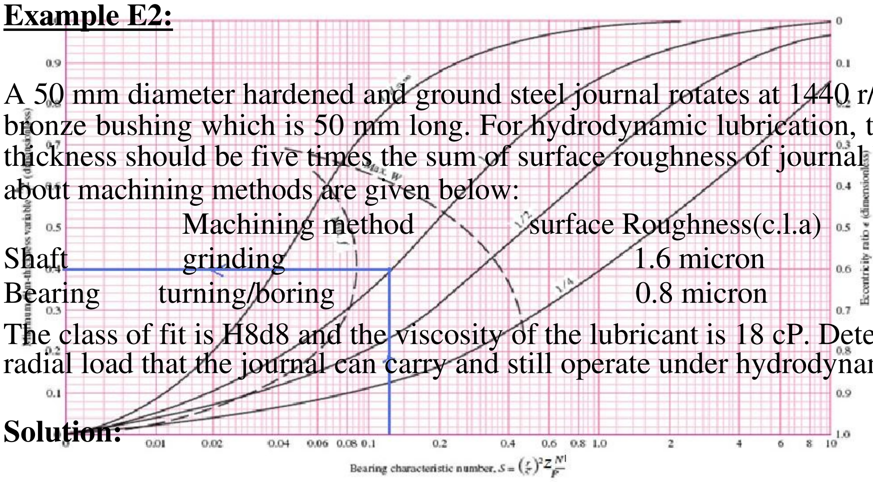
Example E2:

A 50 mm diameter hardened and ground steel journal rotates at 1440 r/min in a lathe turned bronze bushing which is 50 mm long. For hydrodynamic lubrication, the minimum oil film thickness should be five times the sum of surface roughness of journal bearing. The data about machining methods are given below:

	Machining method	surface Roughness(c.l.a)
Shaft	grinding	1.6 micron
Bearing	turning/boring	0.8 micron

The class of fit is H8d8 and the viscosity of the lubricant is 18 cP. Determine the maximum radial load that the journal can carry and still operate under hydrodynamic conditions.

Solution:



Min. film thickness = h_o= 5 [0.8+1.6] = 12 micron = 0.012 mm For H8 d8 fit, referring to table of tolerances,

Ø50 H8 = Min. hole limit = 50.000 mm
Max. hole limit = 50.039 mm
Mean hole diameter= 50.0195 mm

Ø 50 d8 = Max. shaft size = 50- 0.080= 49.920 mm
Min. shaft size = 50- 0.119= 49.881 mm
Mean shaft diameter= 49.9005 mm.

Assuming that the process tolerance is centered,
Diametral clearence= 50.0195- 49.9005= 0.119 mm Radial clearence= 0.119/2= 0.0595 mm

h_o /c = 0.012/ 0.0595 = 0.2
L/d = 50/50= 1
From the graph, Sommerfeld number= 0.045

S= (ZN´/p) (r/c)^2 = 0.045

r/c= 25/0.0595= 420.19

Z= 18 cP= 18x10^{-3} Pa.sec
N′= 1440/60= 24 r/sec

From the above equation, Bearing pressure can be calculated. p= 1.71x10^{6} Pa = 1.71 MPa.

The load that the bearing can carry:
W= pLd = 1.71x 50x 50= 4275 N

Example E3:

The following data are given for a full hydrodynamic journal bearing: Radial load=25kN
Journal speed=900 r/min.
Unit bearing pressure= 2.5 MPa (l/d) ratio= 1:1
Viscosity of the lubricant=20cP
Class of fit=H7e7

Calculate: 1. Dimensions of bearing
2. Minimum film thickness and
3. Requirement of oil flow

Solution:

N′ = 900/60= 15 r/sec
P=W/Ld
2.5= 25000/Ld=25000/d^2 As L=d.
d= 100 mm & L=100 mm

For H7 e7 fit, referring to table of tolerances,
Ø100 H7 = Min. hole limit = 100.000 mm
Max. hole limit = 100.035 mm
Mean hole diameter= 100.0175 mm
Ø 100 e7 = Max. shaft size = 100- 0.072= 99.928 mm
Min. shaft size = 100- 0.107= 99.893 mm
Mean shaft diameter= 99.9105 mm

Assuming that the process tolerance is centered,
Diametral clearence= 100-0175- 99.9105= 0.107 mm Radial clearence= 0.107/2= 0.0525mm

Assume r/c = 1000 for general bearing applications.
C= r/1000=50/1000 = 0.05 mm.

Z= 20 cP= 20x10^{-3} Pa.sec
N^1 = 15 r/sec
P= 2.5 MPa= 2.5 x10^{6} Pa
S= (ZN′/p) $(r/c)^2$ =0.12

For L/d=1 & S=0.12, Minimum Film thickness variable= h_o /c = 0.4

h_o = 0.4x 0.05= 0.02 mm

Example E4:

A journal bearing has to support a load of 6000N at a speed of 450 r/min. The diameter of the journal is 100 mm and the length is 150mm.The temperature of the bearing surface is limited to 50 ºC and the ambient temperature is 32 ºC. Select a suitable oil to suit the above conditions.

Solution:

N^1 = 450/60 =7.5 r/sec, W=6000 N, L=150mm, d=100 mm, t_A = 32 ºC, t_B = 50 ºC.
Assume that all the heat generated is dissipated by the bearing.

Use the Mckee's Equation for the determination of coefficient of friction.

f=Coefficient of friction= K_a (ZN^1 /p) (r/c) 10^{-10} + f

p= W/Ld= 6000/100x150 = 0.4 MPa.
K_a = 0.195x 10^6 for a full bearing
f = 0.002
r/c= 1000 assumed
U= 2πrN^1 = 2x3.14x 50x7.5= 2335 mm/sec= 2.335 m/sec.
f = 0.195x 10^6 x (Z * 7.5 / 0.4) x 1000 x 10^{-10} +0.002
f = 0.365Z+0.002

Heat generated= f *W*U
Heat generated= (0.365Z+ 0.002)x6000x2.335

Heat dissipated from a bearing surface is given by:

H_D= ld $(T+18)^2$/ K_3

Where K_3= 0.2674x 10^6 for bearings of heavy construction and well ventilated
= 0.4743x10^6 for bearings of light construction in still air air

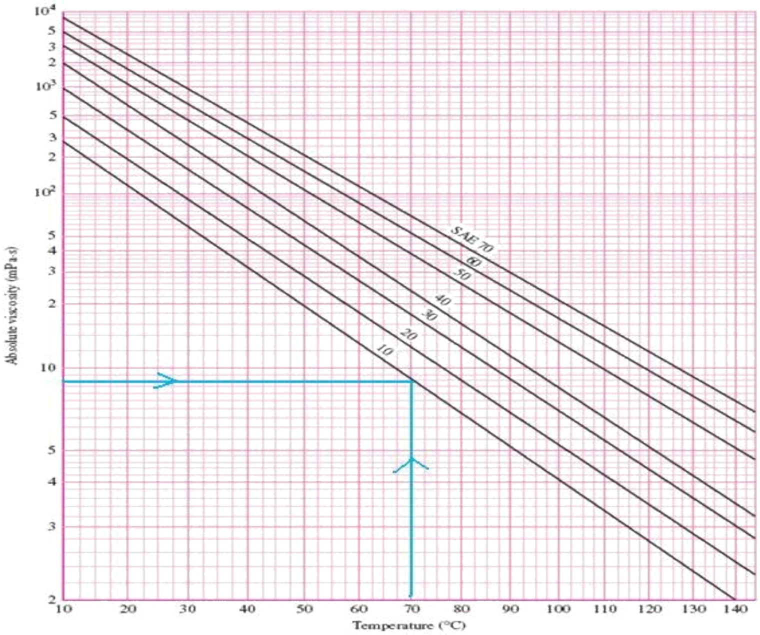
T= t_B - t_A = 50-32 =18ºC
H_D = 150x100($18+18)^2$ / 0.2674x10^6 =72.7 Watt

H_D = Hg for a self contained bearing.

72.7 = (0.365Z+ 0.002)x6000x2.335
Z= 0.0087 Pa.Sec.

Relation between oil temp, Amb. temp, & Bearing surface temperature is given by $t_B - t_A$= ½ (t_O- t_A)
t_O = oil temperature= 68 ºC

Unit 5- Oil Flow and thermal Equilibrium of Bearing



Select SAE 10 Oil for this application

Hydrostatic bearings derive their load capacity not from shear flow driven effects (hydrodynamic wedge and surface sliding) but rather from the combination of pressure versus flow resistance effects through a feed restrictor and in the film lands. Figure 1 depicts thrust and radial hydrostatic bearing configurations for process fluid lubrication turbopumps. Table 1 presents the major advantages and disadvantages of hydrostatic bearings

The hydrostatic stiffness is of unique importance for the centering of high-precision milling machines, gyroscopes, large arena movable seating areas, telescope bearings, and even cryogenic fluid turbo pumps for rocket engines. Note that hydrostatic bearings require an external pressurized supply system and some type of flow restrictor. Also, under dynamic motions, hydrostatic bearings may display a pneumatic hammer effect due to fluid compressibility. However, and most importantly, the load and static stiffness of a hydrostatic bearing are independent of fluid viscosity; thus making this bearing type very attractive for application with non-viscous fluids, including gases and cryogens.

Hydrostatic Bearings: Advantages and Disadvantages

Advantages	Disadvantages
1.Support very large loads. The load support is a function of the pressure drop across the bearing and the area of fluid pressure action. Load does not depend on film thickness or lubricant viscosity. Long life (infinite in theory) without wear of surfaces Provide stiffness and damping coefficients of very large magnitude. Excellent for exact positioning and control.	Require ancillary equipment. Larger installation and maintenance costs. Require ancillary equipment. Larger installation and maintenance costs. High power consumption because of pumping losses. Potential to induce hydrodynamic instability in hybrid mode operation. Potential to show pneumatic hammer instability for highly compressible fluids, i.e. loss of damping at low and high frequencies of operation due to compliance and time lag of trapped fluid volumes.

Consider the fundamental operation of a simple one dimensional hydrostatic bearing [Rowe 1983, San Andrés 2002]. The flow is laminar and fluid inertia effects are not accounted for; i.e. a classical lubrication example. Figure 2 depicts a 1D bearing of very large width (B). A hydrostatic bearing combines two flow restrictions in series, one at the feed or supply port, and the other through the film lands. In the feed restrictor (orifice, capillary, etc.) the fluid drops its pressure from the supply value (Ps) to a magnitude (PR) within a recess or pocket of typically large volume (see Figure 3). Since the recess is deep, the pocket pressure is regarded as uniform over the entire recess area AR=bB. The fluid then flows from the recess into the film lands of small thickness h, and discharges to ambient pressure through the bearing sides, say Pa=0 for simplicity.

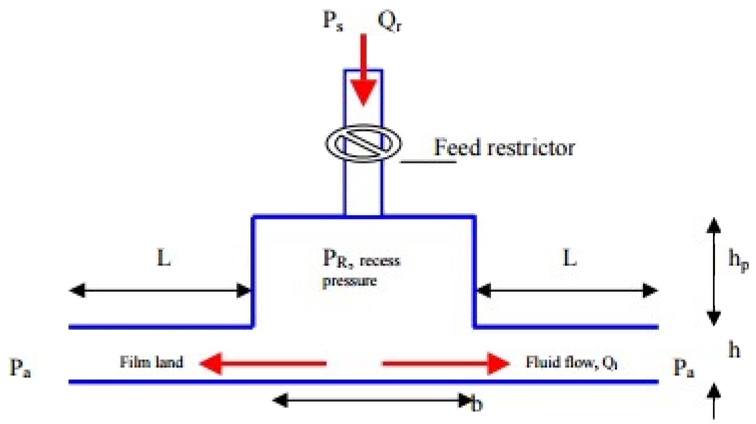


Fig. 2 Geometry of a simple 1-D hydrostatic bearing

The flow rate (Qr) across the restrictor is a function of the pressure drop, Qt=f(Ps-PR). For an orifice and capillary feeding,

$$Q_r = Q_o = A_o C_d \sqrt{\frac{2}{\rho}(P_s - P_R)} \;;\quad Q_r = Q_c = \frac{\pi d^4}{128 \mu \ell_c}(P_s - P_R) \qquad (1)$$

with *Ao* and *Cd* as the orifice area and empirical discharge coefficient, respectively. (*d, ℓc*) are the diameter and length of the capillary tube, typically *ℓc* » 20 *d.* The orifice coefficient (*Cd*) ranges from 0.6 to 1.0, depending on the flow condition (Reynolds number), the orifice geometry and even the film thickness. Under turbulent flow conditions, tests and CFD analysis evidence *Cd* ~0.80. Across the bearing film lands the fluid drops in pressure from (*PR*) to ambient pressure, *Pa*. In the laminar flow of an incompressible fluid, the flow rate is a function of the pressure drop and equals.

$$Q_\ell = -\frac{B h^3}{12\mu}\frac{\partial P}{\partial x} = +\frac{B h^3 (P_R - P_a)}{12 \mu L}$$

where B is the bearing width and L is the film length with thickness h. Presently, no surface motion along the x-axis is accounted for, i.e. the bearing is stationary. Understeady state conditions, the flow through the restrictor equals the flow through the film lands, i.e.

$$Q_r = f(P_s - P_R) = 2C_l(P_R - P_a) = 2Q_l \qquad (3)$$

with $Cl = B\ h3/(12\ \mu\ L)$ as a flow-conductance along the film land. Eqn. (3) permits the determination of the recess pressure (PR) given the film conductance (Cl) and feed restrictor parameters. For bearing design, a value of pocket pressure (PR) is desired, and Eqn. (3) serves to size the diameter of the supply restrictor. For the simple bearing considered, the pressure field on the bearing surface takes the shape shown in Figure 3. Note that the recess pressure is assumed uniform or constant.

Problems

1) A hydrostatic step bearing the following specifications:
   - Diameter of shaft: $D_\circ$ = 150 mm
   - Diameter of pocket: D_i = 100 mm
   - Vertical thrust on bearing:W = 70 KN
   - Shaft speed:N = 1000rpm
   - Viscousity of the lubricant:µ = 0.025 Pa-s
   - Desirable oil film thickness:h = 0.125 mm
   - Determine: (i) Rate of flow through the bearing. (ii) Power loss due to viscous friction. (iii) Cofficient of friction.

Solution: $$Q = \frac{\pi h^3 P_s}{6\mu ln\left(\frac{r_\circ}{r_i}\right)}$$

$$P_s = \frac{2Wln\left(\frac{r_\circ}{r_i}\right)}{\pi\left(r_\circ^4 - r_i^4\right)} = \frac{2\times 70\times 10^3\times \ln\left(\frac{75\times 10^{-3}}{50\times 10^{-3}}\right)}{¿}$$

75×10^{-3} ¿ ¿ 50×10^{-3} ¿ ¿ ¿ π ¿

$$= \mathbf{0.7116} \times 10^9 \frac{N}{m^2}$$

$$Q = \frac{\pi\times(0.125\times 10^{-3})^3\times 0.7116\times 10^9}{6\times 0.025\times \ln\left(\frac{75\times 10^{-3}}{50\times 10^{-3}}\right)} = \mathbf{0.0718}\ \frac{m^3}{sec}$$

P = T $\times \omega$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1000}{60} = 104.71 \frac{rad}{sec}$$

$$75 \times 10^{-3}$$
¿
¿
$$50 \times 10^{-3}$$
¿
¿

T = $¿4 - (¿4¿)$
¿
$\pi \times 0.025 \times 104.71¿$

$$\frac{\pi \times \mu\omega\left(r_{\circ}^{4} - r_{i}^{4}\right)}{2h} = ¿$$

P = $0.8352 \times 104.71 = 0.0874\, KW$

$$\frac{(¿¿\circ - r_i)}{2} = 0.0625\, m$$
r
$r_m = ¿$

f = $\dfrac{F}{W} = \dfrac{T}{r_m \times W} = \dfrac{0.8352}{0.0625 \times 70 \times 10^{3}} = 0.191 \times 10^{-3}$

2. A hydrost circular thrust bearing has the following data :

a) a shaft ϕ = D = 300 mm

b) ϕ of pocket = d 200 mm

c) shaftspee N = 100 rpm

d) Pr at pocket P_s = 500 KN / m^2

e) film thickness h = 0.07 mm

f) viscosity of rub μ = 0. 05 N-s/m^2

Determine load carrying capacity, oil flow rate, power loss due to viscous friction.

d_o = 300 mm = 0.3 mt $\therefore$ r_o = 0.15 mt

d_i = 200 mm = 0.2 mt r_i = 0.1 mt

N = 100 rpm

P_s = 500 X 10^3N/m^2 **WKT, W =** $\dfrac{\pi P_s\left(r_o^2 - r_i^2\right)}{2.\ln\left(\dfrac{r_o}{r_i}\right)}$

h = 0.07 X 10^{-3}mt

μ **= 0.05 N-sa/m^2**

W = ? $= \dfrac{\pi X 500 X 10^3\left(0.15^2 - 0.1^2\right)}{2.\ln\left(\dfrac{0.15}{0.1}\right)}$

Q = ? $= \dfrac{19,635}{0.8109}$

Hp= ? **= 24, 213 N**

Q = $\dfrac{\pi h^3 p_5}{6n - \ln(\dfrac{r_o}{r_i})}$ $= \dfrac{\pi x(0.07_a^{10-3})3X500X103}{6X0.05X\ln\left(\dfrac{0.15}{0.1}\right)}$ $= \dfrac{5.3878X^{10-7}}{0.1216}$

= 4.43 X 10^{-6} m^3/Sec

Hp = $\dfrac{\pi \mu w^2\left(\partial_o^{\ 4} - r_i^{\ 4}\right)}{2h}$ **w =** $\dfrac{2\pi N}{60} = \dfrac{2\pi X 100}{60}$

= 10.47 rad/Sec

$= \dfrac{\pi X 0.05 X 10.47^2\left(0.15^4 - 0.1^4\right)}{2 X 0.07 X 10^{-3}}$

$= \dfrac{6.0079X10^{-3}}{0.14X10^{-3}} = 50 watts$

3.A hydrostatic step bearing for a furbine rotor has the following specifications :

Dia of shaft = 150 mm

Dia of pocket = 100 mm

Thrust on the bearing = 60 KN

Shaft speed = 1500 rpm.

Viscosity of the lubricant at operating cond = 30 cp

Desirable of film thickness h = 0.125 mm

Find 1) rate of oil flow thro the bearing

2) Power loss due to viscous friction

3) Co-efficient of friction.

Rate of flow is given by

$$Q = \frac{\pi h^3 P_s}{6\mu . \ln\left(\frac{r_o}{r_i}\right)}$$

But

$$P_s = \frac{2w\ln\left(\frac{r_o}{r_i}\right)}{\pi\left(r_o^2 - r_i^2\right)}$$

$$r_o = \frac{150}{2} = 75mm = 75\,X\,10^{-3}\,m$$

$$r_i = \frac{150}{2} = 50mm = 50\,X\,10^{-3}\,m$$

$$\therefore P_s = \frac{2X60X10^3 \ \ln\left(\frac{75X10^{-3}}{50X10^{-3}}\right)}{\pi\left[\left(75X10^{-3}\right)^2 - \left(50X10^{-3}\right)\right]}$$

= 4.95X10^6 N/M^2

$$Q = \frac{\pi\,X\,0.125X10^{-3}\,X\,4.95X10^6}{6X0.03\ .\ \ln\left(\frac{75X10^{-3}}{50X10^{-3}}\right)} = 0.41X10^{-3}\ m^3/\sec$$

WKT, Power loss, P = T_f X w

But w = $\frac{2\pi n}{60}$

$$T_f = \frac{\mu . w\pi}{2h}\left(r_o^4 - r_o^4\right)$$

$$= \frac{2\pi X1500}{60}$$

$$= \frac{0.03X157X\pi\left|\left(75X10^{-3}\right)^4 - \left(50X10^{-3}\right)^4\right|}{2X0.125X10^{-3}} \quad = 157\ {rad}/{Sec}$$

= 1.5 N-m

$\therefore$ **power loss, P = 1.5 X 157**

= 235.5 watts

Co-efficient of friction F = $\frac{F}{W}$

But T = FXr

$\therefore$ **F =** $\frac{T}{r}$

$\therefore$ **F =** $\frac{T}{r_n}.\frac{1}{w}$

$$= \frac{1.5}{62.5*10^{-3}*60000}$$

$$= 4*10^{-4}$$

$$\gamma_m = \frac{r_0 + r_1}{2}$$

=0.000625

Bearing Materials

Aluminium alloys

- Good fatigue strength, load bearing capacity, thermal conductivity, and corrosion resistance
- Less expensive than babbitt materials
- Most aluminium allows contain tin as an element which remains in the free state to provide a better bearing surface
- The strongest aluminium alloy used is aluminium-silicon
- Thermal expansion is relatively high and this restricts their usage at high temperatures
- Emeddability, conformability, and compatibility are not very good and these are improved by providing a babbitt overlay

Cadmium and silver alloys

Cadmium:

- Cadmium alloys offer good fatigue resistance and excellent compatibility characteristics
- Their corrosion resistance is poor and are they are expensive

Silver:

- Used as deposited material on steel with an overlay of lead
- The addition of lead improves the embeddability, anti-weld and anti-scoring properties

Babbitt :

Babbitt is usually used in integral bearings. It is coated over the bore, usually to a thickness of 1 to 100 thou (0.025 to 2.540 mm), depending on the diameter. Babbitt bearings are designed to not damage the journal during direct contact and to collect any contaminants in the lubrication.

Bronze

A common plain bearing design utilizes a hardened and polished steel shaft and a softer bronze bushing. The bushing is replaced whenever it has worn too much.

Common bronze alloys used for bearings include: SAE 841, SAE 660 (CDA 932), SAE 863, and CDA 954

Cast iron

A cast iron bearing can be used with a hardened steel shaft because the coefficient of friction is relatively low. The cast iron glazes over therefore wear becomes negligible

Bearing materials- desired characteristics

Load capacity-

- The allowable compressive strength the material can withstand without any appreciable change in shape is the primary deciding factor in deciding a bearing material

- Plain bearings are expected to have the following characteristics for the ease of functioning and satisfying the design criteria
- Strength to take care of load-speed combinations
- Fatigue strength, where bearing materials are subjected to stress cycle as in internal combustion engines
- The retention of strength characteristics of softer bearing materials at temperature of operation which may rise within the design limit
- The material must easily conform to shape of the journal and should be soft enough to allow the particulate contaminants to get embedded

- **Compatibility-**
  - The shaft and bearing materials in rubbing condition should not produce localized welds leading to scoring or seizure.
  - A good bearing-shaft metal combination is necessary
  - **Corrosion resistance-**
  - The oxidised products of oils corrode many bearing alloys.
  - Some protection can be provided by forming a thin layer of anti-corrosion materials on the bearing alloy surface
  - **Conformability-**
  - It helps to accommodate misalignment and increase the pressure bearing area (reduce the localized forcse).
  - Relatively softer bearing alloys are better in this respect
- **Embeddability-**
  - It is the ability of a material to embed dirt and foreign particles to prevent scoring and wear (decrease 3^{rd}. Body abrasion).
  - Materials with high hardness values have poor embeddability characteristics
- **Elasticity-** should be elastic enough to allow the bearing to return to original shape upon relief of stresses that may cause temporary distortion, such as misalignment and overloading
- **Availability-** The material should be readily and sufficiently available, not only for initial installation but also to facilitate replacement in the event of bearing failure
- **Cost-** The economic consideration is the ultimate deciding factor in selecting a bearing material

Unit 8 Behavior of Tribological components

Introduction to Wear

• Plastic deformation at the interface often leads to wear, i.e., deformation induced wear. • Wear can also be caused by chemical processes. • There are many different kinds of wear mechanisms • We have to analyze these wear mechanisms using mechanics, thermodynamics, etc. Tribology is a multi-disciplinary subject.

Types of Wear

1. Adhesive wear
2. Abrasive wear
3. Erosive wear
4. Fretting Wear
5. Corrosive wear

1. Adhesive wear

   Adhesive wear can be found between surfaces during frictional contact and generally refers to unwanted displacement and attachment of wear debris and material compounds from one surface to another. Two separate mechanisms operate between the surfaces

Friccohesity defines actual changes in cohesive forces and their reproduction in form of kinetic or frictional forces in liquid when the clustering of the nano-particles scatter in medium for making smaller cluster or aggregates of different nanometer levels.

Adhesive wear are caused by relative motion, "direct contact" and plastic deformation which create wear debris and material transfer from one surface to another.

Cohesive adhesive forces, holds two surfaces together even though they are separated by a measurable distance, with or without any actual transfer of material.

The above description and distinction between "**Adhesive wear**" and its Counterpart "**cohesive adhesive forces**" are quite common. Usually cohesive surface forces and adhesive energy potentials between surfaces are examined as a special field in physics departments. The adhesive wear and material transfer due to direct contact and plastic deformation are examined in engineering science and in industrial research.

Two aligned surfaces may always cause material transfer and due to overlaps and symbiotic relations between relative motional "wear" and "chemical" cohesive attraction, the wear-categorization have been a source for discussion. Consequently, the definitions and nomenclature must evolve with the latest science and empiric observations.

Generally, adhesive wear occurs when two bodies slide over or are pressed into each other, which promote material transfer. This can be described as plastic deformation of very small fragments within the surface layers. The asperities or microscopic high points or surface roughness found on each surface, define the severity on how fragments of oxides are pulled off and adds to the other surface, partly due to strong adhesive forces between atoms[1] but also due to accumulation of energy in the plastic zone between the asperities during relative motion.

2. Abrasive wear

Abrasive wear occurs when a hard rough surface slides across a softer surface. ASTM International (formerly American Society for Testing and Materials) defines it as the loss of material due to hard particles or hard protuberances that are forced against and move along a solid surface.

Abrasive wear is commonly classified according to the type of contact and the contact environment.The type of contact determines the mode of abrasive wear. The two modes of abrasive wear are known as two-body and three-body abrasive wear. Two-body wear occurs when the grits or hard particles remove material from the opposite surface. The common analogy is that of material being removed or displaced by a cutting or plowing operation. Three-body wear occurs when the particles are not constrained, and are free to roll and slide down a surface. The contact environment determines whether the wear is classified as open or closed. An open contact environment occurs when the surfaces are sufficiently displaced to be independent of one another.

There are a number of factors which influence abrasive wear and hence the manner of material removal. Several different mechanisms have been proposed to describe the manner in which the material is removed. Three commonly identified mechanisms of abrasive wear are:

1. Plowing
2. Cutting
3. Fragmentation

Plowing occurs when material is displaced to the side, away from the wear particles, resulting in the formation of grooves that do not involve direct material removal. The displaced material forms ridges adjacent to grooves, which may be removed by subsequent passage of abrasive particles. Cutting occurs when material is separated from the surface in the form of primary debris, or microchips, with little or no material displaced to the sides of the grooves. This mechanism closely resembles conventional machining. Fragmentation occurs when material is separated from a surface by a cutting process and the indenting abrasive causes localized fracture of the wear material. These cracks then freely propagate locally around the wear groove, resulting in additional material removal by spalling

3.Erosive wear

Erosive wear can be defined as an extremely short sliding motion and is executed within a short time interval. Erosive wear is caused by the impact of particles of solid or liquid against the surface of an object. The impacting particles gradually remove material from the surface through repeated deformations and cutting actions .It is a widely encountered mechanism in industry. Due to the nature of the conveying process, piping systems are prone to wear when abrasive particles have to be transported.

The rate of erosive wear is dependent upon a number of factors. The material characteristics of the particles, such as their shape, hardness, impact velocity and impingement angle are primary factors along with the properties of the surface being eroded. The impingement angle is one of the most important factors and is widely recognized in literature. For ductile materials the maximum wear rate is found when the impingement angle is approximately 30°, whilst for non ductile materials the maximum wear rate occurs when the impingement angle is normal to the surface.

Fretting Wear

Fretting wear is the repeated cyclical rubbing between two surfaces, which is known as fretting, over a period of time which will remove material from one or both surfaces in contact. It occurs typically in bearings, although most bearings have their surfaces hardened to resist the problem. Another problem occurs when cracks in either surface are created, known as fretting fatigue. It is the more serious of the two phenomena because it can lead to catastrophic failure of the bearing. An associated problem occurs when the small particles removed by wear are oxidized in air. The oxides are usually harder than the underlying metal, so wear accelerates as the harder particles abrade the metal surfaces further. Fretting corrosion acts in the same way, especially when water is present. Unprotected bearings on large structures like bridges can suffer serious degradation in behavior, especially when salt is used during winter to deice the highways carried by the bridges. The problem of fretting corrosion was involved in the Silver Bridge tragedy and the Mianus River Bridge accident.

Corrosion and oxidation wear

This kind of wear occurs in a variety of situations both in lubricated and unlubricated contacts. The fundamental cause of these forms of wear is chemical reaction between the worn material and the corroding medium. This kind of wear is a mixture of corrosion, wear and the synergistic term of corrosion-wear which is also called tribocorrosion.