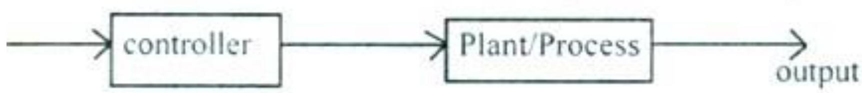
Question Papers Solution

Module 1

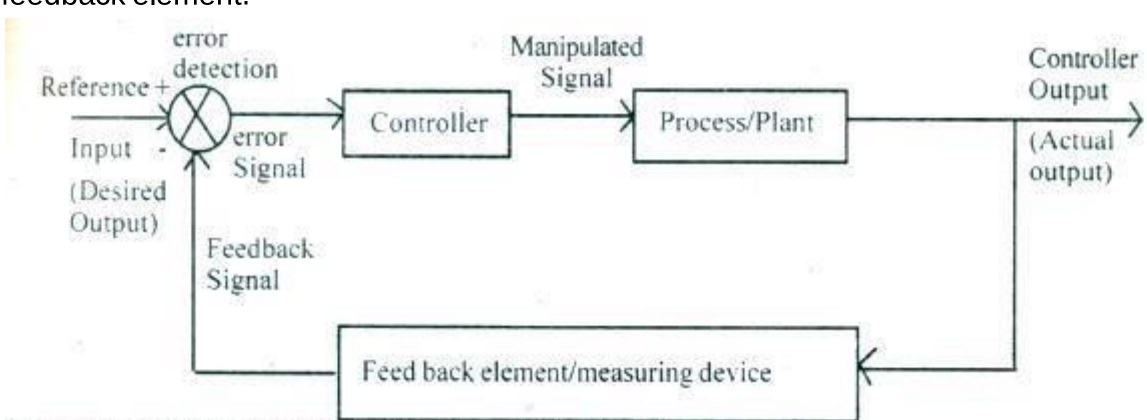
June 2015

1. **A) Define control system. Compare open loop and closed loop control systems with two example{or each type**

Ans: A control system can be defined as an arrangement of physical components connected or related in such a manner as to command, direct or regulate itself or another system. Open-. Loop control system: An open loop system is one in which control action is independent of the desired output. It Means the desired output is neither measured nor compared with the input.

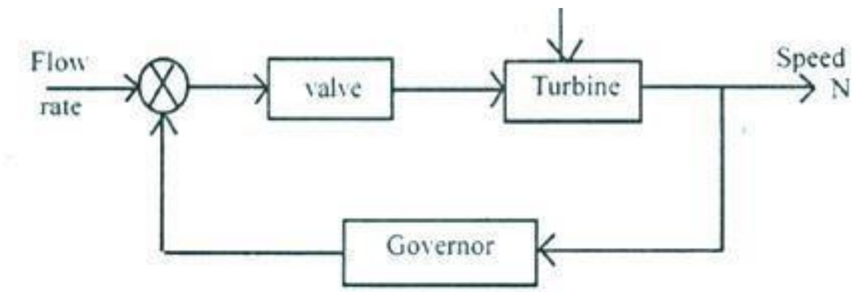


Exmaple (l): Traffic control system - for regulating the flow of traffic at the crossing of two Or more roads. Here red and green lights are put on by a timer mechanism set for predetermined Fixed intervals of time. It is obvious that this system doesn't take into account the varying rates Of traffic flow from time to time on any day. Example (2): Washing machine: Soaking, washing and rinsing in the washing machine are Operated on time basis, hence it is clear that the machine doesn't measure the output signal namely the cleanliness of the cloth. Closed loop control system: A closed loop control system is one in which control action is dependent on the desired output. It means the desired output is measured and compared with input using the feedback element.

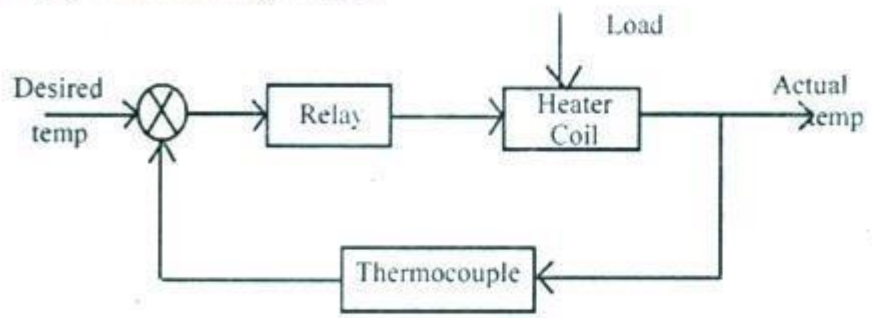


Example (1): Speed regulation of Turbine shaft:

The difference between the desired output and actual output is used as an error signal inturn controls the valve position thereby controlling the output, the desired output is 0btained



:xample (2) : Automatic furnace:

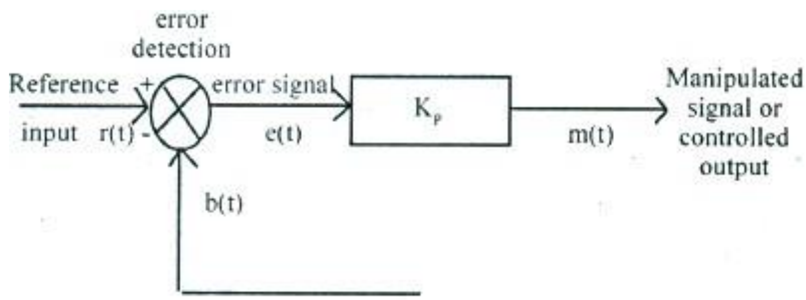


Heater coil is operated by relay. The actual temperature is sensed by thermocouple and compressed with desired temperature. The difference between these two actuates the relay mechanism change the input as per the requirement.

1. b. Name the basic controllers and their good and undesirable characteristics.

Ans: (i) Proportional controller: The system is stable. But the steady state error exists. (ii)Integral controller: The system tends to become unstable. The steady state error is (iii)Proportional plus Integral controller: The system is stable and steady state e zero. (iv) Proportional plus Derivative Controller: The addition of a derivative controller effect on the steady state error directly, but it adds damping to the system and improv stability of the system. (v) Proportional + Derivative + Integral Controller: The combination ofproportionalc action, derivative control action and integral control action istermed proportional derivative plus integral control action. This combined action has the advantages ofea the three individual control actions.

1.c With a block diagram, explain proportional, integral differential controller. Proportional controller



m(t)an error signal e(t) is, met) = Kp e(t). TakingLaplace transform on both sides, we get

M(S) = Kp E(S) M(S). :. Kp = E(S) ,prop0l1ional gain.

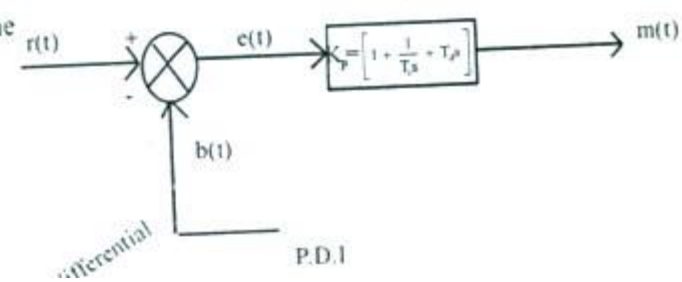
Integral differential controller: Itisthe combination of proportional. integral and differential controlactionssoas.toderivetheadvantagesofallthecontrolaction.Generallyitisknownas PID controllers. Theequation for the PID controller is given by,

$$m(t) = K_p\, e(t) + \frac{K_p}{T_i}\int e(t)dt + k_p t_d \frac{de(t)}{dt}$$

king Laplace transform.

$$M(S) = E(S).K_p\left[1 + \frac{1}{T_i s} + T_d s\right]$$

here, K_p = proportional gain
Ti = Integral time
T_d = differential time



Dec 2014

2. **a) Define open-loop and closed loop control system, mention their merits ami demeti:**
Open loop controlsystem

Any physical system which doesnot automatically correct the variation in its output. An open loop system in which output quantity has no effect upon the input quantity open loop control system. Merits: 1. The open loop systems are simple and economical 2. The open loop systems are easier to construct. 3. They are easy for maintenance 4. They are stable. Demerits: I. .They are inaccurate and unreliable 2. The changes in the output due to external disturbances are not corrected auto Closed loop control system: Control systems in which the output has an effect upon the input quantity in such a to maintain the desired output value are called closed loop control systems.

Merits: I . They are accurate 2. They are accurate even in the presence of nonlinearities 3. They are less affected by noise 4. The sensitivity of the systems may be made small make the system mo

Demerits: 1. They are complex and costlier 2. They may lead to oscillatory response 3. The feedback reduces the overall gain of the system 4. Stability is a major issue and more care is needed to design a stable closed system.

b) What is feedback? Explain the effects of feedback.

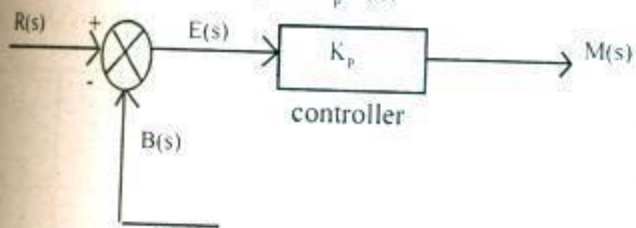
Feedbackistheproperty of a closed loop control system which permits the out Compared with the input to the system so that appropriate control action may be fo Some function of the output and input. The effects of feedback on the control system are, 1. Feedback in control system improves the time response 2. Properdesign and application of feedback, stability of the system can be effectively Controlled. 3. Gainof the system can be controlled by controlling feedback. 4. Feedbackin control system reduces the effect of disturbance (Internal and External) On the system and reduces the sensitivity of the system to variation in parameter. 5. Reducedeffects of nonlinearities and distortions 6. Flexibility in the system 7. Independentof operating conditions

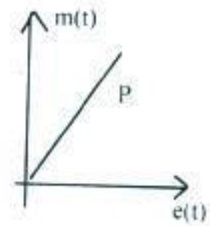
c) Explain proportional and integral controller and derive the closed loop transfer function of PI Controller for a second order system

(10 M)

Ans: *Proportional controller:*

For a controller with proportional control action, the relation between the controller m(t) and the actuating error signal e(t) is $m(t) = K_p e(t)$.



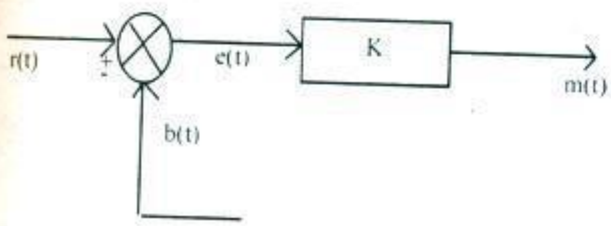


Proportional gain, $K_p = \dfrac{M(S)}{E(S)}$

Here the system is stable but steady state error exists.

Integral controller:

Here output of the controller i.e. manipulated signal is changed at a rate proportional to the input of the controller i.e. error signal.



For a controller with integral control action the relationship between output of the controller

For a controller with integral control action the relationship between output of the controller m(t) and error signal e(t) is $\dfrac{dm(t)}{dt} = K_i e(t)$

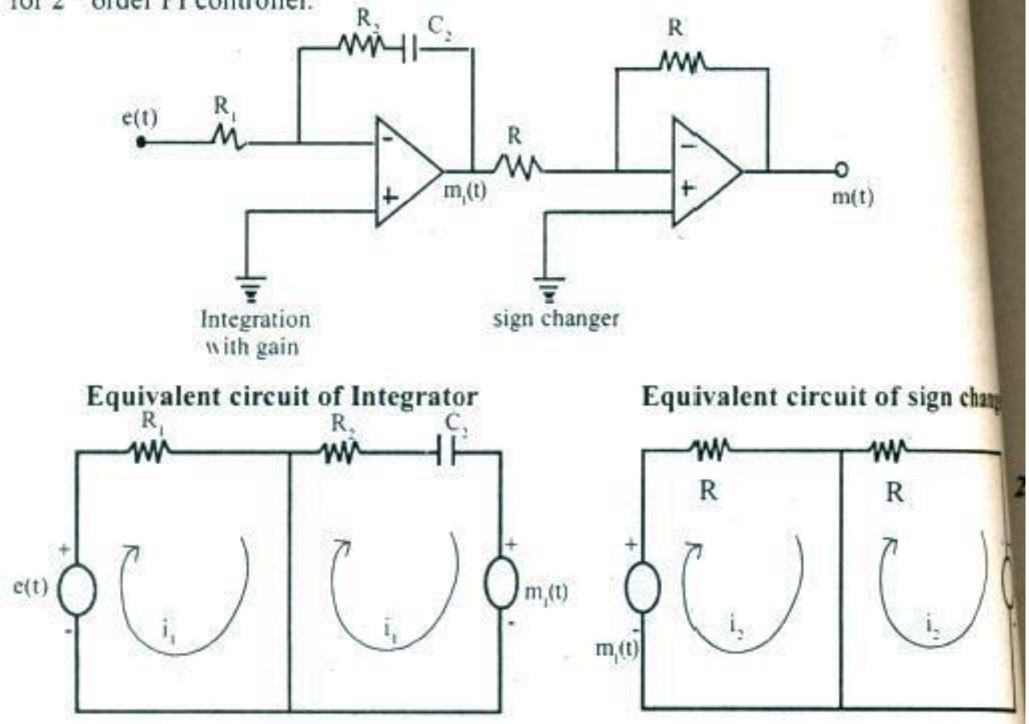
By integrating we get, $m(t) = K_i \int e(t)dt$

Taking Laplace transform and simplifying we get $K_i = \dfrac{sM(S)}{E(S)}$

Here the system tends to become unstable and steady state error is zero.

Proportional plus Integral controller (PI controller):

Consider an operational amplifier integrator with gain follows by a sign changer as a for 2^{nd} order PI controller.



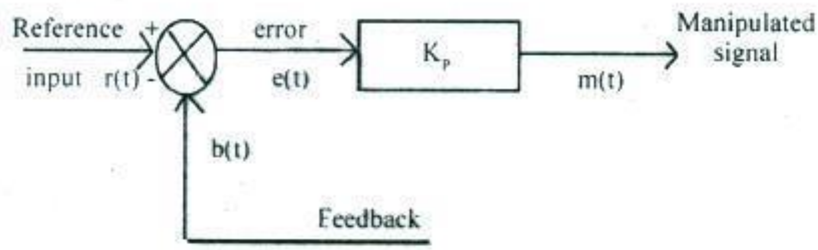
June 2014

3 a) Explain ideal requirements of control system? Explain

stability (2) sensitivity (3) speed (4) Accuracy (5) Disturbance/Noise (6) bandwidth Stability in a control system implies that small charges in the system input, in initial conditions Or in system parameters do not result in large changes in the system behavior. An ideal control system should be insensitive to the variations in parameters of the system but It should be sensitive to the input commands. the control system means how fast the output of the system approaches to the desired value .An ideal system should have good speed. How much the output of the control system is nearer to the input or desired value is accuracy. An ideal system should be highly Accurate. The system should be insensitive to noise and disturbances. Bandwidth means for the range of input, output should be constant.

b) What is control action? Briefly explain proportional, proportional plus derivative and proportional plus derivative plus Integral controllers, with the help of block diagrams.

Proportional controller:



In this output is proportional to the input of the controller. i.e. $m(t) = K_p e(t)$

Taking Laplace transform, $M(s) = K_p E(s)$

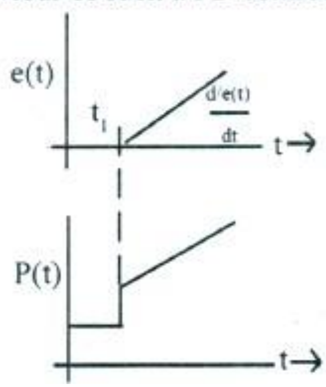
$$\therefore K_p = \frac{M(s)}{E(s)} \text{ Proportional gain}$$

Proportional plus derivative controller:

The series combination of proportional and derivative control modes gives proportional derivative control mode. The mathematical expression for PD composite control is,

$$P(t) = K_p e(t) + K_p K_d \frac{d\ e(t)}{dt} + P(o)$$

The behaviour of such a PD control to a ramp type of input is shown here



P(o) = initial value of output

The various important features of PD control are; improvement in damping, reducing overshoot, reduction in risetime, stability, improvement in bandwidth etc.

Proportional plus derivative plus Integral controller

It is the combination of proportional, integral and differential control actions so as to derive advantages of all the control action. The equation for PID controller is given by,

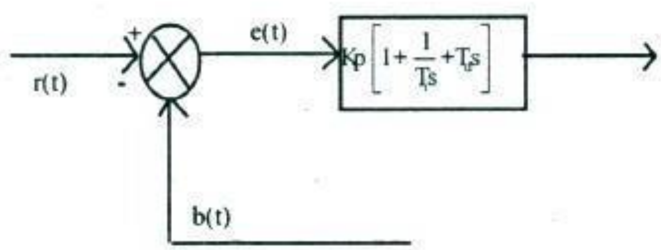
$$m(t) = K_p e(t) + \frac{K_p}{T_i}\int e(t)dt + K_p t_d \frac{de(t)}{dt}$$

$$\text{Or } M(s) = E(s) . Kp\left[1 + \frac{1}{T_i s} + T_d s\right]$$

K_p = proportional gain

T_i = Integral time

T_d = Derivative or differential time

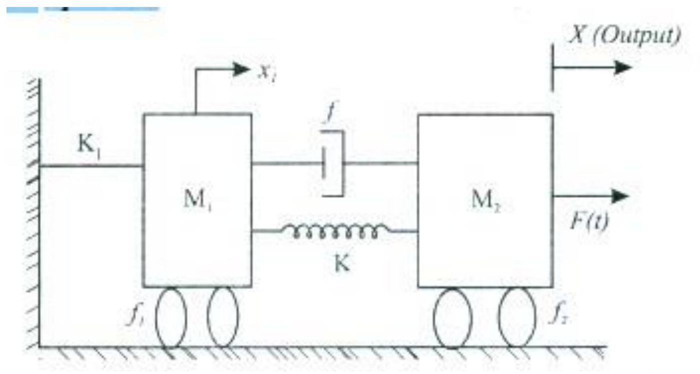


With PID control, there is no offset and system achieves the steady state with len settling time. Thus PID is the ultimate procen composite controller.

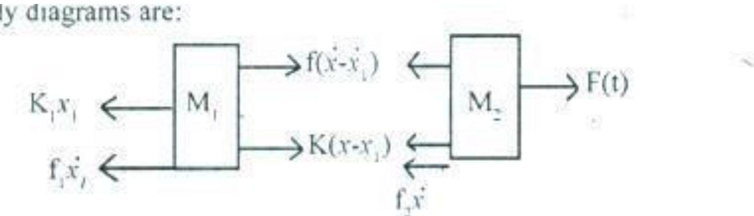
Module 2

June 2015

1. **a. Obtain the transfer function of the mechanical system shown in Fig.Q2(a),write physical system equations.**



ns: Free body diagrams are:



The differential equation of the mass M_2 is,

$$M_2\ddot{x} = f\left(\dot{x} \quad \dot{x}_1\right) \quad f_2\dot{x} \quad K(x \quad x_1) + F(t)$$

i.e., $$M_2\ddot{x} + f\left(\dot{x} - \dot{x}_1\right) - f_2\dot{x} - K(x - x_1) = F(t)$$

Taking Laplace transform.

$$M_2S^2X(S) + fSX(S) - fSX_1(S) + f_2S(S) + KX(S) - KX_1(S) = F(S)$$
$$(M_2S_2 + fS + f_2S + K) \times (S) - (fS + K)X_1(S) = F(S)$$

The differential equation of the mass M^1 is:

$$M_1\ddot{x}_1 = f\left(\dot{x} - \dot{x}_1\right) + K(x - x_1) - f_1\dot{x}_1 - K_1x_1$$

$$M_1\ddot{x}_1 = f\left(\dot{x} - \dot{x}_1\right) - f_1\dot{x}_1 + K_1x_1 - K(x - x_1) = 0$$

Taking Laplace Transform,

$$M_1S^2X_1(S) - fSX(S) + fSX_1(S) + f_1SX_1(S) + K_1X_1(S) - KX(S) + KX_1(S) = ($$

i.e. $$-(fS + K)X(S) + (M_1S^2 - fS + f_1S + K_1 + K)X_1(S) = 0 \quad \text{——(2)}$$

The output of the system X(S) is obtained from equation. (1) & (2) are,

$$X(S) = \frac{\begin{vmatrix} F(S) & -(fS + K) \\ 0 & M_1S^2 + fS + f_1S + K_1 + K \end{vmatrix}}{\begin{vmatrix} M_1S^2 + fS + f_2S + K & -f(S + K) \\ -(fS + K) & M_1S^2 + fS + f_1S + K_1 + K \end{vmatrix}}$$

$$= \frac{F(S)[M_1S^2 + fS + f_1S + K_1 + K_2]}{[(M_2S^2 + fS + f_2S + K)(M_1S^2 + fS + f_1S + K_1 + K) - (fS +}$$

b) Write the differential equations governing the behavior of the mechanical system shown in fig.Q2 (b). Also obtain tile analogous electrical-circuit based on force voltage analogy and Loop equations.

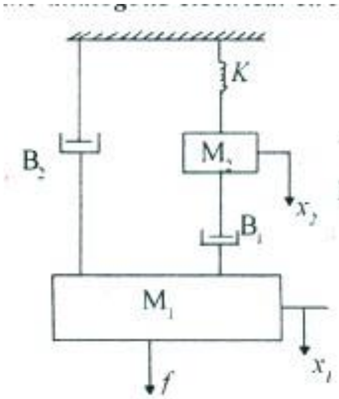
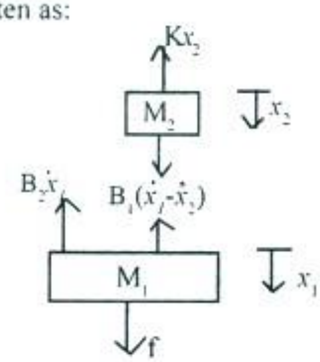


Fig.Q2(b)

Free body diagram can be written as:



The differential equations for mass m_1 is,

$$M_1\ddot{x}_1 = f - B_2\dot{x}_1 - B_1(\dot{x}_1 - \dot{x}_2)$$

i.e $$M_1\ddot{x}_1 + B_2\dot{x}_1 + B_1(\dot{x}_1 - \dot{x}_2) = f$$

Taking Laplace transform,

$$M_1S^2X_1(S) + B_2SX_1(S) + B_1SX_1(S) - B_1SX_2(S) = F(S)$$

$$(M_1S^2 + B_2S + B_1S)\,X_1(S) - B_1SX_2(S) = F(S) \quad \text{——(1)}$$

The differential equation for mass m_2 is,

$$M_2\ddot{x}_2 = B_1(\dot{x}_1 - \dot{x}_2) - Kx_2$$

i.e. $$M_2\ddot{x}_2 = B_1(\dot{x}_1 - \dot{x}_2) - Kx_2 = 0$$

Taking Laplace transform.

$$M_2S^2X_2(S) - B_1SX_1(S) - B_1SX_2(S) + KX_2(S) = 0$$

$$-B_1SX_1(S) + (M_2S^2 + B_1S + K)\,X_2(S) = 0 \quad \text{——(2)}$$

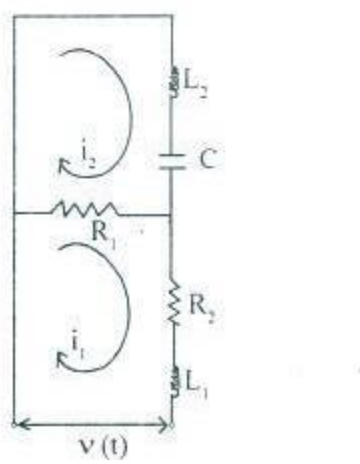
$$\text{i.e. } X_2(S) = \frac{\begin{vmatrix} M_1S^2 + B_2S + B_1S & F(S) \\ -B_1S & 0 \end{vmatrix}}{\begin{vmatrix} M_1S^2 + B_2S + B_1S & -B_1S \\ -B_1S & M_2S^2 + B_1S + K \end{vmatrix}}$$

$$= \frac{-[-B_1S \,.\, F(S)]}{(M_1S^2 + B_2S + B_1S)(M_2S^2 + B_1S + K) - (B_1S)^2}$$

- Transfer function of the system is,

$$\frac{X_1(S)}{F(S)} = \frac{B_1S}{(M_1S^2 + B_2S + B_1S)(M_2S^2 + B_1S + K) - (B_1S)^2}$$

Analogous electrical circuit:



Dec 2014

2. a) Derive the system equation in Laplace form for the system shown in fig.

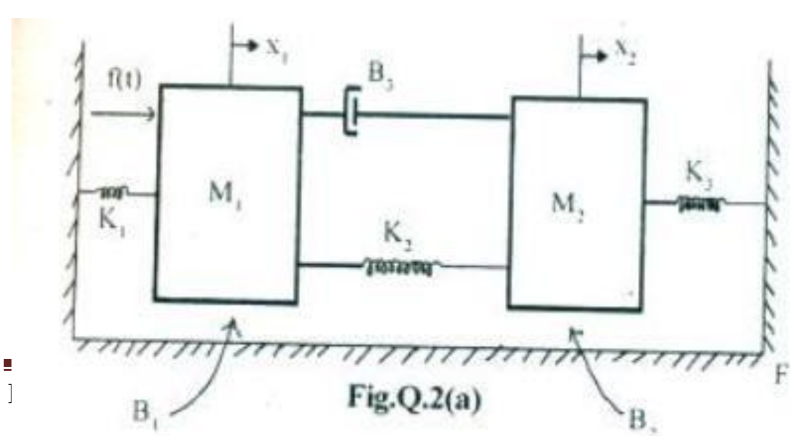
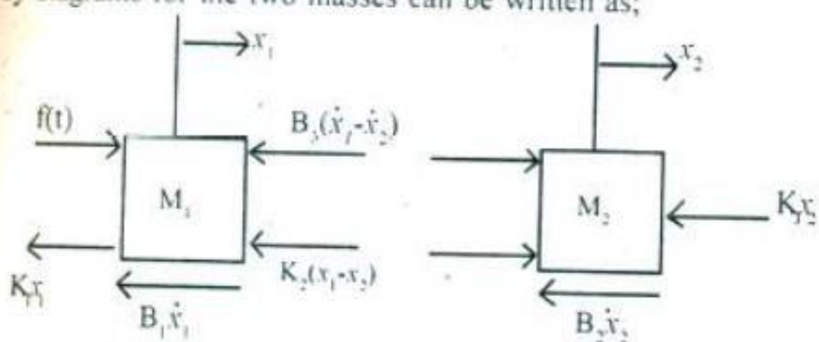


Fig.Q.2(a)

Sol:

Free body diagrams for the two masses can be written as;



Differential equation for mass M_1 is,

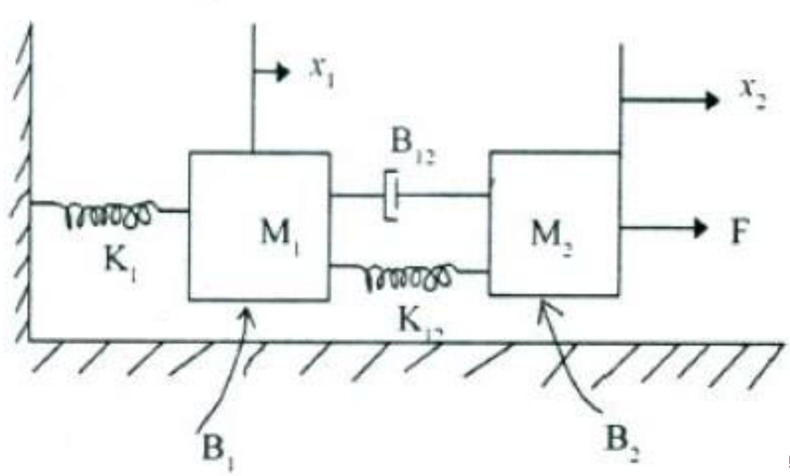
$$f(t) = B_3(\dot{x}_1 - \dot{x}_2) + K_1x_1 + B_1\dot{x}_1 + K_2(x_1 - x_2) + m_1\ddot{x}_1$$

Taking Laplace transform we get,

$$F(s) = M_1S^2X_1(s) + B_3SX_1(s) - B_3SX_2(s) + K_1X_1(s) + B_1SX_1(s) + K_2X_1(s) - K_2X_2(s)$$

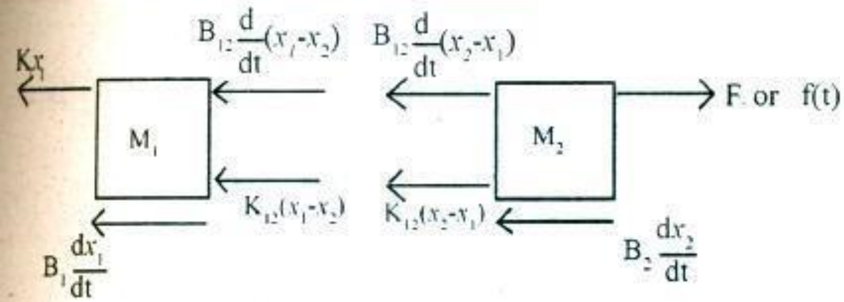
This can be written as:

b) Obtain the force voltage analogy for the given mechanical analogy



Sol:

Free body diagram for mass m_1 and m_2 are as shown.



Differential equation for the mass m_1

$$0 = m_1\frac{d^2x_1}{dt^2} + B_{12}\frac{d}{dt}(x_1 - x_2) + K_{12}(x_1 - x_2) + B_1\frac{dx_1}{dt} + K_1x_1$$

i.e. $0 = m_1S^2X_1(s) + B_{12}SX_1(s) - B_{12}SX_2(s) + K_{12}X_1(s) - K_{12}X_2(s) + B_1SX_1(s) + K_1X_1(s)$

i.e. $0 = (m_1S^2 + B_{12}S + K_{12} + B_1S + K_1)\,X_1(s) - (B_{12}S + K_{12})\,X_2(s)$ ———(1)

Differential equation for mass m_2,

$$f(t) = m_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + B_{12}\frac{d}{dt}(x_2 - x_1) + K_{12}(x_2 - x_1)$$

i.e. $F(s) = m_2S^2X_2(s) + B_2SX_2(s) + B_{12}SX_2(s) - B_{12}SX_1(s) + K_{12}X_2(s) - K_{12}X_1(s)$

$F(s) = (m_2S^2 + B_2S + B_{12}S + K_{12})\,X_2(s) - (B_{12} + K_{12})\,X_1(s)$ ———(2)

Now electrical analogous quantities are replaced in equations (1) and (2) to make an electrical equation.

Mechanical	Electrical
F(s)	V(s)
m	L
B	R
K	1/C
X(s)	Q(s)

Also $Q(s) = \frac{I(s)}{S}$

Replacing the above quantities of electrical in (1)

$$0 = \left(L_1S^2 + R_{12}S + \frac{1}{C_{12}} + R_1S + \frac{1}{C_1}\right)Q_1(s) - \left(R_{12}S + \frac{1}{C_{12}}\right)Q_2(s)$$

i.e. $$0 = \left(L_1S^2 + R_{12}S + \frac{1}{C_{12}} + R_1S + \frac{1}{C_1}\right)\frac{I_1(s)}{S} - \left(R_{12}S + \frac{1}{C_{12}}\right)\frac{I_2(s)}{S}$$

i.e. $$0 = L_1S\,I_1(s) + R_{12}I_1(s) + \frac{1}{C_{12}S}I_1(s) + R_1I_1(s) + \frac{1}{C_1S}I_1(s) - R_{12}I_2(s) - \frac{1}{C_{12}S}I_2(s)$$

low taking inverse Laplace transform.

$$0 = L_1 \frac{di_1}{dt} + R_{12}i_1 + \frac{1}{C_{12}} \int i_1 dt + R_1 i_1 - R_{12}i_2 - \frac{1}{C_{12}} \int i_2 dt + \frac{1}{C_1} \int i_1 dt \quad \text{——(3)}$$

Now replacing electrical analogous quantities in eqn (2)

$$V(s) = \left(L_2S^2 + R_2S + R_{12}S \ \frac{1}{C_{12}} \right) Q_2(s) - \left(R_{12}S + \frac{1}{C_{12}} \right) Q_1(s)$$

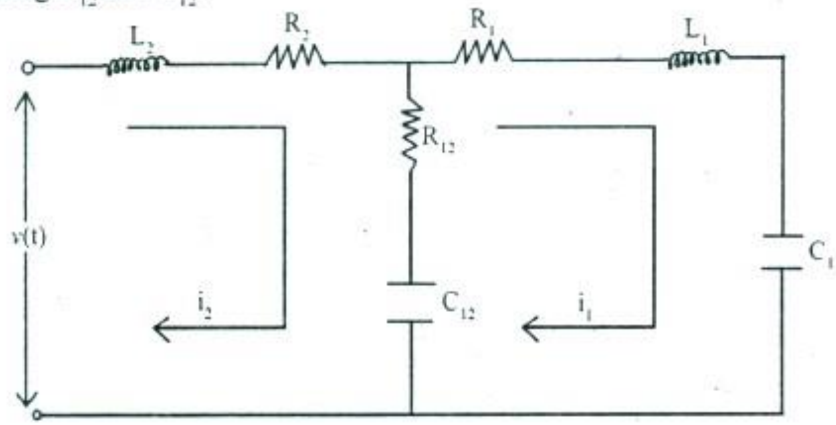
$$\text{i.e. } V(s) = \left(L_2S^2 + R_2S + R_{12}S \ \frac{1}{C_{12}} \right) \frac{I_2(s)}{S} - \left(R_{12}S + \frac{1}{C_{12}} \right) \frac{I_1(s)}{S}$$

$$\text{i.e } V(s) = L_2SI_2(s) + R_2I_2(s) + R_{12}I_2(s) + \frac{1}{C_{12}S} I_2(s) - R_{12}I_1(s) - \frac{1}{C_{12}S} I_1(s)$$

Now taking inverse Laplace transform.

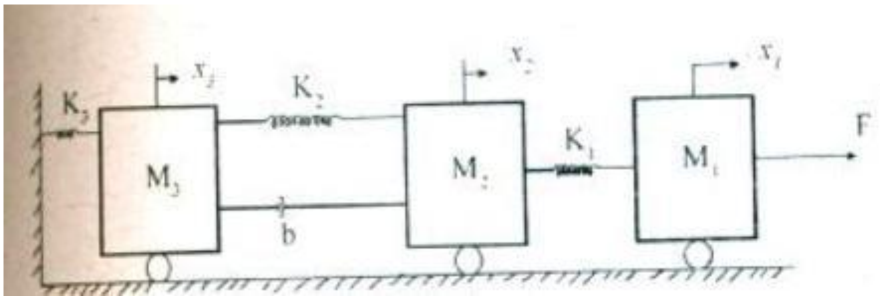
$$V(t) = L_2 \frac{di_2}{dt} + R_2 i_2 + R_{12} i_2 + \frac{1}{C_{12}} \int i_2 dt - R_{12} i_1 - \frac{1}{C_{12}} \int i_1 dt \quad \text{————(4)}$$

Analysing equations (3) & (4) we come to know that there are two loops with com elements being R_{12} and C_{12}.



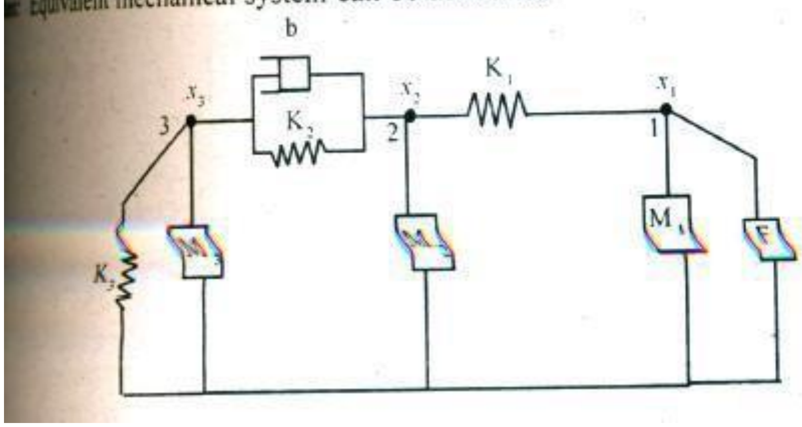
June 2014

1. a) Write governing equation for the mechanical system shown in figure.



Soln:

Equivalent mechanical system can be drawn as.



$\Sigma F = 0$ for each nodes 1, 2 and 3.

At node 1,

$$F = M_1S^2X_1 + K_1(X_1 - X_2) \qquad (1)$$

At node 2,

$$0 = M_2S^2X_2 + K_1(X_2 - X_1) + bs\,(X_2 - X_3) + K_2\,(X_2 - X_3) \qquad (2)$$

At node 3,

$$0 = M_3S^2X_3 + K_3X_3 + bs\,(X_3 - X_2) + K_2\,(X_3 - X_2) \qquad (3)$$

These are the governing equations.

Force-voltage analogy : Analogous quanties are:

$m \rightarrow L$, $b \rightarrow R$, $k \rightarrow 1/c$, $x \rightarrow q$, $F \rightarrow v$

$\therefore$ equations (1), (2) and (3) can be written as,

$$V(s) = L_1S^2q_1 + \left(\frac{1}{C_1}\right)(q_1 - q_2)$$

$$0 = L_2S^2q_2 + \left(\frac{1}{C_1}\right)(q_2 - q_1) + RS(q_2 - q_3) + \left(\frac{1}{C_2}\right)(q_2 - q_3)$$

$$0 = L_3S^2q_3 + \left(\frac{1}{C_3}\right)q_3 + RS(q_3 - q_2) + \left(\frac{1}{C_2}\right)(q_3 - q_2)$$

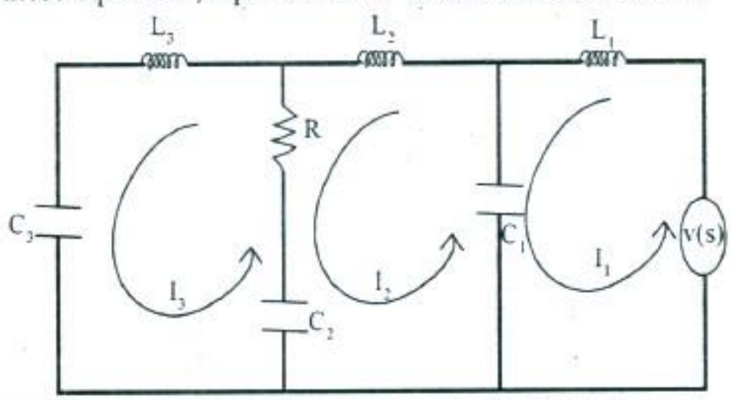
Further replacing I(s) = s Q(s) we get,

$$V(s) = L_1 S I_1(s) + \frac{1}{C_1 S}\left[I_1(s) - I_2(s)\right], \text{ Loop1}$$

$$0 = L_2 S I_2(s) + \frac{1}{C_1 S}\left[I_2(s) - I_1(s)\right] + R\left[I_2(s) - I_3(s)\right] + \frac{1}{C_2 S}\left[I_2(s) - I_3(s)\right], \text{ Loop2}$$

$$0 = L_3 S I_3(s) + \frac{1}{C_3 S} I_3(s) + R\left[I_3(s) - I_2(s)\right] + \frac{1}{C_2 S}\left[I_3(s) - I_2(s)\right], \text{ Loop3}$$

Based on these equations, equivalent F-V sketch can be drawn as.



Force-current analogy:

Analogous quantities are:

$F \to I$, $m \to C$, $b \to 1/R$, $K \to 1/L$, $x \to \phi$

$$I(s) = C_1 S^2 \phi_1 + \frac{1}{L_1}(\phi_1 - \phi_2)$$

$$0 = C_2 S^2 \phi_2 + \frac{1}{L_1}(\phi_2 - \phi_1) + \frac{S}{R_1}(\phi_2 - \phi_3) + \frac{1}{L_2}(\phi_2 - \phi_3)$$

$$0 = C_3 S^2 \phi_3 + \frac{1}{L_3}\phi_3 + \frac{S}{R_1}(\phi_3 - \phi_2) + \frac{1}{L_2}(\phi_3 - \phi_2)$$

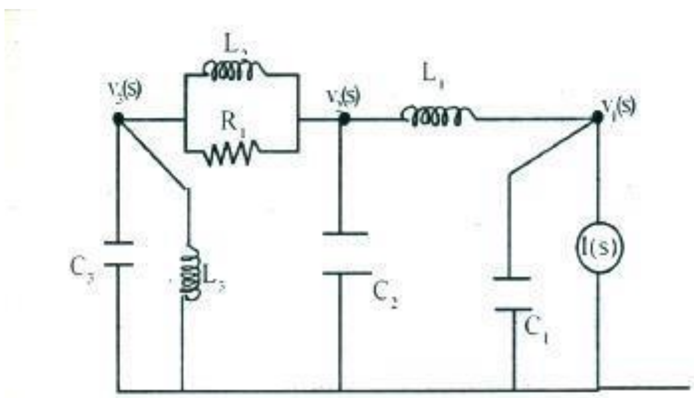
replacing $S\phi(s)$ V(s) we get,

$$I(s) = C_1SV_1(s) + \frac{1}{L_1S}\left[V_1(s) - V_2(s)\right]. \text{ node 1}$$

$$0 = C_2SV_2(s) + \frac{1}{L_1S}\left[V_2(s) - V_1(s)\right] + \frac{1}{R_1}\left[V_2(s) - V_3(s)\right] + \frac{1}{L_2S}\left[V_2(s) - V_3(s)\right]$$

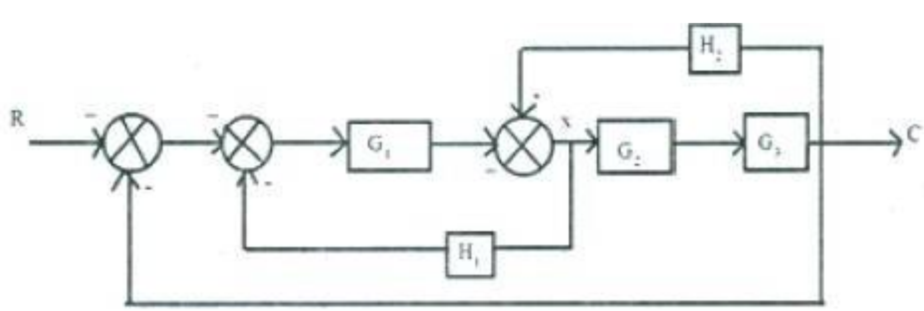
$$0 = C_3SV_3(s) + \frac{1}{L_3S}V_3(s) + \frac{1}{R_1}\left[V_3(s) - V_2(s)\right] + \frac{1}{L_2S}\left[V_3(s) - V_2(s)\right]. \text{ node}$$

Based on these equations we can sketch F – I analogy as.

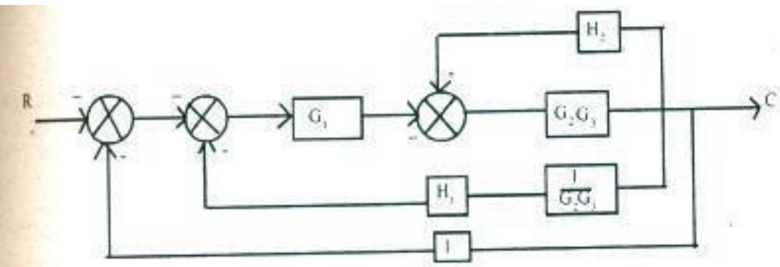


June 2015

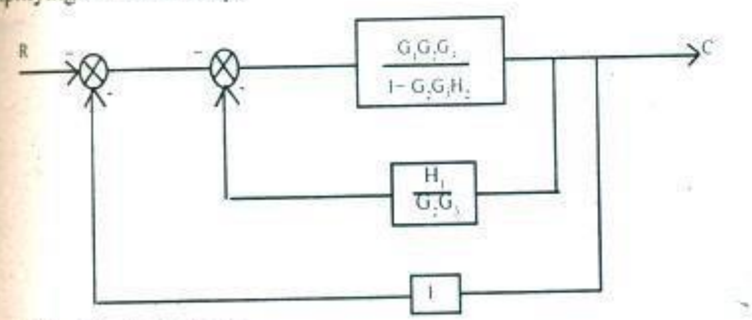
1. Determine the overall Transfer function for the given block diagram



Sol:



Simplifying the inner loop,

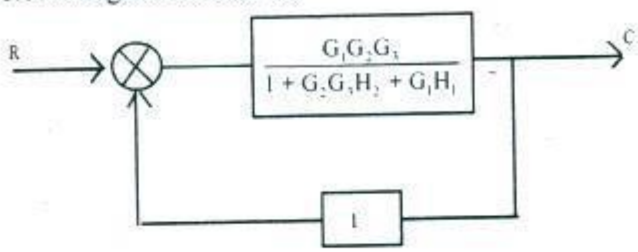


Simplifying the inner loop.

$$\frac{\dfrac{G_1G_2G_3}{(1+G_2G_3H_2)}}{1+\dfrac{G_1G_2G_3}{(1+G_2G_3H_2)}\times\dfrac{H1}{G_2G_3}}$$

$$=\frac{G_1G_2G_3}{1+G_2G_3H_2+G_1H_1}$$

Hence the block diagram reduces to,

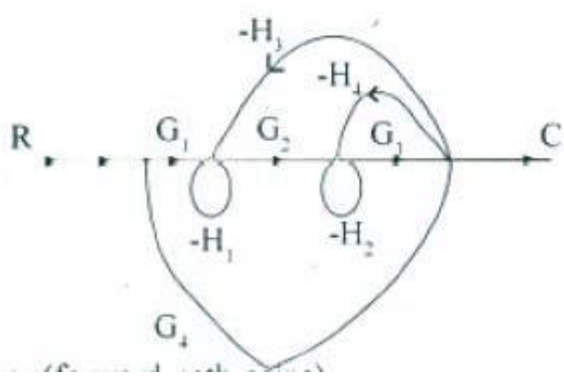


Further simplifying,

$$\frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2+G_1H_1}}{1+\dfrac{G_1G_2G_3}{(1+G_2G_3H_2+G_1H_1)}\times 1}$$

$$=\frac{G_1G_2G_3}{1+G_2G_3H_2+G_1H_1+G_1G_2G_3}$$

b) Determine the Transfer function using Moson's gain formula



$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{1 - \{\text{sum of loop gain } s\} + \left\{\begin{array}{l}\text{sum of gain products of}\\ \text{all possible combinations of}\\ \text{2 non touching loops}\end{array}\right\} - \left\{\begin{array}{l}\text{sum of gain products of}\\ \text{all possible combinations}\\ \text{of 3 non touching loops}\end{array}\right\} + }$$

$$= \frac{P_1\Delta_1 + P_2\Delta_2}{1-\{P_{11}+P_{12}+P_{13}+P_{14}\}+\{P_{13}\times P_{14}+P_{11}\times P_{13}\}}$$

$$= \frac{G_1G_2G_3+G_4(1+H_1+H_2+H_1H_2)}{1-\{-H_1-H_2-G_3H_4-G_2G_3H_4\}+\{H_1H_2+H_1G_3H_4\}}$$

$$= \frac{G_1G_2G_3+G_4(1+H_1+H_2+H_1H_2)}{1+H_1+H_2+G_3H_4+G_2G_3H_4+H_1H_2+H_1G_3H_4}$$

Forward paths are; (forward path gains)

(1) $G_1G_2G_3 = P_1$ $\Delta_1 = 1$

(2) $G_4 = P_2$, $\Delta_2 = (1 + H_1 + H_2 + H_1H_2)$

Loops are I (Loop gains)

(1) $-H_1 = P_{11}$

(2) $-H_2 = P_{12}$

(3) $-G_3H_4 = P_{13}$

(4) $-G_2G_3H_4 = P_{14}$

Non touching loops are:

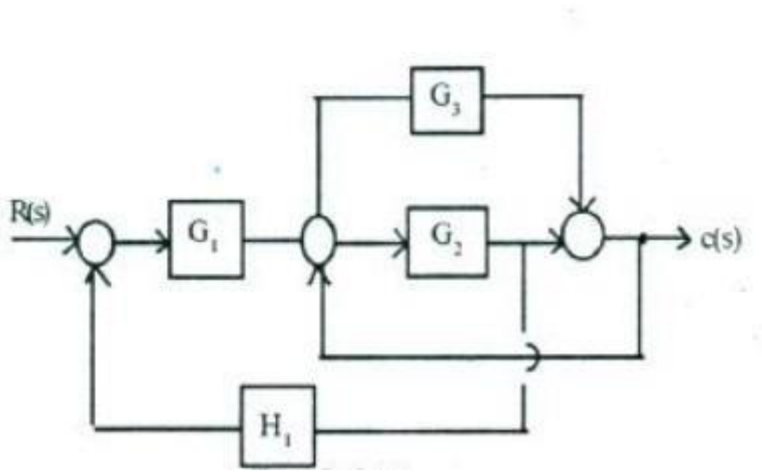
(1) $-H_1$ and $-H_2 = P_{13}\times P_{14}$

(2) $-H_1$ and $-G_3H_4 = P_{11}\times P_{13}$

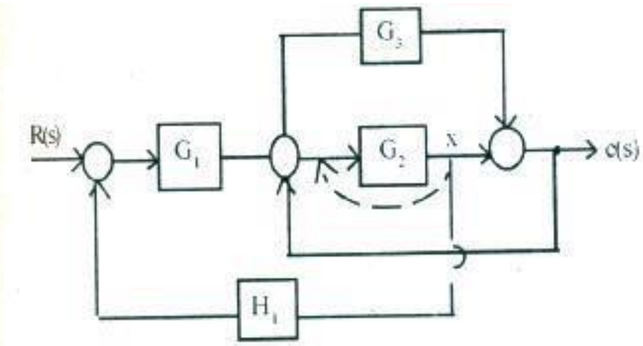
Hence the transfer function is:

Dec 2014

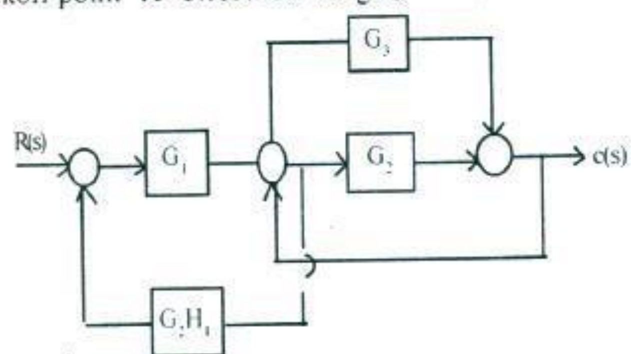
2 a) Reduce the given block diagram and find the transfer function



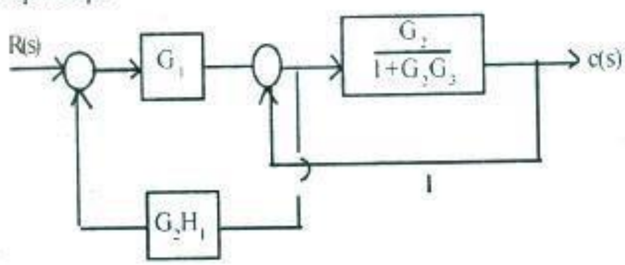
Sol:



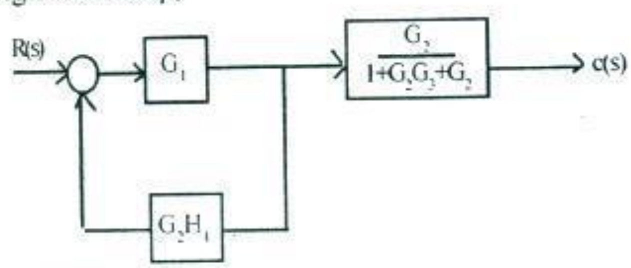
Moving the pickoff point 'X' bifore G2 we get,



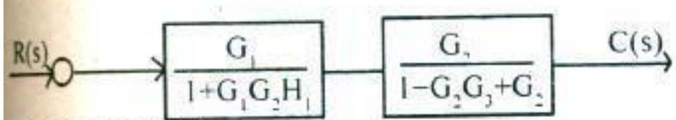
Simplifying the top loop.



implifying the right side loop,

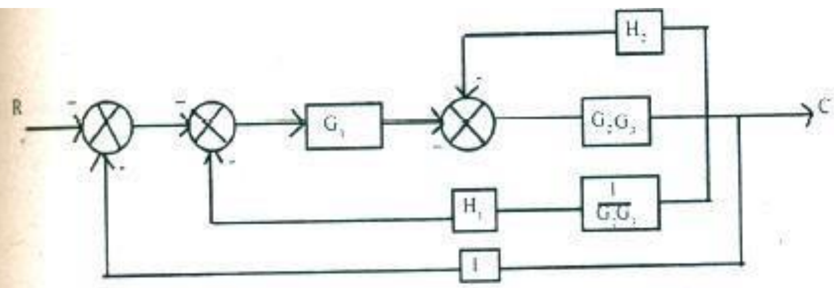


implifying the left loop.

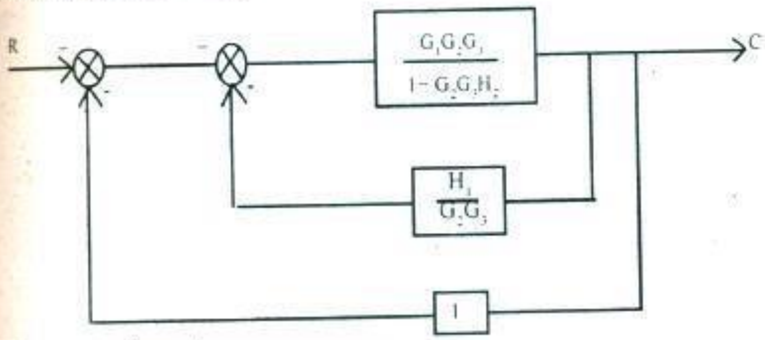


∴ Transfer function,

$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{(1+G_1G_2H_1)(1+G_2G_3+G_2)}$$



Simplifying the inner loop,

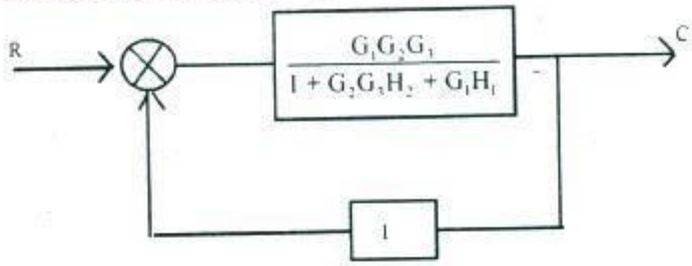


Simplifying the inner loop.

$$\frac{\dfrac{G_1G_2G_3}{(1+G_2G_3H_2)}}{1+\dfrac{G_1G_2G_3}{(1+G_2G_3H_2)}\times\dfrac{H1}{G_2G_3}}$$

$$=\frac{G_1G_2G_3}{1+G_2G_3H_2+G_1H_1}$$

Hence the block diagram reduces to,



Further simplifying,

$$\frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2+G_1H_1}}{1+\dfrac{G_1G_2G_3}{(1+G_2G_3H_2+G_1H_1)}\times 1}$$

$$=\frac{G_1G_2G_3}{1+G_2G_3H_2+G_1H_1+G_1G_2G_3}$$

b)Findthetransferofthesystemshownin Fig.Q.3(b)using Masonsgainformula.

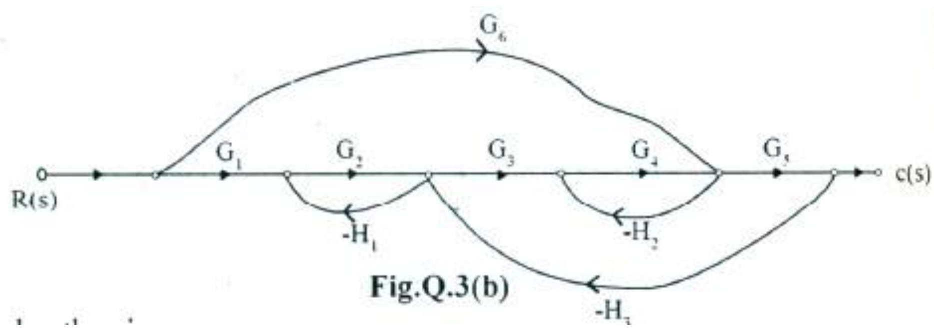


Fig.Q.3(b)

Sol:

-H_3

Ans: Forward path gains are:

$P_1 = 1 \times G_1 \times G_2 \times G_3 \times G_4 \times G_5 \times 1 = G_1G_2G_3G_4G_5$

$P_2 = 1 \times G_6 \times G_5 = G_6 G_5$

2 forward paths i.e K = 2

Loop gains are,

$L_1 = - G_2H_1$

$L_2 = - G_4H_2$

$L_3 = -G_3G_4G_5H_3$

3 individual loops

Combination of two non touching loops are, $L_1L_2 = G_2G_4H_1H_2$

There is no combination of 3 non touching or more non touching loops

Δ = Determinant of the graph

= 1 – (sum of individual loop gain) + sum of gain products of all combination of 2 n[on] touching loops) – sum of gain products of all combinations of 3 non touching loops)

....

i.e. $= 1 - [L_1+L_2+L_3] + [L_1L_2]$

$= 1 - [-G_2H_1 - G_4H_4 - G_3G_4G_5H_3] + [G_2G_4H_1H_2]$

$= 1 + G_2H_1 - G_4H_2 - G_3G_4G_5H_3 + G_2G_4H_1H_2$

K = value of by eliminating all loop gains and anociated products which touching to the forward path

i.e. *For* P_1, $\Delta_1 = L$

For P_2, $\Delta_2 = 1 - L_1 = 1 + G_2H_1$

Thus Masan's gain formula,

$$\text{Gain} = \frac{1}{\Delta} = \sum_{K=1}^{2} P_K \ \Delta_K = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

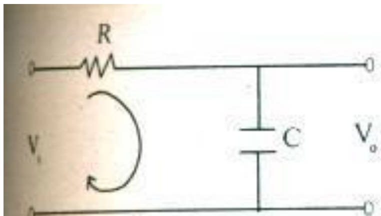
$$\text{i.e.} \quad \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5 + G_6G_5(1+G_2H_1)}{1 + G_2H_1 + G_4H_4 + G_3G_4G_5H_3 + G_2G_4H_1H_2}$$

June 2014

Module 3

June 2015

1. **a. Derive an expression for the unit step response of first order systems and steady state system**



$$V_i = R_i + \frac{1}{C}\int i\,dt$$

Differential equation governing the output

$$V_o = \frac{1}{C}\int i\,dt$$

Taking Laplace transform,

$$V_i(S) = \left[R + \frac{1}{CS}\right] I(S)$$

$$V_o(S) = \frac{1}{CS} I(S)$$

Transfer function, $\frac{V_o(S)}{V_i(S)} = \frac{(1/CS)\, I(S)}{\left(R + \frac{1}{CS}\right) I(S)} = \frac{1}{RCS + 1} = \frac{1}{TS + 1}$

Where T = RC the time constant of the system.

∴ Transfer function in general for 1st order system is,

$$\frac{C(S)}{R(S)} = \frac{1}{TS + 1}$$

For unit step input, R(S) = 1/S then, $C(S) = \frac{1}{S(TS + 1)}$

i.e. $C(S) = \frac{A}{S} + \frac{B}{TS + 1}$

i.e. $1 = A(TS + 1) + B(S)$

put $S = 0 \Rightarrow A = 1$

when $S = \frac{-1}{T}$; $1 = B\left(-\frac{1}{T}\right) \Rightarrow B = -T$

then, $C(S) = \frac{1}{S} - \frac{T}{TS + 1} = \frac{1}{S} = \frac{1}{S + 1/T}$

Taking inverse Laplace for this we get,

$$\boxed{C(t) = 1 - e^{-t/T}}$$

Steady state error:

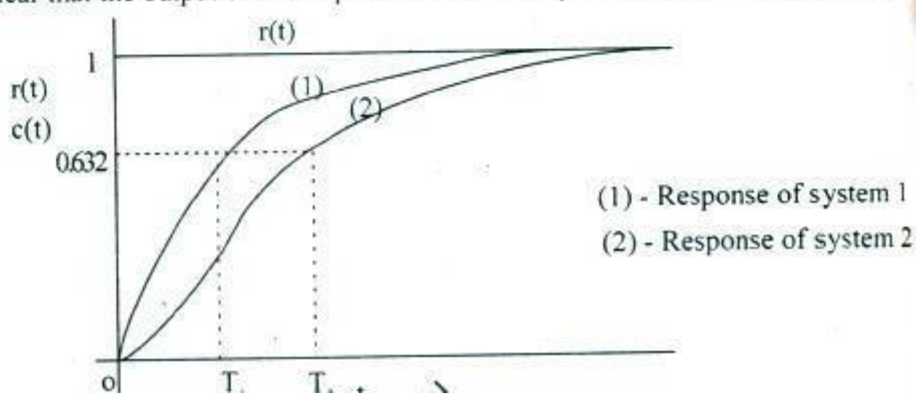
It is given by. $e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$

$= \lim_{t \to \infty} [1 - (1 - e^{-t/T})]$

$= \lim_{t \to \infty} (e^{-t/T})$

$= 0$

It is clear that the output or the response follows the system input with zero steady state



b. A unity feedback system is characterized by an open-loop transfer function G(S) = S(S Determine the gain K, so that, the system will have a damping ratio of 0.5. For this velocity K, determine the settling time, peak overshot ami time to peak overshot for a unit step

Ans: The characteristic equation of the system is given by

$$1 + G(S) = 0$$

i.e. $\Rightarrow 1 + \dfrac{K}{S(S+10)} = 0$

i.e. $S^2 + 10S + K = 0$

comparing with, $S^2 + 2\xi\omega_n S + \omega_n^2 = 0$

$$\omega_n^2 = K$$

$2\xi\omega_n = 10 \Rightarrow \quad \omega_n = \dfrac{5}{0.5} = 10$ rad/s

$$K = 10^2 = 100$$

Peak time, $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi}{10\sqrt{1-0.5^2}} = 0.3627s$

Settling time, $t_s = \dfrac{4}{\xi\omega_n}$

$$= \frac{4}{0.5 \times 10} = 0.8s$$

Peak over shoot, $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$$= e^{-\pi \times 0.5/\sqrt{1-0.5^2}}$$

$= 0.16303$

$\% M_p = 16.303\%$

c. Determine the stability of the system whose characteristic equation is given by S4 + 6SJ +23S1 + 40S + 50 = 0 (04 M)

Ans: $S^4 + 6S^3 + 23S^2 + 40S + 50 = 0$

We can form the Routh's array as,

S^4	1	21	20
S^3	6	36	
	1	6	
S^2	15	20	
	14	4	
S^1	4/3		
S^0	4		

There is no sign change in the first column of Routh's array and hence the system under consideration is stable.

Dec 2014

2a)Definerisetime,peakovershootandsettlingtimeofacontrolsystem.Risetime(T)

It is the time required for the response to reach 100% of the final value. Peaks overshoot (M,): It is the max deviation of the output from the mean value in the transient state. This is the measure of relative stability of any system. More this value more time the system takes to settle.

Settling time (T):

It is the time required for the response to reach and stay within a specified tolerance band say, 2% or 5% of the final value.

1. *The open-loop transfer function of a unity feedback control system is given by* $G(s) = \frac{25}{s(s+5)}$. *Obtain the maximum overshoot, peak time, rise time and settling time.* **(07 M)**

Ans: The characteristic equation of the system is given by,

$$1 + G(s)H(s) = 0$$

i.e. $1 + \frac{25}{s(1+5)} \times 1 = 0 \qquad \because H(s) = 1$

i.e. $s^2 + 5s + 25 = 0$

comparing with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n^2 = 25 \Rightarrow \omega_n = 5 \text{ rad/s}$$

and $2\xi\omega_n = 5 \Rightarrow \xi = \frac{5}{2\omega_n} = \frac{5}{2\times5} = 0.5$

(i) Max-overshoot, $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-\pi\times0.5/\sqrt{1-0.5^2}} = 0.16303$

$\%M_p = 16.303\%$

(ii) Peak time, $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{5\sqrt{1-0.5^2}} = 0.7255\text{s}$

(iii) Settling time, $t_s = \frac{4}{\xi\omega_n}$ (for 2% tolerance band)

$$= \frac{4}{0.5\times5} = 1.6 \text{ sec}$$

Rise time: $t_r = \dfrac{\pi - \theta}{\omega_n \sqrt{1-\xi^2}}$

where $\theta = \tan^{-1}\left(\dfrac{\sqrt{1-\xi^2}}{\xi}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1-0.5^2}}{0.5}\right) \times \frac{\pi}{180}$$

$$= 1.047 \text{ rad}$$

$$\therefore \quad t_r = \frac{\pi - 1.047}{5\sqrt{1-0.5^2}}$$

$$= 0.4837 s$$

b)

The closed loop transfer function of a system is given by $\dfrac{C(s)}{R(s)} = \dfrac{k}{s(s^2+s+1)(s+2)+k}$ *mine the value of k for which the system is stable.*

$$\frac{C(s)}{R(s)} = \frac{k}{s(s^2+s+1)(s+2)+k} = G(s)H(s)$$

Considering the characteristic equation,

$$1 + G(s)H(s) = 0$$

i.e. $\quad 1 + \dfrac{k}{s(s^2+s+1)(s+2)+k} = 0$

i.e. $\quad s(s^2+s+1)(s+2)+k+k = 0$

i.e. $\quad (s^2+s+1)(s^2+2s)+2k = 0$

i.e. $\quad s^4+2s^3+s^3+2s^2+s^2+2s+2k = 0$

i.e. $\quad s^4+3s^3+3s^2+2s+2k = 0$

s^4	1	3	2k
s^3	3	2	0
s^2	7/3	2k	
s^1	$\frac{(14/3 - 6k)}{7/3}$	0	
s^0	$\frac{2k\left(2 - \frac{18k}{7}\right)}{\left(2 - \frac{18k}{7}\right)} = 2k$		

$$\frac{\frac{14}{3}}{7/3} - \frac{6k}{7/3}$$

$$= \left(2 - \frac{18k}{7}\right)$$

For the system to be stable, all the elements in the first column should be +'ve.

i.e. $2k > 0$ and $2 - \frac{18k}{7} > 0$

or $k > 0$ ie $2 > \frac{18k}{7}$

$$\frac{14}{18} > k$$

or $k < \frac{7}{9}$

$\therefore$ $\boxed{0 < k < 7/9}$ condition for stability

June 2015

Module-4

1. Draw the Nyquist stability criterion

Ans: $$GH(S) = \frac{K}{S(1+S)(1+2S)(1+3S)}$$

For this,

$$M(\omega)\underline{|\phi(\omega)} = \frac{1}{\omega\sqrt{1+\omega^2}\ \sqrt{1+(2\omega)^2}\ \sqrt{1+(3\omega)^2}}\ \underline{|-90-\tan^{-1}(\omega)-\tan^{-1}(2\omega)-t}$$

$$\therefore \quad M = \frac{1}{\omega\sqrt{1+w^2}\ \sqrt{1+4w^2}\ \sqrt{1+9w^2}}$$

$$\phi = -90^\circ - \tan^{-1}(w) - \tan^{-1}(2w) - \tan^{-1}(3w)$$

When $\omega = 0$, , $\phi = -90^\circ$

When $\omega = \infty$, $= -360^\circ$

$$G(j\omega) = \frac{K}{j\omega(1+j\omega)(1+2j\omega)(1+3j\omega)}$$

Rationalizing and equating the imaginary part to zero we get,

$$\frac{1}{\omega^2} = 1$$

$$\omega = 0.3 \text{ rad/s}$$

Or

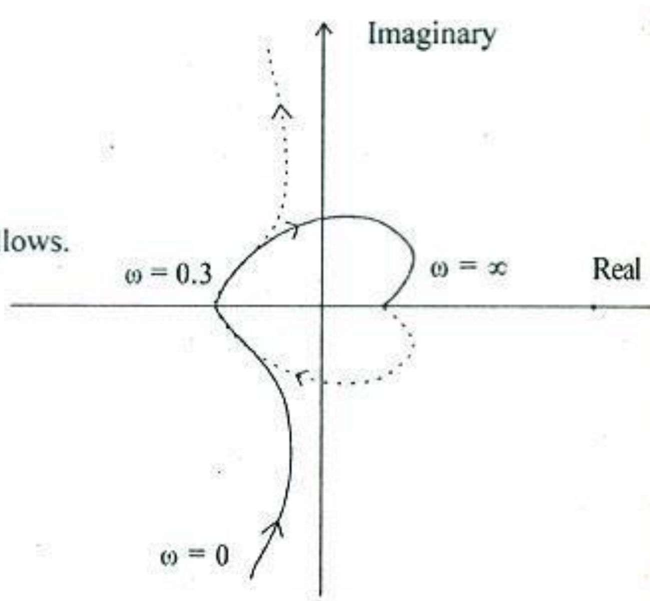
$$M/_{\omega=0.3} = \frac{K}{0.3\sqrt{1+0.3^2}\ \sqrt{1+4\times 0.3^2}\ \sqrt{1+9\times 0.3^2}}$$

$$= \frac{K}{0.5}$$

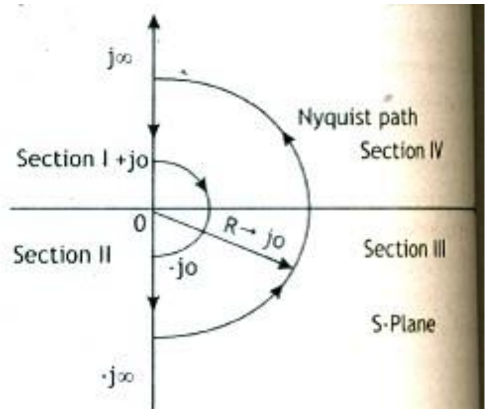
for stability, $\frac{K}{0.5} < 1$

or $K < 0.5$

Nyquist plot can be drawn as follows.



Dec 2014



(4) $$G(j\omega)H(j\omega) = \frac{(1 + 0.5j\omega)}{(j\omega)(j\omega)(1 + 0.1j\omega)(1 + 0.02j\omega)}$$

section I = S = +j∞ to S = +jo

Starting point	$\omega \to \infty$	$0 < \frac{90}{90 .90 .90 .90} = 0\angle -270^\circ$	$-180^\circ - (-270$ ACW rotation
Terminating point	$\omega = +0$	$\infty < \frac{0}{90 .90 .0 .0} = \infty\angle -180^\circ$	

Section II = S = +jo to S = -Jo

Starting point	$\omega = 0$	$\infty\angle -180^\circ$	$180-(-180^\circ)=$ ACW rotation
Terminating point	$\omega \to -0$	$\infty < \frac{0}{-90 . -90 .0 .0} = \infty\angle +180^\circ$	

section III is mirror image of section I

section IV is not required.

(5) $$G(j\omega)H(j\omega) = \frac{(1 + 0.5j\omega)}{(-\omega^2)(1 + 0.1j\omega)(1 + 0.02j\omega)}$$

Rationalizing G(j)H(j) and separating real and imaginary part we get,

$$G(j\omega)H(j\omega) = \frac{(1 + 0.5j\omega)[1 - 0.12j\omega - 0.002\omega^2]}{(-\omega^2)(1 + 0.01j\omega^2)(1 + 0.0004\omega^2)}$$

$$= \frac{(1 + 0.058\omega^2)}{D} = \frac{j\omega[0.38 - 0.001\omega^2]}{D}$$

Module 4

June 2015

Ans: $G(S) = \dfrac{Ke^{-0.1S}}{S(1+S)(1+0.1S)}$

Log Magnitude plot:

Let K = 1, the normalised transfer function is

$$G(S) = \frac{e^{-0.1S}}{S(1+S)(1+0.1S)} = \frac{1}{S} \cdot \frac{1}{(1+S)} \cdot \frac{1}{(1+0.1S)}$$

(The factor $e^{-0.1S}$ is not taken into account as $20\log|e^{-0.1S}|=0$ dB

Factor	Corner frequency rad/s	Individual slope	Cumulative slope
$\frac{1}{S}$	-	-20	-20
1/(1+S)	1	-20	-40
1/(1+0.1S)	10	-20	-60

starting frequency $S = 0.1$ rad/s

starting point S is $20\log\left|\frac{1}{S}\right| = 20\log\left(\frac{1}{0.1}\right) S = 20$ rad/s

last frequency = 100 rd/s

Phase angle plot

$$G(S) = \frac{Ke^{-0.1S}}{S(1+S)(1+0.1S)}$$

$$\therefore \quad G(j\omega) = \frac{(K+j0)(\cos(0.1\omega) - j\sin(0.1\omega))}{(0+j\omega)(1+j\omega)(1+0.1j\omega)}$$

$$\therefore \quad \phi(\omega) = -\left(0.1\omega\frac{180}{\pi}\right) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}(\omega) - \tan^{-1}(0.1\omega)$$

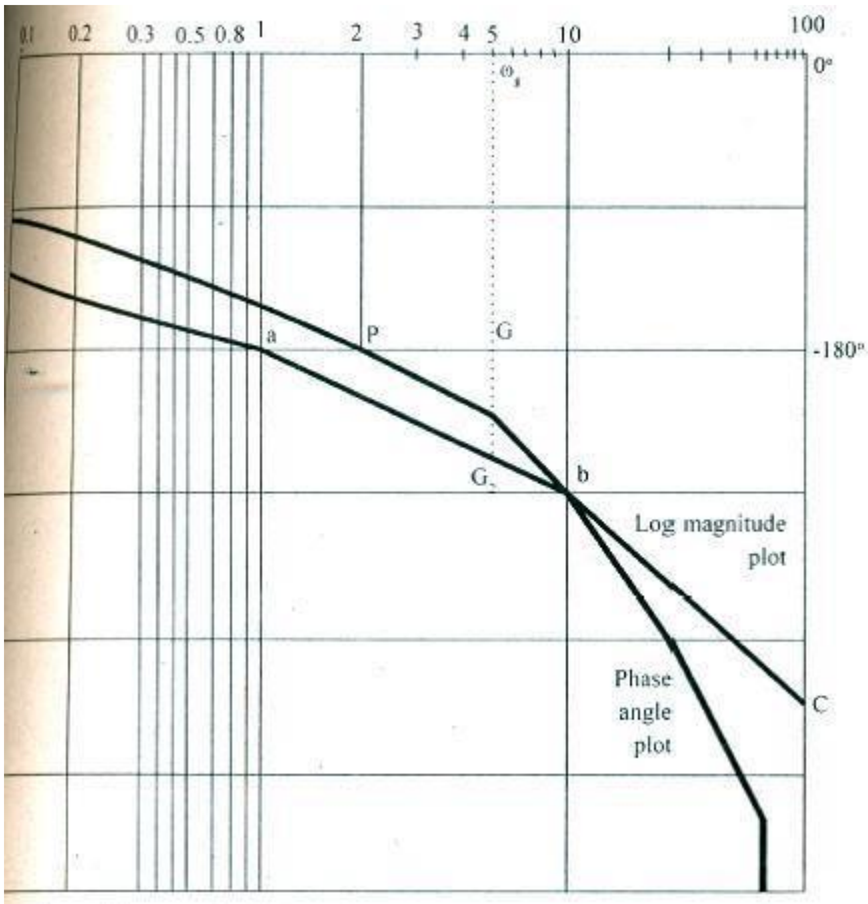
$$\phi(\omega) = 0 - 5.72\omega - 90\tan^{-1}(\omega) - \tan^{-1}(0.1\omega)$$
$$= -90 - 5.72\omega - \tan^{-1}(\omega) - \tan^{-1}(0.1\omega)$$

ω	0.1	0.2	0.5	0.8	1.0	2.0	5.0	8.0	10.0	12.0	50.0
φ(ω)	-97	-104	-122	-138	-147	-176	-224	-257	-277	-354	-543

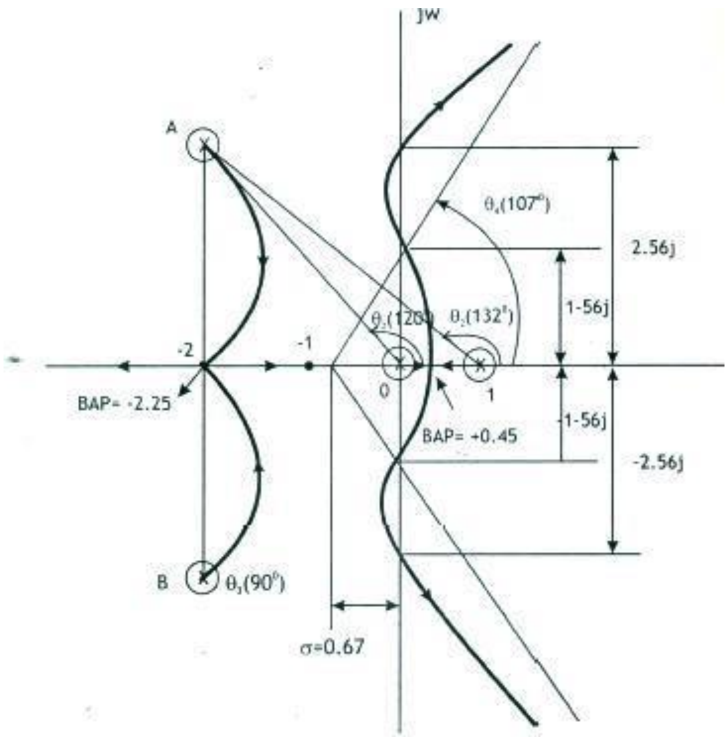
The Bode plots are constructed as follows:

Scale: 1 in 40 db for 20 log |Mω|

1 in = 100^0 for φ(ω)


0.1
0.2
0.3
0.5
0.8
1
2
3
4
5
10
100
0°
-180°
a
P
G
b
Log magnitude plot
Phase angle plot
C

Dec 2014


jw
A
B
θ4(107°)
θ2(120)
θ1(132°)
θ3(90°)
2.56j
1-56j
-2.56j
-2
-1
0
1
BAP= -2.25
BAP= +0.45
σ=0.67

Module-4

June 2015

1. Sketch the Bode plot for the transfer function G(S) = Ke-/S(1 + S) (1 + 0.1S) • Find the K for the crossover frequency = 5 rad/sec

$$G(S) = \frac{e^{-0.1S}}{S(1+S)(1+0.1S)} = \frac{1}{S} \cdot \frac{1}{(1+S)} \cdot \frac{1}{(1+0.1S)}$$

(The factor $e^{-0.1S}$ is not taken into account as $20\log|e^{-0.1S}| = 0$ dB

Factor	Corner frequency rad/s	Individual slope	Cumulative slope
$\frac{1}{S}$	-	-20	-20
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starting frequency $S = 0.1$ rad/s

starting point S is $20 \log \left|\frac{1}{S}\right| = 20 \log \left(\frac{1}{0.1}\right) S = 20$ rad/s

last frequency = 100 rd/s

Phase angle plot

$$G(S) = \frac{Ke^{-0.1S}}{S(1+S)(1+0.1S)}$$

$$\therefore \quad G(j\omega) = \frac{(K + j0)(\cos(0.1\omega) - j\sin(0.1\omega))}{(0 + j\omega)(1 + j\omega)(1 + 0.1j\omega)}$$

$$\therefore \quad \phi(\omega) = -\left(0.1\omega \frac{180}{\pi}\right) - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}(\omega) - \tan^{-1}(0.1\omega)$$

$$\phi(\omega) = 0 - 5.72\omega - 90 \tan^{-1}(\omega) - \tan^{-1}(0.1\omega)$$

$$= -90 - 5.72\omega - \tan^{-1}(\omega) - \tan^{-1}(0.1\omega)$$

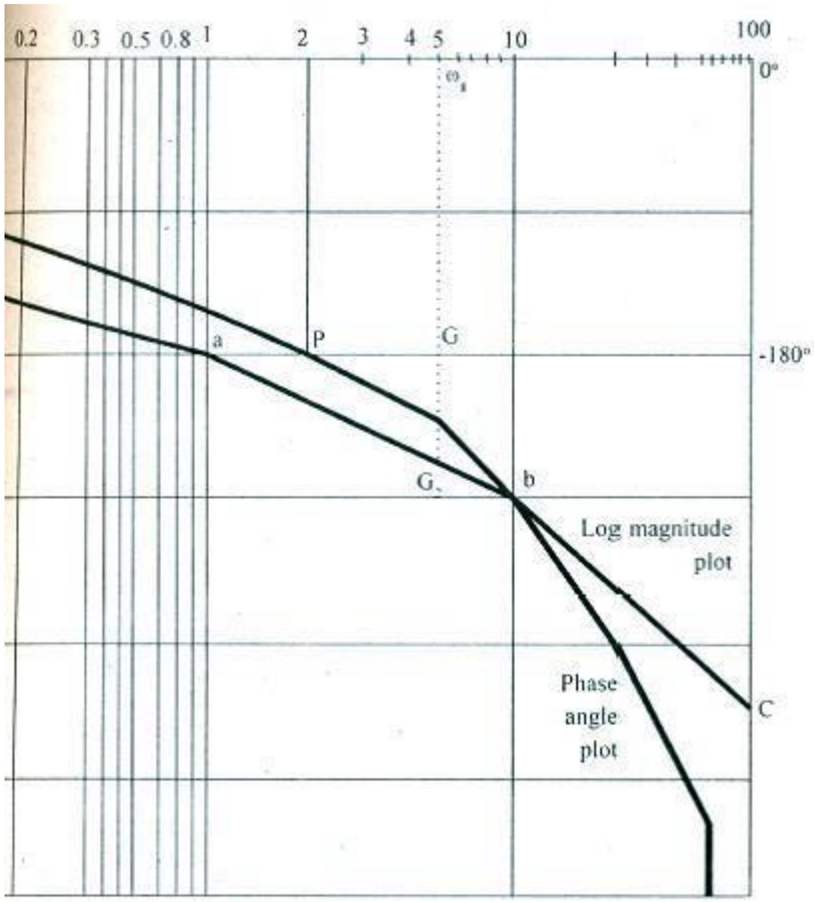
ω	0.1	0.2	0.5	0.8	1.0	2.0	5.0	8.0	10.0	12.0	50.0
φ(ω)	-97	-104	-122	-138	-147	-176	-224	-257	-277	-354	-543

The Bode plots are constructed as follows:

Scale: 1 in 40 db for 20 log |Mω|

1 in = 100° for φ(ω)

Dec 2014



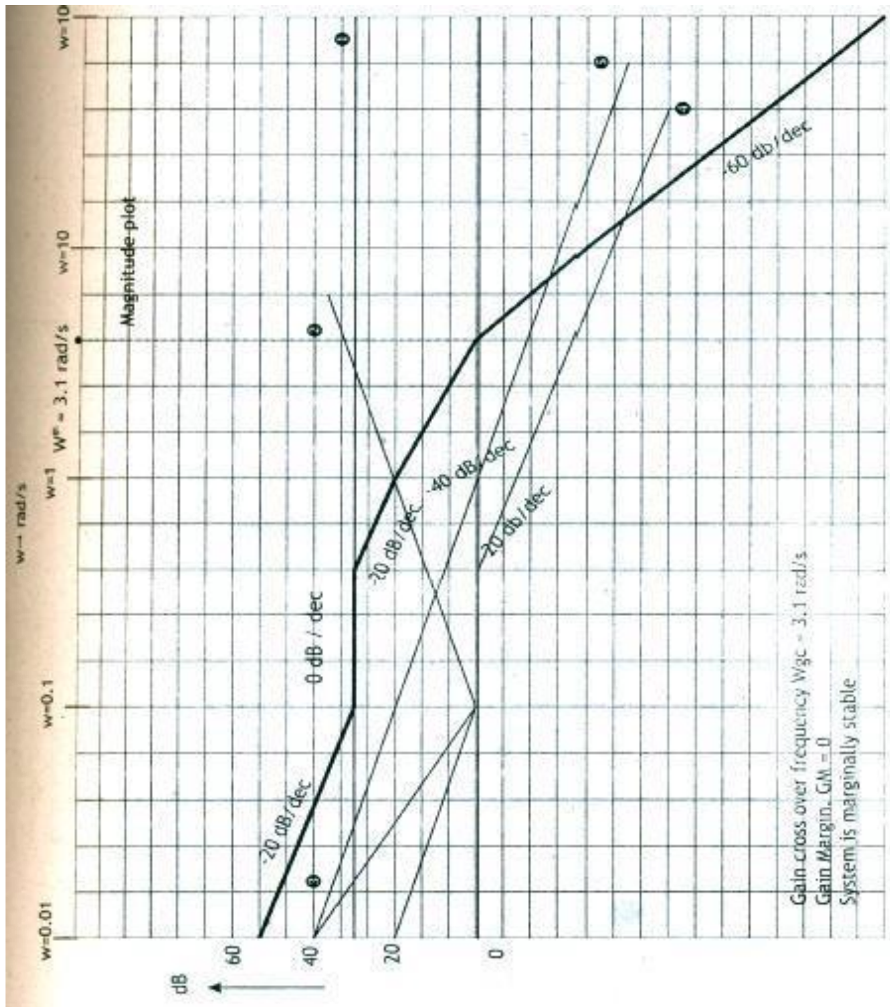
System gain such that (!)~= 5 rad/s. From the Bode plot we get GG2 = 28dB. The log magnitude plot has to be shifted upwards by GG~ dB. Such that ())g = 5 rad/s :. 20 log K = GG~ = 28 :. K = 25.11

Bode plot $GH = \frac{100(10s+1)}{s(s+0.4)(s+1)(s+10)}$

$$GH = \frac{100(1+10s)}{s \times 0.4(1+2.5s)(1+s) \times 10(1+0.1s)}$$

$$= \frac{25(1+10s)}{s(s+2.5s)(1+s)(1+0.1s)}$$

Factor	Details on Magnitude plot	Details of phase angle plot
1. 25	$20\log 25 \simeq 28$	$0°$
2. $(1+10j\omega)$	$20\log\sqrt{1+(10\omega)^2}$ $\Rightarrow 1 = 10\omega$ $\Rightarrow \omega = \omega_c = 0.1$ rad/s	$\phi(\omega) = \tan^{-1}(10\omega)$ $\omega = 0, \phi(\omega) = 0°$ $\omega = \infty, \phi(\omega) = +90°$ $\frac{\omega_c}{5} = 0.02; \omega_c \times 5 = 0.5$
3. $\frac{1}{j\omega}$	-20log	$\phi(\omega) = -90°$
4. $\frac{1}{1+2.5j\omega}$	$-20\log\sqrt{1+(2.5\omega)^2}$ $\Rightarrow 1 = 2.5\omega$ $\Rightarrow \omega = \omega_c = 0.4$ rad/s	$\phi(\omega) = -\tan^{-1}(2.5\omega)$ $\omega = 0, \phi(\omega) = 0°$ $\omega = \infty, \phi(\omega) = -90°$

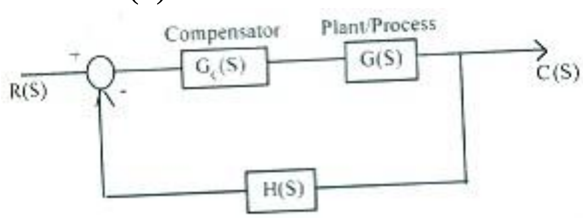


Module-5

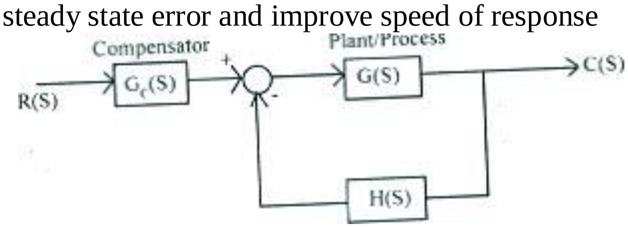
June 2015

1. Discuss various methods of compensation 'in feed back control systems.

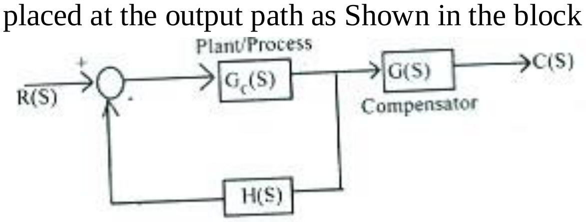
System compensation" is defined as the adjustment/redesigning of a system so as to me!! required specification by altering or by adding an external device to system. There are4 of system compensation. 1. Cascade compensation 2. Feedback compensation 3. Input compensation 4. Output compensation *Cascade compensation:* In cascade compensation the compensations element whose transfer function Ge(S) ispi in series with the forward transfer function O(S) as shown. It is also referred as *compensation.*



feedback **compensation*:*** In a feedback compensation, the compensating device whose transfer function 'Ge(S) is placed In the feed back path and is also termed as parallel compensation. Feedback compensation may be usedto improve system stability, to reduce steady state error and improve speed of response



Output compensation: Here the compensation device whose transfer function is Ge(S) is placed at the output path as Shown in the block diagram.

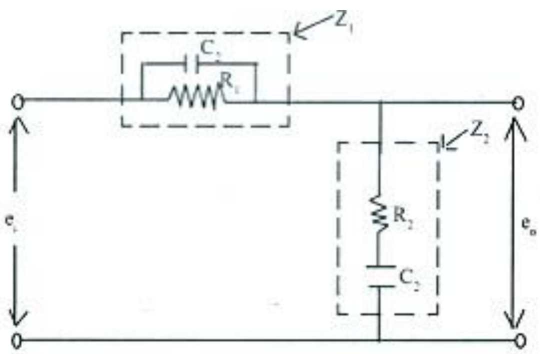


The selection of a particular compensation type depends upon native of the signal~ levels at various points, availability of the components and the cost considerations.

Dec 2014

2. Explain with a block diagram the lag lead compensator:

Lag-Lead compensator Lead compensation increases the bandwidth which improves the system response reduces the amount of overshoot. However, improvement in steady state performances small. Lag compensation results in a large improvement in steady state performance but in slower response due to reduced bandwidth. If improvement in both transient and state response are desired, then both a lead network and a lag network may~ simultaneously. The name .lag - lead compensation comes from the. fact that when the input is output is sinusoidal with a phase shift which is a function of the input frequency. This angle varies from lag to lead as the frequency is increased from zero to infinity. a lag lead compensation is the electrical network shown below.



Here, $Z_1 = \frac{R_1}{R_1C_1S+1}$; $Z_2 = \frac{R_2C_2S+1}{C_2S}$

The transfer function is,

$$\frac{E_o(S)}{E_i(S)} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{R_2C_2S+1}{C_2S}}{\frac{R_1}{R_1C_1S+1}+\frac{R_2C_2S+1}{C_2S}}$$

Let $R_1C_1 = T_1$, $R_2C_2 = T_2$, $R_1C_1 + R_2C_2 + R_1C_2 = \frac{T_1}{\beta} + \beta T_2$, $(\beta > 1)$

substituting and simplifying we get,

$$\frac{E_o(S)}{E_i(S)} = \frac{\left(S+\frac{1}{T_1}\right)\left(S+\frac{1}{T_2}\right)}{\left(S+\frac{\beta}{T_1}\right)\left(S+\frac{1}{\beta T_2}\right)} \quad \text{——— (1)}$$

A lag-lead compensation has a transfer function as given in equation (1) characteristics of Lag-Lead compensation It improves both transient and steady state performance of the system Due to this, control system is more stable and system will have increased bandwidth. Due to increased bandwidth reduced rise time and settling time. It makes the system response more faster.

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