MODULE I

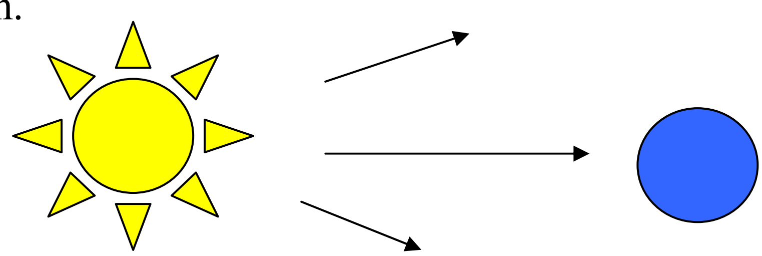
RADIATION HEAT TRANSFER

Radiation

Definition

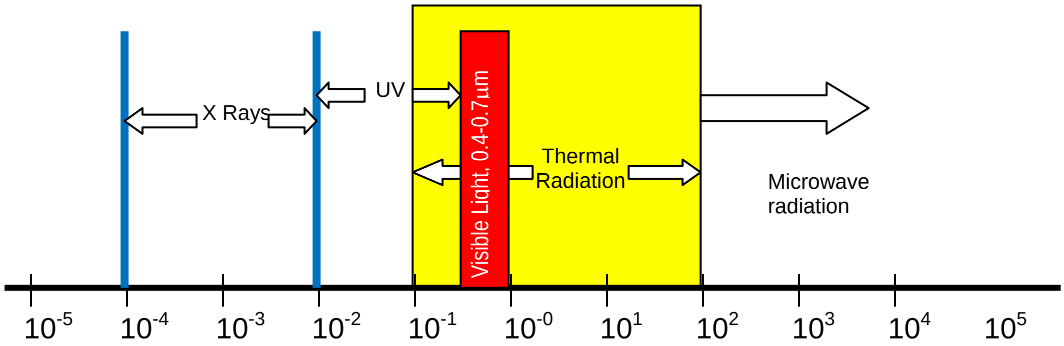
Radiation, energy transfer across a system boundary due to a ΔT, by the mechanism of photon emission or electromagnetic wave emission.

Because the mechanism of transmission is photon emission, unlike conduction and convection, there need be no intermediate matter to enable transmission.



The significance of this is that radiation will be the only mechanism for heat transfer whenever a vacuum is present.

Electromagnetic Phenomena.
We are well acquainted with a wide range of electromagnetic phenomena in modern life. These phenomena are sometimes thought of as wave phenomena and are, consequently, often described in terms of electromagnetic wave length, λ. Examples are given in terms of the wave distribution shown below:



One aspect of electromagnetic radiation is that the related topics are more closely associated with optics and electronics than with those normally found in mechanical engineering courses. Nevertheless, these are widely encountered topics and the student is familiar with them through every day life experiences.

From a viewpoint of previously studied topics students, particularly those with a background in mechanical or chemical engineering, will find the subject of Radiation Heat Transfer a little unusual. The physics background differs fundamentally from that found in the areas of continuum mechanics. Much of the related material is found in courses more closely identified with quantum physics or electrical engineering, i.e. Fields and Waves. At this point, it is important for us to recognize that since the subject arises from a different area of physics, it will be important that we study these concepts with extra care.

Stefan-Boltzman Law

Both Stefan and Boltzman were physicists; any student taking a course in quantum physics will become well acquainted with Boltzman's work as he made a number of important contributions to the field. Both were contemporaries of Einstein so we see that the subject is of fairly recent vintage. (Recall that the basic equation for convection heat transfer is attributed to Newton.)

$$E_b = \sigma \cdot T_{abs}^4$$

where: E_b = Emissive Power, the gross energy emitted from an ideal surface per unit area, time.

σ = Stefan Boltzman constant, $5.67 \cdot 10^{-8}$ W/m$^2 \cdot$K^4

T_{abs} = Absolute temperature of the emitting surface, K.

Take particular note of the fact that absolute temperatures are used in Radiation. It is suggested, as a matter of good practice, to convert all temperatures to the absolute scale as an initial step in all radiation problems.

You will notice that the equation does not include any heat flux term, q". Instead we have a term the emissive power. The relationship between these terms is as follows. Consider two infinite plane surfaces, both facing one another. Both surfaces are ideal surfaces. One surface is found to be at temperature, T_1, the other at temperature, T_2. Since both temperatures are at temperatures above absolute zero, both will radiate energy as described by the Stefan-Boltzman law. The heat flux will be the net radiant flow as given by:

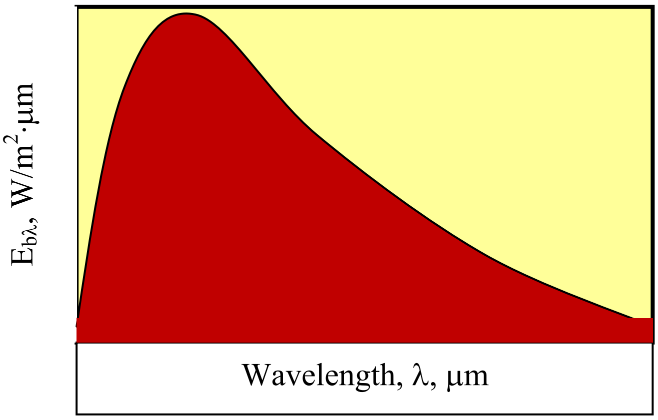
$$q'' = E_{b1} - E_{b2} = \sigma \cdot T_1^4 - \sigma \cdot T_2^4$$

Plank's Law

While the Stefan-Boltzman law is useful for studying overall energy emissions, it does not allow us to treat those interactions, which deal specifically with wavelength, λ. This problem was overcome by another of the modern physicists, Max Plank, who developed a relationship for wave-based emissions.

$$E_{b\lambda} = f(\lambda)$$

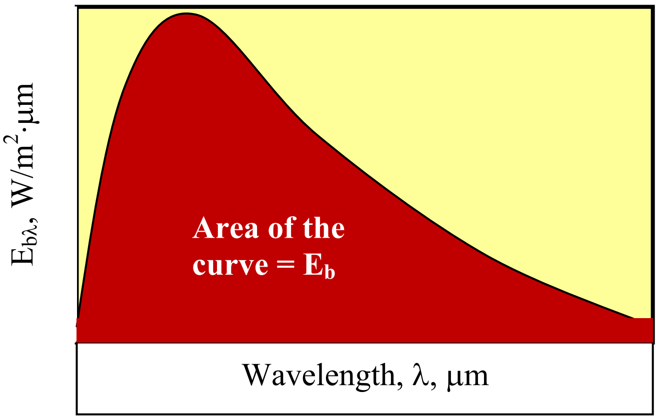
We plot a suitable functional relationship below:



We haven't yet defined the Monochromatic Emissive Power, $E_{b\lambda}$. An implicit definition is provided by the following equation:

$$E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda$$

We may view this equation graphically as follows:



A definition of monochromatic Emissive Power would be obtained by differentiating the integral equation:

$$E_{b\lambda} \equiv \frac{dE_b}{d\lambda}$$

The actual form of Plank's law is:

$$E_{b\lambda} = \frac{C_1}{\lambda^5 \cdot \left[e^{C_2 / \lambda \cdot T} - 1 \right]}$$

where: $C_1 = 2 \cdot \pi \cdot h \cdot c_o^{\ 2} = 3.742 \cdot 10^8\ W \cdot \mu m^4/m^2$
$C_2 = h \cdot c_o/k = 1.439 \cdot 10^4\ \mu m \cdot K$

Where: h, c_o, k are all parameters from quantum physics. We need not worry about their precise definition here.

This equation may be solved at any T, λ to give the value of the monochromatic emissivity at that condition. Alternatively, the function may be substituted into the integral $E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda$ to find the Emissive power for any temperature. While performing this integral by hand is difficult, students may readily evaluate the integral through one of several computer programs, i.e. MathCad, Maple, Mathmatica, etc.

$$E_b = \int_0^\infty E_{b\lambda} \cdot d\lambda = \sigma \cdot T^4$$

Emission Over Specific Wave Length Bands

Consider the problem of designing a tanning machine. As a part of the machine, we will need to design a very powerful incandescent light source. We may wish to know how much energy is being emitted over the ultraviolet band (10^{-4} to 0.4 μm), known to be particularly dangerous.

$$E_b(0.0001 \rightarrow 0.4) = \int_{0.001 \cdot \mu m}^{0.4 \cdot \mu m} E_{b\lambda} \cdot d\lambda$$

With a computer available, evaluation of this integral is rather trivial. Alternatively, the text books provide a table of integrals. The format used is as follows:

$$\frac{E_b(0.001 \to 0.4)}{E_b} = \frac{\int_{0.001\cdot\mu m}^{0.4\cdot\mu m} E_{b\lambda}\cdot d\lambda}{\int_0^{\infty} E_{b\lambda}\cdot d\lambda} = \frac{\int_0^{0.4\cdot\mu m} E_{b\lambda}\cdot d\lambda}{\int_0^{\infty} E_{b\lambda}\cdot d\lambda} - \frac{\int_{0.}^{0.0001\cdot\mu m} E_{b\lambda}\cdot d\lambda}{\int_0^{\infty} E_{b\lambda}\cdot d\lambda} = F(0 \to 0.4) - F(0 \to 0.0001)$$

Referring to such tables, we see the last two functions listed in the second column. In the first column is a parameter, λ·T. This is found by taking the product of the absolute temperature of the emitting surface, T, and the upper limit wave length, λ. In our example, suppose that the incandescent bulb is designed to operate at a temperature of 2000K. Reading from the table:

λ, μm	T, K	$\lambda\cdot T$, $\mu m\cdot K$	$F(0\to\lambda)$
0.0001	2000	0.2	0
0.4	2000	600	0.000014
F(0.4→0.0001μm) = F(0→0.4μm)- F(0→0.0001μm)			**0.000014**

This is the fraction of the total energy emitted which falls within the IR band. To find the absolute energy emitted multiply this value times the total energy emitted:

$$E_{bIR} = F(0.4\to 0.0001\mu m)\cdot\sigma\cdot T^4 = 0.000014\cdot 5.67\cdot 10^{-8}\cdot 2000^4 = \mathbf{12.7\ W/m^2}$$

Solar Radiation

The magnitude of the energy leaving the Sun varies with time and is closely associated with such factors as solar flares and sunspots. Nevertheless, we often choose to work with an average value. The energy leaving the sun is emitted outward in all directions so that at any particular distance from the Sun we may imagine the energy being dispersed over an imaginary spherical area. Because this area increases with the distance squared, the solar flux also decreases with the distance squared. At the average distance between Earth and Sun this heat flux is 1353 W/m^2, so that the average heat flux on any object in Earth orbit is found as:

$$G_{s,o} = S_c \cdot f \cdot \cos\theta$$

Where S_c = Solar Constant, 1353 W/m^2
f = correction factor for eccentricity in Earth Orbit, $(0.97<f<1.03)$
θ = Angle of surface from normal to Sun.

Because of reflection and absorption in the Earth's atmosphere, this number is significantly reduced at ground level. Nevertheless, this value gives us some opportunity to estimate the potential for using solar energy, such as in photovoltaic cells.

Some Definitions

In the previous section we introduced the Stefan-Boltzman Equation to describe radiation from an ideal surface.

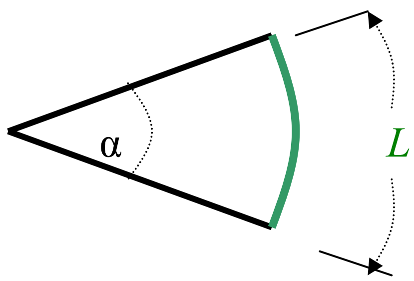
$$E_b = \sigma \cdot T_{abs}^4$$

This equation provides a method of determining the total energy leaving a surface, but gives no indication of the direction in which it travels. As we continue our study, we will want to be able to calculate how heat is distributed among various objects.

For this purpose, we will introduce the radiation intensity, I, defined as the energy emitted per unit area, per unit time, per unit solid angle. Before writing an equation for this new property, we will need to define some of the terms we will be using.

Angles and Arc Length

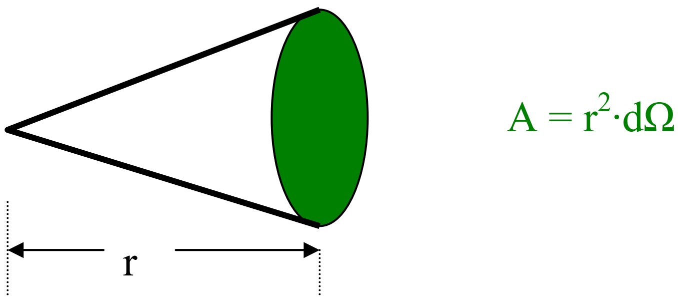
We are well accustomed to thinking of an angle as a two dimensional object. It may be used to find an arc length:



$$L = r \cdot \alpha$$

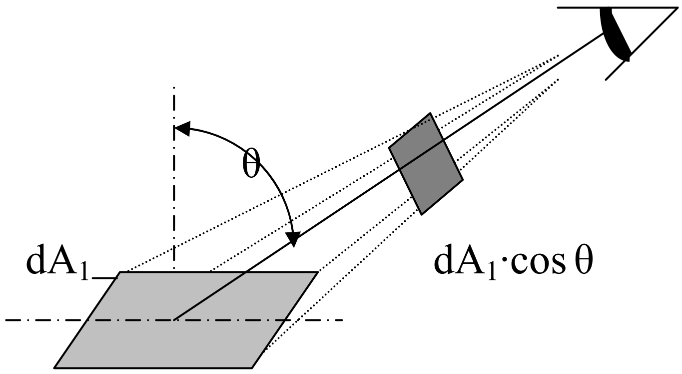
Solid Angle

We generalize the idea of an angle and an arc length to three dimensions and define a solid angle, Ω, which like the standard angle has no dimensions. The solid angle, when multiplied by the radius squared will have dimensions of length squared, or area, and will have the magnitude of the encompassed area.



Projected Area

The area, dA_1, as seen from the prospective of a viewer, situated at an angle θ from the normal to the surface, will appear somewhat smaller, as $\cos\theta \cdot dA_1$. This smaller area is termed the projected area.



$$A_{projected} = \cos\theta \cdot A_{normal}$$

Intensity

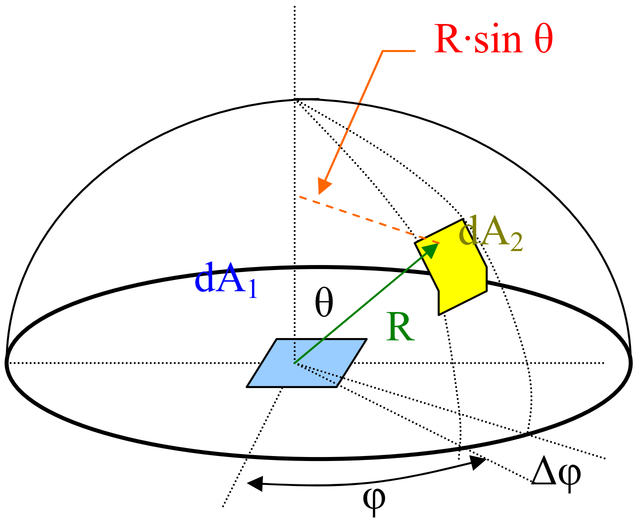
The ideal intensity, I_b, may now be defined as the energy emitted from an ideal body, per unit projected area, per unit time, per unit solid angle.

$$I = \frac{dq}{\cos\theta \cdot dA_1 \cdot d\Omega}$$

Spherical Geometry

Since any surface will emit radiation outward in all directions above the surface, the spherical coordinate system provides a convenient tool for analysis. The three basic coordinates shown are R, φ, and θ, representing the radial, azimuthal and zenith directions.

In general dA_1 will correspond to the emitting surface or the source. The surface dA_2 will correspond to the receiving surface or the target. Note that the area proscribed on the hemisphere, dA_2, may be written as:



$$dA_2 = [(R \cdot \sin\theta) \cdot d\varphi] \cdot [R \cdot d\theta]$$

or, more simply as:

$$dA_2 = R^2 \cdot \sin\theta \cdot d\varphi \cdot d\theta]$$

Recalling the definition of the solid angle,

$$dA = R^2 \cdot d\Omega$$

we find that:

$$d\Omega = R^2 \cdot \sin\theta \cdot d\theta \cdot d\varphi$$

Real Surfaces

Thus far we have spoken of ideal surfaces, i.e. those that emit energy according to the Stefan-Boltzman law:

$$E_b = \sigma \cdot T_{abs}^{\ 4}$$

Real surfaces have emissive powers, E, which are somewhat less than that obtained theoretically by Boltzman. To account for this reduction, we introduce the emissivity, ε.

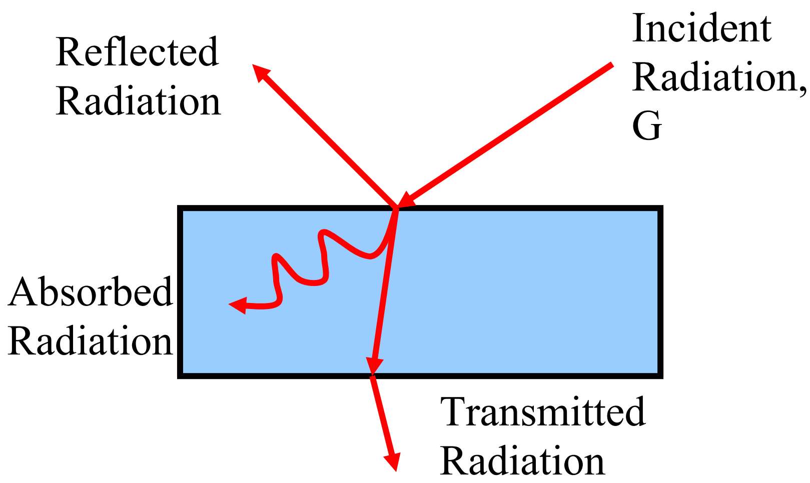
$$\varepsilon \equiv \frac{E}{E_b}$$

so that the emissive power from any real surface is given by:

$$E = \varepsilon \cdot \sigma \cdot T_{abs}^{4}$$

Receiving Properties

Targets receive radiation in one of three ways; they absorption, reflection or transmission. To account for these characteristics, we introduce three additional properties:



- Absorptivity, α, the fraction of incident radiation absorbed.
- Reflectivity, ρ, the fraction of incident radiation reflected.
- Transmissivity, τ, the fraction of incident radiation transmitted.

We see, from Conservation of Energy, that:

$$\alpha + \rho + \tau = 1$$

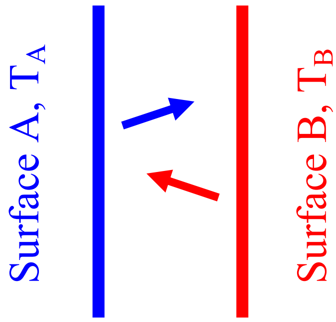
In this course, we will deal with only opaque surfaces, $\tau = 0$, so that:

$$\alpha + \rho = 1$$

Opaque Surfaces

Relationship Between Absorptivity, α, and Emissivity, ε

Consider two flat, infinite planes, surface A and surface B, both emitting radiation toward one another. Surface B is assumed to be an ideal emitter, i.e. $\varepsilon_B = 1.0$. Surface A will emit radiation according to the Stefan-Boltzman law as:



$$E_A = \varepsilon_A \cdot \sigma \cdot T_A^4$$

and will receive radiation as:

$$G_A = \alpha_A \cdot \sigma \cdot T_B^4$$

The net heat flow from surface A will be:

$$q'' = \varepsilon_A \cdot \sigma \cdot T_A^4 - \alpha_A \cdot \sigma \cdot T_B^4$$

Now suppose that the two surfaces are at exactly the same temperature. The heat flow must be zero according to the 2nd law. If follows then that:

$$\boxed{\alpha_A = \varepsilon_A}$$

Because of this close relation between emissivity, ε, and absorptivity, α, only one property is normally measured and this value may be used alternatively for either property.

Let's not lose sight of the fact that, as thermodynamic properties of the material, α and ε may depend on temperature. In general, this will be the case as radiative properties will depend on wavelength, λ. The wave length of radiation will, in turn, depend on the temperature of the source of radiation.

The emissivity, ε, of surface A will depend on the material of which surface A is composed, i.e. aluminum, brass, steel, etc. and on the temperature of surface A.

The absorptivity, α, of surface A will depend on the material of which surface A is composed, i.e. aluminum, brass, steel, etc. and on the temperature of surface B.

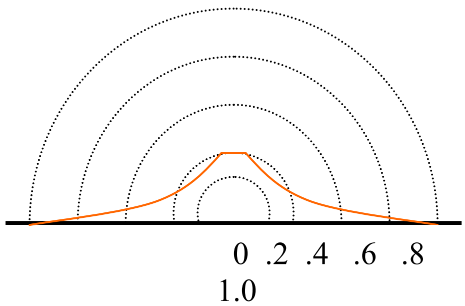
In the design of solar collectors, engineers have long sought a material which would absorb all solar radiation, ($\alpha = 1$, $T_{sun} \sim 5600K$) but would not re-radiate energy as it came to temperature ($\varepsilon << 1$, $T_{collector} \sim 400K$). NASA developed an anodized chrome, commonly called "black chrome" as a result of this research.

Black Surfaces

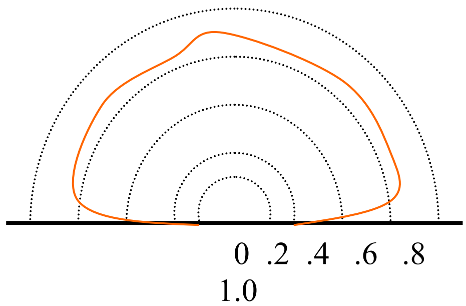
Within the visual band of radiation, any material, which absorbs all visible light, appears as black. Extending this concept to the much broader thermal band, we speak of surfaces with $\alpha = 1$ as also being "black" or "thermally black". It follows that for such a surface, $\varepsilon = 1$ and the surface will behave as an ideal emitter. The terms ideal surface and black surface are used interchangeably.

Lambert's Cosine Law:

A surface is said to obey Lambert's cosine law if the intensity, I, is uniform in all directions. This is an idealization of real surfaces as seen by the emissivity at different zenith angles:



Dependence of Emissivity on Zenith Angle, Typical Metal.



Dependence of Emissivity on Zenith Angle, Typical Non-Metal.

The sketches shown are intended to show is that metals typically have a very low emissivity, ε, which also remain nearly constant, expect at very high zenith angles, θ. Conversely, non-metals will have a relatively high emissivity, ε, except at very high zenith angles. Treating the emissivity as a constant over all angles is generally a good approximation and greatly simplifies engineering calculations.

Relationship Between Emissive Power and Intensity

By definition of the two terms, emissive power for an ideal surface, E_b, and intensity for an ideal surface, I_b.

$$E_b = \int_{hemisphere} I_b \cdot \cos\theta \cdot d\Omega$$

Replacing the solid angle by its equivalent in spherical angles:

$$E_b = \int_0^{2\cdot\pi} \int_0^{\pi/2} I_b \cdot \cos\theta \cdot \sin\theta \cdot d\theta \cdot d\varphi$$

Integrate once, holding I_b constant:

$$E_b = 2\cdot\pi\cdot I_b \cdot \int_0^{\pi/2} \cos\theta \cdot \sin\theta \cdot d\theta$$

Integrate a second time. (Note that the derivative of sin θ is cos θ·dθ.)

$$E_b = 2\cdot\pi\cdot I_b \cdot \left.\frac{\sin^2\theta}{2}\right|_0^{\pi/2} = \pi \cdot I_b$$

$$\boxed{E_b = \pi\cdot I_b}$$

Radiation Exchange

During the previous lecture we introduced the intensity, I, to describe radiation within a particular solid angle.

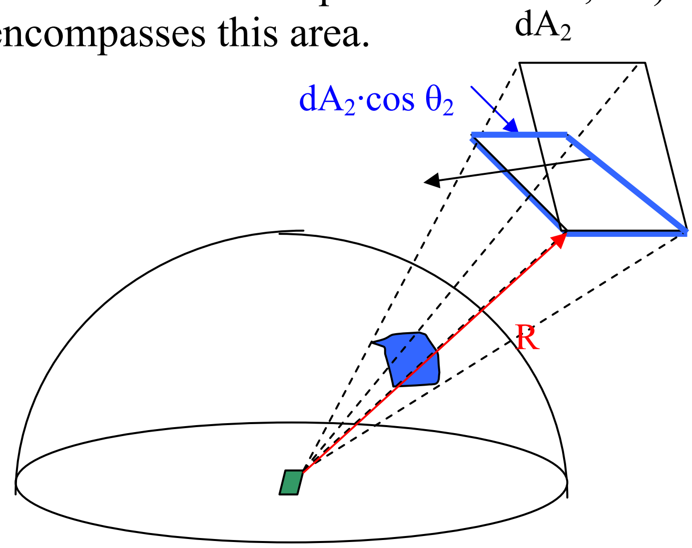
$$I = \frac{dq}{\cos\theta \cdot dA_1 \cdot d\Omega}$$

This will now be used to determine the fraction of radiation leaving a given surface and striking a second surface.

Rearranging the above equation to express the heat radiated:

$$dq = I \cdot \cos\theta \cdot dA_1 \cdot d\Omega$$

Next we will project the receiving surface onto the hemisphere surrounding the source. First find the projected area of surface dA_2, $dA_2 \cdot \cos\theta_2$. (θ_2 is the angle between the normal to surface 2 and the position vector, R.) Then find the solid angle, Ω, which encompasses this area.



Substituting into the heat flow equation above:

$$dq = \frac{I \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 dA_2}{R^2}$$

To obtain the entire heat transferred from a finite area, dA_1, to a finite area, dA_2, we integrate over both surfaces:

$$q_{1\to 2} = \int_{A_2}\int_{A_1} \frac{I \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 dA_2}{R^2}$$

To express the total energy emitted from surface 1, we recall the relation between emissive power, E, and intensity, I.

$$q_{emitted} = E_1 \cdot A_1 = \pi \cdot I_1 \cdot A_1$$

View Factors-Integral Method

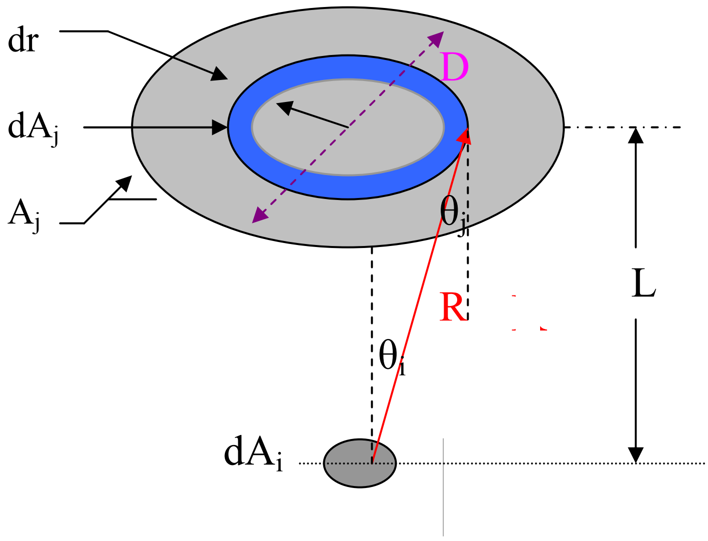
Define the view factor, $F_{1\text{-}2}$, as the fraction of energy emitted from surface 1, which directly strikes surface 2.

$$F_{1\to 2} = \frac{q_{1\to 2}}{q_{emitted}} = \frac{\int_{A_2}\int_{A_1} \frac{I \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 dA_2}{R^2}}{\pi \cdot I \cdot A_1}$$

after algebraic simplification this becomes:

$$F_{1\to 2} = \frac{1}{A_1} \cdot \int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cdot \cos\theta_2 \cdot dA_1 \cdot dA_2}{\pi \cdot R^2}$$

Example Consider a diffuse circular disk of diameter D and area A_j and a plane diffuse surface of area $A_i << A_j$. The surfaces are parallel, and A_i is located at a distance L from the center of A_j. Obtain an expression for the view factor F_{ij}.



The view factor may be obtained from:

$$F_{1\to 2} = \frac{1}{A_1}\cdot\int_{A_2}\int_{A_1}\frac{\cos\theta_1\cdot\cos\theta_2\cdot dA_1\cdot dA_2}{\pi\cdot R^2}$$

Since dA_i is a differential area

$$F_{1\to 2} = \int_{A_1}\frac{\cos\theta_1\cdot\cos\theta_2\cdot dA_1}{\pi\cdot R^2}$$

Substituting for the cosines and the differential area:

$$F_{1\to 2} = \int_{A_1}\frac{\left(L/R\right)^2\cdot 2\pi\cdot r\cdot dr}{\pi\cdot R^2}$$

After simplifying:

$$F_{1\to 2} = \int_{A_1}\frac{L^2\cdot 2\cdot r\cdot dr}{R^4}$$

Let $\rho^2 \equiv L^2 + r^2 = R^2$. Then $2\cdot\rho\cdot d\rho = 2\cdot r\cdot dr$.

$$F_{1\to 2} = \int_{A_1}\frac{L^2\cdot 2\cdot\rho\cdot d\rho}{\rho^4}$$

After integrating,

$$F_{1\to 2} = -2\cdot L^2\cdot\left.\frac{\rho^{-2}}{2}\right|_{A_2} = -L^2\cdot\left[\frac{1}{L^2+\rho^2}\right]_0^{D/2}$$

Substituting the upper & lower limits

$$F_{1\to2} = -L^2 \cdot \left[\frac{4}{4 \cdot L^2 + D^2} - \frac{1}{L^2} \right]_0^{D/2} = \frac{D^2}{4 \cdot L^2 + D^2}$$

This is but one example of how the view factor may be evaluated using the integral method. The approach used here is conceptually quite straight forward; evaluating the integrals and algebraically simplifying the resulting equations can be quite lengthy.

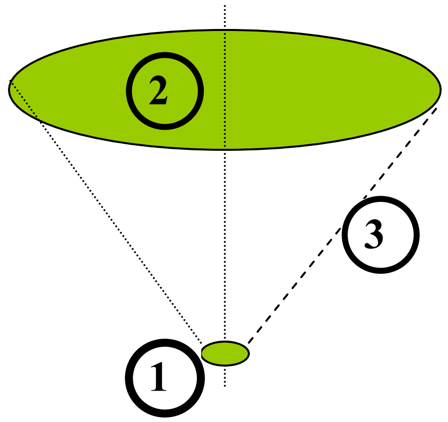
Enclosures

In order that we might apply conservation of energy to the radiation process, we must account for all energy leaving a surface. We imagine that the surrounding surfaces act as an enclosure about the heat source which receive all emitted energy. Should there be an opening in this enclosure through which energy might be lost, we place an imaginary surface across this opening to intercept this portion of the emitted energy. For an N surfaced enclosure, we can then see that:

$$\sum_{j=1}^{N} F_{i,j} = 1$$

This relationship is known as the "Conservation Rule".

Example: Consider the previous problem of a small disk radiating to a larger disk placed directly above at a distance L.



The view factor was shown to be given by the relationship:

$$F_{1\to2} = \frac{D^2}{4 \cdot L^2 + D^2}$$

Here, in order to provide an enclosure, we will define an imaginary surface 3, a truncated cone intersecting circles 1 and 2.

From our conservation rule we have:

$$\sum_{j=1}^{N} F_{i,j} = F_{1,1} + F_{1,2} + F_{1,3}$$

Since surface 1 is not convex $F_{1,1} = 0$. Then:

$$F_{1\to 3} = 1 - \frac{D^2}{4 \cdot L^2 + D^2}$$

Reciprocity

We may write the view factor from surface i to surface j as:

$$A_i \cdot F_{i\to j} = \int_{A_j}\int_{A_i} \frac{\cos\theta_i \cdot \cos\theta_j \cdot dA_i \cdot dA_j}{\pi \cdot R^2}$$

Similarly, between surfaces j and i:

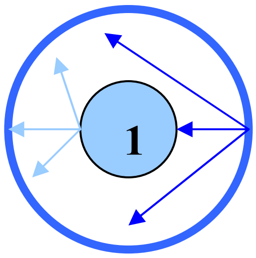
$$A_j \cdot F_{j\to i} = \int_{A_j}\int_{A_i} \frac{\cos\theta_j \cdot \cos\theta_i \cdot dA_j \cdot dA_i}{\pi \cdot R^2}$$

Comparing the integrals we see that they are identical so that:

$$A_i \cdot F_{i\to j} = A_j \cdot F_{j\to i}$$

This relationship is known as "Reciprocity".

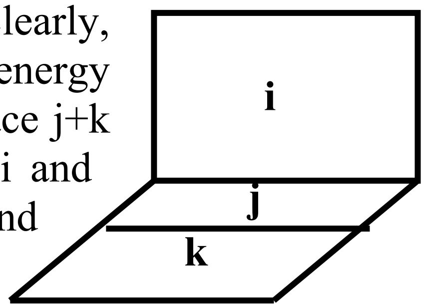
Example: Consider two concentric spheres shown to the right. All radiation leaving the outside of surface 1 will strike surface 2. Part of the radiant energy leaving the inside surface of object 2 will strike surface 1, part will return to surface 2. To find the fraction of energy leaving surface 2 which strikes surface 1, we apply reciprocity:



$$A_2 \cdot F_{2,1} = A_1 \cdot F_{1,2} \Rightarrow F_{2,1} = \frac{A_1}{A_2} \cdot F_{1,2} = \frac{A_1}{A_2} = \frac{D_1}{D_2}$$

Associative Rule

Consider the set of surfaces shown to the right: Clearly, from conservation of energy, the fraction of energy leaving surface i and striking the combined surface j+k will equal the fraction of energy emitted from i and striking j plus the fraction leaving surface i and striking k.



$$F_{i \Rightarrow (j+k)} = F_{i \Rightarrow j} + F_{i \Rightarrow k}$$

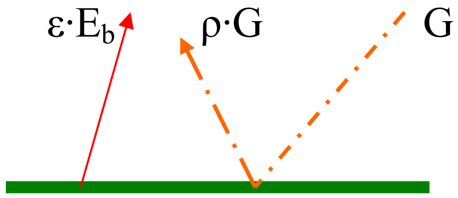
This relationship is known as the "Associative Rule".

Radiosity

We have developed the concept of intensity, I, which let to the concept of the view factor. We have discussed various methods of finding view factors. There remains one additional concept to introduce before we can consider the solution of radiation problems.

Radiosity, J, is defined as the total energy leaving a surface per unit area and per unit time. This may initially sound much like the definition of emissive power, but the sketch below will help to clarify the concept.

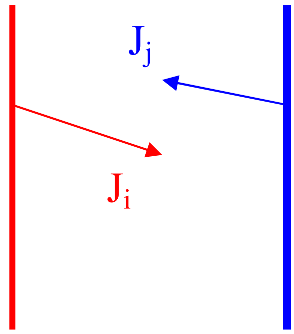
$$J \equiv \varepsilon \cdot E_b + \rho \cdot G$$



Net Exchange Between Surfaces

Consider the two surfaces shown. Radiation will travel from surface i to surface j and will also travel from j to i.

$$q_{i \rightarrow j} = J_i \cdot A_i \cdot F_{i \rightarrow j}$$



likewise,

$$q_{j \rightarrow i} = J_j \cdot A_j \cdot F_{j \rightarrow j}$$

The net heat transfer is then:

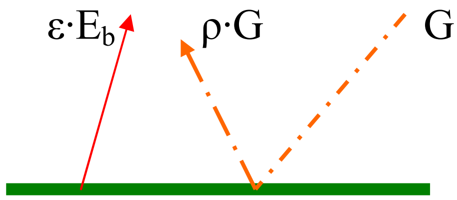
$$q_{j \rightarrow i\ (net)} = J_i \cdot A_i \cdot F_{i \rightarrow j} - J_j \cdot A_j \cdot F_{j \rightarrow j}$$

From reciprocity we note that $F_{1 \rightarrow 2} \cdot A_1 = F_{2 \rightarrow 1} \cdot A_2$ so that

$$q_{j \rightarrow i\ (net)} = J_i \cdot A_i \cdot F_{i \rightarrow j} - J_j \cdot A_i \cdot F_{i \rightarrow j} = A_i \cdot F_{i \rightarrow j} \cdot (J_i - J_j)$$

Net Energy Leaving a Surface

The net energy leaving a surface will be the difference between the energy leaving a surface and the energy received by a surface:



$$q_{1 \rightarrow} = [\varepsilon \cdot E_b - \alpha \cdot G] \cdot A_1$$

Combine this relationship with the definition of Radiosity to eliminate G.

$$J \equiv \varepsilon \cdot E_b + \rho \cdot G \Rightarrow G = [J - \varepsilon \cdot E_b]/\rho$$

$$q_{1\rightarrow} = \{\varepsilon \cdot E_b - \alpha \cdot [J - \varepsilon \cdot E_b]/\rho\} \cdot A_1$$

Assume opaque surfaces so that $\alpha + \rho = 1 \Rightarrow \rho = 1 - \alpha$, and substitute for ρ.

$$q_{1\rightarrow} = \{\varepsilon \cdot E_b - \alpha \cdot [J - \varepsilon \cdot E_b]/(1 - \alpha)\} \cdot A_1$$

Put the equation over a common denominator:

$$q_{1\rightarrow} = \left[\frac{(1-\alpha)\cdot\varepsilon\cdot E_b - \alpha\cdot J + \alpha\cdot\varepsilon\cdot E_b}{1-\alpha}\right]\cdot A_1 = \left[\frac{\varepsilon\cdot E_b - \alpha\cdot J}{1-\alpha}\right]\cdot A_1$$

If we assume that $\alpha = \varepsilon$ then the equation reduces to:

$$q_{1\rightarrow} = \left[\frac{\varepsilon\cdot E_b - \varepsilon\cdot J}{1-\varepsilon}\right]\cdot A_1 = \left[\frac{\varepsilon\cdot A_1}{1-\varepsilon}\right]\cdot (E_b - J)$$

Electrical Analogy for Radiation

We may develop an electrical analogy for radiation, similar to that produced for conduction. The two analogies should not be mixed: they have different dimensions on the potential differences, resistance and current flows.

	Equivalent Current	Equivalent Resistance	Potential Difference
Ohms Law	I	R	ΔV
Net Energy Leaving Surface	$q_{1\rightarrow}$	$\left[\frac{1-\varepsilon}{\varepsilon\cdot A}\right]$	$E_b - J$
Net Exchange Between Surfaces	$q_{i\rightarrow j}$	$\frac{1}{A_1\cdot F_{1\rightarrow 2}}$	$J_1 - J_2$

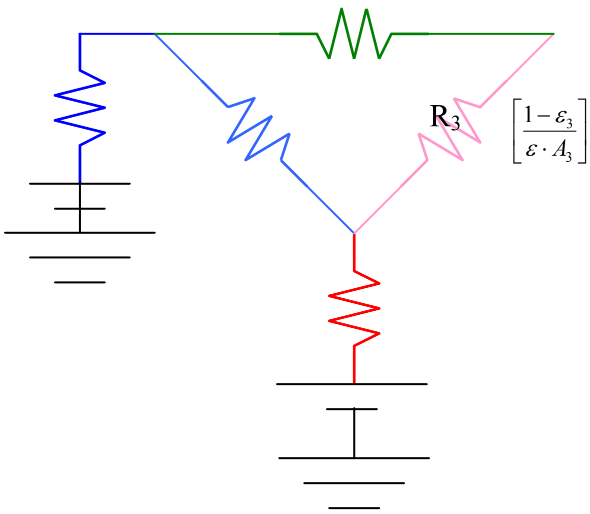
Alternate Procedure for Developing Networks

- Count the number of surfaces. (A surface must be at a “uniform” temperature and have uniform properties, i.e. ε, α, ρ.)
- Draw a radiosity node for each surface.
- Connect the Radiosity nodes using view factor resistances, $1/A_i \cdot F_{i\rightarrow j}$.
- Connect each Radiosity node to a grounded battery, through a surface resistance, $\left[1-\varepsilon / \varepsilon \cdot A\right]$.

This procedure should lead to exactly the same circuit as we obtain previously.

Simplifications to the Electrical Network

- Insulated surfaces. In steady state heat transfer, a surface cannot receive net energy if it is insulated. Because the energy cannot be stored by a surface in steady state, all energy must be re-radiated back into the enclosure. *Insulated surfaces are often termed as re-radiating surfaces.*



Electrically cannot flow through a battery if it is not grounded.

Surface 3 is not grounded so that the battery and surface resistance serve no purpose and are removed from the drawing.

- Black surfaces: A black, or ideal surface, will have no surface resistance:

$$\left[\frac{1-\varepsilon}{\varepsilon \cdot A}\right] = \left[\frac{1-1}{1 \cdot A}\right] = 0$$

In this case the nodal Radiosity and emissive power will be equal.

This result gives some insight into the physical meaning of a black surface. Ideal surfaces radiate at the maximum possible level. Non-black surfaces will have a reduced potential, somewhat like a battery with a corroded terminal. They therefore have a reduced potential to cause heat/current flow.

- Large surfaces: Surfaces having a large surface area will behave as black surfaces, irrespective of the actual surface properties:

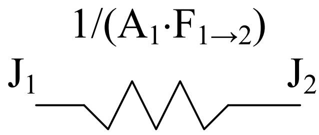
$$\left[\frac{1-\varepsilon}{\varepsilon \cdot A}\right] = \left[\frac{1-\varepsilon}{\varepsilon \cdot \infty}\right] = 0$$

Physically, this corresponds to the characteristic of large surfaces that as they reflect energy, there is very little chance that energy will strike the smaller surfaces; most of the energy is reflected back to another part of the same large surface. After several partial absorptions most of the energy received is absorbed.

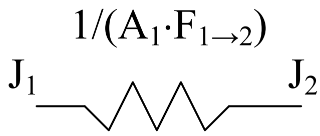
Solution of Analogous Electrical Circuits.

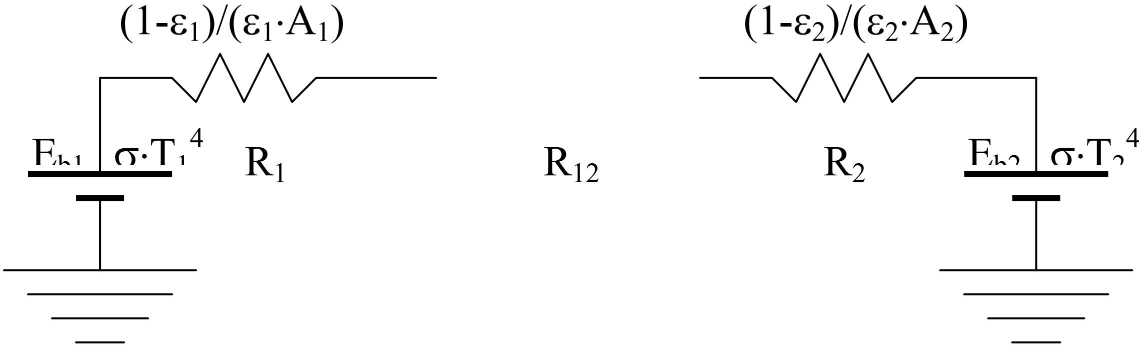
- Large Enclosures

Consider the case of an object, 1, placed inside a large enclosure, 2. The system will consist of two objects, so we proceed to construct a circuit with two radiosity nodes.

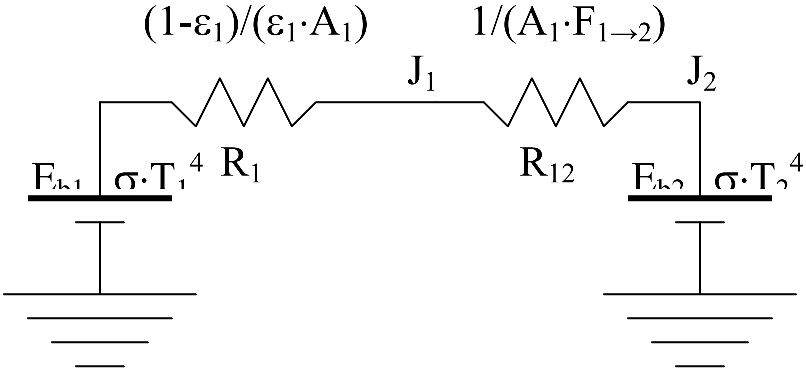


Now we ground both Radiosity nodes through a surface resistance.





Since A_2 is large, $R_2 = 0$. The view factor, $F_{1\to2} = 1$



Sum the series resistances:

$$R_{Series} = (1-\varepsilon_1)/(\varepsilon_1\cdot A_1) + 1/A_1 = 1/(\varepsilon_1\cdot A_1)$$

Ohm's law:

$$i = \Delta V/R$$

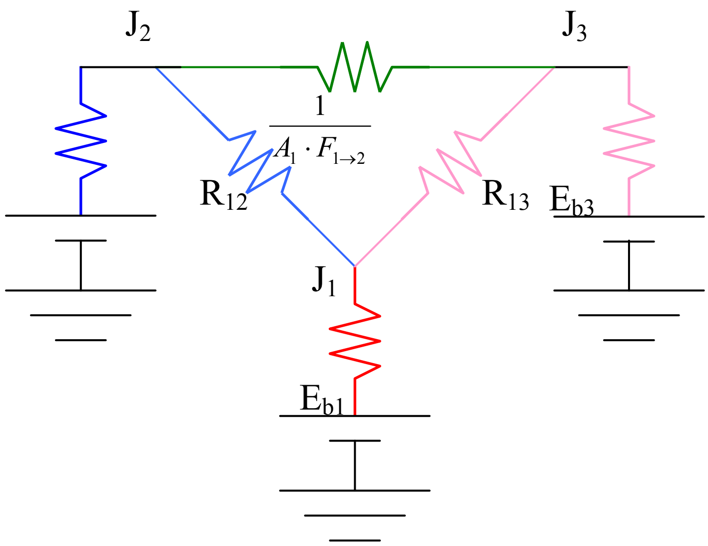
or by analogy:

$$q = \Delta E_b/R_{Series} = \varepsilon_1\cdot A_1\cdot\sigma\cdot(T_1^4 - T_2^4)$$

You may recall this result from Thermo I, where it was introduced to solve this type of radiation problem.

- Networks with Multiple Potentials

Systems with 3 or more grounded potentials will require a slightly different solution, but one which students have previously encountered in the Circuits course.



The procedure will be to apply Kirchoff's law to each of the Radiosity junctions.

$$\sum_{i=1}^{3} q_i = 0$$

In this example there are three junctions, so we will obtain three equations. This will allow us to solve for three unknowns.

Radiation problems will generally be presented on one of two ways:

- The surface net heat flow is given and the surface temperature is to be found.
- The surface temperature is given and the net heat flow is to be found.

Returning for a moment to the coal grate furnace, let us assume that we know (a) the total heat being produced by the coal bed, (b) the temperatures of the water walls and (c) the temperature of the super heater sections.

Apply Kirchoff's law about node 1, for the coal bed:

$$q_1 + q_{2\to1} + q_{3\to1} = q_1 + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$

Similarly, for node 2:

$$q_2 + q_{1\to 2} + q_{3\to 2} = \frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0$$

(Note how node 1, with a specified heat input, is handled differently than node 2, with a specified temperature.

And for node 3:

$$q_3 + q_{1\to 3} + q_{2\to 3} = \frac{E_{b3} - J_3}{R_3} + \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 0$$

The three equations must be solved simultaneously. Since they are each linear in J, matrix methods may be used:

$$\begin{bmatrix} -\frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{12}} & \frac{1}{R_{13}} \\ \frac{1}{R_{12}} & -\frac{1}{R_2} - \frac{1}{R_{12}} - \frac{1}{R_{13}} & \frac{1}{R_{23}} \\ \frac{1}{R_{13}} & \frac{1}{R_{23}} & -\frac{1}{R_3} - \frac{1}{R_{13}} - \frac{1}{R_{23}} \end{bmatrix} \cdot \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} -q_1 \\ -\frac{E_{b2}}{R_2} \\ -\frac{E_{b3}}{R_3} \end{bmatrix}$$

The matrix may be solved for the individual Radiosity. Once these are known, we return to the electrical analogy to find the temperature of surface 1, and the heat flows to surfaces 2 and 3.

Surface 1: Find the coal bed temperature, given the heat flow:

$$q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{\sigma \cdot T_1^4 - J_1}{R_1} \Rightarrow T_1 = \left[\frac{q_1 \cdot R_1 + J_1}{\sigma}\right]^{0.25}$$

Surface 2: Find the water wall heat input, given the water wall temperature:

$$q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{\sigma \cdot T_2^4 - J_2}{R_2}$$

Surface 3: (Similar to surface 2) Find the water wall heat input, given the water wall temperature:

$$q_3 = \frac{E_{b3} - J_3}{R_3} = \frac{\sigma \cdot T_3^4 - J_3}{R_3}$$