ENERGY TRANSFER ACROSS THE ROTOR

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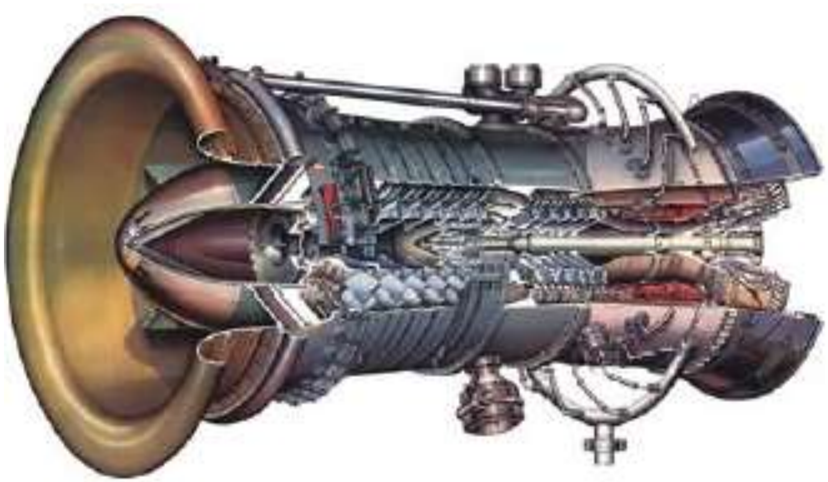
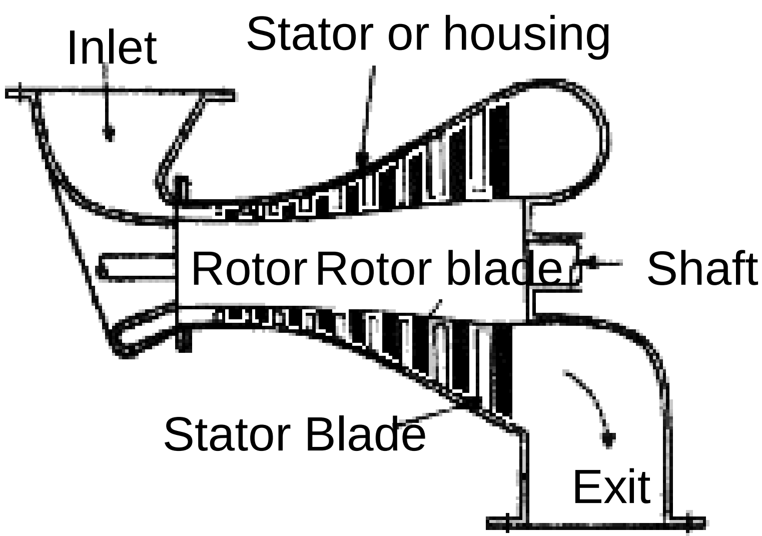


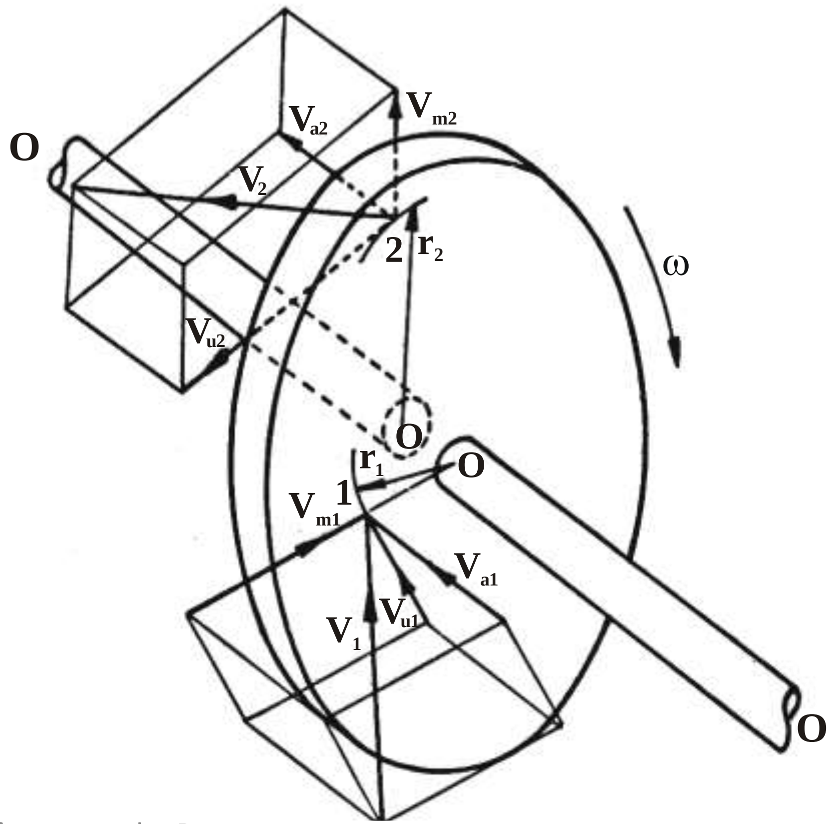


Euler's Turbine Equation

- The basic desired relationship of all turbo machine is only a form of **Newton's law of motion applied to a fluid traversing the rotor**.
- Use of "**Law of conservation of momentum**" applicable to a fluid element

$$F = \frac{d(mV)}{dt}$$

$$F = \frac{m\Delta V}{\Delta t} = \dot{m}\Delta V$$



We assume the following:

i. The flow is steady,

ii. The heat and work interactions between the rotor and its surroundings take place at a constant rate.

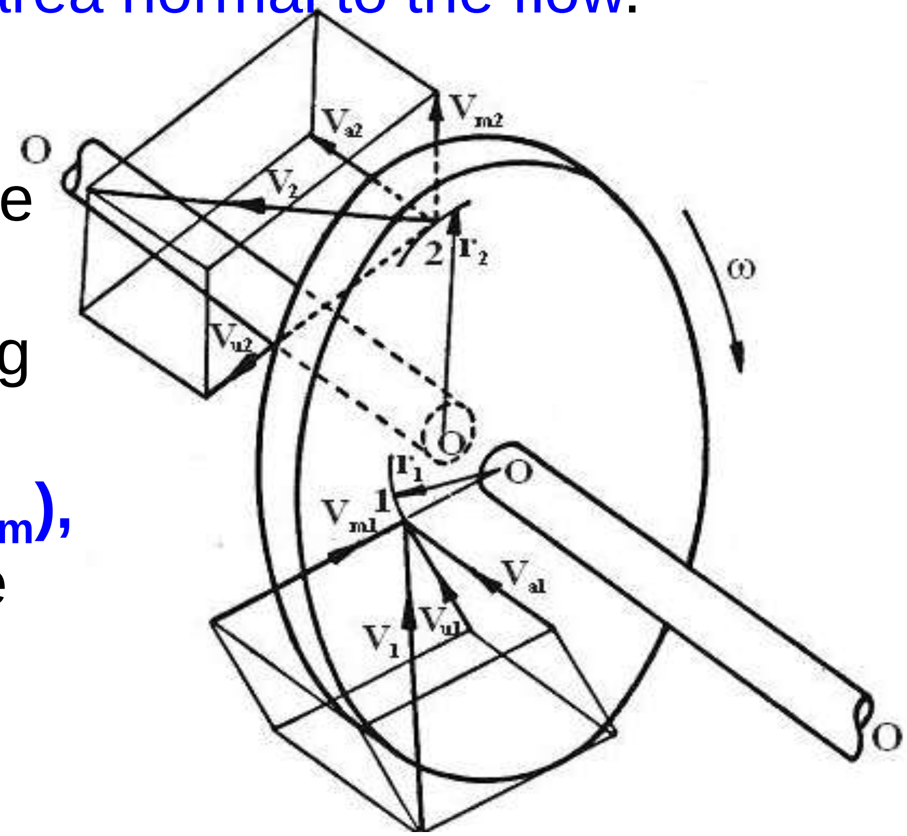
iii. Velocity is uniform over any area normal to the flow.

The **absolute velocity (V)** of the fluid can be resolved into :

a. Axial component (V_a) - along the axis of rotation.

b. Radial component (V_{rd} or V_m), which is perpendicular to the axis of rotation.

c. The tangential component (V_u),- along the tangential direction of the rotor



- The change in magnitude of axial velocity components give rise to a axial thrust, i.e, $\Delta V_{ax} \sim F_a$
- which must be taken up by the thrust bearings.

- The change in magnitude of radial velocity components give rise to a radial thrust, i.e, $\Delta V_{rd} \sim F_{rd}$
- which is to be taken up by the journal bearing

Neither of these forces cause no angular rotation nor has any effect on the torque exerted on the rotor.

- The only velocity component which changes the angular momentum of fluid is the tangential velocity component, i.e, $\Delta V_u \sim F_T$

By **Newton's law of motion**, the **rate of change of angular momentum is equal to the summation of all the applied forces on the rotor**.

Net force acting on the rotor according to Newton's law of motion is given by.

$$F = \frac{d(mV)}{dt}$$

$$F = \frac{M_1}{g_c t} V_{u1} - \frac{M_2}{g_c t} V_{u2}$$

$$\text{Now} \quad \frac{M_1}{t} = \frac{M_2}{t} = \dot{m} \qquad \text{[Since the flow is steady]}$$

$$F = \frac{\dot{m}}{g_c} (V_{u1} - V_{u2}) \qquad (2.1)$$

Applied torque, $T = F \times r$.

$$T = \frac{\dot{m}}{g_c} (V_{u1}\, r_1 - V_{u2}\, r_2) \qquad (2.2)$$

The rate of energy transfer, E_o (i.e., power) is the product of torque and angular velocity i.e., $E_o = \omega T$

$$E_o = \frac{\omega' \dot{m}}{g_c} (V_{u1} r_1 - V_{u2} r_2)$$

But $\omega \times r = U$, Tangential velocity of the rotor.

$$E_o = \frac{\dot{m}}{g_c} (U_1 V_{u1} - U_2 V_{u2}) \quad (2.3)$$

Hence the energy transfer per unit mass of fluid is

$$E = \frac{\dot{E}_o}{\dot{m}} = \text{Energy transfer in J/kg}$$

$$E = \frac{(U_1 V_{u1} - U_2 V_{u2})}{g_c} = \Delta h_o \quad (2.4)$$

The equations (2.2), (2.3) and (2.4) are the forms of general Euler's turbine equation or simply Euler's equation, and these form the basic equations for all kind of turbo machines, which may be Power Generating namely Turbines or Power Absorbing type viz Pumps, Compressors, Fans etc

Note :-

1. For Power Generating Type, the work done to be positive.

i.e., $\mathbf{U_1 V_{u1}} > \mathbf{U_2 V_{u2}}$ in eqns. (2.1) to (2.4).

$$E = gH = \frac{1}{g_c}(U_1 V_{u1} - U_2 V_{u2})$$

2. **For Power Absorbing turbomachines**, the work done to be negative.

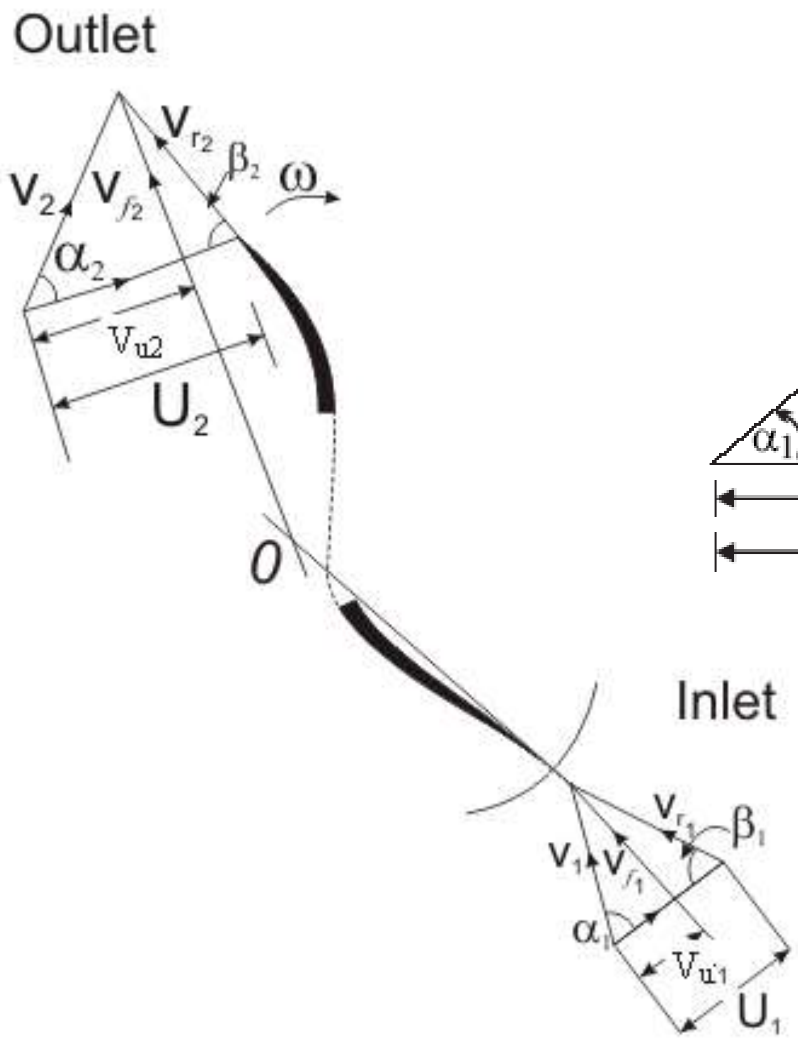
ie., $\mathbf{U_2 V_{u2}} > \mathbf{U_1 V_{u1}}$

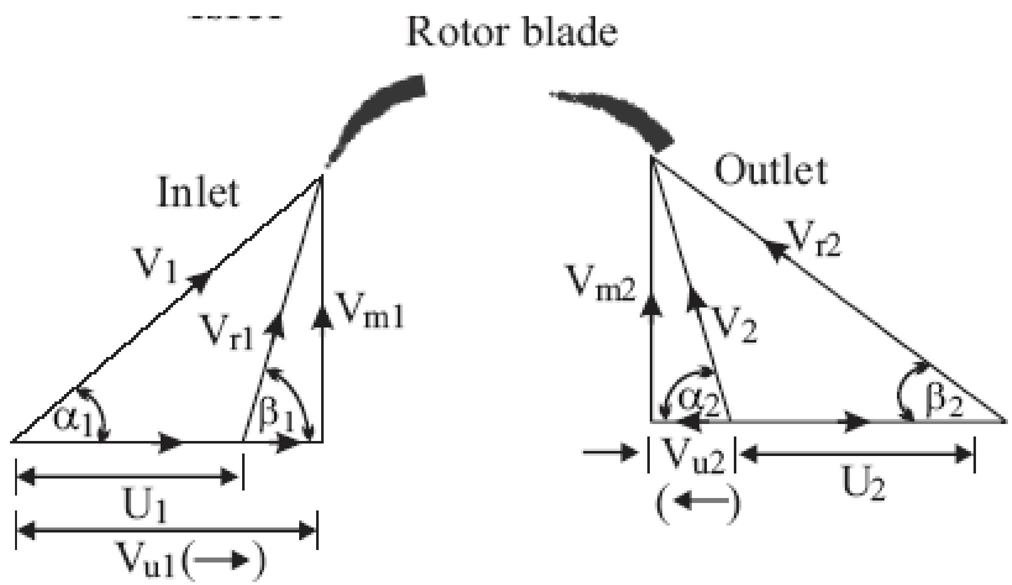
$$-E = \frac{U_2 V_{u2} - U_1 V_{u1}}{g_c}$$

For the simplicity, the energy transfer in compressors, pumps etc., is treated as

$$E = \frac{U_2 V_{u2} - U_1 V_{u1}}{g_c}$$

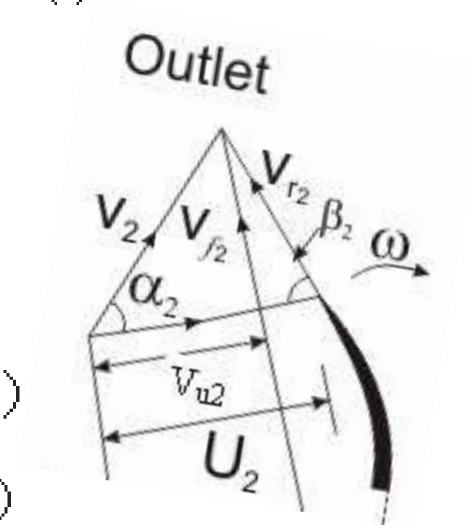
Components of Energy Transfer





Velocity Triangles at Inlet and Outlet of the Rotor

Consider the velocity triangle at outlet. From the geometry of Fig 2.2 (a)



$$V_{m2}^2 = V_2^2 - V_{u2}^2$$

and $$V_{m2}^2 = V_{r2}^2 - (U_2 - V_{u2})^2 = V_{r2}^2 - U_2^2 - V_{u2}^2 + 2U_2V_{u2}$$

$$\therefore \quad V_2^2 - V_{u2}^2 = V_{r2}^2 - U_2^2 - V_{u2}^2 + 2U_2V_{u2}$$

$$U_2V_{u2} = \frac{1}{2}(V_2^2 + U_2^2 - V_{r2}^2) \quad (2.5)$$

Similarly at inlet, $$U_1V_{u1} = \frac{1}{2}(V_1^2 + U_1^2 - V_{r1}^2) \quad (2.6)$$

By Substituting equation (2.5) and (2.6) in the equation (2.4), we get the Euler's equation as

$$E = \frac{1}{2g_c}\left[V_1^2 + U_1^2 - V_{r1}^2 - V_2^2 - U_2^2 + V_{r2}^2\right]$$

For power generating type

$$E = \frac{1}{2g_c}[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)] \quad (2.7)$$

For power absorbing type

$$E = \frac{1}{2g_c}[(V_2^2 - V_1^2) + (U_2^2 - U_1^2) + (V_{r1}^2 - V_{r2}^2)] \quad (2.8)$$

For power generating type

$$E = \frac{1}{2g_c}[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)]$$

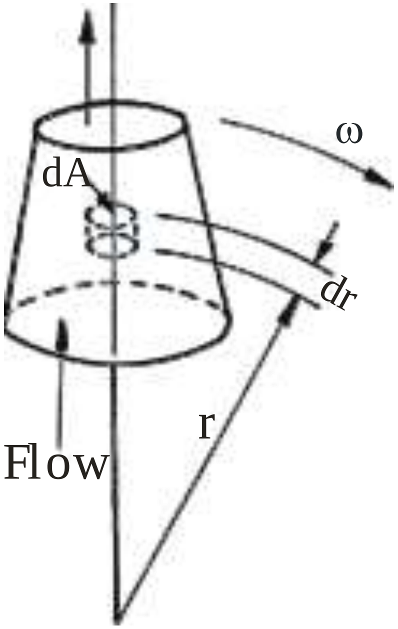
The pair of terms in each bracket, indicates the nature of energy transfer and their relative values which are useful to estimate the performance of the machine

- First term $(V_1^2 - V_2^2)/2g_c$ represent the **change in absolute kinetic energy of the fluid** during its passage through the machine. This is also called **Dynamic Energy**
- Second term $(U_1^2 - U_2^2)/2g_c$ represent the **change in fluid energy due to the movement of rotation of fluid from one radius to another**, i.e., **Centrifugal Energy**. This is also called **Static Energy**
- Third term, $(V_{r2}^2 - V_{r1}^2)/2g_c$ represents a **kinetic energy change due to relative velocity change**. This will result in a change of static head or pressure within the rotor itself. It is also called **Reactive Component** and is also a **Static Energy**.

The term $(U_1^2-U_2^2)/2g_c$ represent the **change in fluid energy due to the movement of rotation of fluid from one radius to another**, i.e., **Centrifugal energy**. This can be understood in better as follows:

$$\text{centrifugal force} \quad dF_c = \frac{dm}{g_c}\omega^2 r.$$

$$= \frac{dA\, dr\, \rho\, \omega^2 r}{g_c}$$



Pressure force on elemental strip dA is

$$dF_p = dp\ dA$$

At equilibrium condition,

$$dF_p = dF_c$$

$$dp\ dA = \frac{dA\ dr\ \rho\,\omega^2 r}{g_c}$$

$$\frac{dp}{\rho} = \frac{\omega^2 r\, dr}{g_c}$$

- For a reversible flow (flow without friction) between two points, say, 1 and 2, the **work done per unit mass of the fluid** (i.e. the flow work) can be written as

$$\int_1^2 \frac{dp}{\rho} = \int_1^2 \omega^2 \, r \, dr = \frac{\omega^2 r_2^2 - \omega^2 r_1^2}{2} = \frac{U_2^2 - U_1^2}{2}$$

$$\therefore \text{ Static enthalpy change,} \quad \Delta h_s = \frac{\Delta p}{\rho} = (h_2 - h_1) = \frac{\omega^2 (r_2^2 - r_1^2)}{2g_c} = \frac{U_2^2 - U_1^2}{2g_c}$$

- Thus **the energy transfer/unit mass of the fluid due to the transfer or movement of the fluid from one radius (r_1) to another radius (r_2)** is equal to **$(U_1^2 - U_2^2)/2g_c$**, which is the **energy transfer by a centrifugal effect**.

- The transfer of energy due to a **change in centrifugal head $(U_1^2 - U_2^2)/2g$**, causes a change in the **static head of the fluid**

- The third term $(V_{r2}^2 - V_{r1}^2)/2g_c$ represents a **change in the static Energy due to a change in fluid velocity relative to the rotor**.
- Regarding the effect of flow area on fluid velocity relative to the rotor **(V_r)**, a **converging passage in the direction of flow through the rotor increases the relative velocity, i.e., $V_{r2} > V_{r1}$** and hence **decreases the static pressure**. This usually happens in case of **Turbines**.
- Similarly, a **diverging passage** in the direction of flow through the rotor **decreases the relative velocity i.e., $V_{r2} < V_{r1}$**. and **increases the static pressure** as occurs in case of **Pumps and Compressors**

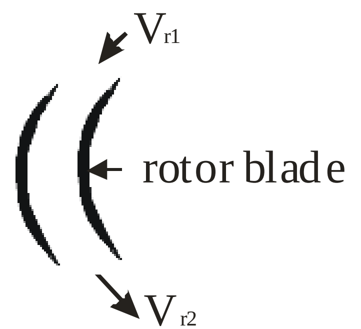


Fig 2.4 Relative velocity at inlet & outlet of rotor blade

Hence the **second** and **third terms** of Eqs. (2.7) & (2.8) correspond to a **change in Static Head/Energy/Enthalpy**

In General, the energy equation is

$$E = gH = \frac{1}{g_c}\left(U_1 V_{u1} \pm U_2 V_{u2}\right)$$

Note :- Depending upon the directions & magnitudes of V_1 and $V_{2,}$ it is possible to have V_{u1} and V_{u2} of the same sign or opposite signs.

- **If V_{u1} and V_{u2} are in opposite directions**, then the quantities **(U_1V_{u1}) and (U_2 V_{u2}) must be just add-up** in the evaluation of energy or power.

  i.e., $$E = gH = \frac{1}{g_c}\left(U_1 V_{u1} + U_2 V_{u2}\right)$$

- **If V_{u1} and V_{u2} are in same direction**, then the eqn. (2.3) & (2.4) are used as it is.

i.e., $$E = gH = \frac{1}{g_c}\left(U_1 V_{u1} - U_2 V_{u2}\right)$$

General Analysis of Turbines

Impulse and Reaction Machines, and Degree of Reaction (R)

- **Impulse type machines are those in which, only the kinetic energy is available at inlet of the machine for the energy transfer,**
- **i.e., the static pressure at the inlet is same as the static pressure at the outlet of the machine**
- **Degree of reaction, R=0**
- **Hence it signifies that $V_{r2} = V_{r1}$ for impulse machine**

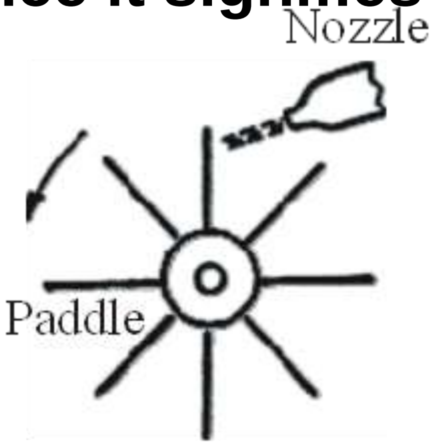
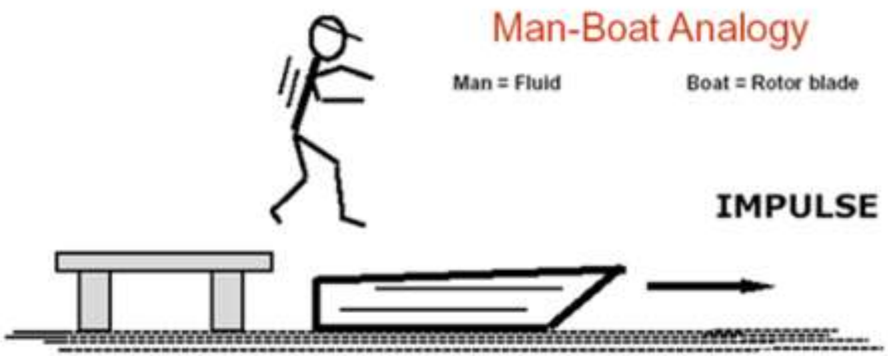
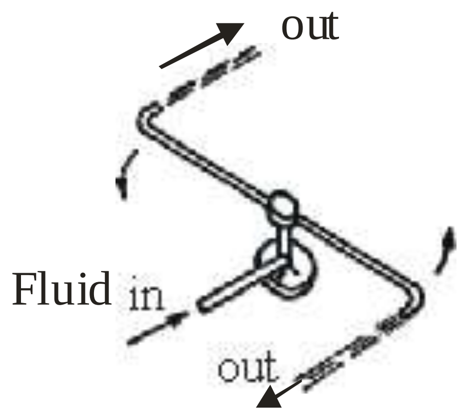


Fig. 2.5 Impulse type peddle wheel



- **Reaction machines** are those in which **both kinetic and pressure energies of the fluid at inlet are available.**
- **Hence in these case, V_{r2} is not equal to V_{r1}.**
- In these machines, $V_{r2} > V_{r1}$
- **Hence Pressure varies across the machine, i.e $p_e < p_i$**



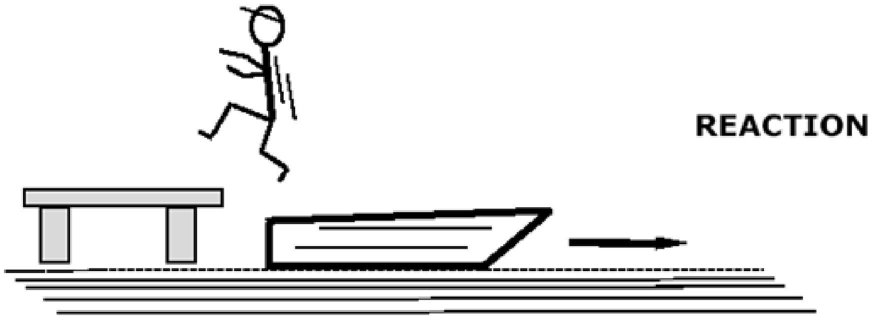


Fig. 2.6. Reaction turbine (Lawn sprinkler)

- **A machine with any degree of reaction must have an enclosed rotor so that the fluid cannot expand freely in all direction.**

Degree of Reaction (R)

- **It is defined as "*the ratio of static energy transfer due to the static pressure change to the total energy transfer due to the total pressure change in a rotor*".**

Mathematically,

$$R = \frac{\text{Static head}}{\text{Total head,}} = \frac{\text{Static enthalpy change in rotor}}{\text{Total enthalpy change in rotor}} = \frac{\Delta h_s}{\Delta h_0}$$

$$= \frac{\frac{1}{2g_c}\left[(U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]}{\frac{1}{2g_c}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]}$$

$$R = \frac{\left[(U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]}{\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]} = \frac{S}{D+S} \qquad (2.12)$$

$$S = \frac{R}{1-R} D \qquad (2.13)$$

- **The value of R may be any value like zero, +ve or -ve.**
- **Zero degree of reaction is the characteristics of Impulse machine i.e $V_{r1} = V_{r2}$**

Utilization Factor (Є):

- **Even for an ideal fluid, *all the energy supplied at inlet of any turbines cannot be converted into useful work due to finite exit velocity or loss of velocity at the machine outlet*, which being wasted without use.**

- **The ratio of ideal work transfer by the rotor to the energy supplied is called as the *diagram efficiency* or *utilization factor* (Є).**

$$\in = \frac{E_{utilised}}{E_{available}}$$

- The energy available to the rotor is the sum of absolute K.E at inlet and pressure energy due to change in relative velocities of the fluid and movement of the fluid

$$E_{avail} = \frac{1}{2g_c}\left[V_1^2 + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]$$

- The energy utilized by the rotor in the absence of fluid friction is

$$E_{utilised} = \frac{1}{2g_c}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]$$

- By the definition of utilization factor

$$\in = \frac{E_{utilised}}{E_{available}} = \frac{\frac{1}{2g_c}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]}{\frac{1}{2g_c}\left[V_1^2 + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]}$$

- It can also be written

$$\in = \frac{\frac{1}{2g_c}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]}{\frac{1}{2g_c}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right] + \frac{V_2^2}{2g_c}}$$

$$\text{Therefore, } \epsilon = \frac{E_{ideal}}{E_{ideal} + \frac{V_2^2}{2g_c}} = \frac{E_{ideal}}{E_{ideal} + Losses}$$

E_{ideal} is obtained by the Euler's turbine equation, i.e.,

$$\epsilon = \frac{E}{E + \frac{V_2^2}{2g_c}} = \frac{(U_1 V_{u1} - U_2 V_{u2})_{id}}{(U_1 V_{u1} - U_2 V_{u2})_{id} + \frac{V_2^2}{2g_c}}$$

- The velocity components or energy values in the above equation are based on the ideal velocity diagram
- In equation (2.19), **if the numerator refers to the actual energy transferred by the rotor** and **the denominator referrers to the ideal energy transfer**, then resulting parameter is known as **Vane Efficiency**.

Thus the **Vane Efficiency** is defined as the "**ratio of actual energy transfer to the ideal energy transfer across the rotor**".

$$\text{i.e., } \eta_{vane} = \frac{(U_1 V_{u1} - U_2 V_{u2})_{actual}}{(U_1 V_{u1} - U_2 V_{u2})_{ideal}} = \frac{W_{act}}{W_{ideal}}$$

The hydraulic efficiency η_h [which is the product of Є and η_{vane}] is defined *as the ratio of actual energy transferred by the rotor in the presence of fluid friction to the energy available to the rotor.*

$$\text{i.e., } \eta_h = \frac{\text{Actual energy output}}{\text{Hydrodynamic energy available from fluid}}$$

When the mechanical efficiency $\eta_m = 1$, then the hydraulic or adiabatic efficiency, η_h is same as the utilization factor $(\in)$

Relation between Degree of Reaction (R) and the Utilization Factor (Є)

General equation for degree of reaction (R) for any turbine is given by the equation (2.12). i.e.,

$$R = \frac{[(U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)]}{[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)]}$$

$$\Rightarrow (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2) = R/(1-R)(V_1^2 - V_2^2) \qquad (2.21)$$

Also, the utilization factor ($\in$), for any type of turbine is given by the equation (2.17), i.e.,

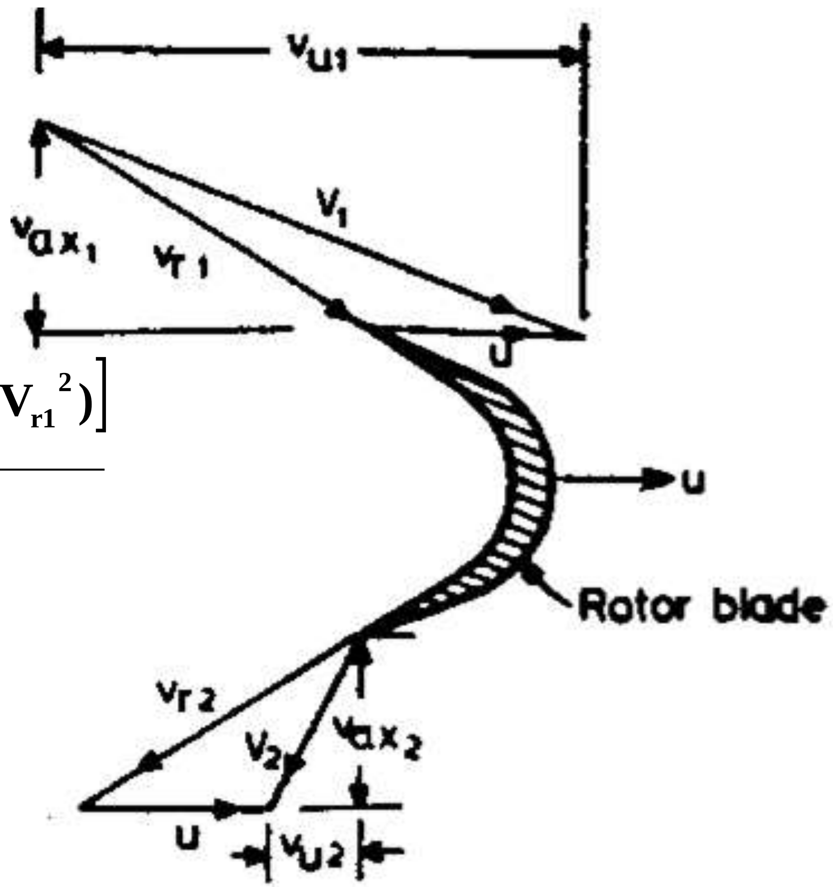
$$\in \; = \frac{[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)]}{[V_1^2 + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)]}$$

Using equation (2.21), the above equation can be expressed as,

$$\in \; = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2} \qquad (2.22)$$

Eqn.(2.22) is the general utilization factor irrespective of any type of turbines whether it is axial or radial type.

Axial Flow Turbines



For axial flow machines, $U_1 = U_2$

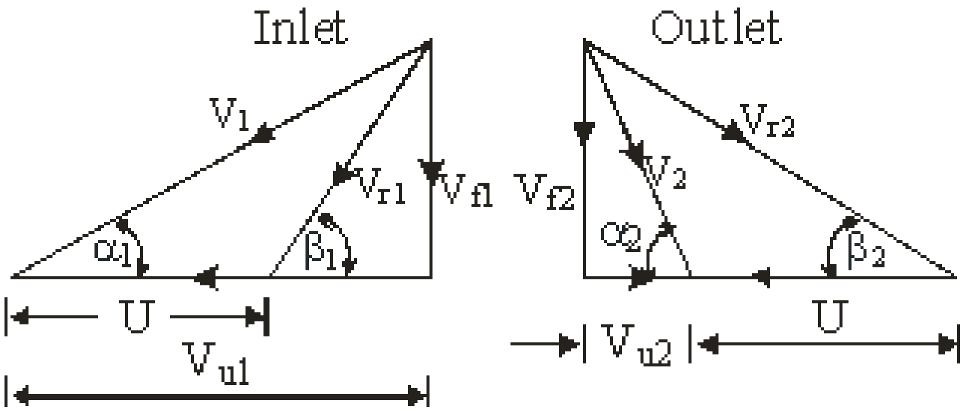
Degree of reaction (R) is

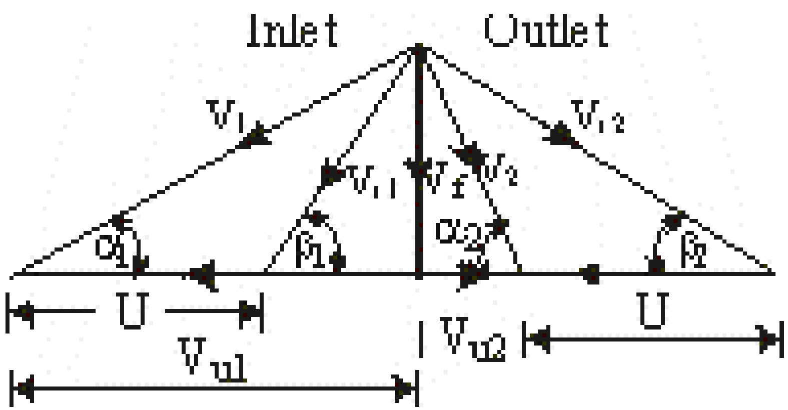
$$R = \frac{\frac{1}{2g_c}\left[(V_{r2}^2 - V_{r1}^2)\right]}{\frac{1}{2g_c}\left[(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)\right]} = \frac{\frac{1}{2g_c}\left[(V_{r2}^2 - V_{r1}^2)\right]}{E}$$

Energy transfer eqn. becomes,

$$E = \frac{1}{2g_c}\left[(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)\right]$$

- Values of R can be -ve, Zero or +ve depending on magnitudes of velocity components for different velocity triangles.
- Energy transfer E is always positive for Turbines





Combined Velocity Triangles

$$R = \frac{\left[(V_{r2}^{2} - V_{r1}^{2})\right]}{\left[(V_{1}^{2} - V_{2}^{2}) + (V_{r2}^{2} - V_{r1}^{2})\right]}$$

Velocity Diagrams for Different Values of R

(i) When R<0 (i.e., R is negative)

V_{r1} should be greater than V_{r2} , i.e., $V_{r1}>V_{r2}$.

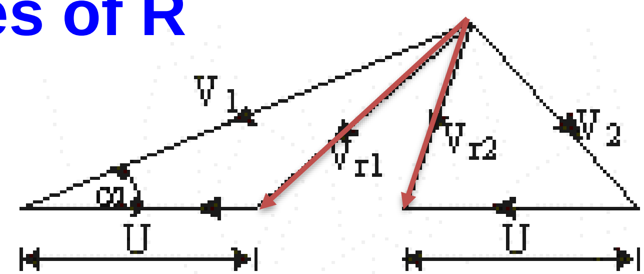


Fig. 2.7 (a). Velocity diagram for R<0

(ii) When R = 0 (i.e, Impulse type)

$V_{r1} = V_{r2}$, and hence $\beta_1 = \beta_2$

- **This is the characteristic of impulse turbine.**
- **This also implies that there is no static pressure change across the rotor.**
- **Energy transfer occurs purely due to the change in absolute kinetic energy.**

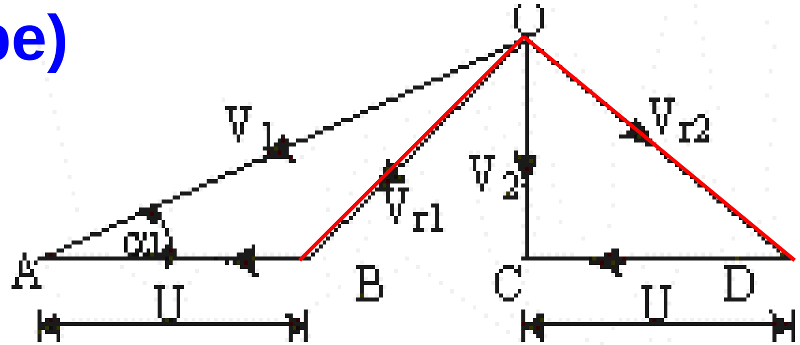


Fig. 2.7 (b). Velocity diagram for R=0

$$R = \frac{\left[(V_{r2}^2 - V_{r1}^2)\right]}{\left[(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)\right]}$$

(iii) When R = 0.5 (i.e., 50% Reaction)

- $V_1^2 - V_2^2 = V_{r2}^2 - V_{r1}^2$
- For symmetric Vel. Δle at I/L & O/L, i.e., $V_1 = V_{r2}$ and $V_2 = V_{r1}$.

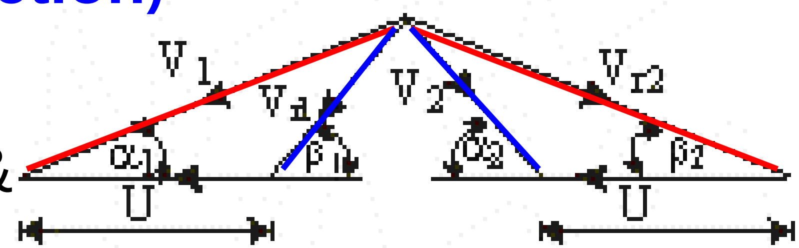


Fig. 2.7(c). Velocity diagram for R=0.5

- **Energy transfer occurs initially by impulse action and then by reaction.**

(iv) When R = 1 (Fully reaction)

When $V_1 = V_2$

Energy transfer (E) occurs purely due to change in relative K.E. of fluid

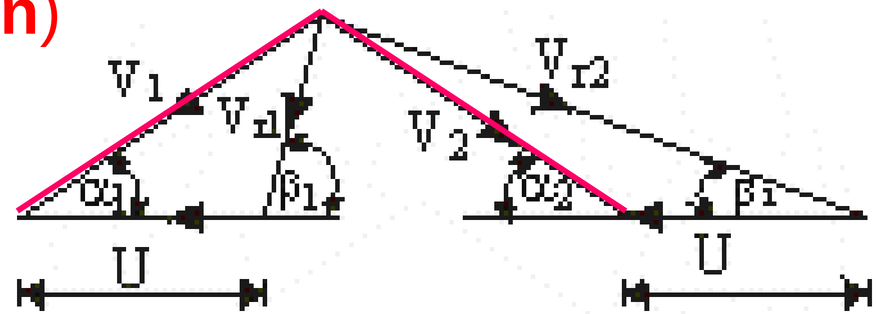


Fig. 2.7(d). Velocity diagram for R=1

$$R = \frac{\left[(V_{r2}^2 - V_{r1}^2)\right]}{\left[(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)\right]}$$

(v) When R > 1

When $V_2 > V_1$

Energy transfer (E) may be negative or positive.

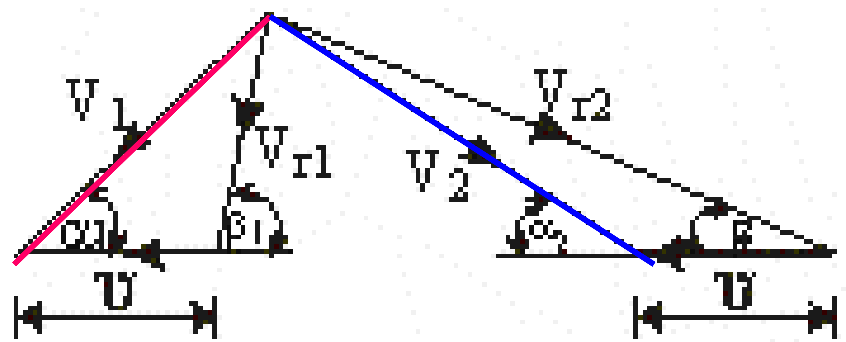
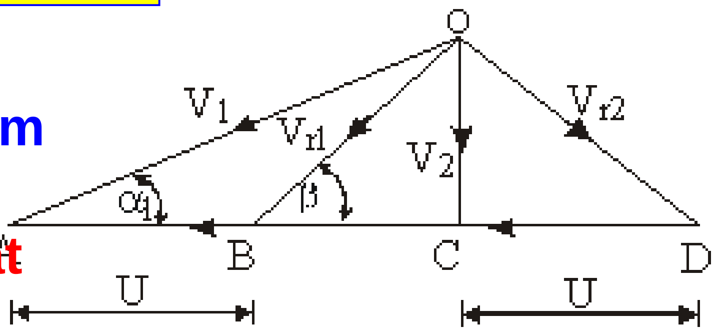


Fig. 2.7(e). Velocity diagram for R>1

Maximum Utilization Factor

- **Value of V_2 should be minimum**
- **It is apparent that V_2 is having minimum value when it is Axial or Radial.**
- **In other words, Whirl velocity is zero at exit, i.e. $V_{u2} = 0$,**



2.8(a). Velocity diagram for maximum utilization.

We have the utilization factor,

$$\in = \frac{(V_1^2 - V_2^2)}{V_1^2 - RV_2^2}$$

From Vel.Δ^{le} for $\in_{max}$, $V_2 = V_1 \sin\alpha_1$

$$\in_{max} = \frac{(V_1^2 - V_1^2 \sin^2\alpha_1)}{V_1^2 - RV_1^2 \sin^2\alpha_1}$$

$$\in_{max} = \frac{V_1^2 (1 - \sin^2\alpha_1)}{V_1^2 (1 - \sin^2\alpha_1)} = \frac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1}$$

- **Є is absolute maximum and equal to unity, when $\alpha_1 = 0$.**

Then **$V_2 = V_1 \sin\alpha_1 = V_1 \sin(0) = 0$** results in **zero - angle turbine**.

- This represents the ideal turbine, which is **impossible to attain**, because **even α_1 can be zero**, but **finite velocity V_2 with an axial component is necessary to provide steady flow**.
- However this shows that the **nozzle angle α_1** should be **as small as** possible.

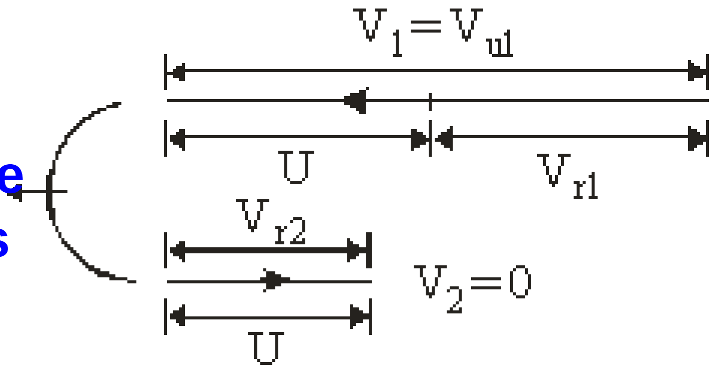


Fig. 2.8(b). Zero-angle turbine.

Condition For ϵ_{max} In Impulse Turbine

We have the

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - R V_2^2} = \frac{V_1^2 - V_1^2 \sin^2\alpha_1}{V_1^2 - R\,(V_1^2 \sin^2\alpha_1)}$$

$$\epsilon_{max} = \frac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1}$$

For $R = 0$, $\quad \epsilon_{max} = \cos^2\alpha_1$

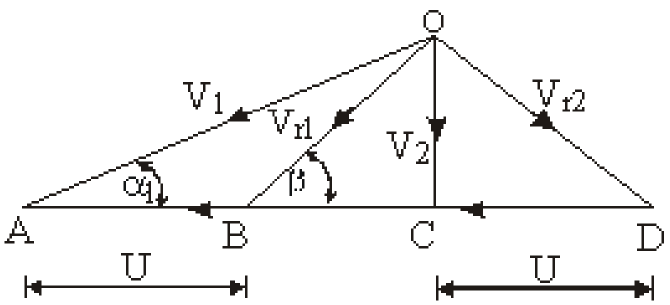


Fig. 2.8(a). Velocity diagram for maximum utilization.

From velocity triangles OBC and OCD are similar, hence BC should be equal to U.

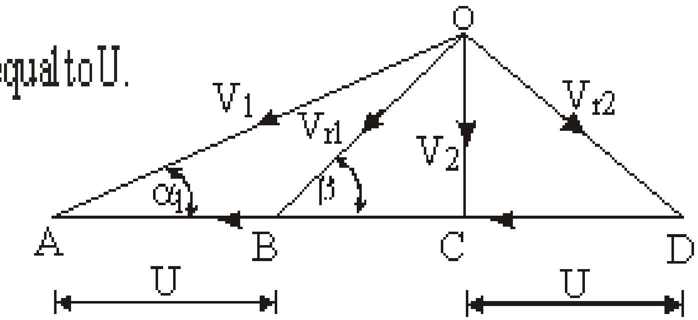


Fig. 2.8(a). Velocity diagram for maximum utilization.

$$\cos\alpha_1 = \frac{U+U}{V_1} = \frac{2U}{V_1} = 2\,\phi$$

Thus for $\in_{max}$, ϕ = Speed ratio = $\frac{\cos\alpha_1}{2}$

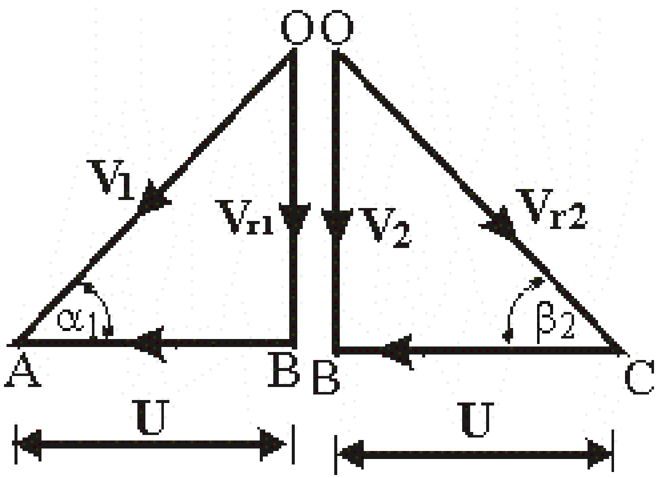
For zero-angle turbine, speed ratio, $\frac{U}{V_1} = \phi = \frac{1}{2}$

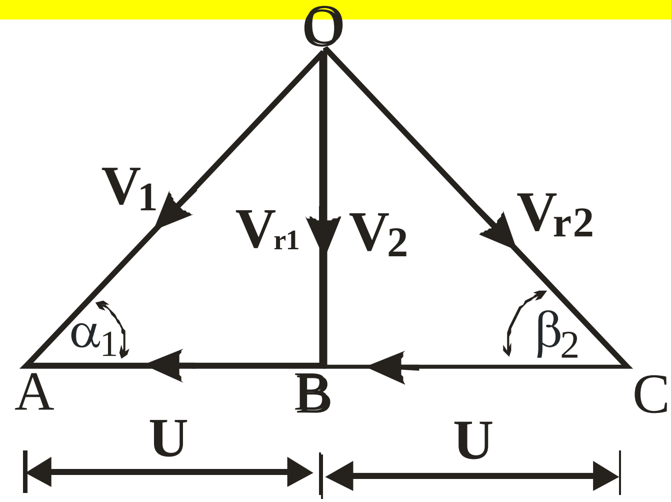
For impulse turbine, the nozzle angle should be small

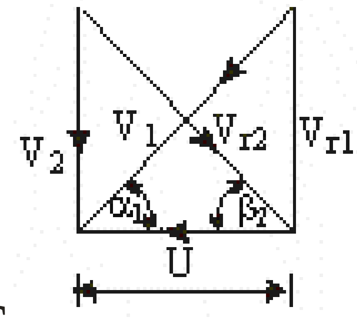
Ex : If $\alpha_1 = 20^\circ$, $\cos\alpha_1 = \cos 20^\circ = 0.94$ & speed ratio $\phi = \frac{U}{V_1} = 0.47$.

The nozzle angle usually between 15° - 20° is therefore employed. The factor $\frac{U}{V_1}$ is an useful parameter to judge the utilization factor.

Condition for Maximum Utilization in 50% Reaction Turbine







(ii) Common base

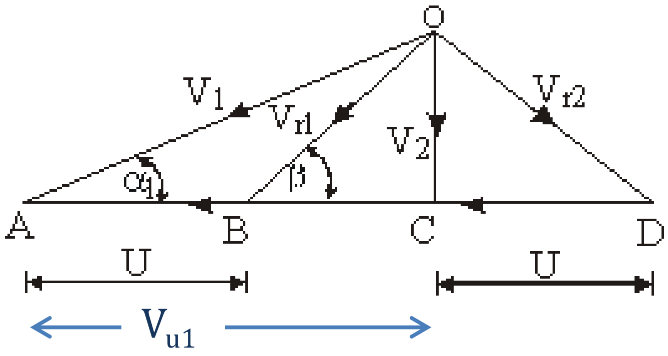
For 50% Reaction, we know that $V_1 = V_{r2}$ and $V_2 = V_{r1}$ (ie., $\alpha_1 = \beta_2$ & $\alpha_2 = \beta_1$) and for maximum utilization, V_2 must be axial. With this condition, the velocity triangles are drawn for maximum utilization as shown in Fig. 2.8(d)(i) &(ii).

$$\epsilon_{max} = \frac{1-\sin^2\alpha_1}{1-R\sin^2\alpha_1} = \frac{\cos^2\alpha_1}{1-R\sin^2\alpha_1} = \frac{\cos^2\alpha_1}{1-0.5\sin^2\alpha_1} = \frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$$

Also from triangle OAB, $\cos\alpha = \dfrac{AB}{OA} = \dfrac{U}{V_1} = \phi$:

For ϵ_{max}, Speed ratio $= \phi = \dfrac{U}{V_1} = \cos\alpha_1$

Comparison of Energy Transfer

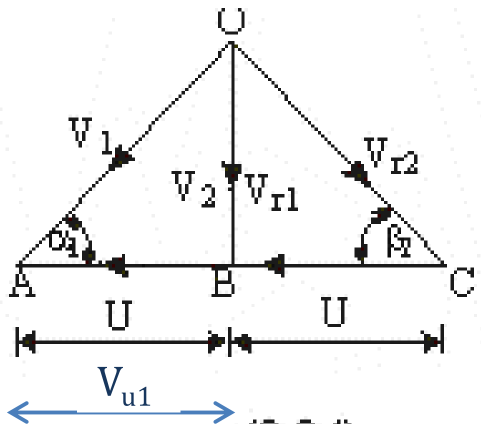


(a) When both have same blade speed,

Energy transfer by Impulse turbine is given by,

$$E_I = \frac{U_1 V_{u1} - U_2 V_{u2}}{g_c} = \frac{U_I (V_{u1} - V_{u2})}{g_c}$$

$$E_I = \frac{U_I V_{u1}}{g_c} = \frac{2U_I^2}{g_c} \quad (\because \; V_{u1} = 2U_I) \qquad (2.33)$$



Energy transfer by the 50% reaction turbine is given by

$$E_R = \frac{U_R V_{u1}}{g_c}$$

From Fig. 2.9.(b), $AB = V_{u1} = U_R$

$$E_R = \frac{U_R \times U_R}{g_c} = \frac{U_R^2}{g_c} \qquad (2.34)$$

By comparing the equation (2.33) and (2.34), it is clear that the **energy transfer per unit mass** of fluid in **Impulse turbine is twice that of 50% reaction turbine** for the same blade speed when the utilization is maximum.

(b) When both have same Energy transfer :

i.e., $E_R = E_I$

$$\frac{U_R^2}{g_c} = \frac{2U_I^2}{g_c} \quad \text{or} \quad U_R = \sqrt{2U_I^2} = 1.414\, U_I \qquad (2.35)$$

In equation (2.35) there will be no restriction on the absolute velocity V_1 and nozzle α_1 i.e., they may different in impulse and 50% Reaction turbine.

(c) When V_1 and α_1 are same in both the machines

Speed ratio for impulse stage for maximum utilization is,

$$\frac{U_I}{V_1} = \phi = \frac{\cos\alpha_1}{2} \quad \text{or} \quad 2U_I = V_1 \cos\alpha_1 \qquad (2.36a)$$

and speed ratio for 50% Reaction stage for maximum utilization is,

$$\frac{U_R}{V_1} = \phi = \cos\alpha_1 \quad \text{or} \quad U_R = V_1 \cos\alpha_1 \qquad (2.36b)$$

By comparing above two equation, we can write,

$$U_I = \frac{V_1 \cos\alpha}{2} = \frac{U_R}{2}$$

$$\text{Hence,} \quad U_R = 2U_I \qquad (2.37)$$

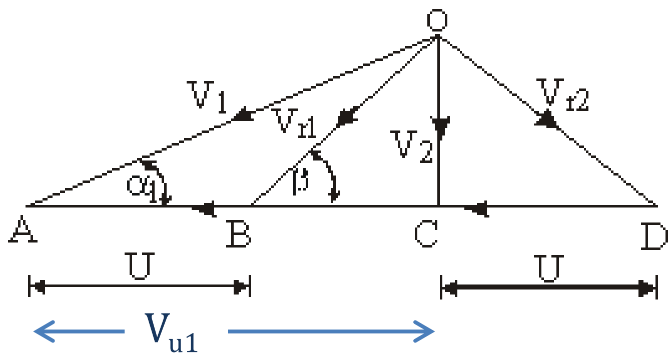
If both the Impulse and 50% Reaction turbine have the same nozzle angle (α_1) and the absolute velocity V_1, then for maximum utilization, the rotational speed (U) for 50% Reaction turbine should be double that of Impulse turbine.

Comparison of Impulse & 50% Reaction Turbine for ϵ_{max}

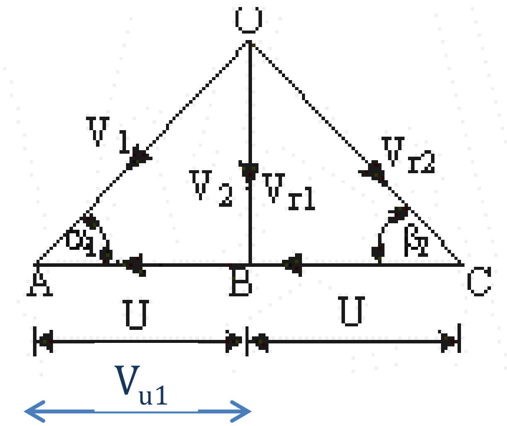
Impulse Turbine | **50% Reaction Turbine**

Vel. Δ^{le}

Impulse Turbine:



50% Reaction Turbine:



	Impulse Turbine	50% Reaction Turbine
Energy Transfer	$E_I = 2U_I^2/g_c$	$E_R = U_R^2/g_c$
Speed Ratio	$\Phi = U/V_1 = \cos\alpha_1/2$	$\Phi = U/V_1 = \cos\alpha_1$
ϵ_{max}	$\epsilon_{max} = \cos^2\alpha_1$	$\epsilon_{max} = 2\cos^2\alpha_1/(1+\cos^2\alpha_1)$

Key Points & Hints To Draw the Velocity Triangles for the Given Condition and Other Aspects Related for Solving Problems

1.If the m/c is of axial flow type, $U_1 = U_2 = U = \frac{\pi D N}{60}$ m/s.

2. If the m/c is of impulse type, $R = 0$
   Then $V_{r1} = V_{r2}$ or $\beta_1 = \beta_2$ for ideal case.

   In real case $V_{r2} < V_{r1}$ due to blade friction and $\beta_1 = \beta_2$ for equiangular rotor blades even for R=0

3. If the m/c is of 50% Reaction,
   The necessary condition is $V_1^2 - V_2^2 = V_{r2}^2 - V_{r1}^2$.

   For Symmetric Vel. Δ^{les} at I/L & O/L,
   $V_1 = V_{r2}$ and $V_2 = V_{r1}$. or $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$

4. For radial flow machines, the fluid enters at radius r_1 and leaves at radius r_2, then $U_1 \neq U_2$

5. For Inward Flow m/cs,

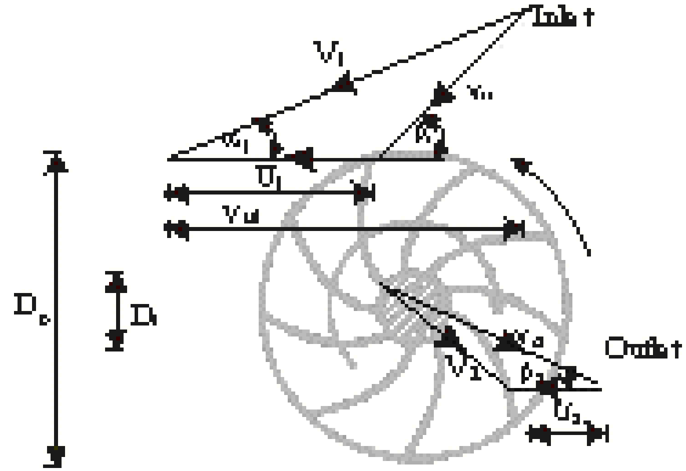


Fig. 2.18. Inward Flow Turbomachine

$U_1 \alpha D_o$

$U_2 \alpha D_i$

6. For Outward Flow m/cs

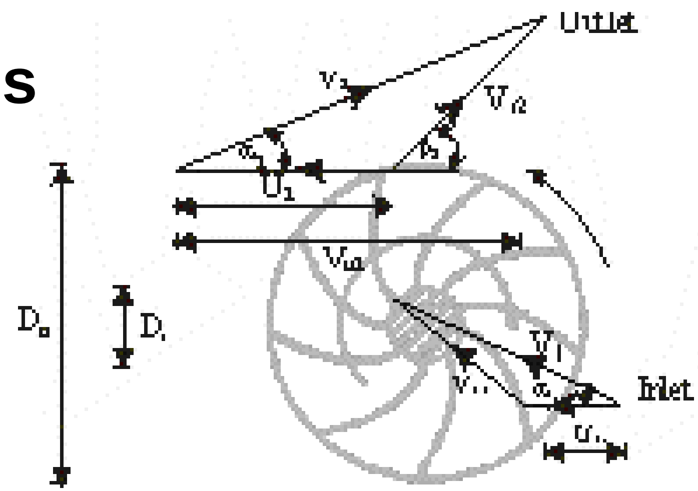
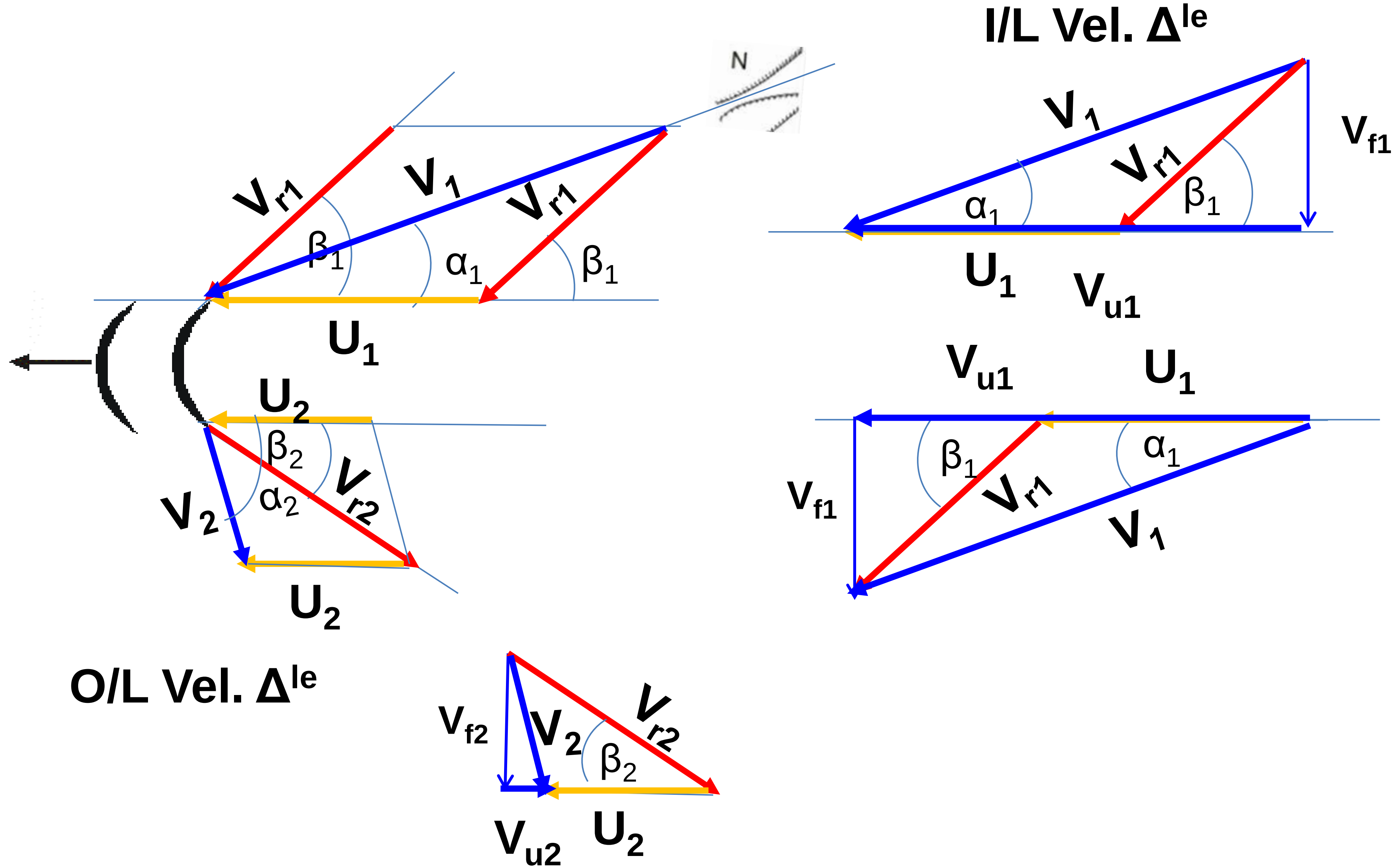


Fig. 2.19. Outward Flow Turbomachine

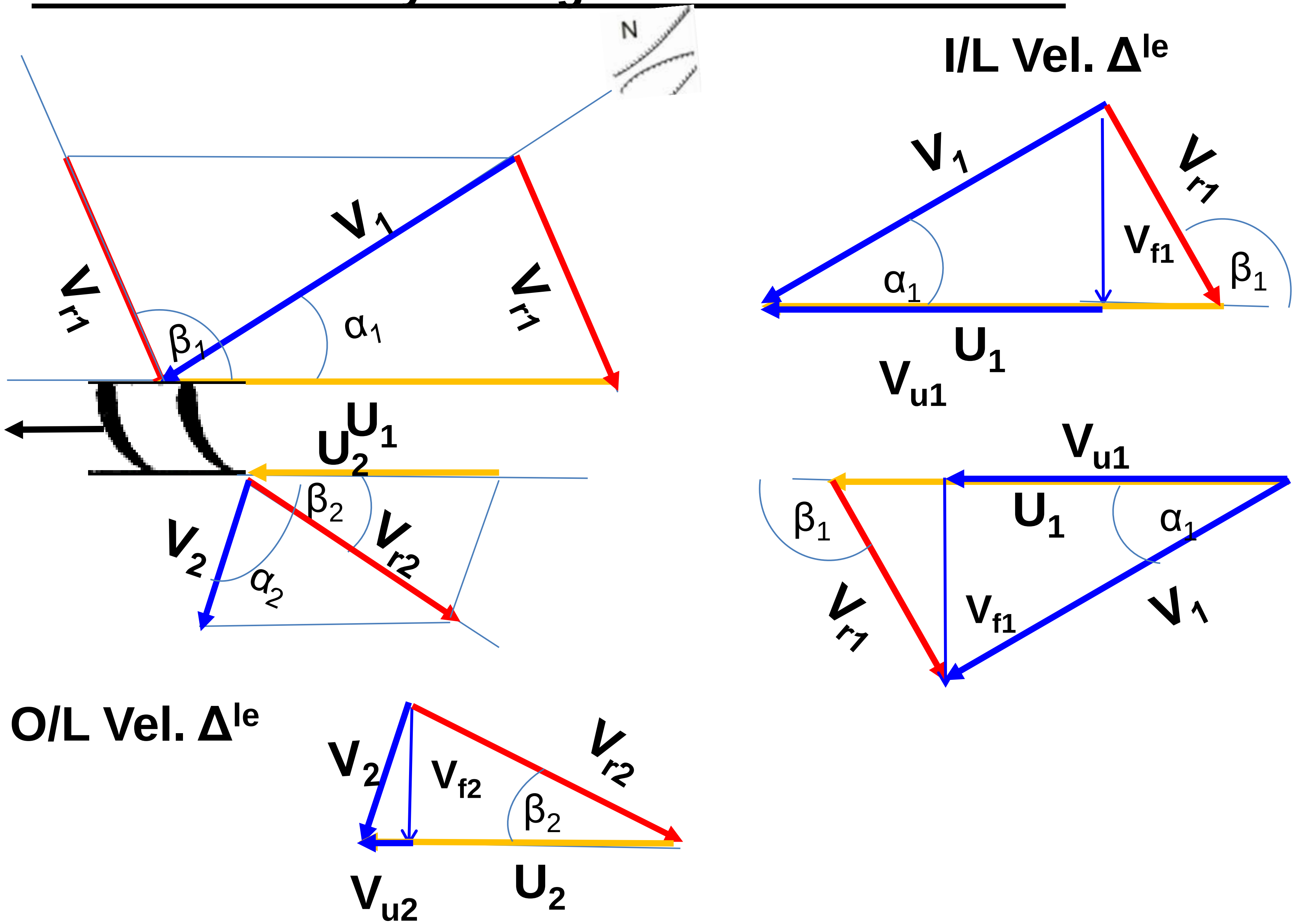
$U_1 \alpha D_i$

$U_2 \alpha D_o$

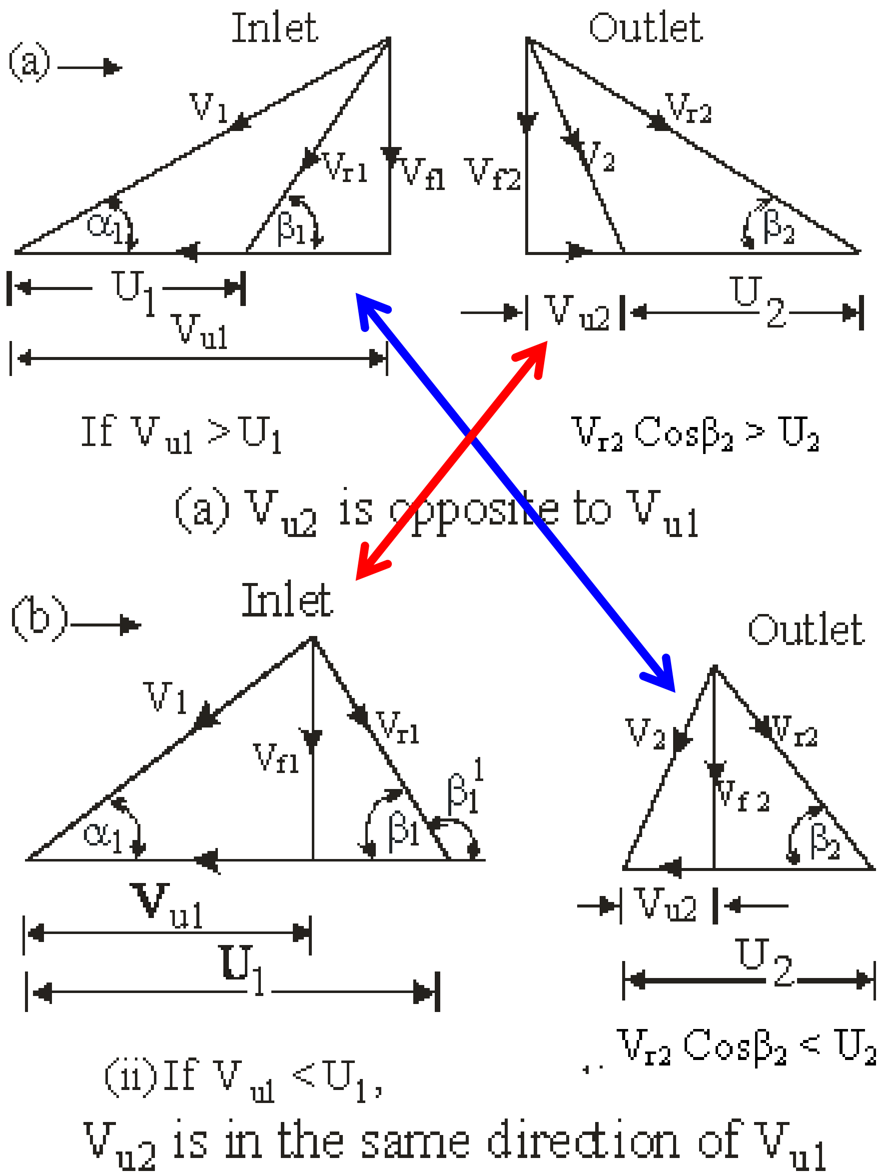
Possible Velocity Triangles at inlet and outlet



Possible Velocity Triangles at inlet and outlet



Possible Velocity Triangles at inlet and outlet



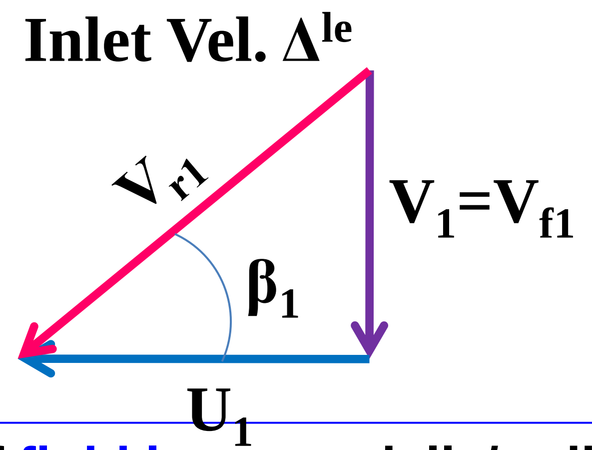
If V_{u1} and V_{u2} are of opposite directions, then.,

$$E = gH = \frac{U_1 V_{u1} + U_2 V_{u2}}{g_c}$$

If V_{u1} and V_{u2} are of the same directions, then

$$E = gH = \frac{U_1 V_{u1} - U_2 V_{u2}}{g_c}$$

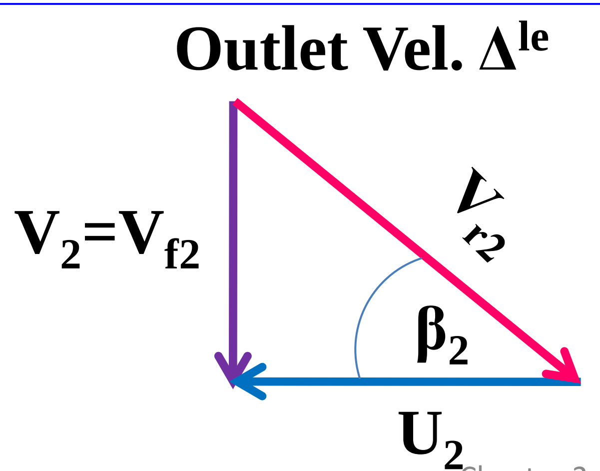
- If the fluid enters axially / radially in case of axial / radial flow machines respectively,
- Then $V_1 = V_{f1}$, $\alpha_1 = 90^0$ or $V_{u1}=0$. Then the I/L Vel. Δ^{le} becomes



Energy transfer , $E = U_2 V_{u2}/g_c$

Ex. Power absorbing m/c,

- If fluid leaves axially/radially in an axial/radial flow type machines,
- Then $V_2 = V_{f2}$, $\alpha_2 = 90^0$ & $V_{u2} = 0$. Then the O/L Vel. Δ^{le} becomes

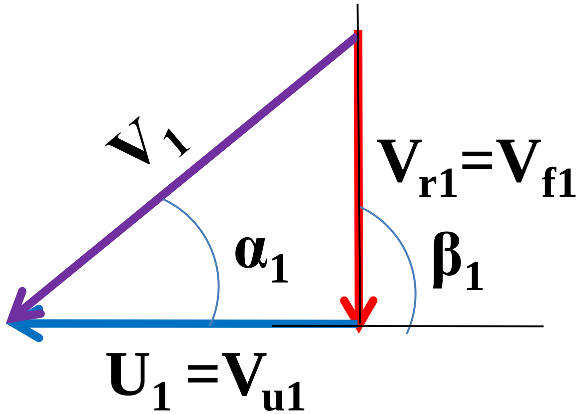


Energy transfer , $E = U_1 V_{u1}/g_c$

Ex. Power generating m/c,

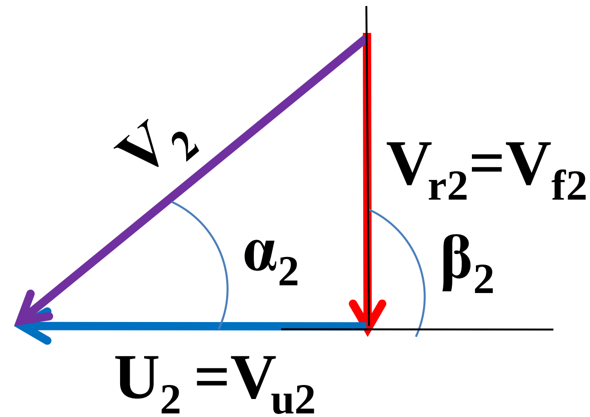
- If the blades are radial at inlet, ie., $\beta_1 = 90^0$, then $V_{r1} = V_{f1}$, & velocity triangle at inlet is

Inlet Vel. Δ^{le}



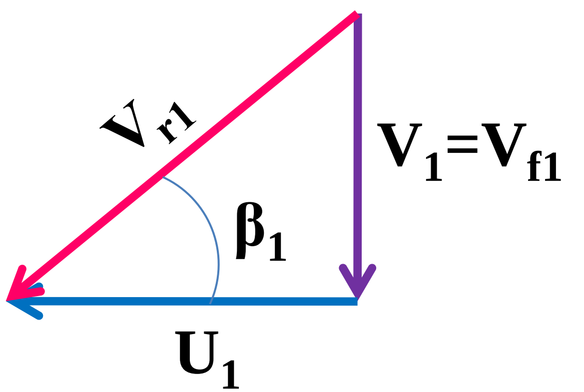
- If the blades are radial at outlet, ie., $\beta_2 = 90^0$, then $V_{r2} = V_{f2}$, & velocity triangle at outlet is

Outlet Vel. Δ^{le}

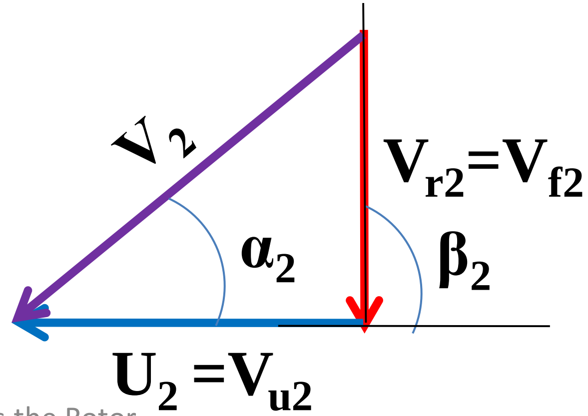


- If the fluid enters axially/radially at inlet and the blades are radial at outlet, then the velocity triangles

Inlet Vel. Δ^{le}

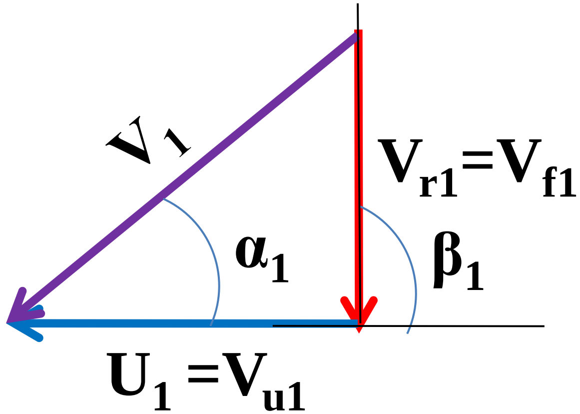


Outlet Vel. Δ^{le}

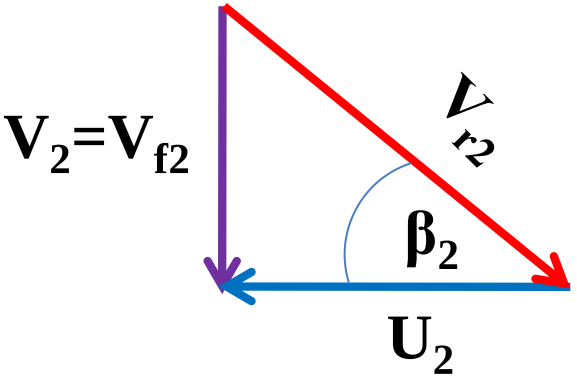


- If the **blades are radial at Inlet** and the **fluid leaves axially/radially**, then the velocity triangles becomes

Inlet Vel. Δ^{le}



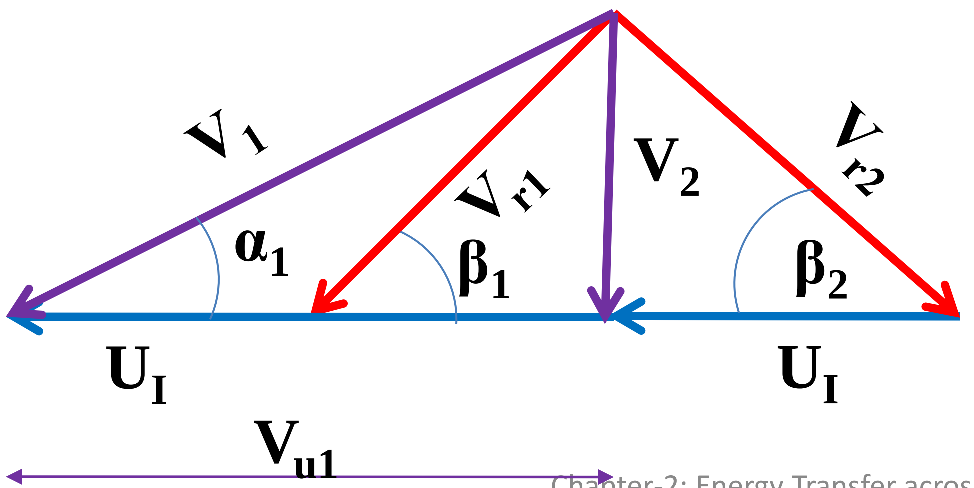
Outlet Vel. Δ^{le}



- For maximum Utilization, **V_2 must be axial/radial**, ie., $\alpha_2=90^0$ & $V_{u2}=0$

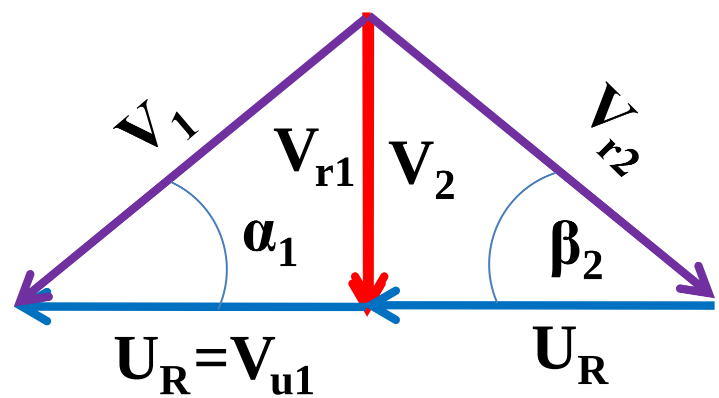
Impulse Turbine

Combined Vel. Δ^{le}



50% Reaction Turbine

Combined Vel. Δ^{le}



Important Formulae for Turbines

- **Tangential force,** $$F_T = \frac{\dot{m}}{g_c}\left(V_{u1} \pm V_{u2}\right) = \frac{\dot{m}}{g_c}\Delta V_u$$
- **Axial or Radial thrust** $$F_a \text{ or } F_{rd} = \frac{\dot{m}}{g_c}\left(V_{f1} - V_{f2}\right) = \frac{\dot{m}}{g_c}\Delta V_a$$
- **Torque Exerted,** $$T = \frac{\dot{m}}{g_c}(V_{u1}r_1 \pm V_{u2}r_2)$$
- **Energy Transfer,**

$$E = gH = W = \frac{U_1V_{u1} \pm U_2V_{u2}}{g_c}$$

$$= \frac{1}{2g_c}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)\right]$$

$$E = W = \frac{(V_1^2 - V_2^2)}{2(1-R)}$$

- **Power output,** $$P = \dot{m}E = \dot{m}W = \dot{m}\,gH_e = \dot{m}\,\Delta h$$

Important Formulae for Turbines

- **Degree of Reaction,**

$$R = \frac{[(U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)]}{[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)]} = \frac{E - \left(V_1^2 - V_2^2\right)/2}{E}$$

- **Utilization factor,**

$$\in = \frac{E_{utiilsed}}{E_{Avail}} = \frac{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)}{V_1^2 + (U_1^2 - U_2^2) - (V_{r1}^2 - V_{r2}^2)}$$

$$\in = \frac{E}{E + V_2^2/2} = \frac{E}{E + Losses}$$

$$\in = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

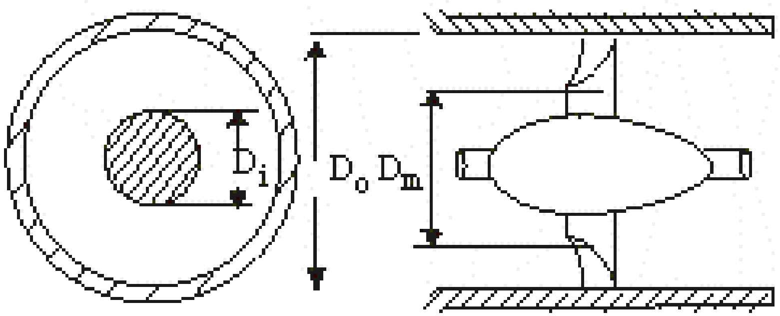
Flow Area

(a) **Axial flow machine** :

$$A_f = \pi (D_0^2 - D_i^2)/4$$

Where D_0 & D_i = Tip & hub dia. of the rotor

Then the flow, $Q = A_f V_f$

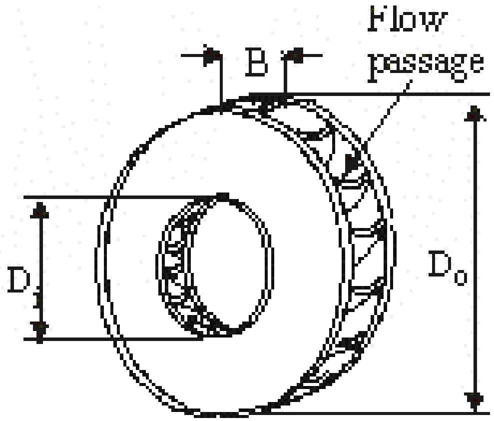


(b) Radial flow machine:

$$A_{f1} = \pi D_i B_1 \ \& \ A_{f2} = \pi D_0 B_2$$

Where D_0 & D_i =Tip & hub dia. of the impeller and B_1&B_2= Width of the impeller at I/L & O/L respectively

$$Q = \pi D_i B_1 V_{f1} = \pi D_0 B_2 V_{f2}.$$



Power Absorbing Turbomachines

- Compressors, blowers, fans and pumps are of power absorbing type of turbo machines
- In these m/cs, the static pressure of fluid increases from inlet to outlet due to the mechanical energy input.
- These machines can also be classified as axial, radial and mixed flow type depending on the direction of flow
- The quantity of interest is to the evaluate the stagnation enthalpy rise of fluid when flows through machine
- The turning angle of fluid in case of compressors is very small up to 20^{O} due to separation of fluid, but it was about 150^{O} to 170^{O} in case of turbines for a given amount of energy transfer .
- As a result, small values of V_{u1} & V_{u2} and hence it requires more number of stages for a particular pressure rise.
- Each compressor stage generally consisting of a rotor and stator,
- There will be a diffuser at the exit to recover the part of the exit kinetic energy of the fluid and to produce an increase in static pressure.

Axial Flow Compressors, Blowers, Pumps

Energy Transfer

Enthalpy rise/kg of fluid is

$$\Delta h_0 = \frac{U}{g_c}\left(V_{u2} - V_{u1}\right) \qquad (2.42)$$

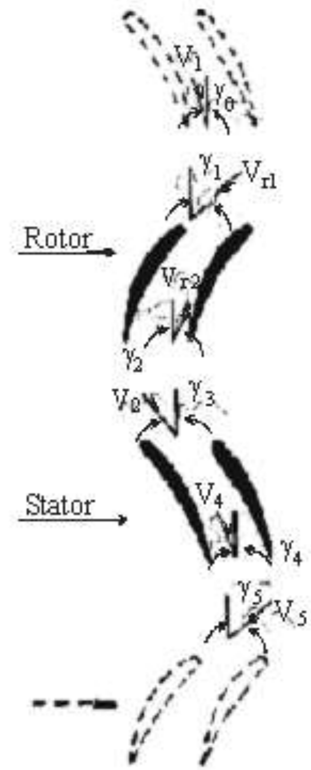
consider a Vel. Δ^{le} OAC,

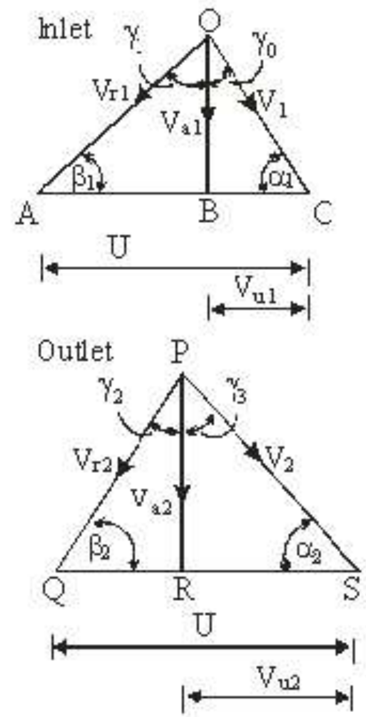
$$U = AB + BC = V_a \tan\gamma_1 + V_a \tan\gamma_0$$

$$U = V_a\left(\tan\gamma_1 + \tan\gamma_0\right) \qquad (2.43)$$

& from Vel. Δ^{le} PQS,

$$U = QR + RS = V_a\left(\tan\gamma_2 + \tan\gamma_3\right) \qquad (2.44)$$





$$U = V_a\left(\tan\gamma_1 + \tan\gamma_0\right) = V_a\left(\tan\gamma_2 + \tan\gamma_3\right)$$

$$\tan\gamma_0 + \tan\gamma_1 = \tan\gamma_2 + \tan\gamma_3$$

$$\tan\gamma_1 - \tan\gamma_2 = \tan\gamma_3 - \tan\gamma_0 \qquad (2.45)$$

Also, $V_{u1} = \tan\gamma_0\, V_a$ & $V_{u2} = V_a \tan\gamma_3$

In eqn. (2.42), $\Delta h_0 = \frac{U}{g_c} V_a (\tan\gamma_3 - \tan\gamma_0)$ (2.46)

Where γ_0 & γ_3 are the angles made by the absolute inlet and outlet velocities w.r.t to be axial direction and more direct information is obtained if the energy transfer is expressed in terms of blade angles γ_1 & γ_2 (sometimes they are called as air angles).

Eqn. for Energy expression in terms of blade angles with axial direction is

$$E = \frac{U V_a}{g_c} (\tan\gamma_1 - \tan\gamma_2) \qquad (2.47)$$

In terms of blade angles β_1 & β_2 (which are w.r.t tangential speed), we can also express the energy equation as

$$\Delta h_0 = E = \frac{U\ V_a}{g_c} (\cot\beta_1 - \cot\beta_2) \qquad [\because \tan\gamma_1 = \tan(90-\beta_1) = \cot\beta_1]$$

$$= \frac{U\ V_a}{g_c}\left(\frac{1}{\tan\beta_1} - \frac{1}{\tan\beta_2}\right)$$

$$\Delta h_0 = \frac{UV_a}{g_c}\left(\frac{\tan\beta_2 - \tan\beta_1}{\tan\beta_1 \tan\beta_2}\right) \qquad (2.48)$$

Change in (V_{u2}-V_{u1}) is only a fraction of U, i.e., about 10% to 15% of U

Degree of Reaction (R)

The static head in the rotor is simply due to change of relative K.E.,

$$\text{static head} = \frac{1}{2g_c}\left(V_{r1}^2 - V_{r2}^2\right)$$

The degree of reaction,

$$R = \frac{\text{Static enthalpy rise in rotor}}{\text{Total enthalpy rise across the stage}} = \frac{\frac{1}{2g_c}\left(V_{r1}^2 - V_{r2}^2\right)}{\Delta h_0} \qquad (2.49)$$

From triangle OAB of Fig. 2.13,

$$OA^2 = AB^2 + OB^2$$
$$= OB^2 + OB^2 \tan^2\gamma_1$$

$$\frac{AB}{OB} = \tan\gamma_1$$

$$AB = OB\tan\gamma_1$$

$$V_{r1}^2 = V_a^2 + V_a^2\tan^2\gamma_1$$

$$V_{r1}^2 - V_{r2}^2 = \left(V_a^2 + V_a^2\tan^2\gamma_1\right) - \left(V_a^2 + V_a^2\tan^2\gamma_2\right)$$

$$V_{r1}^2 - V_{r2}^2 = V_a^2\left(\tan^2\gamma_1 - \tan^2\gamma_2\right)$$

Also, from eqn. (2.47),

$$\Delta h_0 = \frac{UV_a}{g_c}\left(\tan\gamma_1 - \tan\gamma_2\right)$$

$$R = \frac{\frac{1}{2g_c}\left(V_{r1}^2 - V_{r2}^2\right)}{\frac{UV_a}{g_c}\left(\tan\gamma_1 - \tan\gamma_2\right)} = \frac{V_a^2\left(\tan^2\gamma_1 - \tan^2\gamma_2\right)}{2UV_a\left(\tan\gamma_1 - \tan\gamma_2\right)} = \frac{V_a}{U}\left(\frac{\tan\gamma_1 + \tan\gamma_2}{2}\right)$$

$$R = \frac{V_a}{U} \tan\gamma_m \tag{2.51}$$

where $\tan\gamma_m = \left(\frac{\tan\gamma_1 + \tan\gamma_2}{2}\right)$

In terms of β_1 & β_2, eqn. (2.50) can also be expressed as,

$$R = \frac{V_a}{2U}(\cot\beta_1 + \text{Cot}\,\beta_2) \qquad (\because \tan\gamma_1 = \tan(90 - \beta_1) = \cot\beta_1)$$

$$R = \frac{V_a}{2U}\left(\frac{1}{\tan\beta_1} + \frac{1}{\tan\beta_2}\right) = \frac{V_a}{2U}\left(\frac{\tan\beta_1 + \tan\beta_2}{\tan\beta_1 \tan\beta_2}\right) \tag{2.52}$$

Radial Flow Machines

Ex: **Centrifugal Pump, Compressor, Blower etc.**

Energy Transfer

Generally **V_{u1} is assumed as zero** (i.e.,no whirl at inlet or fluid enters radially)

The total energy transferred is,

$$\Delta h_0 = \frac{U_2 V_{u2}}{g_c}$$

From Vel. Δ^{le} OAB, AC = AB - CB

$$AC = AB - OC \cot \beta_2$$

i.e., $$V_{u2} = U_2 - V_{f2} \cot \beta_2$$

$$\Delta h_0 = \frac{U_2}{g_c}\left(U_2 - V_{f2} \cot \beta_2\right) \qquad (2.53)$$

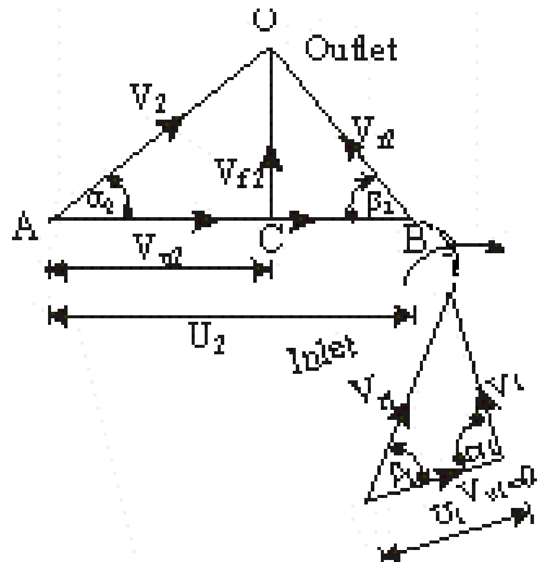


Fig. 2.14 Velocity triangles for radial flow machines for $\beta_2 < 90°$

Degree of Reaction (R):

For radial inlet, $\alpha_1 = 90^\circ, V_{u1} = 0$ & $V_{f1} = V_1 = V_{f2}$

$$\text{Then} \quad R = \frac{\dfrac{U_2 V_{u2}}{g_c} - \dfrac{(V_2^2 - V_1^2)}{2g_c}}{\dfrac{U_2 V_{u2}}{g_c}}$$

$$= \frac{U_2 V_{u2} - (V_2^2 - V_1^2)/2}{U_2 V_{u2}}$$

From outlet Velocity Δ^{le} OAB,

$$R = \frac{U_2 V_{u2} - [(V_{f2}^2 + V_{u2}^2) - V_{f2}^2]/2}{U_2 V_{u2}} = 1 - \frac{V_{u2}^2}{2 U_2 V_{u2}} = 1 - \frac{V_{u2}}{2U_2}$$

$$= 1 - \frac{(U_2 - V_{f2} \cot \beta_2)}{2U_2}$$

$$= 1 - \frac{1}{2}\left(1 - \frac{V_{f2} \cot \beta_2}{2U_2}\right)$$

$$R = \frac{1}{2}\left(1 + \frac{V_{f2} \cot \beta_2}{U_2}\right)$$

Head – Capacity (H-Q) Curve for Radial Outward Flow Devices

$$E = \Delta h_0 = gH = \frac{U_2 V_{u2}}{g_c}$$

From outlet velocity triangle

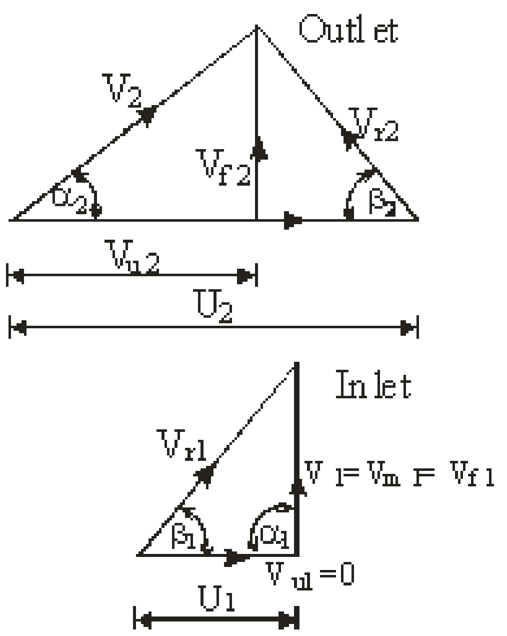
$$V_{u2} = U_2 - V_{f2} \cot\beta_2$$

$$gH = \frac{U_2(U_2 - V_{f2}\cot\beta_2)}{g_c}$$

$$gH = \frac{U_2^2 - U_2 V_{f2} \cot\beta_2)}{g_c} \qquad (2.56)$$

$$= \frac{U_2^2}{g_c} - \frac{U_2}{g_c} \cot\beta_2 \frac{Q}{\pi D_2 B_2}$$

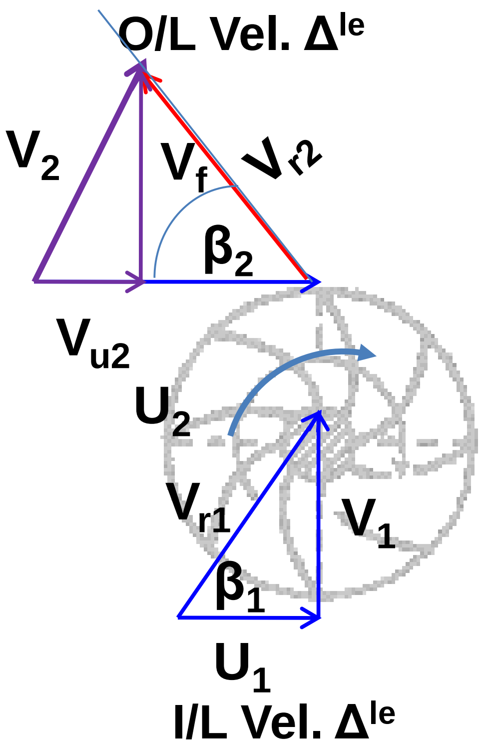
$$H = \frac{U_2^2}{g} - \left(\frac{U_2 \operatorname{Cot}\beta_2}{\pi D_2 B_2 g} \right) Q$$



Hence $\qquad H = K_1 - K_2 Q$

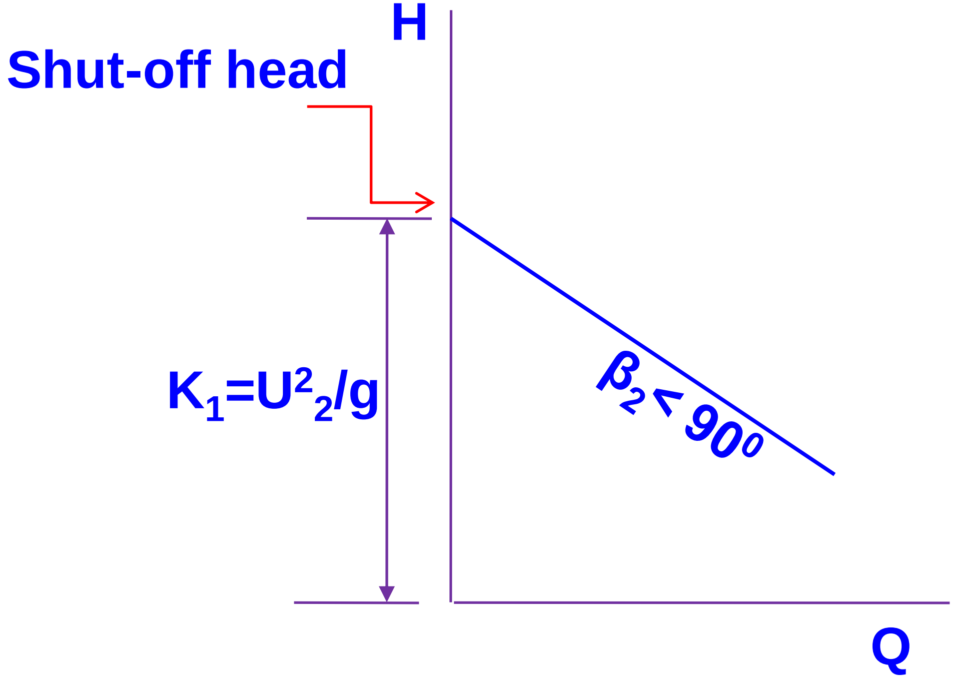
Types of Centrifugal Pump Impeller

Backward Curved Vanes (i.e, $\beta_2 < 90^0$)



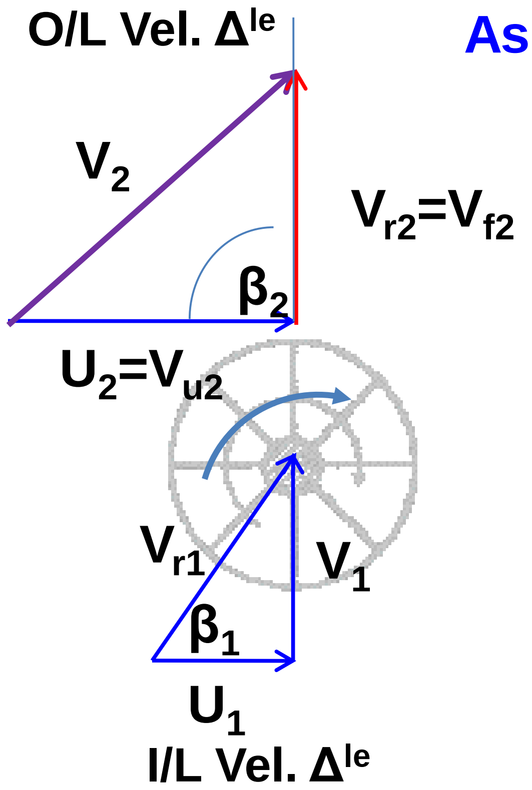
As cot β_2 is +ve for $\beta_2 < 90^0$

$H = K_1 - K_2\, Q$



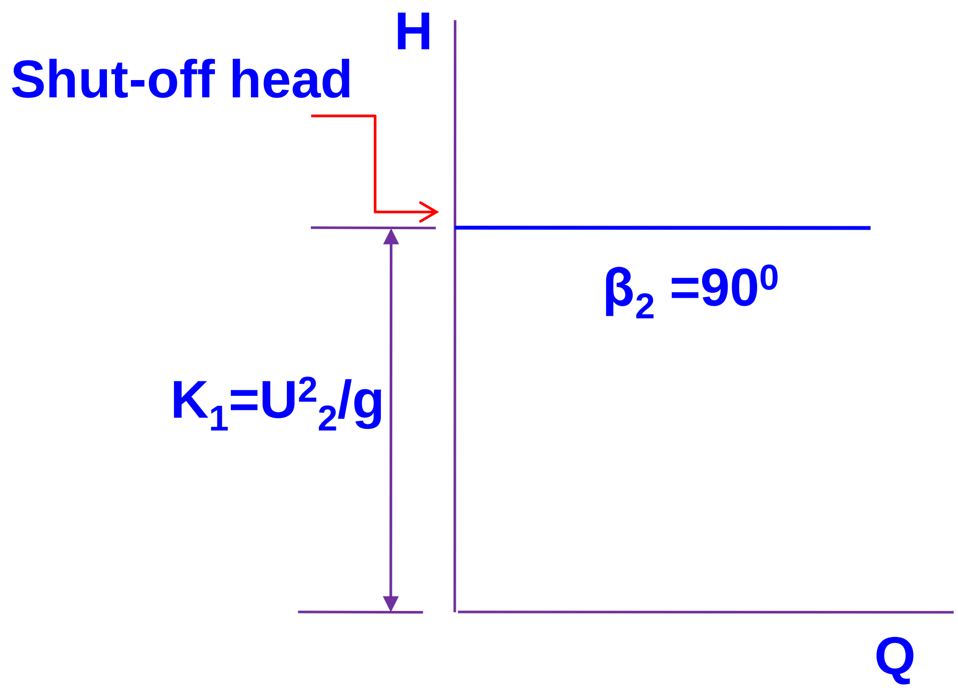
Type of Centrifugal Pump Impeller

Radial Vanes (i.e, $\beta_2 = 90^0$)



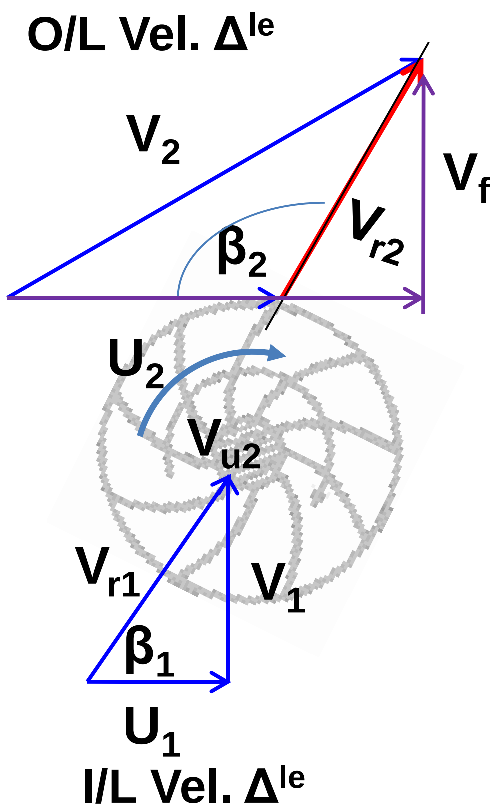
As cot β_2 is Zero for $\beta_2=90^0$

$H=K_1$ for all values of Q



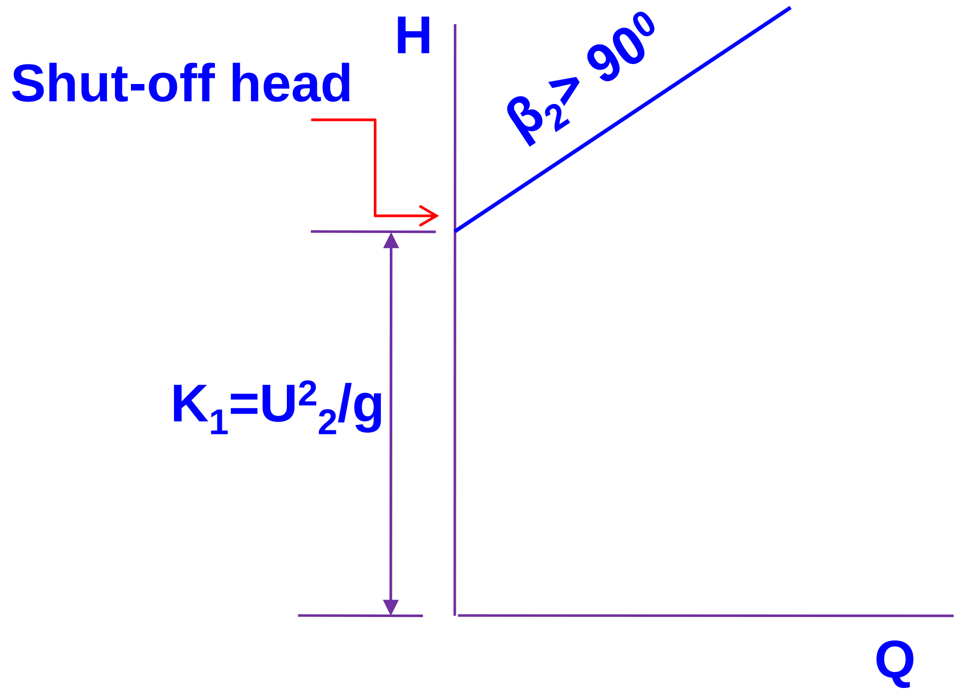
Type of Centrifugal Pump Impeller

Forward Curved Vanes ($\beta_2 > 90^0$)

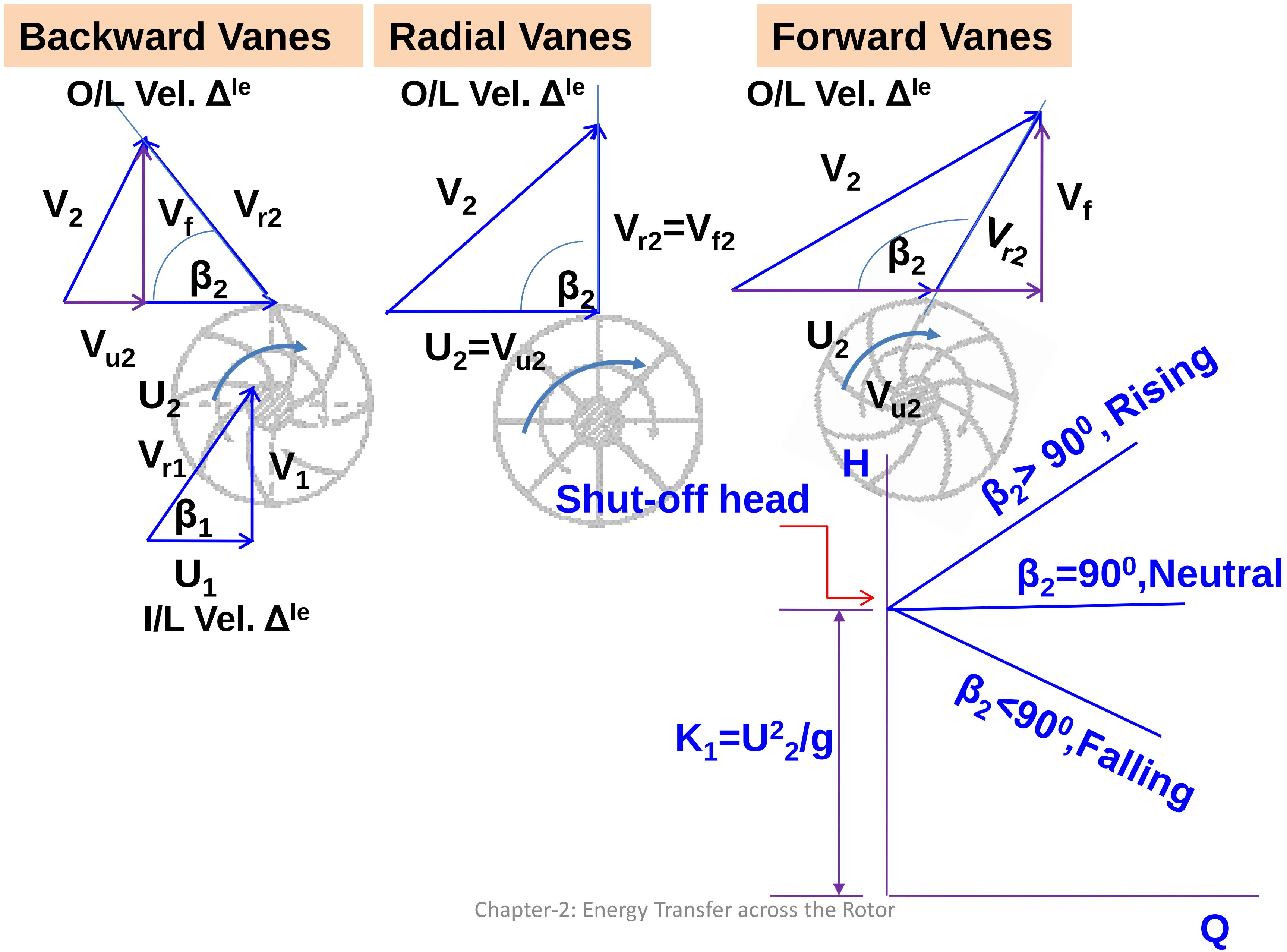


As $\cot \beta_2$ is -ve for $\beta_2 > 90^0$

$$H = K_1 + K_2 Q$$



Head – Capacity (H-Q) Curve for Radial Outward Flow Devices



Important Formulae for Axial Flow Compressor/Blower

- **Tangential Force,** $$F_T = \frac{\dot{m}}{g_c}\left(V_{u2} - V_{u1}\right) = \frac{\dot{m}}{g_c}\Delta V_u$$
- **Axial Thrust** $$F_a = \frac{\dot{m}}{g_c}\left(V_{f1} - V_{f2}\right) = \frac{\dot{m}}{g_c}\Delta V_a$$
- **Torque Exerted,** $$T = \frac{\dot{m}}{g_c} r_m \Delta V_u$$
- **Energy Transfer,**

$$E = \Delta h = \frac{U\,\Delta V_u}{g_c} = \frac{UV_a\left(\tan\gamma_1 - \tan\gamma_2\right)}{g_c} = \frac{UV_a\left(\cot\beta_1 - \cot\beta_2\right)}{g_c}$$

$$= \frac{1}{2g_c}\left[(V_2{}^2 - V_1{}^2) + (V_{r1}{}^2 - V_{r2}{}^2)\right]$$

$$E = W = \frac{(V_2{}^2 - V_1{}^2)}{2(1-R)}$$

- **Power output,** $$P = \dot{m}E = \dot{m}W = \dot{m}\,gH_e = \dot{m}\,\Delta h$$

Important Formulae for Axial Flow Compressor/Blower

- **Tangential Force,** $$F_T = \frac{\dot{m}}{g_c}\left(V_{u2} - V_{u1}\right) = \frac{\dot{m}}{g_c}\Delta V_u$$
- **Axial Thrust** $$F_a = \frac{\dot{m}}{g_c}\left(V_{f1} - V_{f2}\right) = \frac{\dot{m}}{g_c}\Delta V_a$$
- **Torque Exerted,** $$T = \frac{\dot{m}}{g_c} r_m \Delta V_u$$
- **Energy Transfer,**

$$E = \Delta h = \frac{U\,\Delta V_u}{g_c} = \frac{UV_a\left(\tan\gamma_1 - \tan\gamma_2\right)}{g_c} = \frac{UV_a\left(\cot\beta_1 - \cot\beta_2\right)}{g_c}$$

$$= \frac{1}{2g_c}\left[(V_2^{\,2} - V_1^{\,2}) + (V_{r1}^{\,2} - V_{r2}^{\,2})\right]$$

$$E = W = \frac{(V_2^{\,2} - V_1^{\,2})}{2(1-R)}$$

- **Power output,** $$P = \dot{m}E = \dot{m}W = \dot{m}\,gH_e = \dot{m}\,\Delta h$$

Important Formulae for Axial Flow Compressor/Blower

- **Degree of Reaction,**

$$R = \frac{\left[\left(V_{r1}^{2} - V_{r2}^{2}\right)\right]}{\left[\left(V_{2}^{2} - V_{1}^{2}\right) + \left(V_{r1}^{2} - V_{r2}^{2}\right)\right]} = \frac{E - \left(V_{2}^{2} - V_{1}^{2}\right)/2}{E}$$

$$= \frac{V_a}{U}\frac{\left(\tan\gamma_1 + \tan\gamma_2\right)}{2} = \frac{V_a}{U}\tan\gamma_m$$

$$= \frac{V_a}{U}\frac{\left(\cot\beta_1 + \cot\beta_2\right)}{2} = \frac{V_a}{U}\cot\beta_m$$

Important Formulae for Radial Flow Compressor/Blower and Pumps

- **Tangential Force,** $$F_T = \frac{\dot{m}}{g_c}\left(V_{u2} - V_{u1}\right) = \frac{\dot{m}}{g_c}\Delta V_u$$
- **Radial Thrust** $$F_{rd} = \frac{\dot{m}}{g_c}\left(V_{rd1} - V_{rd2}\right) = \frac{\dot{m}}{g_c}\Delta V_{rd}$$
- **Torque Exerted,** $$T = \frac{\dot{m}}{g_c}\left[r_2 V_{u2} - r_1 V_{u1}\right]$$
- **Energy Transfer,**

$$E = gH_e = \Delta h = \frac{U_2 V_{u2}}{g_c} = \frac{U_2\left[U_2 - V_{f2}\cot\beta_2\right]}{g_c}$$

$$H = \frac{U_2}{g} - \frac{U_2 \cot\beta_2}{g\,\pi D_2 b_2} Q \;= K_1 - K_2 Q$$

$$E = \frac{1}{2g_c}\left[\left(V_2^{\,2} - V_1^{\,2}\right) + \left(U_2^{\,2} - U_1^{\,2}\right) + \left(V_{r1}^{\,2} - V_{r2}^{\,2}\right)\right]$$

- **Power output,** $$P = \dot{m}E = \dot{m}W = \dot{m}\,gH_e = \dot{m}\,\Delta h$$

Important Formulae for Radial Flow Compressor/Blower and Pumps

- **Degree of Reaction,**

$$\mathbf{R} = \frac{\left[\left(\mathbf{U_2}^2 - \mathrm{U_1}^2\right) + \left(\mathbf{V_{r1}}^2 - \mathbf{V_{r2}}^2\right)\right]}{\left[\left(\mathbf{V_2}^2 - \mathbf{V_1}^2\right) + \left(\mathbf{U_2}^2 - \mathrm{U_1}^2\right) + \left(\mathbf{V_{r1}}^2 - \mathbf{V_{r2}}^2\right)\right]} = \frac{\mathbf{E} - \left(\mathbf{V_2}^2 - \mathbf{V_1}^2\right)/2}{\mathbf{E}}$$

$$\mathrm{R} = \frac{1}{2}\left[1 + \frac{\mathbf{V_{f2}\cot\beta_2}}{\mathbf{U_2}}\right]$$