Theory of Machines:

.

It is defined as that branch of Engineering science, which deals with the study of relative motion between the various parts of a machine, and forces which act on them

Kinematics of Machines

It is defined as that branch of Theory of machines which deals with the relative motion between the various parts of the machines.

Dynamics of Machines

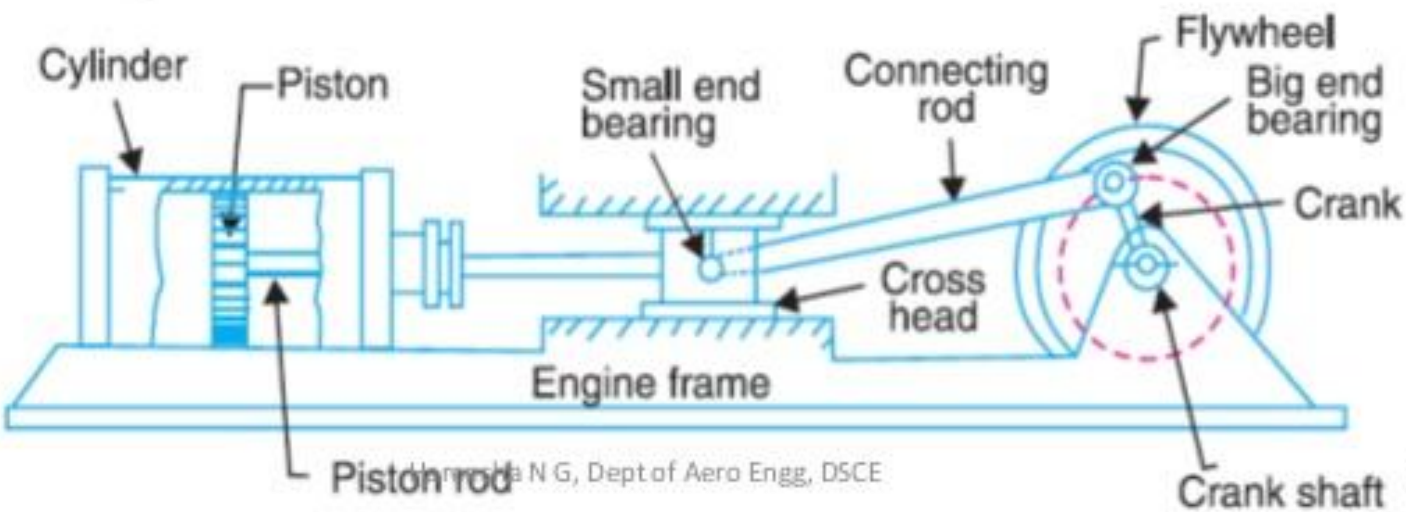
It is defined as that branch of Theory of machines which deals with the Forces and their effects, the various parts of the machines.

Kinematics of Machines:

- It is defined as that branch Theory of machines which deals with the relative motion between the various parts of the machines.

Kinematic Link or Element

- Each part of a machine, which moves relative to some other part, is known as a *kinematic link (or simply link) or element.*
- *A link may consist of several parts, which are rigidly* fastened together, so that they do not move relative to one another.
- For example, in a reciprocating steam engine, as shown in Fig. 1, piston, piston rod and crosshead constitute one link ; connecting rod with big and small end bearings constitute a second link ; crank, crank shaft and flywheel a third link and the cylinder, engine frame and main bearings a fourth link.

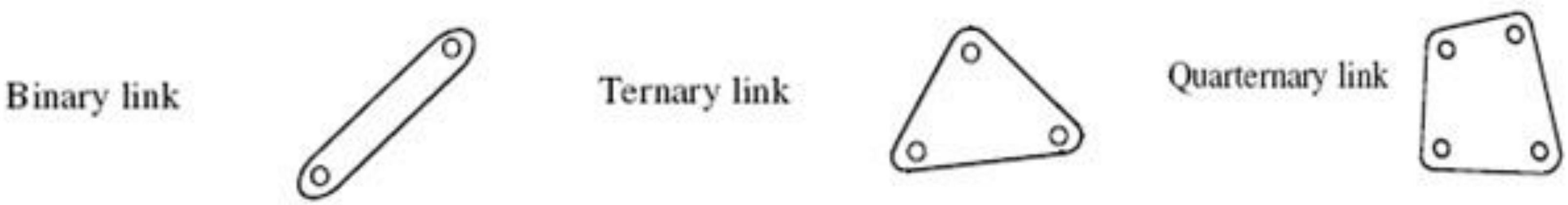


Types of Links: Depending upon its ends on which revolute joint can be placed for pairing with other; Links can be classified in to

Binary,

Ternary

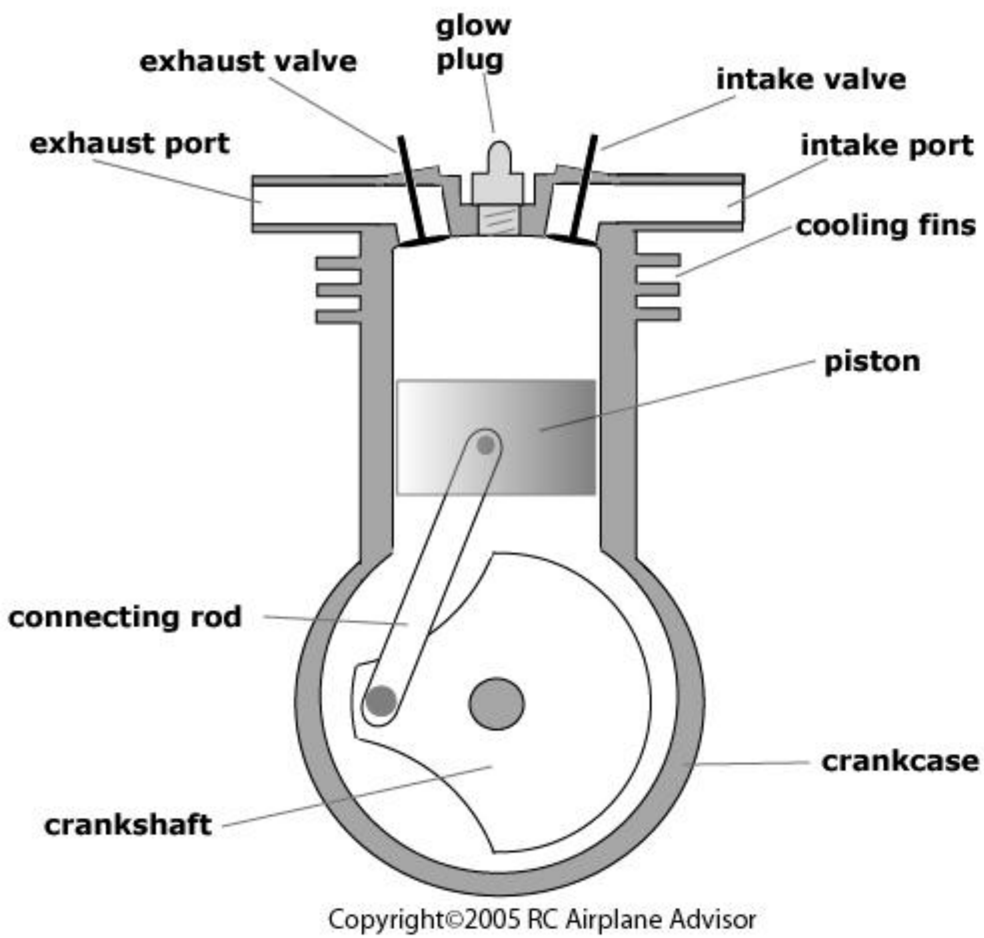
Quaternary link, etc.,



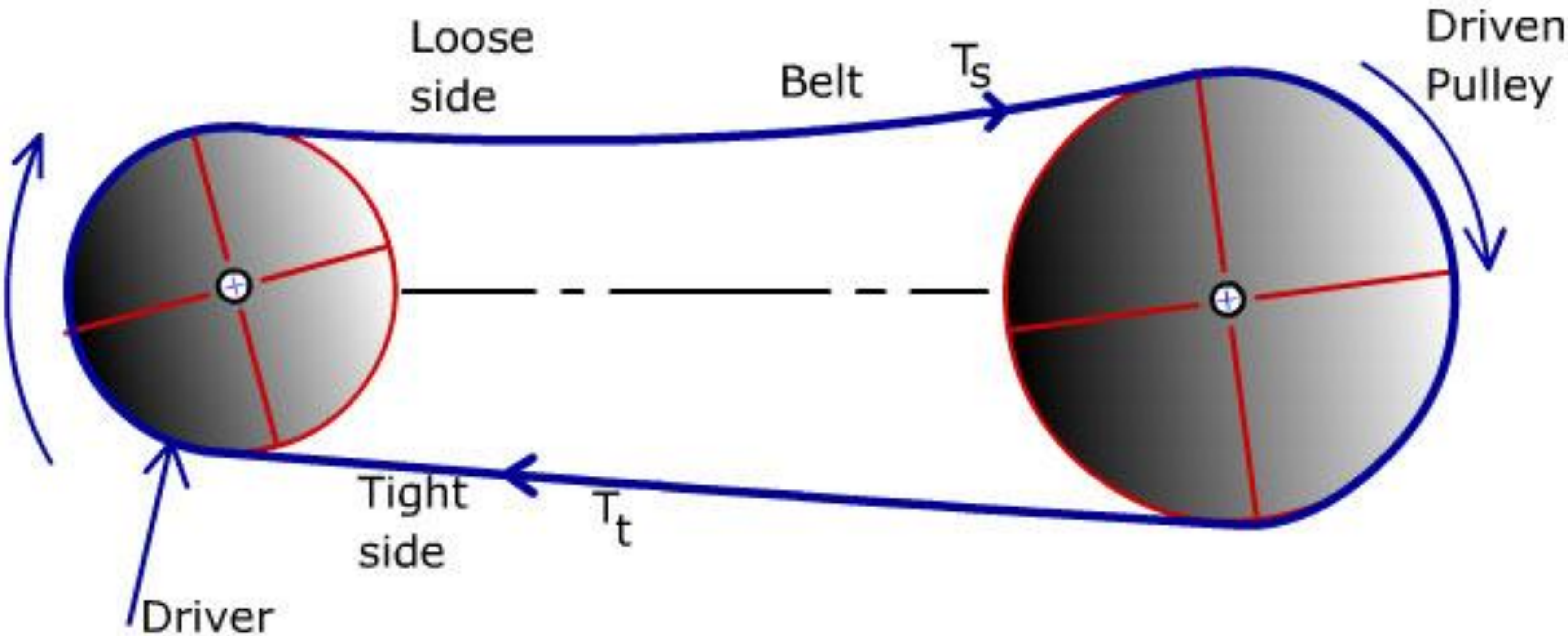
Types of Links: In order to transfer the motion the driver and the follower may be connected by the following three types of links:

1.Rigid link:
2.Flexible link:.
3.Fluid link:

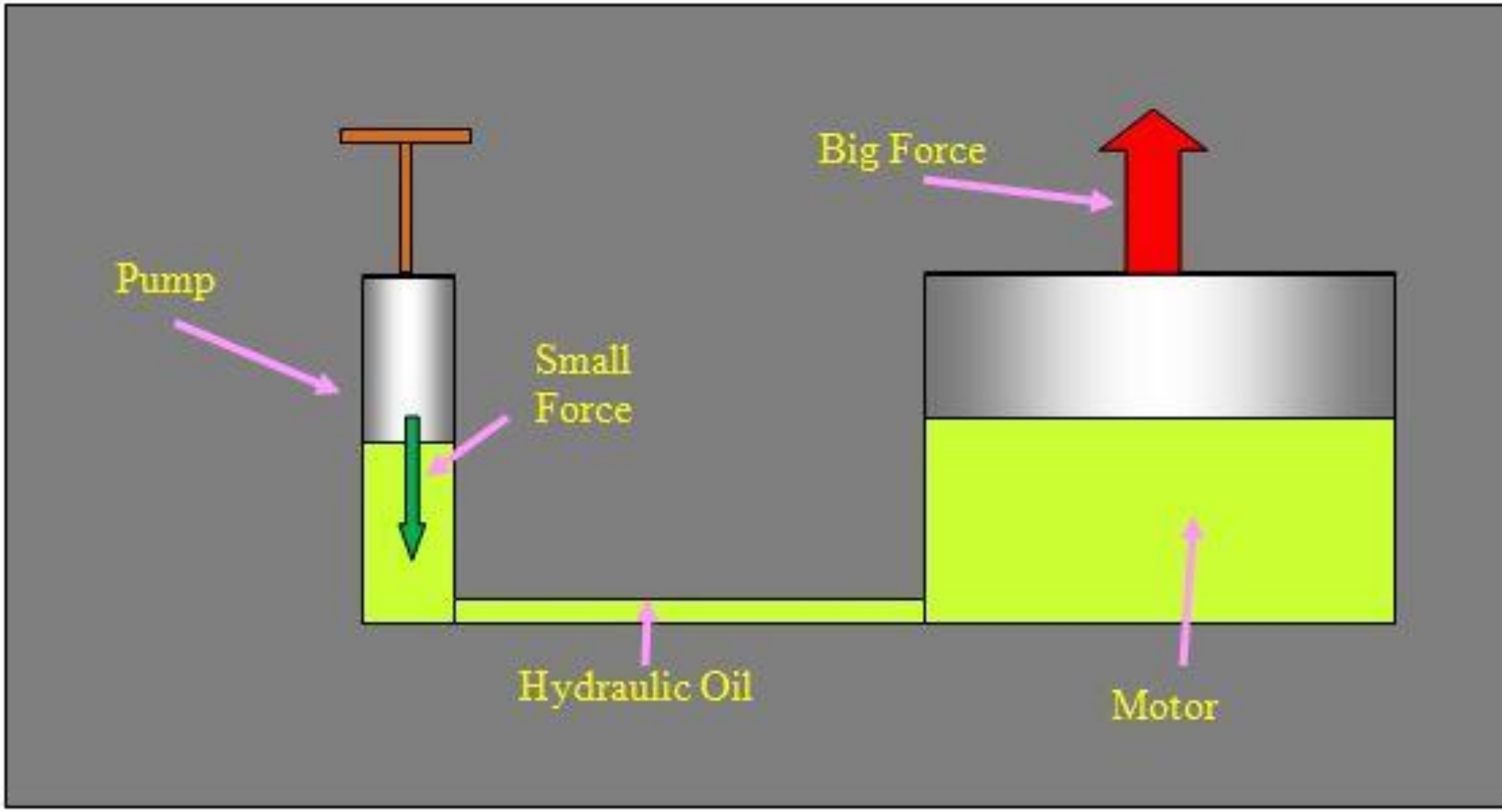
1.Rigid link: Does not undergo any deformation while transmitting motion. Ex: Crank shaft, Connecting rod.



2.Flexible link: Partly deformed in a manner not to affect the transmission of motion. Ex:Belts, ropes .



3.Fluid link: Motion is transmitted through the fluid by pressure or compression only. Ex: Hydraulic presses, Jacks and brakes



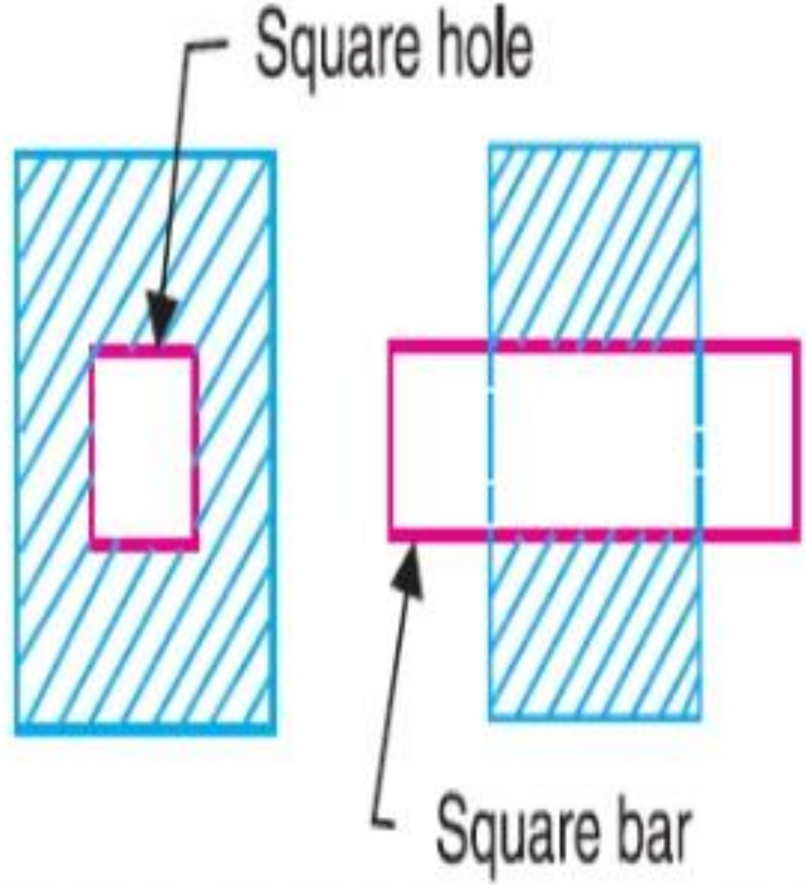
Types of constrained motion

- 1. Completely constrained motion
- 2. Incompletely Constrained motion
- 3. Successfully constrained motion

Types of 'Constrain Motion'

1. Completely constrained motion

- *When the motion between a pair is limited to a definite* direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.
- The motion of a square bar in a square hole, as shown in Fig. 2, and the motion of a shaft with collars at each end in a circular hole, as shown in Fig.



2. Incompletely Constrained motion: If the motion between a pair of links is not confined to a definite direction, then it is incompletely constrained motion.

E.g.: Circular shaft in a circular hole may either rotate or slide in the hole

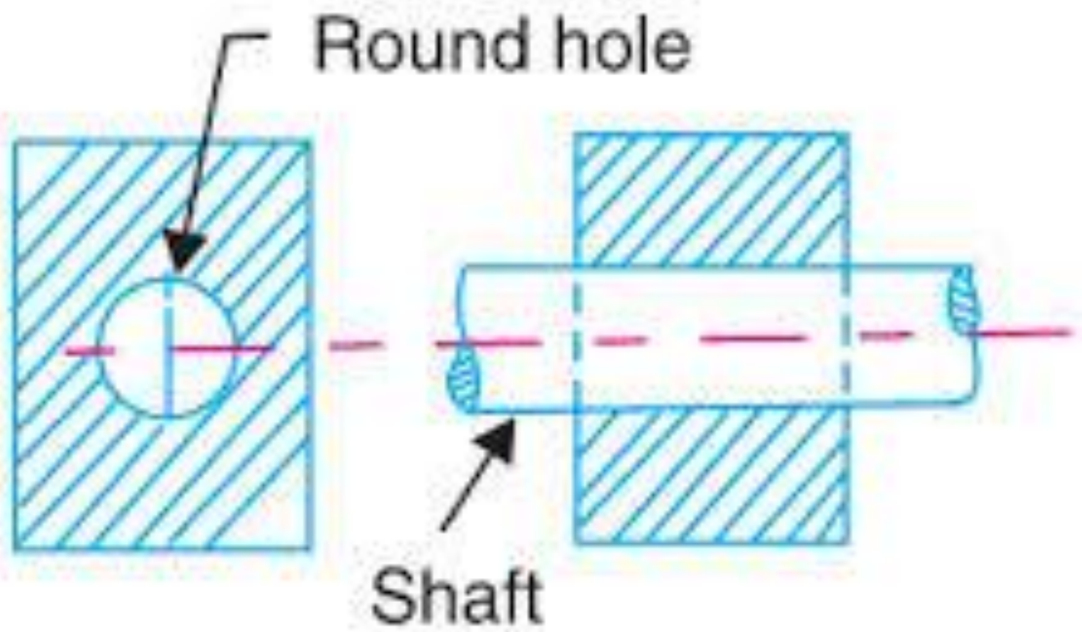


Fig. 5.4. Shaft in a circular hole.

Successfully constrained motion

If the motion in a definite direction is not brought about by itself but by some other means, then it is known as successfully constrained motion.

E.g.: Foot step Bearing.

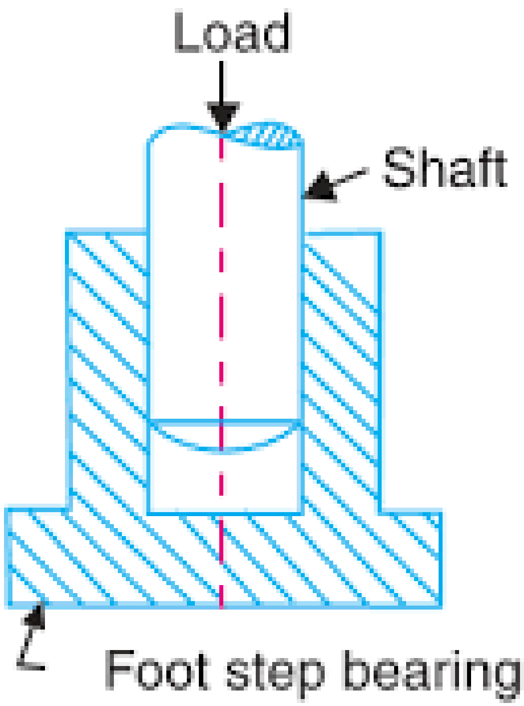
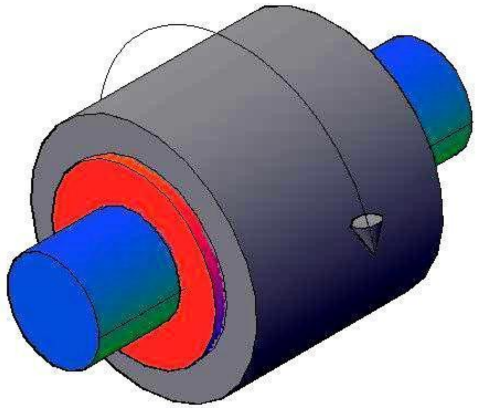
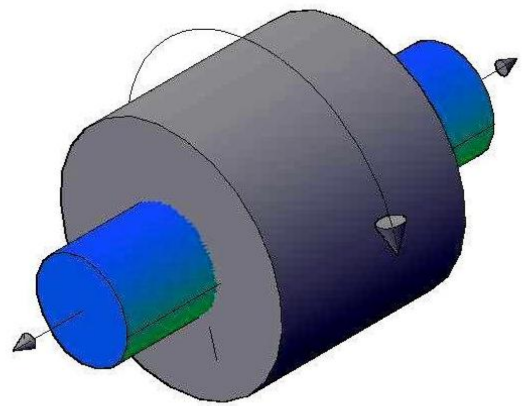
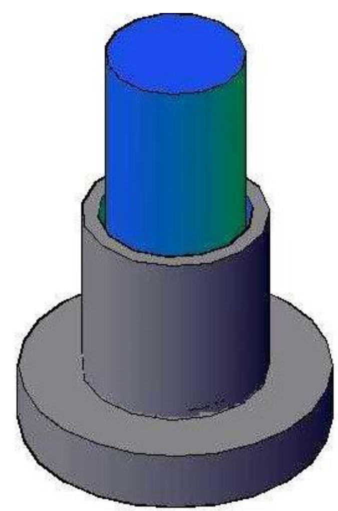


Fig. 5.5. Shaft in a foot step bearing.

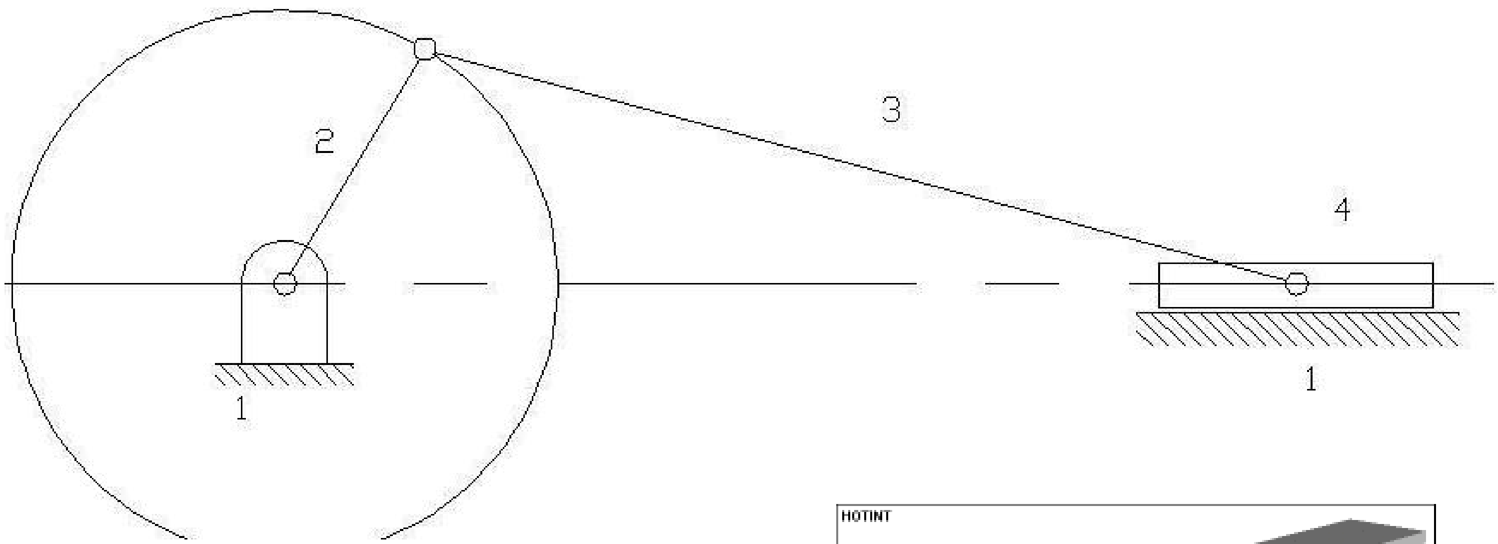


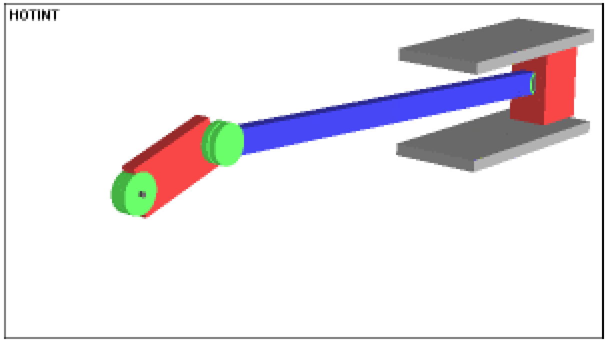




Kinematic Pair:

When two links are connected together in such a way that their relative motion is constrained, form a kinematic pair.





link 2 & link 1 = turning pair.
link 2 & link 3 = turning pair.
link 3 & link 4 = turning pair.
link 4 & link 1 = sliding pair.

Classification of Kinematic pair:

Kinematic pairs may be classified according to the following consideration:

1. Nature of contact.
2. Nature of mechanical constraint.
3. Nature of relative motion.

i) Kinematic pairs according to nature of contact:

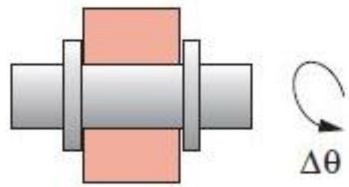
- ***Lower Pair***
- ***Higher Pair***

Lower Pair:

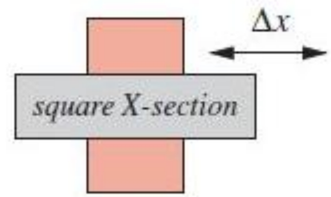
A pair of links having surfaced or area contact between the members is known as a lower pair.

The contact surfaces of the two links are similar.

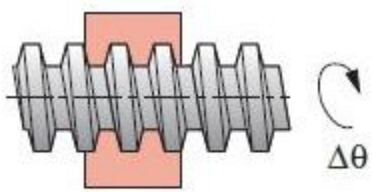
Examples: Nut turning on a screw, shaft rotating in a bearing,



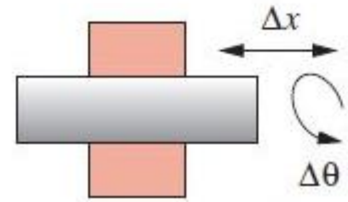
Revolute (R) joint



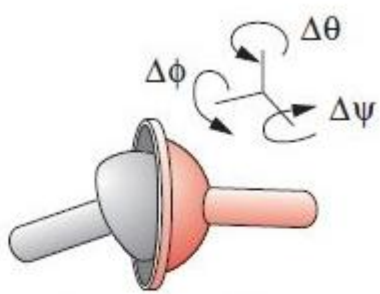
Prismatic (P) joint



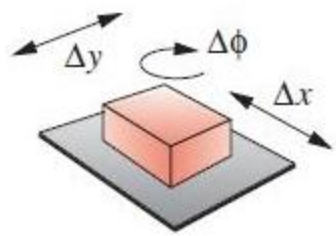
Helical (H) joint



Cylindrical (C) joint



Spherical (S) joint

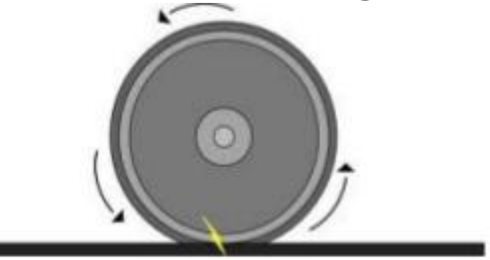


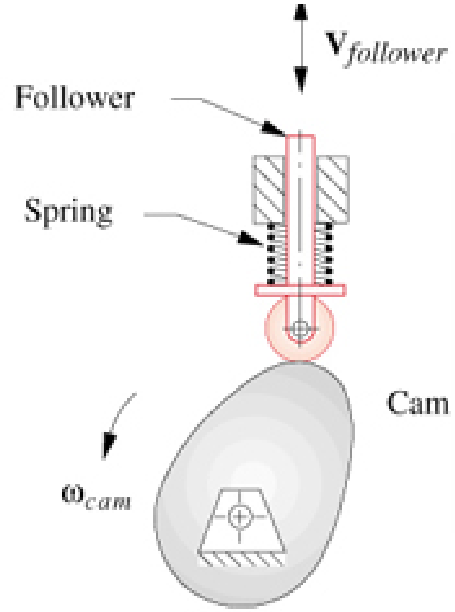
Flat (F) joint

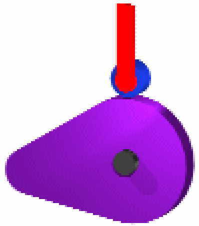
Higher Pair : When a pair has a point or line contact between the links, it is known as a higher pair.

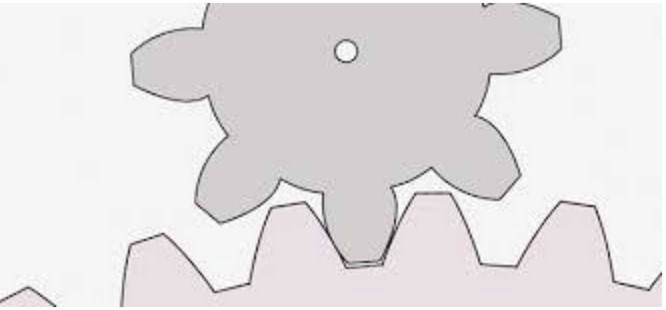
The contact surfaces of the two links are dissimilar.

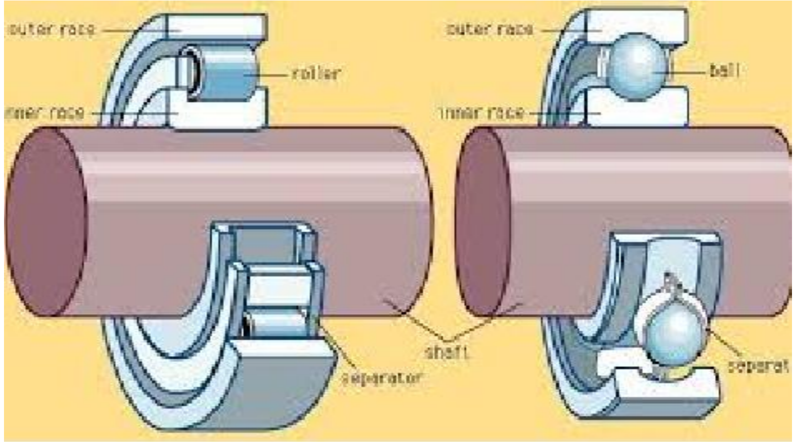
Examples: Wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings, etc

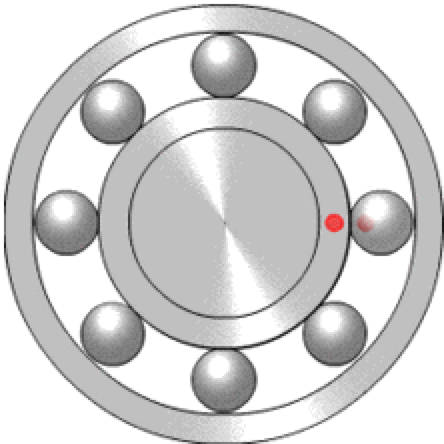












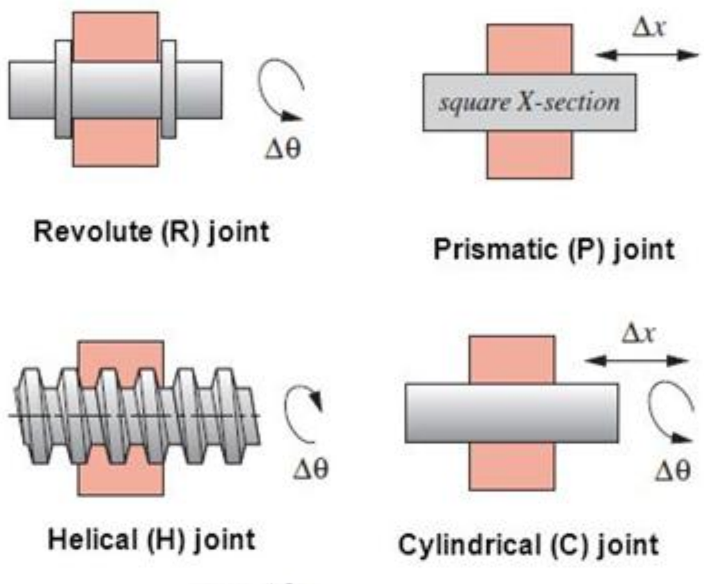
ii) Kinematic pairs according to nature of mechanical constraint.

- ***Closed pair***
- ***Unclosed pair***

Closed pair: When the elements of a pair are held together mechanically, it is known as a closed pair.

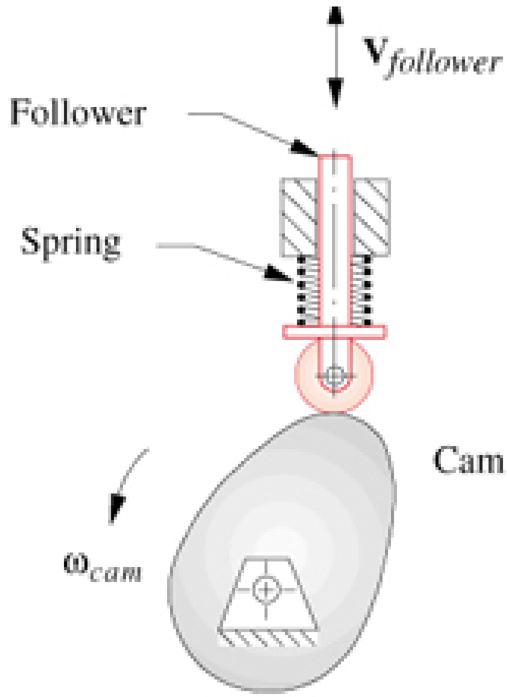
The contact between the two can only be broken only by the destruction of at least one of the members.

All the lower pairs and some of the higher pairs are closed pairs.



Unclosed pair: When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this the links are not held together mechanically.

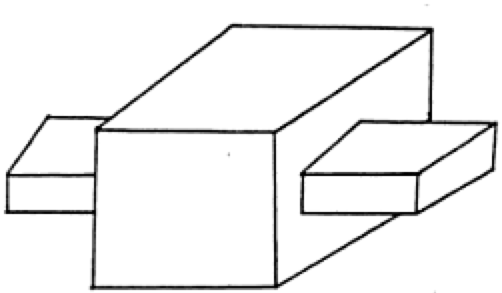
Ex.: Cam and follower pair.

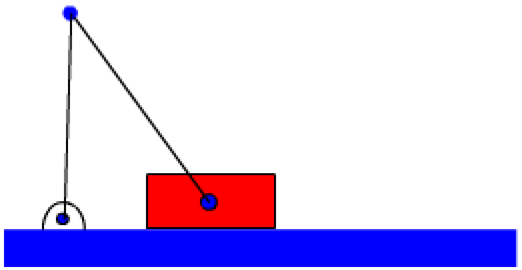


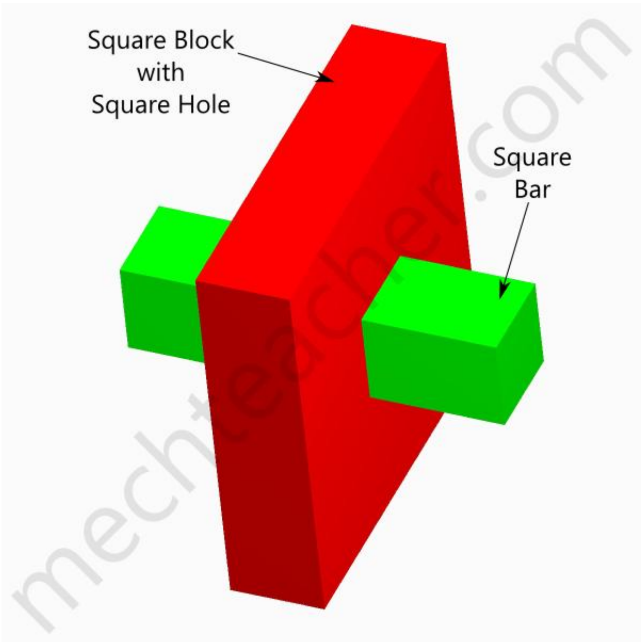
iii) Kinematic pairs according to nature of relative motion.

- ***Sliding pair***
- ***Turning Pair***
- ***Rolling pair***
- ***Screw pair (Helical Pair)***
- ***Spherical pair***

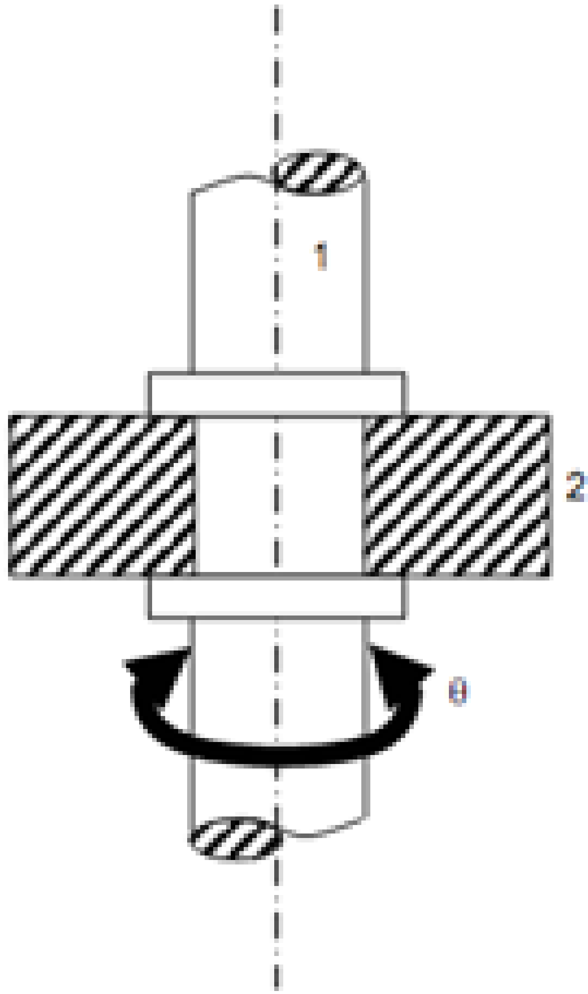
Sliding pair: If two links have a sliding motion relative to each other, they form a sliding pair.
A rectangular rod in a rectangular hole in a prism is an example of a sliding pair.

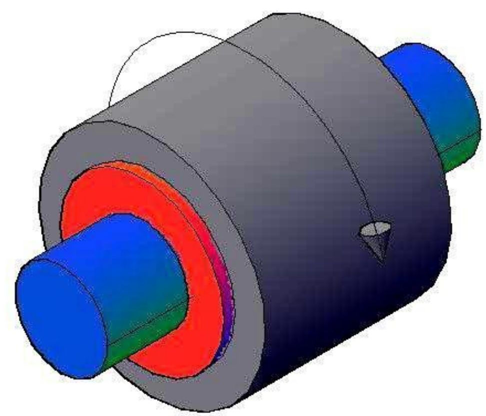






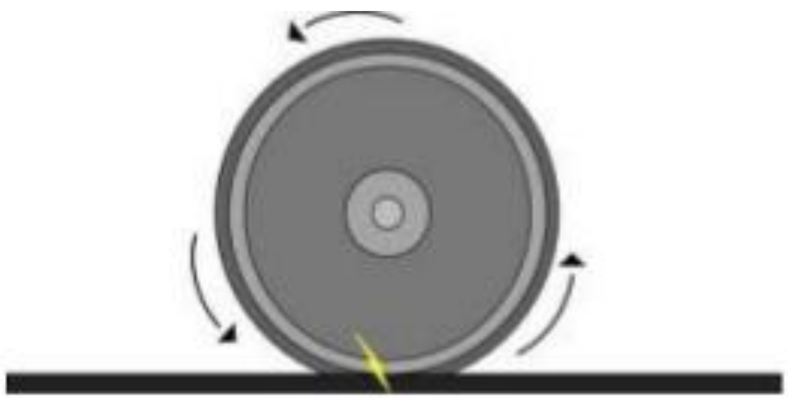
Turning Pair: When one link has a turning or revolving motion relative to the other, they constitute a turning pair or revolving pair.



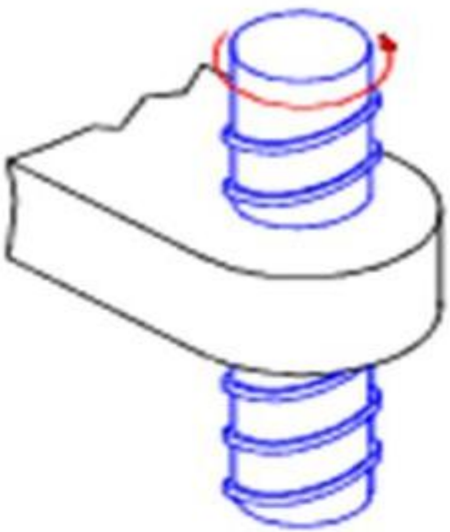


Rolling pair: When the links of a pair have a rolling motion relative to each other, they form a rolling pair.

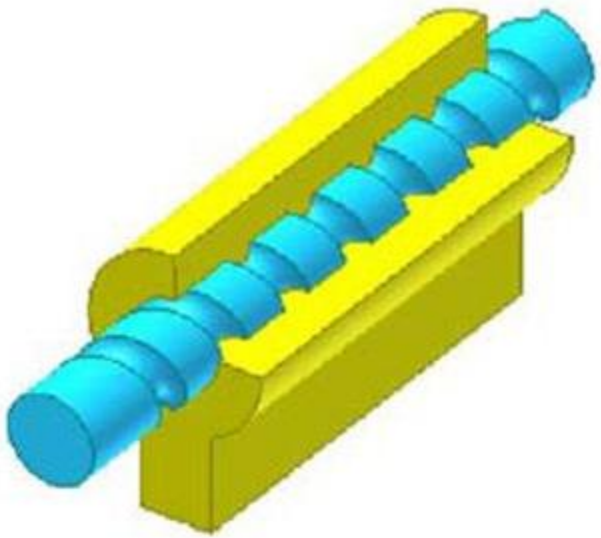
A rolling wheel on a flat surface, ball and roller bearings, etc. are some of the examples for a Rolling pair.



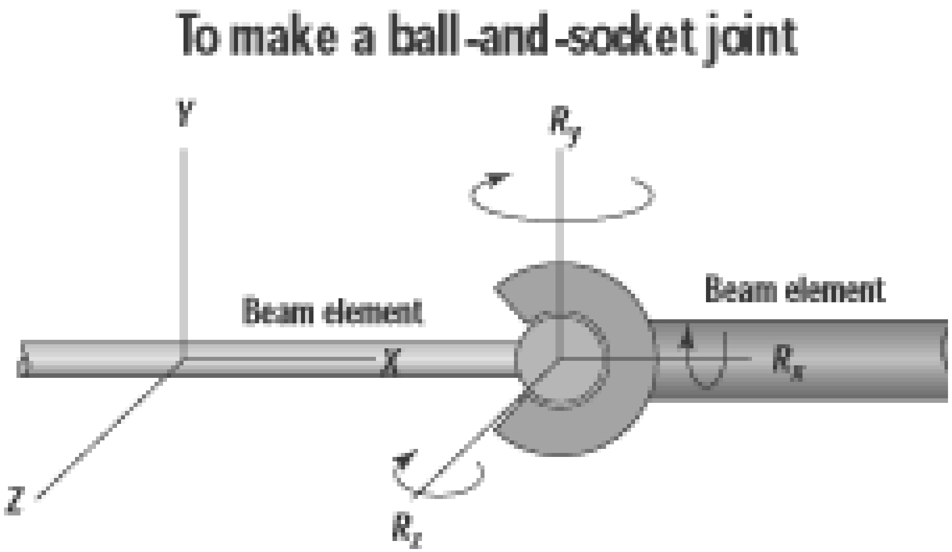
Screw pair (Helical Pair): If two mating links have a turning as well as sliding motion between them, they form a screw pair
. This is achieved by cutting matching threads on the two links. The lead screw and the nut of a lathe is a screw Pair

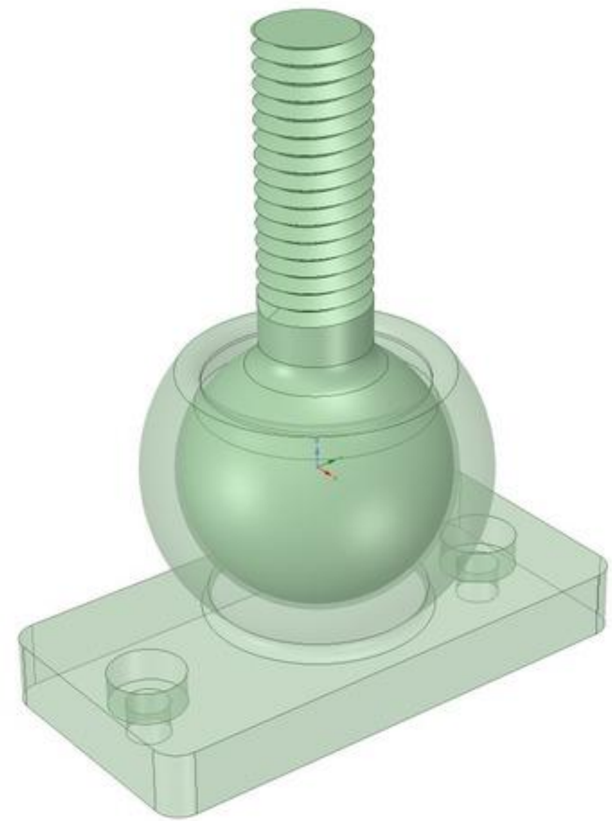


Screw Pair ...1-DOF

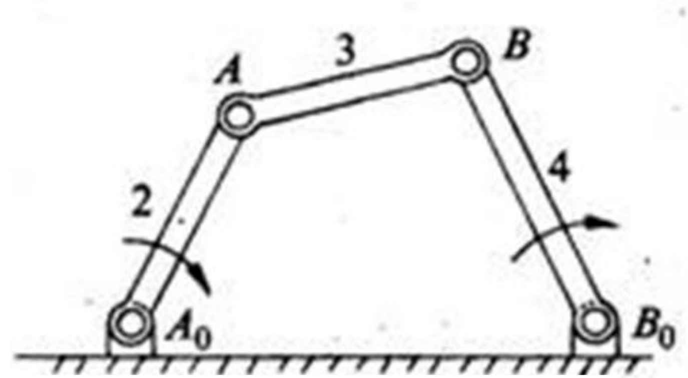


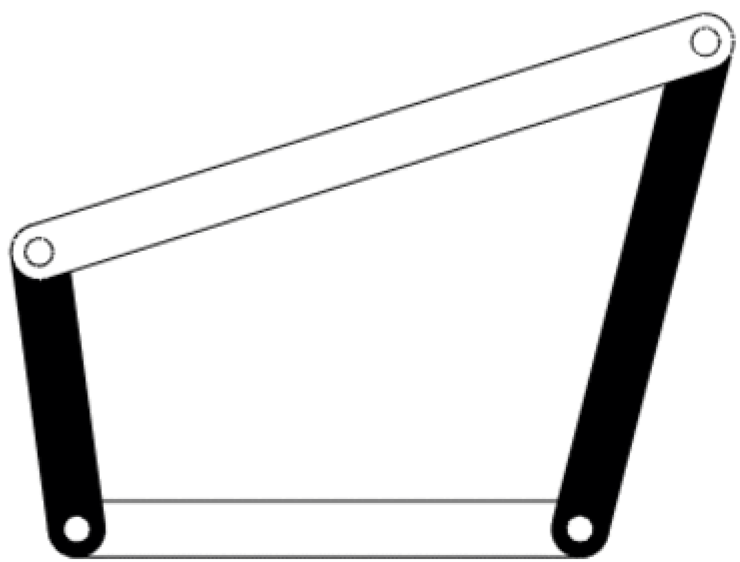
Spherical pair: When one link in the form of a sphere turns inside a fixed link, it is a spherical pair. The ball and socket joint is a spherical pair.



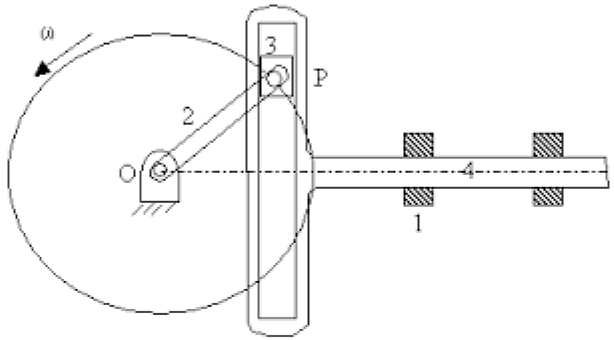


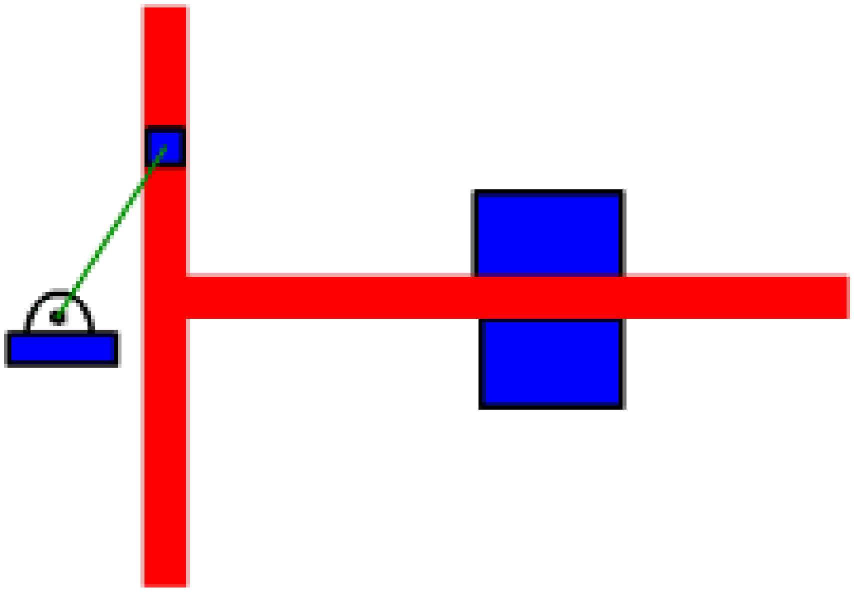
Kinematic chain: When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion, it is called Kinematic chain.





Mechanism: By fixing one of the links of a kinematic chain the arrangement may be used to transmit or transform motion and this arrangement is known as Mechanism.

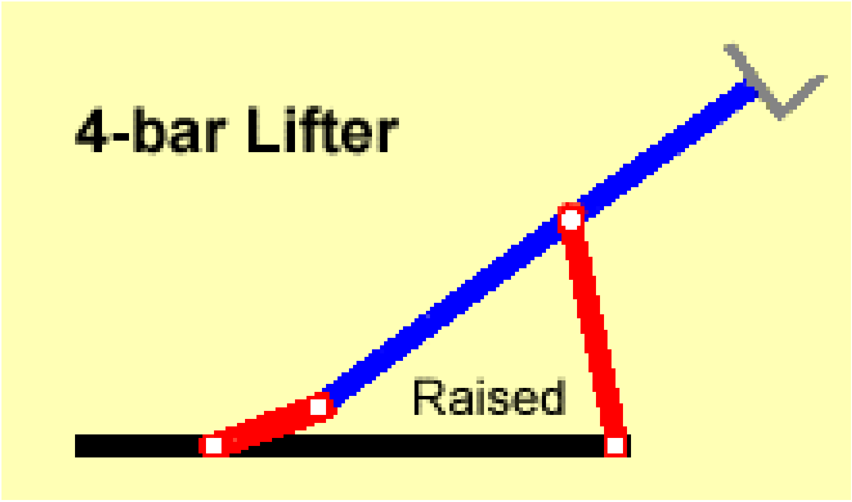




Machine: It is a device which receives energy and transforms it into some useful work.

When a mechanism is used to transmit power or to do some particular type of work, it then becomes a machine.

Ex: Lathe, Shaper, Planer, Roller, etc.

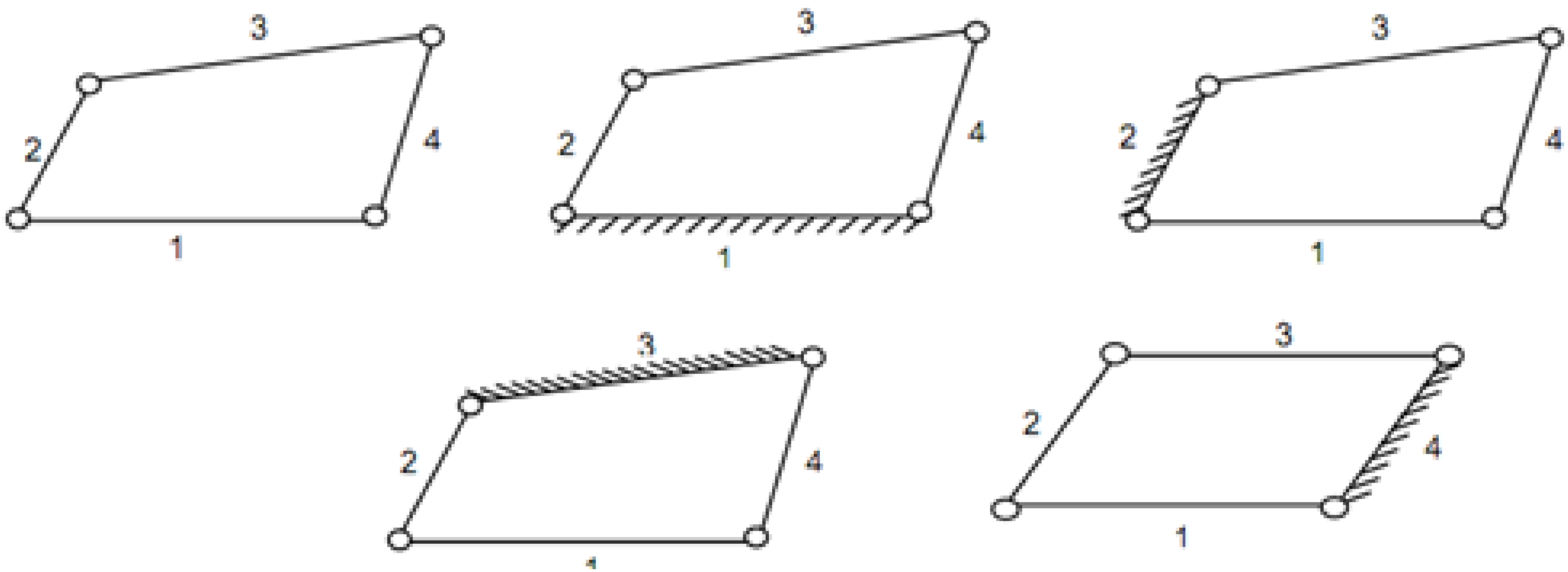


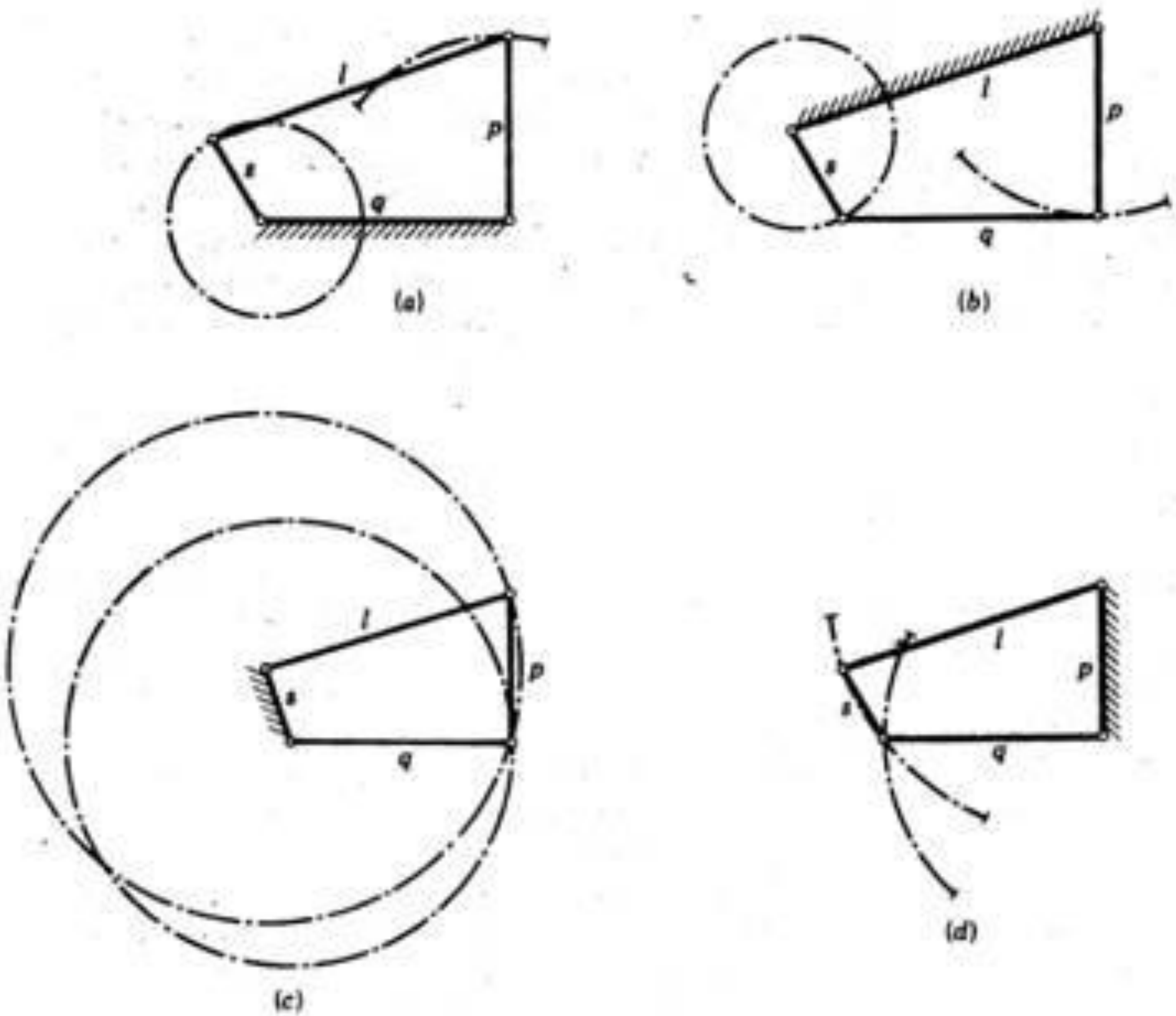
Structure: It is an assemblage of a number of resistant bodies having no relative motion between them. These are meant for taking loads only.

The degree of freedom of a structure is zero or less. A structure with negative degrees of freedom is known as a Superstructure.

Ex: Railway bridges, Roof trusses.

Inversion: By fixing each link at a time we get as many mechanisms as the number of links, then each mechanism is called „Inversion‟ of the original Kinematic Chain.

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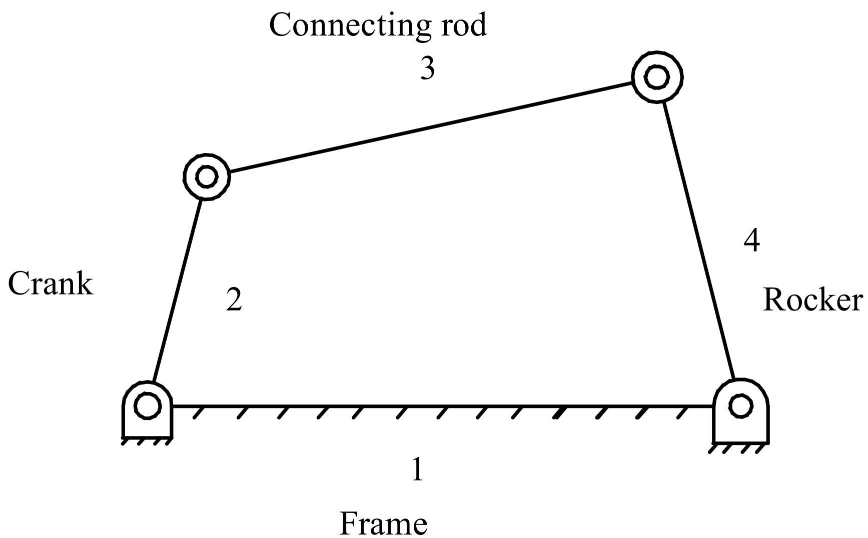
Types of Kinematic Chain:

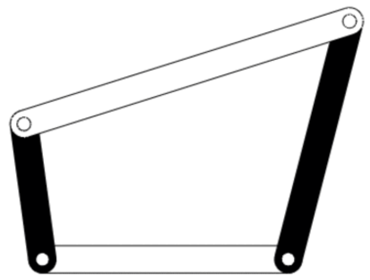
Four bar chain

Single slider crank chain

Double Slider crank chain

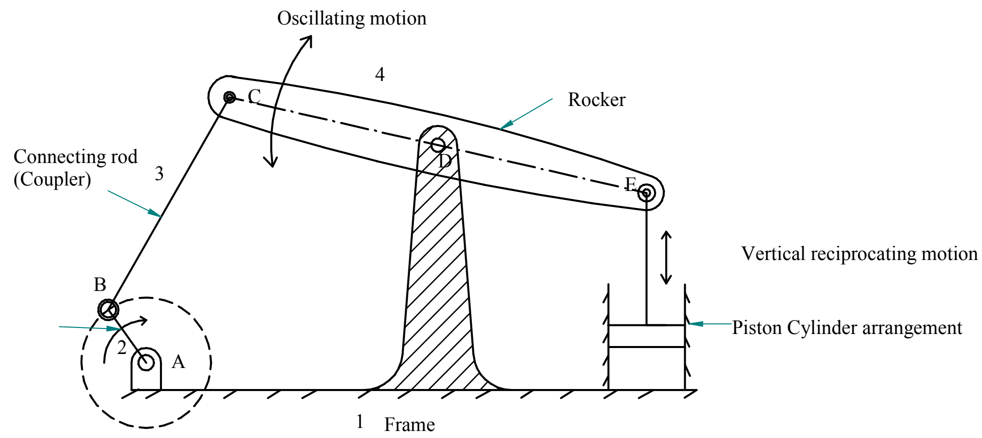
1. Four bar chain(Quadric chain)

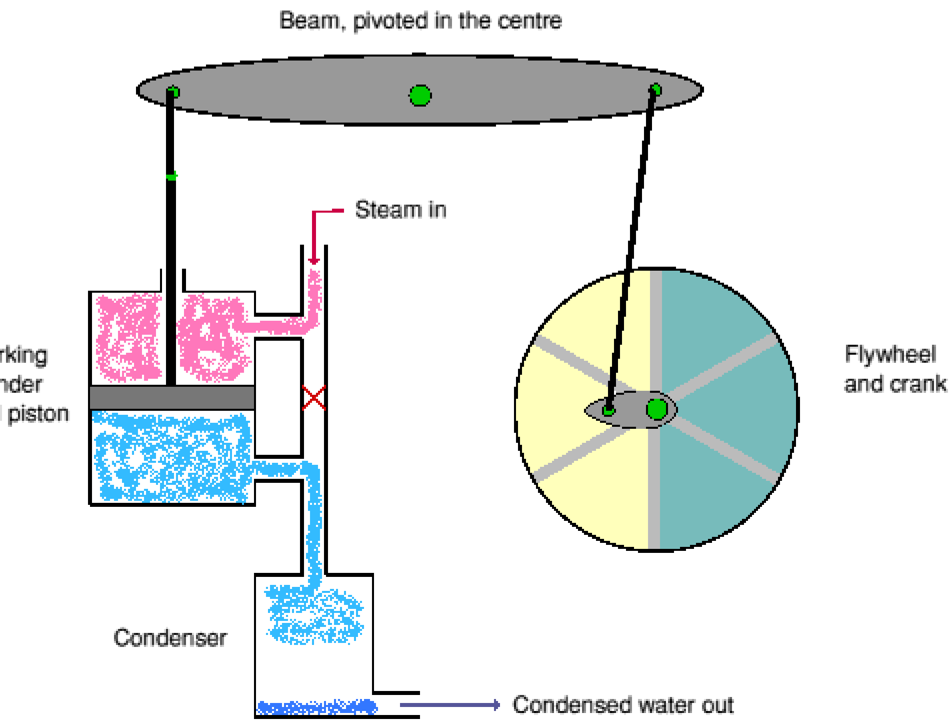




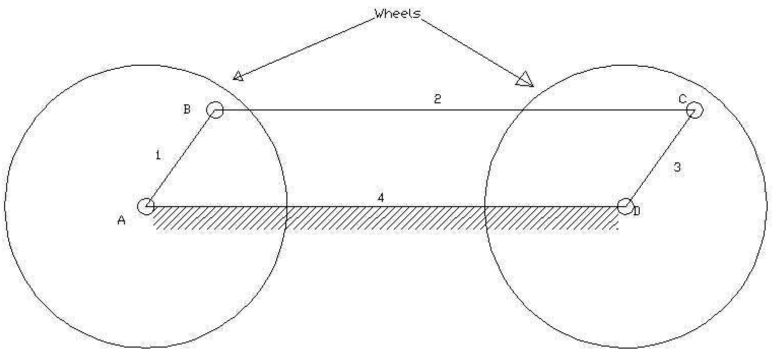
- ***Inversions of four bar chain mechanism:***
  - Beam Engine or Crank and lever mechanism.
  - Coupling rod of locomotive or double crank mechanism.
  - Watt‟s straight line mechanism or double lever mechanism.

Beam Engine or Crank and lever mechanism

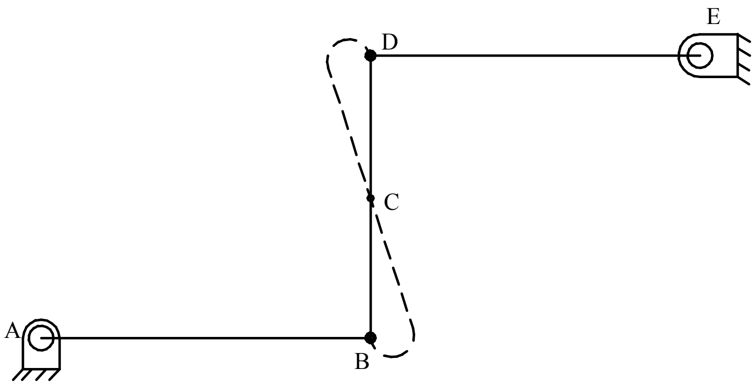


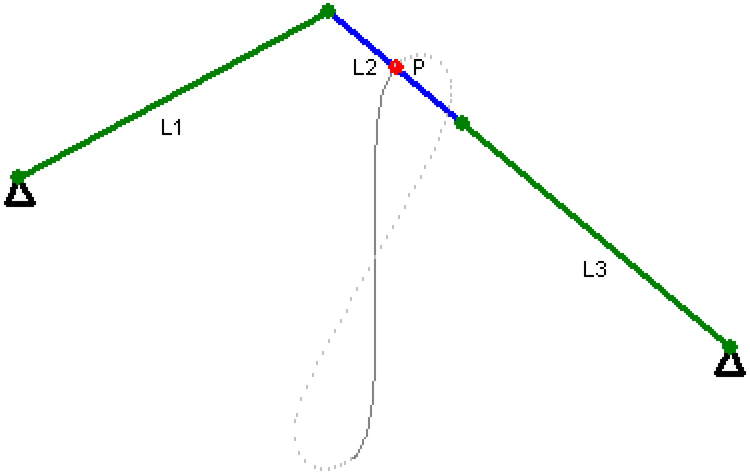


Coupling rod of locomotive



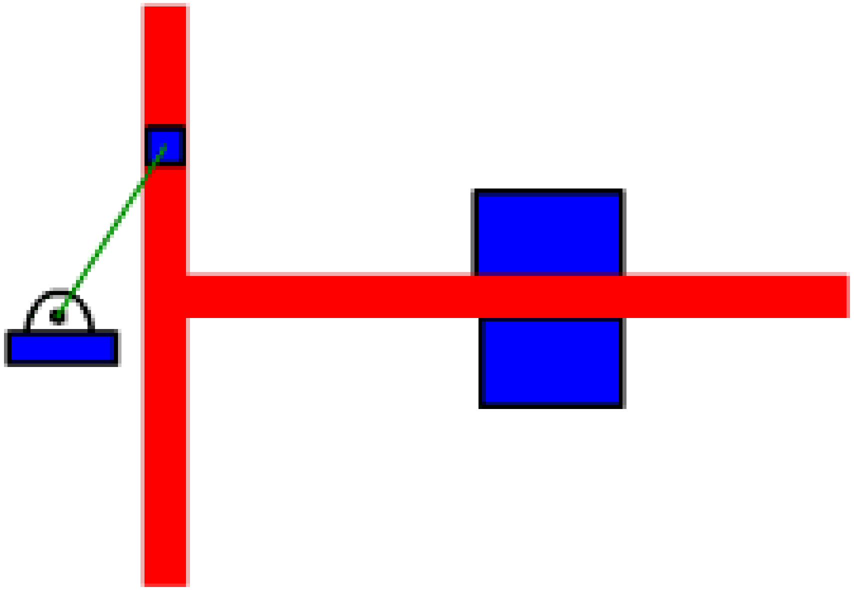
Watt‟s straight line mechanism or Double lever mechanism.





Double slider crank chain:

A four bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as double slider crank chain.



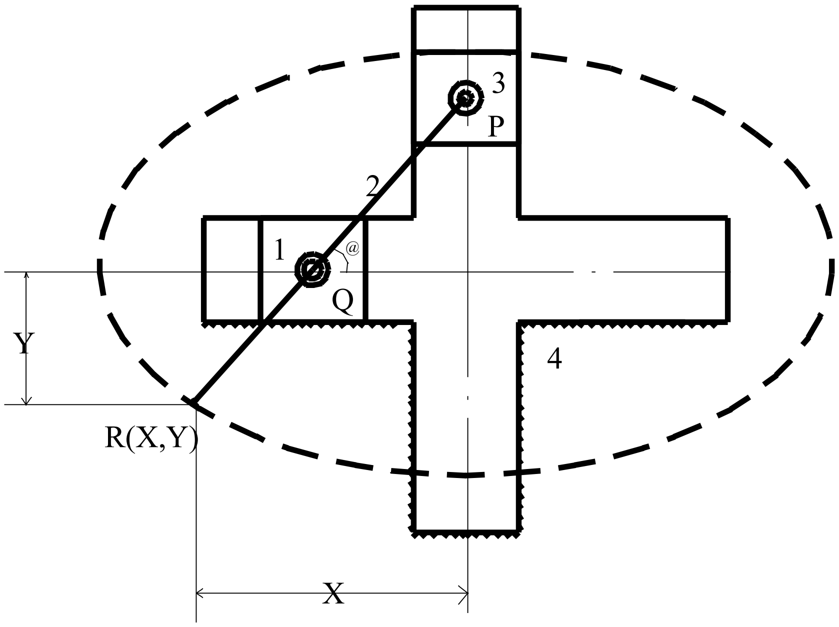
Inversions of Double slider Crank chain:

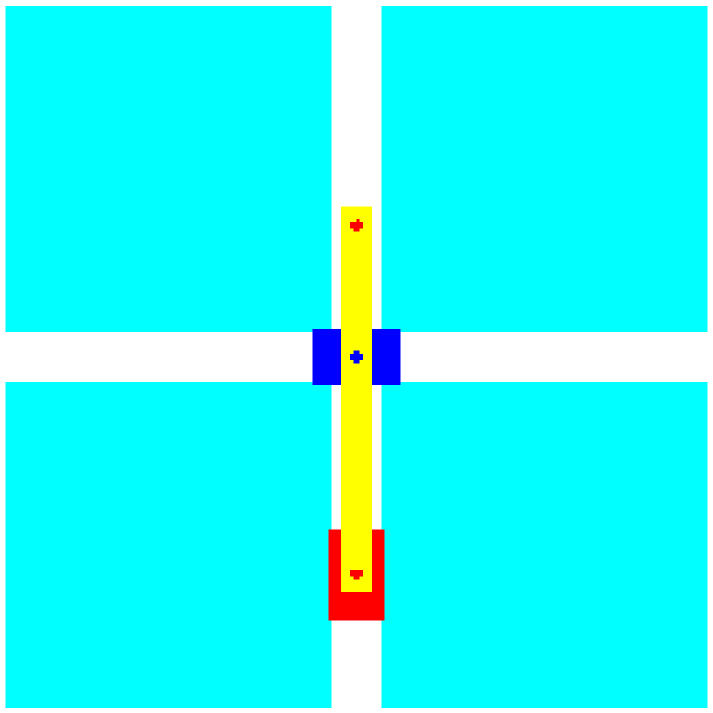
*Elliptical trammel.

*Scotch yoke mechanism.

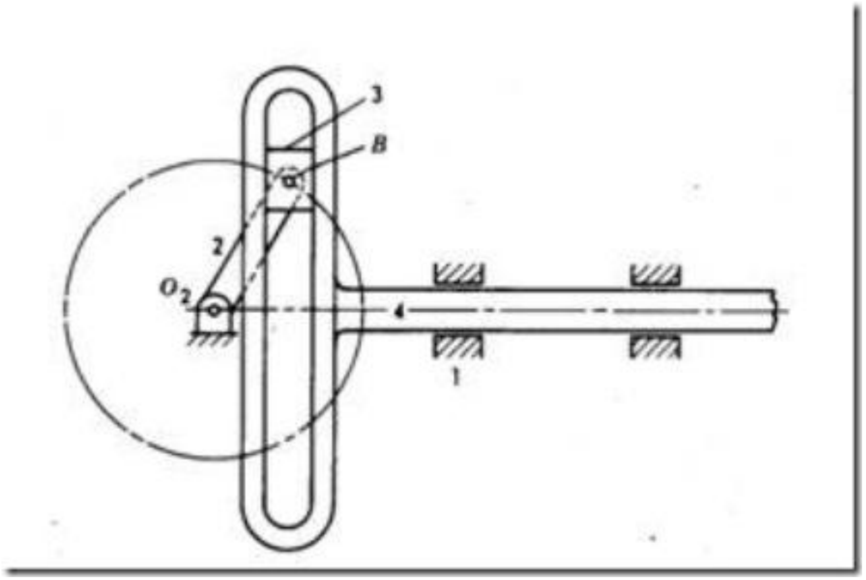
*Oldham‟s Coupling.

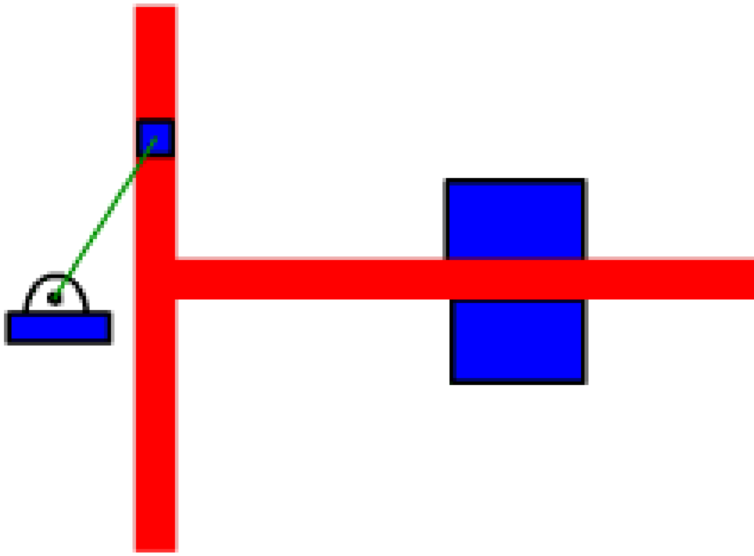
Elliptical Trammel:



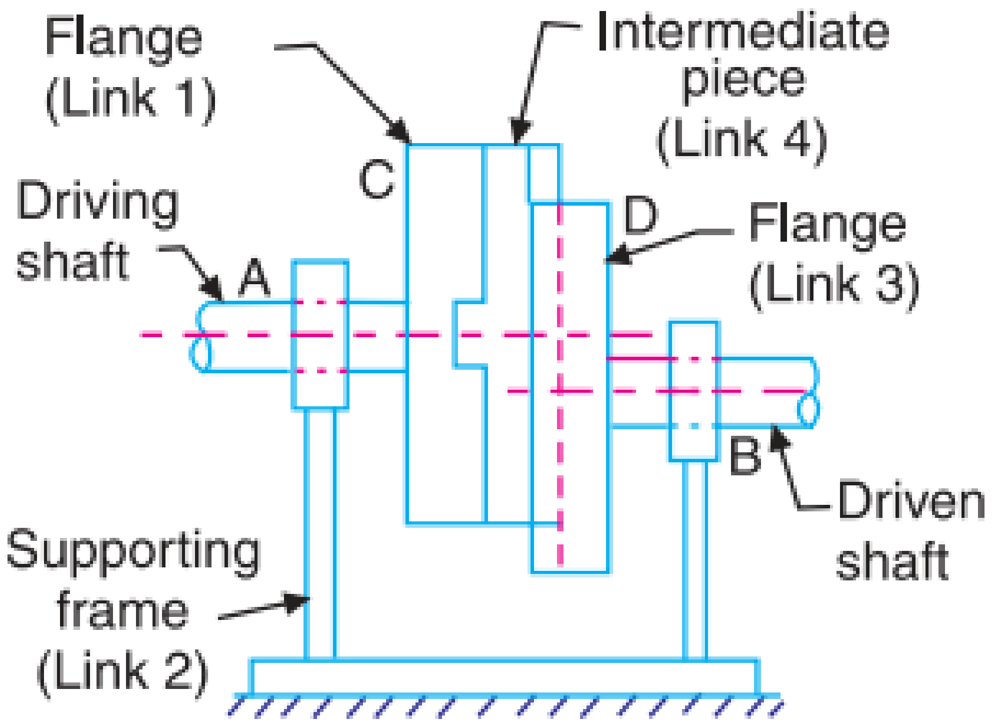


Scotch yoke mechanism:





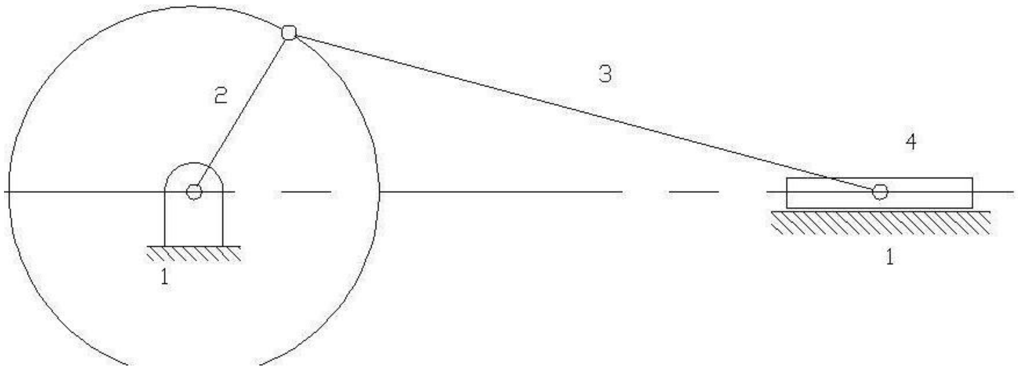
Oldham's coupling:

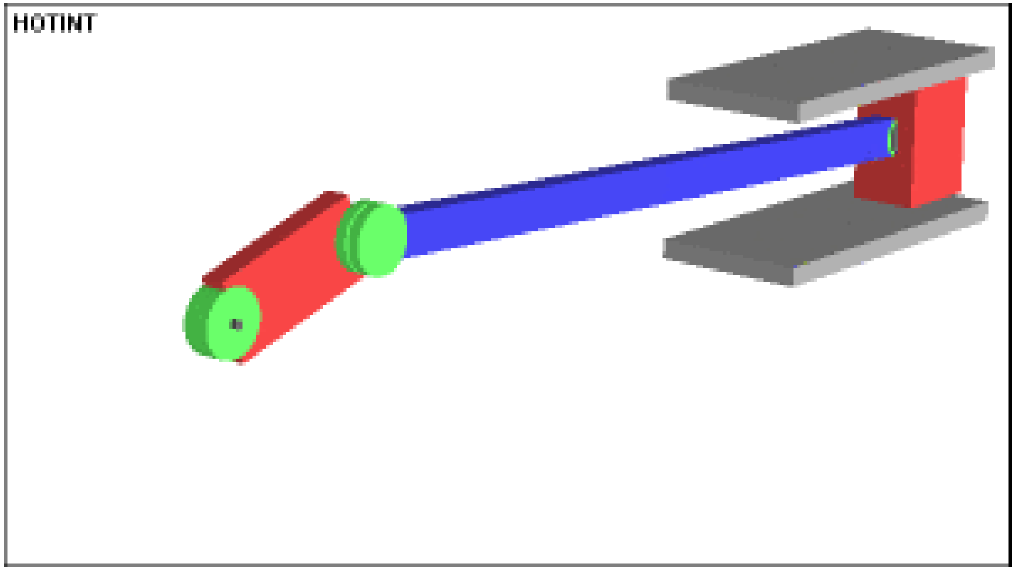


(a)

Slider crank Chain Mechanism:

It is a four bar chain having one sliding pair and three turning pairs.





Inversions of a Slider crank chain:

There are four inversions in a single slider chain mechanism. They are:

Reciprocating engine mechanism (1^{st} inversion)

Oscillating cylinder engine mechanism (2^{nd} inversion)

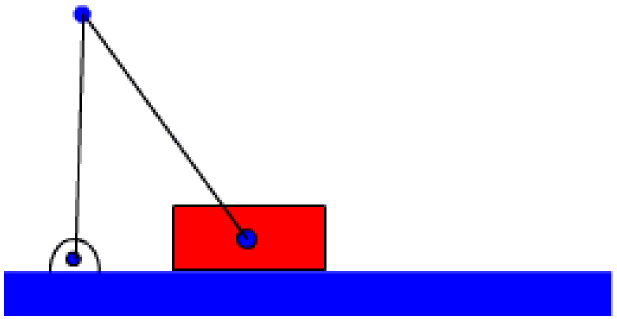
Crank and slotted lever mechanism (2^{nd} inversion)

Whitworth quick return motion mechanism (3^{rd} inversion)

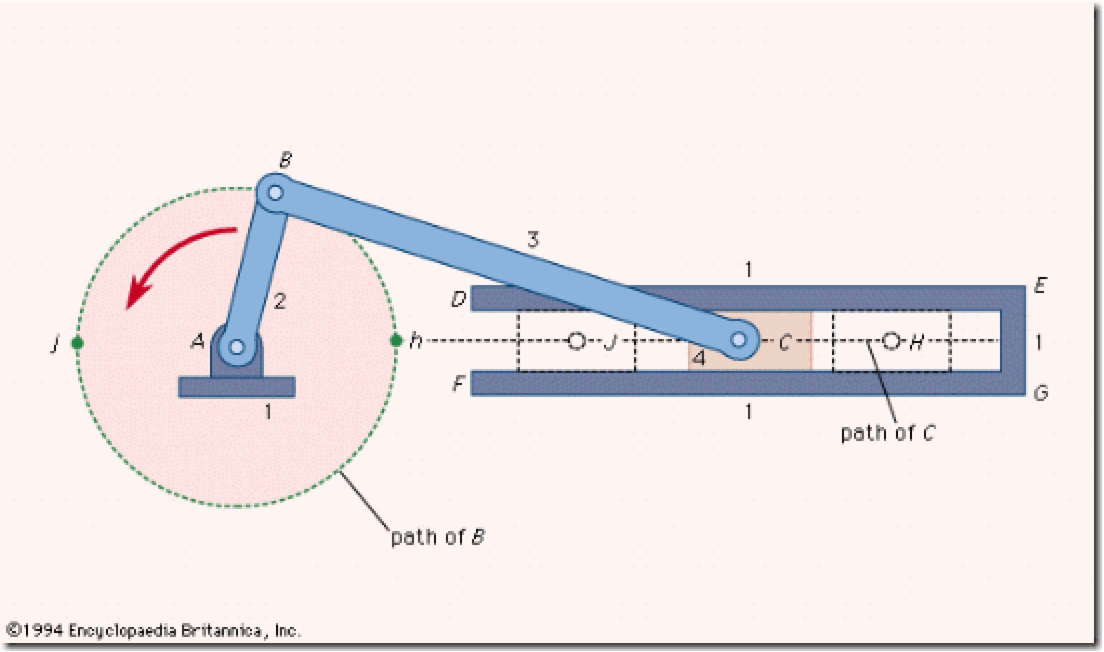
Rotary engine mechanism (3^{rd} inversion)

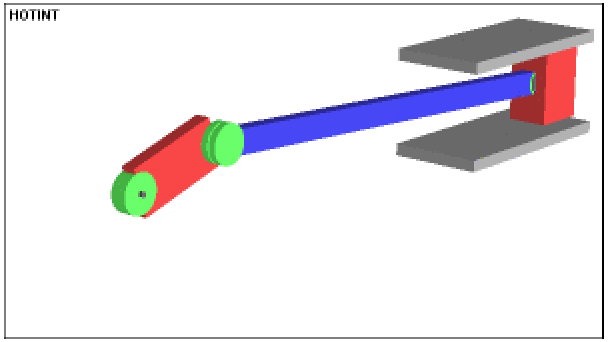
Bull engine mechanism (4^{th} inversion)

Hand Pump (4^{th} inversion)

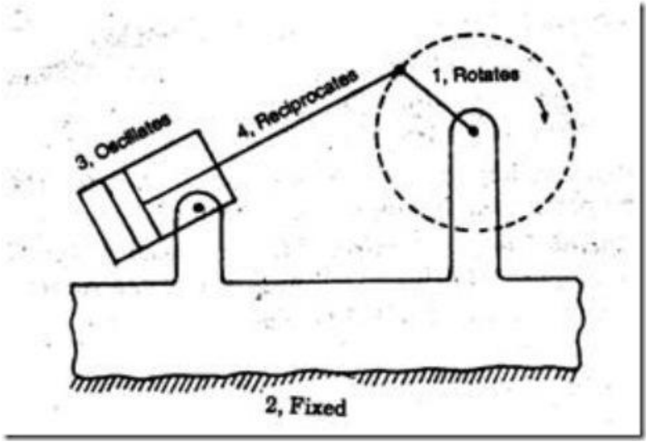


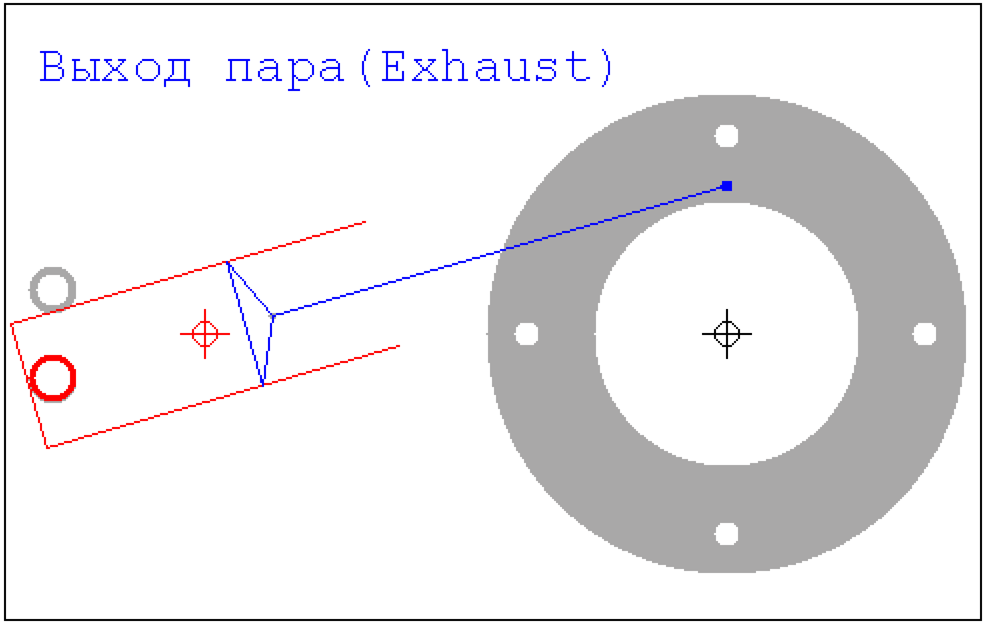
Reciprocating engine mechanism (1st inversion)



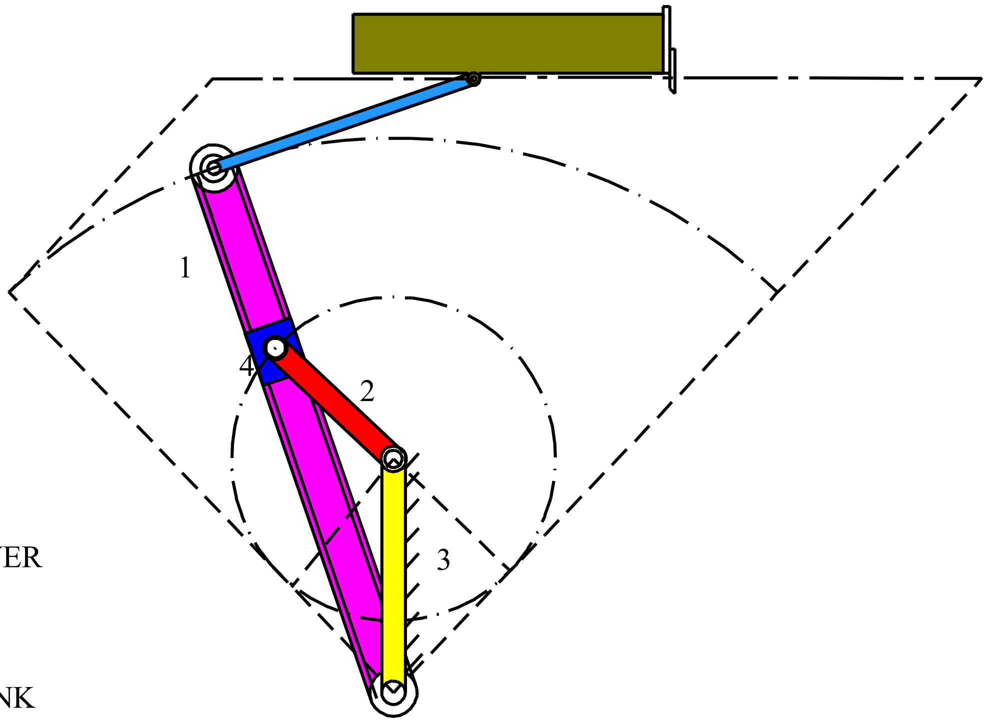


Oscillating cylinder engine mechanism (2nd inversion)

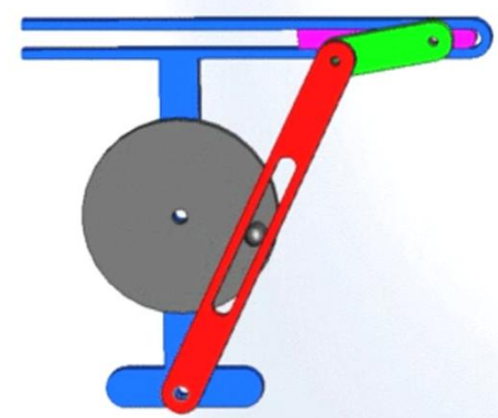




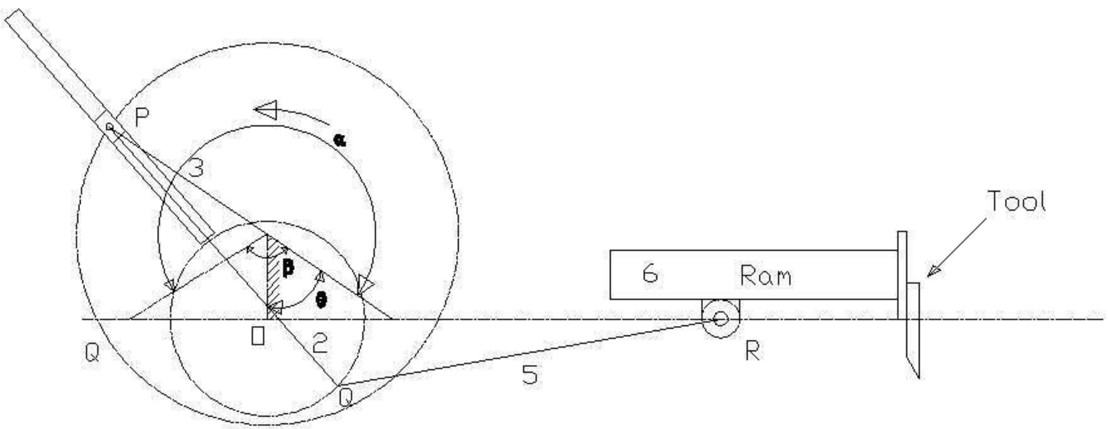
Crank and slotted lever mechanism (2nd inversion)



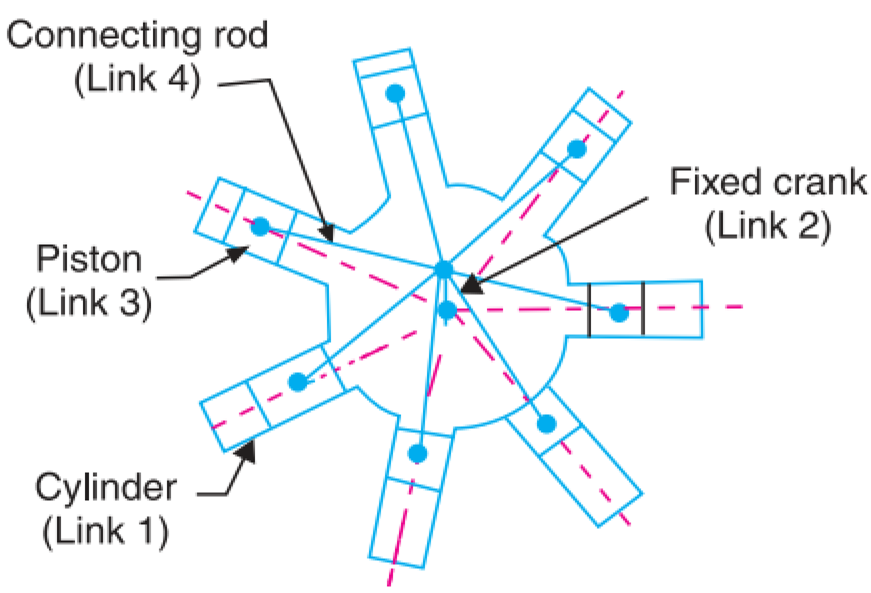
1-SLOTTED LEVER
2-CRANK
3-FIXED LINK
4-SLIDER
5-FLOATING LINK
6- TOOL HOLDER

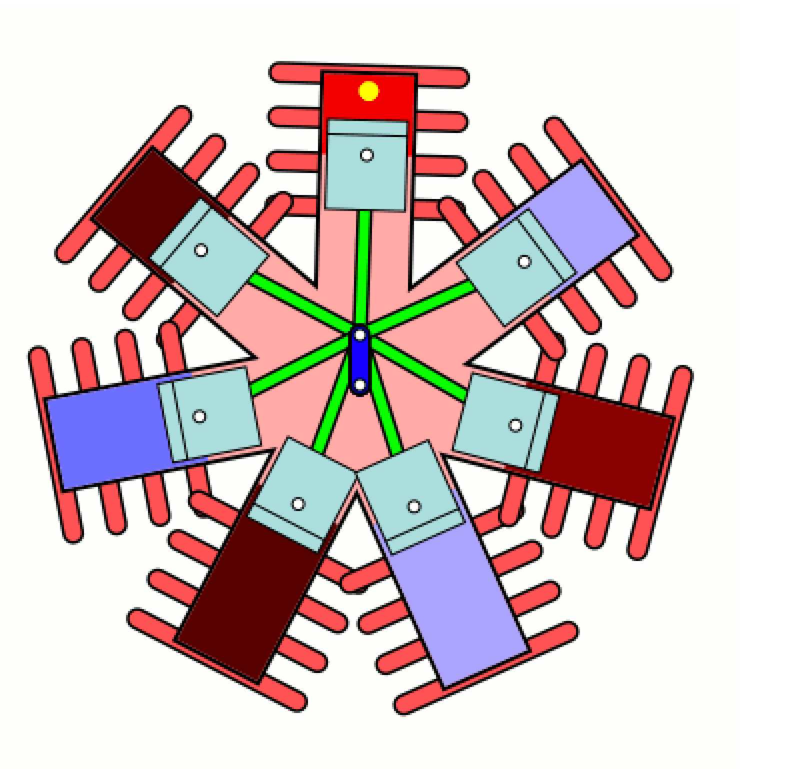


Whitworth quick return motion mechanism (3rd inversion)

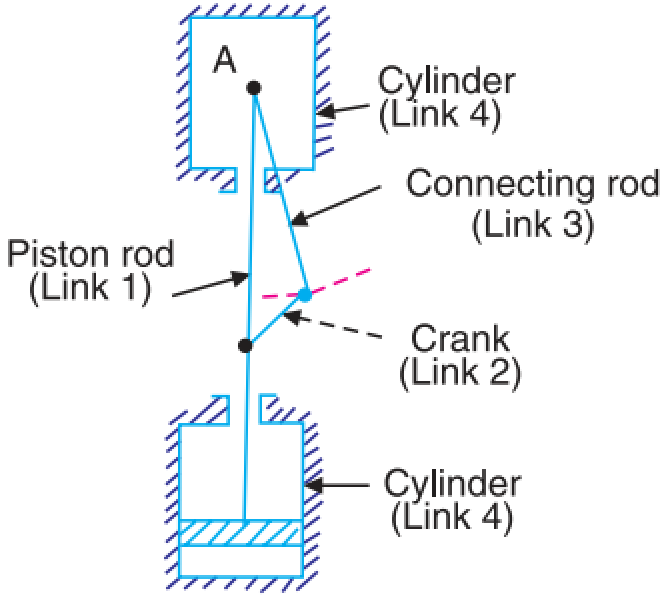


Rotary engine mechanism (3^{rd} inversion)

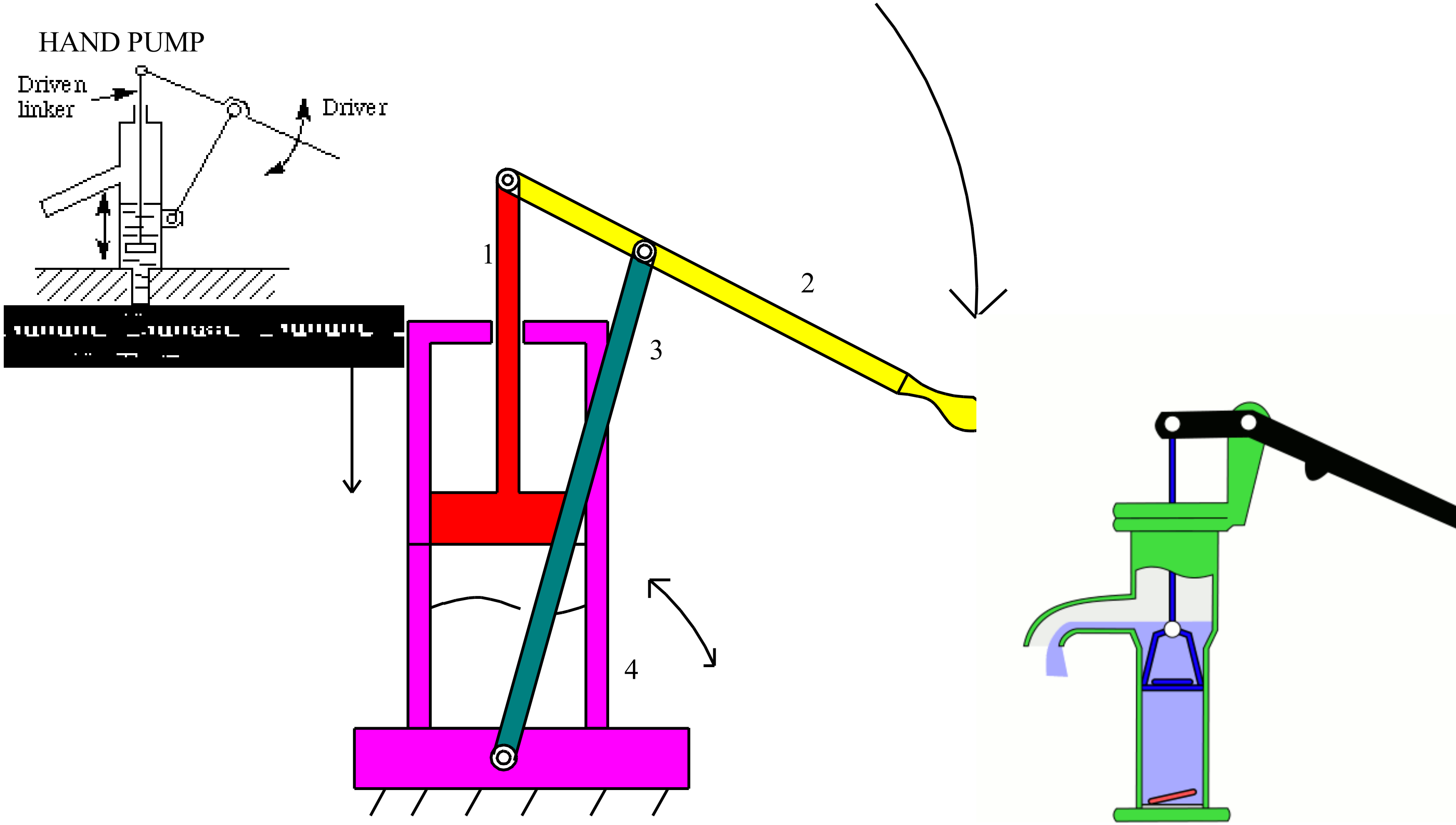




Bull engine mechanism (4th inversion)



Hand Pump (4th inversion)



Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

$$\boldsymbol{F = 3\,(n - 1) - 2\,j_1 - j_2}$$

F = Mobility or number of degrees of freedom

n = Number of links including frame.

j_1 = Joints with single (one) degree of freedom.

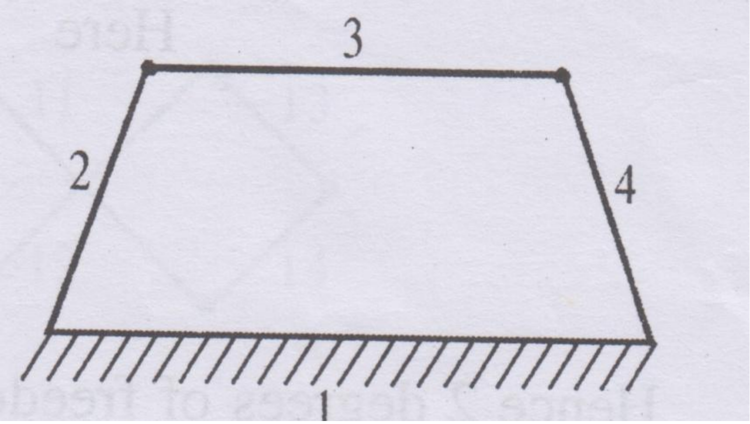
j_2 = Joints with two degrees of freedom.

If $F > 0$, results a mechanism with ‘F’ degrees of freedom.

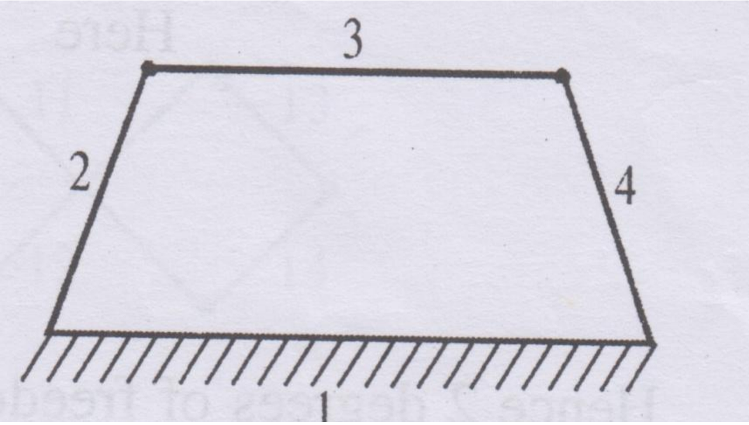
$F = 0$, results in a statically determinate structure.

$F < 0$, results in a statically indeterminate structure.

Determine the mobility of the mechanisms



Determine the mobility of the mechanisms



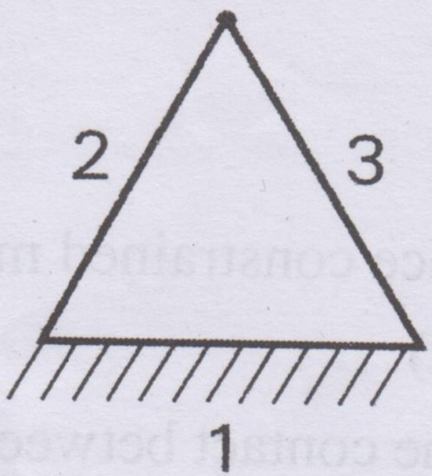
$$\text{Here } j_1 = n + l - 1 = 4 + 1 - 1 = 4$$

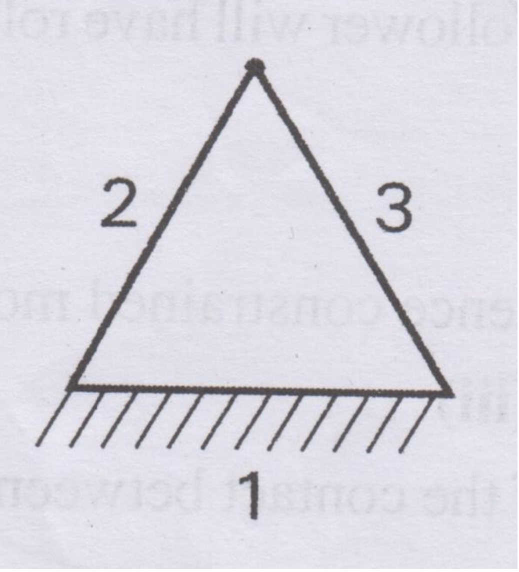
$$\text{We have } F = 3(n-1) - 2j_1 - j_2$$

$$F = 3(4-1) - 2 \times 4 - 0$$

$$\therefore F = 1 > 0$$

Hence the mechanism has constrained motion i.e., one degree of freedom.


2
3
1



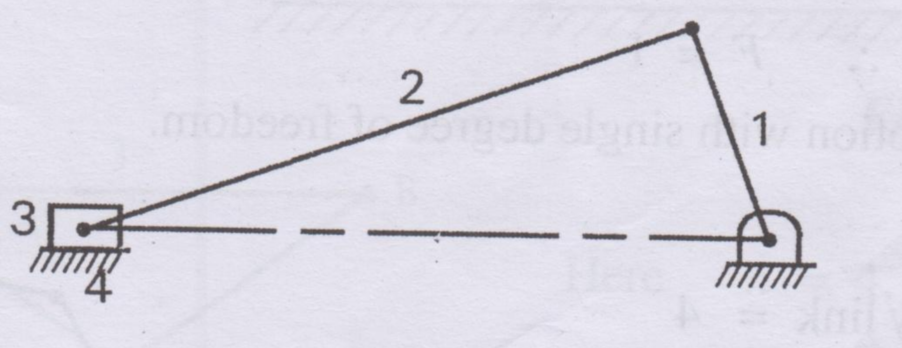
Here $j_1 = n+l-1 = 3+1-1=3$

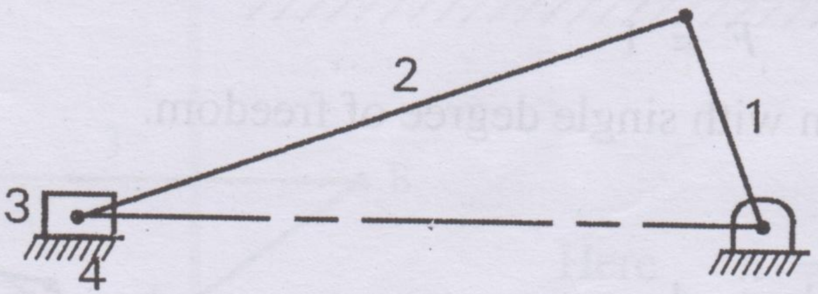
$j_2 = 0$ and $n = 3$

$F = 3\ (3-1) - 2 \times 3 - 0$

$\therefore \quad F = 0$

Hence the mechanism is statically determinate structure.


2
1
3
4



Here $j_1 = n+l-1 = 4+1-1 = 4$

$j_2 = 0, n = 4$

$F = 3(4-1) - 2 \times 4 - 0$

$\therefore \quad F = 1$

Hence the mechanism has constrained motion i.e., one degree of freedom.

Case (i) Follower will have rolling and sliding

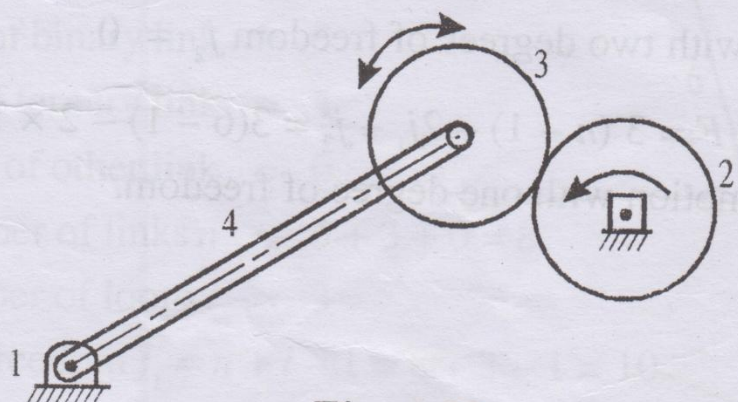
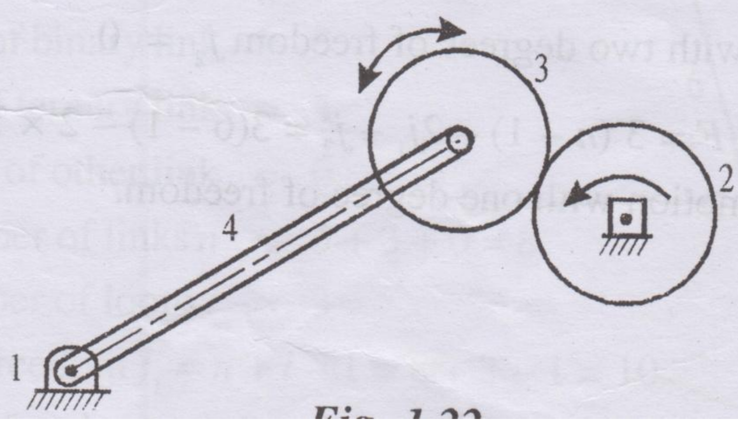


Fig. 1.22



Case (i) Follower will have rolling and sliding

Here $j_1 = 3, j_2 = 1, n = 4$

$$F = 3(4-1) - 2 \times 3 - 1$$
$$= 2 > 0$$

Hence 2 degrees of freedom.

Case (ii)

If Links 4 and 3 constitute one Link (By welding or by some other means)

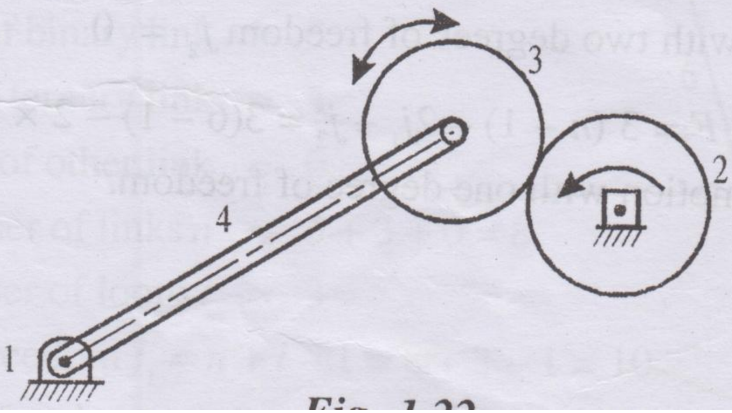
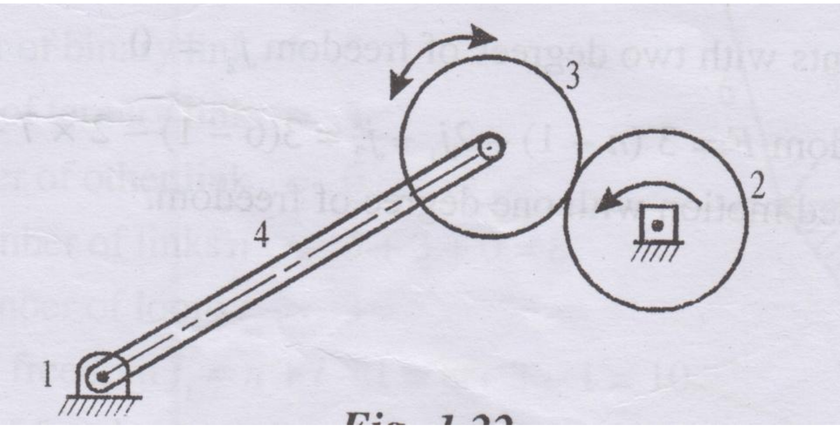


Fig. 1.22



Case (ii)

If Links 4 and 3 constitute one Link (By welding or by some other means)

then $n = 3, j_1 = 2, j_2 = 1,$

(Follower will have rolling and sliding)

$$F = 3\,(3-1) - 2 \times 2 - 1$$

$$F = 1$$

Hence constrained motion with single degree of Freedom.

Case (iii)

If the contact between Cam and Follower is Pure Rolling (No sliding)

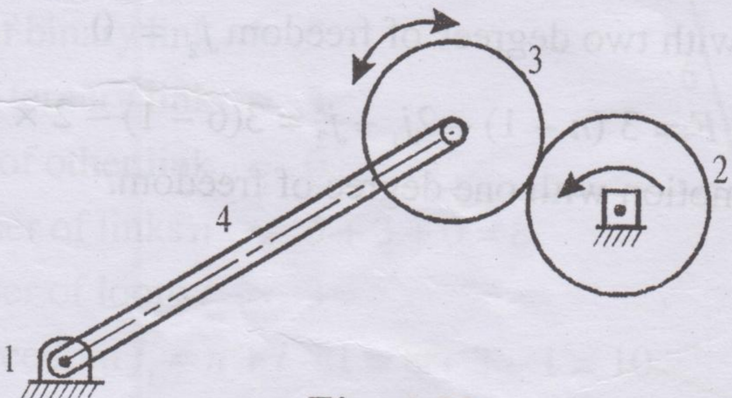
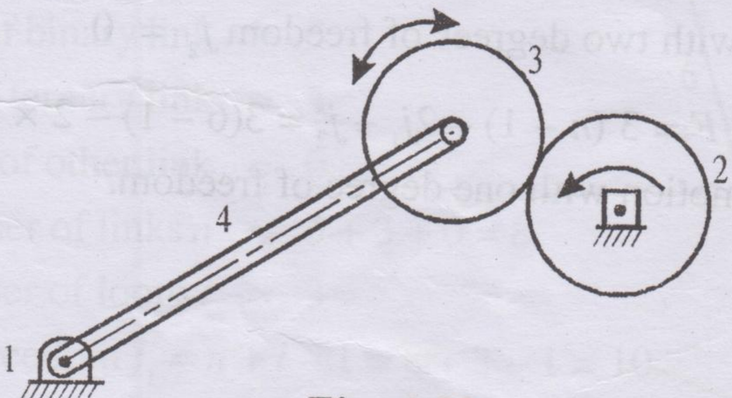


Fig. 1.22



Case (iii)

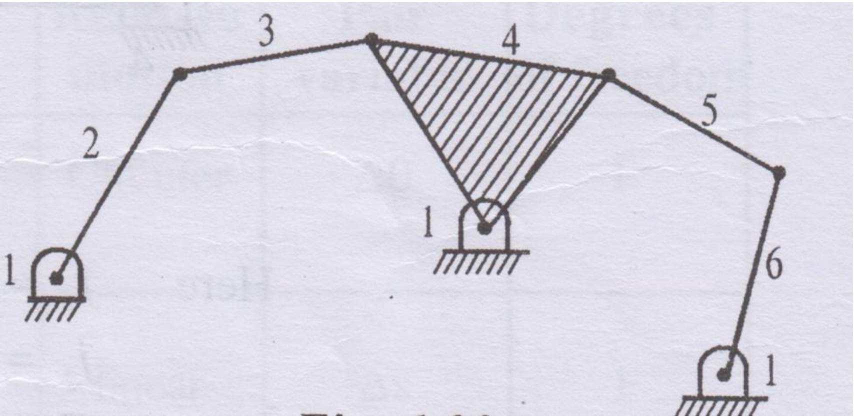
If the contact between Cam and Follower is Pure Rolling (No sliding)

$$n = 4,\ j_1 = 4,\ j_2 = 0,$$

$$F = 3\,(4-1) - 2 \times 4 - 0$$

$$\therefore \quad F = 1$$

Hence constrained motion with single degree of freedom.



3
4
2
5
1
1
6
1

Number of binary link = 4

Number of ternary link = 2

Number of other link = 0

$\therefore$ Total number of links $n = 4 + 2 + 0 = 6$

Number of loops $l = 2$

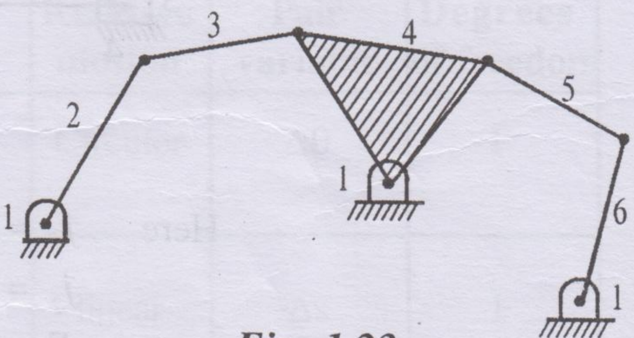


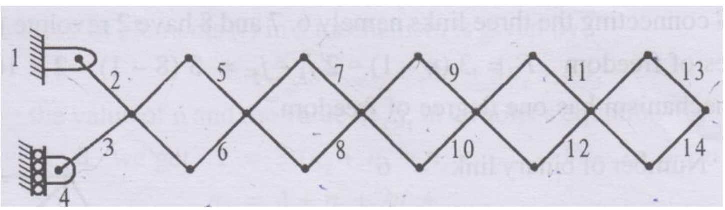
Fig. 1.23

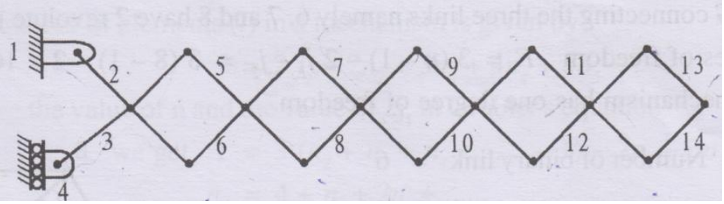
Joints with one degree of freedom $j_1 = n + l - 1 = 6 + 2 - 1 = 7$

Joints with two degrees of freedom $j_2 = 0$

Degrees of freedom $F = 3\,(n - 1) - 2j_1 - j_2 = 3(6 - 1) - 2 \times 7 - 0 = 1$

Hence constrained motion with one degree of freedom.


1
2
3
4
5
6
7
8
9
10
11
12
13
14



Here $n = 14, j_l = n + l - 1 = 13 + 6 - 1 = 18$,

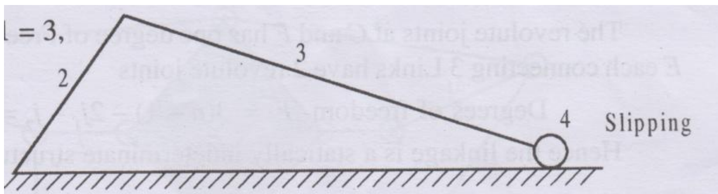
Link 4 will have sliding and Rolling

$\therefore \quad j_2 = 1$ (joint with 2 degrees of freedom)

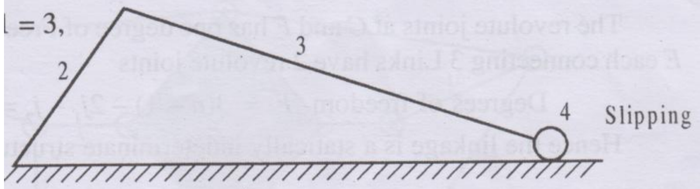
$F = 3(14 - 1) - 2 \times 18 - 1$

$\therefore \quad F = 2$

Hence two degrees of freedom.


= 3,
2
3
4
Slipping

l = 3,



Here $n = 4, j_1 = n + l - 1 = 3 + 1 - 1 = 3,$

$j_2 = 1$ (Rolling and sliding)

$F = 3(4-1) - 2 \times 3 - 1$

$F = 2$

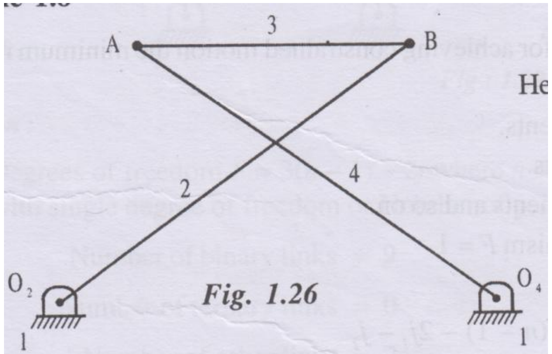


Fig. 1.26

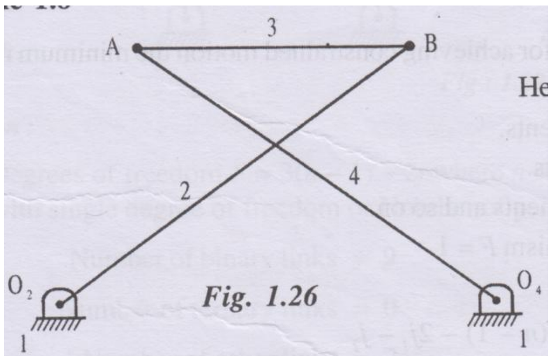


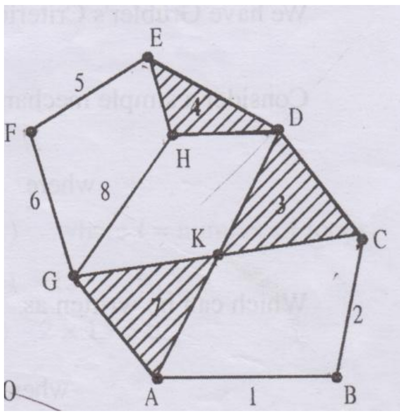
Fig. 1.26

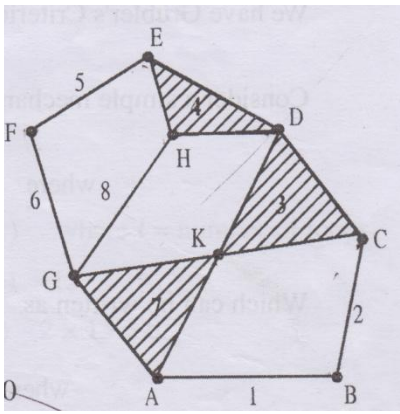
Here $n = 4,\ j_1 = n + 1 - 1 = 4 + 1 - 1 = 4,$

$$j_2 = 0,$$

$$F = 3\,(4 - 1) - 2 \times 4 - 0$$

$$\therefore F = 1$$


E
5
4
D
F
H
6
8
3
K
C
G
7
2
A
1
B



Number of binary link $=$ 5

Number of ternary link $=$ 3

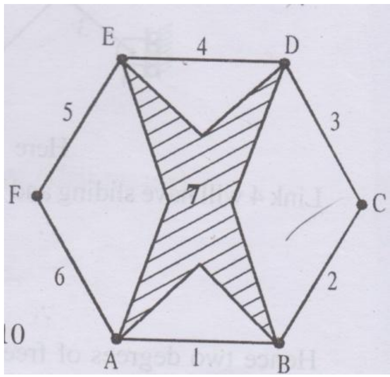
Number of other link $=$ 0

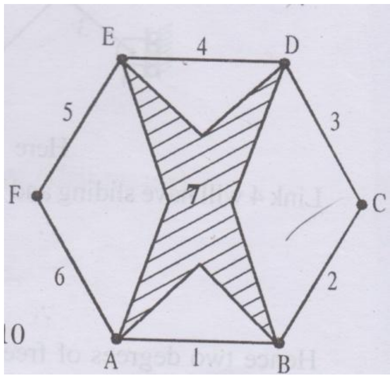
Total number of links $n = 5 + 3 + 0 = 8$

Number of loops $l = 3$

Joints with one degree of freedom $j_1 = n + l - 1 = 8 + 3 - 1 = 10$

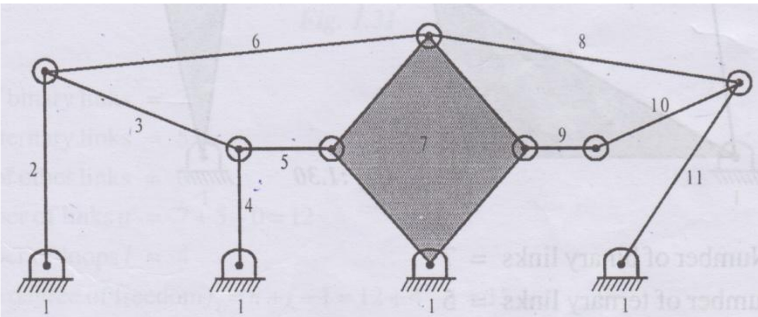
Joints with two degrees of freedom $j_2 = 0$

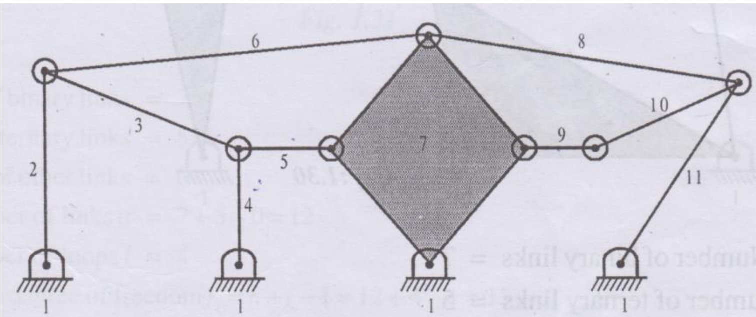

E
4
D
5
3
7
F
C
6
2
A
1
B



Degrees of freedom $F = 3(n-1) - 2j_1 - j_2 = 3\,(7-1) - 2 \times 10 - 0 = -2$

Hence the linkage is a statically indeterminate structure or super structure.


6
8
10
3
7
9
5
2
4
11
1
1
1
1



Number of binary links $= 9$

Number of ternary links $= 0$

Number of other links $= 2$

$\therefore$ Total number of links $n = 9 + 0 + 2 = 11$

Number of loops $l = 5$

Number of joints with one degree of freedom $j_1 = (n + l - 1)$ where l = number of loops

$$= 11 + 5 - 1 = 15$$

$$\therefore F = 3(11 - 1) - 2 \times 15 = 0$$

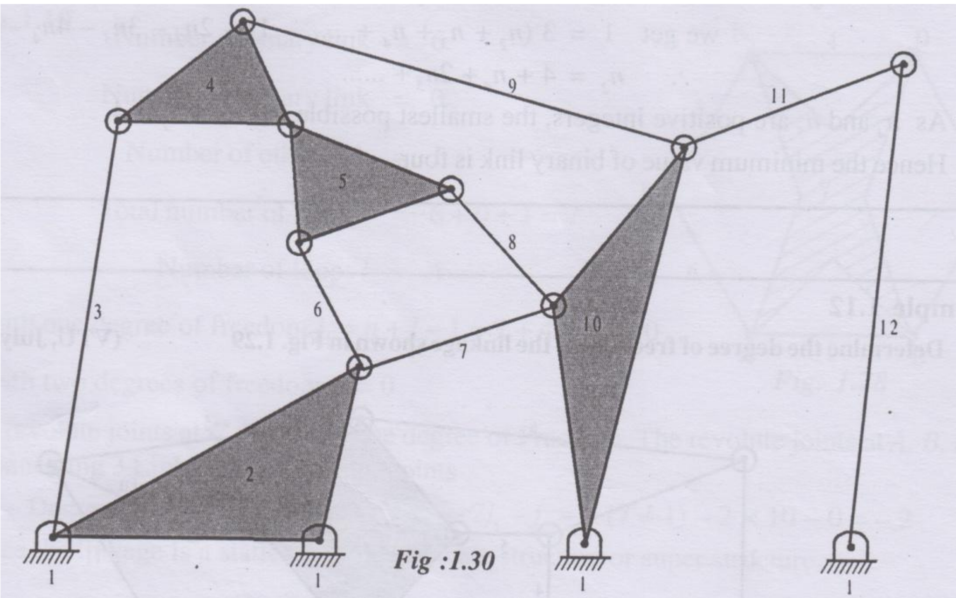


Fig :1.30

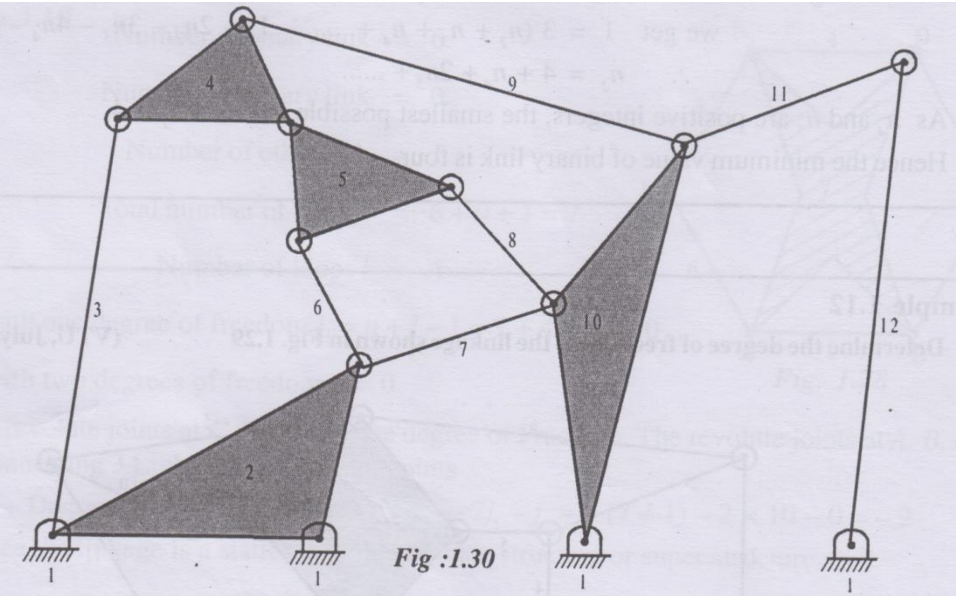


Fig :1.30

Number of binary links = 7

Number of ternary links = 5

Number of other links = 0

$\therefore$ Total number of links n = 7 + 5 + 0 = 12

Number of loops l = 5

Number of joints or pairs j_1 = $n + l - 1 = 12 + 5 - 1 = 16$

Degrees of freedom $F = 3(n-1) - 2j = 3(12-1) - 2 \times 16 = 1$

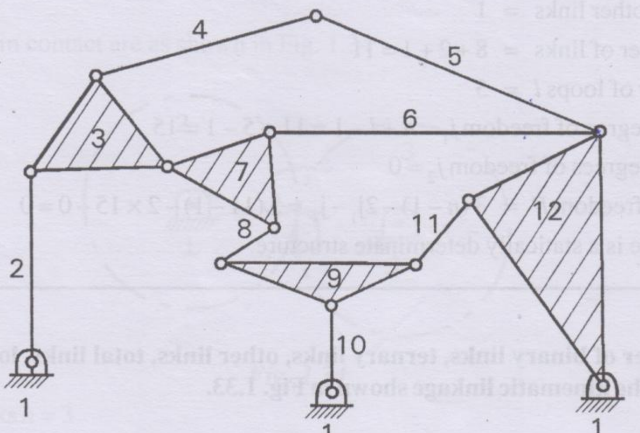
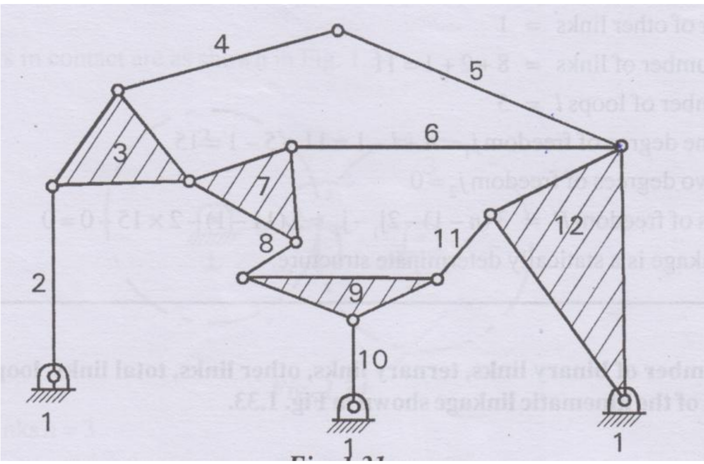


Fig. 1.31



Number of binary links $= 7$

Number of ternary links $= 5$

Number of other links $= 0$

$\therefore$ Total number of links $n = 7 + 5 + 0 = 12$

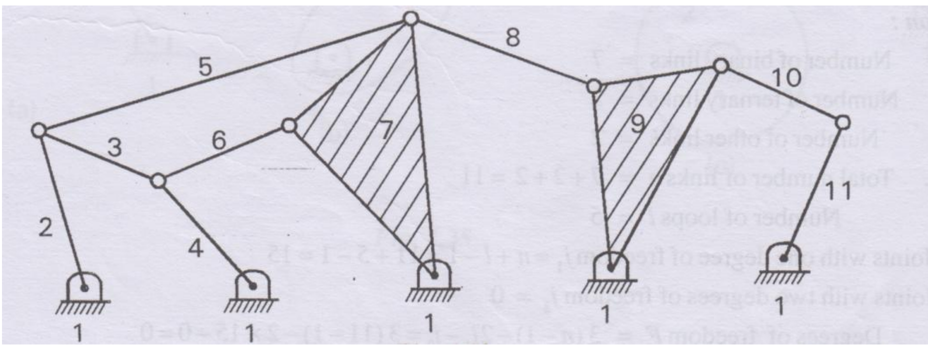
Number of loops $l = 4$

Joints with one degree of freedom $j_1 = n + l - 1 = 12 + 4 - 1 = 15$

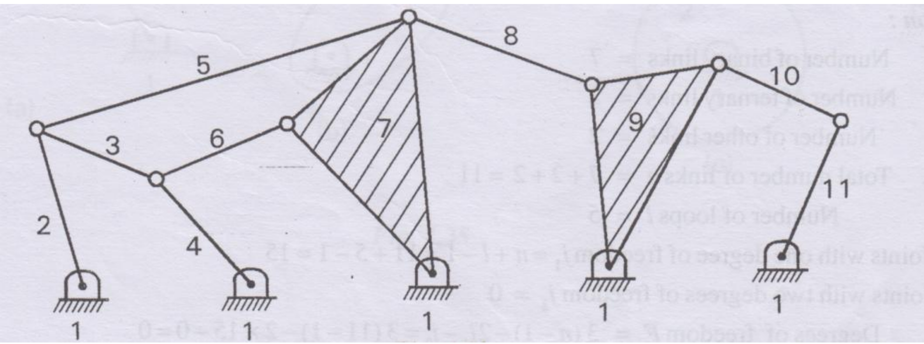
Joints with two degrees of freedom $j_2 = 0$

Degrees of freedom $F = 3(n-1) - 2j_1 - j_2 = 3(12-1) - 2 \times 15 - 0 = 3$

Hence it is a mechanism with three degrees of freedom.



5
8
10
7
9
3
6
11
2
4
1
1
1
1
1



Number of binary links = 8

Number of ternary links = 2

Number of other links = 1

$\therefore$ Total number of links = $8 + 2 + 1 = 11$

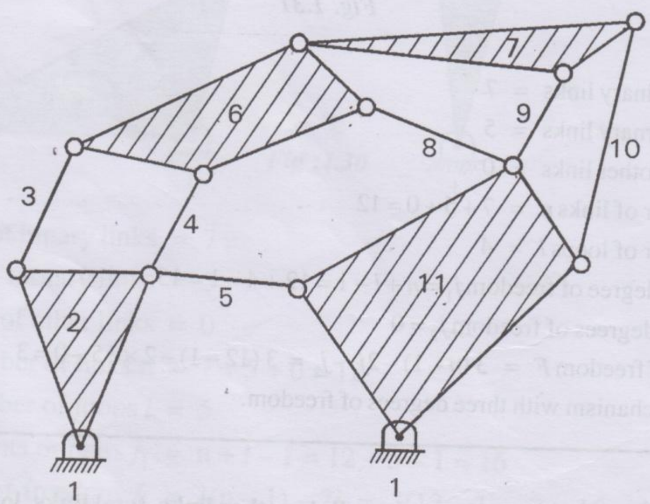
Number of loops l = 5

Joints with one degree of freedom $j_1 = n + l - 1 = 11 + 5 - 1 = 15$

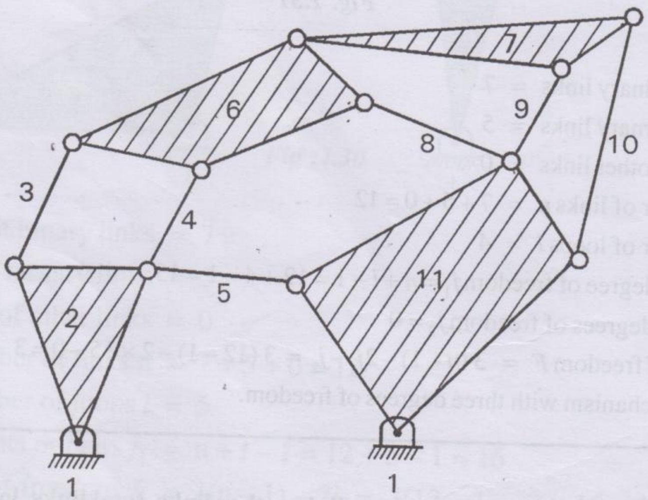
Joints with two degrees of freedom $j_2 = 0$

Degrees of freedom $F = 3(n-1) - 2j_1 - j_2 = 3(11-1) - 2 \times 15 - 0 = 0$

Hence the linkage is a statically determinate structure.



1
2
3
4
5
6
7
8
9
10
11
1



Number of binary links = 7

Number of ternary links = 2

Number of other links = 2

∴ Total number of links $n = 7 + 2 + 2 = 11$

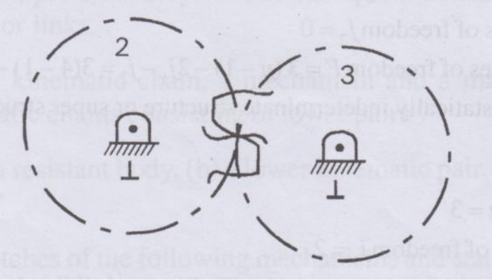
Number of loops $l = 5$

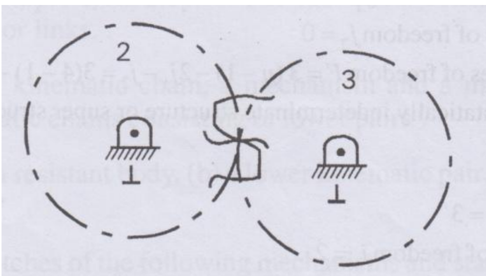
Joints with one degree of freedom $j_1 = n + l - 1 = 11 + 5 - 1 = 15$

Joints with two degrees of freedom $j_2 = 0$

Degrees of freedom $F = 3(n-1) - 2j_1 - j_2 = 3(11-1) - 2 \times 15 - 0 = 0$

Hence the linkage is a statically determinate structure.


2
3



Total number of links $n = 3$

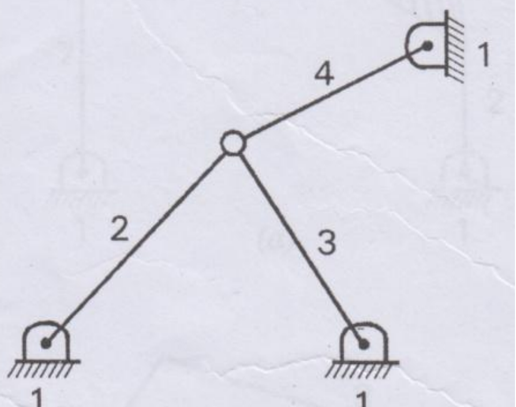
Joints with one degree of freedom $j_1 = 2$ (i.e., 1 and 2, 1 and 3)

Joints with two degree of freedom $j_2 = 1$ (i.e., 2 and 3, Rolling and sliding)

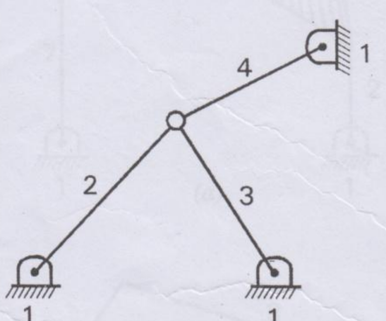
Degrees of freedom $F = 3(n-1) - 2j_1 - j_2 = 3(3-1) - 2 \times 2 - 1 = 1$

$\therefore$ Mobility of two spur gears in contact = 1.

Hence it is a mechanism with one degree of freedom.



(a)



(a)

Total number of links $n = 4$

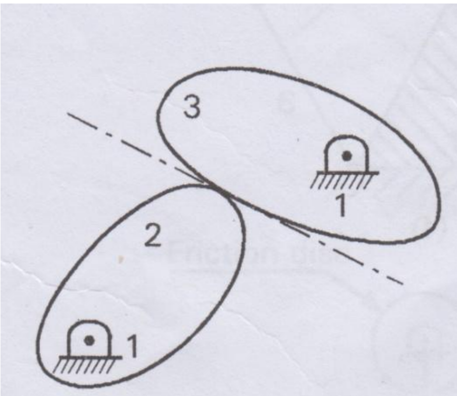
Number of loops $l = 2$

Joints with one degree of freedom $j_1 = n + l - 1 = 4 + 2 - 1 = 5$

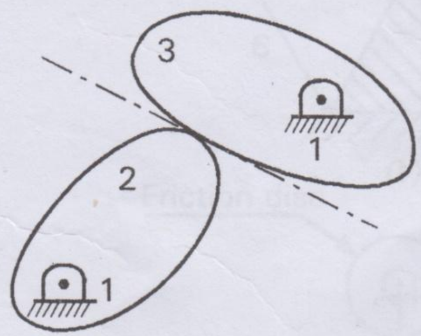
Joints with two degrees of freedom $j_2 = 0$

$\therefore$ Mobility or Degrees of freedom $F = 3(n-1) - 2j_1 - j_2 = 3(4-1) - 2 \times 5 - 1 = -1$

Hence the linkage is a statically indeterminate structure or super structrue.



3
1
2
1



Total number of links $n = 3$

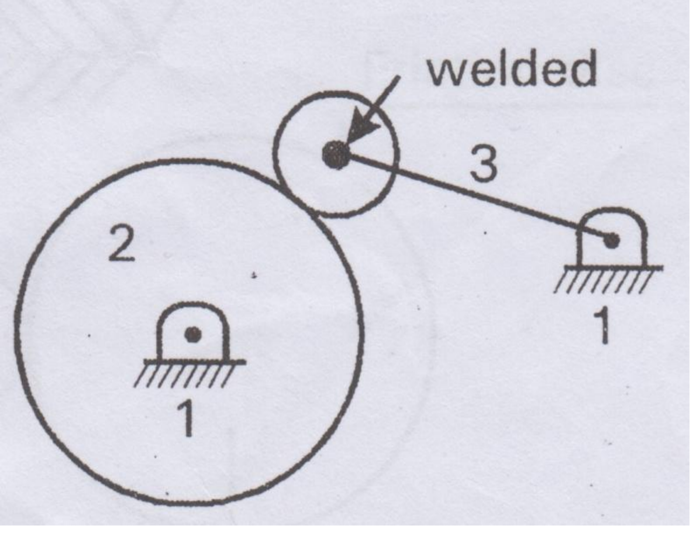
Joints with one degree of freedom $j_1 = 2$

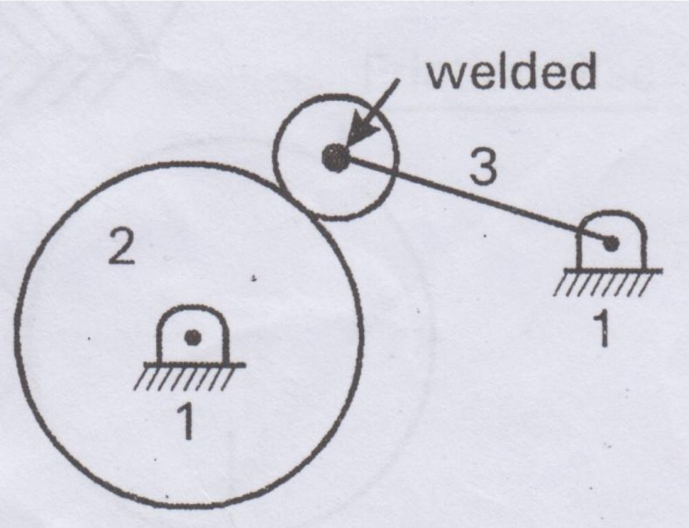
Joints with two dregrees of freedom $j_2 = 1$ (i.e., Rolling and sliding between 2 and 3)

$\therefore$ Mobility or Degree of freedom $F = 3(n-1) - 2j_1 - j_2$

$$= 3(3-1) - 2 \times 2 - 1 = 1$$

Hence it is a mechanism with one degree of freedom.


welded
3
2
1
1



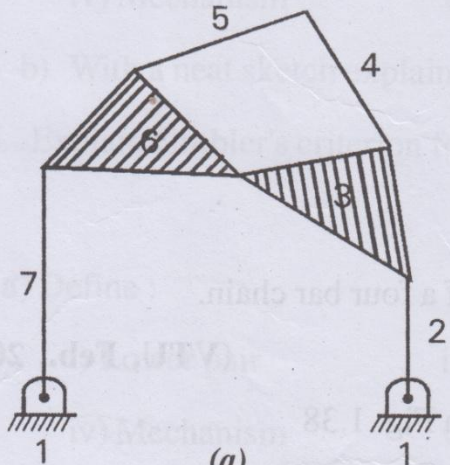
Total number of links $n = 3$

Joints with one degree of freedom $j_1 = 2$

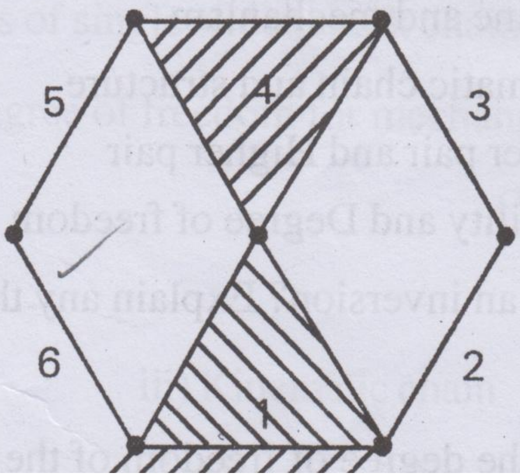
Joints with two degrees of freedom $j_2 = 1$

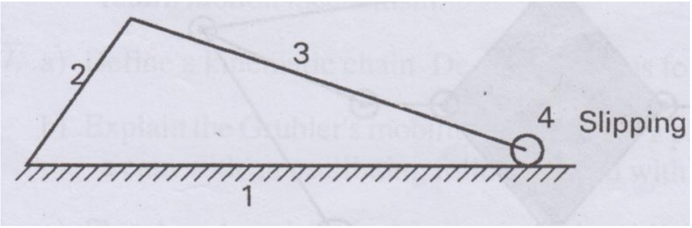
$\therefore$ Mobility or Degrees of freedom $F = 3(n-1) - 2j_1 - j_2 = 3(3-1) - 2 \times 2 - 1 = 1$

Hence it is a mechanism with one degree of freedom.



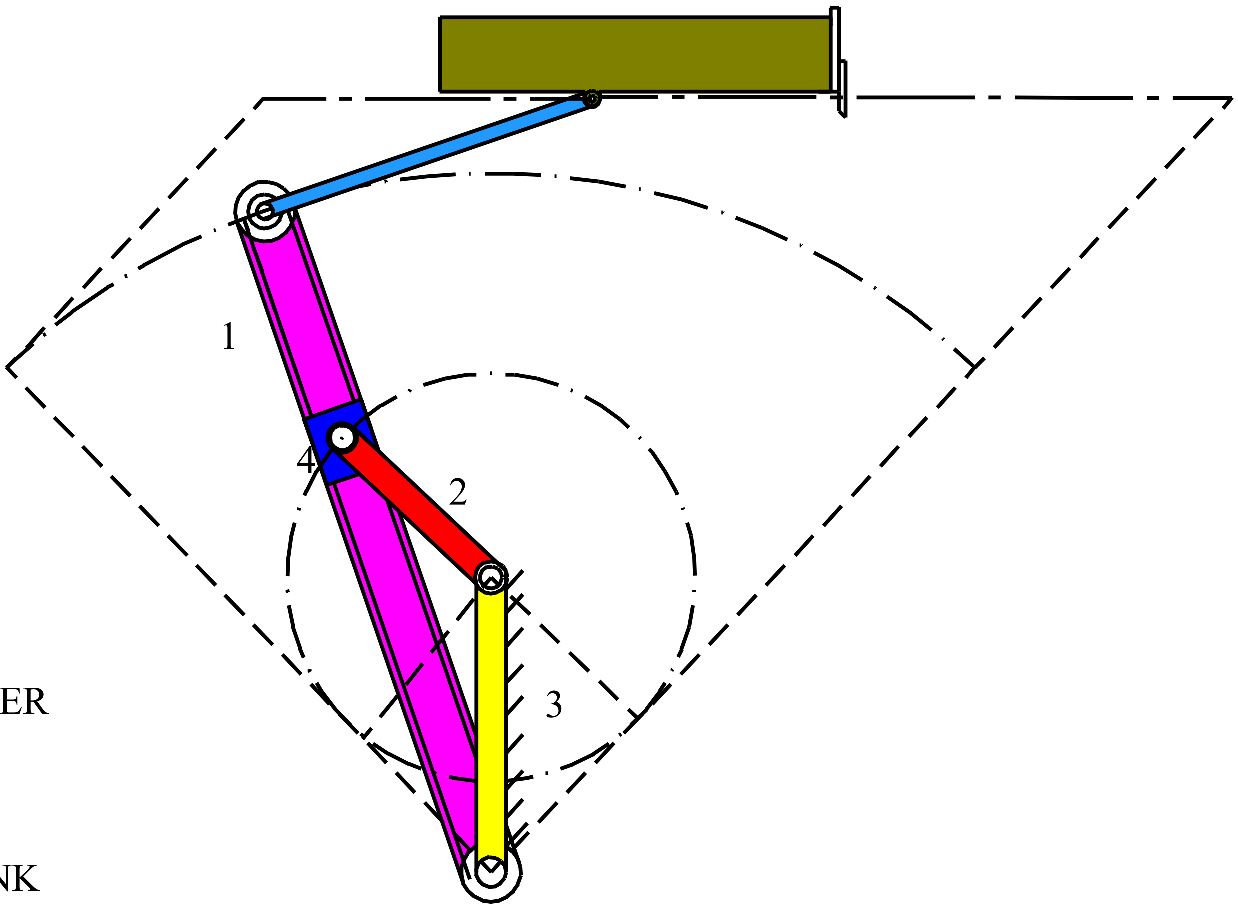
(a)


5
4
3
6
1
2

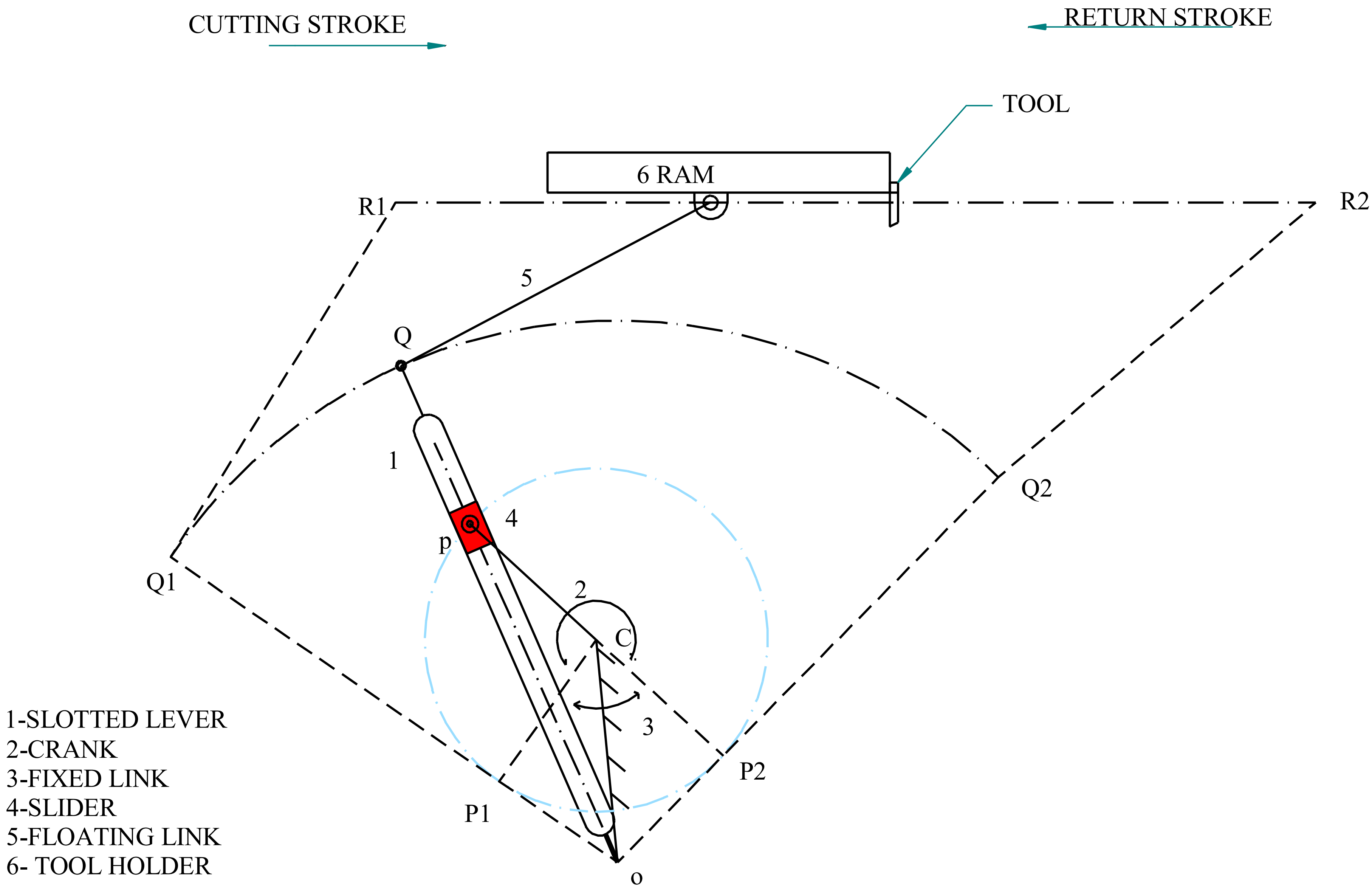


2
3
4
Slipping
1

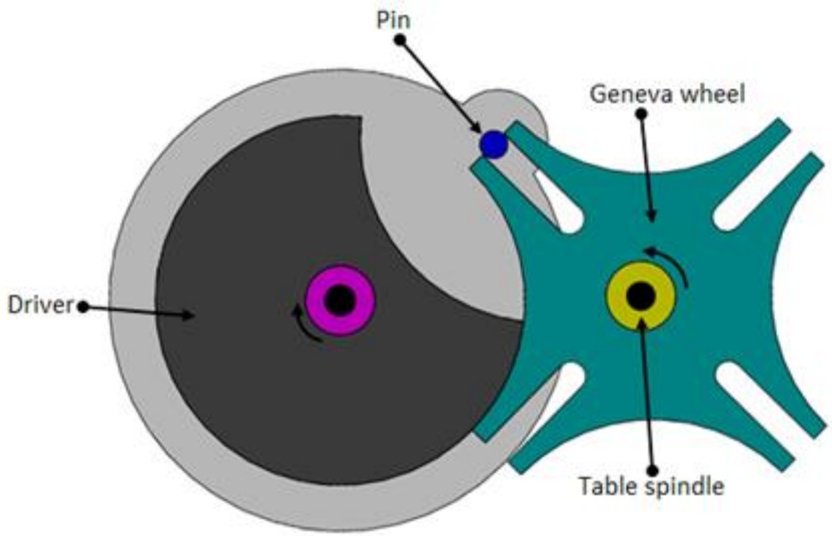
Crank and slotted lever mechanism

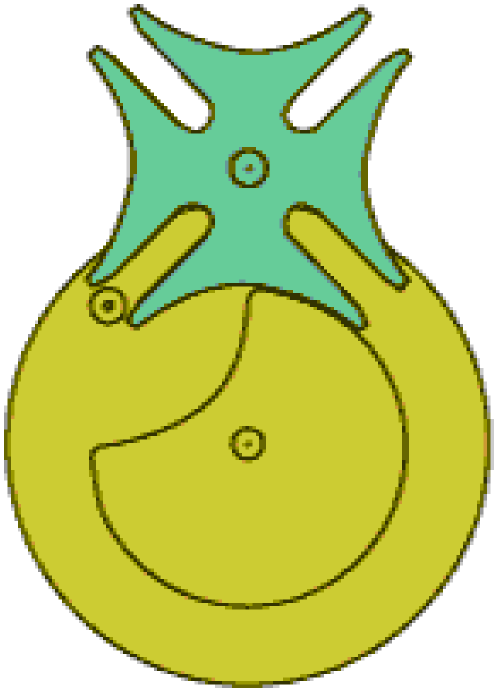


1-SLOTTED LEVER
2-CRANK
3-FIXED LINK
4-SLIDER
5-FLOATING LINK
6- TOOL HOLDER

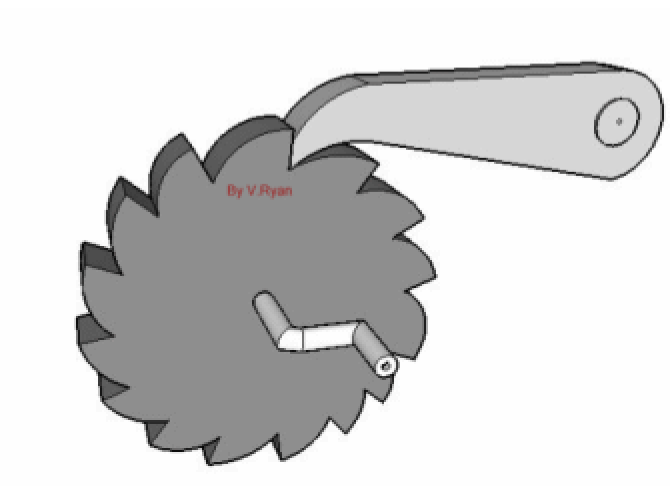

CUTTING STROKE
RETURN STROKE
TOOL
6 RAM
R1
R2
5
Q
Q1
Q2
1
4
p
2
C
3
P1
P2
o
1-SLOTTED LEVER
2-CRANK
3-FIXED LINK
4-SLIDER
5-FLOATING LINK
6- TOOL HOLDER

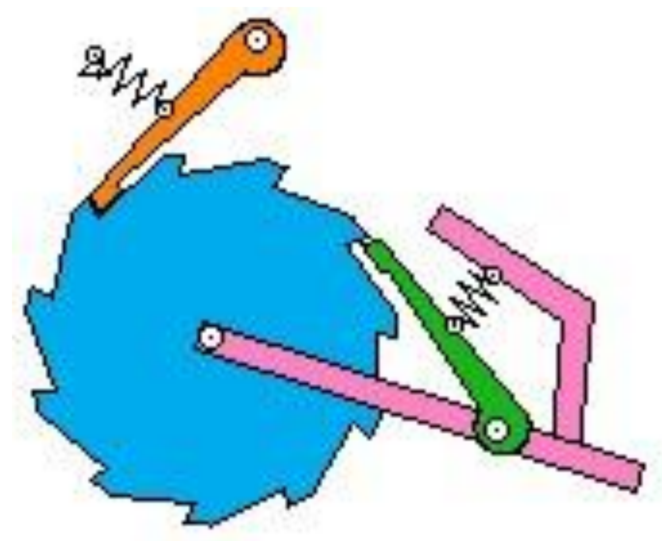
Geneva Mechanism

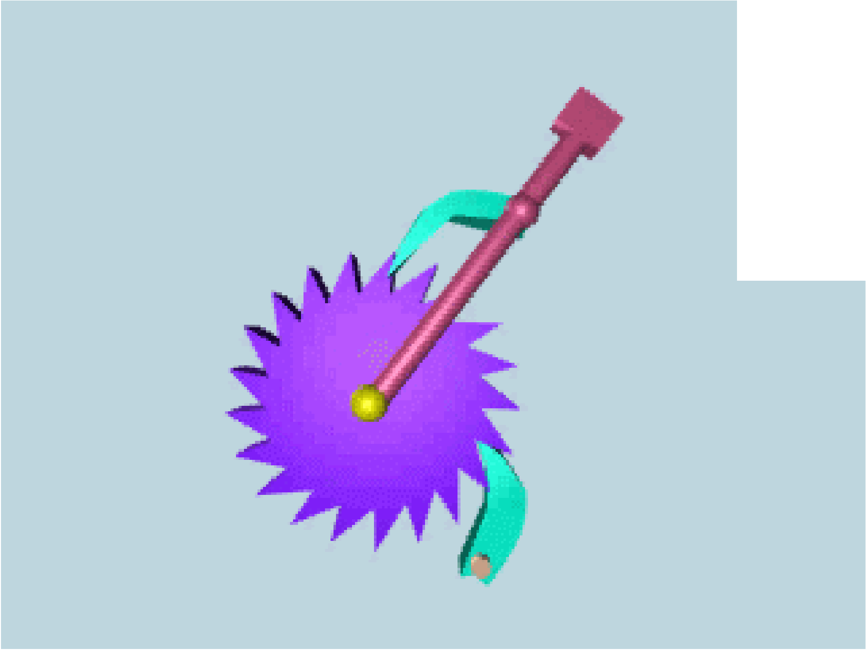


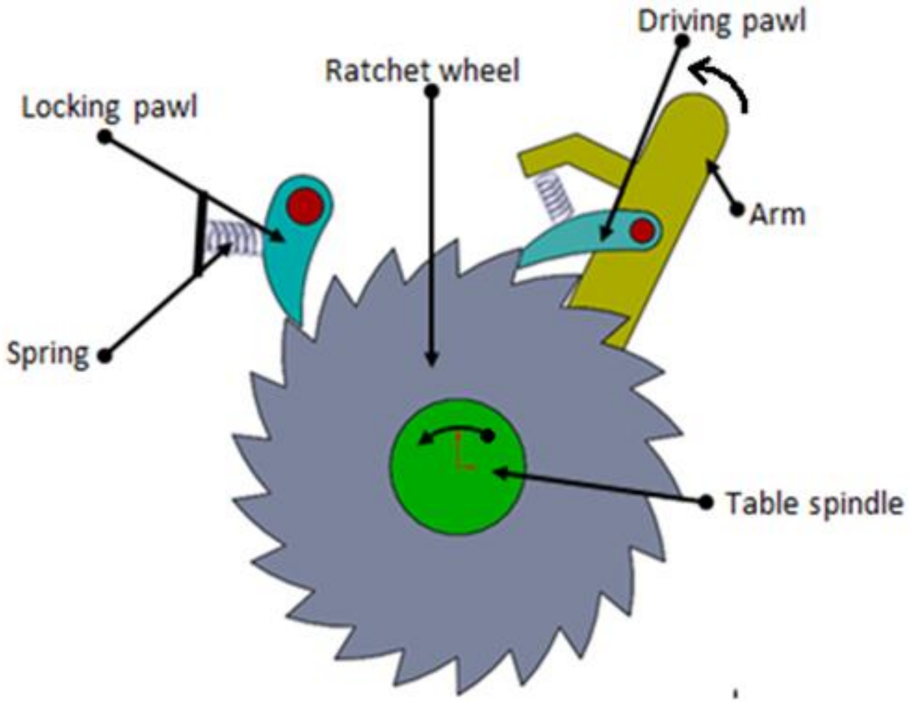


Ratchet and Pawl Mechanism

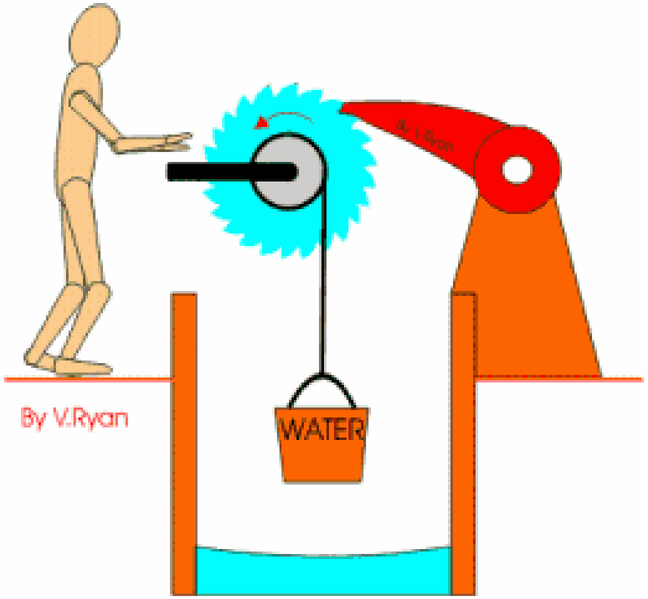








Driving pawl
Ratchet wheel
Locking pawl
Arm
Spring
Table spindle



WATER
By V.Ryan

Toggle Mechanism

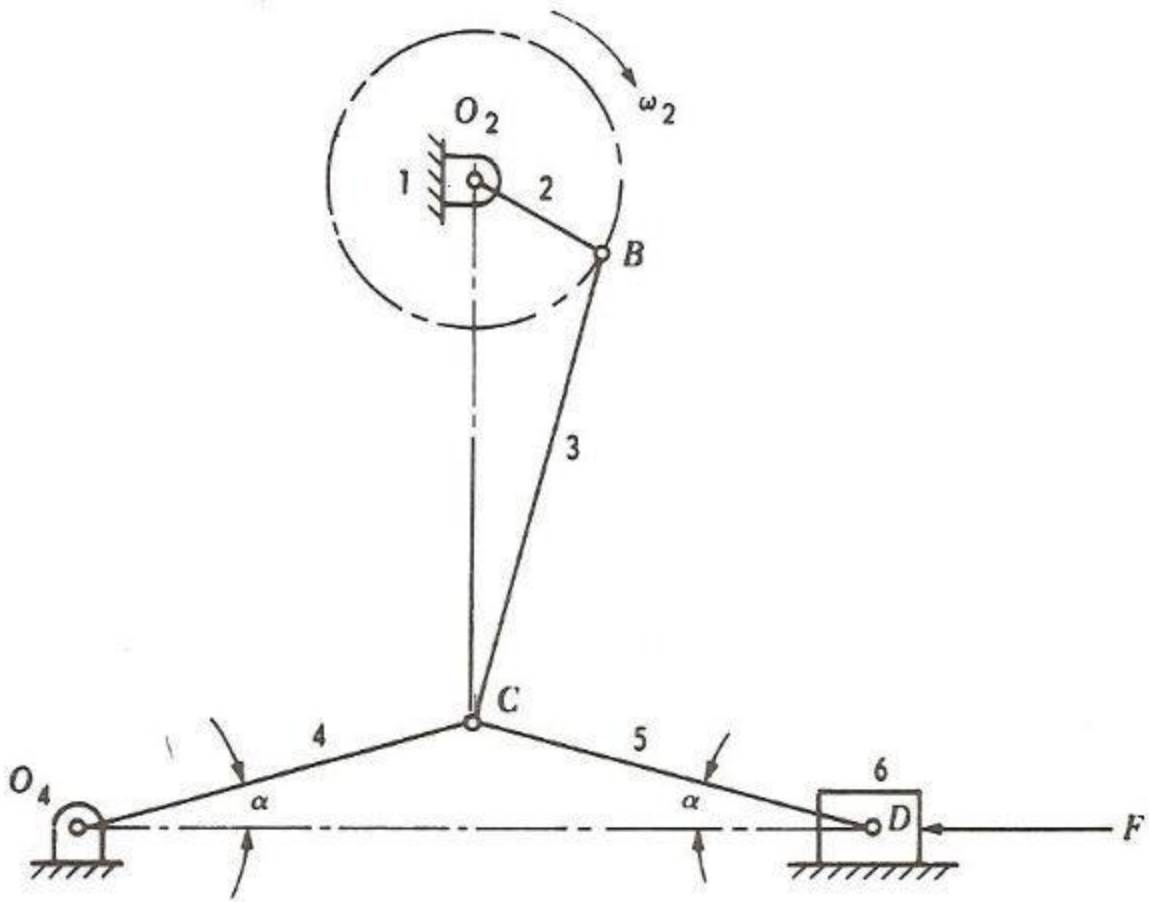
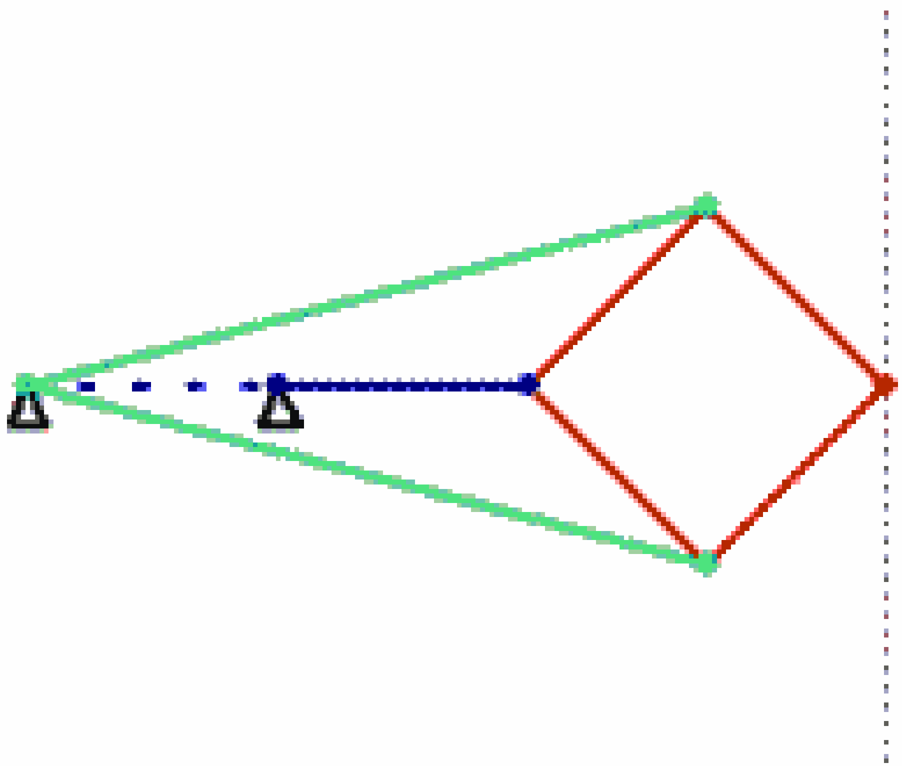
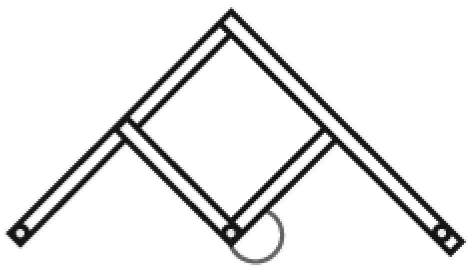
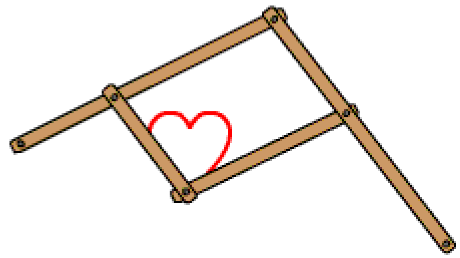
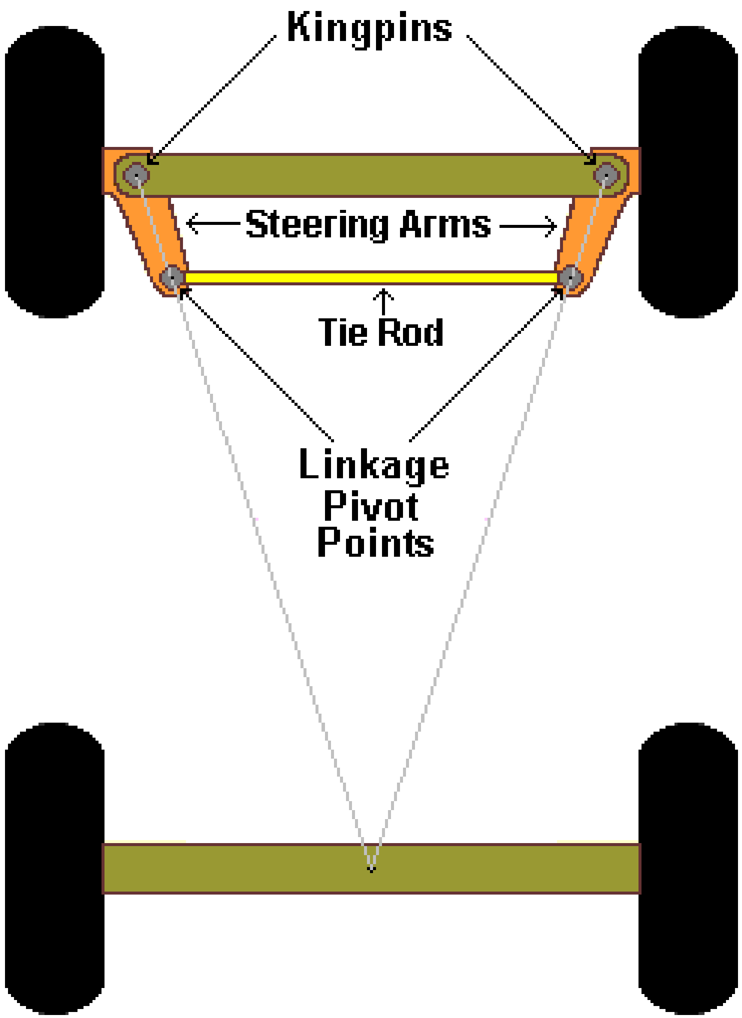


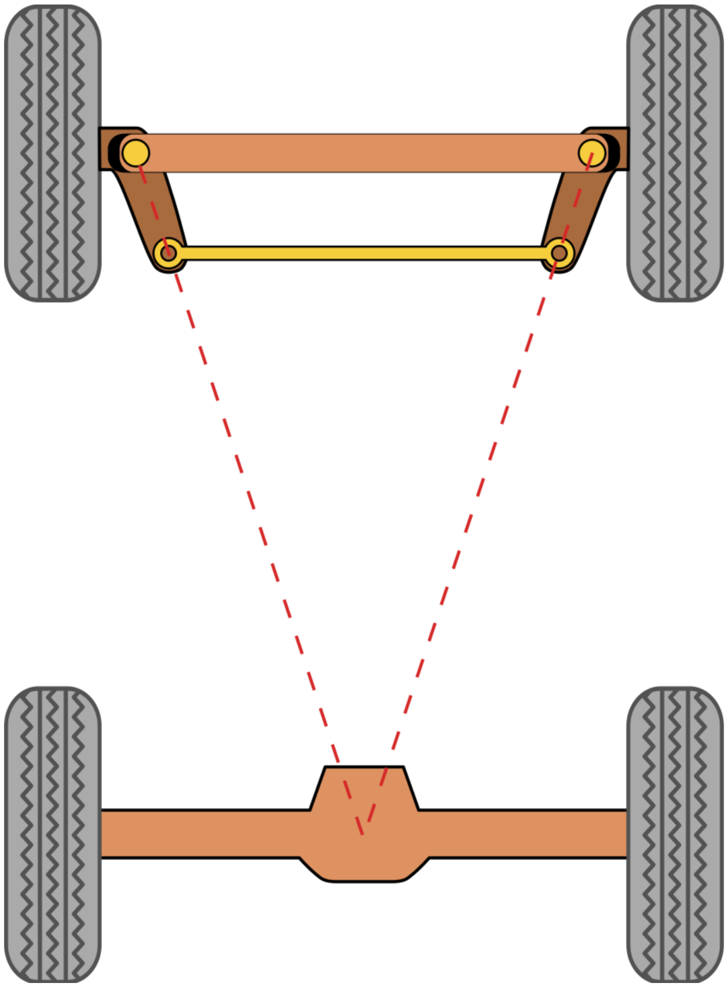
Figure 3-21

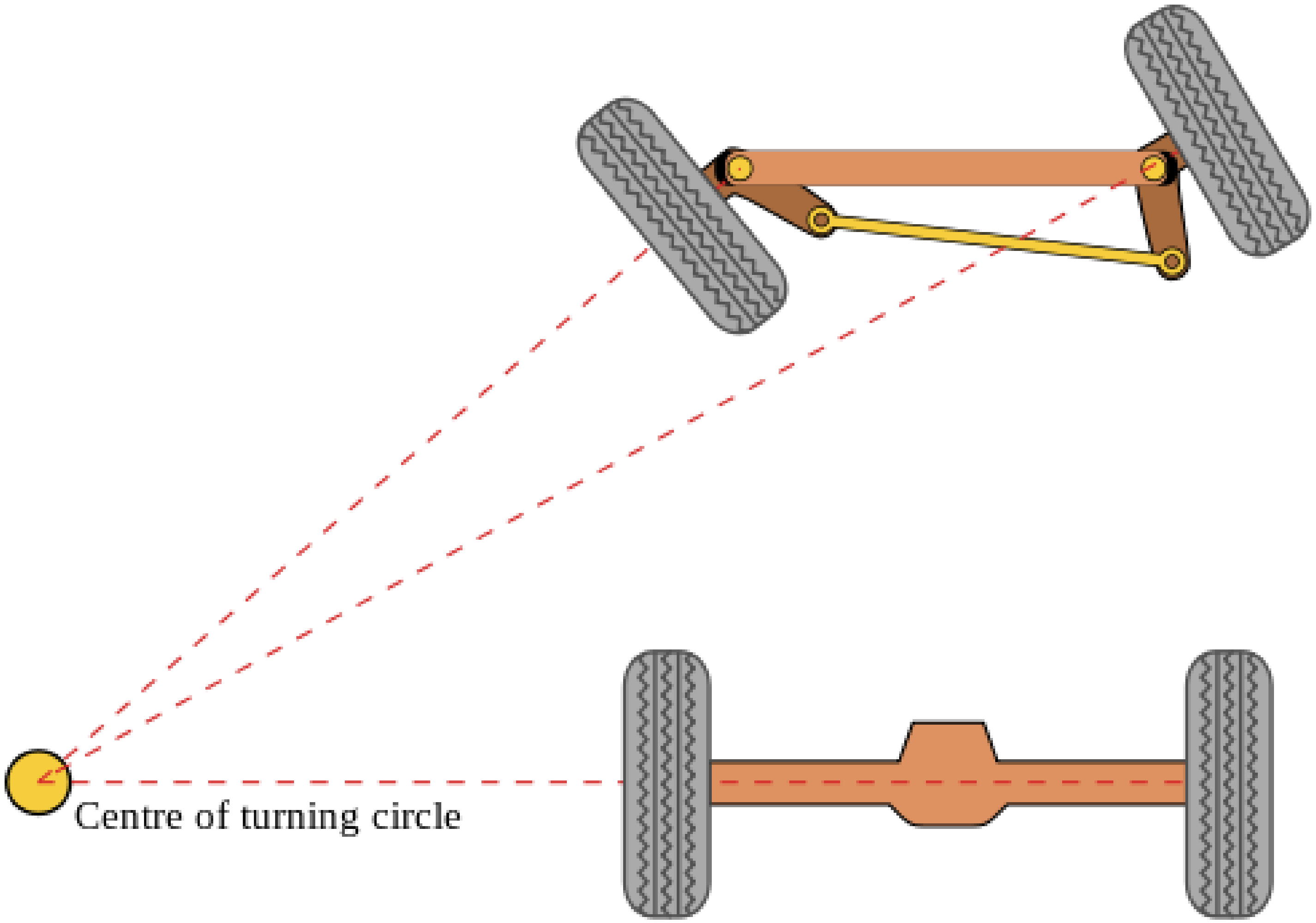


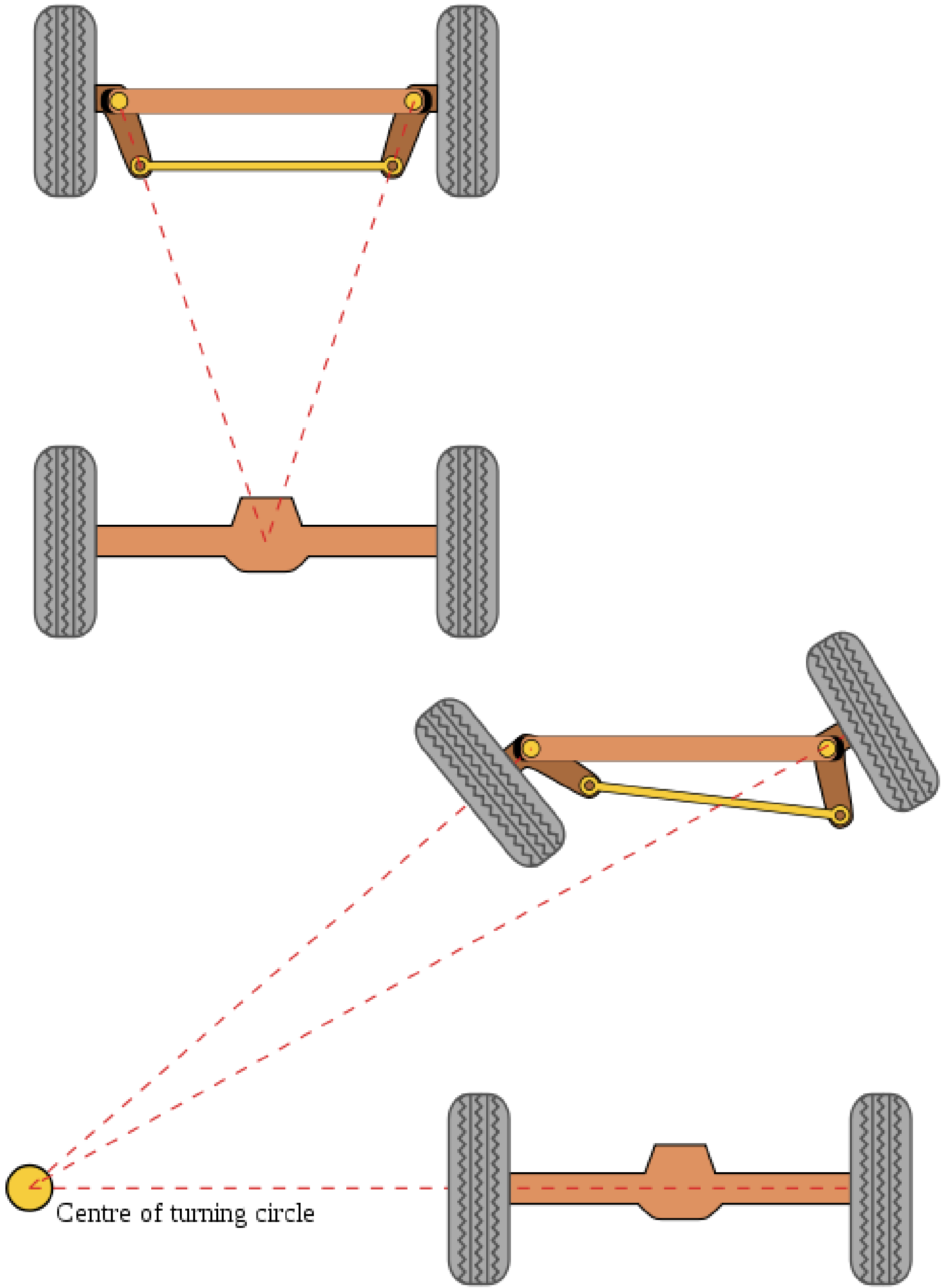





Kingpins
Steering Arms
Tie Rod
Linkage
Pivot
Points




Centre of turning circle


Centre of turning circle

INTRODUCTION TO KOM

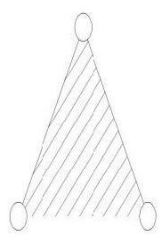
Link or element:

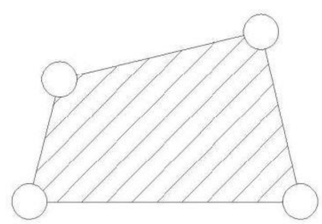
A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a link. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them, thus a link may consist of one or more resistant bodies. A link is also known as *Kinematic link* or an *element*.

Links can be classified into

+ Binary
+ Ternary
+ Quarternary, etc.

Binary Link ***Ternary Link*** ***Quarternary Link***

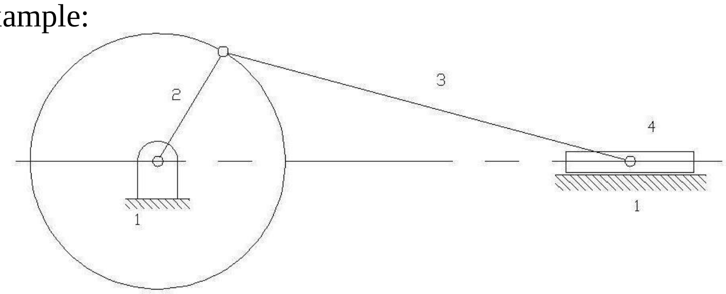




Kinematic Pair:

A Kinematic Pair or simply a pair is a joint of two links having relative motion between them.

Example:



In the above given Slider crank mechanism, link 2 rotates relative to link 1 and constitutes a revolute or turning pair. Similarly, links 2, 3 and 3, 4 constitute turning pairs. Link 4 (Slider) reciprocates relative to link 1 and it‟s a sliding pair.

Types of Kinematic Pairs:

Kinematic pairs can be classified according to

i) Nature of contact.

ii) Nature of mechanical constraint.

iii) Nature of relative motion.

i) Kinematic pairs according to nature of contact :

a) Lower Pair: A pair of links having surfaced or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar.

Examples: Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint.

b) Higher Pair: When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

Examples: Wheel rolling on a surface cam and follower pair, tooth gears, ball and roller bearings, etc.

Kinematic pairs according to nature of mechanical constraint.

Closed pair: When the elements of a pair are held together mechanically, it is known as a closed pair. The contact between the two can only be broken only by the destruction of at least one of the members. All the lower pairs and some of the higher pairs are closed pairs.

Unclosed pair: When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this the links are not held together mechanically. Ex.: Cam and follower pair.

iii) Kinematic pairs according to nature of relative motion.

a) Sliding pair: If two links have a sliding motion relative to each other, they form a sliding pair. A rectangular rod in a rectangular hole in a prism is an example of a sliding pair.
b) Turning Pair: When on link has a turning or revolving motion relative to the other, they constitute a turning pair or revolving pair.
c) Rolling pair: When the links of a pair have a rolling motion relative to each other, they form a rolling pair. A rolling wheel on a flat surface, ball ad roller bearings, etc. are some of the examples for a Rolling pair.

d) Screw pair (Helical Pair): if two mating links have a turning as well as sliding motion between them, they form a screw pair. This is achieved by cutting matching threads on the two links. The lead screw and the nut of a lathe is a screw Pair

e) Spherical pair: When one link in the form of a sphere turns inside a fixed link, it is a spherical pair. The ball and socket joint is a spherical pair.

<u>Degrees of Freedom:</u>

An unconstrained rigid body moving in space can describe the following independent motions.

1. Translational Motions along any three mutually perpendicular axes x, y and z,

2. Rotational motions along these axes.

Thus a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

Degrees of freedom = 6 – no. of restraints.

To find the number of degrees of freedom for a plane mechanism we have an equation known as Grubler‟s equation and is given by

$$F = 3(n-1) - 2j_1 - j_2$$

F = Mobility or number of degrees of freedom n = Number of links including frame.

j_1 = Joints with single (one) degree of freedom. j_1 = Joints with two degrees of freedom.

If $F > 0$, results a mechanism with „F‟ degrees of freedom.
F = 0, results in a statically determinate structure.

F < 0, results in a statically indeterminate structure.

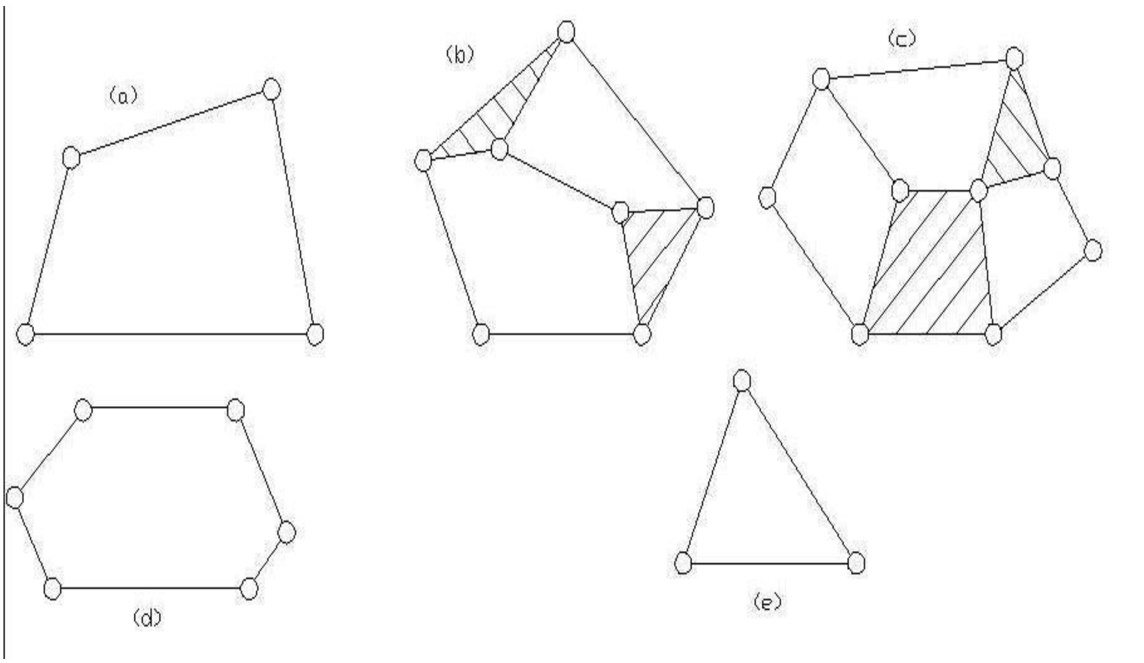
The degrees of freedom for various joints are given by:

Type of joint	Nature of Motion.	Degrees of freedom.
Hinges (Revolute)	Pure rolling	1
Slider (prismatic)	Pure Sliding	1
Cylindrical, Cam, Gear, Ball Bearings	Rolling and Sliding	2
Rolling Contact	Pure Rolling	1
Spherical		3

Note: A revolute joint connecting m links at the same point must be considered as (*m*-1) joints.

Kinematic Chain:

A Kinematic chain is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the others is definite (fig. a, b, and c.)



In case, the motion of a link results in indefinite motions of other links, it is a nonkinematic chain. However, some authors prefer to call all chains having relative motions of the links as kinematic chains.

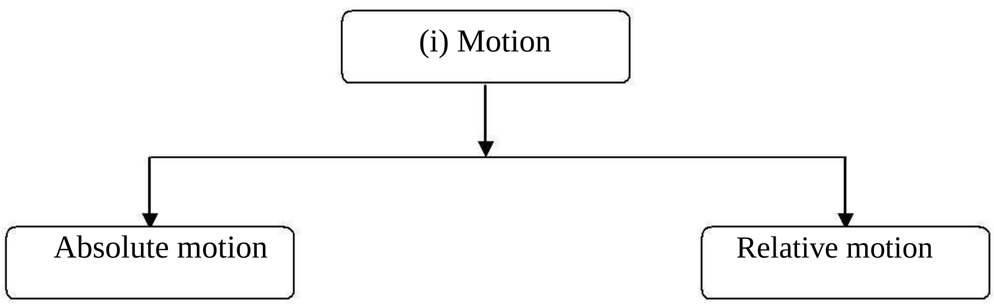
<u>Linkage, Mechanism and structure:</u>

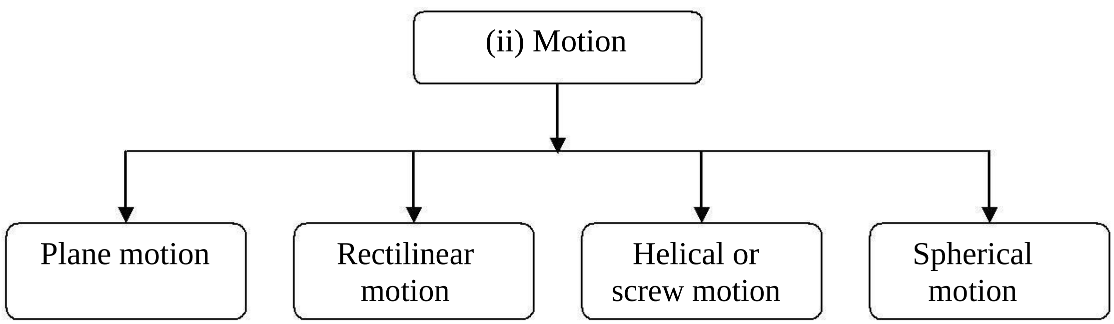
A linkage is obtained if one of the links of kinematic chain is fixed to the ground. If motion of each link results in definite motion of the others, the linkage is known as *mechanism*.

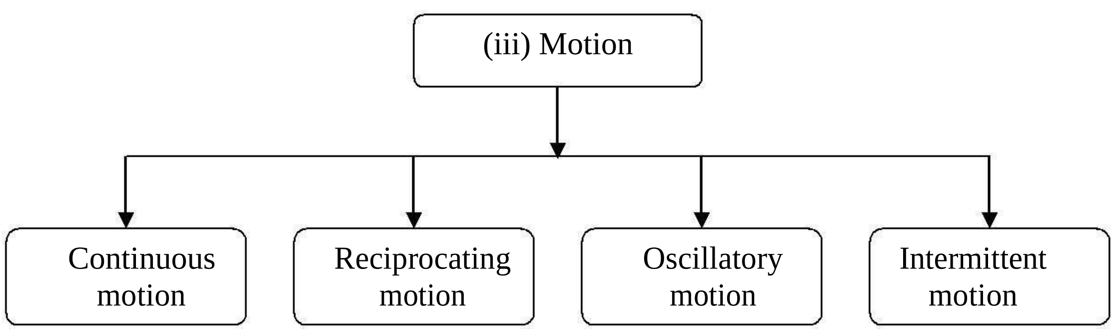
If one of the links of a redundant chain is fixed, it is known as a *structure*.

To obtain constrained or definite motions of some of the links of a linkage, it is necessary to know how many inputs are needed. In some mechanisms, only one input is necessary that determines the motion of other links and are said to have one degree of freedom. In other mechanisms, two inputs may be necessary to get a constrained motion of the other links and are said to have two degrees of freedom and so on.

<u>Motion and its types</u>:







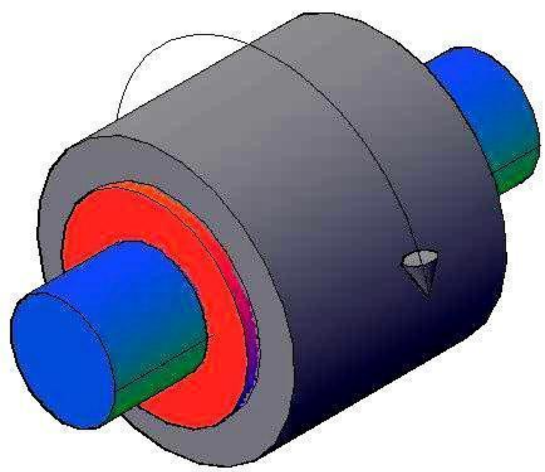
(iv) Motion

The tree main types of constrained motion in kinematic pair are,

(i) **Completely constrained motion** :

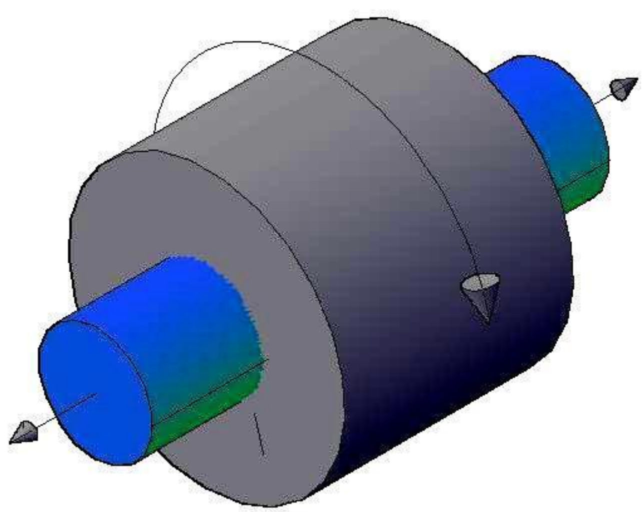
If the motion between a pair of links is limited to a definite direction, then it is completely constrained motion.

E.g.: Motion of a shaft or rod with collars at each end in a hole as shown in fig.



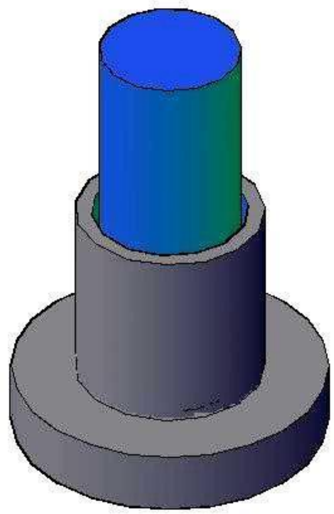
(ii) Incompletely Constrained motion :

If the motion between a pair of links is not confined to a definite direction, then it is incompletely constrained motion. E.g.: A spherical ball or circular shaft in a circular hole may either rotate or slide in the hole as shown in fig.



(iii) **Successfully constrained motion or partially constrained motion.**

If the motion in a definite direction is not brought about by itself but by some other means, then it is known as successfully constrained motion.
E.g.: Foot step Bearing.



<u>Inversions</u>:

By fixing each link at a time we get as many mechanisms as the number of links, then each mechanism is called „Inversion" of the original Kinematic Chain.

<u>Machine</u>:

It is a combination of resistant bodies with successfully constrained motion which is used to transmit or transform motion to do some useful work.

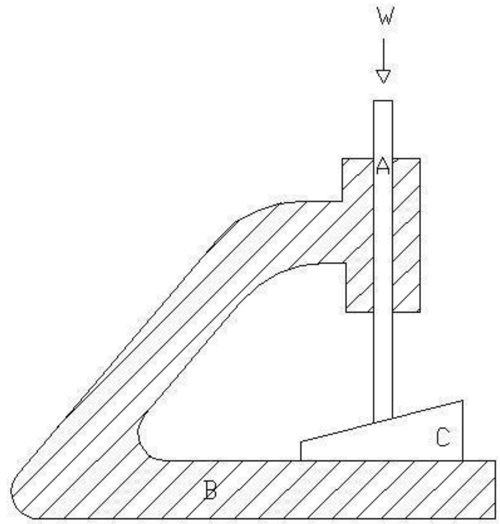
E.g.: Lathe, Shaper, Steam Engine, etc.

1.2 Kinematic chains and Inversions:

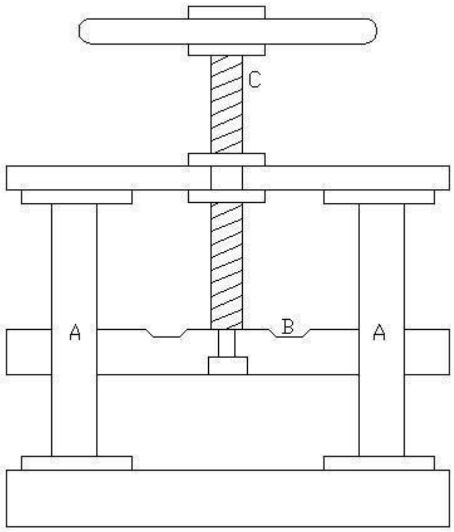
<u>Kinematic chain with three lower pairs</u>

It is impossible to have a kinematic chain consisting of three turning pairs only. But it is possible to have a chain which consists of three sliding pairs or which consists of a turning, sliding and a screw pair.

The figure shows a kinematic chain with three sliding pairs. It consists of a frame B, wedge C and a sliding rod A. Hence the three sliding pairs are, one between the wedge C and the frame B, second between wedge C and sliding rod A and the frame B.



This figure shows the mechanism of a fly press. The element B forms a sliding with A and turning pair with screw rod C which in turn forms a screw pair with A. When link A is fixed, the required fly press mechanism is obtained.

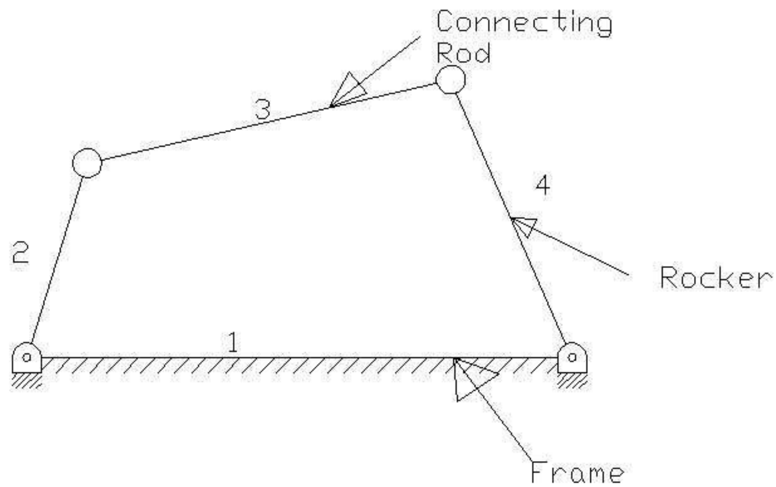


Types of Kinematic Chain:

1) Four bar chain
2) Single slider chain
3) Double Slider chain

1) Four bar Chain:

The chain has four links and it looks like a cycle frame and hence it is also called *quadric cycle chain*. It is shown in the figure. In this type of chain all four pairs will be turning pairs.



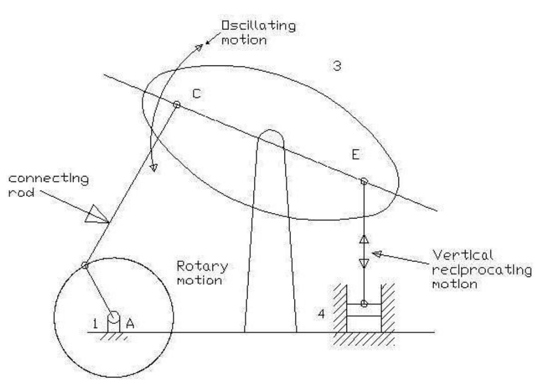
Inversions of four bar chain mechanism:

There are three inversions:

1) Beam Engine or Crank and lever mechanism.
2) Coupling rod of locomotive or double crank mechanism.
3) Watt‟s straight line mechanism or double lever mechanism.

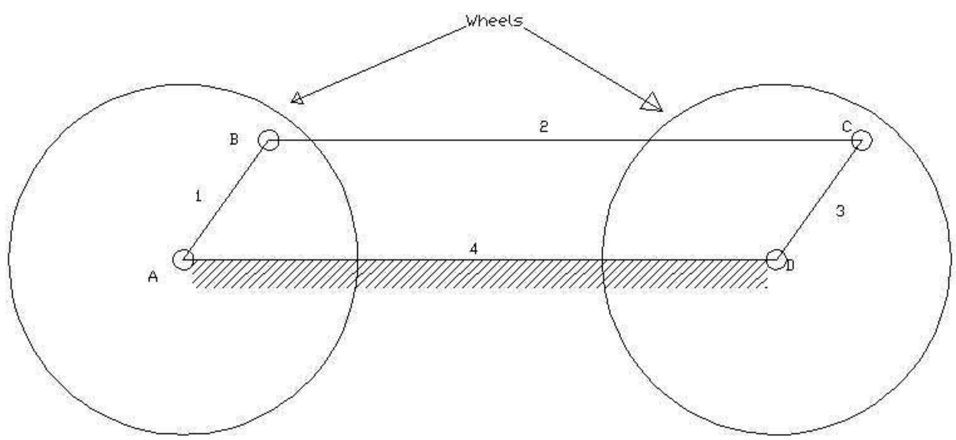
1) Beam Engine:

When the crank AB rotates about A, the link CE pivoted at D makes vertical reciprocating motion at end E. This is used to convert rotary motion to reciprocating motion and vice versa. It is also known as Crank and lever mechanism. This mechanism is shown in the figure below.



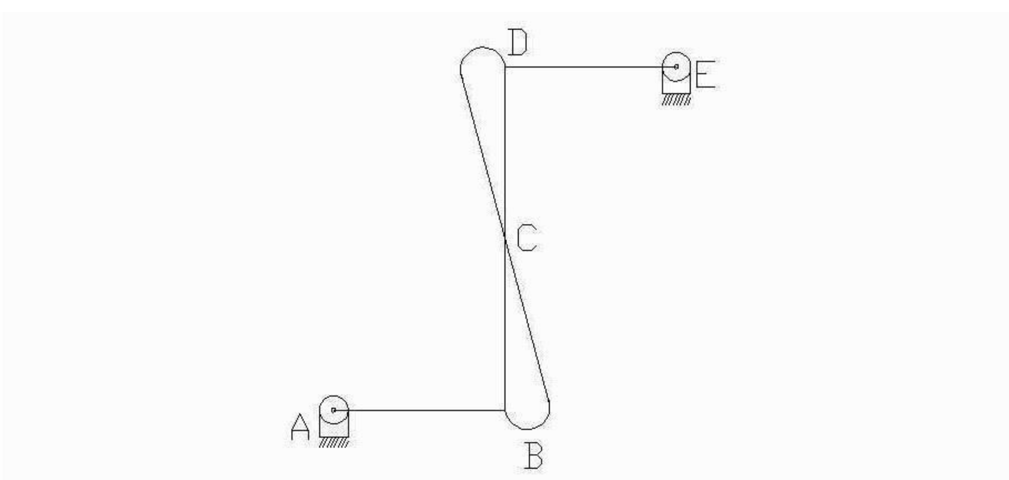
2) Coupling rod of locomotive

In this mechanism the length of link AD = length of link C. Also length of link AB = length of link CD. When AB rotates about A, the crank DC rotates about D. this mechanism is used for coupling locomotive wheels. Since links AB and CD work as cranks, this mechanism is also known as double crank mechanism. This is shown in the figure below.



1) Watt‟s straight line mechanism or Double lever mechanism. In this mechanism, the links AB & DE act as levers

at the ends A & E of these levers are fixed. The AB & DE are parallel in the mean position of the mechanism and coupling rod BD is perpendicular to the levers AB & DE. On any small displacement of the mechanism the tracing point „C‟ traces the shape of number „8‟, a portion of which will be approximately straight. Hence this is also an example for the approximate straight line mechanism. This mechanism is shown below.



UNIT-02

MECHANISMS

Slider crank Chain Mechanism:

It is a four bar chain having one sliding pair and three turning pairs. It is shown in the figure below the purpose of this mechanism is to convert rotary motion to reciprocating motion and vice versa.

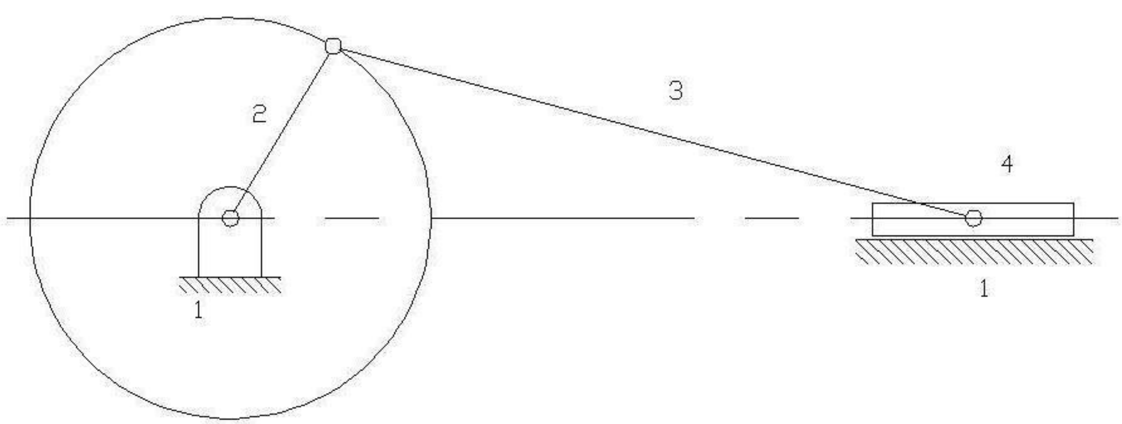
Inversions of a Slider crank chain:

There are four inversions in a single slider chain mechanism. They are:

1) Reciprocating engine mechanism (1^{st} inversion)
2) Oscillating cylinder engine mechanism (2^{nd} inversion)
3) Crank and slotted lever mechanism (2^{nd} inversion)
4) Whitworth quick return motion mechanism (3^{rd} inversion)
5) Rotary engine mechanism (3^{rd} inversion)
6) Bull engine mechanism (4^{th} inversion)
7) Hand Pump (4^{th} inversion)

1) Reciprocating engine mechanism :

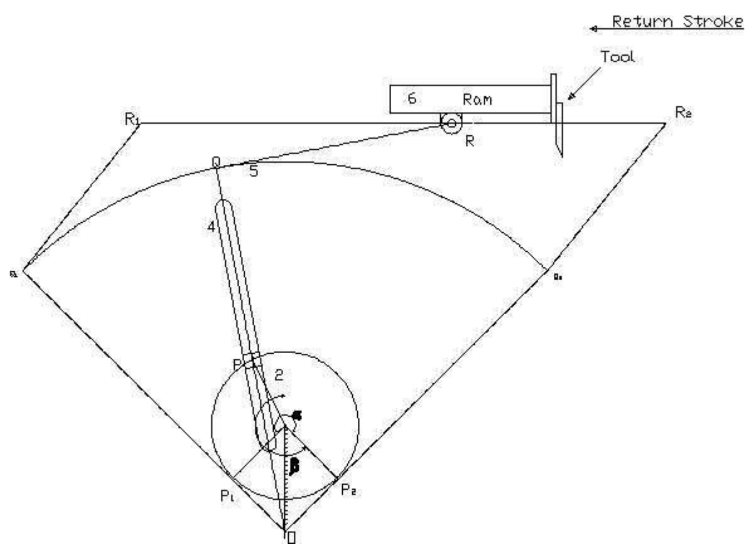
In the first inversion, the link 1 i.e., the cylinder and the frame is kept fixed. The fig below shows a reciprocating engine.



A slotted link 1 is fixed. When the crank 2 rotates about O, the sliding piston 4 reciprocates in the slotted link 1. This mechanism is used in steam engine, pumps, compressors, I.C. engines, etc.

2) Crank and slotted lever mechanism:

It is an application of second inversion. The crank and slotted lever mechanism is shown in figure below.



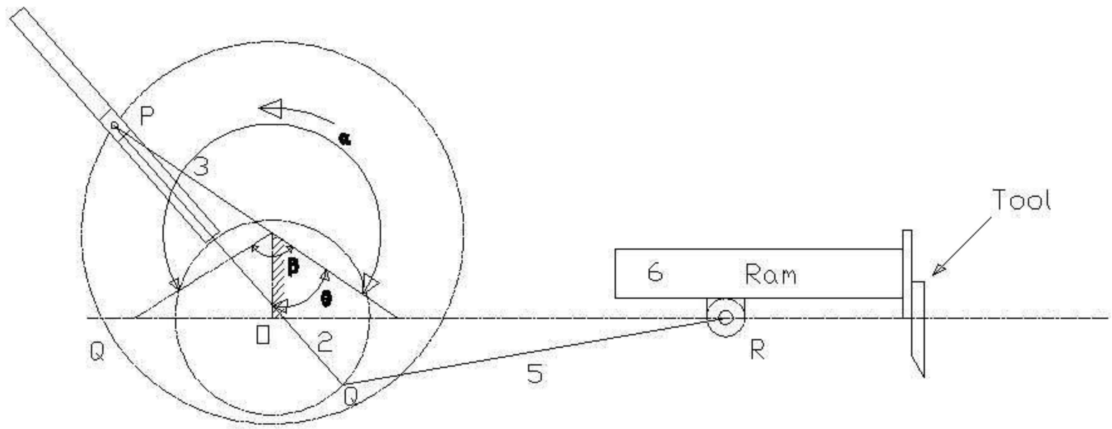
In this mechanism link 3 is fixed. The slider (link 1) reciprocates in oscillating slotted lever (link 4) and crank (link 2) rotates. Link 5 connects link 4 to the ram (link 6). The ram with the cutting tool reciprocates perpendicular to the fixed link 3. The ram with the tool reverses its direction of motion when link 2 is perpendicular to link 4. Thus the cutting stroke is executed during the rotation of the crank through angle α and the return stroke is executed when the crank rotates through angle β or 360 – α. Therefore, when the crank rotates uniformly, we get

This mechanism is used in shaping machines, slotting machines and in rotary engines.

2) Whitworth quick return motion mechanism

Third inversion is obtained by fixing the crank i.e. link 2. Whitworth quick return mechanism is an application of third inversion. This mechanism is shown in the figure below. The crank OC is fixed and OQ rotates about O. The slider slides in the slotted link and generates a circle of radius CP. Link 5 connects the extension OQ provided on the opposite side of the link 1 to the ram (link 6). The rotary motion of P is taken to the ram

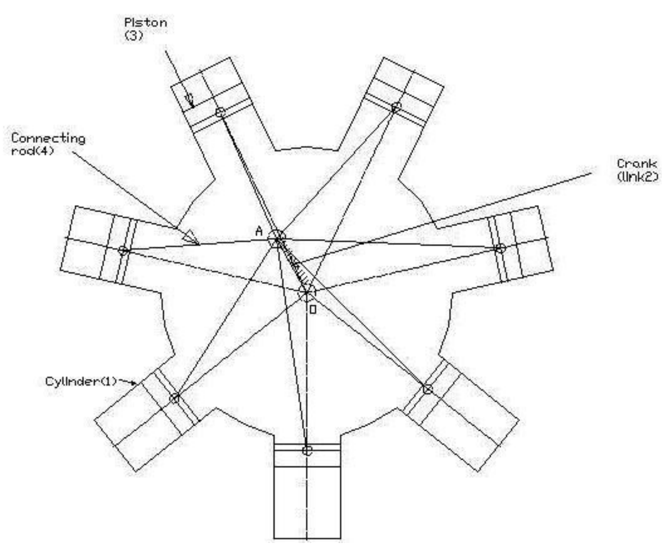
R which reciprocates. The quick return motion mechanism is used in shapers and slotting machines.



The angle covered during cutting stroke from P_1 to P_2 in counter clockwise direction is α or 360 - 2θ. During the return stroke, the angle covered is 2θ or β.

4) Rotary engine mechanism or Gnome Engine

Rotary engine mechanism or gnome engine is another application of third inversion. It is a rotary cylinder V – type internal combustion engine used as an aero – engine. But now Gnome engine has been replaced by Gas turbines. The Gnome engine has generally seven cylinders in one plane. The crank OA is fixed and all the connecting rods from the pistons are connected to A. In this mechanism when the pistons reciprocate in the cylinders, the whole assembly of cylinders, pistons and connecting rods rotate about the axis O, where the entire mechanical power developed, is obtained in the form of rotation of the crank shaft. This mechanism is shown in the figure below.



Double slider crank chain:

A four bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as double slider crank chain.

Inversions of Double slider Crank chain:

It consists of two sliding pairs and two turning pairs. They are three important inversions of double slider crank chain.

1) Elliptical trammel.
2) Scotch yoke mechanism.
3) Oldham‟s Coupling.

1) Elliptical Trammel:

This is an instrument for drawing ellipses. Here the slotted link is fixed. The sliding block P and Q in vertical and horizontal slots respectively. The end R generates an ellipse with the displacement of sliders P and Q.

The co-ordinates of the point R are x and y.

From the fig. $\cos\theta = \frac{x}{PR}$

and $\sin\theta = \frac{y}{QR}$

Squaring and adding (i) and (ii) we get

$$\frac{x^2}{(PR)^2} + \frac{y^2}{(QR)^2} = \cos^2\theta + \sin^2\theta$$

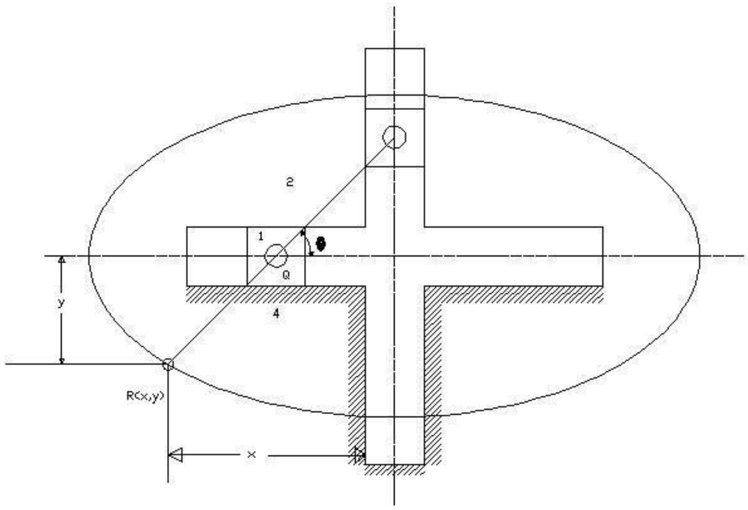
$$\frac{x^2}{(PR)^2} + \frac{y^2}{(QR)^2} = 1$$

The equation is that of an ellipse, Hence the instrument traces an ellipse.

Path traced by mid-point of PQ is a circle. In this case, PR = PQ.

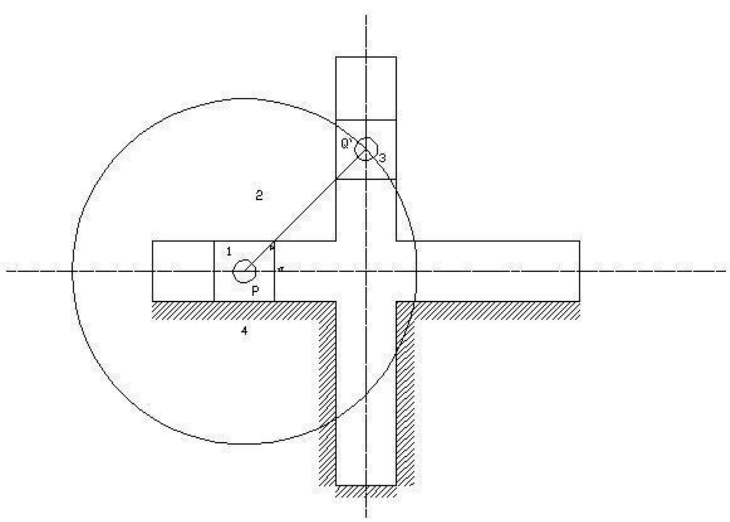
$$\frac{x^2}{(PR)^2} + \frac{y^2}{(QR)^2} = 1$$

Its an equation of circle with PR = QR = radius of a circle.



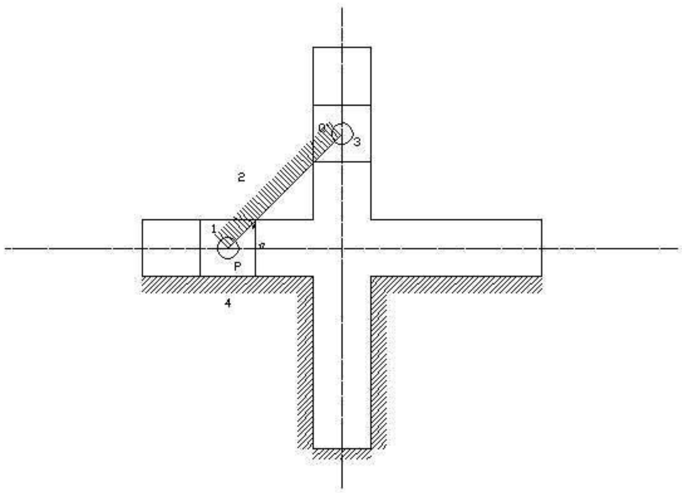
2) Scotch yoke mechanism:

This mechanism, the slider P is fixed. When PQ rotates above P, the slider Q reciprocates in the vertical slot. The mechanism is used to convert rotary to reciprocating mechanism.



3) Oldham‟s coupling:

The third inversion of obtained by fixing the link connecting the 2 blocks P & Q. If one block is turning through an angle, the frame and the other block will also turn through the same angle. It is shown in the figure below.



An application of the third inversion of the double slider crank mechanism is Oldham‟s coupling shown in the figure. This coupling is used for connecting two parallel shafts when the distance between the shafts is small. The two shafts to be connected have flanges at their ends, secured by forging. Slots are cut in the flanges. These flanges form 1 and 3. An intermediate disc having tongues at right angles and opposite sides is fitted in between the flanges. The intermediate piece forms the link 4 which slides or reciprocates in flanges 1 & 3. The link two is fixed as shown. When flange 1 turns, the intermediate disc 4 must turn through the same angle and whatever angle 4 turns, the flange 3 must turn through the same angle. Hence 1, 4 & 3 must have the same angular velocity at every instant. If the distance between the axis of the shaft is x, it will be the diameter if the circle traced by the centre of the intermediate piece. The maximum sliding speed of each tongue along its slot is given by

v = x
where,

= angular velocity of each shaft in rad/sec v = linear velocity in m/sec

1.3 Mechanisms:

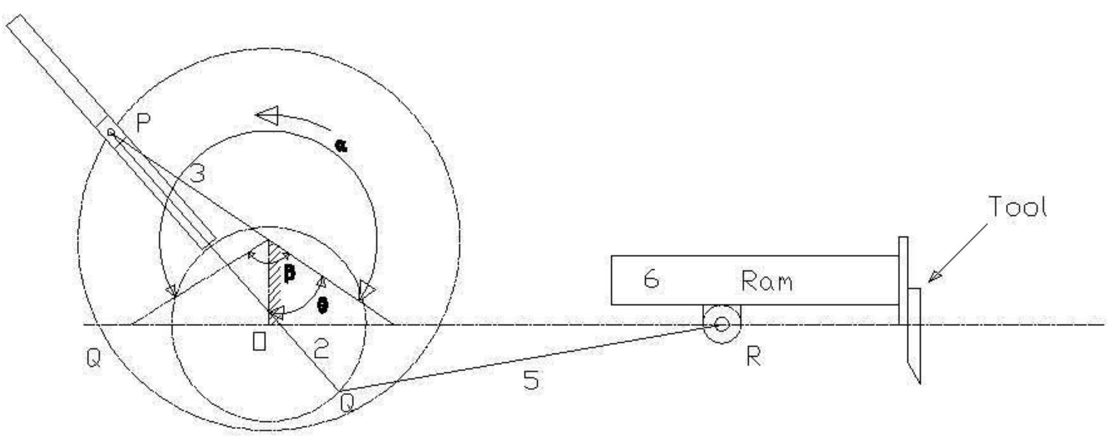
i) Quick return motion mechanisms:

Many a times mechanisms are designed to perform repetitive operations. During these operations for a certain period the mechanisms will be under load known as working stroke and the remaining period is known as the return stroke, the mechanism returns to repeat the operation without load. The ratio of time of working stroke to that of the return stroke is known a time ratio. Quick return mechanisms are used in machine tools to give a slow cutting stroke and a quick return stroke. The various quick return mechanisms commonly used are

Whitworth
Drag link.
Crank and slotted lever mechanism

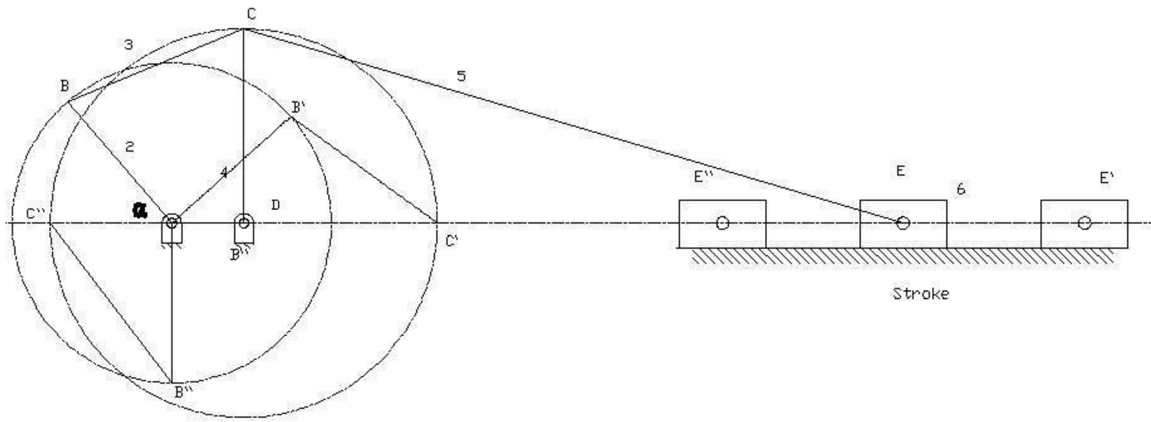
o Whitworth quick return mechanism:

Whitworth quick return mechanism is an application of third inversion of the single slider crank chain. This mechanism is shown in the figure below. The crank OC is fixed and OQ rotates about O. The slider slides in the slotted link and generates a circle of radius CP. Link 5 connects the extension OQ provided on the opposite side of the link 1 to the ram (link 6). The rotary motion of P is taken to the ram R which reciprocates. The quick return motion mechanism is used in shapers and slotting machines.



The angle covered during cutting stroke from P_1 to P_2 in counter clockwise direction is α or 360 - 2θ. During the return stroke, the angle covered is 2θ or β.

ii) Drag link mechanism :

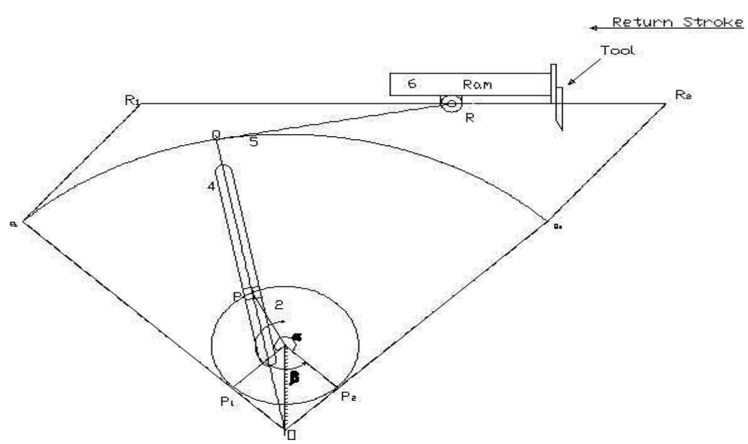


This is four bar mechanism with double crank in which the shortest link is fixed. If the crank AB rotates at a uniform speed, the crank CD rotate at a non-uniform speed. This rotation of link CD is transformed to quick return reciprocatory motion of the ram E by the link CE as shown in figure. When the crank AB rotates through an angle α in Counter clockwise direction during working stroke, the link CD rotates through 180^{o} . We can observe that /̲ α > /̲ β. Hence time of working stroke is α /β times more or the return stroke is α /β times quicker.

Shortest link is always stationary link. Sum of the shortest and the longest links of the four links 1, 2, 3 and 4 are less than the sum of the other two. It is the necessary condition for the drag link quick return mechanism.

3) Crank and slotted lever mechanism:

It is an application of second inversion. The crank and slotted lever mechanism is shown in figure below.



In this mechanism link 3 is fixed. The slider (link 1) reciprocates in oscillating slotted lever (link 4) and crank (link 2) rotates. Link 5 connects link 4 to the ram (link 6). The ram with the cutting tool reciprocates perpendicular to the fixed link 3. The ram with the tool reverses its direction of motion when link 2 is perpendicular to link 4. Thus the cutting stroke is executed during the rotation of the crank through angle α and the return stroke is executed when the crank rotates through angle β or 360 – α. Therefore, when the crank rotates uniformly, we get,

$$\frac{\text{Time to cutting}}{\text{Time of return}} = \frac{\alpha}{\beta} = \frac{\alpha}{360 - \alpha}$$

This mechanism is used in shaping machines, slotting machines and in rotary engines.

(ii) Straight line Motion machines:

The easiest way to generate a straight line motion is by using a sliding pair but in precision machines sliding pairs are not preferred because of wear and tear. Hence in such cases different methods are used to generate straight line motion mechanisms:

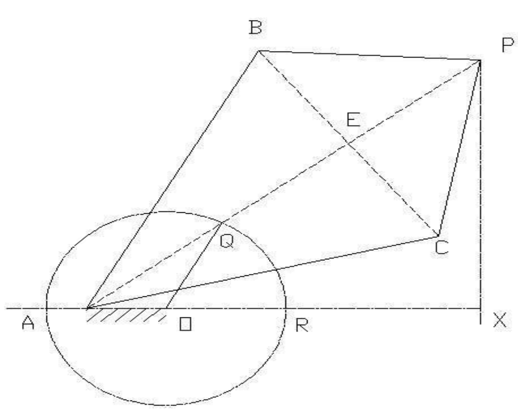
1) Exact straight line motion mechanism. Ex.: Peaucellier mechanism. Hart mechanism.

Scott Russell mechanism.

2) Approximate straight line motion mechanisms. Ex.: Watt mechanism

Grasshopper‟s mechanism Robert‟s mechanism Tchebicheff‟s mechanism

Peaucillier mechanism :



The pin Q is constrained to move long the circumference of a circle by means of the link OQ. The link OQ and the fixed link are equal in length. The pins P and Q are on opposite corners of a four bar chain which has all four links QC, CP, PB and BQ of equal length to the fixed pin A. i.e., link AB = link AC. The product AQ x AP remain constant as the link OQ rotates may be proved as follows:

Join BC to bisect PQ at F; then, from the right angled triangles AFB, BFP, we have

$AB^2 = AF^2 + FB^2$ and $BP^2 = BF^2 + FP^2$

Subtracting,

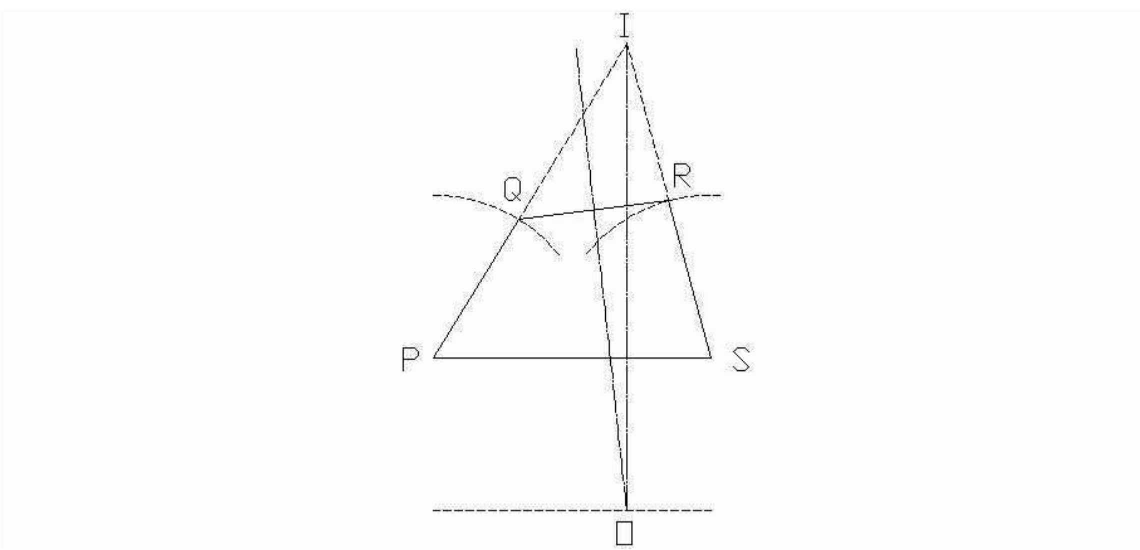
$AB^2 - BP^2 = AF^2 - FP^2 = (AF – FP) (AF+FP) = AQ \times AP$

Since AB and BP are links of a constant length, the product AQ x AP is constant. Therefore the point P traces out a straight path normal to AR.

- Robert‟s mechanism :

This is also a four bar chain. The link PQ and RS are of

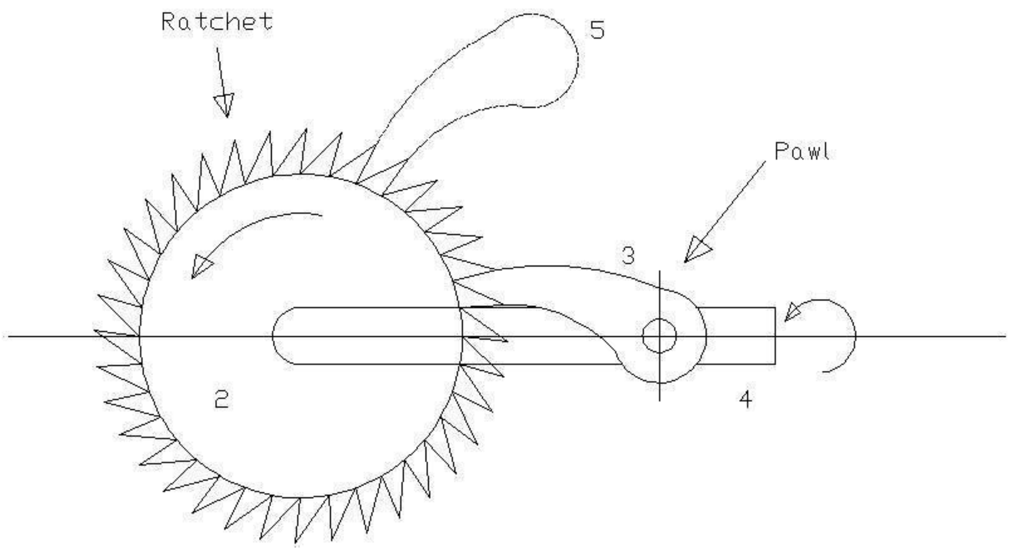
equal length and the tracing pint „O‟ is rigidly attached to the link QR on a line which bisects QR at right angles. The best position for O may be found by making use of the instantaneous centre of QR. The path of O is clearly approximately horizontal in the Robert‟s mechanism.



iii) Intermittent motion mechanism:

1) Ratchet and Pawl mechanism:

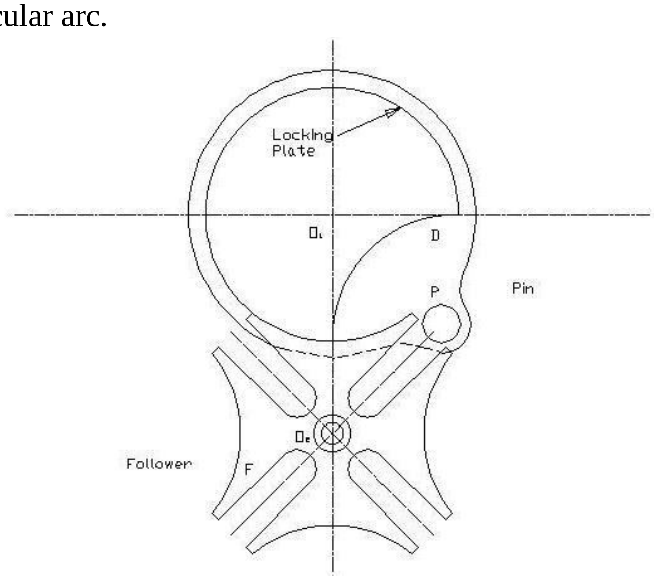
This mechanism is used in producing intermittent rotary Motion member. A ratchet and Pawl mechanism consists of a ratchet wheel 2 and a pawl 3 as shown in the figure. When the lever 4 carrying pawl is raised, the ratchet wheel rotates in the counter clock wise direction (driven by pawl). As the pawl lever is lowered the pawl slides over the ratchet teeth. One more pawl 5 is used to prevent the ratchet from reversing. Ratchets are used in feed mechanisms, lifting jacks, clocks, watches and counting devices.



2) Geneva mechanism:

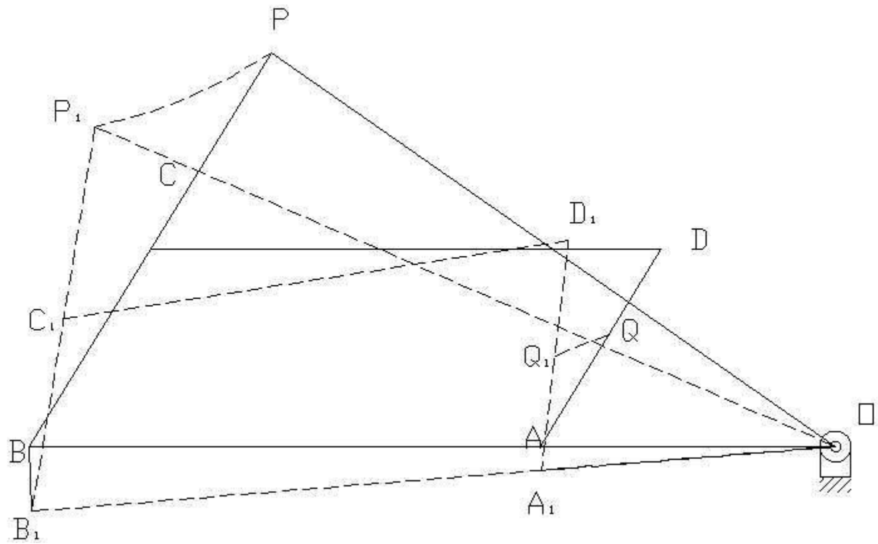
Geneva mechanism is an intermittent motion mechanism.

It consists of a driving wheel D carrying a pin P which engages in a slot of follower F as shown in figure. During one quarter revolution of the driving plate, the Pin and follower remain in contact and hence the follower is turned by one quarter of a turn. During the remaining time of one revolution of the driver, the follower remains in rest locked in position by the circular arc.



3) Pantograph:

Pantograph is used to copy the curves in reduced or enlarged scales. Hence this mechanism finds its use in copying devices such as engraving or profiling machines.



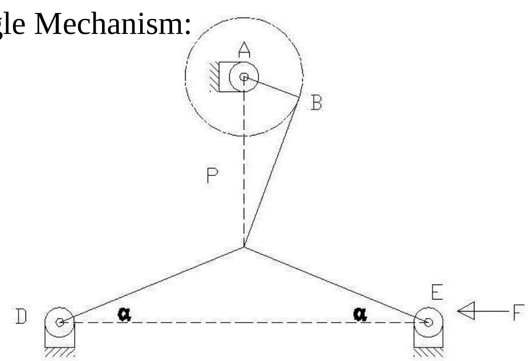
This is a simple figure of a Pantograph. The links are pin jointed at A, B, C and D. AB is parallel to DC and AD is parallel to BC.

Link BA is extended to fixed pin O. Q is a point on the link AD. If the motion of Q is to be enlarged then the link BC is extended to P such that O, Q and P are in a straight line.

Then it can be shown that the points P and Q always move parallel and similar to each other over any path straight or curved. Their motions will be proportional to their distance from the fixed point.

Let ABCD be the initial position. Suppose if point Q moves to Q_1 , then all the links and the joints will move to the new positions (such as A moves to A_1 , B moves to Q_1 , C moves to Q_1 , D moves to D_1 and P to P_1) and the new configuration of the mechanism is shown by dotted lines. The movement of Q (Q $Q_{1)}$ will be enlarged to PP1 in a definite ratio.

4. Toggle Mechanism:



In slider crank mechanism as the crank approaches one of its dead centre position, the slider approaches zero. The ratio of the crank movement to the slider movement approaching infinity is proportional to the mechanical advantage. This is the principle used in toggle mechanism. A toggle mechanism is used when large forces act through a short distance is required. The figure below shows a toggle mechanism. Links CD and CE are of same length.

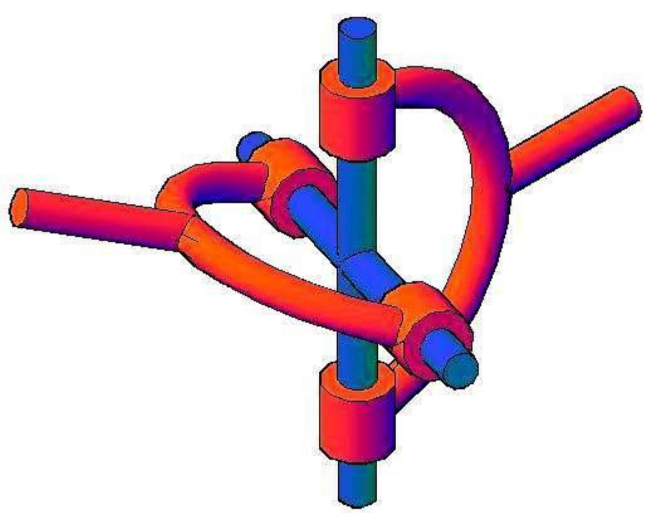
Resolving the forces at C vertically

$$\frac{F \sin \alpha}{\cos \alpha} = \frac{P}{2}$$

Therefore, $F = \frac{P}{2 \tan \alpha}$ (because Sin α/Cos α = Tan α)

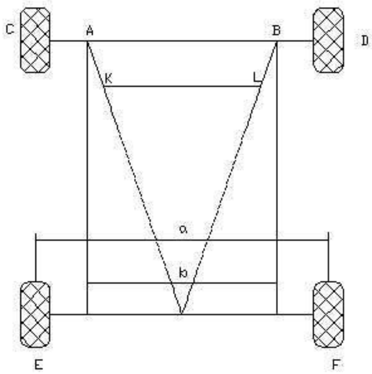
Thus for the given value of P, as the links CD and CE approaches collinear position (α O), the force F rises rapidly.

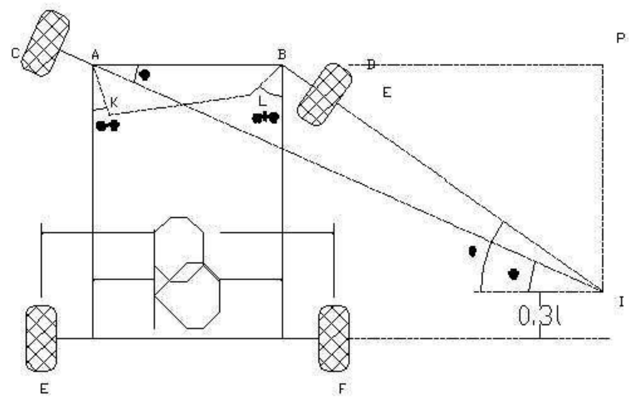
4) Hooke"s joint:



Hooke"s joint used to connect two parallel intersecting shafts as shown in figure. This can also be used for shaft with angular misalignment where flexible coupling does not serve the purpose. Hence Hooke"s joint is a means of connecting two rotating shafts whose axes lie in the same plane and their directions making a small angle with each other. It is commonly known as Universal joint. In Europe it is called as Cardan joint.

5) Ackermann steering gear mechanism:





This mechanism is made of only turning pairs and is made of only turning pairs wear and tear of the parts is less and cheaper in manufacturing. The cross link KL connects two short axles AC and BD of the front wheels through the short links AK and BL which forms bell crank levers CAK and DBL respectively as shown in fig, the longer links AB and KL are parallel and the shorter links AK and BL are inclined at an angle α.

When the vehicles steer to the right as shown in the figure, the short link BL is turned so as to increase α, where as the link LK causes the other short link AK to turn so as to reduce α.

The fundamental equation for correct steering is,
$\text{Cot}\ \Phi - \text{Cos}\ \theta = b / l.$

In the above arrangement it is clear that the angle Φ through which AK turns is less than the angle θ through which the BL turns and therefore the left front axle turns through a smaller angle than the right front axle.

For different angle of turn θ, the corresponding value of Φ and (Cot Φ – Cos θ) are noted. This is done by actually drawing the mechanism to a scale or by calculations. Therefore for different value of the corresponding value of and are tabulated. Approximate value of b/l for correct steering should be between 0.4 and 0.5. In an Ackermann steering gear mechanism, the instantaneous centre I does not lie on the axis of the rear axle but on a line parallel to the rear axle axis at an approximate distance of 0.3l above it.

Three correct steering positions will be:

When moving straight

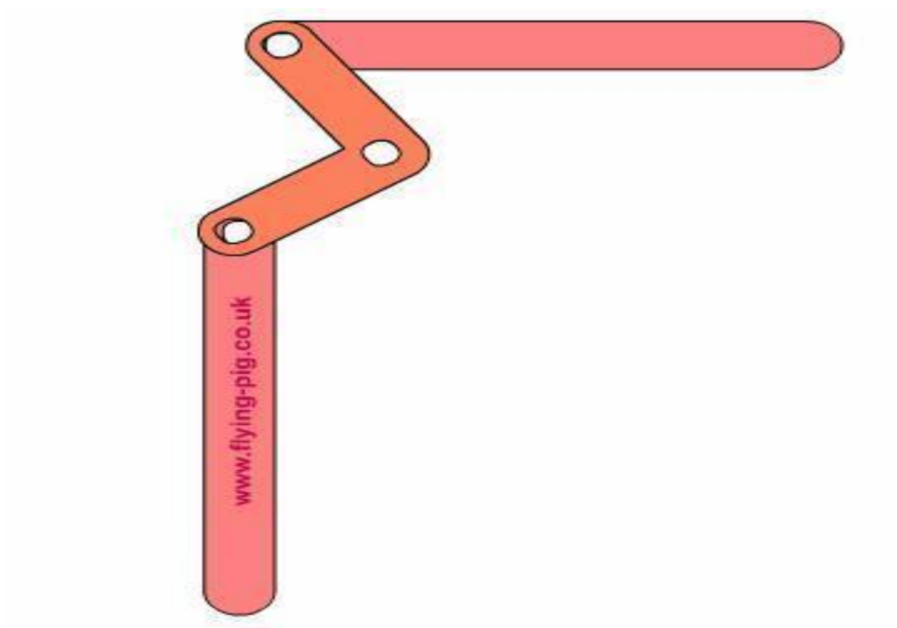
When moving one correct angle to the right corresponding to the link ratio AK/AB and angle α.

Similar position when moving to the left.

(In all other positions pure rolling is not obtainable.)

Some Of The Mechanisms Which Are Used In Day To Day Life.

Bell Crank:



The bell crank is used to convert the direction of reciprocating movement. By varying the angle of the crank piece it can be used to change the angle of movement from 1 degree to 180 degrees.

The bell crank was originally used in large house to operate the servant‟s bell, hence the name.

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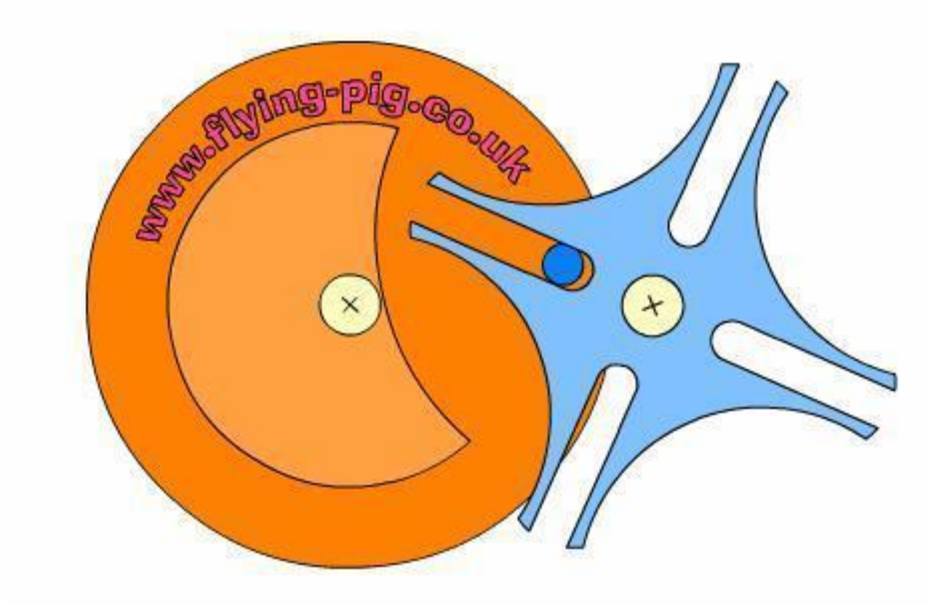
GENEVA STOP:

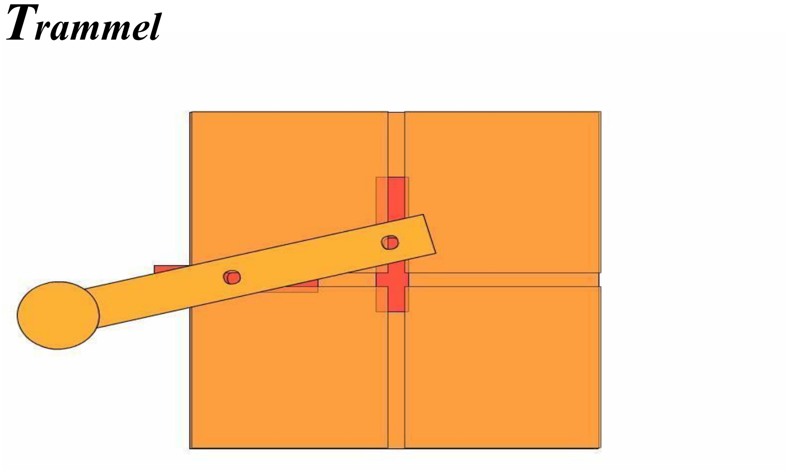


The Geneva stop is named after the Geneva cross, a similar shape to the main part of the mechanism.

The Geneva stop is used to provide intermittent motion, the orange wheel turns continuously, the dark blue pin then turns the blue cross quarter of a turn for each revolution of the drive wheel. The crescent shaped cut out in dark orange section lets the points of the cross past, then locks the wheel in place when it is stationary.

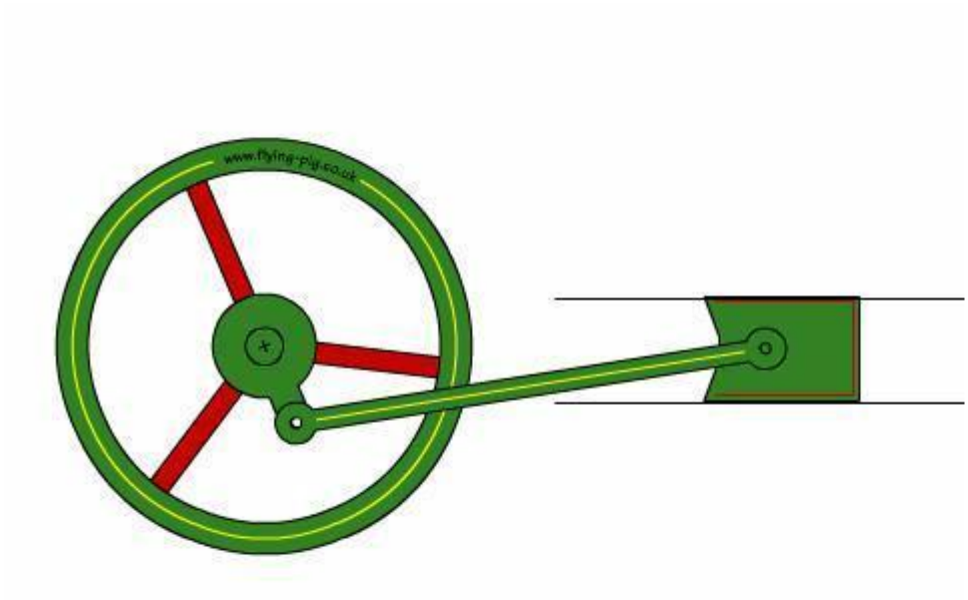
The Geneva stop mechanism is used commonly in film

Elliptical Trammel



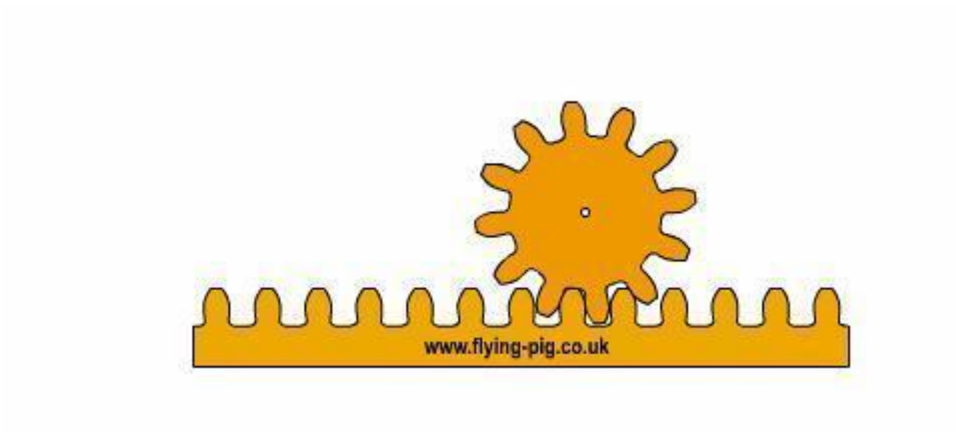
This fascinating mechanism converts rotary motion to reciprocating motion in two axis.Notice that the handle traces out an ellipse rather than a circle. A similar mechanism is used in ellipse drawing tools.

Piston Arrangement



This mechanism is used to convert between rotary motion and reciprocating motion, it works either way. Notice how the speed of the piston changes. The piston starts from one end, and increases its speed. It reaches maximum speed in the middle of its travel then gradually slows down until it reaches the end of its travel.

Rack And Pinion

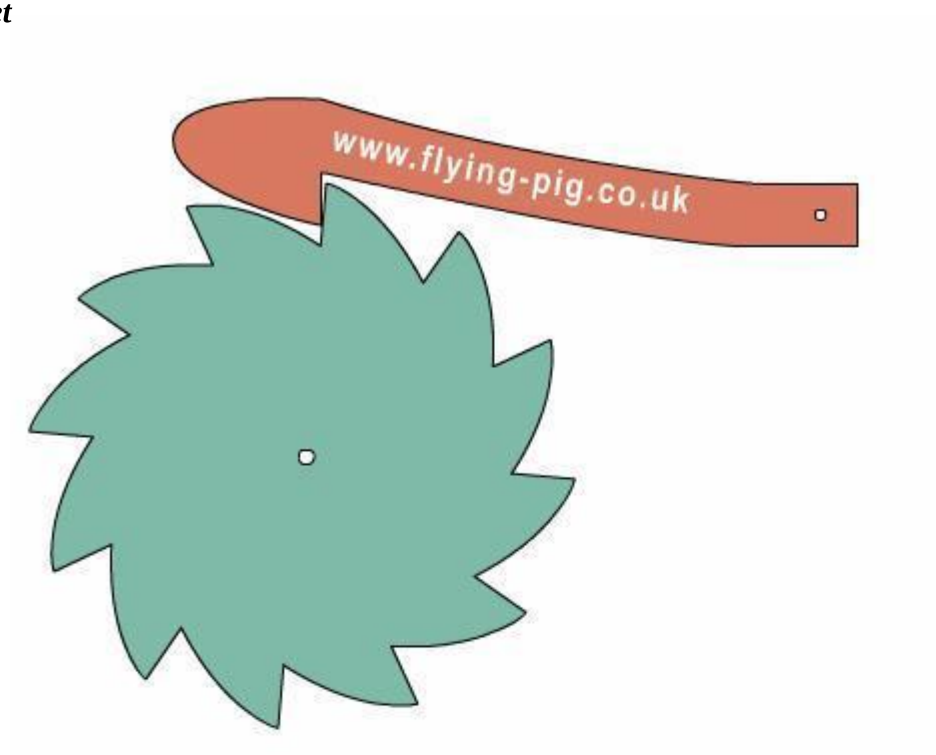


The rack and pinion is used to convert between rotary and linear motion. The rack is the flat, toothed part, the pinion is the gear. Rack and pinion can convert from rotary to linear of from linear to rotary.

The diameter of the gear determines the speed that the rack moves as the pinion turns. Rack and pinions are commonly used in the steering system of cars to convert the rotary motion of the steering wheel to the side to side motion in the wheels.

Rack and pinion gears give a positive motion especially compared to the friction drive of a wheel in tarmac. In the rack and pinion railway a central rack between the two rails engages with a pinion on the engine allowing the train to be pulled up very steep slopes.

Ratchet



The ratchet can be used to move a toothed wheel one tooth at a time. The part used to move the ratchet is known as the pawl.

The ratchet can be used as a way of gearing down motion. By its nature motion created by a ratchet is intermittent. By using two pawls simultaneously this intermittent effect can be almost, but not quite, removed.

Ratchets are also used to ensure that motion only occurs in only one direction, useful for winding gear which must not be allowed to drop. Ratchets are also used in the freewheel mechanism of a bicycle.

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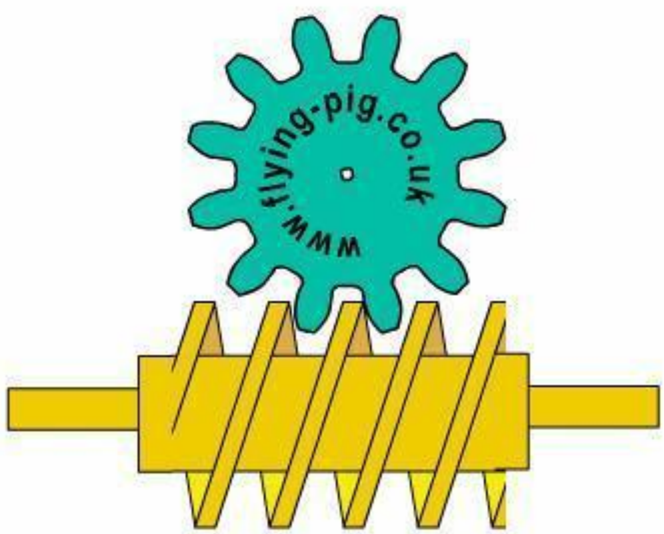
Worm Gear



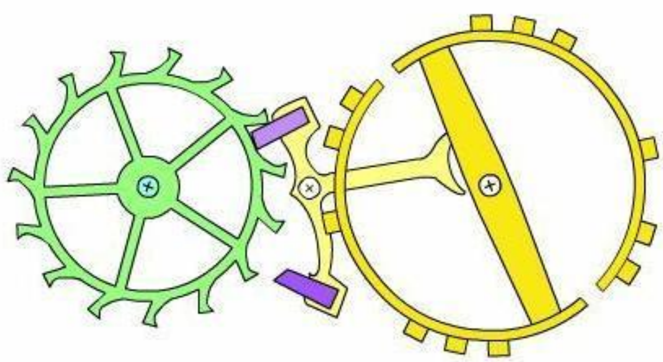
A worm is used to reduce speed. For each complete turn of the worm shaft the gear shaft advances only one tooth of the gear.

In this case, with a twelve tooth gear, the speed is reduced by a factor of twelve. Also, the axis of rotation is turned by 90 degrees.

Unlike ordinary gears, the motion is not reversible, a worm can drive a gear to reduce speed but a gear cannot drive a worm to increase it.

As the speed is reduced the power to the drive increases correspondingly. Worm gears are a compact, efficient means of substantially decreasing speed and increasing power. Ideal for use with small electric motors.

Watch Escapement.

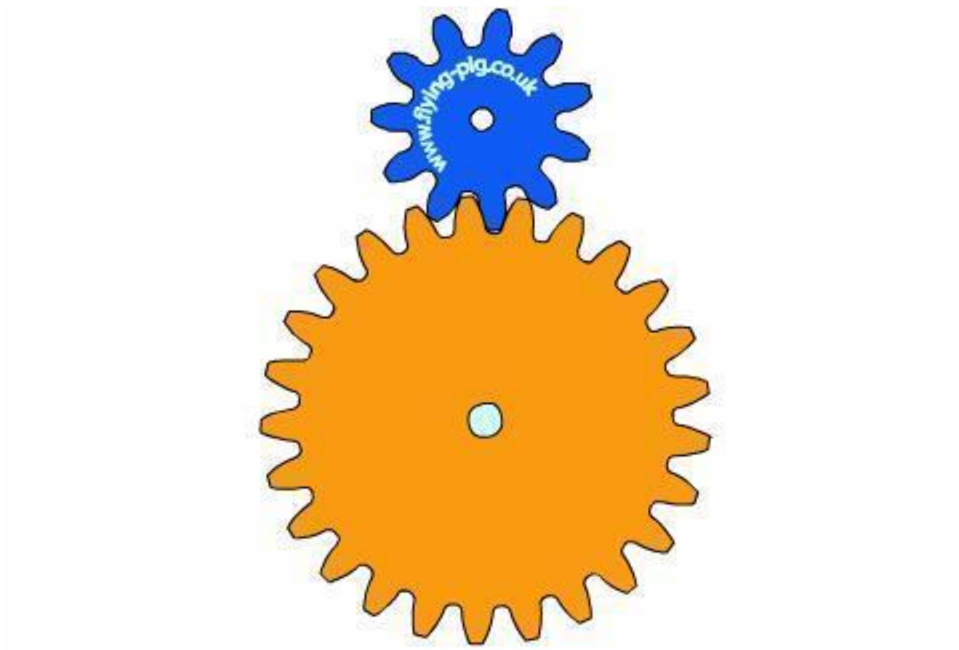


The watch escapement is the centre of the time piece. It is the escapement which divides the time into equal segments.

The balance wheel, the gold wheel, oscillates backwards and forwards on a hairspring (not shown) as the balance wheel moves the lever is moved allowing the escape wheel (green) to rotate by one tooth.

The power comes through the escape wheel which gives a small 'kick' to the palettes (purple) at each tick.

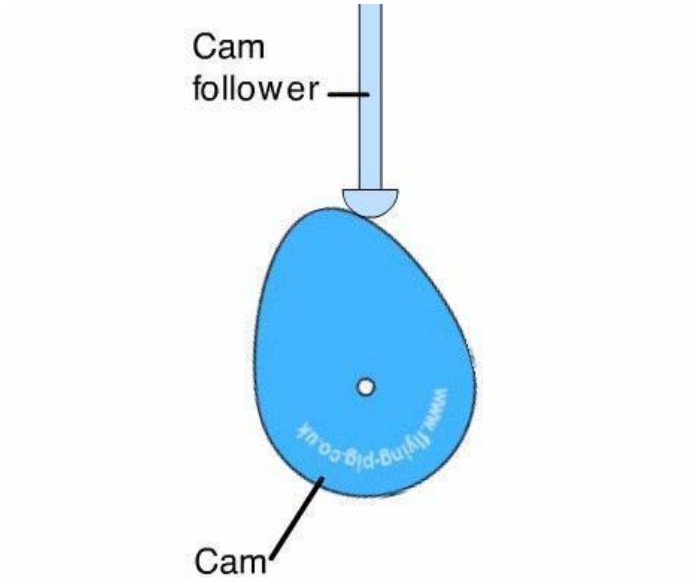
Gears.



Gears are used to change speed in rotational movement. In the example above the blue gear has eleven teeth and the orange gear has twenty five. To turn the orange gear one full turn the blue gear must turn 25/11 or 2.2727r turns. Notice that as the blue gear turns clockwise the orange gear turns anti-clockwise.

In the above example the number of teeth on the orange gear is not divisible by the number of teeth on the blue gear. This is deliberate. If the orange gear had thirty three teeth then every three turns of the blue gear the same teeth would mesh together which could cause excessive wear. By using none divisible numbers the same teeth mesh only every seventeen turns of the blue gear.

Cam Follower.

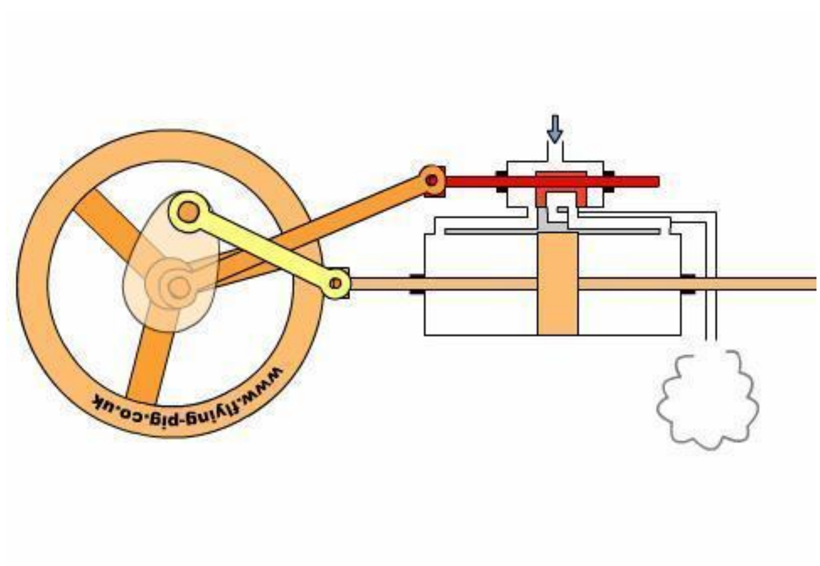


Cams are used to convert rotary motion into reciprocating motion. The motion created can be simple and regular or complex and irregular.

As the cam turns, driven by the circular motion, the cam follower traces the surface of the cam transmitting its motion to the required mechanism.

Cam follower design is important in the way the profile of the cam is followed. A fine pointed follower will more accurately trace the outline of the cam. This more accurate movement is at the expense of the strength of the cam follower.

Steam Engine.



Steam engines were the backbone of the industrial revolution. In this common design high pressure steam is pumped alternately into one side of the piston, then the other forcing it back and forth. The reciprocating motion of the piston is converted to useful rotary motion using a crank.

As the large wheel (the fly wheel) turns a small crank or cam is used to move the small red control valve back and forth controlling where the steam flows.

In this animation the oval crank has been made transparent so that you can see how the control valve crank is attached.

UNIT 03

VELOCITY AND ACCELERATION ANALYSIS

Introduction [redacted]

Kinematics deals with study of relative motion between the various parts of the machines. Kinematics does not involve study of forces. Thus motion leads study of displacement, velocity and acceleration of a part of the machine.

Study of Motions of various parts of a machine is important for determining their velocities and accelerations at different moments.

As dynamic forces are a function of acceleration and acceleration is a function of velocities, study of velocity and acceleration will be useful in the design of mechanism of a machine. The mechanism will be represented by a line diagram which is known as configuration diagram. The analysis can be carried out both by graphical method as well as analytical method.

Some important Definitions [redacted]

Displacement: All particles of a body move in parallel planes and travel by same distance is known, linear displacement and is denoted by „x‟.

A body rotating about a fired point in such a way that all particular move in circular path angular displacement and is den[redacted]d by „ ‟.

Velocity: Rate of change of displacement is velocity. Velocity can be linear velocity of angular velocity.

Linear velocity is Rate of change of linear displacement= $V = \frac{dx}{dt}$

Angular velocity is Rate of change of angular displacement = $\omega = \frac{d\theta}{dt}$

Relation between linear velocity and angular velocity.

$$x = r\theta$$

$$\frac{dx}{dt} = r\frac{d\theta}{dt}$$

$$V = r\omega$$

$$\omega = \frac{d\theta}{dt}$$

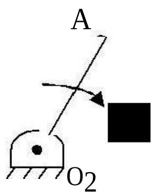
<u>Acceleration</u>: Rate of change of velocity

$f = \frac{dv}{dt}$ [redacted] $\frac{d^2x}{dt^2}$ Linear Acceleration (Rate of change of linear velocity)

Thirdly [redacted] d[redacted]/dt [redacted] d^2[redacted]/dt^2 Angular Acceleration (Rate of change of angular velocity)

We also have,

Absolute velocity: Velocity of a point with respect to a fixed point (zero velocity point).

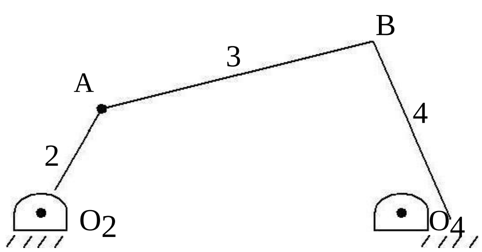


V_a = [redacted] x r

V_a = [redacted] x O_2 A

Ex: V_{ao_2} is absolute velocity.

Relative velocity: Velocity of a point with respect to another point „x‟

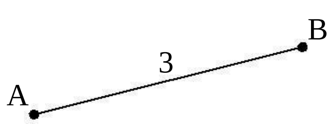


Ex: V_{ba} → Velocity of point B with respect to A

<u>Note</u>: Capital letters are used for configuration diagram. Small letters are used for velocity vector diagram.

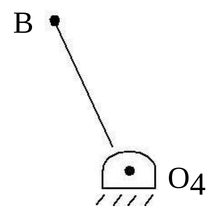
This is absolute velocity

Velocity of point A with respect to O_2 fixed point, zero velocity point.

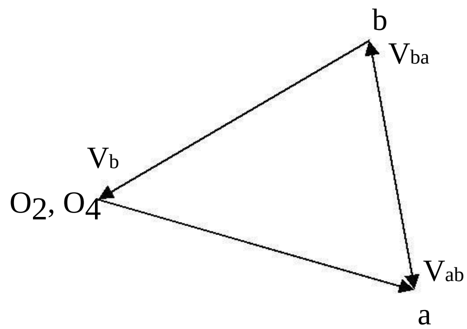


V_{ba} = or V_{ab}

V_{ba} = or V_{ab} Equal in magnitude but opposite in direction.



V_b → Absolute velocity is velocity of B with respect to O_4 (fixed point, zero velocity point)



Velocity vector diagram

Vector $\overrightarrow{O_2a} = V_a$= Absolute velocity

Vector $\overrightarrow{ab} = V_{ab}$ } Relative velocity

ba = V_a

V_{ab} is equal magnitude with V_{ba} but is apposite in direction.

Vector $\overrightarrow{O_4\,b} = V_b$ absolute velocity.

To illustrate the difference between absolute velocity and relative velocity. Let, us consider a simple situation.

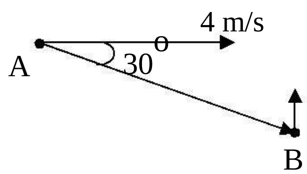
A link AB moving in a vertical plane such that the link is inclined at 30^o to the horizontal with point A is moving horizontally at 4 m/s and point B moving vertically upwards. Find velocity of B.

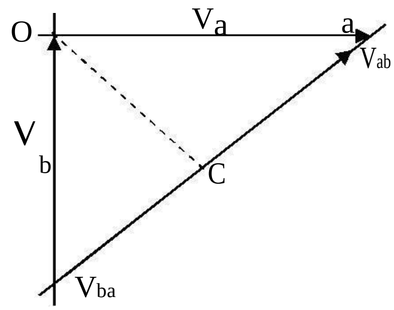
V_a = 4 m/s ab Absolute velocity Horizontal direction

(known in magnitude and directors)

→

V_b = ? $\overrightarrow{ab}$ Absolute velocity Vertical direction (known in directors only)





Velocity of B with respect to A is equal in magnitude to velocity of A with respect to B but opposite in direction.

[redacted] **Relative Velocity Equation** [redacted]

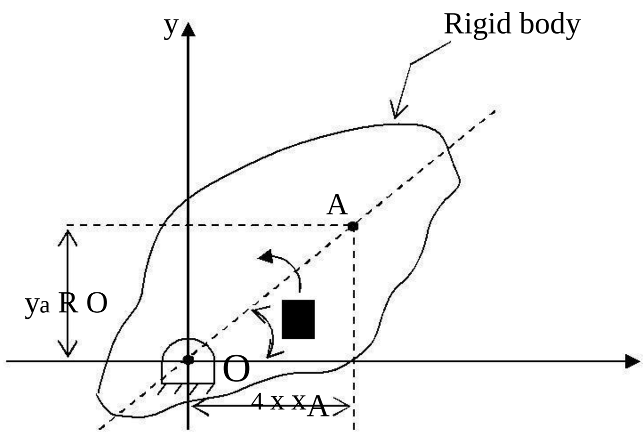


Fig. 1 Point O is fixed and End A is a point on rigid body.

Rotation of a rigid link about a fixed centre.

Consider rigid link rotating about a fixed centre O, as shown in figure. The distance between O and A is R and OA makes and angl[redacted] ‘‘ with x-axis next link

x_A = R cos[redacted] y_A = R sin [redacted]

Differentiating x_A with respect to time gives velocity.

d

xA R sin θ d θ dt dt

= - R sin

Similarly, $\frac{dy_A}{dt}$ [redacted] $d\theta$

$= -$ R [redacted] os [redacted]

Let, $\frac{d\,x_A}{dt}$ ___

[redacted] $\frac{d\theta}{\ }$ = angular velocity of OA dt [redacted]

[redacted]

[redacted] $_A{}^x = -R\omega \sin$ [redacted]

$V_A{}^y = -R\omega \cos$ [redacted]

[redacted]otal velocity of point A is given by

$V_A = \sqrt{}$ [redacted] n [redacted] s θ [redacted]

$\mathbf{V_A = R\omega}$

[redacted] Relative Velocity Equation of Two Points on a Rigid link [redacted]

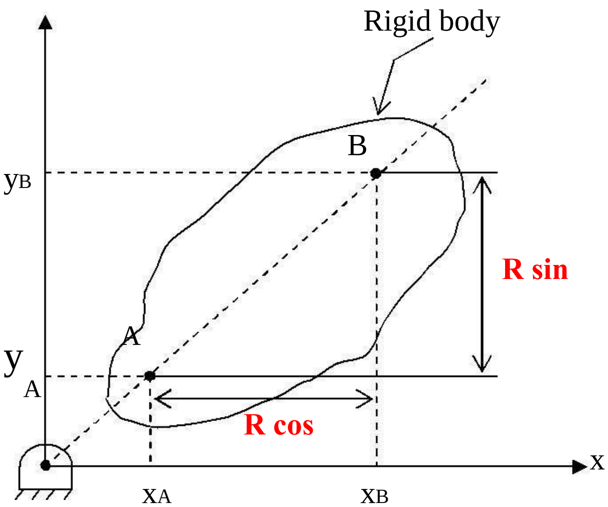


Fig. 2 Points A and B are located on rigid body

From Fig. 2

$x_B = x_A + R\cos$ $y_B = y_A + R\sin$

Differentiating x_B and y_B with respect to time we get,

$\frac{d}{dt} x_B$ [redacted] V_B^x [redacted] $\frac{d}{dt} x_A$ [redacted] [redacted] in [redacted] $\frac{d\theta}{dt}$

[redacted] $\frac{d}{dt} x_A$ [redacted] R [redacted] in θ [redacted] V_A^x [redacted] R [redacted] n [redacted]

Similarly, $\frac{d}{dt} y_B$ [redacted] V_B^y [redacted] $\frac{d}{dt} y_A$ [redacted] R [redacted] os [redacted] $\frac{d\theta}{dt}$

[redacted] $\frac{d}{dt} y_A$ [redacted] R [redacted] cos θ [redacted] V_A^y [redacted] A [redacted] os θ

$V_A = V_A^x \rightarrow V_A^y$ = Total velocity of point A

Similarly, $V_B = V_B^x \rightarrow V_B^y$ = Total velocity of point B

$= V_A^x \rightarrow$ (Rω sin [redacted] $\rightarrow V_A^y \rightarrow$ Rω cos [redacted]

$= (V_A^x \rightarrow V_A^y) \rightarrow$ (Rω sin [redacted] + R ωcos [redacted]

$= (V_A^x \rightarrow V_A^y) V_A$ Similarly, (R ωsin [redacted] + Rω cos [redacted] = Rω

[redacted] $B = V_A$ Rω = $V_A \rightarrow V_{BA}$

[redacted] **$BA = V_B - V_A$**

Velocity analysis of any mechanism can be carried out by various methods.

1. By graphical method
2. By relative velocity method
3. By instantaneous method

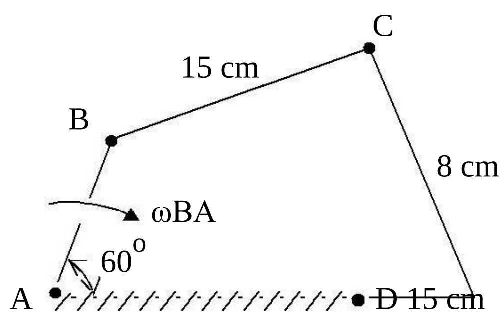
[redacted] **By Graphical Method** [redacted]

The following points are to be considered while solving problems by this method.

1. Draw the configuration design to a suitable scale.
2. Locate all fixed point in a mechanism as a common point in velocity diagram.
3. Choose a suitable scale for the vector diagram velocity.
4. The velocity vector of each rotating link is [redacted] the link.
5. Velocity of each link in mechanism has both magnitude and direction. Start from a point whose magnitude and direction is known.
6. The points of the velocity diagram are indicated by small letters.

To explain the method let us take a few specific examples.

1. *Four – Bar Mechanism*: In a four bar chain ABCD link AD is fixed and in 15 cm long. The crank AB is 4 cm long rotates at 180 rpm (cw) while link CD rotates about D is 8 cm long BC = AD and | BAD = 60^o. Find angular velocity of link CD.



Configuration Diagram

Velocity vector diagram

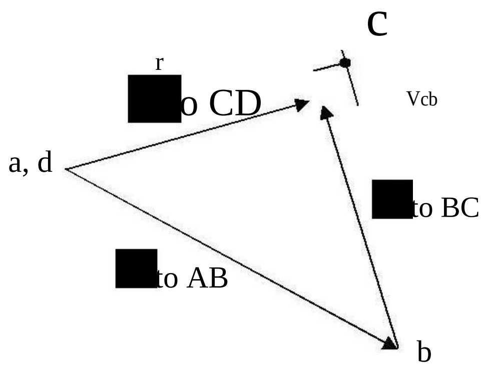
[redacted]

2πx120

V_b = r = $_{ba}$ x AB = x4 = 50.24 cm/sec 60

Choose a suitable scale

1 cm = 20 m/s = $\overrightarrow{ab}$



$V_{cb} = \overrightarrow{bc}$

$V_c = \overrightarrow{dc}$ = 38 cm/sec = V_{cd}

We know that V =ω R

V_{cd} = [redacted]D x CD

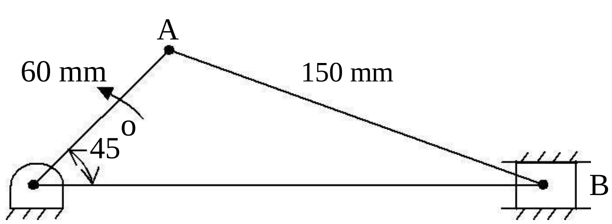
$\omega_{cD} = \frac{v_{cd}}{CD}$ [redacted] $\frac{38}{8}$ [redacted].75 rad/sec (cw)

2. *Slider Crank Mechanism*:

In a crank and slotted lever mechanism crank rotates of 300 rpm in a counter clockwise direction. Find

(i) Angular velocity of connecting rod and

(ii) Velocity of slider.



Configuration diagram

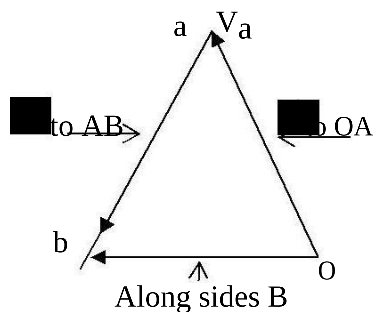
Step 1: Determine the magnitude and velocity of point A with respect to

[redacted] 0, 2π x 300

V_A = O1A x O_2A = x 60 60

= 600 [redacted]m/sec

Step 2: Choose a suitable scale to draw velocity vector diagram.



Velocity vector diagram

$V_{ab} = \overrightarrow{ab} = 1300\text{mm/sec}$

$[redacted]_{ba} = \frac{V_{ba}}{[redacted]} \frac{1300}{[redacted]}$ 8.66 rad/sec BA 150

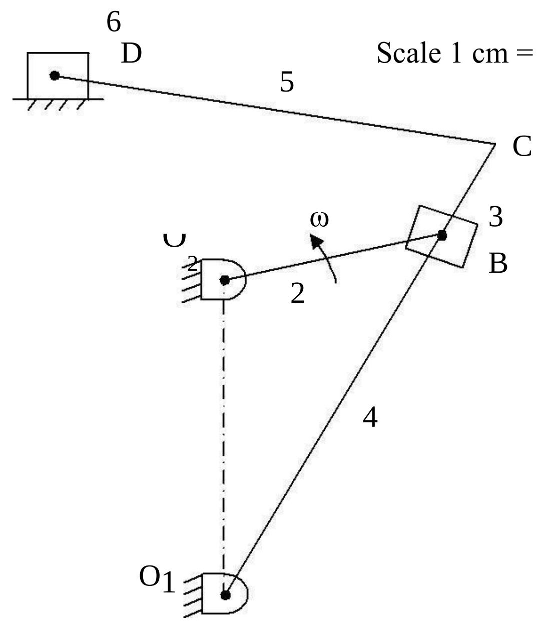
$V_b = \overrightarrow{ob}$ velocity of slider

Note: Velocity of slider is along the line of sliding.

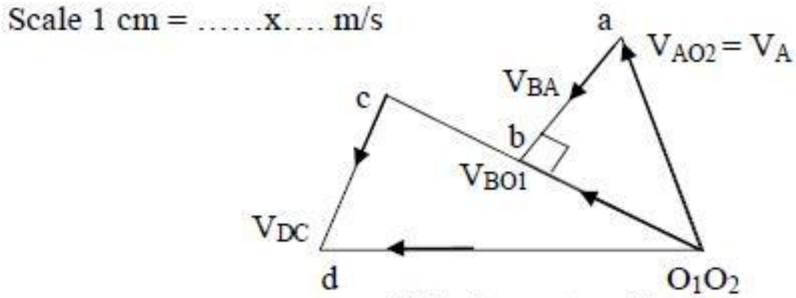
3. Shaper Mechanism:

In a crank and slotted lever mechanisms crank O_2A rotates at [redacted]d/sec in CCW direction. Determine the velocity of slider.

Scale 1 cm = ……x…. m



Configuration diagram



Velocity vector diagram

$V_a = \omega_2 \times O_2A$

$$\frac{\overrightarrow{O_1b}}{O_1B} = \frac{\overrightarrow{O_1c}}{O_1C}$$

To locate point C

$$\therefore \overrightarrow{O_1c} = \overrightarrow{O_1b}\left(\frac{O_1C}{O_1B}\right)$$

To Determine Velocity of Rubbing

Two links of a mechanism having turning point will be connected by pins. When the links are motion they rub against pin surface. The velocity of rubbing of pins depends on the angular velocity of links relative to each other as well as direction.

For example: In a four bar mechanism we have pins at points A, B, C and D.

[redacted]ra = [redacted] x ratios of pin A (r_{pa}) [redacted]

+ sign is used [redacted]s CW and W_{bc} is CCW i.e. when angular velocities are in opposite directions use + sign when angular velocities are in some directions use

-ve sign.

V_{rb} = [redacted] + [redacted] radius r_{pb}

V_{rC} = [redacted] + [redacted] radius r_{pc}

VrD = [redacted] cd rpd

Problems on velocity by velocity vector method (Graphical solutions)

Problem 1:

In a four bar mechanism, the dimensions of the links are as given below:

AB = 50 mm, BC = 66 mm

CD = 56 mm and AD = 100 mm

At a given instant when | DAB [redacted]0^{o} the angular velocity of link AB is 10.5 rad/sec in CCW direction.

Determine,

i) Velocity of point C

ii) Velocity of point E on link BC when BE = 40 mm

iii) The angular velocity of link BC and CD

iv) The velocity of an offset point F on link BC, if BF = 45 mm, CF = 30 mm and BCF is read clockwise.

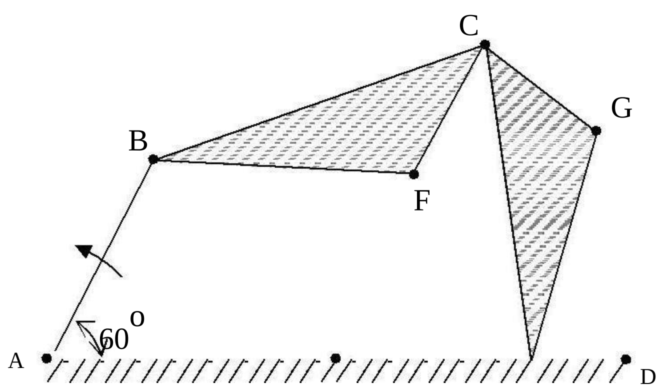
v) The velocity of an offset point G on link CD, if CG = 24 mm, DG = 44 mm and DCG is read clockwise.

vi) The velocity of rubbing of pins A, B, C and D. The ratio of the pins are 30 mm, 40 mm, 25 mm and 35 mm respectively.

Solution:

Step -1: Construct the configuration diagram selecting a suitable scale.

Scale: 1 cm = 20 mm



Step – 2: Given the angular velocity of link AB and its direction of rotation determine velocity of point with respect to A (A is fixed hence, it is zero velocity point).

V_{ba} = [redacted]A x BA

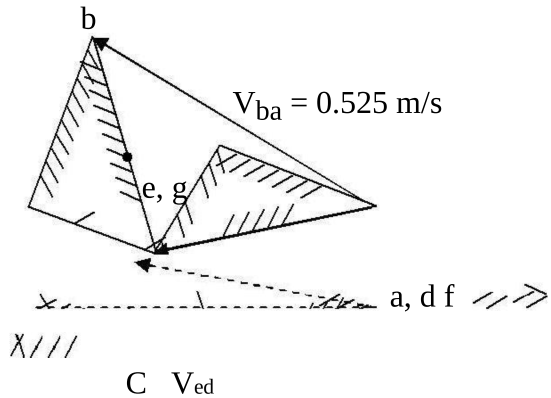
= 10.5 x 0.05 = 0.525 m/s

Step – 3: To draw velocity vector diagram choose a suitable scale, say 1 cm = 0.2 m/s.

[redacted] First locate zero velocity points. [redacted]

[redacted]

[redacted] Draw a line [redacted] link AB in the direction of rotation of link AB (CCW) equal to 0.525 m/s. [redacted]



[redacted] From b draw a line [redacted]o BC and from d. Draw d line [redacted]to CD to interest at C. [redacted]

[redacted] V_{cb} is given vector bc V_{bc} = 0.44 m/[redacted]

[redacted] V_{cd} is given vector dc V_{cd} = 0.39 m/[redacted]

Step – 4: To determine velocity of point E (Absolute velocity) on link BC, first locate the position of point E on velocity vector diagram. This can be done by taking corresponding ratios of lengths of links to vector distance i.e.

$\frac{be}{bc}$ [redacted] $\frac{BE}{BC}$

[redacted]e = $\frac{BE}{BC} \times V_{cb} = \frac{0.04}{0.066} \times 0.44 = 0.24$ m/s

Join e on velocity vector diagram to zero velocity points a, d / vector de = V_e = 0.415 m/s.

Step 5: To determine angular velocity of links BC and CD, we know V_{bc} and V_{cd}. [redacted]

$V_{bc} = \omega_{BC} \times BC$

$$\omega_{BC} = \frac{V_{bc}}{BC} = \frac{0.44}{0.066} = 6.6\ r/s\ .\ (cw)$$

Similarly, $V_{cd} = \omega_{CD} \times CD$

$$[\text{redacted}]_{CD} = \frac{V_{cd}}{CD} [\text{redacted}] \frac{0.39}{0.056} [\text{redacted}] 6.96 \; r/s \; (CCW)$$

Step – 6: To determine velocity of an offset point F

- Draw a line [redacted]o CF from C on velocity vector diagram. [redacted]
- Draw a line [redacted]to BF from b on velocity vector diagram to intersect the previously drawn line at „f". [redacted]
- From the point f to zero velocity point a, d and measure vector fa to get [redacted] V_f = 0.495 m/s.

Step – 7: To determine velocity of an offset point.

- Draw a line [redacted]o GC from C on velocity vector diagram. [redacted]
- Draw a line [redacted]o DG from d on velocity vector diagram to intersect previously drawn line at g. [redacted]
- Measure vector dg to get velocity of point G. [redacted]

$$V_g = \vec{dg} [\text{redacted}].305 \; m/s$$

Step – 8: To determine rubbing velocity at pins

- Rubbing velocity at pin A will be [redacted]

  V_{pa} = [redacted] x r of pin A [redacted]

  V_{pa} = 10.5 x 0.03 = 0.315 m/s [redacted]

- Rubbing velocity at pin B will be V_{pb} = [redacted] + cb)

  [redacted]r_{pb} of point at B. [redacted]

  [[redacted] CCW and [redacted]CW] [redacted]

  V_{pb} = (10.5 + 6.6) x 0.04 = 0.684 m/s. [redacted]

- Rubbing velocity at point C will be = 6.96 x 0.035 = 0.244 m/s [redacted]

Problem 2:

In a slider crank mechanism the crank is 200 mm long and rotates at 40 rad/sec in a CCW direction. The length of the connecting rod is 800 mm. When the crank turns through 60^{o} from Inner-dead centre.

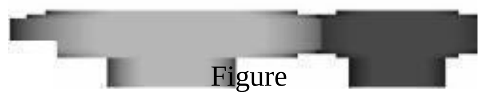
UNIT-06

GEARS

Introduction:

The slip and creep in the belt or rope drives is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slip is to reduce the velocity ratio of the drive. In precision machine, in which a definite velocity ratio is importance (as in watch mechanism, special purpose machines.etc), the only positive drive is by means of gears or toothed wheels.

Friction Wheels: Kinematiclly, the motion and power transmitted by gears is equivalent to that transmitted by friction wheels or discs in contact With sufficient friction between them. In order to



Figure

understand motion transmitted by two toothed 4.1 wheels, let us consider the two discs placed together

as shown in the figure 4.1.

When one of the discs is rotated, the other disc will be rotate as long as the tangential force exerted by the driving disc does not exceed the maximum frictional resistance between the two discs. But when the tangential force exceeds the frictional resistance, slipping will take place between the two discs. Thus the friction drive is not positive a drive, beyond certain limit.

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers. Gears are highly efficient (nearly 95%) due to primarily rolling contact between the teeth, thus the motion transmitted is considered as positive.

Gears essentially allow positive engagement between teeth so high forces can be transmitted while still undergoing essentially rolling contact. Gears do not depend on friction and do best when friction is minimized.

act as a brake for the conveyor when the motor is not turning. One other very interesting usage of worm gears is in the Torsion differential, which is used on some high-performance cars and trucks.

4.3 Terminology for Spur Gear

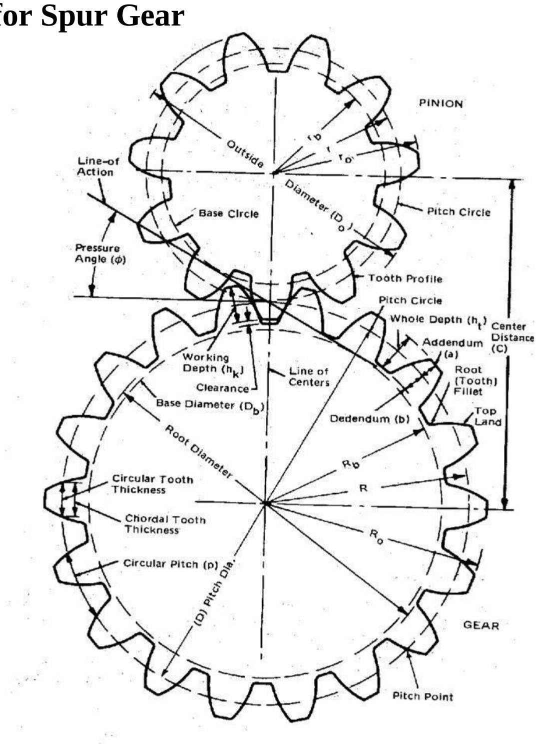
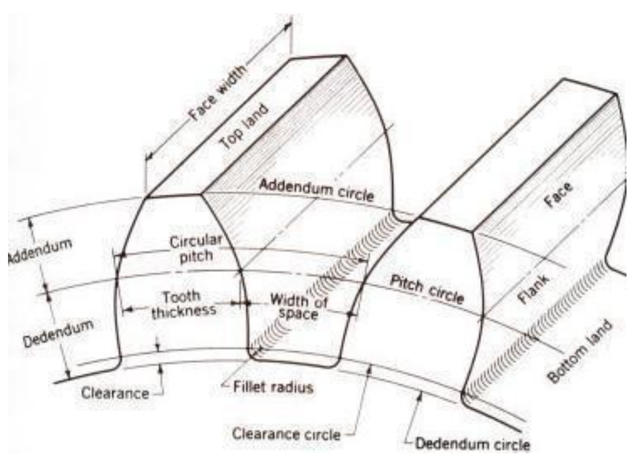


Figure 4-4 Spur Gear

Terminology:



Addendum: The radial distance between the Pitch Circle and the top of the teeth.

Arc of Action: Is the arc of the Pitch Circle between the beginning and the end of the engagement of a given pair of teeth.

Arc of Approach: Is the arc of the Pitch Circle between the first point of contact of the gear teeth and the Pitch Point.

Arc of Recession: That arc of the Pitch Circle between the Pitch Point and the last point of contact of the gear teeth.

Backlash: Play between mating teeth.

Base Circle: The circle from which is generated the involute curve upon which the tooth profile is based.

Center Distance: The distance between centers of two gears.

Chordal Addendum: The distance between a chord, passing through the points where the Pitch Circle crosses the tooth profile, and the tooth top.

Chordal Thickness: The thickness of the tooth measured along a chord passing through the points where the Pitch Circle crosses the tooth profile.

Circular Pitch: Millimeter of Pitch Circle circumference per tooth.

Circular Thickness: The thickness of the tooth measured along an arc following the Pitch Circle

Clearance: The distance between the top of a tooth and the bottom of the space into which it fits on the meshing gear.

Contact Ratio: The ratio of the length of the Arc of Action to the Circular Pitch. **Dedendum:** The radial distance between the bottom of the tooth to pitch circle **Diametral Pitch:** Teeth per mm of diameter.

Face: The working surface of a gear tooth, located between the pitch diameter and the top of the tooth.

Face Width: The width of the tooth measured parallel to the gear axis.

Flank: The working surface of a gear tooth, located between the pitch diameter and the bottom of the teeth

Gear: The larger of two meshed gears. If both gears are the same size, they are both called "gears".

Land: The top surface of the tooth.

Line of Action: That line along which the point of contact between gear teeth travels, between the first point of contact and the last.

Module: Millimeter of Pitch Diameter to Teeth.

Pinion: The smaller of two meshed gears.

Pitch Circle: The circle, the radius of which is equal to the distance from the center of the gear to the pitch point.

Diametral pitch: Teeth per millimeter of pitch diameter.

Pitch Point: The point of tangency of the pitch circles of two meshing gears, where the Line of Centers crosses the pitch circles.

Pressure Angle: Angle between the Line of Action and a line perpendicular to the Line of Centers.

Profile Shift: An increase in the Outer Diameter and Root Diameter of a gear, introduced to lower the practical tooth number or acheive a non-standard Center Distance.

Ratio: Ratio of the numbers of teeth on mating gears.

Root Circle: The circle that passes through the bottom of the tooth spaces.

Root Diameter: The diameter of the Root Circle.

Working Depth: The depth to which a tooth extends into the space between teeth on the mating gear.

4.2 Gear-Tooth Action

4.2.1 Fundamental Law of Gear-Tooth Action

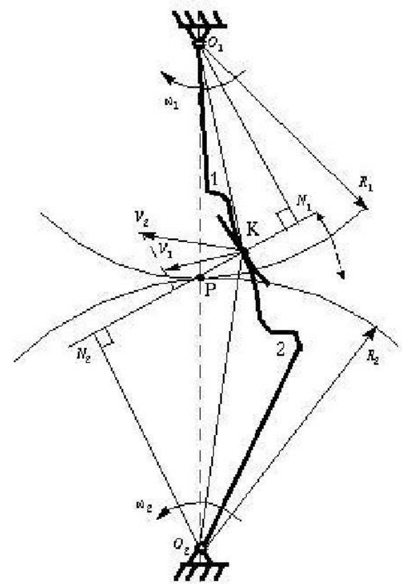


Figure shows two mating gear teeth, in which

- Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point K.
- N_1N_2 is the common normal of the two profiles.
- N_1 is the foot of the perpendicular from O_1 to N_1N_2
- N_2 is the foot of the perpendicular from O_2 to N_1N_2.

Although the two profiles have different velocities V_1 and V_2 at point K, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

$$O_1N_1\,\omega_1 = O_2N_2\,\omega_2 \qquad (4.1)$$

$$\frac{\omega_1}{\omega_2}=\frac{O_2N_2}{O_1N_1} \qquad (4.2)$$

We notice that the intersection of the tangency N_1N_2 and the line of center O_1O_2 is point P, and from the similar triangles,

$$\Delta O_1N_1\,P=\Delta O_2N_2\,P \qquad (4.3)$$

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or **velocity ratio**, of a pair of mating teeth is

$$\frac{\omega_1}{\omega_2}=\frac{O_2P}{O_1P} \qquad (4.4)$$

Point P is very important to the velocity ratio, and it is called the **pitch point**. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the **fundamental law of gear-tooth action**.

From the equations 4.2 and 4.4, we can write,

$$\frac{\omega_1}{\omega_2}=\frac{O_2P}{O_1P}=\frac{O_2N_2}{O_1N_1} \qquad (4.5)$$

which determines the ratio of the radii of the two base circles. The radii of the base circles is given by:

$$O_1N_1=O_1P\cos\phi \qquad and \qquad O_2N_2=O_2P\cos\phi \qquad (4.6)$$

Also the centre distance between the base circles:

$$O_1O_2=O_1P+O_2P=\frac{O_1N_1}{\cos\phi}+\frac{O_2N_2}{\cos\phi}=\frac{O_1N_1+O_2N_2}{\cos\phi} \qquad (4.7)$$

where ϕ is the pressure angle or the angle of obliquity. It is the angle which the common normal to the base circles make with the common tangent to the pitch circles.

Although the two profiles have different velocities $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$ at point K, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

We notice that the intersection of the tangency N_1N_2 and the line of center O_1O_2 is point P, and from the similar triangles,

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or **velocity ratio**, of a pair of mating teeth is Point P is very important to the velocity ratio, and it is called the **pitch point**. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the **fundamental law of gear-tooth action**. where is the pressure angle or the angle of obliquity. It is the angle which the common normal to the base circles make with the common tangent to the pitch circles

4.2.2 Constant Velocity Ratio

For a constant velocity ratio, the position of *P* should remain unchanged. In this case, the motion transmission between two gears is equivalent to the motion transmission between two imagined slip-less cylinders with radius R_1 and R_2 or diameter D_1 and D_2. We can get two circles whose centers are at O_1 and O_2, and through pitch point *P*. These two circles are termed **pitch circles**. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

The **fundamental law of gear-tooth action** may now also be stated as follow (for gears with fixed center distance)

A common normal (the line of action) to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point

Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves, and the relative rotation speed of the gears will be constant(constant velocity ratio).

4.2.3 Conjugate Profiles

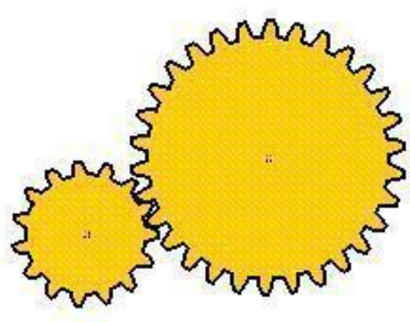
To obtain the expected velocity ratio of two tooth profiles, the normal line of their profiles must pass through the corresponding pitch point, which is decided by the velocity ratio. The two profiles which satisfy this requirement are called **conjugate profiles**. Sometimes, we simply termed the tooth profiles which satisfy the fundamental law of gear-tooth action the conjugate profiles.

Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the cycloidal and involute profiles. The involute has important advantages; it is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required when using the involute profile. The most commonly used conjugate tooth curve is the involute curve. (Erdman & Sandor). **conjugate action** : It is essential for correctly meshing gears, the size of the teeth (the module) must be the same for both the gears.

Another requirement - the shape of teeth necessary for the speed ratio to remain constant during an increment of rotation; this behavior of the contacting surfaces (ie. the teeth flanks) is known as **conjugate action**

4.3 Involute Curve

The following examples are involute spur gears. We use the word involute because the contour of gear teeth curves inward. Gears have many terminologies, parameters and principles. One of the important concepts is the velocity ratio, which is the ratio of the rotary velocity of the driver gear to that of the driven gears.



4.1 Generation of the Involute Curve

The curve most commonly used for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string, which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the **base circle**

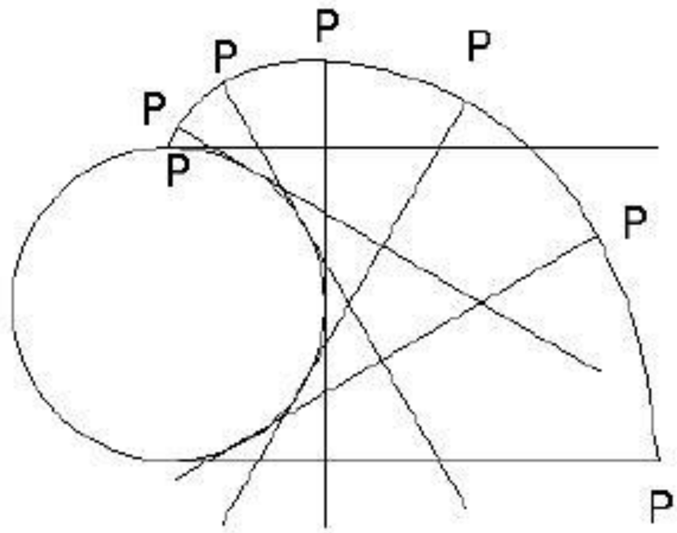


Figure Involute curve

1. The line rolls without slipping on the circle.

2. For any instant, the *instantaneous center* of the motion of the line is its point of tangent with the circle.

Note: We have not defined the term *instantaneous center* previously. The **instantaneous center** or **instant center** is defined in two ways.

1. When two bodies have planar relative motion, the instant center is a point on one body about which the other rotates at the instant considered.

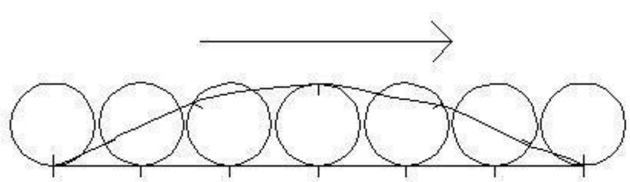
2. When two bodies have planar relative motion, the instant center is the point at which the bodies are relatively at rest at the instant considered.

3. The normal at any point of an involute is tangent to the base circle. Because of the property

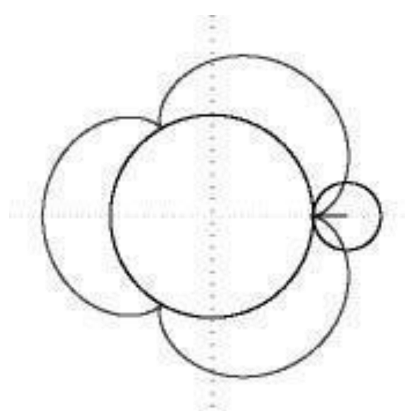
(2) of the involute curve, the motion of the point that is tracing the involute is perpendicular to the line at any instant, and hence the curve traced will also be perpendicular to the line at any instant.

There is no involute curve within the base circle.

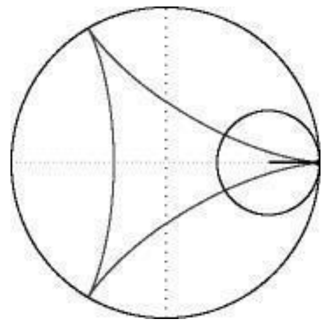
Cycloidal profile:



Epicycliodal Profile:



Hypocycliodal Profile:



The involute profile of gears has important advantages;

• It is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required. The most commonly used *conjugate* tooth curve is the *involute curve.*

(Erdman & Sandor).

2. In involute gears, the pressure angle, remains constant between the point of tooth engagement and disengagement. It is necessary for smooth running and less wear of gears.

But in cycloidal gears, the pressure angle is maximum at the beginning of engagement,

reduces to zero at pitch point, starts increasing and again becomes maximum at the end of

engagement. This results in less smooth running of gears.

3. The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively. Thus the involute teeth are easy to manufacture than cycloidal teeth.
In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

Advantages of Cycloidal gear teeth:

1. Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the

involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially

for cast teeth.

2. In cycloidal gears, the contact takes place between a convex flank and a concave surface, where as in involute gears the convex surfaces are in contact. This condition results in less wear

in cycloidal gears as compared to involute gears. However the difference in wear is negligible

3. In cycloidal gears, the interference does not occur at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

Properties of involute teeth:

1. A normal drawn to an involute at pitch point is a tangent to the base circle.
2. Pressure angle remains constant during the mesh of an involute gears.
3. The involute tooth form of gears is insensitive to the centre distance and depends only on the dimensions of the base circle.
4. The radius of curvature of an involute is equal to the length of tangent to the base circle.
5. Basic rack for involute tooth profile has straight line form.
6. The common tangent drawn from the pitch point to the base circle of the two involutes is the line of action and also the path of contact of the involutes.
7. When two involutes gears are in mesh and rotating, they exhibit constant angular velocity ratio and is inversely proportional to the size of base circles. (Law of Gearing or conjugate action)
8. Manufacturing of gears is easy due to single curvature of profile.

System of Gear Teeth

The following four systems of gear teeth are commonly used in practice:

- $14\frac{1}{2}^{O}$ Composite system
- $14\frac{1}{2}^{O}$ Full depth involute system
- 20^{O} Full depth involute system
- 20^{O} Stub involute system

The $14\frac{1}{2}^{O}$ *composite system* is used for general purpose gears.

It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion.The teeth are produced by formed milling cutters or hobs

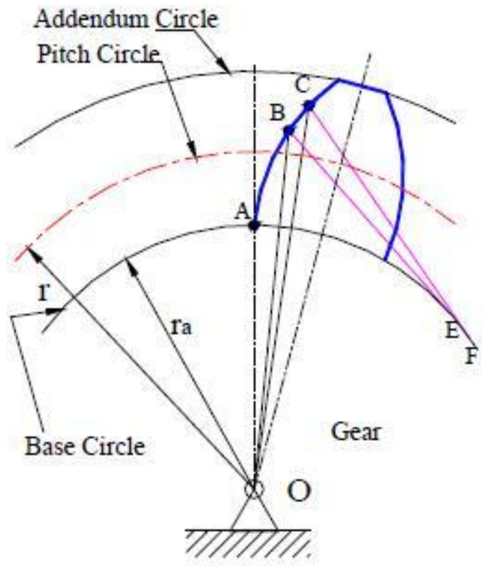
The tooth profile of the $14\frac{1}{2}^{O}$ *full depth involute system* was developed using gear hobs for spur and helical gears.

The tooth profile of the 20^{O} *full depth involute system* may be cut by hobs.

The increase of the pressure angle from $14\frac{1}{2}^{o}$ to 20^{o} results in a stronger tooth, because the tooth acting as a beam is wider at the base.

The 20^{o} *stub involute system* has a strong tooth to take heavy loads.

Involutometry



The study of the geometry of the involute profile for gear teeth is called involumetry. Consider an involute of base circle radius ra and two points B and C on the involute as shown in figure. Draw normal to the involute from the points B and C. The normal BE and CF are tangents to the Base circle.

Let

r_a= base circle radius of gear

r_b= radius of point B on the involute

r_c= radius of point C on the involute

and

[redacted]= pressure angle for the point B

[redacted]= pressure angle for the point C

t_b= tooth thickness along the arc at B

t_c= tooth thickness along the arc at C

From [redacted]*BE and*

OCF

$r_a \quad r_b \cos{}_b$ (1)

$r_a \quad r_c \cos{}_c$ 2

Therefore

$r_b \cos{}_b \quad r_c \cos{}_c$

From the properties of the Involute:

Arc AE = Length BE and

Arc AF = Length CF

[redacted] OE [redacted] $\frac{ArcAE}{OE}$ [redacted] $\frac{BE}{OE}$ [redacted] an [redacted] $_b$

[redacted] AOB [redacted] AOE [redacted] $_b$ [redacted] tan $_b$ [redacted] $_b$ [redacted]

[redacted] *inv.* $_b$ [redacted] tan $_b$ [redacted] $_b$ [redacted]

[redacted] *Expression* [redacted] an [redacted] *s* [redacted]

[redacted] *alled involute function* [redacted]

Similarly:

[redacted] OF [redacted] $\frac{ArcAF}{OF}$ [redacted] $\frac{BE}{OF}$ [redacted] an [redacted] $_c$

[redacted] OC [redacted] OF [redacted] an [redacted]

[redacted] *nv.* [redacted] an [redacted]

At the po int *B* [redacted]

[redacted] OD [redacted] OB [redacted] $\frac{t_b}{2r}$

[redacted] an [redacted] $\frac{t_b}{2r_b}$

At the po int *C*

[redacted] OD [redacted] OC [redacted] $\frac{t_c}{2r}$

[redacted] an [redacted] $\frac{t_c}{2r_c}$

Equating the above equations :

tan $_b$ [redacted] $_b$ $\frac{t_b}{2r_b}$ [redacted] an [redacted] $_c$ [redacted] $_c$ [redacted] $\frac{t_c}{2r_c}$

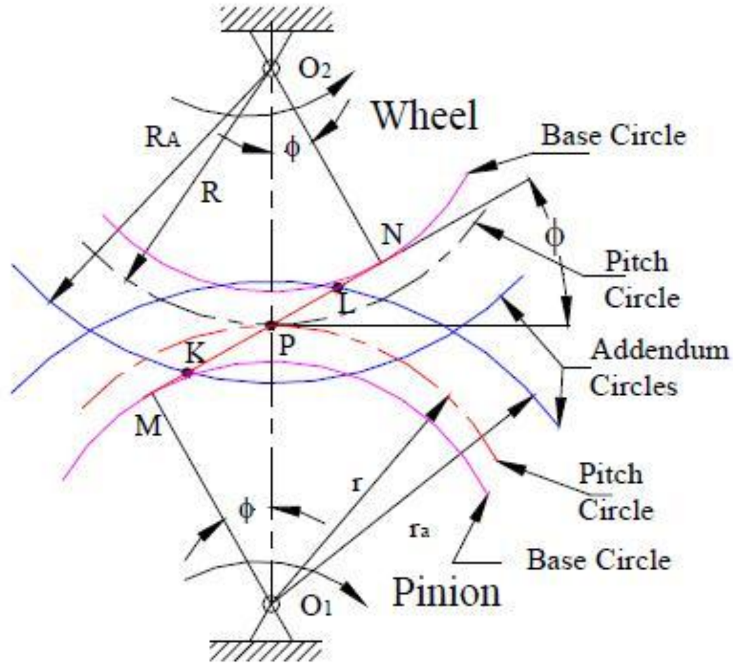
inv. [redacted] $_b$ $\frac{t_b}{2r_b}$ [redacted] *nv.* [redacted] $_c$ [redacted] $\frac{t_c}{2r_c}$

t_c [redacted] *nv.* [redacted] *nv.* $_b$ [redacted] c [redacted] $\frac{t_b}{b}$ [redacted] $2r_c$ [redacted]

[redacted] *ooth thickness at C*

Using this equation and knowing tooth thickness at any point on the tooth, it is possible to calculate the thickness of the tooth at any point

Path of contact:



Consider a pinion driving wheel as shown in figure. When the pinion rotates in clockwise, the contact between a pair of involute teeth begins at *K* (on the near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at *L* (outer end of the tooth face on the pinion or on the flank near the base circle of wheel).

MN is the common normal at the point of contacts and the common tangent to the base circles. The point *K* is the intersection of the addendum circle of wheel and the common tangent. The point *L* is the intersection of the addendum circle of pinion and common tangent.

The length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion. Thus the length of part of contact is *KL* which is the sum of the parts of path of contacts *KP* and *PL*. Contact length *KP* is called as **path of approach** and contact length *PL* is called as **path of recess**.

$r_a = O_1L$ = Radius of addendum circle of pinion,

and

$R_A = O_2K$ = Radius of addendum circle of

wheel $r = O_1P$ = Radius of pitch circle of pinion,

and

$R = O_2P$ = Radius of pitch circle of wheel.

Radius of the base circle of pinion = $O_1M = O_1P \cos$ [redacted] $= r \cos$ [redacted]

And radius of the base circle of wheel = $O2N = O2P \cos$ [redacted] $= R \cos$ [redacted]

From right angle triangle O_2KN

KN [redacted] O_2 [redacted] O_2 [redacted]

[redacted] A [redacted] 2 [redacted] R [redacted] os [redacted] 2 [redacted]

PN [redacted] $O_2 P \sin$ [redacted] $R \sin$ [redacted]

Path of approach: KP

KP [redacted] KN [redacted] PN

[redacted] A [redacted] 2 [redacted] os 2 [redacted] $R \sin$ [redacted] Similarly from right angle triangle O_1ML

ML [redacted] L [redacted] M [redacted]

[redacted] a [redacted] os [redacted]

MP [redacted] $O_1P \sin$ [redacted] $r \sin$ [redacted]

Path of recess: PL

PL [redacted] ML [redacted] MP

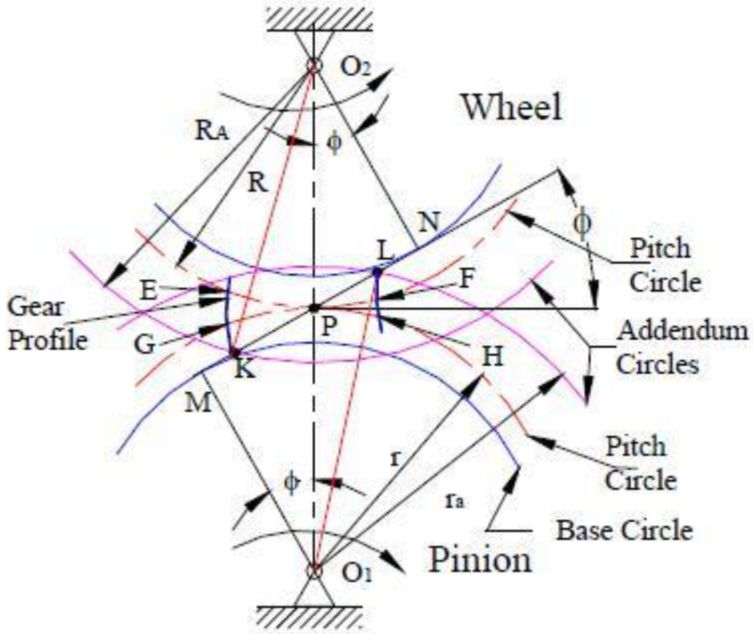
[redacted] a [redacted] cos [redacted] sin

[redacted] Length of path of contact = KL

KL [redacted] KP [redacted] PL

[redacted] A [redacted] 2 R [redacted] cos [redacted] a [redacted] 2 r [redacted] cos [redacted] R r sin [redacted]

Arc of contact: Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Figure, the arc of contact is *EPF* or *GPH*.



Considering the arc of contact *GPH.*

The arc *GP* is known as *arc of approach* and the arc *PH* is called *arc of recess*. The angles subtended by these arcs at O_1 are called *angle of approach* and *angle of recess* respectively.

Length of arc of approach = arc $GP = \dfrac{\text{Lenght of path of approach}}{\cos\phi} = \dfrac{KP}{\cos\phi}$

Length of arc of recess = arc $PH = \dfrac{\text{Lenght of path of recess}}{\cos\phi} = \dfrac{PL}{\cos\phi}$

Length of arc contact = arc *GPH = arc GP + arc PH*

$$= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi} = \frac{\text{Length of path of contact}}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

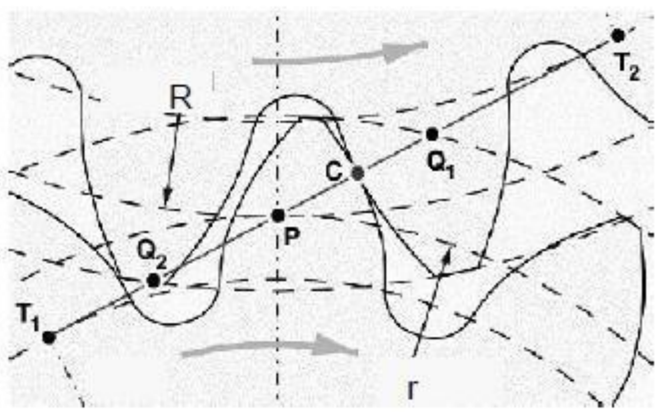
The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

Mathematically, *Contat ratio* [redacted] $\frac{\textit{Length of the arc of contact}}{c}$

Where: P_C [redacted] *Circular pitch* [redacted] *n* and *m* = Module.

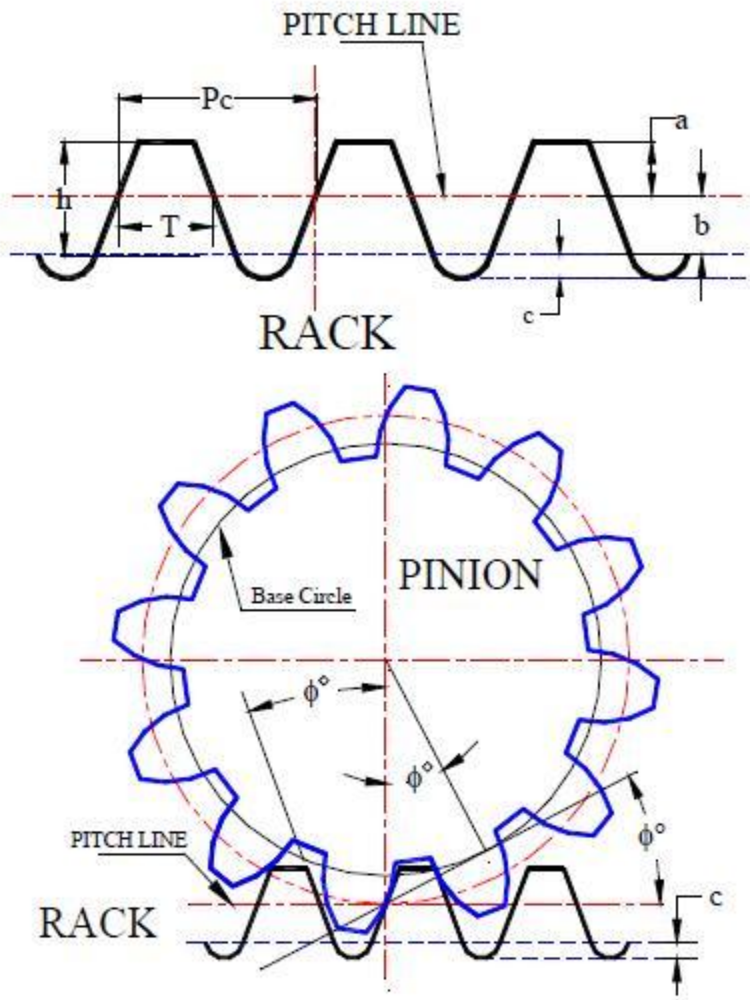
Number of Pairs of Teeth in Contact

Continuous motion transfer requires **two pairs of teeth in contact at the ends of the path of contact**, though there is only one pair in contact in the middle of the path, as in Figure.



The average number of teeth in contact is an important parameter - if it is too low due to the use of inappropriate profile shifts or to an excessive centre distance.The manufacturing inaccuracies may lead to loss of kinematic continuity - that is to impact, vibration and noise.The average number of teeth in contact is also a guide to load sharing between teeth; it is termed the ***contact ratio***

Length of path of contact for Rack and Pinion:



Let

[redacted] = Pressure angle

r_a. = Addendu m radius of the pinion

a = Addendum of rack

EF = Length of path of contact

***EF* = Path of approach EP + Path of recess PF**

$$\sin \quad \frac{AP}{EP} \quad \frac{a}{EP}$$

$$Path\ of\ approach \quad EP \quad \frac{a}{\sin}$$

Path of recess [redacted] *PF* [redacted] *NF* [redacted] *NP*

From triangle O_1 *NP* :

NP [redacted] $_1P$ sin [redacted] *r* sin [redacted]

O_1 *N* [redacted] $_1P$ cos [redacted] *r* cos [redacted]

From triangle O_1NF:

NF([redacted] $_1$ *N* ^{2} $^{\frac{1}{2}}$ [redacted] $_r$ [redacted] ^{2} cos ^{2} [redacted] $^{\frac{1}{2}}$

Substituting NP and NF values in the equation (3)

Path of racess [redacted] *PF* [redacted] $_a$ [redacted] ^{2} cos ^{2} [redacted] $_{sin}$ [redacted] $^{\frac{1}{2}}$

[redacted] *ath of length of contact* [redacted] *F* [redacted] *EP* [redacted] *PF*

a

[redacted] sin [redacted] [redacted] $_a$ ^{2} [redacted] *r* cos [redacted] ^{2} ^{2} $^{\frac{1}{2}}$ [redacted] sin [redacted]

Exercise problems refer presentation slides

Interference in Involute Gears

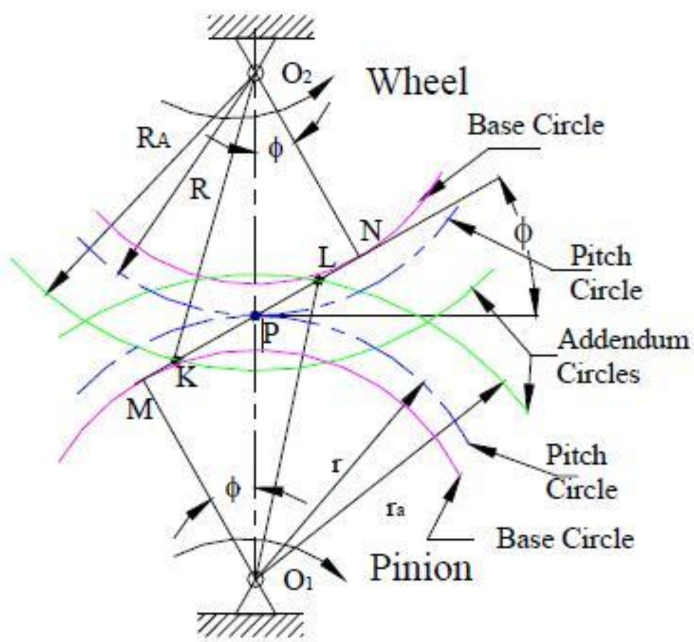


Figure shows a pinion and a gear in mesh with their center as O_1 and O_2 respectively. *MN* is the common tangent to the basic circles and *KL* is the path of contact between the two mating teeth.

Consider, the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will moves from L to N. If this radius is further increased, the point of contact L will be inside of base circle of wheel and not on the involute profile of the pinion.

The tooth tip of the pinion will then undercut the tooth on the wheel at the root and damages part of the involute profile. This effect is known as *interference*, and occurs when the teeth are being cut and weakens the tooth at its root.

In general, the phenomenon, when the tip of tooth undercuts the root on its mating gear is known as interference

Similarly, if the radius of the addendum circles of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called interference points.

Interference may be avoided if the path of the contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel is O_2M.

The interference may only be prevented, if the point of contact between the two teeth is always on the involute profiles and if the addendum circles of the two mating gears cut the common tangent to the base circles at the points of tangency.

When interference is just prevented, the maximum length of path of contact is MN.

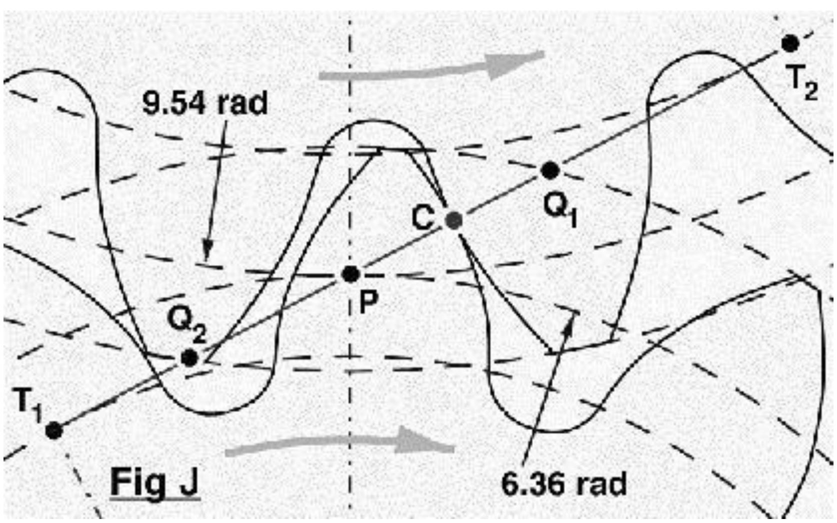


Fig J

Maximum path of approach [redacted]P [redacted] sin [redacted]

Maximum path of recess [redacted]N [redacted]sin [redacted]

Maximum length of path of contact [redacted]MN

MN [redacted]P [redacted]r [redacted]

Maximum length of arc of contact [redacted] ([redacted]in [redacted]) / (cos[redacted]) [redacted]n [redacted]

Methods to avoid Interference

1. Height of the teeth may be reduced.

Under cut of the radial flank of the pinion.

Centre distance may be increased. It leads to increase in pressure angle.

By tooth correction, the pressure angle, centre distance and base circles remain unchanged, but tooth thickness of gear will be greater than the pinion tooth thickness.

Minimum number of teeth on the pinion avoid Interference

The pinion turns clockwise and drives the gear as shown in Figure.

Points M and N are called interference points. i.e., if the contact takes place beyond M and N, interference will occur.

The limiting value of addendum circle radius of pinion is O_1N and the limiting value of addendum circle radius of gear is O_2M. Considering the critical addendum circle radius of gear, the limiting number of teeth on gear can be calculated.

Let

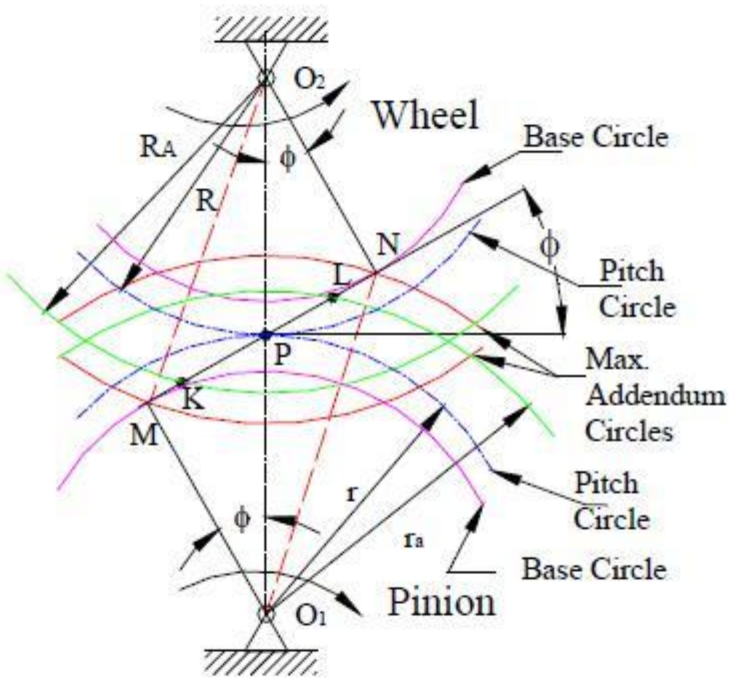
Φ = pressure angle

R = pitch circle radius of gear

r = pitch circle radius of pinion = ½ mt

T & t = number of teeth on gear & pinion

m = module



N = speed in rpm

a_w = Addendum constant of gear (or) wheel

$a_p \cdot m$ = Addendum of pinion

a_p = Addendum constant of pinion

$a_w \cdot m$ = Addendum of gear

G = Gear ratio = T/t

From triangle O_1NP, Applying cosine rule

O_1N^2 [redacted] O_1P^2 [redacted] NP^2 [redacted] 2 [redacted] O_1P [redacted] $PN \cos O_1PN$

[redacted] 2 [redacted] $R^2 \sin^2$ [redacted] 2 $r R \sin$ [redacted] cos [redacted] 0 [redacted]

[redacted] 2 [redacted] $R^2 \sin^2$ [redacted] 2 $r R \sin^2$ [redacted]

[redacted] 2 [redacted] $\frac{R^2 \sin^2}{r^2}$ [redacted] $\frac{2R \sin^2}{r}$ [redacted] r [redacted] $\frac{R}{r}$ [redacted] $\frac{R}{r}$ [redacted] 2 sin 2 [redacted]

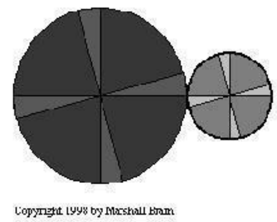
[redacted] PN [redacted] 2 $P \sin$ [redacted] $R \sin$ [redacted]

GEARS TRAINS

A gear train is two or more gear working together by meshing their teeth and turning each other in a system to generate power and speed. It reduces speed and increases torque. To create large gear ratio, gears are connected together to form gear trains. They often consist of multiple gears in the train.

The most common of the gear train is the gear pair connecting parallel shafts. The teeth of this type can be spur, helical or herringbone. The angular velocity is simply the reverse of the tooth ratio.

Any combination of gear wheels employed to transmit motion from one shaft to the other is called a gear train. The meshing of two gears may be idealized as two smooth discs with their edges touching and no slip between them. This ideal diameter is called the Pitch Circle Diameter (PCD) of the gear.



Simple Gear Trains

The typical spur gears as shown in diagram. The direction of rotation is reversed from one gear to another. It has no affect on the gear ratio. The teeth on the gears must all be the same size so if gear A advances one tooth, so does B and C.

t = number of teeth on the gear,

D = Pitch circle diameter,

m = module = $\frac{D}{t}$

and

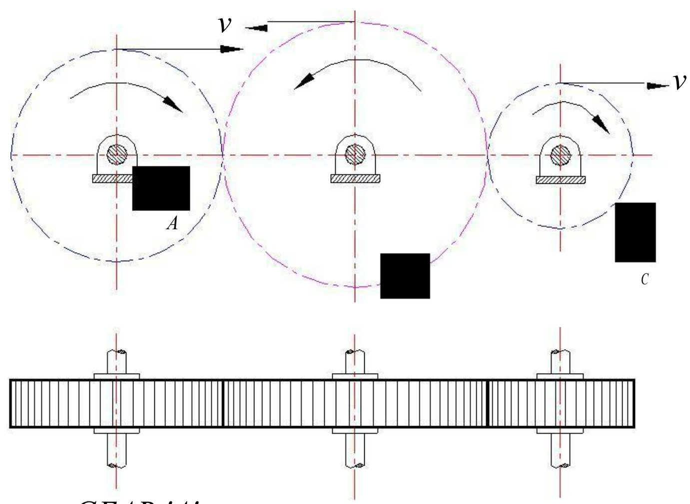
module *must be the same for all gears otherwise they would not mesh.*

$$m = \frac{D_A}{t_A} = \frac{D_B}{t_B} = \frac{D_C}{t_C}$$

$D_A = m\,t_A$; $D_B = m\,t_B$ and $D_C = m\,t_C$

[redacted] = *angular velocity.*

v = linear velocity on the circle. v = [redacted] $\frac{D}{2}$ = [redacted]



GEAR 'A' GEAR 'B' GEAR 'C' (Idler gear)

The velocity v of any point on the circle must be the same for all the gears, otherwise they would be

slipping.

v [redacted] $_A \frac{D_A}{2}$ [redacted] $_B \frac{D_B}{2}$ [redacted] $_C \frac{D_C}{2}$

[redacted] [redacted] $_A D_A$ [redacted] D_{BC} [redacted] D_C [redacted]

[redacted] $_A\, m\, t_A$ [redacted] $m\, t_B$ [redacted] $m\, t_C$ [redacted]

[redacted] $_A\, t_A$ [redacted] t_B [redacted] t_C [redacted]

or in terms of *rev* / min

$N_A\ t_A$ [redacted] $N_B\ t_B$ [redacted] $N_C\ t_C$

Application:

1.3 to connect gears where a large center distance is required

1.4 to obtain desired direction of motion of the driven gear (CW or CCW)

1.5 to obtain high speed ratio

Torque & Efficiency

The power transmitted by a torque T N-m applied to a shaft rotating at N rev/min is given by:

P [redacted] $\frac{2 \text{[redacted]} T}{60}$

In an ideal gear box, the input and output powers are the same so;

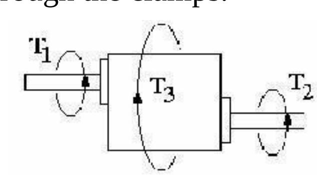
P [redacted] $\frac{\text{[redacted]} T_1}{60}$ [redacted] $\frac{2 \text{[redacted]} T_2}{60}$

$N_1 T_1$ [redacted] $N_2 T_2$ [redacted] $\frac{T_2}{T_1}$ [redacted] $\frac{N_1}{N_2}$ [redacted] GR

It follows that if the speed is reduced, the torque is increased and vice versa. In a real gear box, power is lost through friction and the power output is smaller than the power input. The efficiency is defined as:

[redacted] $\frac{Power\ out}{Power\ In}$ [redacted] $\frac{\text{[redacted]} N_2 T_2 \text{[redacted]} 0}{\text{[redacted]} T_1 \text{[redacted]} 60}$ [redacted] $\frac{N_2 T_2}{N_1 T_1}$

Because the torque in and out is different, a gear box has to be clamped in order to stop the case or body rotating. A holding torque T_3 must be applied to the body through the clamps.



The total torque must add up to zero.

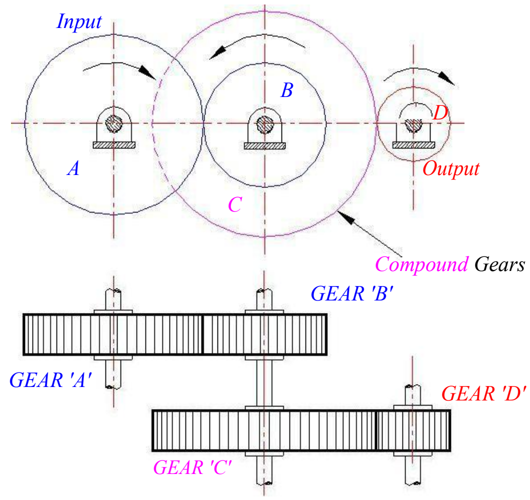
$$T1 + T2 + T3 = 0$$

If we use a convention that anti-clockwise is positive and clockwise is negative we can determine the holding torque. The direction of rotation of the output shaft depends on the design of the gear box.

Compound Gear train

Compound gears are simply a chain of simple gear trains with the input of the second being the output of the first. A chain of two pairs is shown below. Gear B is the output of the first pair and gear C is the input of the second pair. Gears B and C are locked to the same shaft and revolve at the same speed.

For large velocities ratios, compound gear train arrangement is preferred.



The velocity of each tooth on A and B are the same so: [redacted] t_A = [redacted] t_B -as they are simple gears.

Likewise for C and D, *C* t [redacted] *D* t [redacted]

[redacted] $\frac{A}{B}$ [redacted] $\frac{B}{A}$ and [redacted] $\frac{D}{C}$

[redacted] $\frac{t_B}{t_A}$ [redacted] and [redacted] $\frac{t_D}{t_C}$ [redacted]

[redacted] $\frac{t_B}{t_A}$ [redacted] $\frac{t_D}{t_C}$ [redacted]

[redacted] $\frac{t_B}{t_A}$ [redacted] $\frac{D}{t_C}$

Since gear B and C are on the same shaft

[redacted] B [redacted] $\frac{t_A\, t_B\, t_D}{t_D\, t_C\, t_A}$ [redacted] GR [redacted]

Since [redacted]

The gear ratio may be written as:

$\frac{N}{N}$ [redacted] $\frac{t_B}{t_A}$ [redacted] $\frac{t_D}{t_C}$ [redacted] GR

Reverted Gear train

The driver and driven axes lies on the same $\frac{t_B}{t_A}$ [redacted] $\frac{t_D}{t_C}$ line. These are used in speed reducers, clocks and machine tools.

GR [redacted] $\frac{N_A}{N_D}$ [redacted]

If R and T=Pitch circle radius & number of teeth of the gear

$$R_A + R_B = R_C + R_D \quad and \quad t_A + t_B = t_C + t_D$$

Epicyclic gear train:

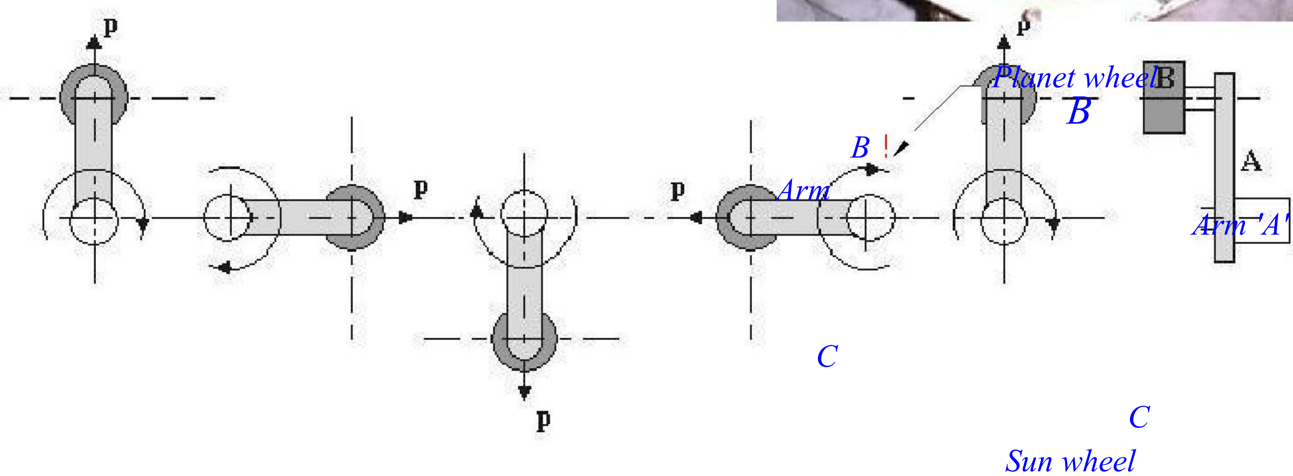
Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun. Here is a picture of a typical gear box.

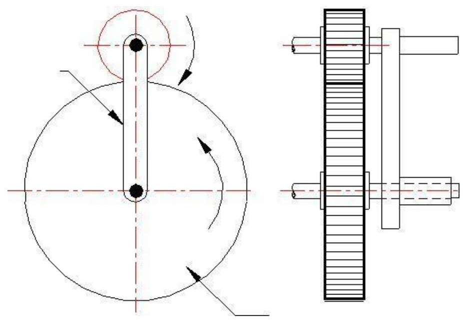
This design can produce large gear ratios in a small space and are used on a wide range of applications from marine gearboxes to electric screwdrivers.

Basic Theory

The diagram shows a gear B on the end of an arm. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun.

First consider what happens when the planet gear orbits the sun gear.





Observe point p and you will see that gear *B* also revolves once on its own axis. Any object orbiting around a center must rotate once. Now consider that *B* is free to rotate on its shaft and meshes with C. Suppose the arm is held stationary and gear *C* is rotated once. *B* spins about its own center and number of revolutions it makes is the ratio $\frac{t_C}{t_B}$. *B* will rotate by this number for every complete revolution of *C*.

Now consider that C is unable to rotate and the arm *A* is revolved once. Gear *B* will revolve 1 ■ $\frac{t_C}{t_B}$ because of the orbit. It is this extra rotation that causes confusion. One way to get round this is to

magine that the whole system is revolved once. Then identify the gear that is fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up. The following tabular method makes it easy.

Suppose gear *C* is fixed and the arm *A* makes one revolution. Determine how many revolutions the planet gear *B* makes.

Step 1 is to revolve everything once about the center.

Step 2 identify that *C* should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions of *B*.

Step 3 is simply add them up and we find the total revs of *C* is zero and for the arm is 1.

Step	***Action***	***A***	***B***	***C***
1	Revolve all once	1	1	1
2	Revolve *C* by –1 revolution, keeping the arm fixed	0	■ $\frac{t_C}{t_B}$	-1
3 —	Add	1	1 ■ $\frac{t_C}{t_B}$	0

The number of revolutions made by B is $\left(1+\frac{t_C}{t_B}\right)$ Note that if C revolves -1, then the direction of B is opposite so $+\frac{t_C}{t_B}$.

Example: A simple epicyclic gear has a fixed sun gear with 100 teeth and a planet gear with 50 teeth. If the arm is revolved once, how many times does the planet gear revolve?

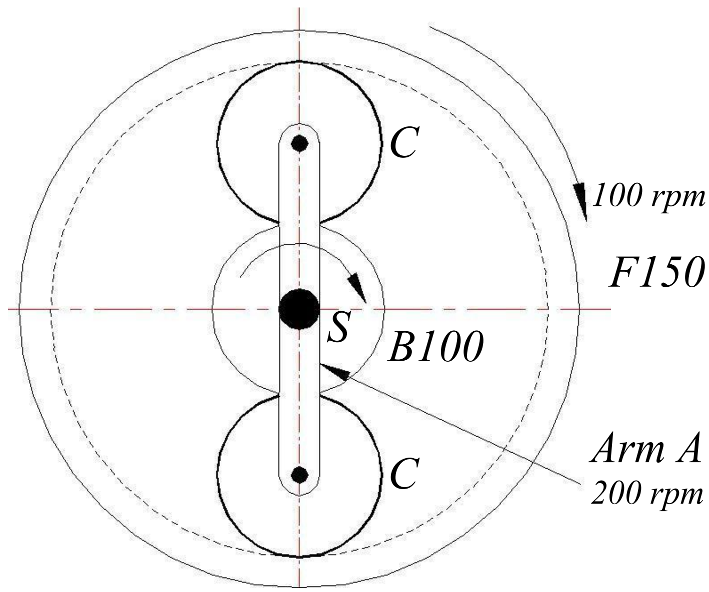
Solution:

Step	***Action***	***A***	***B***	***C***
1	Revolve all once	1	1	1
2	Revolve *C* by –1 revolution, keeping the arm fixed	0	■ $\frac{100}{50}$	-1
3	Add	**1**	**3**	**0**

Gear B makes 3 revolutions for every one of the arm.

The design so far considered has no identifiable input and output. We need a design that puts an input and output shaft on the same axis. This can be done several ways.

Problem 1: In an ecicyclic gear train shown in figure, the arm A is fixed to the shaft S. The wheel B having 100 teeth rotates freely on the shaft S. The wheel F having 150 teeth driven separately. If the arm rotates at 200 rpm and wheel F at 100 rpm in the same direction; find (a) number of teeth on the gear C and (b) speed of wheel B.



Solution:

$T_B=100;$ $T_F=150;$ $N_A=200rpm;$ $N_F=100rpm:$

Since the mod *ule is same for all gears* :

the number of teeth on the gears is proportional to the pitch cirlce :

[redacted] r_F [redacted] $_B$ [redacted] r_C [redacted]

[redacted] T_F [redacted] $_B$ [redacted] T_C [redacted]

[redacted] 150 [redacted] 00 [redacted] $_C$ [redacted]

T [redacted]

$_C$ 25 *Number of teeth on gears C*

The gear B and gear F rotates in the opposite directions:

Train value $\frac{T_B}{T_F}$

also TV $\frac{N_L \quad N_{Arm}}{N_F \quad N_{Arm}}$ $\frac{N_F \quad N_A}{N_B \quad N_A}$ (*general* exp *ression for epicyclic gear train*)

$T_B \quad N_F \quad N_A$

$\frac{100}{150} \quad \frac{100 \quad 200}{N_B \quad 200}$ $N_E \quad 350$

The Gear B rotates at 350 rpm in the same direction of gears F and Arm A.

$$m \quad n \qquad 1000$$

$$Gear\ C\ is \quad fixed; \qquad n \quad \frac{1}{4} m \quad 0$$

$$1000 \quad m \;\; 0.25m \quad 0$$

[redacted]

m [redacted] $\frac{1000}{1.25}$ [redacted] 00

n [redacted] 1000 [redacted] 800 [redacted] 00

Speed of F [redacted] $\frac{5}{16}m$

[redacted] 00 [redacted] 00 $\frac{5}{16}$ [redacted] 50

Speed of the output shaft F [redacted] 0 rpm (CW)

Input power [redacted] P [redacted] $(2$ [redacted] $N_B T_B)/60$

7.5 [redacted] 000 [redacted] $(2$ [redacted] 000 [redacted] $B)/60$

T_B [redacted] $(7500$ [redacted] $60)/(2$ [redacted] $000)$ [redacted] 1.59 Nm

From the energy equation;

$T_B N_B$ [redacted] $T_F N_F$ [redacted] $T_C N_C$ [redacted] 0

Since C is fixed : N_C [redacted] T_B

N_B [redacted] $T_F N_F$ [redacted] 0

[redacted] 1.59 [redacted] 000 [redacted]

T_F [redacted] 431.8 Nm

From the torque equation :

$$T_B \quad T_F \quad T_C \quad 0$$

$$71.59 \quad 1431.8 \quad T_C \quad 0$$

The Torque required to hold the wheel C = 1360.21 Nm in the same direction of wheel

Problem 8: Find the velocity ratio of two co-axial shafts of the epicyclic gear train as shown in figure 6. S_1 is the driver. The number of teeth on the gears are $S_1 = 40$, $A_1 = 120$, $S_2 = 30$, $A_2 = 100$ and the sun wheel S_2 is fixed. Determine also the magnitude and direction of the torque required to fix S_2, if a torque of 300 N-m is applied in a clockwise direction to S_1

Solution: Consider first the gear train S_1, A_1 and A_2 for which A_2 is the arm, in order to find the speed ratio of S_1 to A_2, when A_1 is fixed.

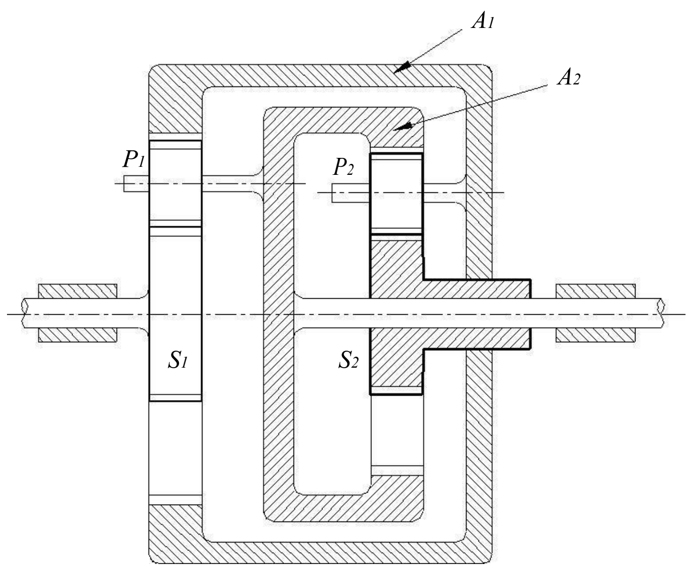


Figure 6

(a) Consider gear train S_1, A_1 and A_2:

Operation	***A₂*** (100)	***A₁*** (120)	***S₁*** (40)
A_2 is fixed & wheel A_1 is given +1 revolution	0	+1	[redacted] $\frac{120}{40}$ [redacted]
Multiply by *m* (A_1 rotates through *m* revolution)	0	+*m*	[redacted] *m*
Add *n* revolutions to all elements	*n*	*n*+ *m*	*n* [redacted] *m*

A_1 is fixed: *m* [redacted] *n*

$\frac{}{}$ $_{S1}$ [redacted] $\frac{n \text{ [redacted] } n}{n}$ [redacted]

$_{A2}$

[redacted] $_{S1}$ [redacted] N_{A2}

(b) Consider complete gear train:

Operation	***A₁*** (120)	***A₂*** (100)	***S₁*** (40)	***S₂*** (30)
A_1 is fixed & wheel S_2 is given +1 revolution	0	$\frac{30}{100}$ $\frac{3}{10}$	$\frac{3}{10}$ 4 $\frac{6}{5}$	+1
Multiply by *m* (A_1 rotates through *m* revolution)	0	$\frac{3m}{10}$	$-\frac{6m}{5}$	+*m*
Add *n* revolutions to all elements	*n*	$n \; \frac{3m}{10}$	$n \; \frac{6m}{5}$	*n*+ *m*

S_2 is fixed $m = -n$

$$\frac{n_{S1}}{n_{A2}} \text{[redacted]} \frac{n \text{[redacted]} \frac{6}{5} n}{n \text{[redacted]} \frac{3}{10} n} \text{[redacted]} \frac{11}{5} \text{[redacted]} \frac{10}{13} \text{[redacted]} \frac{22}{13}$$

Input torque on S_1 = T_{S1} = 300 N-m, in the direction of rotation.

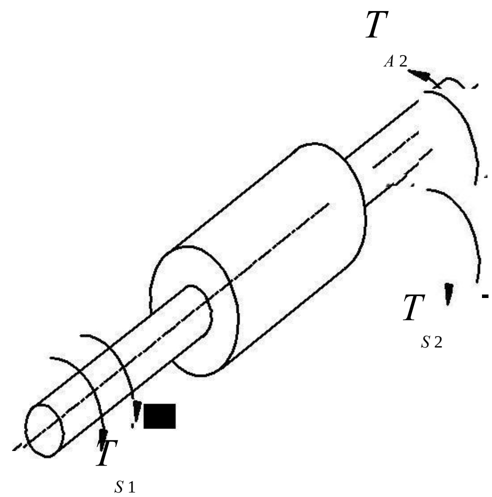
[redacted]esisting torque on A_2;

T_{A2}[redacted]300 5[redacted]7.7 $N\,m$ $\frac{22}{13}$ [redacted]

[redacted]*pposite to directiojn of rotation*

[redacted]eferring to the figure:

T_{S2} [redacted]507.7[redacted]00 [redacted]07.7 N [redacted] (CW)



UNIT-08

CAMS

INTRODUCTION

A cam is a mechanical device used to transmit motion to a follower by direct contact. The driver is called the cam and the driven member is called the follower. In a cam follower pair, the cam normally rotates while the follower may translate or oscillate. A familiar example is the camshaft of an automobile engine, where the cams drive the push rods (the followers) to open and close the valves in synchronization with the motion of the pistons.

Types of cams

Cams can be classified based on their physical shape.

a) Disk or plate cam (Fig. 6.1a and b): The disk (or plate) cam has an irregular contour to impart a specific motion to the follower. The follower moves in a plane perpendicular to the axis of rotation of the camshaft and is held in contact with the cam by springs or gravity.

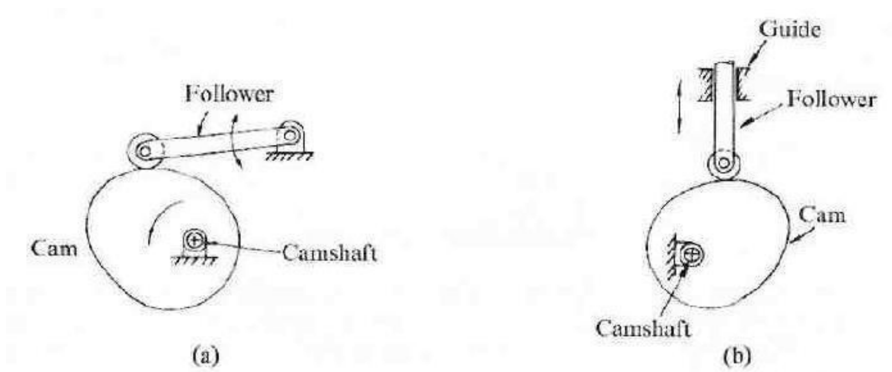


Fig. 6.1 Plate or disk cam.

b) Cylindrical cam (Fig. 6.2): The cylindrical cam has a groove cut along its cylindrical surface. The roller follows the groove, and the follower moves in a plane parallel to the axis of rotation of the cylinder.

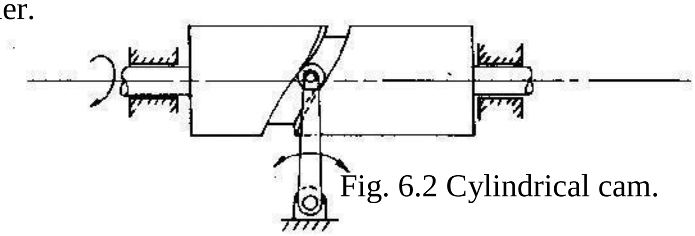


Fig. 6.2 Cylindrical cam.

c) Translating cam (Fig. 6.3a and b). The translating cam is a contoured or grooved plate sliding on a guiding surface(s). The follower may oscillate (Fig. 6.3a) or reciprocate (Fig. 6.3b). The contour or the shape of the groove is determined by the specified motion of the follower.

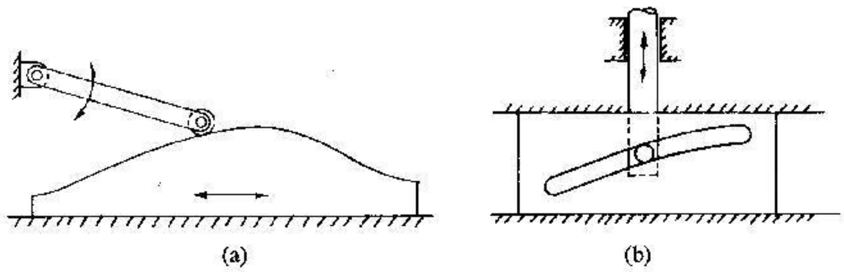


Fig. 6.3 Translating cam

Types of followers:

1.7 Based on surface in contact. **(Fig.6.4)**

Knife edge follower

Roller follower

Flat faced follower

Spherical follower

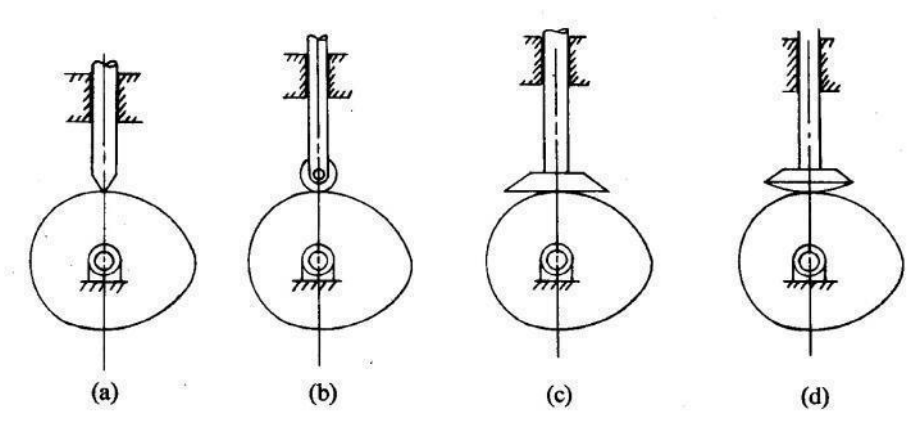


Fig. 6.4 Types of followers

(ii) Based on type of motion: **(Fig.6.5)**

(a) Oscillating follower

(b) Translating follower

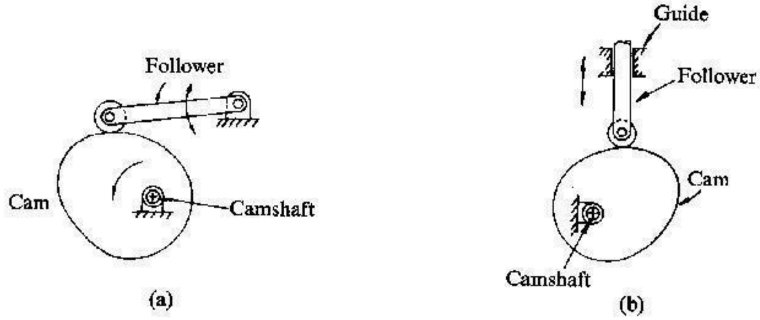


Fig.6.5

(iii) Based on line of motion:

(a) Radial follower: The lines of movement of in-line cam followers pass through the centers of the camshafts (Fig. 6.4a, b, c, and d).

(b) Off-set follower: For this type, the lines of movement are offset from the centers of the camshafts (Fig. 6.6a, b, c, and d).

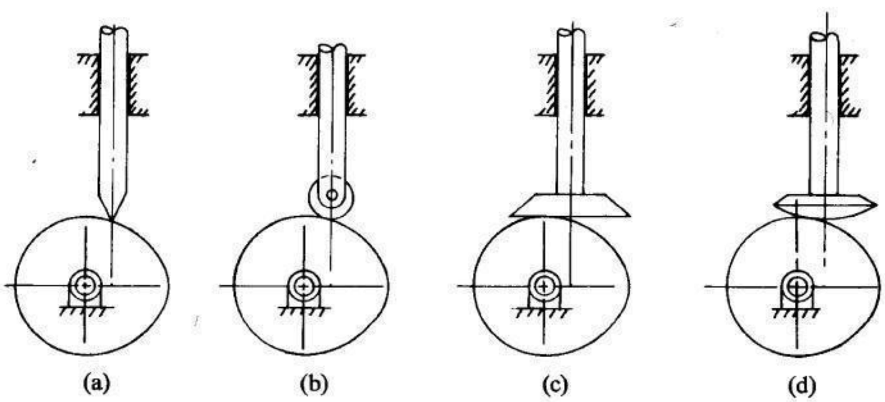


Fig.6.6 Off set followers

Cam nomenclature (Fig. 6.7):

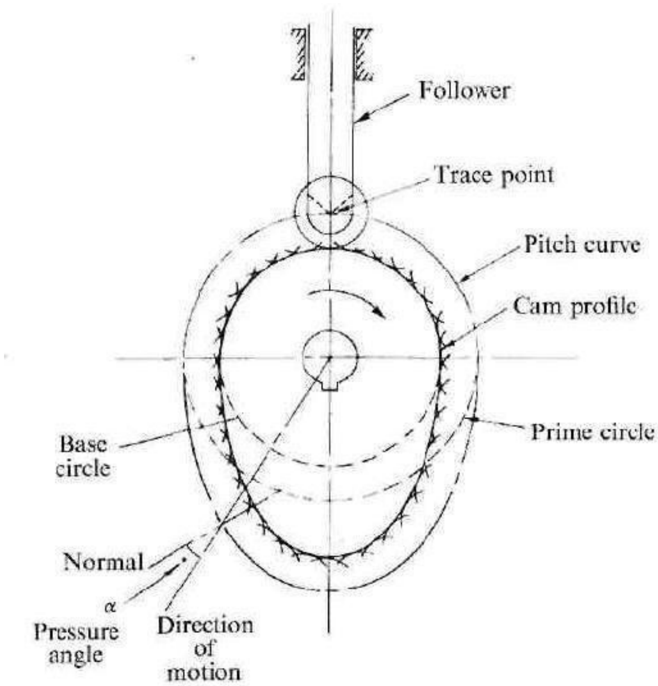


Fig.6.7

Cam Profile The contour of the working surface of the cam.

Tracer Point The point at the knife edge of a follower, or the center of a roller, or the center of a spherical face

Pitch Curve The path of the tracer point.

Base Circle The smallest circle drawn, tangential to the cam profile, with its center on the axis of the camshaft. The size of the base circle determines the size of the cam.

Prime Circle The smallest circle drawn, tangential to the pitch curve, with its center on the axis of the camshaft.

Pressure Angle The angle between the normal to the pitch curve and the direction of motion of the follower at the point of contact.

Types of follower motion:

Cam follower systems are designed to achieve a desired oscillatory motion. Appropriate displacement patterns are to be selected for this purpose, before designing the cam surface. The cam is assumed to rotate at a constant speed and the follower raises, dwells, returns to its original position and dwells again through specified angles of rotation of the cam, during each revolution of the cam.

Some of the standard follower motions are as follows:

They are, follower motion with,

p Uniform velocity

q Modified uniform velocity

r Uniform acceleration and deceleration

s Simple harmonic motion

t Cycloidal motion

Displacement diagrams: In a cam follower system, the motion of the follower is very important. Its displacement can be plotted against the angular displacement θ of the cam and it is called as the displacement diagram. The displacement of the follower is plotted along the y-axis and angular displacement θ of the cam is plotted along x-axis. From the displacement diagram, velocity and acceleration of the follower can also be plotted for different angular displacements θ of the cam. The displacement, velocity and acceleration diagrams are plotted for one cycle of operation i.e., one rotation of the cam. Displacement diagrams are basic requirements for the construction of cam profiles. Construction of displacement diagrams and calculation of velocities and accelerations of followers with different types of motions are discussed in the following sections.

(a) Follower motion with Uniform velocity:

Fig.6.8 shows the displacement, velocity and acceleration patterns of a follower having uniform velocity type of motion. Since the follower moves with constant velocity, during rise and fall, the displacement varies linearly with θ. Also, since the velocity changes from zero to a finite value, within no time, theoretically, the acceleration becomes infinite at the beginning and end of rise and fall.

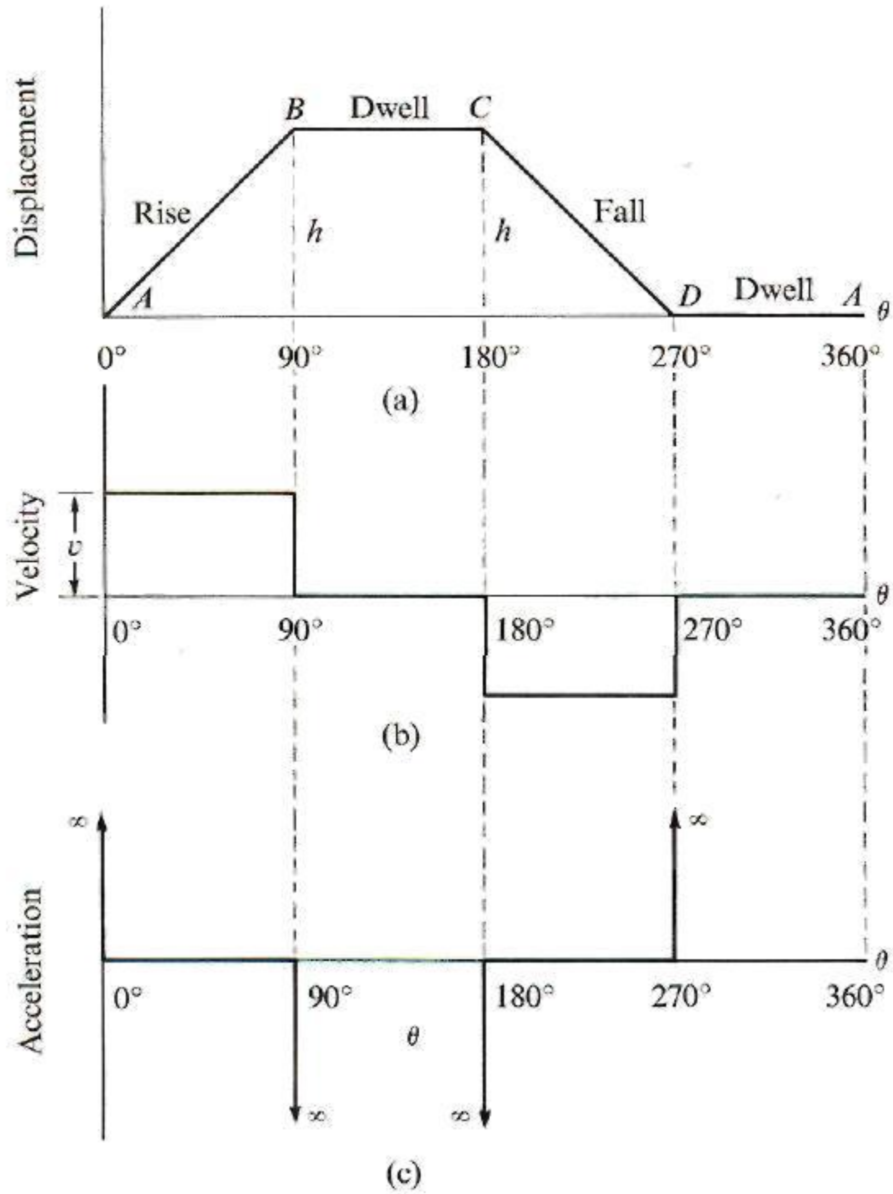


Fig.6.8

(b) Follower motion with modified uniform velocity:

It is observed in the displacement diagrams of the follower with uniform velocity that the acceleration of the follower becomes infinite at the beginning and ending of rise and return strokes. In order to prevent this, the displacement diagrams are slightly modified. In the modified form, the velocity of the follower changes uniformly during the beginning and end of each stroke. Accordingly, the displacement of the follower varies parabolically during these periods. With this modification, the acceleration becomes constant during these periods, instead of being infinite as in the uniform velocity type of motion. The displacement, velocity and acceleration patterns are shown in **fig.6.9**.

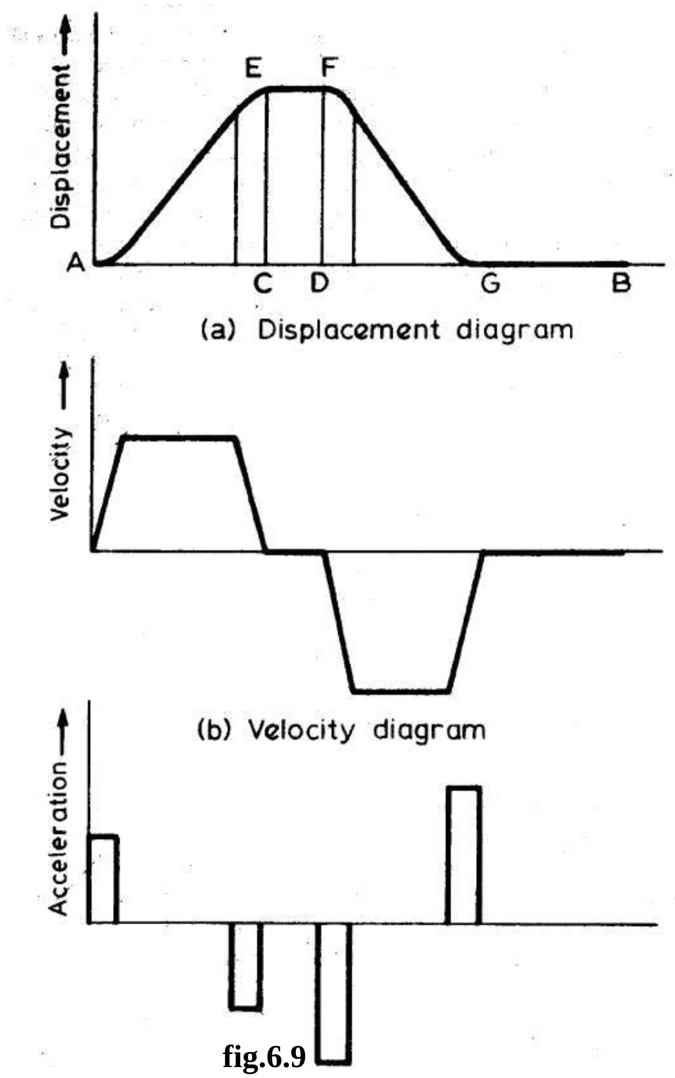


fig.6.9

(c) Follower motion with uniform acceleration and retardation (UARM):

Here, the displacement of the follower varies parabolically with respect to angular displacement of cam. Accordingly, the velocity of the follower varies uniformly with respect to angular displacement of cam. The acceleration/retardation of the follower becomes constant accordingly. The displacement, velocity and acceleration patterns are shown in **fig. 6.10**.

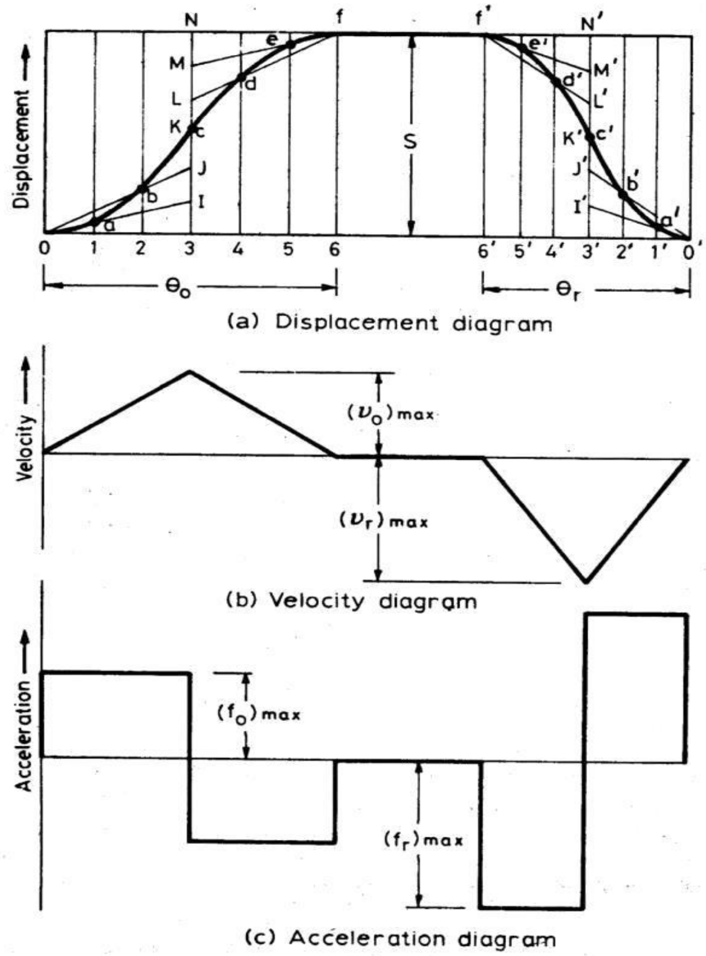


Fig.6.10

s = Stroke of the follower

θ_o and θ_r = Angular displacement of the cam during outstroke and return stroke.

ω = Angular velocity of cam.

Time required for follower outstroke = $t_o = \dfrac{\theta_o}{\omega}$

Time required for follower return stroke = $t_r = \dfrac{\theta_r}{\omega}$

Average velocity of follower = $\dfrac{s}{t}$

Average velocity of follower during outstroke = $\dfrac{s/2}{t_o/2} = \dfrac{s}{t_o} = \dfrac{vo_{min} + vo_{max}}{2}$

$vo_{min} = 0$

$\therefore vo_{max} = \dfrac{2s}{t_o} = \dfrac{2\omega s}{\theta_o}$ = Max. velocity during outstroke.

Average velocity of follower during return stroke = $\dfrac{s/2}{t_r/2} = \dfrac{s}{t_r} = \dfrac{vr_{min} + vr_{max}}{2}$

$vr_{min} = 0$

$\therefore vr_{max} = \dfrac{2s}{t_r} = \dfrac{2\omega s}{\theta_r}$ = Max. velocity during return stroke.

Acceleration of the follower during outstroke = $a_o = \dfrac{vo_{max}}{t_o/2} = \dfrac{4\omega^2 s}{\theta_o^{\,2}}$

Similarly acceleration of the follower during return stroke = $a_r = \dfrac{4\omega^2 s}{\theta_r^{\,2}}$

(d) Simple Harmonic Motion: In fig.6.11, the motion executed by point P^1, which is the projection of point P on the vertical diameter is called simple harmonic motion. Here, P moves with uniform angular velocity ω_p, along a circle of radius r (r = s/2).

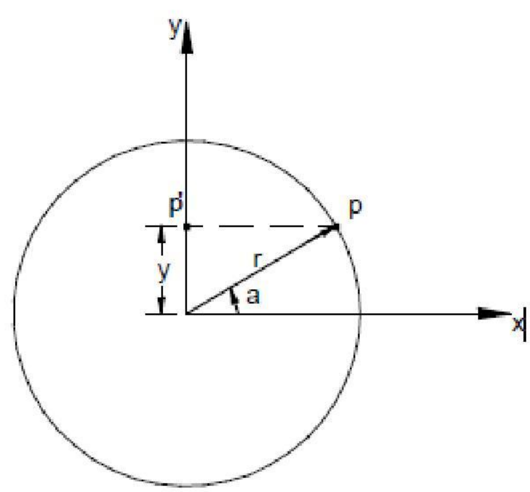


Fig.6.11

Displacement = $y = r\sin\alpha = r\sin\omega_p t$; $y_{max} = r$ [d1]

Velocity = $\dot{y} = \omega_p r\cos\omega_p t$; $\dot{y}_{max} = r\omega_p$ [d2]

Acceleration = $\ddot{y} = -\omega_p{}^2 r\sin\omega_p t = -\omega_p{}^2 y$; $\ddot{y}_{max} = -r\omega_p{}^2$ [d3]

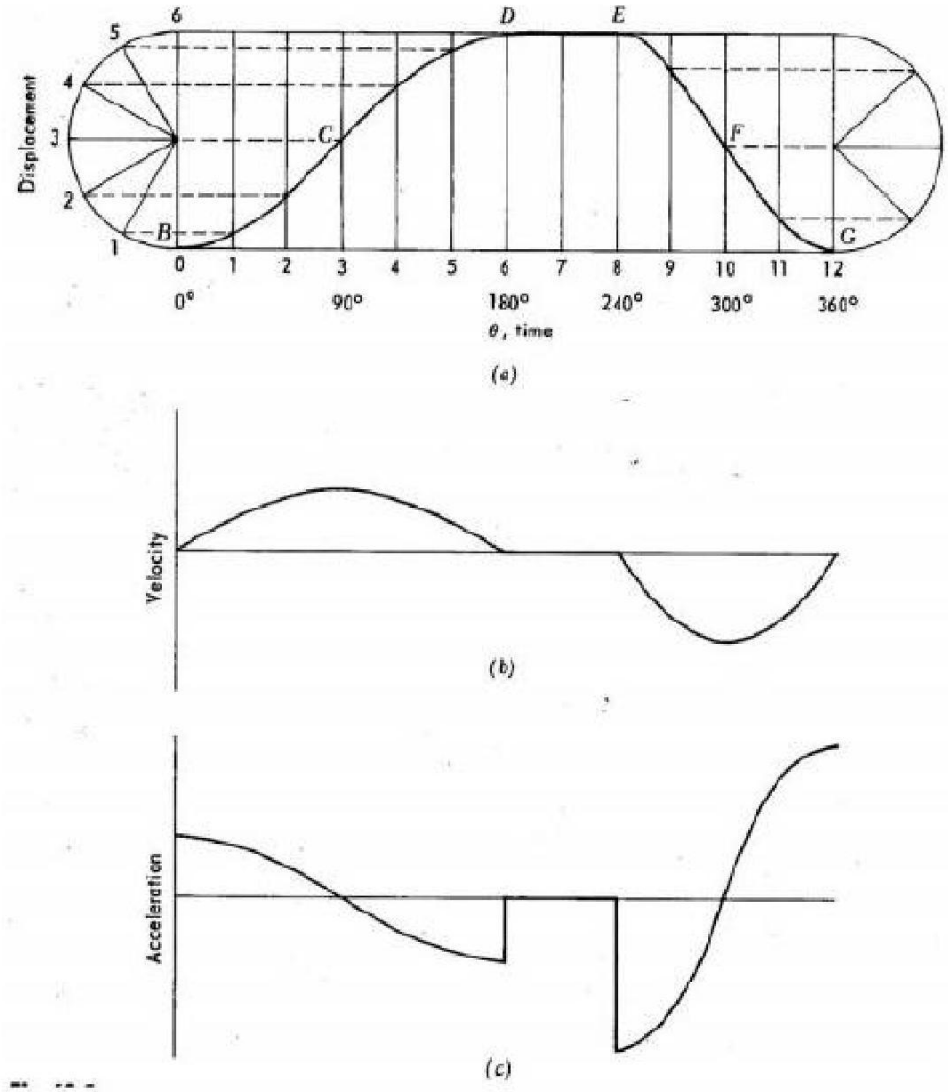


Fig.6.11

s= Stroke or displacement of the follower.

θ_o = Angular displacement during outstroke.

θ_r = Angular displacement during return stroke ω = Angular velocity of cam.

t_o = Time taken for outstroke = $\frac{\theta_o}{\omega}$

t_r = Time taken for return stroke = $\frac{\theta_r}{\omega}$

Max. velocity of follower during outstroke = $vo_{max} = r\omega_p$ (from d2)

$$vo_{max} = \frac{s}{2}\frac{\pi}{t_o} = \frac{\pi\omega s}{2\theta_o}$$

Similarly Max. velocity of follower during return stroke = , $vr_{max} = \frac{s}{2}\frac{\pi}{t_r} = \frac{\pi\omega s}{2\theta_r}$

Max. acceleration during outstroke = $ao_{max} = r\omega^2_p$ (from d3) = $\frac{s}{2}\left(\frac{\pi}{t_o}\right)^2 = \frac{\pi^2\omega^2 s}{2\theta_o^{\ 2}}$

Similarly, Max. acceleration during return stroke = $ar_{max} = \frac{s}{2}\left(\frac{\pi}{t_r}\right)^2 = \frac{\pi^2\omega^2 s}{2\theta^2{}_r}$

(e) Cycloidal motion:

Cycloid is the path generated by a point on the circumference of a circle, as the circle rolls without slipping, on a straight/flat surface. The motion executed by the follower here, is similar to that of the projection of a point moving along a cyloidal curve on a vertical line as shown in figure 6.12.

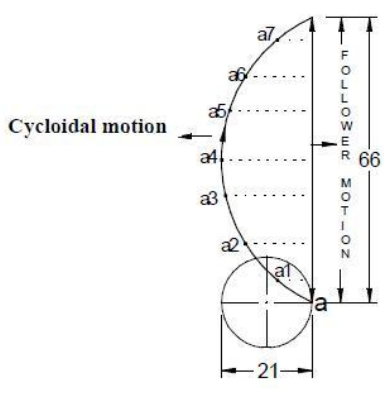


Fig.6.12

The construction of displacement diagram and the standard patterns of velocity and acceleration diagrams are shown in fig.6.13. Compared to all other follower motions, cycloidal motion results in smooth operation of the follower.

The expressions for maximum values of velocity and acceleration of the follower are shown below.

s = Stroke or displacement of the follower.

d = dia. of cycloid generating circle = $\frac{s}{\pi}$

θ_o = Angular displacement during outstroke.

θ_r = Angular displacement during return stroke

ω = Angular velocity of cam.

t_o = Time taken for outstroke = $\frac{\theta_o}{\omega}$

t_r = Time taken for return stroke = $\frac{\theta_r}{\omega}$

vo_{max} = Max. velocity of follower during outstroke = $\frac{2\omega s}{\theta_o}$

vr_{max} = Max. velocity of follower during return stroke = $\frac{2\omega s}{\theta_r}$

ao_{max} = Max. acceleration during outstroke = $\frac{2\pi\omega^2 s}{\theta_o^{\ 2}}$

ar_{max} = Max. acceleration during return stroke = $\frac{2\pi\omega^2 s}{\theta^2{}_r}$

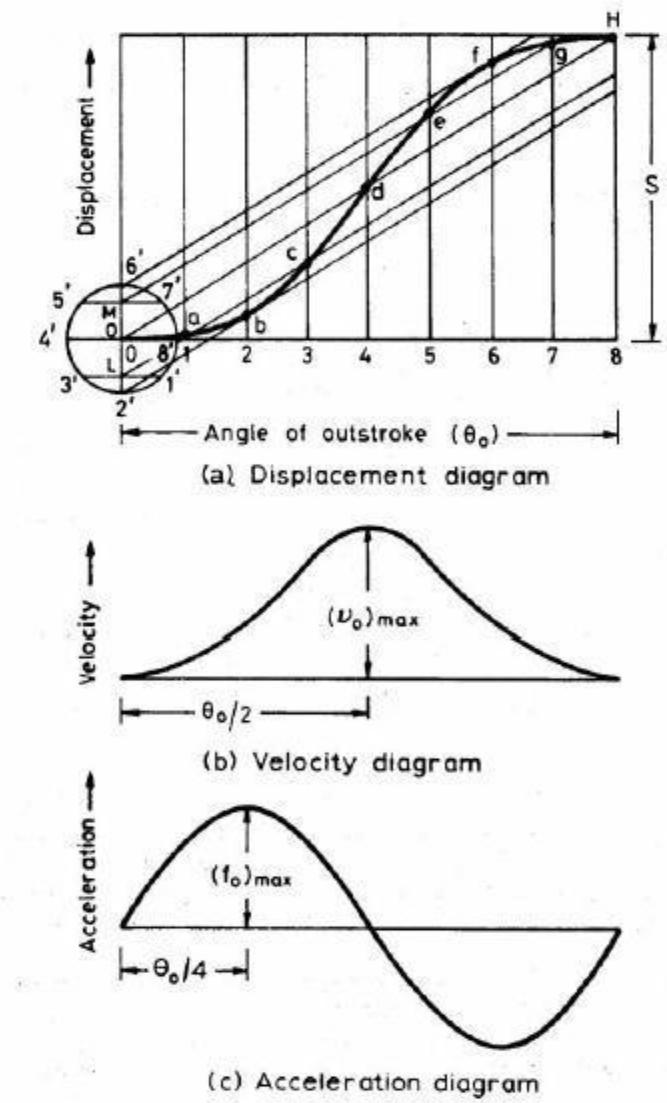


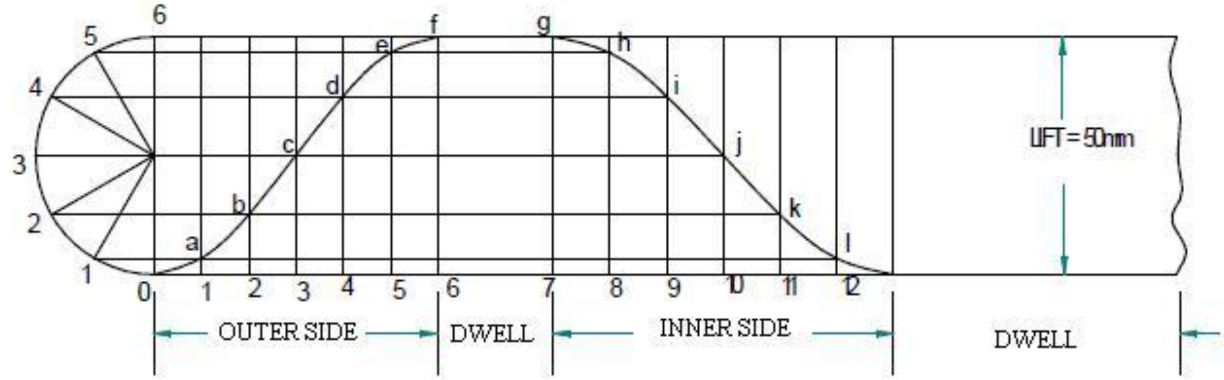
Fig. 6.13

Solved problems

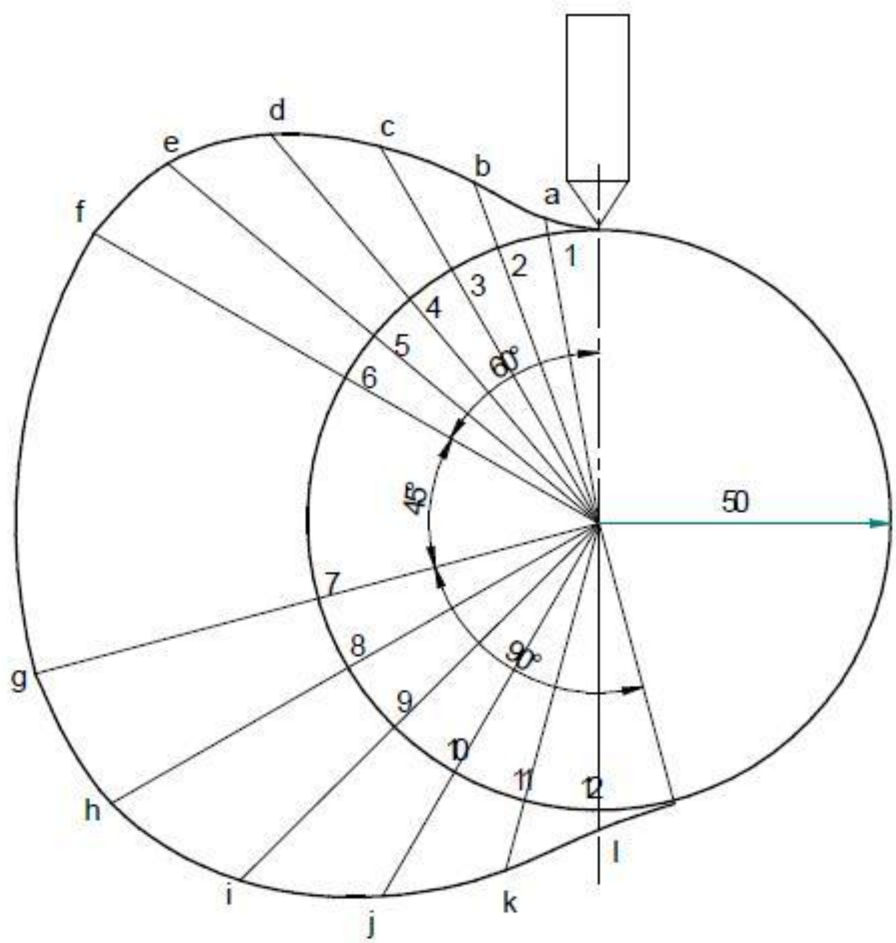
p Draw the cam profile for following conditions:

Follower type = Knife edged, in-line; lift = 50mm; base circle radius = 50mm; out stroke with SHM, for 60^0 cam rotation; dwell for 45^0 cam rotation; return stroke with SHM, for 90^0 cam rotation; dwell for the remaining period. Determine max. Velocity and acceleration during out stroke and return stroke if the cam rotates at 1000 rpm in clockwise direction.

Displacement diagram:



Cam profile: Construct base circle. Mark points 1,2,3…..in direction opposite to the direction of cam rotation. Transfer points a,b,c…..l from displacement diagram to the cam profile and join them by a smooth free hand curve. This forms the required cam profile.



2) Draw the cam profile for the same operating conditions of problem (1), with the follower off set by 10 mm to the left of cam center.

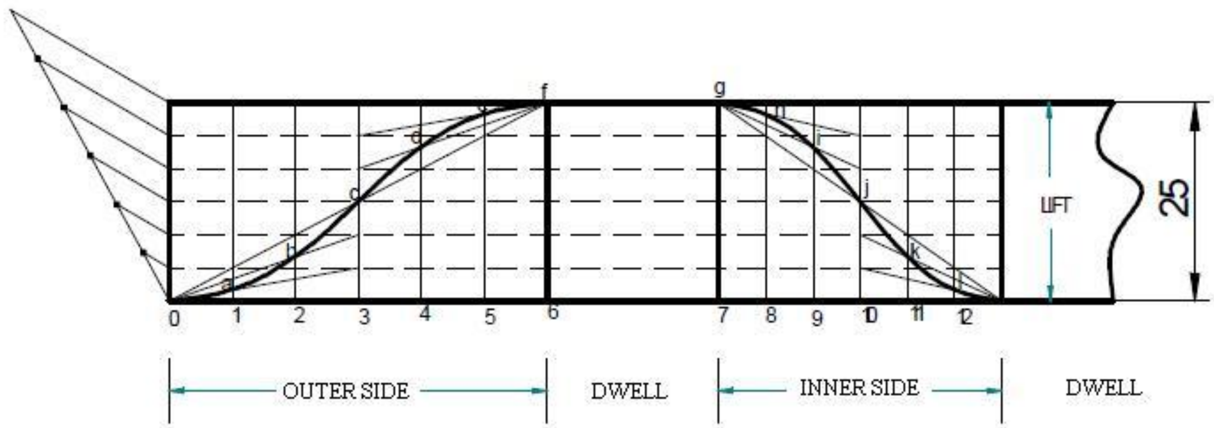
Displacement diagram: Same as previous case.

Cam profile: Construction is same as previous case, except that the lines drawn from 1, 2,3…. are tangential to the offset circle of 10mm dia. as shown in the fig.

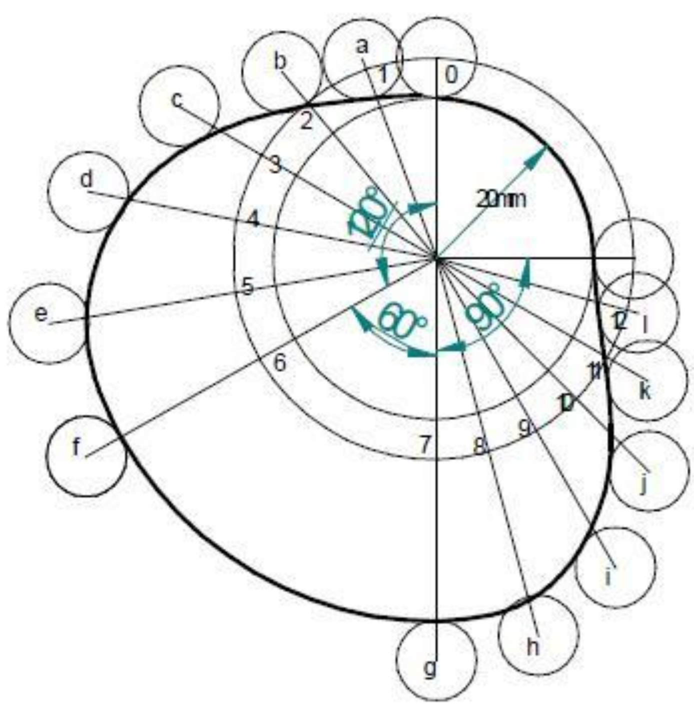
Draw the cam profile for following conditions:

Follower type = roller follower, in-line; lift = 25mm; base circle radius = 20mm; roller radius = 5mm; out stroke with UARM, for 120^0 cam rotation; dwell for 60^0 cam rotation; return stroke with UARM, for 90^0 cam rotation; dwell for the remaining period. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 1200 rpm in clockwise direction.

Displacement diagram:



Cam profile: Construct base circle and prime circle (25mm radius). Mark points 1,2,3…..in direction opposite to the direction of cam rotation, on prime circle. Transfer points a,b,c…..l from displacement diagram. At each of these points a,b,c… draw circles of 5mm radius, representing rollers. Starting from the first point of contact between roller and base circle, draw a smooth free hand curve, tangential to all successive roller positions. This forms the required cam profile.



Calculations:

Angular velocity of the cam = $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1200}{60} = \mathbf{125.71 rad/sec}$

Max. velocity during outstroke $= vo_{\max} = \frac{2s}{t_o} = \frac{2\omega s}{\theta_o} =$

$= \frac{2 \times 125.71 \times 25}{2 \times \pi/3} = 2999.9\text{mm/sec} = \mathbf{2.999 m/sec}$

Max. velocity during return stroke $= vr_{\max} = \frac{2s}{t_r} = \frac{2\omega s}{\theta_r} = \frac{2 \times 125.71 \times 25}{\pi/2} =$

$= 3999.86\text{mm/sec} = \mathbf{3.999 m/sec}$

Acceleration of the follower during outstroke $= a_o = \frac{vo_{\max}}{t_o/2} = \frac{4\omega^2 s}{\theta_o^{\,2}} =$

$= \frac{4 \times (125.71)^2 \times 25}{\left(2 \times \pi/3\right)^2} = 359975\text{mm/sec}^2 = \mathbf{359.975 m/sec^2}$

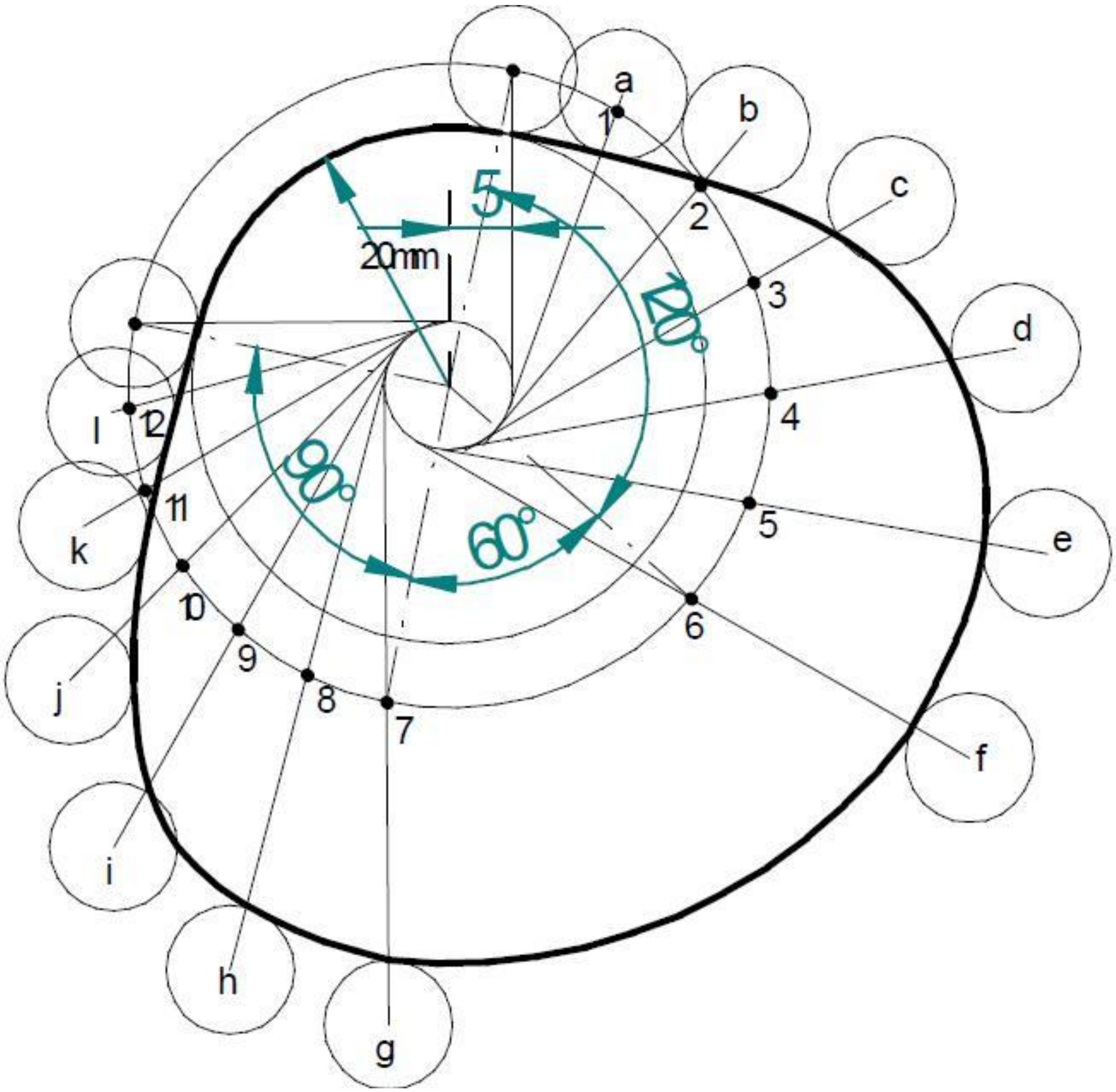
Similarly acceleration of the follower during return stroke $= a_r = \frac{4\omega^2 s}{\theta_r^{\,2}} =$

$= \frac{4 \times (125.71)^2 \times 25}{\left(\pi/2\right)^2} = 639956\text{mm/sec}^2 = \mathbf{639.956 m/sec^2}$

(4) Draw the cam profile for conditions same as in (3), with follower off set to right of cam center by 5mm and cam rotating counter clockwise.

Displacement diagram: Same as previous case.

Cam profile: Construction is same as previous case, except that the lines drawn from 1,2,3…. are tangential to the offset circle of 10mm dia. as shown in the fig.



Draw the cam profile for following conditions:

Follower type = roller follower, off set to the right of cam axis by 18mm; lift = 35mm; base circle radius = 50mm; roller radius = 14mm; out stroke with SHM in 0.05sec; dwell for 0.0125sec; return stroke with UARM, during 0.125sec; dwell for the remaining period. During return stroke, acceleration is 3/5 times retardation. Determine max. Velocity and acceleration during out stroke and return stroke if the cam rotates at 240 rpm.

Calculations:

Cam speed = 240rpm. Therefore, time for one rotation = $\dfrac{60}{240} = 0.25\,\text{sec}$

Angle of out stroke = $\theta_o = \dfrac{0.05}{0.25} \times 360 = 72^0$

Angle of first dwell = $\theta_{w1} = \dfrac{0.0125}{0.25} \times 360 = 18^0$

Angle of return stroke = $\theta_r = \dfrac{0.125}{0.25} \times 360 = 180^0$

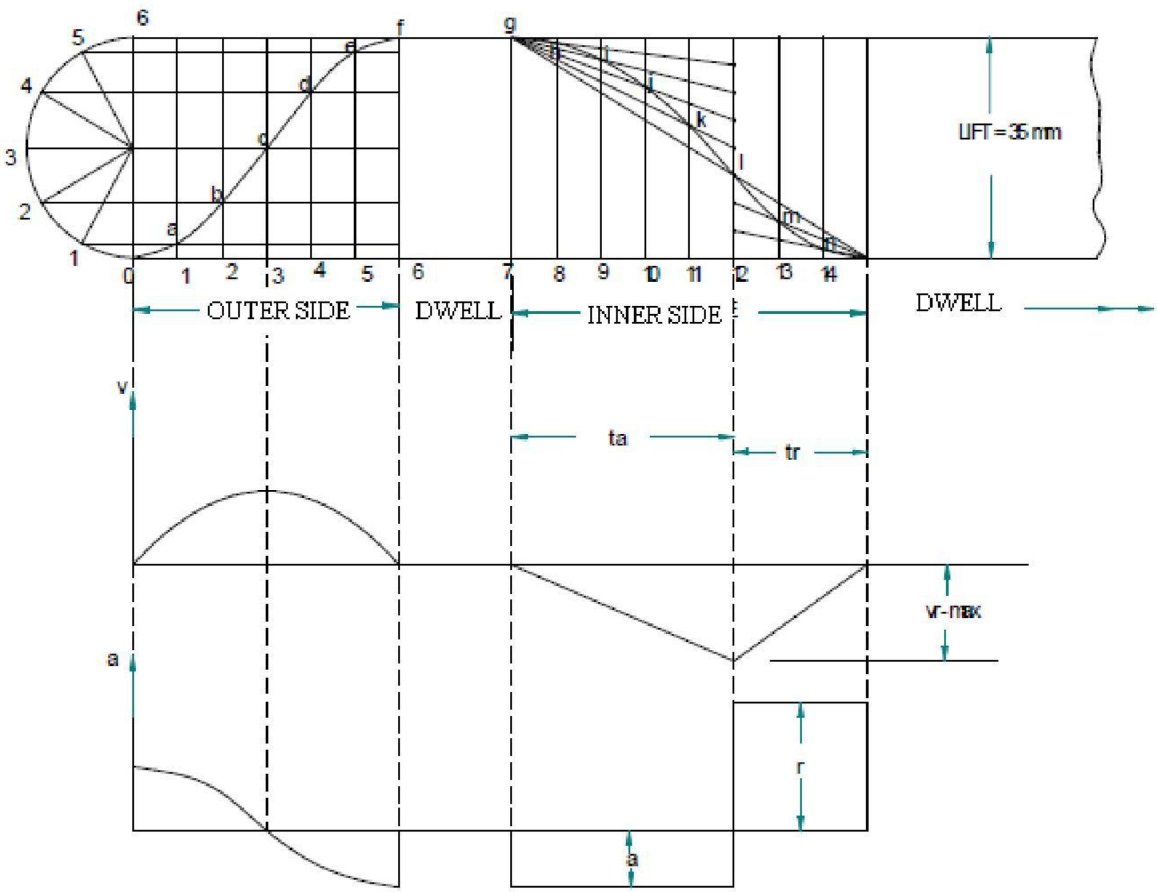
Angle of second dwell = $\theta_{w2} = 90^0$

Since acceleration is 3/5 times retardation during return stroke,

$a = \dfrac{3}{5} r$ (from acceleration diagram) $\therefore \dfrac{a}{r} = \dfrac{3}{5}$

But $a = \dfrac{v_{\max}}{t_a}; r = \dfrac{v_{\max}}{t_r} \therefore \dfrac{a}{r} = \dfrac{t_r}{t_a} = \dfrac{3}{5}$

Displacement diagram is constructed by selecting t_a and t_r accordingly.



Angular velocity of cam = $\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 240}{60}$ =**25.14 rad/sec**

Max. velocity of follower during outstroke = $vo_{max} = \frac{\pi\omega s}{2\theta_o} =$

$= \frac{\pi\times 25.14\times 35}{2\times\left(2\times \pi/5\right)}$ = 1099.87mm/sec =**1.1m/sec**

Similarly Max. velocity during return stroke = $vr_{max} = \frac{2\omega s}{\theta_r} = \frac{2\times 25.14\times 35}{\pi} =$

= 559.9 mm/sec = **0.56m/sec**

Max. acceleration during outstroke = $ao_{max} = r\omega^2_p$ (from d3) = $\frac{\pi^2\omega^2 s}{2\theta_o^{\ 2}}$ =

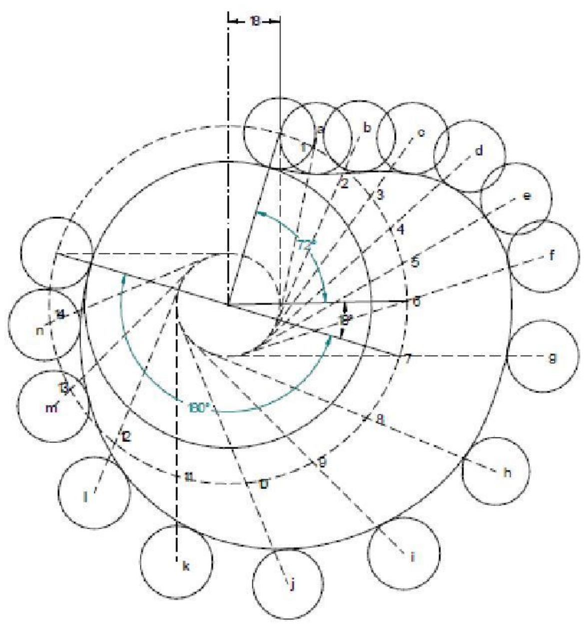
$= \frac{\pi^2\times(25.14)^2\times 35}{2\times\left(2\times\pi/5\right)^2}$ = 69127.14mm/sec^2 = **69.13m/sec^2**

acceleration of the follower during return stroke =

$$a_r = \frac{vr_{max}}{t_a} = \frac{2\omega s/\theta_r}{5\times\pi/8\times\omega} = \frac{16\times\omega^2\times s}{5\times\pi\times\theta_r} = \frac{16\times(25.14)^2\times 35}{5\times\pi\times\pi}$$ = 7166.37 mm/sec^2 = **7.17m/sec^2**

similarly retardation of the follower during return stroke =

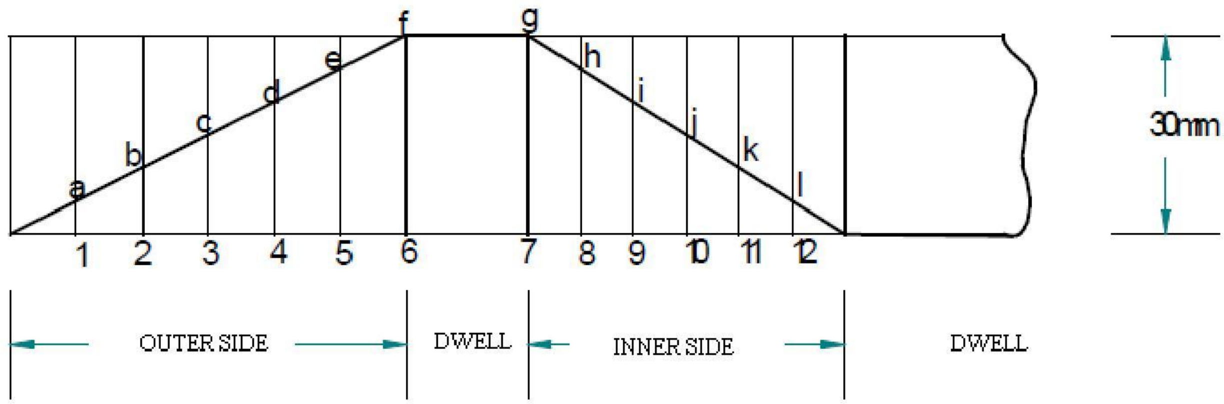
$$r_r = \frac{vr_{max}}{t_r} = \frac{2\omega s/\theta_r}{3\times\pi/8\times\omega} = \frac{16\times\omega^2\times s}{3\times\pi\times\theta_r} = \frac{16\times(25.14)^2\times 35}{3\times\pi\times\pi}$$ = 11943.9 mm/sec^2 = **11.94m/sec^2**

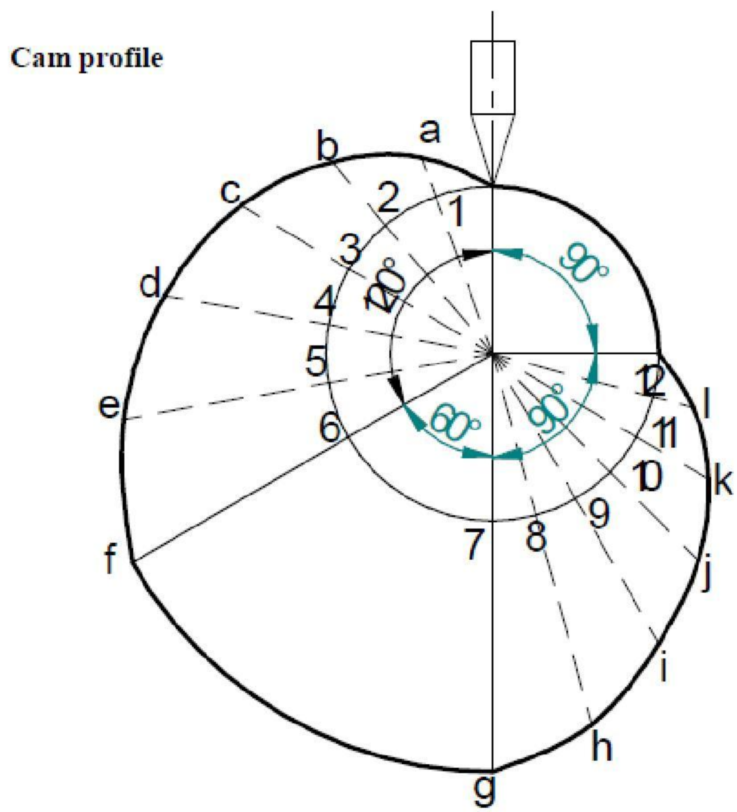


3. Draw the cam profile for following conditions:

Follower type = knife edged follower, in line; lift = 30mm; base circle radius = 20mm; out stroke with uniform velocity in 120^0 of cam rotation; dwell for 60^0; return stroke with uniform velocity, during 90^0 of cam rotation; dwell for the remaining period.

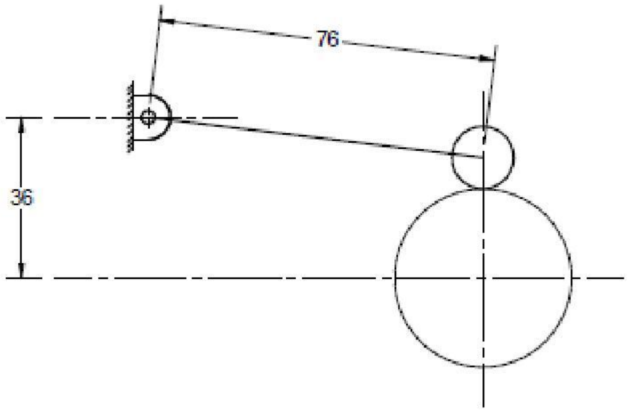
Displacement diagram:

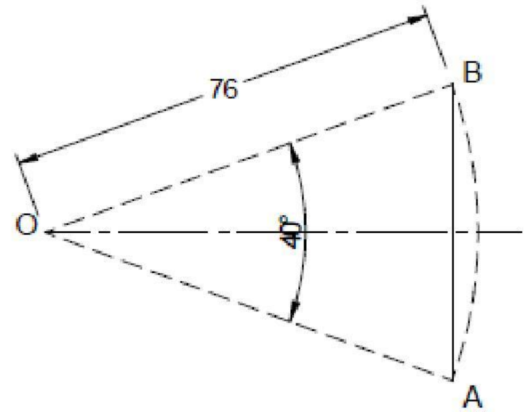




2. Draw the cam profile for following conditions:

Follower type = oscillating follower with roller as shown in fig.; base circle radius = 20mm; roller radius = 7mm; follower to rise through 40^0 during 90^0 of cam rotation with cycloidal motion; dwell for 30^0; return stroke with cycloidal motion during 120^0 of cam rotation; dwell for the remaining period. Also determine the max. velocity and acceleration during outstroke and return stroke, if the cam rotates at 600 rpm.

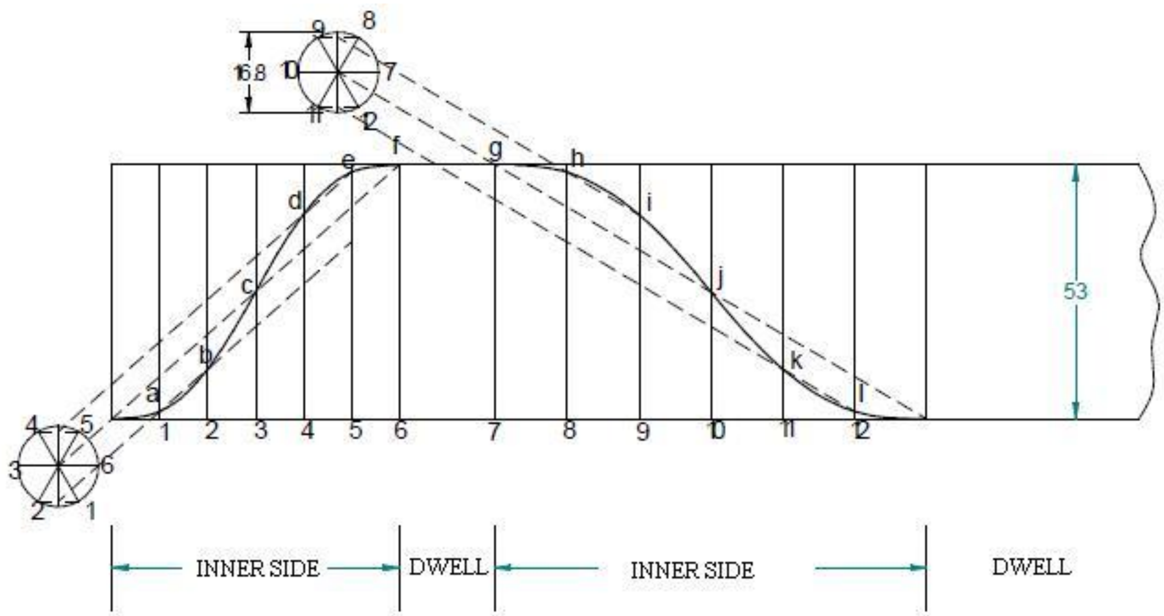




$$\text{Lift of the follower} = S = \text{length AB} \approx \text{arc AB} = OA \times \theta = 76 \times 40 \times \frac{\pi}{180} = 53 \text{ mm.}$$

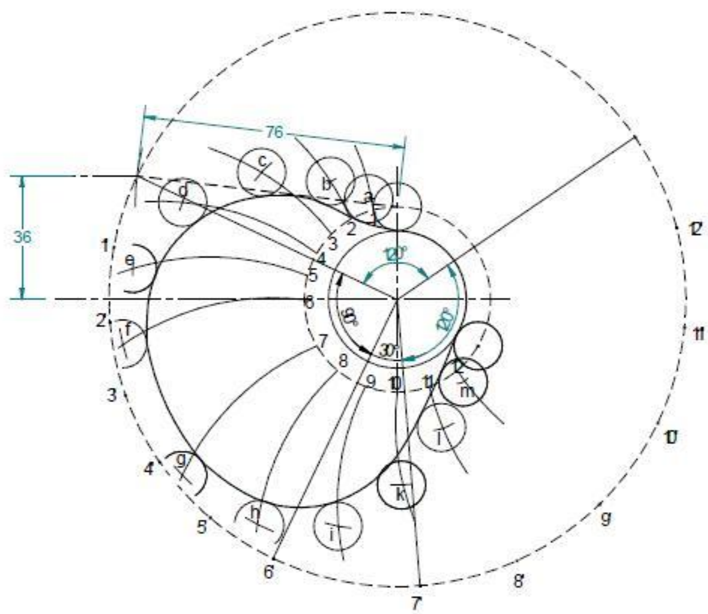
$$\text{Radius of cycloid generating circle} = \frac{53}{2 \times \pi} = 8.4 \text{ mm}$$

Displacement diagram



$$\text{Angular velocity of cam} = \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60} = 62.86 \text{ rad/sec}$$

Cam profile: Draw base circle and prime circle. Draw another circle of radius equal to the distance between cam center and follower pivot point. Take the line joining cam center and pivot point as reference and draw lines indicating successive angular displacements of cam. Divide these into same number of divisions as in the displacement diagram. Show points 1'', 2'', 3''… on the outer circle. With these points as centers and radius equal to length of follower arm, draw arcs, cutting the prime circle at 1,2,3…. Transfer points a,b,c.. on to these arcs from displacement diagram. At each of these points a,b,c… draw circles of 7mm radius, representing rollers. Starting from the first point of contact between roller and base circle, draw a smooth free hand curve, tangential to all successive roller positions. This forms the required cam profile.



4. **Draw the cam profile for following conditions:**

Follower type = knife edged follower, in line; follower rises by 24mm with SHM in 1/4 rotation, dwells for 1/8 rotation and then raises again by 24mm with UARM in 1/4 rotation and dwells for 1/16 rotation before returning with SHM. Base circle radius = 30mm.

$$\text{Angle of out stroke (1)} = \theta_{01} = \frac{1}{4} \times 360^0 = 90^0$$

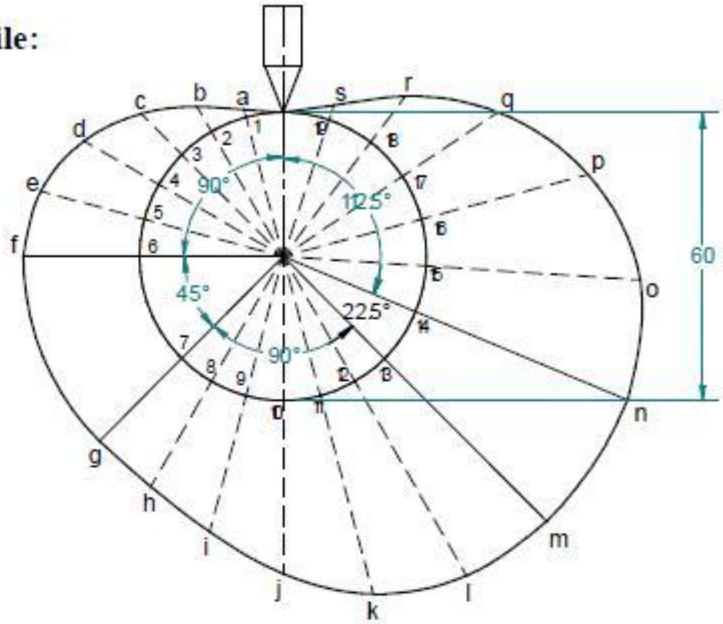
$$\text{Angle of dwell (1)} = \frac{1}{8} \times 360^0 = 45^0$$

$$\text{Angle of out stroke (2)} = \theta_{02} = \frac{1}{4} \times 360^0 = 90^0$$

$$\text{Angle of dwell (2)} = \frac{1}{16} \times 360^0 = 22.5^0$$

$$\text{Angle of return stroke} = \theta_r = \left[1 - \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{4} + \frac{1}{16}\right)\right] \times 360 = \frac{5}{16} \times 360^0 = 112.5^0$$

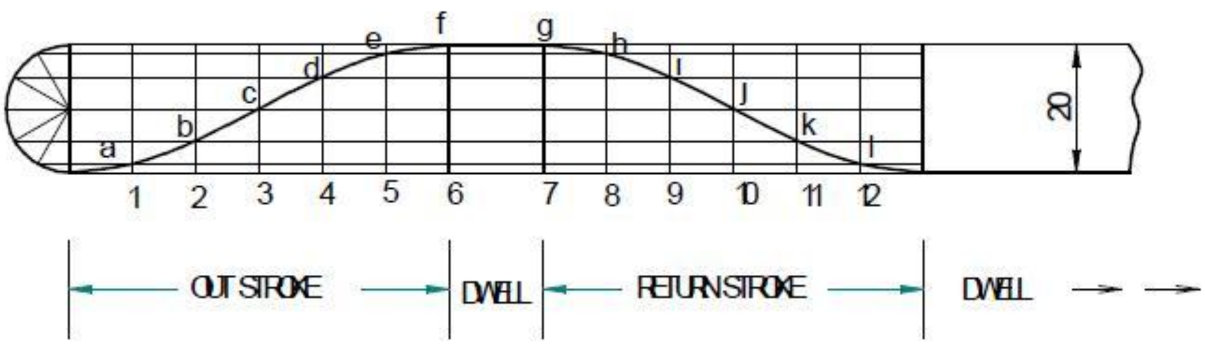
Cam profile:



(9) Draw the cam profile for following conditions:

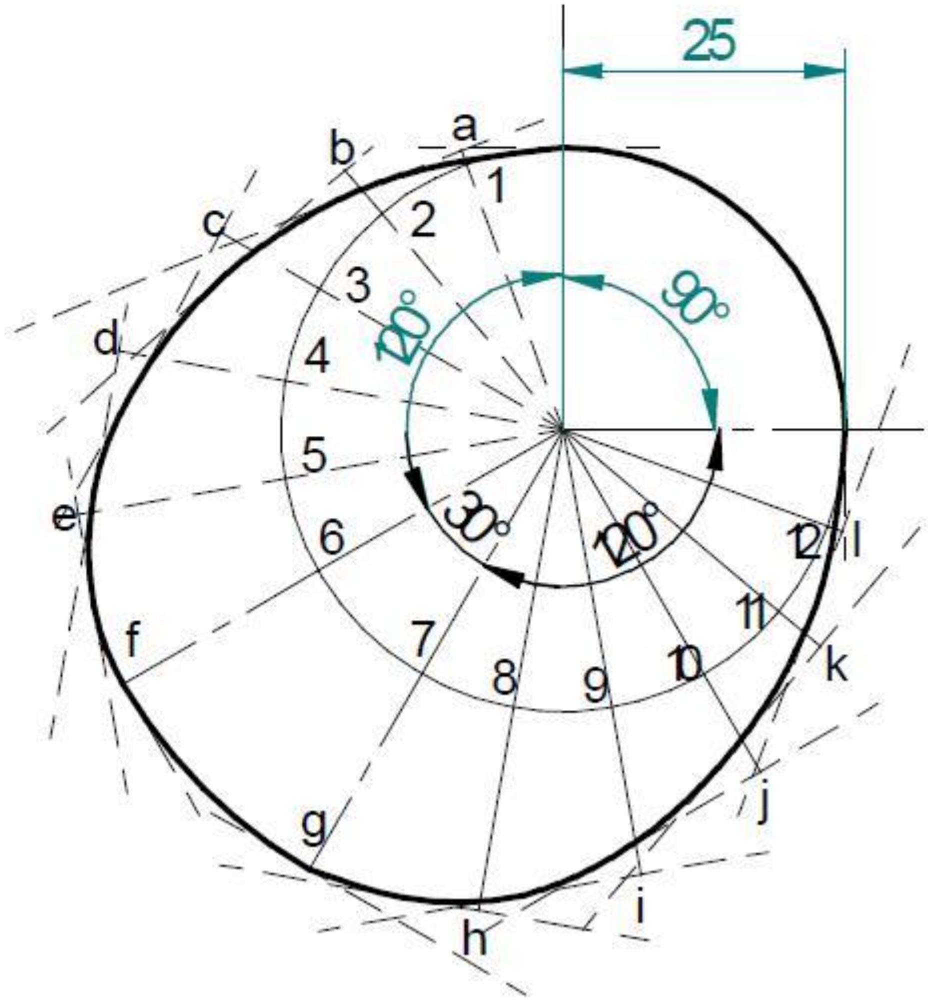
Follower type = flat faced follower, in line; follower rises by 20mm with SHM in 120^0 of cam rotation, dwells for 30^0 of cam rotation; returns with SHM in 120^0 of cam rotation and dwells during the remaining period. Base circle radius = 25mm.

Displacement diagram:



Cam profile: Construct base circle. Mark points 1,2,3…..in direction opposite to the direction of cam rotation, on prime circle. Transfer points a,b,c…..l from displacement diagram. At each of these points a,b,c… draw perpendicular lines to the radials, representing flat faced followers.

Starting from the first point of contact between follower and base circle, draw a smooth free hand curve, tangential to all successive follower positions. This forms the required cam profile.



(10) Draw the cam profile for following conditions:

Follower type = roller follower, in line; roller dia. = 5mm; follower rises by 25mm with SHM in 180^0 of cam rotation, falls by half the distance instantaneously; returns with Uniform velocity in 180^0 of cam rotation. Base circle radius = 20m.

Displacement diagram: