



UNIT-1

COMBUSTION THERMODYNAMICS

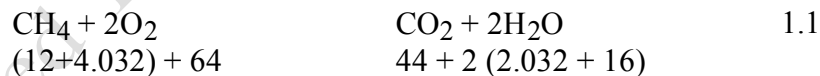
1.1 Introduction

All conventional fossil fuels, whether, solid, liquid or gaseous, contain basically carbon and hydrogen which invariably react with the oxygen in the air forming carbon dioxide, carbon monoxide or water vapour. The heat energy released as a result of combustion can be utilized for heating purposes or for generation of high pressure steam in a boiler or as power from an engine or a gas turbine.

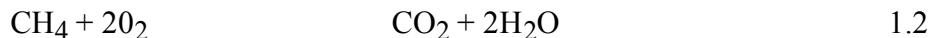
The solid fuels are burned in beds or in pulverised form suspended in the air stream. The liquid fuels are burned either by vaporising and mixing with air before ignition, when they behave like a gaseous fuel. The gaseous fuels are either burned in burners when the fuel and air are premixed or the fuel and air flow separately into a burner or a furnace and simultaneously mix together as combustion proceeds.

The Kg-mole or gram-mole is widely used in combustion calculations as a unit of weight. The molecular weight of any substance in kg represents one kilogram mole or 1K mole. 1Kmol of hydrogen has a mass of 2.016Kg and 1Kmol of carbon has a mass of 12Kg.

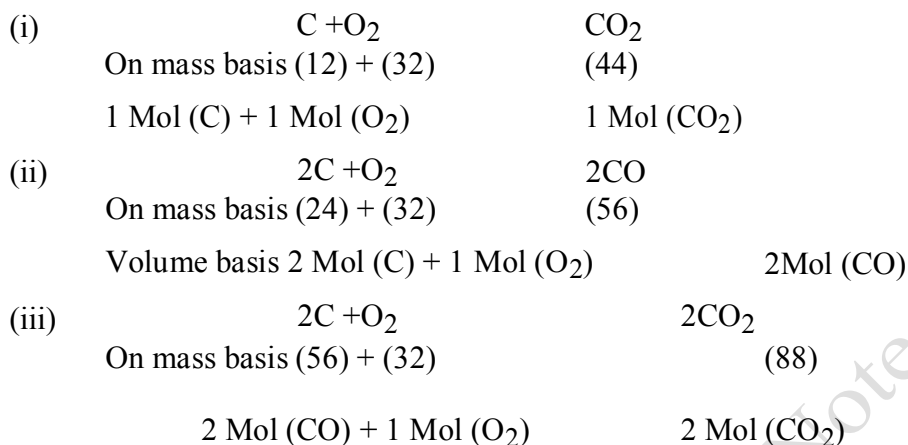
Consider a reaction



16.032kg of methane reacts with 64Kg of oxygen to form 44kg of carbon dioxide and 36.032kg of water. We can also simply state that 1Kmol of methane reacts with 2Kmol of oxygen to form 1Kmol of carbon dioxide and 2K mol of water, this has advantage of permitting easy conversion between the mass and volumetric quantities for the gaseous fuel and the product of combustion. If the gases are considered ideal then according to Avogadro hypothesis, all gases contain the same number of molecules per unit volume. It implies that 1K mole of any gaseous substance occupies the volume of 22.4m^3 at NTP i.e., 1.013bar and 273K.



1 volume of methane reacts with 2 volume of oxygen to form one volume of CO₂ and two volumes of H₂O. Therefore in any reactions, the mass is confirmed but the no. of mol or volumes may not be considered.



1.2 Combustion Stoichiometry

A balanced chemical equation for complete Combustion of the reactions with no excess air in the product is known as a stiochiometric equation. A stiochiometric mixture of the reactants is one in which the molar proportions of the reactants are exactly as given by the stiochiometric coefficients, so that no excess of any constituent is present. In general a chemical reaction may be written as



Where the reactants A and B react to form the products C and D. The small letters a, b, c and d are known as the stiochiometric coefficients.

For the combustion of any fuel the most common oxidizer is air which is a mixture of 21% O₂ and 79% N₂ (on volume basis). One mol of oxygen is accompanied by 79/21 (3.76) mol of Nitrogen. The Chemical equation for the stiochiometric combustion of carbon with air is written as



The minimum amount of air required for the complete combustion of a fuel is known as theoretical air. However in practice it is difficult to achieve complete combustion with theoretical

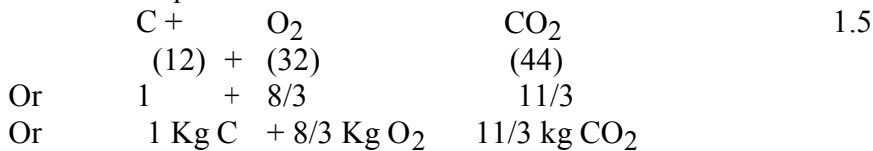


air. Therefore fuel requires some excess air for different application and may vary from 5% ~ 20% and in gas turbine it may go up to 400% of theoretical quantity.

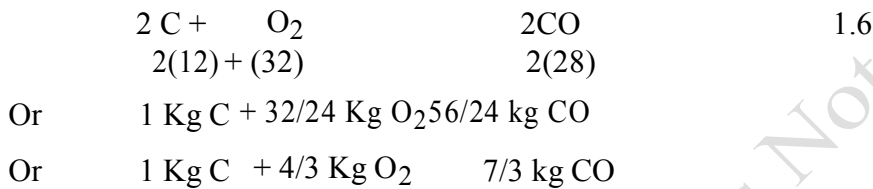
1.3 Theoretical air required for complete combustion.

If the fuel composition is known, the requirement of oxygen or air can be calculated either by mass balance or by mole method.

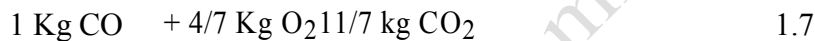
Consider a equation



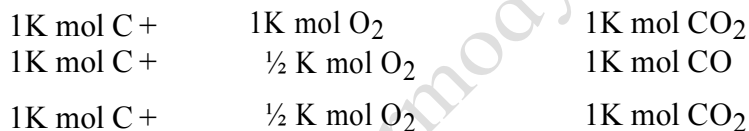
Similarly



Similarly



On molal basis



1.4 Conversion of Gravimetric analysis to volumetric basis and vice versa

If the composition of fuel is given on gravimetric (or weight) basis it can be converted to volumetric (or mole) basis as follows. Divide the weight of each constituents of the mixture by its molecular weight. This will give the relative volume (or mole) of each constituents. Add all the relative volumes of the constituents then,

$$\frac{\text{Individual (relative) volume of the constituents}}{\text{Total volume of all the constituents}} \times 100$$

will give the %age by volume of each constituents in the fuel.

If the volumetric composition of a fuel is given, it can be converted to gravimetric (or weight) basis



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as follows. Multiply the individual volume of each constituent by its molecular weight. This will give relative weight of each constituent. Add all the relative weights of the constituents then

$$\frac{\text{Individual weight of the constituents} \times 100}{\text{Total (relative) weights of the constituents}}$$

will give the %age by weight of each constituent in the fuel.

1.4.1 Calculation of the minimum amount of air for a fuel of known composition.

Example 1

Calculate the minimum volume of air required to burn one Kg of coal having the following composition by weight

C = 72.4%, H₂ 5.3%, N₂ = 1.81, O₂ = 8.5%, moisture 7.2%
S = 0.9% and ash 3.9%

On weight basis:

Taking 1kg coal as basis weight of oxygen required to burn 1kg of coal

C +	O ₂	CO ₂
0.724 x 32/12	=	1.93 kg
0.53 x 16/2	=	0.424 kg
0.009 x 32/32	=	<u>0.009 kg</u>
Total O ₂		= 2.363 kg per kg of coal

But 0.085kg O₂ is available in coal, therefore O₂ required = 2.363 – 0.085 = 2.278kg per Kg of coal.

Air contains 23% of oxygen by weight. Therefore the weight of the air supplied is

$$2.278 \times 100/23 = 9.9 \text{ kg per kg of coal Density of air required at NTP}$$

$$P v = mRT$$

$$P = \frac{m}{v} RT = RT,$$

$$= \frac{\text{Molecular weight}}{\text{Volume}}$$

$$= P/RT = 1.013 \times 10^5 / 287 \times 273 = 1.29 \text{ kg/m}^3$$

$$\text{Therefore volume of air required} = 9.9(\text{kg})/1.29(\text{kg}) = 7.67 \text{ m}^3$$

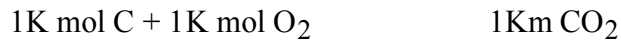


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On mole basis

Consider 100kg of coal

$$\begin{aligned} \text{C} &= 72.4/12 = 6.03\text{K mol}, & \text{O}_2 &= 8.5/32 = 0.265\text{K mol} \\ \text{H}_2 &= 5.3/2 = 2.65\text{K mol}, & \text{H}_2\text{O} &= 7.2/18 = 0.4\text{K mol} \\ \text{N}_2 &= 1.8/28 = 0.064\text{K mol}, & \text{S} &= 0.9/32 = 0.028\text{K mol} \end{aligned}$$



Therefore 6.03 K mol of carbon requires 6.03 K mol of oxygen

$$1\text{K mol H}_2 + \frac{1}{2}\text{K mol O}_2 \rightarrow 1\text{K mol H}_2\text{O} \quad \text{H}_2 - 2.65 \times \frac{1}{2} = 1.325\text{K mol}$$

$$\text{S} - 0.028 \times 1 = 0.028$$

$$\text{Total O}_2 \text{ required } 6.03 + 1.325 + 0.028 = 7.383$$

The oxygen present in coal 0.265K mol

$$\text{Net O}_2 \text{ required} = 7.383 - 0.265 = 7.118\text{K mol}$$

Air required

$$7.118 \times 100/21 = 33.89\text{K mol} / 100\text{kg of coal} = 0.3389\text{K mol} / 1\text{kg coal}$$

Volume of air supplied

$$0.3389\text{K mol/kg} \times 22.4\text{m}^3 = 7.59\text{m}^3/\text{kg of coal}$$

Example 2

Calculate the volumetric analysis of the flue gases when coal burns with 20% excess air from the previous calculation the actual air required 33.89K mol/100kg coal. Therefore the actual air is

$$33.89 \times 120/100 = 40.67\text{K mol} / 100 \text{ kg coal}$$

The amount of N₂ associated with this

$$40.67 \times 79/100 = 32.13\text{K mol}$$

$$\text{The amount of O}_2 \text{ present } 40.67 \times 21/100 = 8.54\text{K mol}$$

The actual amount of O₂ required was 7.118K mol excess O₂ will appear in exhaust gas = 8.54 – 7.118 = 1.422K mol.

Therefore:

$$\text{CO}_2 = 6.03\text{K mol}$$



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$$\begin{aligned} \text{SO}_2 &= 0.028\text{K mol} \\ \text{N}_2 &= 32.13\text{K mol (air)} + 0.064 \text{ (fuel)} \\ &= 32.194\text{K mol} \\ \text{O}_2 &= 1.422\text{K mol os excess oxygen.} \end{aligned}$$

$$\begin{aligned} \text{Therefore the Total volume} &= (6.03 + 0.028 + 32.194 + 1.422) \\ &= 39.674\text{K mol} \end{aligned}$$

The volumetric composition of the gas

$$\begin{aligned} \text{CO}_2 &= (6.03/39.674) \times 100 = 15.12\% \\ \text{SO}_2 &= (0.028/39.674) \times 100 = 0.07\% \\ \text{N}_2 &= (32.13/39.674) \times 100 = 81.15\% \\ \text{O}_2 &= (1.422/39.674) \times 100 = 3.58\% \end{aligned}$$

1.5 Calculation of the composition of fuel and excess air supplied from the exhaust gas analysis:

Some times the composition of fuel is unknown and it becomes necessary to judge whether the amount of air supplied is sufficient or not, or excess. This can be obtained by analyzing the sample of exhaust gases.

Example 3

The composition of dry flue gases obtained by burning a liquid fuel containing only hydrogen and carbon is CO_2 10.7%, O_2 5.1%, N_2 84.2%. Calculate the composition of fuel by weight and excess air used.

Solution: consider 100K mol of dry flue gases. They will contain 10.7K mol of O_2 (from CO_2) + 5.1K mole of (as max. oxygen) = 15.8K mol

Using nitrogen balance the actual air used $84.2 \times 100/79 = 106.58\text{K mol}$ of dry flue gases and oxygen in the air supplied $106.58 \times 21/100 = 22.38\text{K mol}$. Therefore the amount of O_2 present in the water produced by the combustion of H_2 is $22.38 - 15.8 = 6.58\text{K mol O}_2$. We know that 1 K mole of H_2 combines with $\frac{1}{2}$ K mol O_2 to produce water. Therefore the amount of hydrogen present is $6.58 \times 2 = 13.16\text{K mol}/100\text{K mol}$ of dry flue gases, and the carbon present is $12 \times 10.7 = 128.4\text{kg}/100\text{K mol}$ of dry flue gas. Therefore the composition of fuel (by weight) is 128.4kgC and 26.32Kg H_2 on the %age basis.



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$$C = (128.4 / (128.4 + 26.32)) \times 100 = 82.99\%$$

$$H = (26.32 / (128.4 + 26.32)) \times 100 = 17.01\%$$

Excess air supplied

The amount of O₂ required to burn 10.7K mol C is 10.7K mol and to burn 13.16K mol H₂ is 13.16 X ½ = 6.58

Total O₂ required = 10.7 + 6.58 = 17.28K mol/100K mol of dry flue gases %age of excess air =
(22.38 – 17.28)/(17.28) x 100 = 29.5%

1.6 Dew point of products:

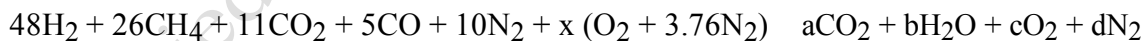
The product of combustion containing water vapour are known as wet products. The water vapour present in combustion product is cooled down to a point of condensation the vapour turn in to liquid and volume will be reduced. Knowing the partial pressure exerted by the water before condensing, it is possible to find the saturation temp. corresponding to partial pressure from the steam tables.

1.7 Flue gas analysis

A fuel has the following %age volumetric analysis H₂ = 48 CH₄ : 26, CO₂ : 11, CO : 5, N₂ = 10

The %age volumetric analysis of the dry exhaust gases in CO₂:8.8, O₂: 5.5, N₂: 85.7 Determine the air/fuel ratio by volume if air contains 21% O₂ by volume

Solution: the chemical equation for the reaction of 100 moles of fuel gas with air may be written as



Carbon balance	(C)	26 + 11 + 5	=	a	= H ₂
	H ₂	48 + 52	=	b	= 100
	O ₂	11 + 2.5 + x	=	a + c	(i)
	N ₂	10 + 3.76x	=	d	(ii)

Solving (i) and (ii) we, have



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From (i) $11 + 2.5 + x = 100/2 + a + c$

$$13.5 + x = 50 + a + c$$

Adding $\underline{10 + 3.76x = d}$

$$23.5 + 4.76x = 50 + a + c + d$$

Or $a + c + d = 4.76x - 26.5$

% CO₂ by volume in dry gas

$$(a/a+c+d) \times 100 = 8.8$$

or $(42/4.76x - 26.5) = 0.088$

$$4.76x = 503.77$$

$$\text{AF} = \frac{\text{Total mol air}}{\text{Total mol fuel}} = \frac{503.77}{100} = 5.038\%$$

Example 4

A blast furnace gas has the following volumetric analysis H₂ CO-24%, CH₄ - 2%, CO₂-6%, O₂-3% and N₂-56%

Determine the Ultimate gravimetric analysis

Given volumetric analysis, H₂ - 9%, CO-24%, CH₄ - 2%, CO₂-6%, O₂-3% and N₂-56%

Solution: The volumetric analysis may be converted into mass or granite metric analysis by completing the table as follows:

Constituent	Volume in 1m ³ of flue gas (a)	Molecular mass (b)	Proportional mass (c)=(a)x(b)	Mass in kg per kg of the gas (d)=(c)/ ©	% by mass = (d)x100
CO	0.24	28	6.72	6.72/18.48 = 0.36	36%
CH ₄	0.02	16	0.32	0.32/18.48 = 0.0173	1.73%
CO ₂	0.06	44	2.64	264/18.48 = 0.142	14.2%
O ₂	0.03	32	0.96	0.96/18.48 = 0.0519	5.19%
N ₂	0.56	14	7.84	7.84/18.48 = 0.42	42%
			c = 18.48	(d) = 1	100

The volumetric analysis of flue gas components becomes



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CO-0.36, CH₄ – 0.0173, CO₂- 0.142, O₂-0.0519 and N₂-0.42

Applied Thermodynamics Notes, HIT, NDS



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Example 5

Determine the fuel gas analysis and air fuel ratio by weight when fuel oil with 84.9% carbon, 11.4% hydrogen, 3.2% sulphur, 0.4% oxygen and 0.1% ash by weight is burnt with 20% excess air, assume complete combustion.

Solution: Consider 1kg of fuel

Oxygen required / Kg of fuel

For burning of 1kg C - $0.849 \times 32/12$

For burning of 1kg H - $0.114 \times 16/2$

For burning of 1kg S - $0.032 \times 32/32$

Total O₂ required is 3.208 kg.

Amount of O₂ contained in the fuel = 0.004Kg Net O₂ supplied / kg of fuel = $3.208 - 0.004$
= 3.204 kg O₂

Net air supplied = $3.204 \times 100/23 = 13.93$ kg/kg of fuel

When 20% excess air supplied

Total air supplied = $13.93 \times 1.2 = 16.716$ kg/kg of fuel.

N₂ actually supplied = $16.716 \times 77/100 = 12.871$ kg/kg of fuel

O₂ actually supplied = $16.716 \times 23/100 = 3.845$ kg/kg of fuel

Total free O₂ in fuel gas = $3.845 - 6.204$

= 0.641 kg/kg of fuel

Total free N₂ in fuel gas = 12.87 kg/kg of fuel

Flue gas analysis:

C converted to CO₂ = $0.849 \times 44/12 = 3.113$ kg CO₂

H converted to H₂O = $0.114 \times 18/2 = 1.026$ kg H₂O

S converted to SO₂ = $0.032 \times 64/32 = 0.064$ kg SO₂

Flue gas / kg of fuel:



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$$= 3.113 + 1.26 + 0.064 + 0.641 + 12.871$$
$$\text{CO}_2 \text{ H}_2\text{O SO}_2 \text{ O}_2 \text{ N}_2 = 17.715\text{kg.}$$

Therefore:

$$\text{CO}_2 = (3.113/17.715) \times 100 = 17.573\%$$

$$\text{SO}_2 = (0.064/17.715) \times 100 = 0.36\%$$

$$\text{O}_2 = (0.641/17.715) \times 100 = 3.618\%$$

$$\text{H}_2\text{O} = (1.026/17.715) \times 100 = 5.79\%$$

$$\text{N}_2 = (12.871/17.715) \times 100 = 72.656\%$$

Air fuel mixture ratio is = 16.716 : 1

Example 6

A blast furnace gas has the following volumetric analysis.

$\text{H}_2 = 9\%$, $\text{CO} = 24\%$, $\text{CH}_4 = 2\%$, $\text{CO}_2 = 6\%$, $\text{O}_2 = 3\%$ and $\text{N}_2 = 56\%$

Determine the ultimate gravimetric analysis.

Solution:

Total H₂ in the blast furnace gas. % volumetric analysis = $9\text{H}_2 + 2\text{H}_4$

Proportional mass = % volumetric analysis X mol. Mass of element = $(9 \times 2) + (2 \times 4) = 18 + 8 = 26$
kg.

Total 'C' in the blast furnace gas.

% of volumetric analysis = $24\text{C} + 2\text{C} + 6\text{C}$

Proportional mass = $(24+2+6) \times 12$
= 384 kg

Total O₂ in the blast furnace gas



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$$\% \text{ of volumetric analysis} = 24xO + 6O_2 + 3O_2$$

$$\text{Proportional mass} = (24+16) \times 9 \text{ (32)}$$

$$= 672 \text{ kg}$$

Total N₂ in the blast furnace gas

$$\% \text{ of volumetric analysis} = 56 \text{ N}_2$$

$$\text{Proportional mass of N}_2 = 56 \times 28 = 1568 \text{ Kg.}$$

Total weight of blast furnace gas:

$$= 384\text{kg C} + 26\text{kg H}_2 + 672\text{kg O}_2 + 1568\text{kg N}_2$$

$$= 2650\text{kgs}$$

Gravimetric %age composition:

$$\text{C} = (384/2650) \times 100 = 14.49\%$$

$$\text{H}_2 = (26/2650) \times 100 = 0.98\%$$

$$\text{O}_2 = (672/2650) \times 100 = 25.36\%$$

$$\text{N}_2 = (1568/2650) \times 100 = 59.17\%$$

Example 7

The analysis of coal used in a boiler trial is as follows. 82% carbon, 6% hydrogen, 4% oxygen, 2% moisture and 8% ash. Determine the theoretical air required for complete combustion of 1kg of coal. If the actual air supplied is 18kg per kg of coal the hydrogen is completely burned & 80% carbon burned to CO₂, the remainder is CO, Determine the volumetric analysis of the dry products of combustion.

Solution: For complete combustion.

O₂ required is

$$\text{For carbon} \quad - \quad 0.82 \quad = \quad 2.186 \text{ kg of O}_2$$



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$$\begin{aligned} \text{For hydrogen -} & \quad 0.006 & = & \quad 0.48 \text{ kg of O}_2 \\ \text{Total O}_2 \text{ required} & = & 2.666\text{kg.} \\ \text{Net O}_2 \text{ supplied} & = & \text{Total O}_2 \text{ required} - \text{O}_2 \text{ present in the fuel} \\ & = & 2.66 - 0.004 \\ & = & 2.662 \text{ kg/kg of coal} \end{aligned}$$

Theoretical minimum air required for complete combustion [C burns to CO₂ totally]

$$\text{Air supplied} = 2.626 \times 100/23 = 11.417 \text{ kg/kg of coal}$$



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Flue gas analysis:

But actually only 80% carbon is burns to CO₂

$$\text{CO}_2 = 0.8 \times 0.82 \times 44/12 = 2.405 \text{ kg of CO}_2$$

20% carbon is burnt to CO

$$\text{CO} = 0.2 \times 0.82 \times 28/12 = 0.383 \text{ kg of CO}$$

O₂ actually required for 80% carbon burnt to CO₂

$$= 0.8 \times 0.82 \times 32/12 = 1.749 \text{ kg of O}_2$$

O₂ actually required for 20% carbon burnt to CO

$$= 0.2 \times 0.82 \times 16/12 = 0.219 \text{ kg of O}_2$$

O₂ required by Hydrogen:

$$= 0.06 \times 8 = 0.48 \text{ kg of O}_2.$$

H₂O produced = 0.06 x 9 = 0.54 kg of H₂O But actual air supplied = 18kg

Actually O₂ supplied = 18 x 23/100 = 4.14 kg of O₂ Free O₂ in the flue gas = 4.14

N₂ in the flue gas

$$+ 0.04 - 1.749 - 0.219 - 0.48$$

$$= 1.732 \text{ kg of O}_2/\text{kg of coal}$$

$$= 18 \times 77/100 = 13.86 \text{ kg/kg of coal}$$

Volumetric analysis of the dry products of combustion.

$$\text{CO}_2 = (2.405/44) \times 100 = 0.0546 \text{ m}^3/\text{K.mol}$$

$$\text{CO} = (0.383/28) \times 100 = 0.0137 \text{ m}^3/\text{K.mol}$$

$$\text{O}_2 = (1.732/32) \times 100 = 0.0541 \text{ m}^3/\text{K.mol}$$

$$\text{N}_2 = (13.86/28) \times 100 = 0.495 \text{ m}^3/\text{K.mol}$$

In % of volume:

$$\text{CO}_2 = (0.0546/0.6174) \times 100 = 8.84\%$$

$$\text{CO} = (0.0137/0.6174) \times 100 = 2.22\%$$

$$\text{O}_2 = (0.0541/0.6174) \times 100 = 8.76\%$$



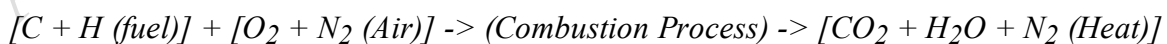
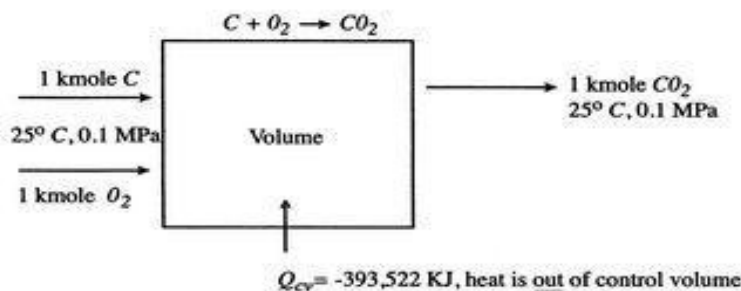
$$N_2 = (0.495/0.6174) \times 100 = 80.70\%$$

1.8 Enthalpy of reaction

Enthalpy of a reaction is defined as the difference between the enthalpy of the products at a specified state and the enthalpy of the reactants at the same state for a complete reaction. For combustion process, the enthalpy of a reaction is usually referred to as the “enthalpy of combustion” it is obviously a very useful property for analyzing the combustion processes of fuels. However there are so many different fuels and fuel mixtures that is not practical to list enthalpy of combustion values for all possible cases. Besides, the enthalpy of combustion is not of much use when the combustion is incomplete. Therefore a more practical approach would be have a more fundamentally property to represent the chemical energy of an element or compound at some reference state. This property is the “enthalpy of formation” which can be viewed as the enthalpy of a substance at a specified state due to its chemical composition. To establish a starting point it is assigned the enthalpy of formation for all stable elements such as O₂, N₂, H₂ and C a value of zero at standard reference state of 25^oC and 1 atm. For all stable compounds.

In a chemical reaction bonds are broken in the reactants and new bonds formed in the products. Energy is required to break bonds and energy is released when bonds are formed. The energy associated with a chemical reaction depends on the number and type of bonds broken and/or formed.

Every chemical species has a certain amount of "heat content," or enthalpy, H, which cannot be measured. However, differences in enthalpy can be measured. The net energy change for a reaction performed at constant pressure is the enthalpy change for the reaction. This enthalpy change, H, has units kJ/mol and is defined:



where

C = Carbon, H = Hydrogen, O = Oxygen, N = Nitrogen



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$$(1) \quad H = H(\text{products}) - H(\text{reactants})$$

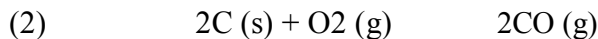
If energy is given off during a reaction, such as in the burning of a fuel, the products have less heat content than the reactants and H will have a negative value; the reaction is said to be exothermic. If energy is consumed during a reaction, H will have a positive value; the reaction is said to be endothermic.

The enthalpy change for a chemical change is independent of the method or path by which the change is carried out as long as the initial and final substances are brought to the same temperature. This observation, known as HESS'S LAW, has important practical utility.

Thermochemical equations may be treated as algebraic equations: they may be written in the reverse direction with a change in the sign of H – even though the reverse reaction may not actually occur; they may be added and subtracted algebraically; the equation and associated

H value may be multiplied or divided by factors. Hess's Law allows the calculation of enthalpy changes that would be difficult or impossible to determine directly, i.e. by experiment.

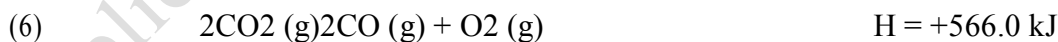
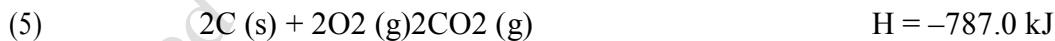
The enthalpy change for the reaction:



cannot be determined directly because carbon dioxide will also form. However, H can be measured for:



Multiplying equation (3) by 2 gives equation (5), and reversing equation (4) gives equation (6):



Adding equations (5) and (6) gives the desired information:



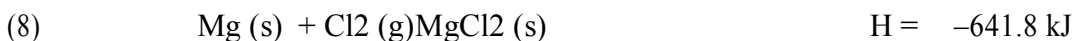
For a reaction in which a compound is formed from the elements, the enthalpy change is called the heat of formation, H_f° , for the compound. The superscript "o" indicates standard conditions of one

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atmosphere pressure. Equation (2) and (3) are such reactions. Some others:



In reactions (2), (3), (7), and (8) H for the reaction is H_f° for the compound. For the reaction:



the heat of reaction is associated with the formation of two moles of SO_3 . But heat of formation is per mole of compound, so H_f° for SO_3 is half of -790.4 , or -395.2 kJ .

Extensive listings of heats of formation are available in handbooks. With these values of H_f° , you can calculate virtually any heat of reaction. The heat of a reaction is the sum of H_f° values for the products minus the sum of H_f° values for the reactants. Expressed as a formula:

$$(10) \quad \Delta H_{\text{rxn}} = \sum \Delta H_{\text{f products}}^\circ - \sum \Delta H_{\text{f reactants}}^\circ$$

Heats of formation for several compounds are given below. Note that the phase of the compound is important when choosing a H_f for a free element is zero.

STANDARD HEATS OF FORMATION, $^\circ H_f$, kJ/mole, at $^\circ 25 \text{ C}$

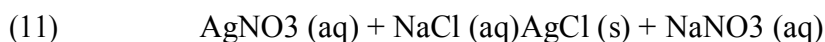
AgCl (s)	-127.1	Ca(OH) ₂ (s)	-986.1	K ₃ PO ₄ (aq)	-2002.9
AgNO ₃ (aq)	-100.7	Ca(OH) ₂ (aq)	-1002.9	K ₂ SO ₄ (aq)	-1409.2
AlCl ₃ (s)	-695.4	HCl (g)	-92.3	MgCl ₂ (aq)	-797.1
AlCl ₃ (aq)	-1027.2	HCl (aq)	-167.4	Mg(NO ₃) ₂ (aq)	-875.1
Al(OH) ₃ (s)	-1272.8	H ₂ O (g)	-241.8	NaCl (aq)	-407.1
Al ₂ (SO ₄) ₃ (aq)	-3753.5	H ₂ O (l)	-285.8	NaHCO ₃ (s)	-947.7
BaCl ₂ (aq)	-873.2	H ₃ PO ₄ (aq)	-1294.1	NaNO ₃ (aq)	-446.2
Ba(NO ₃) ₂ (aq)	-951.4	H ₂ SO ₄ (l)	-814.0	NaOH (aq)	-469.4
BaSO ₄ (s)	-1473.2	H ₂ SO ₄ (aq)	-888.0	Na ₂ SO ₄ (aq)	-1387.0



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EXAMPLE: Using H^f data calculate the heat of reaction for: o



$$\begin{aligned}
 H &= [H_f^o \text{ AgCl (s)} + H_f^o \text{ NaNO}_3 \text{ (aq)}] - [H_f^o \text{ AgNO}_3 \text{ (aq)} + H_f^o \text{ NaCl (aq)}] \\
 &= [(-127.0) + (-446.2)] - [(-100.7) + (-407.1)] \\
 &= [-573.2] - [-507.8] = -573.2 + 507.8 = -65.4 \text{ kJ}
 \end{aligned}$$

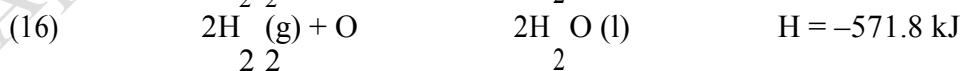
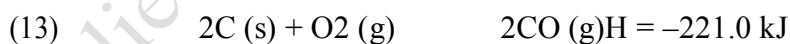
EXAMPLE Using H^f data calculate the heat of reaction for o



$$\begin{aligned}
 H &= [2 H_f^o \text{ AgCl (s)} + H_f^o \text{ Mg(NO}_3)_2 \text{ (aq)}] - [2 H_f^o \text{ AgNO}_3 \text{ (aq)} + H_f^o \text{ MgCl}_2 \text{ (aq)}] \\
 &= [2(-127.0) + (-875.1)] - [2(-100.7) + (-797.1)] \\
 &= [-1129.1] - [-998.5] = -1129.1 + 998.5 = -130.6 \text{ kJ}
 \end{aligned}$$

Note: the values of H^f are multiplied by the stoichiometric coefficients from the balanced chemical equation.

are multiplied by the stoichiometric coefficients from the balanced



Using Hess' Law with appropriate equations from (13)-(18), above, calculate H for each of the
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following reactions:

- 1) $\text{H}_2\text{O} (\text{l}) \rightarrow \text{H}_2\text{O} (\text{g})$
- 2) $\text{C} (\text{s}) + \text{H}_2\text{O} (\text{g}) \rightarrow \text{CO} (\text{g}) + \text{H}_2 (\text{g})$
- 3) $\text{Ca} (\text{s}) + \text{H}_2\text{O} (\text{g}) \rightarrow \text{CaO} (\text{s}) + \text{H}_2 (\text{g})$
- 4) $\text{CO} (\text{g}) + \frac{1}{2} \text{O}_2 (\text{g}) \rightarrow \text{CO}_2 (\text{g})$
- 5) $2\text{Ca} (\text{s}) + 2\text{C} (\text{s}) + 3\text{O}_2 (\text{g}) \rightarrow 2\text{CaCO}_3 (\text{s})$

Using heats of formation values from page T-56 calculate H for each of the following reactions:

- 6) $2\text{Al} (\text{s}) + 3\text{Cl}_2 (\text{g}) \rightarrow 2\text{AlCl}_3 (\text{s})$
- 7) $2\text{Al} (\text{s}) + 3\text{H}_2\text{SO}_4 (\text{aq}) \rightarrow \text{Al}_2(\text{SO}_4)_3 (\text{aq}) + 3\text{H}_2 (\text{g})$
- 8) $2\text{Al} (\text{s}) + 3\text{ZnCl}_2 (\text{aq}) \rightarrow 2\text{AlCl}_3 (\text{aq}) + 3\text{Zn} (\text{s})$
- 9) $3\text{BaCl}_2 (\text{aq}) + \text{Al}_2(\text{SO}_4)_3 (\text{aq}) \rightarrow 3\text{BaSO}_4 (\text{s}) + 2\text{AlCl}_3 (\text{aq})$
- 10) $\text{Na}_2\text{SO}_4 (\text{aq}) + \text{BaCl}_2 (\text{aq}) \rightarrow \text{BaSO}_4 (\text{s}) + 2\text{NaCl} (\text{aq})$
- 11) $\text{BaCl}_2 (\text{aq}) + 2\text{AgNO}_3 (\text{aq}) \rightarrow 2\text{AgCl} (\text{s}) + \text{Ba}(\text{NO}_3)_2 (\text{aq})$
- 12) $\text{Ca}(\text{OH})_2 (\text{aq}) + 2\text{HCl} (\text{aq}) \rightarrow \text{CaCl}_2 (\text{aq}) + 2\text{H}_2\text{O} (\text{l})$
- 13) $2\text{Al}(\text{OH})_3 (\text{s}) + 3\text{H}_2\text{SO}_4 (\text{aq}) \rightarrow \text{Al}_2(\text{SO}_4)_3 (\text{aq}) + 6\text{H}_2\text{O} (\text{l})$
- 14) $\text{AlCl}_3 (\text{aq}) + 3\text{NaOH} (\text{aq}) \rightarrow \text{Al}(\text{OH})_3 (\text{s}) + 3\text{NaCl} (\text{aq})$
- 15) $3\text{KOH} (\text{aq}) + \text{H}_3\text{PO}_4 (\text{aq}) \rightarrow \text{K}_3\text{PO}_4 (\text{aq}) + 3\text{H}_2\text{O} (\text{l})$

Answers to Problems

- | | | | |
|----|------------|-----|----------|
| 1) | +44.1 kJ | 9) | -76.9 kJ |
| 2) | +131.3 kJ | 10) | -19.2 kJ |
| 3) | -393.7 kJ | 11) | -130.8 |
| 4) | -283.0 kJ | 12) | -111.7 |
| 5) | -2414.2 kJ | 13) | -258.7 |
| 6) | -1390.8 kJ | 14) | -58.7 kJ |
| 7) | -1089.5 kJ | 15) | -122.6 |
| 8) | -592.2 kJ | | |

1.9 Internal Energy of Combustion: It is defined as the difference between the internal energy of the products and the internal energy of the reactants when complete

combustion occurs at a given temperature and pressure. $U_c = U_p - U_R$

$$= \sum_p n_e (h_f + h - pv) - \sum_R n_i (h_f + h - pv)$$

1.10 Combustion efficiency

It is defined as the ratio of ideal fuel-air to the actual fuel-air ratio



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$$\text{comb} = \frac{(F/A)_{\text{ideal}}}{(F/A)_{\text{actl}}}$$

Example 8

Consider the following reaction, which occurs in a steady state, steady flow processes.



The reactants and products are each at total pressure of 0.1Mpa and 25°C. Determine the heat transfer for per K mol of fuel entering the combustion chamber.

Solution : using the values of enthalpy of formation

$$Q = h_f = p n_e h_f - R n_i h_f$$

$$R n_i h_f = (h_f) \text{CH}_4 = -74873 \text{KJ}$$

$$p n_e h_f = (h_f) \text{CO}_2 + 2 (h_f) \text{H}_2\text{O} (l) \\ = -393522 + 2(-285283) = -965182 \text{KJ}$$

$$\text{Therefore } Q = -965182 - (-74873) = -890309 \text{KJ}$$

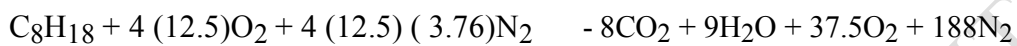


Example 9

A small gas turbine uses C_8H_{18} as fuel and 400% theoretical air. The air and fuel enters at $25^\circ C$ and the products of combustion leaves at $900K$. The output of the engine and the fuel consumption are measured and it is found that the specific fuel consumption is 0.25kg/Sec of fuel per MW output. Determine the heat transfer from the engine per K mol of fuel. Assume complete combustion

Solution:

The combustion equation is



By first law

$$Q + R n_i (h_f + h) = W + p n_e (h_f + h)$$

$$R n_i (h_f + h) = (h_f) C_8H_{18} = 250105\text{KJ/K mol fuel at } 25^\circ C$$

Considering the products

$$p n_e (h_f + h) = n_{CO_2} (h_f + h)_{CO_2} + n_{H_2O} (h_f + h)_{H_2O} + n_{O_2} (h_f + h)_{O_2} + n_{N_2} (h_f + h)_{N_2}$$

h_f of $O_2, N_2 = 0$ $h =$ Enthalpy of formation from $298^\circ K$ to $900K$

$$\text{Therefore } p n_e (h_f + h) = 8(-393522 + 288030) + 9(-241826 + 21892) + 37.5(19249) + 188(18222) = -755769\text{KJ/K mol fuel.}$$

$$W = \frac{1000(\text{KW})}{0.25} \times \frac{114 \text{ Kg}}{\text{K mol}} = 456920\text{KJ/K mol}$$

$$\text{Therefore } Q = -755769 + 456920 - (-250105) = -48744\text{KJ/K mol fuel}$$



UNIT-2

GAS POWER CYCLES

1.1 Theoretical Analysis

The accurate analysis of the various processes taking place in an internal combustion engine is a very complex problem. If these processes were to be analyzed experimentally, the analysis would be very realistic no doubt. It would also be quite accurate if the tests are carried out correctly and systematically, but it would be time consuming. If a detailed analysis has to be carried out involving changes in operating parameters, the cost of such an analysis would be quite high, even prohibitive. An obvious solution would be to look for a quicker and less expensive way of studying the engine performance characteristics. A theoretical analysis is the obvious answer.

A theoretical analysis, as the name suggests, involves analyzing the engine performance without actually building and physically testing an engine. It involves *simulating* an engine operation with the help of thermodynamics so as to formulate mathematical expressions which can then be solved in order to obtain the relevant information. The method of solution will depend upon the complexity of the formulation of the mathematical expressions which in turn will depend upon the assumptions that have been introduced in order to analyze the processes in the engine. The more the assumptions, the simpler will be the mathematical expressions and the easier the calculations, but the lesser will be the accuracy of the final results.

The simplest theoretical analysis involves the use of the air standard cycle, which has the largest number of simplifying assumptions.

1.2 A Thermodynamic Cycle

In some practical applications, notably steam power and refrigeration, a thermodynamic cycle can be identified.



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A thermodynamic cycle occurs when the working fluid of a system experiences a number of processes that eventually return the fluid to its initial state.

In steam power plants, water is pumped (for which work W_P is required) into a boiler and evaporated into steam while heat Q_A is supplied at a high temperature. The steam flows through a turbine doing work W_T and then passes into a condenser where it is condensed into water with consequent rejection of heat Q_R to the atmosphere. Since the water is returned to its initial state, the net change in energy is zero, assuming no loss of water through leakage or evaporation.

An energy equation pertaining only to the system can be derived. Considering a system with one entering and one leaving flow stream for the time period t_1 to t_2

$$\Delta Q - \Delta W + \Delta E_{fin} - \Delta E_{fout} = \Delta E_{system} \quad (1)$$

Q is the heat transfer across the boundary, +ve for heat *added to* the system and -ve for heat *taken from* the system.

W is the work transfer across the boundary, +ve for work *done by* the system and -ve for work *added to* the system

ΔE_{fin} is the energy of all forms *carried* by the fluid across the boundary *into* the system

ΔE_{fout} is the energy of all forms *carried* by the fluid across the boundary *out of* system

E_{system} is the energy of all forms *stored* within the system, +ve for energy *increase* -ve for energy *decrease*

In the case of the steam power system described above

$$Q_A + Q_R = Q = W = W_T + W_P \quad (2)$$

All thermodynamic cycles have a heat rejection process as an invariable characteristic and the net work done is always less than the heat supplied, although, as shown in Eq. 2, it is



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equal to the sum of heat added and the heat rejected (Q_R is a negative number).

The thermal efficiency of a cycle, η_{th} , is defined as the fraction of heat supplied to a thermodynamic cycle that is converted to work, that is

$$\eta_{th} = \frac{W}{Q_A} = \frac{Q_A + Q_R}{Q_A} \quad (3)$$

This efficiency is sometimes confused with the enthalpy efficiency, η_e , or the fuel conversion efficiency, η_f

$$\eta_e = \frac{W}{m_f Q_c} \quad (4)$$

This definition applies to combustion engines which have as a source of energy the chemical energy residing in a fuel used in the engine.

Any device that operated in a thermodynamic cycle, absorbs thermal energy from a source, rejects a part of it to a sink and presents the difference between the energy absorbed and energy rejected *as work to the surroundings* is called a heat engine.

A heat engine is, thus, a device that produces work. In order to achieve this purpose, the heat engine uses a certain working medium which undergoes the following processes:

1. A compression process where the working medium absorbs energy as work.
2. A heat addition process where the working medium absorbs energy as heat from a source.
3. An expansion process where the working medium transfers energy as work to the surroundings.
4. A heat rejection process where the working medium rejects energy as heat to a sink.

If the working medium does not undergo any change of phase during its passage through



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the cycle, the heat engine is said to operate in a non-phase change cycle. A phase change cycle is one in which the working medium undergoes changes of phase. The air standard cycles, using air as the working medium are examples of non-phase change cycles while the steam and vapor compression refrigeration cycles are examples of phase change cycles.

1.3 Air Standard Cycles

The air standard cycle is a cycle followed by a heat engine which uses air as the working medium. Since the air standard analysis is the simplest and most idealistic, such cycles are also called *ideal cycles* and the engine running on such cycles are called *ideal engines*.

In order that the analysis is made as simple as possible, certain assumptions have to be made. These assumptions result in an analysis that is far from correct for most actual combustion engine processes, but the analysis is of considerable value for indicating the upper limit of performance. The analysis is also a simple means for indicating the relative effects of principal variables of the cycle and the relative size of the apparatus.

Assumptions

1. The working medium is a perfect gas with constant specific heats and molecular weight corresponding to values at room temperature.
2. No chemical reactions occur during the cycle. The heat addition and heat rejection processes are merely heat transfer processes.
3. The processes are reversible.
4. Losses by heat transfer from the apparatus to the atmosphere are assumed to be zero in this analysis.
5. The working medium at the end of the process (cycle) is unchanged and is at the same condition as at the beginning of the process (cycle).

In The selecting an idealized process one is always faced with the fact that the simpler the assumptions, the easier the analysis, but the farther the result from reality. The air cycle has the advantage of being based on a few simple assumptions and of lending itself to rapid and easy mathematical handling without recourse to thermodynamic charts or tables or complicated



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calculations. On the other hand, there is always the danger of losing sight of its limitations and of trying to employ it beyond its real usefulness.

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Equivalent Air Cycle

A particular air cycle is usually taken to represent an approximation of some real set of processes which the user has in mind. Generally speaking, the air cycle representing a given real cycle is called an *equivalent air cycle*. The equivalent cycle has, in general, the following characteristics in common with the real cycle which it approximates:

1. A similar sequence of processes.
2. Same ratio of maximum to minimum volume for reciprocating engines or maximum to minimum pressure for gas turbine engines.
3. The same pressure and temperature at a given reference point.
4. An appropriate value of heat addition per unit mass of air.

1.4 The Carnot Cycle

This cycle was proposed by Sadi Carnot in 1824 and has the highest possible efficiency for any cycle. Figures 1 and 2 show the P-V and T-s diagrams of the cycle.

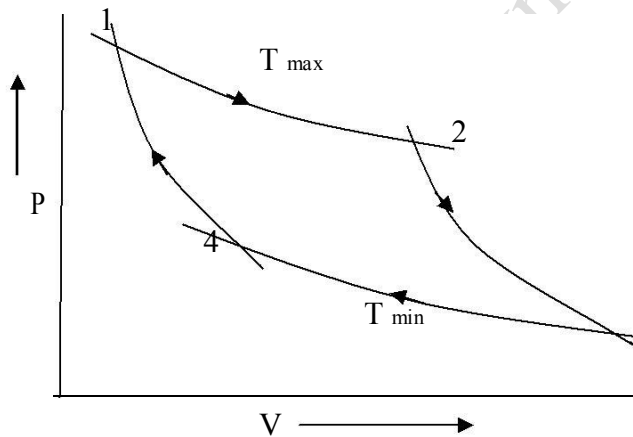


Fig.1: P-V Diagram of Carnot Cycle.

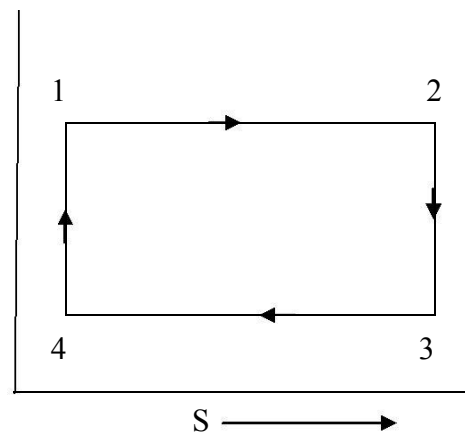


Fig.2: T-S Diagram of Carnot Cycle.

Assuming that the charge is introduced into the engine at point 1, it undergoes isentropic compression from 4 to 1. The temperature of the charge rises from T_{\min} to T_{\max} . At point 2, heat is added isothermally. This causes the air to expand, forcing the piston forward, thus doing work on the piston. At point 3, the source of heat is removed at constant temperature. At point 4, a



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cold body is applied to the end of the cylinder and the piston reverses, thus compressing the air isothermally; heat is rejected to the cold body. At point 1, the cold body is removed and the charge is compressed isentropically till it reaches a temperature T_{\max} once again. Thus, the heat addition and rejection processes are isothermal while the compression and expansion processes are isentropic.

From thermodynamics, per unit mass of charge

$$\text{Heat supplied from point 1 to 2} = p^2 v^2 \ln \frac{v_2}{v_1} \quad (5)$$

$$\text{Heat rejected from point 3 to 4} = p^3 v^3 \ln \frac{v_4}{v_3} \quad (6)$$

$$\text{Now } p_2 v_2 = RT_{\max} \quad (7)$$

$$\text{And } p_4 v_4 = RT_{\min} \quad (8)$$

Since Work done, per unit mass of charge, $W = \text{heat supplied} - \text{heat rejected}$

$$\begin{aligned} W &= RT_{\max} \ln \frac{v_3}{v_2} - RT_{\min} \ln \frac{v_1}{v_4} \\ &= R \ln(r)(T_{\max} - T_{\min}) \quad (9) \end{aligned}$$

We have assumed that the compression and expansion ratios are equal, that is



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$$\frac{v_3}{v_2} = \frac{v_1}{v_4} \quad (10)$$

$$\text{Heat supplied } Q_s = R T_{\max} \ln(r)$$

Hence, the thermal efficiency of the cycle is given by

$$\begin{aligned} \eta_{th} &= \frac{R \ln(r)(T_{\max} - T_{\min})}{R \ln(r) T_{\max}} \\ &= \frac{T_{\max} - T_{\min}}{T_{\max}} \end{aligned}$$

From Eq. 12 it is seen that the thermal efficiency of the Carnot cycle is only a function of the maximum and minimum temperatures of the cycle. The efficiency will increase if the minimum temperature (or the temperature at which the heat is rejected) is as low as possible.

According to this equation, the efficiency will be equal to 1 if the minimum temperature is zero, which happens to be the absolute zero temperature in the thermodynamic scale.

This equation also indicates that for optimum (Carnot) efficiency, the cycle (and hence the heat engine) must operate between the limits of the highest and lowest possible temperatures. In other words, the engine should take in all the heat at as high a temperature as possible and should reject the heat at as low a temperature as possible. For the first condition to be achieved, combustion (as applicable for a real engine using fuel to provide heat) should begin at the highest possible temperature, for then the irreversibility of the chemical reaction would be reduced. Moreover, in the cycle, the expansion should proceed to the lowest possible temperature in order to obtain the maximum amount of work. These conditions are the aims of all designers of modern heat engines. The conditions of heat rejection are governed, in practice, by the temperature of the atmosphere.

It is impossible to construct an engine which will work on the Carnot cycle. In such an engine, it would be necessary for the piston to move very slowly during the first part of the



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forward stroke so that it can follow an isothermal process. During the remainder of the forward stroke, the piston would need to move very quickly as it has to follow an isentropic process. This variation in the speed of the piston cannot be achieved in practice. Also, a very long piston stroke would produce only a small amount of work most of which would be absorbed by the friction of the moving parts of the engine.

Since the efficiency of the cycle, as given by Eq. 11, is dependent only on the maximum and minimum temperatures, it does not depend on the working medium. It is thus independent of the properties of the working medium.

1.5 The Otto Cycle

The Otto cycle, which was first proposed by a Frenchman, Beau de Rochas in 1862, was first used on an engine built by a German, Nicholas A. Otto, in 1876. The cycle is also called a *constant volume* or *explosion* cycle. This is the equivalent air cycle for reciprocating



piston engines using spark ignition. Figures 5 and 6 show the P-V and T-s diagrams respectively.

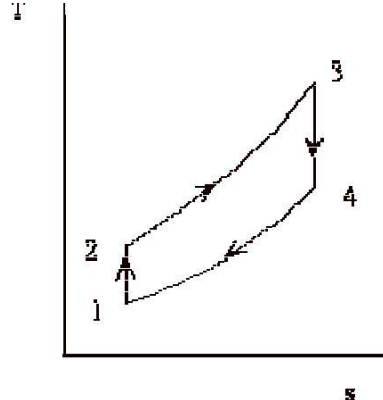
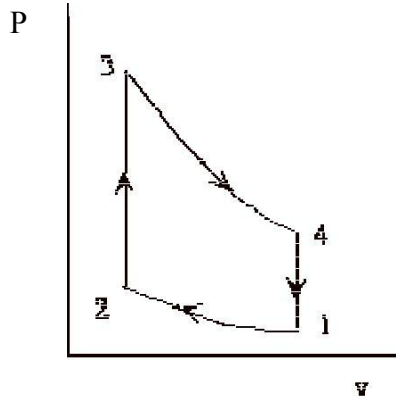


Fig.3: P-V Diagram of Otto Cycle.

Fig.4: T-S Diagram of Otto Cycle.

At the start of the cycle, the cylinder contains a mass M of air at the pressure and volume indicated at point 1. The piston is at its lowest position. It moves upward and the gas is compressed isentropically to point 2. At this point, heat is added at constant volume which raises the pressure to point 3. The high pressure charge now expands isentropically, pushing the piston down on its expansion stroke to point 4 where the charge rejects heat at constant volume to the initial state, point 1.

The isothermal heat addition and rejection of the Carnot cycle are replaced by the constant volume processes which are, theoretically more plausible, although in practice, even these processes are not practicable.

The heat supplied, Q_s , per unit mass of charge, is given by

$$c_v(T_3 - T_2) \quad (13)$$

the heat rejected, Q_r per unit mass of charge is given by

$$c_v(T_4 - T_1) \quad (14)$$

and the thermal efficiency is given by

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$



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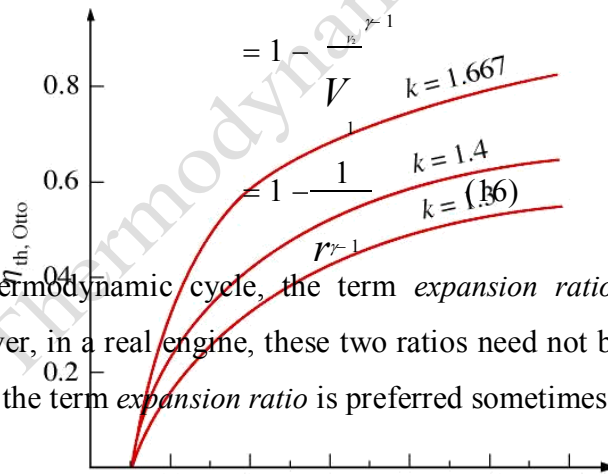
$$= 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} \quad (15)$$

Now $\frac{T_1}{T_2} = \frac{V_2^{\gamma-1}}{V_1^{\gamma-1}} = \frac{V_4^{\gamma-1}}{V_3^{\gamma-1}} = \frac{T_4}{T_3}$

And since $\frac{T_1}{T_2} = \frac{T_4}{T_3}$ we have $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

Hence, substituting in Eq. 15, we get, assuming that r is the compression ratio V_1/V_2

$$\eta_{th} = 1 - \frac{T_1}{T_2}$$



In a true thermodynamic cycle, the term *expansion ratio* and *compression ratio* are synonymous. However, in a real engine, these two ratios need not be equal because of the valve timing and therefore the term *expansion ratio* is preferred sometimes.

Equation 16 shows that the thermal efficiency of the theoretical Otto cycle increases with increase in compression ratio and specific heat ratio but is independent of the heat added (independent of load) and initial conditions of pressure, volume and temperature.

Figure 5 shows a plot of thermal efficiency versus compression ratio for an Otto cycle. It is seen that the increase in efficiency is significant at lower compression ratios. This is also seen in Table 1 given below.



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Fig.5: variation of efficiency with compression ratio

Applied Thermodynamics Notes, HIT, NDS



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Table 1: compression ratio and corresponding thermal efficiency for Otto cycle

R	η
1	0
2	0.242
3	0.356
4	0.426
5	0.475
6	0.512
7	0.541
8	0.565
9	0.585
10	0.602
16	0.67
20	0.698
50	0.791

From the table it is seen that if:

CR is increased from 2 to 4, efficiency increase is 76%

CR is increased from 4 to 8, efficiency increase is only 32.6%

CR is increased from 8 to 16, efficiency increase is only 18.6%

Mean effective pressure:

It is seen that the air standard efficiency of the Otto cycle depends only on the compression ratio. However, the pressures and temperatures at the various points in the cycle and the net work done, all depend upon the initial pressure and temperature and the heat input from point 2 to point 3, besides the compression ratio.

A quantity of special interest in reciprocating engine analysis is the mean effective pressure. Mathematically, it is the net work done on the piston, W , divided by the piston displacement volume, $V_1 - V_2$. This quantity has the units of pressure. Physically, it is that constant pressure which, if exerted on the piston for the whole outward stroke, would yield work equal to the work of the cycle. It is given by



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$$\begin{aligned}
 mep &= \frac{W}{V_1 - V_2} \\
 &= \frac{\eta Q_{2-3}}{V_1 - V_2} \quad (17)
 \end{aligned}$$

where Q_{2-3} is the heat added from points 2 to 3.

Work done per kg of air

$$\begin{aligned}
 W &= \frac{P_3 V_3 - P_4 V_4}{\nu - 1} - \frac{P_2 V_2 - P_1 V_1}{\nu - 1} = mep V_s = P_m (V_1 - V_2) \\
 mep &= \frac{1}{(V_1 - V_2)} \left[\frac{P_3 V_3 - P_4 V_4}{\nu - 1} - \frac{P_2 V_2 - P_1 V_1}{\nu - 1} \right] \quad (17A)
 \end{aligned}$$

The pressure ratio P_3/P_2 is known as explosion ratio r_p

$$\begin{aligned}
 \frac{P_2}{P_1} &= r^{\nu} \quad P_2 = P_1 r^{\nu} \\
 P_3 &= P_2 r_p^{\nu} = P_1 r^{\nu} r_p^{\nu} \\
 \frac{P_4}{P_1} &= \frac{r_p^{\nu}}{r^{\nu}} = P_1 r^{\nu} \frac{r_p^{\nu}}{r^{\nu}} = P_1 r_p^{\nu}
 \end{aligned}$$



$$\begin{aligned} V &= V_1 + V_2 \\ &= V_c + rV_2 \\ \therefore V_3 &= V_c(r-1) \end{aligned}$$

Substituting the above values in Eq 17A

$$mep = P_1 \frac{r(r_p-1)(r^{\gamma-1}-1)}{(r-1)(\gamma-1)}$$

Now

$$\begin{aligned} V_1 - V_2 &= V_1 \left(1 - \frac{V_2}{V_1}\right) \\ &= V_1 \left(1 - \frac{1}{r}\right) \end{aligned} \quad (18)$$

Here r is the compression ratio, V_1/V_2

From the equation of state:

$$V_1 = M \frac{R T_1}{m p_1} \quad (19)$$



R_0 is the universal gas constant

Substituting for V_1 from Eq. 3 in Eq. 2 and then substituting for $V_1 - V_2$ in Eq. 1 we get

$$mep = \eta \frac{Q_{2-3} \frac{p_1 m}{MR_0 T_1}}{1 - \frac{1}{r}} \quad (20)$$

The quantity Q_{2-3}/M is the heat added between points 2 and 3 *per unit mass* of air (M is the mass of air and m is the molecular weight of air); and is denoted by Q' , thus

$$mep = \eta \frac{Q' \frac{p_1 m}{RT_1}}{1 - \frac{1}{r}} \quad (21)$$

We can non-dimensionalize the mep by dividing it by p_1 so that we can obtain the following equation

$$\frac{mep}{p_1} = \eta \frac{Q'}{m} \frac{1}{1 - \frac{1}{r}} \frac{T_1}{T_1} \quad (22)$$

Since $\frac{R_0}{m} = c_v(\gamma - 1)$, we can substitute it in Eq. 25 to get

$$\frac{mep}{p_1} = \frac{Q'}{c_v(\gamma - 1) m} \frac{1}{1 - \frac{1}{r}} \quad (23)$$

The dimensionless quantity mep/p_1 is a function of the heat added, initial temperature, compression ratio and the properties of air, namely,



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c_v and γ . We see that the mean effective pressure is directly proportional to the heat added and inversely proportional to the initial (or ambient) temperature.

We can substitute the value of Q' from Eq. 20 in Eq. 26 and obtain the value of mep/p_1 for the Otto cycle in terms of the compression ratio and heat added.

In terms of the pressure ratio, p_3/p_2 denoted by r_p we could obtain the value of mep/p_1 as follows:

$$\frac{mep}{p_1} = \frac{r(r_p-1)(r^{\gamma-1}-1)}{(r-1)(\gamma-1)}$$

We can obtain a value of r_p in terms of Q' as follows:

$$r_p = \frac{Q'}{c_v T_1 r^{\gamma-1}} + 1$$



Choice of Q'

We have said that

$$Q' = M \quad (26)$$

M is the mass of charge (air) per cycle, kg.

Now, in an actual engine

$$\begin{aligned} Q_{2-3} &= M Q_{f \ c} \\ &= F M_a Q_c \text{ in kJ / cycle} \end{aligned} \quad (27)$$

M_f is the mass of fuel supplied per cycle, kg

Q_c is the heating value of the fuel, kJ/kg

M_a is the mass of air taken in per cycle

F is the fuel air ratio = M_f/M_a

Substituting for Eq. (B) in Eq. (A) we get

$$Q' = \frac{F M_a Q_c}{M} \quad (28)$$

$$\text{Now } \frac{M_a}{M} \approx \frac{V_1 - V_2}{V_1}$$

$$\text{And } \frac{V_1 - V_2}{V_1} = 1 - \frac{1}{r} \quad (29)$$

So, substituting for M_a/M from Eq. (33) in Eq. (32) we get

$$Q' = F Q_c \left(1 - \frac{1}{r} \right) \quad (30)$$

For isooctane, $F Q_c$ at stoichiometric conditions is equal to 2975 kJ/kg, thus



$$Q' = 2975(r - 1)/r \quad (31)$$

At an ambient temperature, T_1 of 300K and c_v for air is assumed to be 0.718 kJ/kgK, we get a value of $Q'/c_v T_1 = 13.8(r - 1)/r$.

Under fuel rich conditions, $\gamma = 1.2$, $Q'/c_v T_1 = 16.6(r - 1)/r$. (32)

Under fuel lean conditions, $\gamma = 0.8$, $Q'/c_v T_1 = 11.1(r - 1)/r$ (33)

1.6 The Diesel Cycle

This cycle, proposed by a German engineer, Dr. Rudolph Diesel to describe the processes of his engine, is also called the constant pressure cycle. This is believed to be the equivalent air cycle for the reciprocating slow speed compression ignition engine. The P-V and T-s diagrams are shown in Figs 6 and 7 respectively.

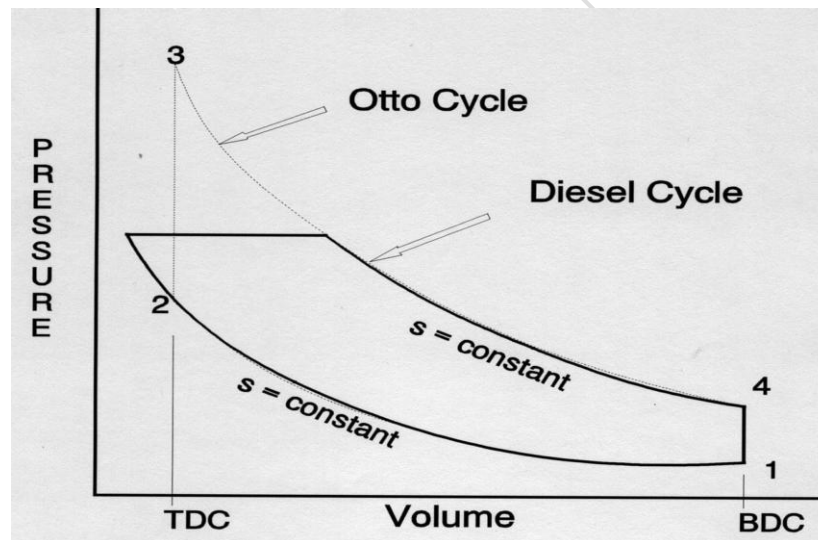


Fig.6: P-V Diagram of Diesel Cycle.

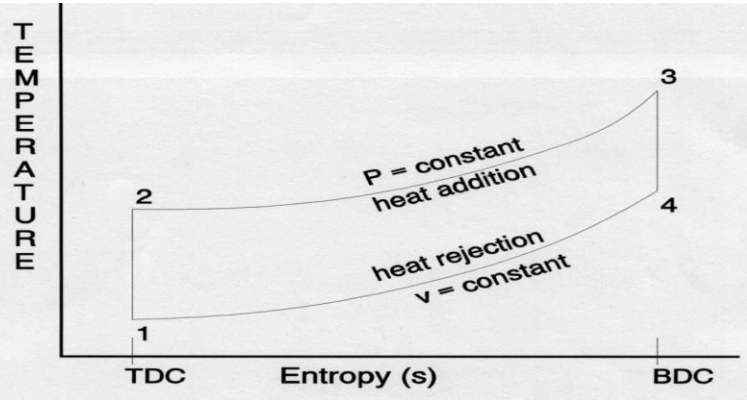


Fig.7: T-S Diagram of Diesel Cycle.

The cycle has processes which are the same as that of the Otto cycle except that the heat is added at constant pressure.

The heat supplied, Q_s is given by

$$c_p(T_3 - T_2) \quad (34)$$

whereas the heat rejected, Q_r is given by

$$c_v(T_4 - T_1) \quad (35)$$

and the thermal efficiency is given by

$$\eta_{th} = 1 - \frac{c_v (T_4 - T_1)}{c_p (T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma} \frac{T_4^{\frac{\gamma-1}{\gamma}} T_1^{\frac{1-\gamma}{\gamma}}}{T_3^{\frac{\gamma-1}{\gamma}} T_2^{\frac{1-\gamma}{\gamma}}} \quad (36)$$



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From the T-s diagram, Fig. 7, the difference in enthalpy between points 2 and 3 is the same as that between 4 and 1, thus

$$\begin{aligned} \Delta s_{2-3} &= \Delta s_{4-1} \\ \therefore c_v \ln \frac{T_3}{T_2} &= c_p \ln \frac{T_4}{T_1} \\ \therefore \ln \frac{T_3}{T_2} &= \gamma \ln \frac{T_4}{T_1} \\ \therefore \frac{T_3}{T_2} &= \left(\frac{T_4}{T_1} \right)^\gamma \quad \text{and} \quad \frac{T_4}{T_2} = \frac{V_2^{\gamma-1}}{V_1^{\gamma-1}} = \frac{1}{r} \end{aligned}$$

Substituting in eq. 36, we get

$$\eta_{th} = 1 - \frac{1}{\gamma r} \frac{T_2^{\gamma-1} T_3}{T_2^{\gamma-1} T_4} \quad (37)$$

$$\text{Now } \frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c = \text{cut-off ratio}$$

$$\frac{1}{r} = \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}}$$

$$\eta = 1 - r^{1-\gamma} \gamma (r_c - 1) \quad (38)$$

When Eq. 38 is compared with Eq. 20, it is seen that the expressions are similar except for the term in the parentheses for the Diesel cycle. It can be shown that this term is always greater than unity.

$$\text{Now } r_c = \frac{V_3}{V_2} = \frac{V_3}{V_4} \frac{V_4}{V_2} = \frac{V_3}{V_1} \frac{V_2}{V_1} = r \quad \text{where } r \text{ is the compression ratio and } r_c \text{ is the expansion ratio}$$

Thus, the thermal efficiency of the Diesel cycle can be written as



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$$\eta = 1 - \frac{1 - \left(\frac{r_e}{r}\right)^\gamma}{\gamma \left(\frac{r_e}{r}\right)^\gamma} \quad (39)$$

Let $r_e = r - \Delta$ since r is greater than r_e . Here, Δ is a small quantity. We therefore have

$$\frac{r_e}{r} = \frac{r - \Delta}{r} = 1 - \frac{\Delta}{r}$$

We can expand the last term binomially so that

$$\left(1 - \frac{\Delta}{r}\right)^{-1} = 1 + \frac{\Delta}{r} + \frac{\Delta^2}{r^2} + \frac{\Delta^3}{r^3} + \dots$$



$$\text{Also } \frac{r}{r_e}^\gamma = \frac{r^\gamma}{(r-\Delta)^\gamma} = \frac{r^\gamma}{r^\gamma \left(1 - \frac{\Delta}{r}\right)^\gamma} = 1 - \frac{\Delta}{r}^{-\gamma}$$

We can expand the last term binomially so that

$$1 - \frac{\Delta}{r}^{-\gamma} = 1 + \gamma \frac{\Delta}{r} + \frac{\gamma(\gamma+1)\Delta^2}{2! r^2} + \frac{\gamma(\gamma+1)(\gamma+2)\Delta^3}{3! r^3}$$

Substituting in Eq. 39, we get

$$\eta = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{\Delta}{r} + \frac{(\gamma+1)\Delta^2}{2! r^2} + \frac{(\gamma+1)(\gamma+2)\Delta^3}{3! r^3} \right] +$$

+

(40)



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Since the coefficients of $\frac{\Delta}{r}$, $\frac{\Delta^2}{r^2}$, $\frac{\Delta^3}{r^3}$, etc are greater than unity, the quantity in the brackets in Eq. 40 will be greater than unity. Hence, for the Diesel cycle, we subtract $\frac{1}{r^{\gamma-1}}$ times a quantity greater than unity from one, hence for the same r , the Otto cycle efficiency is greater than that for a Diesel cycle.

If $\frac{\Delta}{r}$ is small, the square, cube, etc of this quantity becomes progressively smaller, so the thermal efficiency of the Diesel cycle will tend towards that of the Otto cycle.

From the foregoing we can see the importance of cutting off the fuel supply early in the forward stroke, a condition which, because of the short time available and the high pressures involved, introduces practical difficulties with high speed engines and necessitates very rigid fuel injection gear.

In practice, the diesel engine shows a better efficiency than the Otto cycle engine because the compression of air alone in the former allows a greater compression ratio to be employed. With a mixture of fuel and air, as in practical Otto cycle engines, the maximum temperature developed by compression must not exceed the self ignition temperature of the mixture; hence a definite limit is imposed on the maximum value of the compression ratio.

Thus Otto cycle engines have compression ratios in the range of 7 to 12 while diesel cycle engines have compression ratios in the range of 16 to 22.

$$mep = \frac{1}{V_s} r_2 (V_3 - V_2) + \frac{P V_3 - P V_4}{\nu - 1} - \frac{P V_2 - P V_1}{\nu - 1} \quad (42)$$

The pressure ratio P_3/P_2 is known as explosion ratio r_p

$$\frac{P_2}{P_1} = r^{\nu} \quad P_2 = P_1 r^{\nu}$$

$$P_3 = P_2 r_p^{\nu} = P_1 r^{\nu} r_p^{\nu}$$

$$P_4 = P_3 \frac{V_3}{V_4} = P_1 r^{\nu} \frac{r_p^{\nu}}{r} = P_1 r_c^{\nu}$$



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$$V_4 = V_1, V_2 = V_c,$$

$$\frac{V_1 + V_2}{V_c} = r = r V_2$$

$$\therefore V_3 = V_c(r - 1)$$

Substituting the above values in Eq 42 to get Eq (42A)

In terms of the cut-off ratio, we can obtain another expression for mep/p_1 as follows

$$mep = P_1 \frac{\gamma r^\gamma (r_c - 1) - r (r_c^\gamma - 1)}{(r - 1)(\gamma - 1)} \quad (42 A)$$

We can obtain a value of r_c for a Diesel cycle in terms of Q' as follows:

$$r_c = \frac{Q'}{c_p T_1 r^{\gamma-1}} + 1 \quad (41)$$

We can substitute the value of r_c from Eq. 38 in Eq. 26, reproduced below and obtain the value of mep/p_1 for the Diesel cycle.



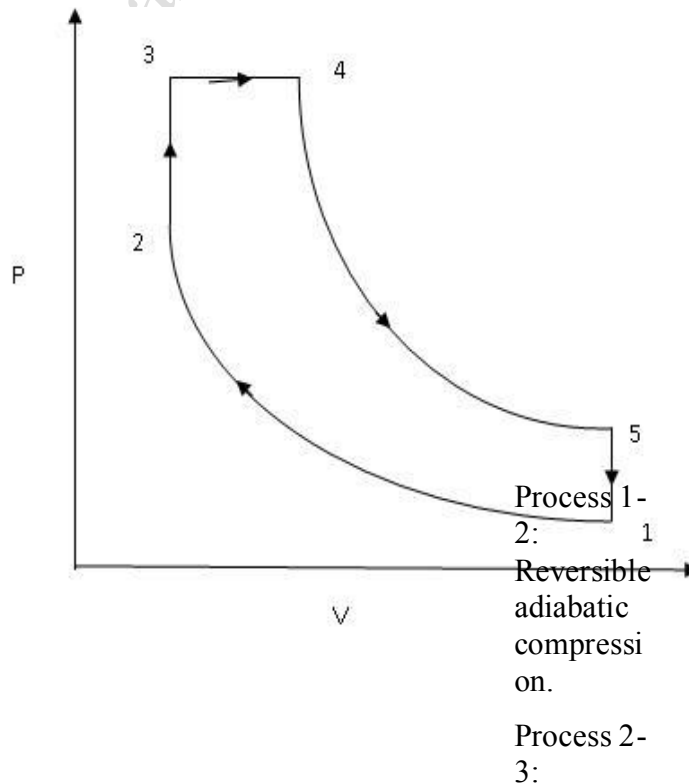
$$\frac{mep}{p_1} = \eta \frac{Q'}{c_v T_1} \frac{1}{1 - \frac{1}{r} [\gamma - 1]}$$

For the Diesel cycle, the expression for mep/p_3 is as follows:

$$\frac{mep}{p_3} = \frac{mep}{p_1} \frac{1}{r^\gamma} \quad (43)$$

Modern high speed diesel engines do not follow the Diesel cycle. The process of heat addition is partly at constant volume and partly at constant pressure. This brings us to the dual cycle.

1.7 The Dual Cycle





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Constant volume heat addition.

Process 3-4: Constant pressure heat addition.

Process 4-5: Reversible adiabatic expansion.

Process 5-1: Constant volume heat reject

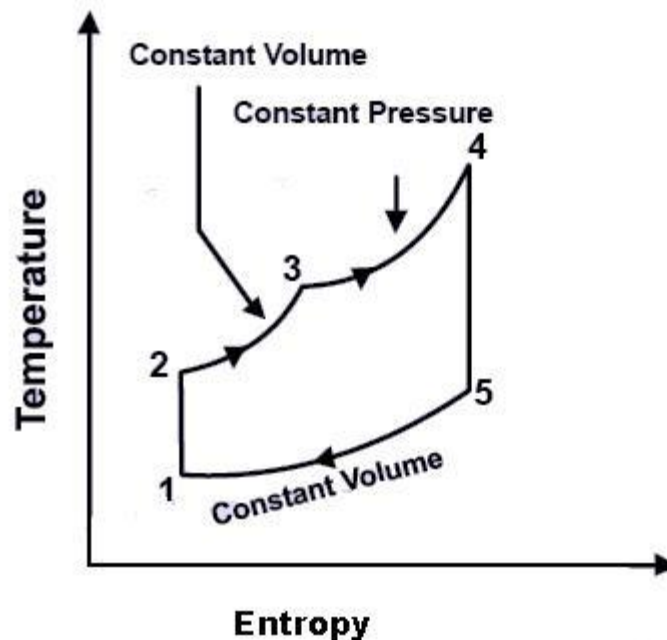


Fig.9: T-S Diagram of Carnot Cycle.

An important characteristic of real cycles is the ratio of the mean effective pressure to the maximum pressure, since the mean effective pressure represents the useful (average) pressure acting on the piston while the maximum pressure represents the pressure which chiefly affects the strength required of the engine structure. In the constant-volume cycle, shown in Fig. 8,

it is seen that the quantity mep/p_3 falls off



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rapidly as the compression ratio increases, which means that for a given mean effective pressure the maximum pressure rises rapidly as the compression ratio increases. For example, for a mean effective pressure of 7 bar and $Q'/c_v T_1$ of 12, the maximum pressure at a compression ratio of 5 is 28 bar whereas at a compression ratio of 10, it rises to about 52 bar. Real cycles follow the same trend and it becomes a practical necessity to limit the maximum pressure when high compression ratios are used, as in diesel engines. This also indicates that diesel engines will have to be stronger (and hence heavier) because it has to withstand higher peak pressures.

Constant pressure heat addition achieves rather low peak pressures unless the compression ratio is quite high. In a real diesel engine, in order that combustion takes place at constant pressure, fuel has to be injected very late in the compression stroke (practically at the top dead center). But in order to increase the efficiency of the cycle, the fuel supply must be cut off early in the expansion stroke, both to give sufficient time for the fuel to burn and thereby increase combustion efficiency and reduce after burning but also reduce emissions. Such situations can be achieved if the engine was a slow speed type so that the piston would move sufficiently slowly for combustion to take place despite the late injection of the fuel. For modern high speed compression ignition engines it is not possible to achieve constant pressure combustion. Fuel is

inj
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somewhat
earlier in
the
compressi
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and has to go through the various stages of combustion. Thus it is seen that combustion is nearly at constant volume (like in a spark ignition engine). But the peak pressure is limited because of strength considerations so the rest of the heat addition is believed to take place at constant pressure in a cycle. This has led to the formulation of the dual combustion cycle. In this cycle, for high compression ratios, the peak pressure is not allowed to increase beyond a certain limit and to account for the total addition, the rest of the heat is assumed to be added at constant pressure. Hence the name *limited pressure cycle*.

The cycle is the equivalent air cycle for reciprocating high speed compression ignition engines. The P-V and T-s diagrams are shown in Figs.8 and 9. In the cycle, compression and expansion processes are isentropic; heat addition is partly at constant volume and partly at constant pressure while heat rejection is at constant volume as in the case of the Otto and Diesel cycles.

The heat supplied, Q_s per unit mass of charge is given by

$$c_v(T_3 - T_2) + c_p(T_3' - T_2) \quad (44)$$

whereas the heat rejected, Q_r per unit mass of charge is given by

$$c_v(T_4 - T_1)$$

and the thermal efficiency is given by

$$\eta_{th} = 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3 - T_2) + c_p(T_3' - T_2)}$$



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$$= 1 - \frac{\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} - 1}{\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} - 1 + \gamma T_2 \left(\frac{T_3}{T_2}\right)^{\frac{\gamma}{\gamma-1}} - 1}$$

$$= 1 - \frac{\frac{T_4}{T_1} - 1}{\frac{T_2}{T_1} \frac{T_3}{T_2} - 1 + \frac{\gamma T_3}{T_2} \frac{T_3}{T_1} - 1}$$

From thermodynamics

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = r_p \quad (45)$$

the explosion or pressure ratio

and



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$$\frac{T_3}{T_4} = \frac{V_3}{V_4} = r_c \quad (46)$$

the cut-off ratio.

$$\text{Now, } \frac{T_4}{T_1} = \frac{p_4}{p_1} = \frac{p_4}{p_3} \frac{p_3}{p_2} \frac{p_2}{p_1}$$

$$\text{Also } \frac{p_4}{p_3} = \frac{V_3}{V_4} = \frac{V_3}{V_2} \frac{V_2}{V_1} = \frac{1}{r_c}$$

$$\text{And } p_2 = p_1 r_c^\gamma$$

$$\text{Thus } \frac{T_4}{T_1} = r_p r_c^\gamma$$

$$\text{Also } \frac{T_2}{T_1} = \frac{V_1}{V_2} = r_p^{\gamma-1}$$

Therefore, the thermal efficiency of the dual cycle is

$$\eta = 1 - \frac{1}{r_p r_c^\gamma} \frac{r_p r_c^\gamma - 1}{(r_p - 1) + \gamma r_p (r_c - 1)} \quad (46)$$

We can substitute the value of from Eq. 46 in Eq. 26 and obtain the value of mep/p_1 for the dual cycle.

In terms of the cut-off ratio and pressure ratio, we can obtain another expression for mep/p_1 as follows:

$$\frac{mep}{p_1} = \frac{\gamma r_p r_c^\gamma (r_c - 1) + r^\gamma (r_p - 1) - (r_p r_c^\gamma - 1)}{(r - 1)(\gamma - 1)} \quad (47)$$

For the dual cycle, the expression for mep/p_3 is as follows:



$$\frac{mep}{p_3} = \frac{mep}{p_1} \frac{p_1}{p_3} \quad (48)$$

Since the dual cycle is also called the limited pressure cycle, the peak pressure, p_3 , is usually specified. Since the initial pressure, p_1 , is known, the ratio p_3/p_1 is known. We can correlate r_p with this ratio as follows:

$$r_p = \frac{p_3}{p_1} = \frac{1}{r^\gamma} \quad (49)$$

We can obtain an expression for r_c in terms of Q' and r_p and other known quantities as follows:

$$r_c = \frac{1}{\gamma} \frac{Q'}{c_v T_1 r^{(\gamma-1)/\gamma}} \frac{1}{r_p} \quad (50)$$

We can also obtain an expression for r_p in terms of Q' and r_c and other known quantities as follows:

$$r_p = \frac{\frac{Q'}{c_v T_1 r^{(\gamma-1)/\gamma}} + 1}{1 + \gamma} r_c^{-\gamma} \quad (51)$$

1.8 Stirling cycle

When a confined body of gas (air, helium, whatever) is heated, its pressure rises. This increased pressure can push on a piston and do work. The body of gas is then cooled, pressure drops, and the piston can return. The same cycle repeats over and over, using the same body of gas. That is all there is to it. No ignition, no carburetion, no valve train, no explosions. Many people have a hard time understanding the Stirling because it is so much simpler than conventional internal combustion engines.

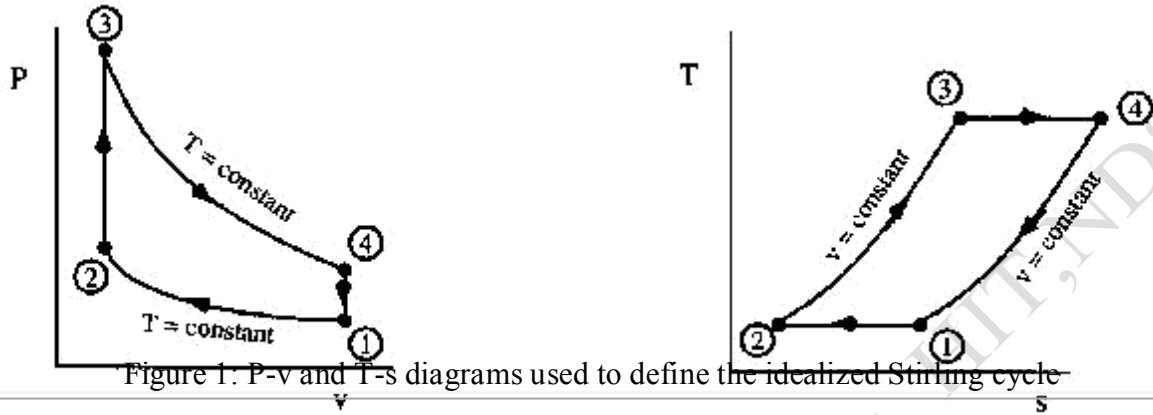
The Stirling cycle is described using the pressure-volume (P-v) and temperature-entropy (T-s)



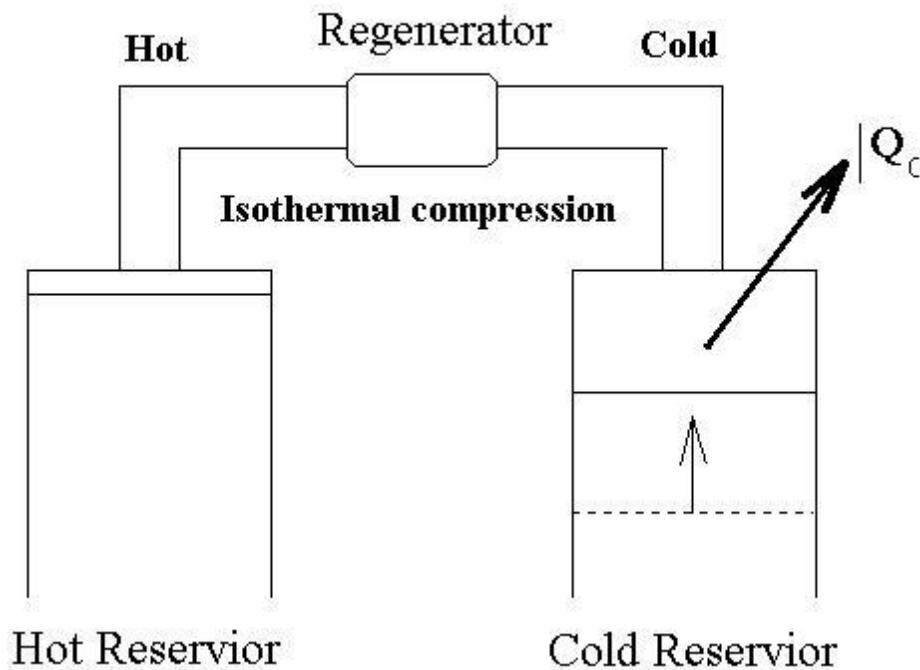
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diagrams shown in Figure 1. The P-v and T-s diagrams show the state of a "working fluid" at any point during the idealized cycle. The working fluid is normally a gas...in the Stirling engines being produced to us, the working fluid is air.

In the idealized Stirling cycle heat (i.e., energy) is transferred to the working fluid during the segment 2-3-4. Conversely, heat (energy) is extracted from the working fluid during the segment 4-1-2. During segment 2-3 heat is transferred to the fluid internally via regeneration of the energy transferred from the fluid during segment 4-1. This means that (ideally) heat is added from an external source only during segment 3-4, and that heat is rejected to the surrounding environment only during segment 1-2. Note that this is the idealized cycle.



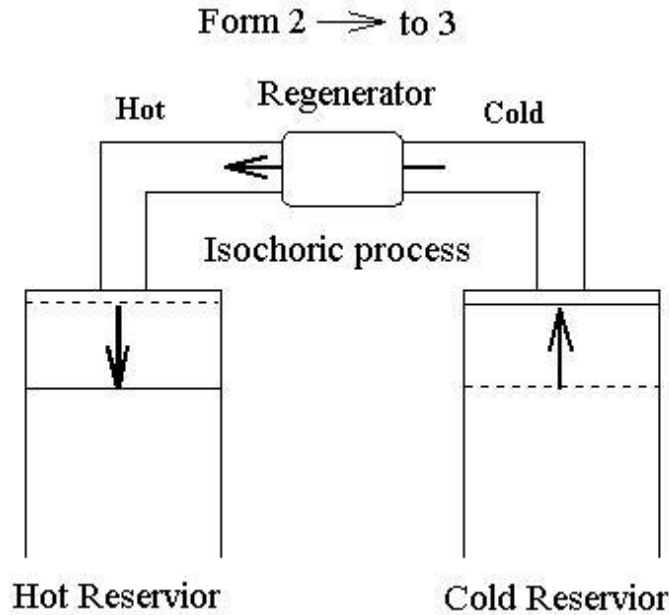
Form 1 \rightarrow to 2



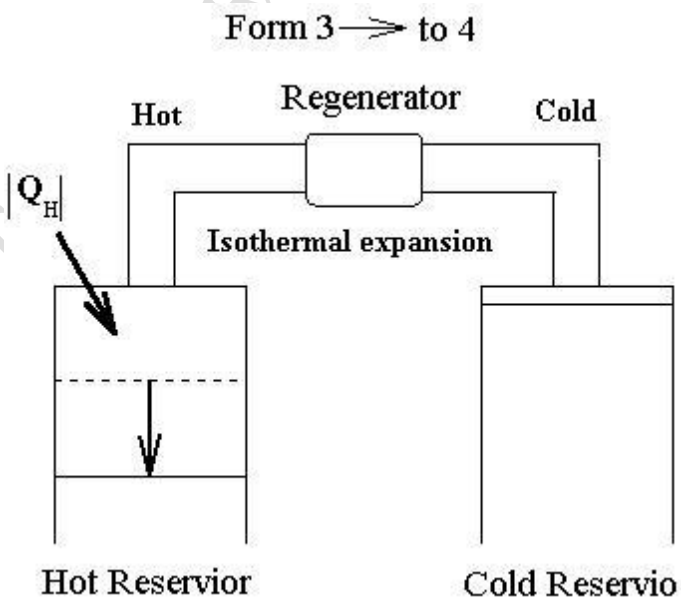
1 2 It is an isothermal process, the piston in contact with cold reservoir is compressed isothermally, hence heat $|Q_c|$ has been rejected, and (isothermal compression $dU = 0$, W is positive and Q_c is negative) the heat rejected is



$$Q_c = P_1 V_1 \ln \frac{V_1}{V_2} = RT_c \ln(r_c) \quad 52$$



2 3 It is an isochoric process, the left piston moves down while the right piston moves up. The volume of system is kept constant, thus no work has been done by the system, but heat Q_R has been input to the system by the regenerator which causes temperature to raise to θ_1 .



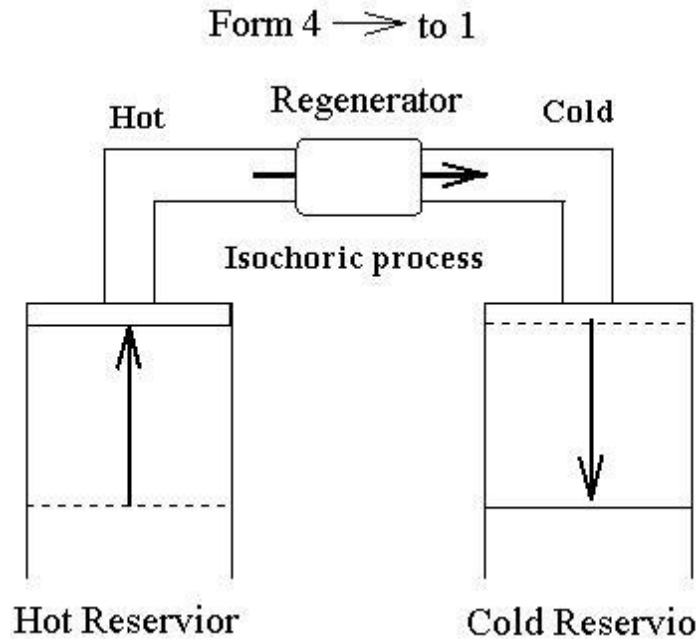
3 4 It is an isothermal expansion process, the left piston in contact with hot reservoir



expanded isothermally at temperature θ_H . Therefore

$$Q_H = P_3 V_3 \ln \frac{V_4}{V_3} = RT_H \ln(r_e)$$

V_3



4 1 It is an isochoric process which is a reversed process of 2 3, but from θ_H to θ_C . The efficiencies of Stirling engine is

$$\eta = 1 - \frac{Q_C}{Q_H} = 1 - \frac{RT_C \ln(r_c)}{RT_H \ln(r_e)} = 1 - \frac{T_C}{T_H}$$

$$\eta_{\text{carnot}} = \eta_{\text{stirling}}$$

Consider regenerator efficiency η_r

$$Q_H = RT_H \ln(r_c) + (1 - \eta_r)(C_v [T_H - T_C])$$

$$Q_C = RT_C \ln(r_c) + (1 - \eta_r)(C_v [T_H - T_C])$$

$$\eta_c = \frac{R \ln(r_c)(T_H - T_C)}{RT_C \ln(r_c) + (1 - \eta_r)C_v(T_H - T_C)} \quad \text{if } \eta_r = 1$$



$$\eta_{st} = \frac{T_H - T_c}{T_H}$$

1.8 Comparison of Otto, Dual and Diesel cycles

In the previous articles we studied about Otto cycle, diesel cycle and dual cycle and looked at their thermal efficiency. In this article we will take a collective look at these three cycles in order to compare and contrast them, so that we can come to know the relative advantages and disadvantages of these cycles.

1.8.1 Comparison based on same maximum pressure and heat rejection

In this article we will focus on **peak pressure, peak temperature and heat rejection**. The P-V and T-S diagrams of these three cycles for such a situation are drawn simultaneously as described below.

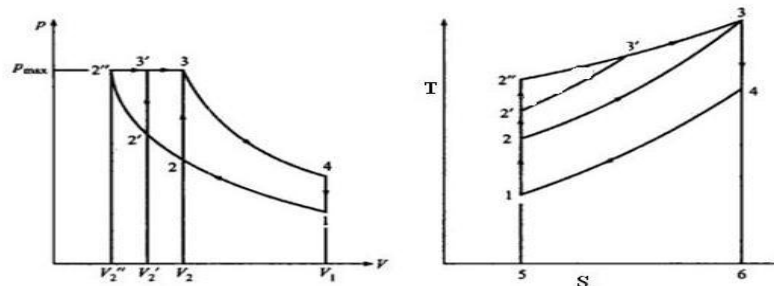


Figure 10 P- V and T- S diagram showing the comparison of Otto, Diesel and Dual cycles



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In the above diagrams the following are the cycles

- Otto cycle: 1 – 2 – 3 – 4 – 1
- Dual cycle: 1 – 2' – 3' – 3 – 4 – 1
- Diesel cycle: 1 – 2'' – 3 – 4 – 1

Remember that we are assuming the same peak pressure denoted by P_{max} on the P-V diagram. And from the T-S diagram we know that T_3 is the highest of the peak temperature which is again same for all three cycles under consideration. Heat rejection given by the area under 4 – 1 – 5 – 6 in the T-S diagram is also same for each case.

In this case the compression ratio is different for each cycle and can be found by dividing V_1 with the respective V_2 volumes of each cycle from the P-V diagram. The heat supplied or added in each cycle is given by the areas as follows from the T-S diagram

- Otto cycle: Area under 2 – 3 – 6 – 5 say q_1
- Dual cycle: Area under 2' – 2' – 3 – 6 – 5 say q_2
- Diesel cycle: Area under 2'' – 3 – 6 – 5 say q_3

It can also be seen from the same diagram that $q_3 > q_2 > q_1$

We know that thermal efficiency is given by $1 - \text{heat rejected/heat supplied}$

Since heat rejected is

Thermal efficiency of these engines under given circumstances is of the following order

Diesel > Dual > Otto

Hence in this case it is the diesel cycle which shows greater thermal efficiency.

1.8.2 Comparison based on same compression ratio and heat rejection



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In this article we will focus on constant compression ratio and constant heat rejection. The P-V and T-S diagrams of these three cycles for such a situation are drawn simultaneously as described below.

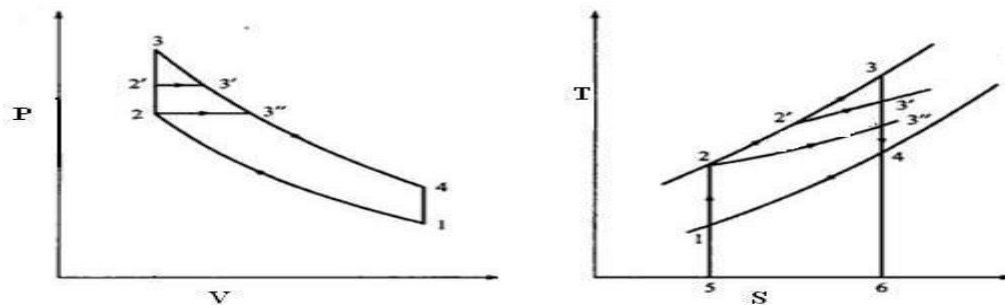


Figure 20 P- V and T- S diagram showing the comparison of Otto, Diesel and Dual cycles

In the above diagrams the following are the cycles

- Otto cycle: 1 – 2 – 3 – 4 – 1
- Dual cycle: 1 – 2 – 2' – 3' – 4 – 1
- Diesel cycle: 1 – 2 – 3'' – 4 – 1

Remember that we are assuming constant compression ratio for all three cycles which is given by V_1/V_2

The other parameter which is constant is the heat rejected from the cycle which is given by the following in each case as per the T-S diagram

All cycles: Area under 4 – 1 – 5 – 6 in the T-S diagram

The heat supplied is different in each case and can be established from the T-S diagram as follows



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- Otto cycle: Area under 2 – 3 – 6 – 5 say q_1
- Dual cycle: Area under 2 – 2' – 3' – 6 – 5 say q_2
- Diesel cycle: Area under 2 – 3'' – 6 – 5 say q_3

It can also be seen from the same diagram that $q_3 < q_2 < q_1$

We know that thermal efficiency is given by $1 - \text{heat rejected/heat supplied}$

Since heat rejected is same and we know the order of magnitude of heat supplied, we can combine this information to conclude that

Thermal efficiency

Otto > Dual > Diesel

Hence we see that in this case as well the Otto cycle shows higher thermal efficiency than a dual cycle and even better than the diesel cycle



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Unit-3

Testing of I.C.Engines

1.1. Introduction: - The basic task in the design and development of I.C.Engines is to reduce the cost of production and improve the efficiency and power output. In order to achieve the above task, the engineer has to compare the engine developed by him with other engines in terms of its output and efficiency. Hence he has to test the engine and make measurements of relevant parameters that reflect the performance of the engine. In general the nature and number of tests to be carried out depend on a large number of factors. In this chapter only certain basic as well as important measurements and tests are described.

1.2. Important Performance Parameters of I.C.Engines:- The important performance parameters of I.C. engines are as follows:

- (i) Friction Power,
- (ii) Indicated Power,
- (iii) Brake Power,
- (iv) Specific Fuel Consumption,
- (v) Air – Fuel ratio
- (vi) Thermal Efficiency
- (vii) Mechanical Efficiency,
- (viii) Volumetric Efficiency,
- (ix) Exhaust gas emissions,
- (x) Noise

1.3. Measurement of Performance Parameters in a Laboratory

1.3.1. Measurement of Friction Power:- Friction power includes the frictional losses and the pumping losses. During suction and exhaust strokes the piston must move against a gaseous pressure and power required to do this is called the “pumping losses”. The friction



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loss is made up of the energy loss due to friction between the piston and cylinder walls, piston rings and cylinder walls, and between the crank shaft and camshaft and their bearings, as well as by the loss incurred by driving the essential accessories, such as water pump, ignition unit etc.

Following methods are used in the laboratory to measure friction power:

- (i) Willan's line method;
- (ii) From the measurement of indicated power and brake power;
- (iii) Motoring test;
- (iv) Retardation test;
- (v) Morse Test.



1.3.1.1. Willan's Line Method:- This method is also known as fuel rate extrapolation method. In this method a graph of fuel consumption (vertical axis) versus brake power (horizontal axis) is drawn and it is extrapolated on the negative axis of brake power (see Fig. 1). The intercept of the negative axis is taken as the friction power of the engine at

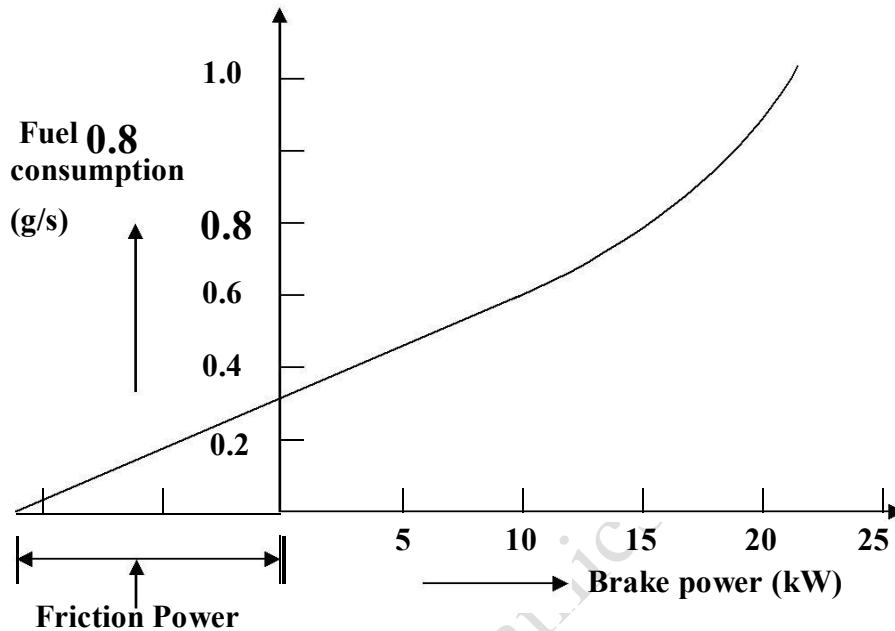


Figure.1 Willan's line method

that speed. As shown in the figure, in most of the power range the relation between the fuel consumption and brake power is linear when speed of the engine is held constant and this permits extrapolation. Further when the engine does not develop power, i.e. brake power = 0, it consumes a certain amount of fuel. This energy in the fuel would have been spent in overcoming the friction. Hence the extrapolated negative intercept of the horizontal axis will be the work representing the combined losses due to friction, pumping and as a whole is termed as the frictional loss of the engine. This method of measuring friction power will hold good only for a particular speed and is applicable mainly for compression ignition engines.

The main draw back of this method is the long distance to be extrapolated from data between 5 and 40 % load towards the zero line of the fuel input. The directional margin of error is rather wide because the graph is not exactly linear.

1.3.1.2. From the Measurement of Indicated Power and Brake Power:- This is an ideal method by which friction power is obtained by computing the difference between the indicated power and brake power. The indicated power is obtained from an indicator diagram and brake power is obtained by a brake dynamometer. This method requires

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elaborate equipment to obtain accurate indicator diagrams at high speeds.

1.3.1.3. Morse Test:- This method can be used only for multi – cylinder IC engines. The Morse test consists of obtaining indicated power of the engine without any elaborate equipment. The test consists of making, in turn, each cylinder of the engine inoperative and noting the reduction in brake power developed. In a petrol engine (gasoline engine), each cylinder is rendered inoperative by “shorting” the spark plug of the cylinder to be made inoperative. In a Diesel engine, a particular cylinder is made inoperative by cutting off the supply of fuel. It is assumed that pumping and friction are the same when the cylinder is inoperative as well as during firing.

In this test, the engine is first run at the required speed and the brake power is measured. Next, one cylinder is cut off by short circuiting the spark plug if it is a petrol engine or by cutting of the fuel supply if it is a diesel engine. Since one of the cylinders is cut of from producing power, the speed of the engine will change. The engine speed is brought to its original value by reducing the load on the engine. This will ensure that the frictional power is the same.

If there are k cylinders, then

Total indicated power
when all the cylinders are working = $ip_1 + ip_2 + ip_3 + \dots + ip_k = \sum_{j=1}^k ip_j$

We can write $\sum_{j=1}^k ip_j = B_t + F_t \dots \dots \dots (1)$

where ip_j is the indicated power produced by j th cylinder, k is the number of cylinders,

B_t is the total brake power when all the cylinders are producing power and F_t is the total frictional power for the entire engine.

If the first cylinder is cut – off, then it will not produce any power, but it will have frictional losses. Then

we can write $\sum_{j=2}^k ip_j = B_1 - F_t \dots \dots \dots (2)$

where B_1 = total brake power when cylinder 1 is cut - off and

F_t = Total frictional power.

Subtracting Eq. (2) from Eq. (1) we have the indicated power of the cut off cylinder. Thus



$$ip_1 = B_t - B_1 \dots\dots\dots(3).$$

Similarly we can find the indicated power of all the cylinders, viz., ip_2, ip_3, \dots, ip_k .
 Then the total indicated power is calculated as

$$(ip)_{total} = \sum_{j=1}^k ip_j \dots\dots\dots(4)$$

The frictional power of the engine is therefore given by

$$F_t = (ip)_{total} - B_t \dots\dots\dots(5)$$

The procedure is illustrated by some examples worked out at the end of the chapter.

1.4. MEASUREMENT OF INDICATED POWER

The power developed in the cylinder is known as Indicated Horse Power and is designated as IP.

The IP of an engine at a particular running condition is obtained from the indicator diagram. The indicator diagram is the $p-v$ diagram for one cycle at that load drawn with the help of indicator fitted on the engine. The construction and use of mechanical indicator for obtaining $p-v$ diagram is already explained.

A typical $p-v$ diagram taken by a mechanical indicator is shown in Figure 2.

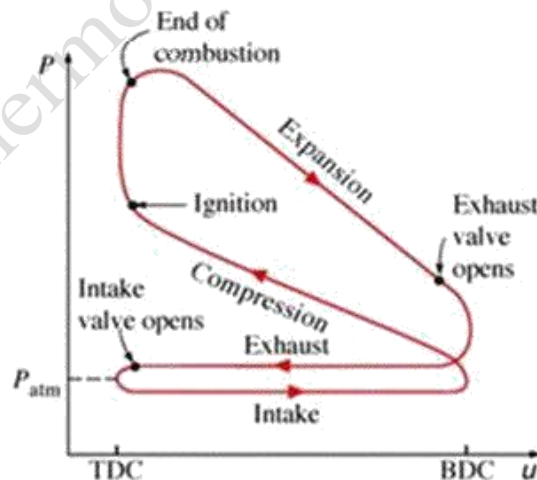


Figure.2 $p-v$ diagram taken by mechanical indicator

The areas, the positive loop and negative loop, are measured with the help of a planimeter
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and let these be A_p and A_n cm² respectively, the net positive area is $(A_p - A_n)$. Let the actual length of the diagram as measured be L cm, then the average height of the net positive area is given by

$$h = (A_p - A_n) / L \quad \text{in centimetre}$$

The height multiplied by spring-strength (or spring number) gives the indicated mean effective pressure of the cycle.

$$I_{mep} = (A_p - A_n) * S / L \quad \dots\dots(6)$$

Where S is spring scale and it is defined as a force per unit area required to compress the spring through a height of one centimeter (N/m²/cm).

Generally the area of negative loop A_n is negligible compared with the positive loop and it cannot be easily measured especially when it is taken with the spring used for taking positive loop. Special light springs are used to obtain the negative loop. When two different springs are used for taking the $p-v$ diagram of positive and negative loop, then the net indicated mean effective pressure is given by

$$P_m = A_p * S_p / L - A_n * S_n / L \quad \dots\dots(7)$$

Where S_p = Spring strength used for taking $p-v$ diagram of positive loop, (N/m² per cm)

S_n = Spring strength used for taking $p-v$ diagram of negative loop, (N/m² per cm)

A_p = Area in Cm² of positive loop taken with spring of strength S_p

A_n = Area in Cm² of positive loop taken with spring of strength S_n

Sometimes spring strength is also noted as spring constant.

The IP developed by the engine is given by $IP = P_m L A_n / L$

$$\dots\dots(8)$$

Where ' n ' is the number of working strokes per second.

The explanation of this expression is already given in the last chapter.

1.5. MEASUREMENT OF B.P

Part of the power developed in the engine cylinder is used to overcome the internal friction. The net power available at the shaft is known as brake power and it is denoted by B.P. The arrangement used for measuring the BP of the engine is described below:

- (a) Prony Brake. The arrangement of the braking system is shown in Figure 3. It consists of brake shoes made of wood and these are clamped on to the rim of the



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brake wheel by means of the bolts. The pressure on the rim is adjusted with the help of nut and springs as shown in Fig 2. A load bar extends from top of the brake and a load carrier is attached to the end of the load bar. Weight kept on this load carrier is balanced by the torque reaction in the shoes. The load arm is kept horizontal to keep the arm length constant.

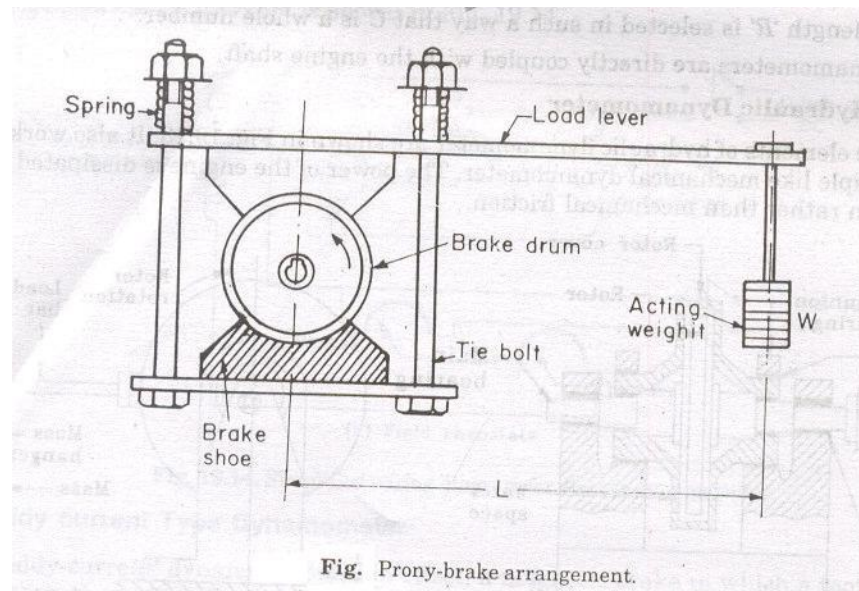


Figure.3

The energy supplied by engine to the brake is eventually dissipated as heat. Therefore, most of the brakes are provided with a means of supply of cooling water to the inside rim of the brake drum.

The BP of the engine is given by

$$\text{B.P (brake power)} = \frac{2 \cdot \pi \cdot N \cdot T}{60} \text{ watts} = \frac{2 \cdot \pi \cdot N \cdot T}{60} \cdot 1000 \text{ Kw} \dots (9)$$

Where T = $(W \cdot L)$ (N-m)

Where W = Weight on load carrier, (N)

And L = Distance from the centre of shaft to the point of load-meter in meters.



The prony brake is inexpensive, simple in operation and easy to construct. It is, therefore, used extensively for testing of low speed engines. At high speeds, grabbing and chattering of the band occur and lead to difficulty in maintaining constant load. The main disadvantage of the prony brake is its constant torque at any one band pressure and therefore its inability to compensate for varying conditions.

1.5.1 Hydraulic Dynamometer.

The BP of an engine coupled to the dynamometer is given by

$$\text{B.P (brake power)} = 2 \cdot \pi \cdot N \cdot W \cdot R / 60 \cdot 1000 = \frac{WN(2 \cdot \pi \cdot R / 60 \cdot 1000)}{K} \text{ Kw}$$

The working of a prony brake dynamometer is shown in figure 4

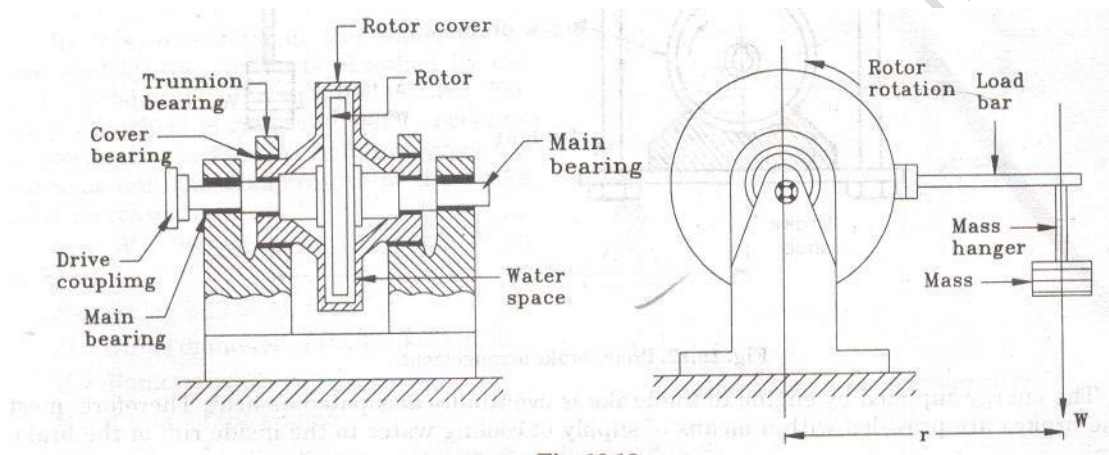


Figure.4 Hydraulic dynamometer

In the hydraulic dynamometer, as the arm length (R) is fixed, the factor $[2 R / (60 \times 1000)]$ is constant and its value is generally given on the name plate of the dynamometer by the manufacturer and is known as brake or dynamometer constant. Then the BP measured by the dynamometer is given by

$$\text{B.P} = \frac{WN}{K} \quad \dots(10)$$

Where W = Weight measured on the dynamometer, N
 K = Dynamometer constant ($60 \cdot 1000 / 2 \cdot \pi \cdot R$)
 and N = RPM of the engine.

The arm length ' R ' is selected in such a way that K is a whole number.

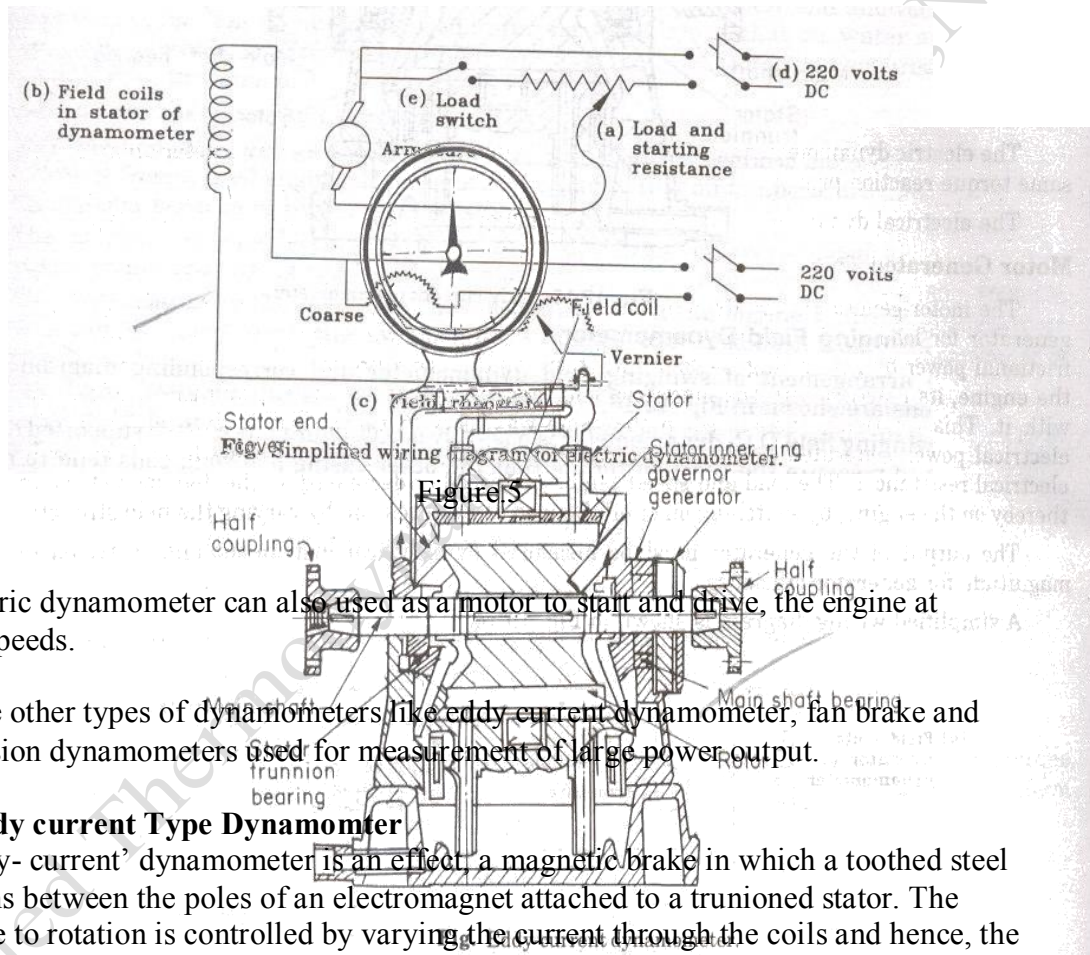
These dynamometers are directly coupled with the engine shaft.

1.5.2 Electric Dynamometer:

The electric generator can also be used for measured BP of the engine. The output of the



generator must be measured by electrical instruments and corrected for generator efficiency. Since the efficiency of the generator depends upon load, speed and temperature, this device is rather inconvenient to use in the laboratory for obtaining precise measurement. To overcome these difficulties, the generator stator may be supported in ball bearing trunnions and the reaction force exerted on the stator of the generator may be measured by a suitable balance. The tendency to rotate or the reaction of the stator will be equal and opposite to the torque exerted on the armature, which is driven by the engine which is shown in Figure 5.



The electric dynamometer can also be used as a motor to start and drive the engine at various speeds.

There are other types of dynamometers like eddy current dynamometer, fan brake and transmission dynamometers used for measurement of large power output.

1.5.3 Eddy current Type Dynamometer

The 'eddy-current' dynamometer is an effect, a magnetic brake in which a toothed steel rotor turns between the poles of an electromagnet attached to a trunnioned stator. The resistance to rotation is controlled by varying the current through the coils and hence, the strength of the magnetic field. The flux tends to follow the smaller air gaps at the ends of the rotor teeth and eddy currents are set up within the metal of the pole pieces, resulting in heating the stator. The heat energy is removed by circulating water through a water jacket formed in the stator. Figure 6 shows the "Heenan eddy-current dynamometer"



Figure.6

The power output of eddy-current dynamometer is given by the equation where C is eddy-current dynamometer constant.

The advantages of eddy-current dynamometer are listed below:

1. High absorbing power per unit weight of dynamometer.
2. Level of field excitation is below 1% of the total power handled by the dynamometer.
3. The torque development is smooth as eddy current developed smooth.
4. Relatively higher torque is provided under low speed conditions.
5. There is no limit to the size of dynamometer.

1.5.4 Swinging Field Dynamometer

The arrangement of swinging field dynamometer and corresponding diagram of electric connections are shown in Figure 7.

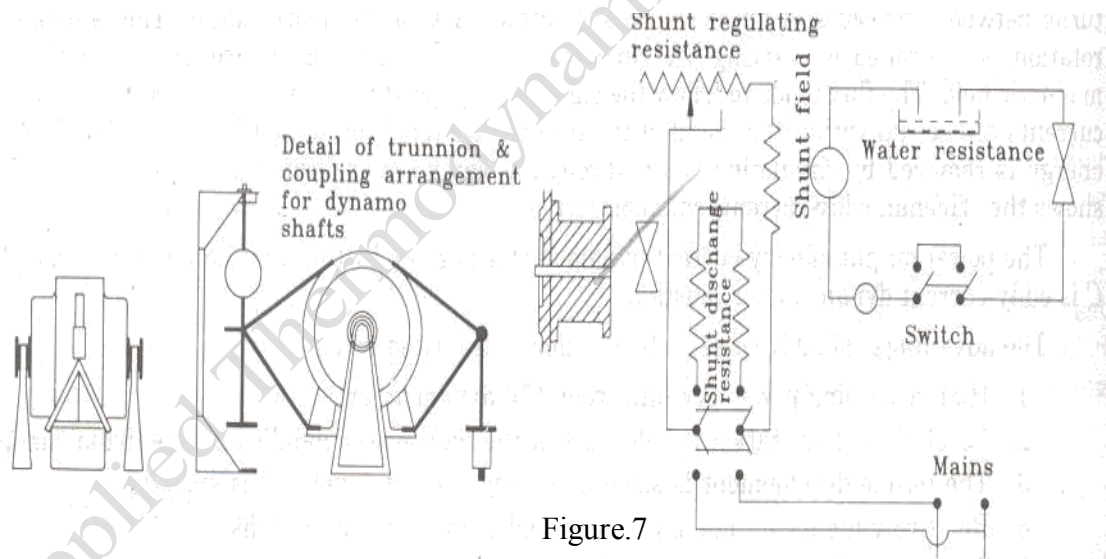


Figure.7

A swinging field DC dynamometer is basically a DC shunt motor. It is supported on trunnion bearings to measure the reaction torque that the outer casing and field coils tend to rotate with the magnetic drag. Therefore, it is named as “Swinging field”. The Torque is measured with an arm and weighting equipment in the usual manner.

The choice of dynamometer depends on the use for which the machine is purchased. An electric dynamometer is preferred as it can operate as motor used for pumping or



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generator for testing the engine. Also, engine friction power can also be measured by operating the dynamometer in the motoring mode.

An eddy-current or hydraulic dynamometer may be used because of low initial cost and an ability to operate at high speeds. The armature of the electric dynamometer is large and heavy compared with eddy-current dynamometer and requires strong coupling between dynamometer and engine.

1.6 MEASUREMENT OF I.P OF MULTI-CYLINDER ENGINE (MORSE TEST)

This method is used in multi-cylinder engines to measure I.P with out the use of indicator. The BP of the engine is measured by cutting off each cylinder in turn. If the engine consists of 4-cylinders, then the BP of the engine should be measured four times cutting each cylinder turn by turn. This is applicable to petrol as well as for diesel engines. The cylinder of a petrol engine is made inoperative by “shorting” the spark plug whereas in case of diesel engine, fuel supply is cut-off to the required cylinder.

If there are ‘ n ’ cylinders in an engine and all are working, then

$$(B.P)_n = (I.P)_n - (F.P)_n \dots\dots(11)$$



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Where F.P is the frictional power per cylinder.

If one cylinder is inoperative then the power developed by that cylinder (IP) is lost and the speed of the engine will fall as the load on the engine remains the same. The engine speed can be resorted to its original value by reducing the load on the engine by keeping throttle position same. This is necessary to maintain the FP constant, because it is assumed that the FP is independent of load and depends only on speed of the engine.

When cylinder "1" is cut off; then

$$(B.P)_{n-1} = (I.P)_{n-1} - (F.P)_n \dots\dots(12)$$

By subtracting Eq. (23.7) from Eq.(23.6), we obtain the IP of the cylinder which is

not firing i.e., $(B.P)_n - (B.P)_{n-1} = (IP)_n - (IP)_{n-1} = I.P_1$

Similarly IP of all other cylinders can be measured one by one then the sum of IPs of all cylinders will be the total IP of the engine.

This method of obtaining IP of the multicylinder engine is known as 'Morse Test'.

1.7 MEASUREMENT OF AIR-CONSUMPTION

The method is commonly used in the laboratory for measuring the consumption of air is known as 'Orifice Chamber Method'. The arrangement of the system is shown in Figure 8.

It consists of an air-tight chamber fitted with a sharp-edged orifice of known coefficient of discharge. The orifice is located away from the suction connection to the engine.

Due to the suction of engine, there is pressure depression in the chamber which causes the flow through orifice for obtaining a steady flow, the volume of chamber should be sufficiently large compared with the swept volume of the cylinder; generally 500 to 600 times the swept volume. A rubber diaphragm is provided to further reduce the pressure pulsations.

It is assumed that the intermittent suction of the engine will not affect the air pressure in the air box as the volume of the box is sufficiently large, and pressure in the box remains constant.

The pressure difference causing the flow through the orifice is measured with the help of a water monometer. The pressure difference should be limited to 10cm of water to make the compressibility effect negligible. Let

A_o = Area orifice in m^2 ; h_w = Head of water in cm causing the flow.

C_d = Coefficient of discharge for orifice. ; d = Diameter of orifice in cm.

ρ_a = Density of air in kg/m^3 under atmospheric conditions.

Head in terms of meters of air is given by

$$H. \rho_a = \frac{h_w}{\rho_w} \rho_w; \therefore H = \frac{h_w}{\rho_a} \cdot \rho_w = \frac{h_w}{\rho_a} \times \frac{1000}{1} = \frac{1000 h_w}{\rho_a} \text{ m of air}$$



$$100 \quad 100 \rho_a \quad 100 \quad \rho_a \quad \rho_a$$

The velocity of air passing through the orifice is given by

$$v = \sqrt{2gH} \text{ m/Sec} = \sqrt{2g \cdot \frac{10h_w}{\rho_a}} \text{ m/Sec}$$

The volume of air passing through the orifice is given by

$$v_a = A \cdot v \cdot C_d \cdot \sqrt{\frac{2g \cdot 10h_w}{\rho_a}} = 14.01 \cdot A \cdot C_d \cdot \sqrt{\frac{h_w}{\rho_a}} \text{ cu. m/Sec}$$

$$840.428 \cdot A \cdot C_d \cdot \sqrt{\frac{h_w}{\rho_a}} \text{ m}^3 / \text{min}$$

The volumetric efficiency of the engine

$$= \frac{\text{Actual volume of air taken in as measure}}{\frac{\pi D^2}{4} \cdot L \cdot \frac{N}{60} \cdot n} = \frac{14.01 A_0 \cdot C_d \cdot \sqrt{\frac{h_w}{\rho_a}}}{\frac{\pi D^2}{4} \cdot L \cdot \frac{N}{60} \cdot n}$$

Displacement volume

Where N is RPM of the engine and n is number of cylinders. D & L are diameter and stroke of each cylinder.

Mass of air passing through the orifice is given by

$$m_a = V_a \cdot \rho_a = 14.01 \times \frac{\pi d^2}{4 \times 100} \cdot C_d \cdot \sqrt{\frac{10h_w}{\rho_a}} \cdot \rho_a = 11.003 \times 10^{-4} C_d \cdot d^2 \sqrt{\rho_a h_w}$$

$$= 0.0011 C_d \cdot d^2 \sqrt{h_w \rho_a} \text{ Kg / Sec} = 0.066 \cdot C_d \cdot d^2 \sqrt{h_w \rho_a} \text{ kg / min} \dots\dots(13)$$

Where d is in cm; h_w is in cm of water and P_a is in kg/m^3

The density of atmospheric air is given by

$$\rho_a = \frac{p_a \times 10^5}{287 \times T_a}$$

Where P_a is the atmospheric pressure in bar and T_a is the atmospheric temperature in K.

Substituting the value of ρ_a in Eq. (13)



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$$m_a = 0.066 \cdot C_d \cdot d^2 \sqrt{h_w \frac{p_a \times 10^5}{287 \cdot T_a}}$$
$$= 1.23 \cdot C_d \cdot d^2 \sqrt{\frac{p_a \times h_w}{T_a}} \text{ kg / min}$$

Where d is in cm, h_w is in cm of water, P_a is in bar and T_a is in K.

The measurement of air consumption by the orifice chamber method is used for:

- (a) The determination of the actual A : F ratio of the engine at running condition.
- (b) The weight of exhaust gases produced, and
- (c) The volumetric efficiency of the engine at the running condition.

The mass of air supplied per kg of fuel used can also be calculated by using the following formula if the volumetric analysis of the exhaust gases is known.



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$$m_a / \text{Kg of fuel} = \frac{N \times C}{33(C_1 + C_2)} \quad \dots(14)$$

Where N = Percentage of nitrogen by volume in exhaust gases.
 C_1 = Percentage of carbon dioxide by volume in exhaust gases.
 C_2 = Percentage of carbon monoxide by volume in exhaust gases.
 C = Percentage of carbon in fuel by weight.

If $C_2 = 0$ then; $m_a = \frac{N \times C}{33 \cdot C_1} \quad \dots(15)$



1.8 MEASUREMENT OF FUEL CONSUMPTION

Two glass vessels of 100cc and 200cc capacity are connected in between the engine and main fuel tank through two, three- way cocks. When one is supplying the fuel to the engine, the other is being filled. The time for the consumption of 100 or 200cc fuel is measured with the help of stop watch.

A small glass tube is attached to the main fuel tank as shown in figure. When fuel rate is to be measured, the valve is closed so that fuel is consumed from the burette. The time for a known value of fuel consumption can be measured and fuel consumption rate can be calculated.

$$\text{Fuel consumption kg/hr} = \frac{X_{cc} \times \text{Sp. gravity of fuel}}{1000 \times t}$$

1.9 MEASUREMENT OF HEAT CARRIED AWAY BY COOLING WATER

The heat carried away by cooling water is generally measured by measuring the water flow rate through the cooling jacket and the rise in temperatures of the water during the flow through the engine.

The inlet and out let temperatures of the water are measured by the thermometers inserting in the pockets provided at inlet to and outlet from the engine. The quantity of water flowing is measured by collecting the water in a bucket for a specified period or directly with the help of flow meter in case of large engine. The heat carried away by cooling water is given by

Where

Q_w	=	$C_p m_w (T_{wo} - T_{wi})$ kJ/min.
M_w	=	mass of water/min.
T_{wi}	=	Inlet temperature of water, °C
T_{wo}	=	Out let temperature of water, °C
C_p	=	Specific heat of water.

1.10 MEASUREMENT OF HEAT CARRIED AWAY BY EXHAUST GASES

The mass of air supplied per kg of fuel used can be calculated by using the equation if the exhaust analysis is made

$$m_a = \frac{N \times C}{33 (C_1 + C_2)}$$

And heat carried away by the exhaust gas per kg of fuel supplied can be calculated as

$Q_g = (m_a + 1) C_{pg} (T_{ge} - T_a)$ kJ/kg of fuel ... (16) Where $(m_a + 1)$ = mass of exhaust gases formed per kg of fuel supplied to engine



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C_{pg} = Specific heat of exhaust gases

T_{ge} = Temperature of exhaust gases coming out from the engine

$^{\circ}\text{C}$. T_a = Ambient temperature $^{\circ}\text{C}$ or engine room temperature.

The temperature of the exhaust gases is measured with the help of suitable thermometer or thermocouple.

Another method used for measuring the heat carried away by exhaust gases is to measure the fuel supplied per minute and also to measure the air supplied per minute with the help of air box method. The addition of fuel and air mass will be equal to the mass of exhaust gases.

And exhaust gas calorimeter is commonly used in the laboratory for the measurement of heat carried by exhaust gases.

1.10.1 Exhaust Gas Calorimeter

The exhaust gas calorimeter is a simple heat exchanger in which, part of the heat of the exhaust gases is transferred to the circulating water. This calorimeter helps to determine the mass of exhaust gases coming out of the engine.

The arrangement of the exhaust gas calorimeter is shown in fig. 23.5.

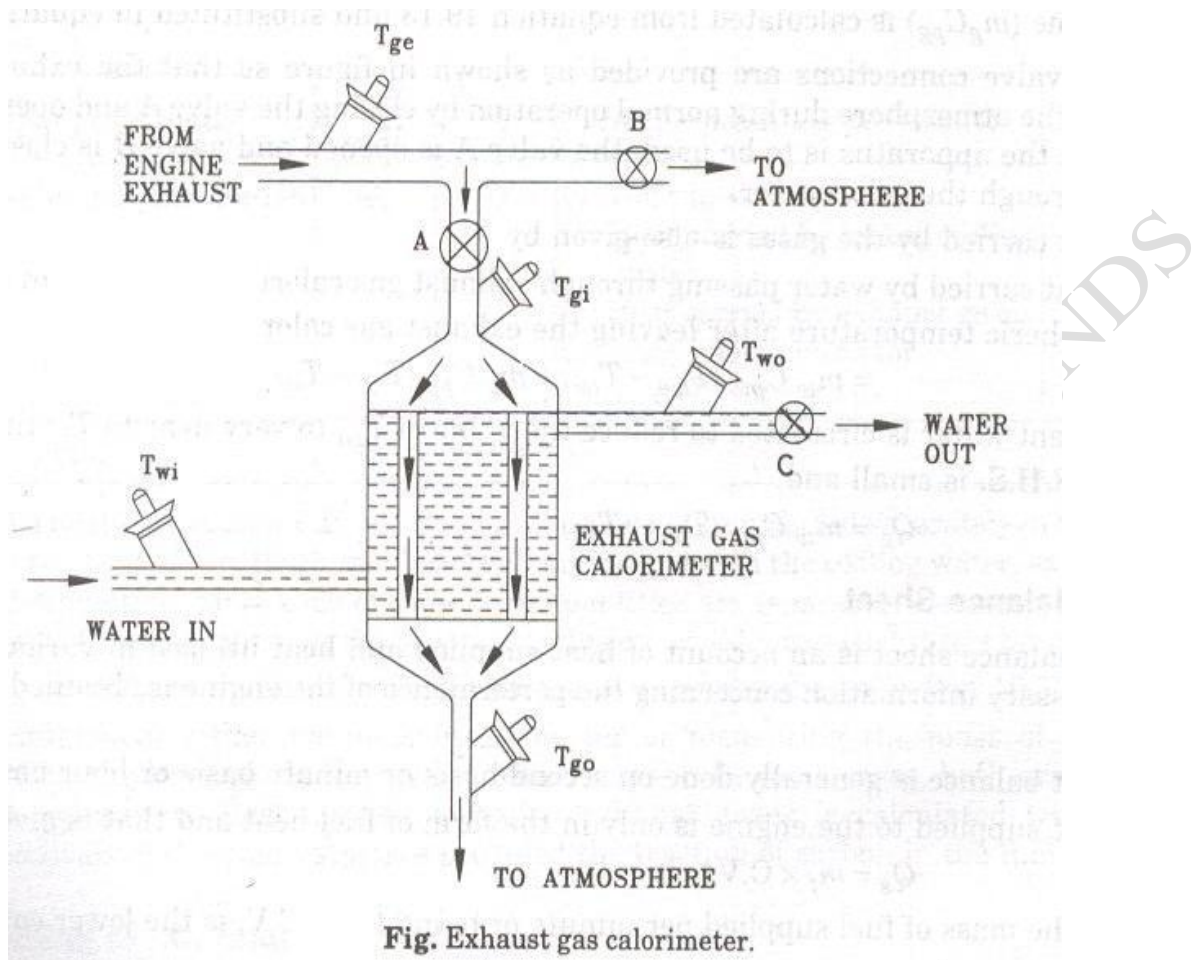


Figure.8

The exhaust gases from the engine exhaust are passed through the exhaust gas calorimeter by closing the valve *B* and opening the valve *A*. The hot gases are cooled by the water flow



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rate is adjusted with the help of valve of 'C' to give a measurable temperature rise to water circulated.

If it is assumed that the calorimeter is well insulated, there is no heat loss except by heat transfer from the exhaust gases to the circulating water, then

Heat lost by exhaust gases = Heat gained by circulating water.

Therefore $m_g C_{pg} (T_{gi} - T_{go}) = m_w C_{pw} (T_{wo} - T_{wi})$

Where T_{gi} = The temperature of the exhaust gases entering the calorimeter, °C

T_{go} = The temperature of the exhaust gases leaving the calorimeter, °C

T_{wi} = The temperature of water entering the calorimeter, °C

T_{wo} = The temperature of water leaving the calorimeter, °C

m_w = Mass of water circulated through the exhaust gas calorimeter, generally measured.

m_g = Mass of exhaust gases (unknown)

C_{pg} = specific heat of exhaust gases.

C_{pw} = Specific heat of water.

$$\therefore m_g = \frac{C_{pw} T_{wo} - T_{wi}}{C_{pg} T_{gi} - T_{go}} m_w \quad \dots(17)$$

As all the quantities on the RHS are known the gas flow rate can be determined.

Then the heat carried away by the exhaust gases is given by

$Q_g = m_g C_{pg} (T_{ge} - T_a) \dots(18)$ Where T_{ge} = Temperature of exhaust gases just leaving the engine exhaust valve, °C

T_a = Ambient temperature, °C

Usually valve connections are provided as shown in figure so that the exhaust gases are exhausted to the atmosphere during normal operation by closing the valve A and opening the valve B. Only when the apparatus is to be used, the valve A is opened and valve B is closed so that the gases pass through the calorimeter.

The heat carried by the gases is also given by

Q_g = Heat carried by water passing through exhaust gas calorimeter + Heat in exhaust gases above atmospheric temperature after leaving the exhaust gas calorimeter.

$$= m_w C_{pw} (T_{wo} - T_{wi}) + m_g C_{pg} (T_{go} - T_a) \quad \dots(19)$$

If sufficient water is circulated to reduce the value of T_{go} to very near to T_a , then the second term on the RHS is small and,

$$Q_g = m_w C_{pw} (T_{wo} - T_{wi}) \quad \dots(20)$$

1.11 HEAT BALANCE SHEET

A heat balance sheet is an account of heat supplied and heat utilized in various ways in the system. Necessary information concerning the performance of the engine is obtained from



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the heat balance.

The heat balance is generally done on second basis or minute basis or hour basis. The heat supplied to the engine is only in the form of fuel-heat and that is given by

$$Q_s = m_f \times CV$$

Where m_f is the mass of fuel supplied per minute or per sec. and CV is the lower calorific value of the fuel.

The various ways in which heat is used up in the system is given by

(a) Heat equivalent of BP = kW = kJ/sec. = 0 kJ/min.

(b) Heat carried away by cooling water

$$= C_{pw} \times m_w (T_{wo} - T_{wi}) \text{ kJ/min.}$$

Where m_w is the mass of cooling water in kg/min or kg/sec circulated through the cooling jacket and $(T_{wo} - T_{wi})$ is the rise in temperature of the water passing through the cooling jacket of the engine and C_{pw} is the specific heat of water in kJ/kg-K.

(c) Heat carried away by exhaust gases

$$= m_g C_{pg} (T_{ge} - T_a) \text{ (kJ/min.) or (kJ/sec)}$$

Where m_g is the mass of exhaust gases in kg/min. or kg/sec and it is calculated by using one of the methods already explained.

T_g = Temperature of burnt gases coming out of the engine. T_a = Ambient Temperature.

C_{pg} = Sp. Heat of exhaust gases in (kJ/kg-K)

(d) A part of heat is lost by convection and radiation as well as due to the leakage of gases. Part of the power developed inside the engine is also used to run the accessories as lubricating pump, cam shaft and water circulating pump. These cannot be measured precisely and so this is known as unaccounted 'losses'. This unaccounted heat energy is calculated by the different between heat supplied Q_s and the sum of (a) + (b) (c).

The results of the above calculations are tabulated in a table and this table is known as "Heat Balance Sheet". It is generally practice to represent the heat distribution as percentage of heat supplied. This is also tabulated in the same heat balance sheet.



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<i>Heat input per minute</i>	<i>kcal (kj)</i>	<i>%</i>	<i>Heat expenditure per minute</i>	<i>kcal (kj)</i>	<i>%</i>
Heat supplied by the combustion fuel	Q_s	100%	(a) Heat in BP. (b) Heat carried by jacket cooling water (c) Heat Carried by exhaust gases (d) Heat unaccounted for $= Q_s - (a + b + c)$	-- -- -- --	-- -- -- --
Total	Q_s	100%			100%

A sample tabulation which is known as a heat balance sheet for particular load condition is shown below:

NOTE: The heat in frictional FP (IP – BP) should not be included separately in heat balance sheet because the heat of FP (frictional heat) will be dissipated in the cooling water, exhaust gases and radiation and convection. Since each of these heat quantities are separately measured and heat in FP is a hidden part of these quantities; the separate inclusion would mean that it has been included twice.

The arrangement either for measuring the air or measuring the mass of exhaust gas is sufficient to find the heat carried away by exhaust gases. In some cases, both arrangements are used for cross-checking. Heat carried away by exhaust gases is calculated with the help



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of volumetric analysis of the exhaust gases provided the fraction of carbon in the fuel used is known.

1.12 . Indicated Specific Fuel Consumption: This is defined as the mass of fuel consumption per hour in order to produce an indicated power of one kilo watt.

$$\text{Thus, indicated specific fuel consumption} = \text{isfc} = \frac{3600 \dot{m}}{\text{ip}} \text{ kg/kWh} \dots(13)$$

1.13.Brake Specific fuel consumption:- This defined as the mass of fuel consumed per hour, in order to develop a brake power of one kilowatt.

$$\text{Thus, brake specific fuel consumption} = \text{bsfc} = \frac{3600 \dot{m}}{\text{Bp}} \text{ kg/kWh} \dots\dots(14)$$



1.14. Thermal Efficiency : There are two definitions of thermal efficiency as applied to IC engines. One is based on indicated power and the other on brake power. The one based on indicated power is called as *‘indicated thermal efficiency’*, and the one based on brake power is known as *‘brake thermal efficiency’*.

Indicated thermal efficiency is defined as the ratio of indicated power to the energy available due to combustion of the fuel.

Thus $\eta_{ith} =$

$$\frac{\text{Indicated Power in kW}}{\text{Mass of water fuel in kg} \times \text{Calorific value of fuel in kJ/kg}}$$

Or $\eta_{ith} =$ -----

$$\frac{ip}{m \times CV} \dots \dots \dots (15)$$

Similarly brake thermal efficiency is defined as the ratio of brake power to energy available due to combustion of the fuel.

Or $\eta_{bth} = \frac{bp}{m \times CV} \dots \dots \dots (16)$



1.15. Mechanical Efficiency: Mechanical efficiency takes into account the mechanical losses in an engine. The mechanical losses include (i) frictional losses, (ii) power absorbed by engine auxiliaries like fuel pump, lubricating oil pump, water circulating pump, magneto and distributor, electric generator for battery charging, radiator fan etc., and (iii) work required to charge the cylinder with fresh charge and work for discharging the exhaust gases during the exhaust stroke. It is defined as the ratio of brake power to indicated power. Thus

$$\eta_{\text{mech}} = \frac{\text{bp}}{\text{ip}} \dots \dots \dots (17)$$

1.16. Volumetric efficiency: Volumetric efficiency is the ratio of the actual mass of air drawn into the cylinder during a given period of time to the theoretical mass which should have been drawn in during the same interval of time based on the total piston displacement, and the pressure and temperature of the surrounding atmosphere.

Thus

$$\eta_v = \frac{V_{\text{actual}}}{V_{\text{th}}} \dots \dots \dots (18)$$

where n is the number of intake strokes per minute and V_s is the stroke volume of the piston.

2. Illustrative examples:

Example 1:- The following observations have been made from the test of a four cylinder, two – stroke petrol engine. Diameter of the cylinder = 10 cm; stroke = 15 cm; speed = 1600 rpm; Area of indicator diagram = 5.5 cm²; Length of the indicator diagram = 55 mm; spring constant = 3.5 bar/cm; Determine the indicated power of the engine.

Given:- $d = 0.1$ m; $L = 0.15$ m ; No. of cylinders = $K = 4$; $N = 1600$ rpm; $n = N$ (two – stroke); $a = 5.5$ cm²; length of the diagram = $l_d = 5.5$ cm; spring constant = $k_s = 3.5$ bar/cm ;

To find: indicated power, ip.



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Solution: Indicated mean effective pressure = $p_{im} = \frac{a k_s}{l d}$

or $p_{im} = \frac{5.5 \times 3.5}{5.5} = 3.5 \text{ bar} = 3.5 \times 10^5 \text{ N/m}^2$

Indicated power = $i_p = \frac{p_{im} L A n K}{60,000} = \frac{3.5 \times 10^5 \times 0.15 \times (\pi/4) \times 0.1^2 \times 1600 \times 4}{60,000}$
 $= 43.98 \text{ kW}$

Example 2:- A gasoline engine (petrol engine) working on Otto cycle consumes 8 litres of petrol per hour and develops 25 kW. The specific gravity of petrol is 0.75 and its calorific value is 44,000 kJ/kg. Determine the indicated thermal efficiency of the engine

Given:- Volume of fuel consumed/hour = $y/t = 8 \times 10^{-3} / 3600 \text{ cc/s}$;

$i_p = 25 \text{ kW}$; $CV = 44,000 \text{ kJ/kg}$;

Specific gravity of petrol = $s = 0.75$

To find: i_{th} ;

Solution: Mass of fuel consumed = $m = \frac{y s}{1000 t} = \frac{8 \times 10^{-3} \times 0.75}{1000 \times 3600} = 1.67 \times 10^{-5} \text{ kg/s}$.

Indicated thermal efficiency = $i_{th} = \frac{i_p}{m CV} = \frac{25}{1.67 \times 10^{-5} \times 44000}$
 $= 0.3402 = 34.02 \%$.

Example 2.3:- The bore and stroke of a water cooled, vertical, single-cylinder, four stroke diesel engine are 80 mm and 110 mm respectively. The torque is 23.5 N-m. Calculate the brake mean effective pressure.

What would be the mean effective pressure and torque if the engine rating is 4 kW at 1500 rpm?

Given:- Diameter = $d = 80 \times 10^{-3} = 0.08 \text{ m}$; stroke = $L = 0.110 \text{ m}$; $T = 23.5 \text{ N-m}$;



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To find (i) bmep ; (ii) bmep if bp = 4 kw and N= 1500 rpm.

Solution: (i) Relation between brake power (bp) and brake mean effective pressure (bmep) is given by

$$bp = \frac{2 NT}{60,000} = \frac{(bmep)LAN}{60,000}$$

Hence $bmep = (2 NT) / (LAN) = (2 NT) / \{(L d^2 / 4) N/2\}$

$$= \frac{16T}{d^2 L} = \frac{16 \times 23.5}{0.08^2 \times 0.11} = 5.34 \times 10^5 \text{ N/m}^2 = 5.34 \text{ bar}$$

(ii) when bp = 4 kw and N = 1500 rpm, we have

$$bmep = \frac{60,000 bp}{LAN} = \frac{60,000 \times 4}{0.110 \times (\pi/4) \times 0.08^2 \times (1500 / 2)}$$

$$= 5.79 \times 10^5 \text{ N/m}^2 = 5.79 \text{ bar.}$$

$$\text{Also } bp = 2 NT / 60,000 \text{ or } T = \frac{60,000 bp}{2 N} = \frac{60,000 \times 4}{2 \times 1500} = 25.46 \text{ N - m.}$$



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Example 4:- Find the air fuel ratio of a four stroke, single cylinder, air cooled engine with fuel consumption time for 10 cc is 20.4 s and air consumption time for 0.1 m³ is 16.3 s. The load is 7 N at the speed of 3000 rpm. Find also the brake specific fuel consumption in kg/kWh and brake thermal efficiency. Assume the density of air as 1.175 kg/m³ and specific gravity of the fuel to be 0.7. The lower heating value of the fuel is 43 MJ/kg and the dynamometer constant is 5000.

Given:- $y = 10 \text{ cc}$; $t = 20.4 \text{ s}$; $V_a = 0.1 \text{ m}^3$; $t_a = 16.3 \text{ s}$; $W = 7 \text{ N}$; $N = 3000 \text{ rpm}$;

$\rho_a = 1.175 \text{ kg/m}^3$; $s = 0.7$; $CV = 43 \times 10^3 \text{ kJ/kg}$; Dynamometer constant = $C = 5000$.

To find:- (i) m_a / m_f ; (ii) bsfc ; (iii) η_{bth} .

Solution: (i) Mass of air consumed = $m_a = \frac{0.1 \times 1.175}{16.3} = 7.21 \times 10^{-3} \text{ kg/s}$.

Mass of fuel consumed = $m_f = \frac{y \times s}{1000 \times t} = \frac{10 \times 0.7}{1000 \times 20.4} = 0.343 \times 10^{-3} \text{ kg/s}$



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$$\text{Air fuel ratio} = \frac{m_a}{m_f} = \frac{7.2}{0.343} = 21$$

$$(ii) \text{ Brake power} = bp = \frac{WN}{C} = \frac{7 \times 3000}{5000} = 4.2 \text{ kW}$$

$$\text{bsfc} = \frac{m_f \times 3600}{bp} = \frac{0.343 \times 10^{-3} \times 3600}{4.2} = 0.294 \text{ kg/kWh}$$

$$(iii) \text{ b}_{ith} = \frac{bp}{m_f CV} = \frac{4.2}{0.343 \times 10^{-3} \times 43 \times 10^3} = 0.2848 = 28.48 \%$$

Example 2.5:- A six cylinder, gasoline engine operates on the four stroke cycle. The bore of each cylinder is 80 mm and the stroke is 100 mm. The clearance volume in each cylinder is 70 cc. At a speed of 4000 rpm and the fuel consumption is 20 kg/h. The torque developed is 150 N-m. Calculate (i) the brake power, (ii) the brake mean effective pressure, (iii) brake thermal efficiency if the calorific value of the fuel is 43000 kJ/kg and (iv) the relative efficiency if the ideal cycle for the engine is Otto cycle.

Given:- $K = 6$; $n = N / 2$; $d = 8 \text{ cm}$; $L = 10 \text{ cm}$; $V_c = 70 \text{ cc}$; $N = 4000 \text{ rpm}$; $m_f = 20$

kg/h ; $T = 150 \text{ N-m}$; $CV = 43000 \text{ kJ/kg}$;

To find:- (i) bp ; (ii) b_{mep} ; (iii) b_{th} ; (iv) Relative.

Solution:

$$bp = \frac{2 \pi T n}{60,000} = \frac{2 \times 150 \times 2000}{60,000} = 62.8 \text{ kW}$$

$$(ii) \text{ b}_{mep} = \frac{60,000 bp}{L A n K} = \frac{60,000 \times 62.8}{0.1 \times (\pi/4) \times 0.08^2 \times (4000/2) \times 6} = 6.25 \times 10^5 \text{ N/m}^2 = 6.25 \text{ bar}$$



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$$(iii) \text{ bth} = \frac{\text{bp}}{\text{mf CV}} = \frac{62.8}{(20 / 3600) \times 43,000} = 0.263 = 26.3 \%$$

$$(iv) \text{ Stroke volume} = V_s = \left(\frac{\pi}{4}\right) d^2 L = \left(\frac{\pi}{4}\right) \times 8^2 \times 10 = 502.65 \text{ cc}$$

$$\text{Compression Ratio of the engine} = R_c = \frac{V_{s+V_c}}{V_c} = \frac{502.65 + 70}{70} = 8.18$$

$$\begin{aligned} \text{Air standard efficiency of Otto cycle} = \eta_{\text{Otto}} &= 1 - \left(\frac{1}{R_c}\right)^{\gamma-1} \\ &= 1 - \frac{1}{8.18^{0.4}} = 0.568 = 56.8 \% \end{aligned}$$

$$\text{Hence Relative efficiency} = \eta_{\text{Relative}} = \eta_{\text{bth}} / \eta_{\text{Otto}} = 0.263 / 0.568 = 0.463 = 46.3 \%$$

Example 2.6:- An eight cylinder, four stroke engine of 9 cm bore, 8 cm stroke and with a compression ratio of 7 is tested at 4500 rpm on a dynamometer which has 54 cm arm. During a 10 minute test, the dynamometer scale beam reading was 42 kg and the engine consumed 4.4 kg of gasoline having a calorific value of 44,000 kJ/kg. Air at 27 C and 1 bar was supplied to the carburetor at a rate of 6 kg/min. Find (i) the brake power, (ii) the brake mean effective pressure, (iii) the brake specific fuel consumption, (iv) the brake specific air consumption, (v) volumetric efficiency, (vi) the brake thermal efficiency and (vii) the air fuel ratio.

Given:- K = 8 ; Four stroke hence n = N/2 ; d = 0.09 m; L = 0.08 m; R_c = 7; N = 4500

rpm; Brake arm = R = 0.54 m ; t = 10 min ; Brake load = W = (42 x 9.81) N

mf = 4.4 kg ; CV = 44,000 kJ/kg ; T_a = 27 + 273 = 300 K ; p_a = 1 bar; m_a = 6 kg/min;

To find:- (i) bp ; (ii) bmep ; (iii) bsfc ; (iv) bsac ; (v) η_v ; (vi) η_{bth} ; (vii) m_a / mf

Solution:

$$\begin{aligned} (i) \text{ bp} &= \frac{2 \text{ NT}}{60,000} = \frac{2 \text{ NWR}}{60,000} = \frac{2 \times 8 \times 4500 \times (42 \times 9.81) \times 0.54}{60,000} \\ &= 104.8 \text{ kW} \end{aligned}$$



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$$(ii) \text{ bmep} = \frac{60,000 \text{ bp}}{L A n K} = \frac{60,000 \times 104.8}{0.08 \times (\pi/4) \times 0.09^2 \times (4500/2) \times 8}$$

$$= 6.87 \times 10^5 \text{ N/m}^2 = 6.87 \text{ bar.}$$

$$(iii) \text{ mass of fuel consumed per unit time} = \dot{m}_f = m_f / t = 4.4 \times 60 / 10 \text{ kg/h}$$

$$= 26.4 \text{ kg/h}$$

$$\text{Brake specific fuel consumption} = \text{bsfc} = \frac{\dot{m}_f}{\text{bp}} = \frac{26.4}{104.8} = 0.252 \text{ kg/kWh}$$

$$(iv) \text{ brake specific air consumption} = \text{bsac} = \frac{\dot{m}_a}{\text{bp}} = \frac{6 \times 60}{104.8}$$

$$= 3.435 \text{ kg/kWh}$$

$$(v) \text{ bth} = \frac{\text{bp}}{\dot{m}_f \text{ CV}} = \frac{104.8}{(26.4 / 3600) \times 44,000} = 0.325 = 32.5 \%$$

$$(vi) \text{ Stroke volume per unit time} = \dot{V}_s = \left(\frac{d^2}{4}\right) L n K$$

$$= \frac{\pi}{4} \times (0.09^2) \times 0.08 \times (4500/2) \times 8$$

$$= 9.16 \text{ m}^3 / \text{min.}$$

$$\text{Volume flow rate of air per minute} = \dot{V}_a = \frac{\dot{m}_a R_a T_a}{p_a} = \frac{6 \times 286 \times 300}{1 \times 10^5}$$

$$= 5.17 \text{ m}^3 / \text{min}$$

$$\text{Volumetric efficiency} = \eta_v = \dot{V}_a / \dot{V}_s = 5.17 / 9.16 = 0.5644 = 56.44 \%$$

$$(vii) \text{ Air fuel ratio} = \dot{m}_a / \dot{m}_f = 6 / (4.4 / 10) = 13.64$$



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Example 2.7:- A gasoline engine working on four- stroke develops a brake power of 20.9 kW. A Morse test was conducted on this engine and the brake power (kW) obtained when each cylinder was made inoperative by short circuiting the spark plug are 14.9, 14.3, 14.8 and 14.5 respectively. The test was conducted at constant speed. Find the indicated power, mechanical efficiency and brake mean effective pressure when all the cylinders are firing. The bore of the engine is 75mm and the stroke is 90 mm. The engine is running at 3000 rpm.

Given:- brake power when all cylinders are working = $B_t = 20.9 \text{ kW}$;

Brake power when cylinder 1 is inoperative = $B_1 = 14.9 \text{ kW}$;

Brake power when cylinder 2 is inoperative = $B_2 = 14.3 \text{ kW}$;

Brake power when cylinder 3 is inoperative = $B_3 = 14.8 \text{ kW}$;

Brake power when cylinder 4 is inoperative = $B_4 = 14.5 \text{ kW}$;

$N = 3000 \text{ rpm}$; $d = 0.075 \text{ m}$; $L = 0.09 \text{ m}$;

To find:- (i) $(ip)_{\text{total}}$; (ii) η_{mech} ; (iii) b_{mep} ;

Solution:

$$\begin{aligned} \text{(i) } (ip)_{\text{total}} &= ip_1 + ip_2 + ip_3 + ip_4 = (B_t - B_1) + (B_t - B_2) + (B_t - B_3) + (B_t - B_4) \\ &= 4B_t - (B_1 + B_2 + B_3 + B_4) = 4 \times 20.9 - (14.9 + 14.3 + 14.8 + 14.5) \\ &= 25.1 \text{ Kw} \end{aligned}$$

$$\text{(ii) } \eta_{\text{mech}} = \frac{B_t}{(ip)_{\text{total}}} = \frac{20.9}{25.1} = 0.833 = 83.3\%$$

$$\begin{aligned} \text{(iii) } b_{\text{mep}} &= \frac{60,000 B_t}{L A n K} \\ &= \frac{60,000 \times 20.9}{0.09 \times (4) \times 0.075^2 \times (3000 / 2) \times 4} \\ &= 5.25 \times 10^5 \text{ N / m}^2 = 5.25 \text{ bar.} \end{aligned}$$



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Example 2.8:- The following observations were recorded during a trial of a four – stroke, single cylinder oil engine.

Duration of trial = 30 min ; oil consumed = 4 litres ; calorific value of oil = 43 MJ/kg ; specific gravity of fuel = 0.8 ; average area of the indicator diagram = 8.5 cm² ; length of the indicator diagram = 8.5 cm ; Indicator spring constant = 5.5 bar/cm ; brake load = 150 kg ; spring balance reading = 20 kg ; effective brake wheel diameter = 1.5 m ; speed = 200 rpm ; cylinder diameter = 30 cm ; stroke = 45 cm ; jacket cooling water = 10 kg/min ; temperature rise of cooling water = 36 C. Calculate (i) indicated power, (ii) brake power, (iii) mechanical efficiency, (iv) brake specific fuel consumption, (v) indicated thermal efficiency, and (vi) heat carried away by cooling water.

Given:- t = 30 min ; y = 4000 cc ; CV = 43 x 10³ kJ/kg ; s = 0.8 ; area of the diagram = a = 8.5 cm² ; length of the diagram = l_d = 8.5 cm ; indicator spring constant = k_s = 5.5 bar / cm ; W = 150 x 9.81 N ; Brake radius = R = 1.5 / 2 = 0.75 m ; N = 200 rpm ; d = 0.3 m ; L = 0.45 m ; m_w = 10 kg/min ; T_w = 36 C ; Spring Balance Reading = S = 20 x 9.81 N

To find:- (i) ip ; (ii) bp ; (iii) mech ; (iv) bsfc ; (v) ith ; (vi) Q_w

Solution:

$$(i) p_{im} = \frac{a}{l_d} \times k_s = \frac{8.5}{8.5} \times 5.5 = 5.5 \text{ bar} = 5.5 \times 10^5 \text{ N/m}^2$$

$$ip = \frac{p_{im} L A n K}{60,000} = \frac{5.5 \times 10^5 \times 0.45 \times (\pi/4) \times 0.3^2 \times (200/2) \times 1}{60,000}$$

$$= 29.16 \text{ kW}$$

$$(ii) bp = \frac{2 N(W - S) R}{60,000} = \frac{2 \times 200 \times (150 - 20) \times 9.81 \times 0.75}{60,000}$$

$$= 20.03 \text{ kW}$$

$$(iii) \text{ mech} = bp / ip = 20.03 / 29.16 = 0.687 = 68.7 \%$$

$$y s \quad 4000 \times 0.8$$



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$$(iv) \text{ Mass of fuel consumed per hour} = m_f = \frac{\dots}{1000 \text{ t}} \times 60 = \frac{\dots}{1000 \times 30}$$

$$= 6.4 \text{ kg / h.}$$

$$bsfc = m_f / bp = \frac{6.4}{20.03} = 0.3195 \text{ kg/kWh}$$

$$(v) \text{ } i_{th} = \frac{i_p}{m_f \text{ CV}} = \frac{29.16}{(6.4 / 3600) \times 43 \times 10^3} = 0.3814 = 38.14 \%$$

$$(vi) \dot{Q}_w = m C_p T_w = (10 / 60) \times 4.2 \times 36 = 25.2 \text{ kW}$$

Example 2.9:- A four stroke gas engine has a cylinder diameter of 25 cm and stroke 45 cm. The effective diameter of the brake is 1.6 m. The observations made in a test of the engine were as follows.

Duration of test = 40 min; Total number of revolutions = 8080 ; Total number of explosions = 3230; Net load on the brake = 80 kg ; mean effective pressure = 5.8 bar; Volume of gas used = 7.5 m³; Pressure of gas indicated in meter = 136 mm of water (gauge); Atmospheric temperature = 17 C; Calorific value of gas = 19 MJ/ m³ at NTP; Temperature rise of cooling water = 45 C; Cooling water supplied = 180 kg. Draw up a heat balance sheet and find the indicated thermal efficiency and brake thermal efficiency. Assume atmospheric pressure to be 760 mm of mercury.

Given:- d = 0.25 m ; L = 0.45 m; R = 1.6 / 2 = 0.8 m; t = 40 min ; N_{total} = 8080 ;

Hence N = 8080 / 40 = 202 rpm n_{total} = 3230 ;

Hence n = 3230 / 40 = 80.75 explosions / min; W = 80 x 9.81 N; p_{im} = 5.8 bar ;

V_{total} = 7.5 m³; hence V = 7.5 / 40 = 0.1875 m³/min; p_{gauge} = 136 mm of water (gauge);

T_{atm} = 17 + 273 = 290 K; (CV)_{NTP} = 19 x 10³ kJ/ m³ ; T_w = 45 C;

m_w = 180 / 40 = 4.5 kg/min; p_{atm} = 760 mm of mercury

To find:- (i) i_{th} ; (ii) b_{th} ; (iii) heat balance sheet

Solution:

$$(i) \text{ } i_p = \frac{p_{im} L A n K}{60,000} = \frac{5.8 \times 10^5 \times (\pi / 4) \times 0.25^2 \times 0.45 \times 80.75}{60,000}$$



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$$= 17.25 \text{ kW.}$$

$$b_p = \frac{2 \text{ N W R}}{60,000} = \frac{2 \times 202 \times (80 \times 9.81) \times 0.8}{60,000}$$

$$= 13.28 \text{ kW}$$

Pressure of gas supplied = $p = p_{\text{atm}} + p_{\text{gauge}} = 760 + 136 / 13.6 = 770$ mm of mercury

Volume of gas supplied as measured at NTP = $\dot{V}_{\text{NTP}} = \dot{V} (T_{\text{NTP}} / T)(p / p_{\text{NTP}})$

$$= \frac{0.1875 \times 273 \times 770}{290 \times 760} = 0.17875 \text{ m}^3 / \text{min}$$

Heat supplied by fuel = $Q_f = \dot{V}_{\text{NTP}} (CV)_{\text{NTP}} = 0.17875 \times 19 \times 10^3 = 3396.25$ kJ/min

Heat equivalent of b_p in kJ/min = $13.28 \times 60 = 796.4$ kJ/min

Heat lost to cooling water in kJ/min = $\dot{m}_w C_p T_w = 4.5 \times 4.2 \times 45 = 846.5$ kJ/min

Friction power = $i_p - b_p = 17.25 - 13.28 = 3.97$ kW

Hence heat loss due to friction, pumping etc. = $3.97 \times 60 = 238.2$ kJ/min

Heat lost in exhaust, radiation etc (by difference) = $3396.25 - (896.4 + 796.4 + 238.2)$

$$= 1465.15 \text{ kJ/min}$$



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Heat Balance Sheet:

Item No.		Heat Energy Input		Heat Energy spent	
		(kJ/min)	(percent)	(kJ/min)	(percent)
1	Heat supplied by fuel	3396.25	100.00		
2	Heat equivalent of bp			896.4	26.4
3	Heat lost to cooling Water			796.4	23.4
4	Heat equivalent of fp			238.2	7.0
5	Heat unaccounted (by difference)			1465.15	43.2
Total		3396.25	100.0	3396.25	100.0

Example 2.10:- A test on a two-stroke engine gave the following results at full load.

Speed = 350 rpm; Net brake load = 65 kg ; mean effective pressure = 3 bar ; Fuel consumption = 4 kg/h ; Jacket cooling water flow rate = 500 kg/h ; jacket water temperature at inlet = 20 C ; jacket water temperature at outlet = 40 C ; Test room temperature = 20 C ; Temperature of exhaust gases = 400 C ; Air used per kg of fuel = 32 kg ; cylinder diameter = 22 cm ; stroke = 28 cm ; effective brake diameter = 1 m ; Calorific value of fuel = 43 MJ/kg ; Mean specific heat of exhaust gases = 1 kJ/kg –K. Find indicated power, brake power and draw up a heat balance for the test in kW and in percentage.



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Given:- Two stroke engine. Hence $n = N$; $N = 350$ rpm ; $W = (65 \times 9.81)$ N ;

$p_{im} = 3$ bar ; $\dot{m}_f = 4$ kg/h ; $\dot{m}_w = 500$ kg/h ; $T_{wi} = 20$ C ; $T_{wo} = 40$ C ; $T_{atm} = 20$ C ;

$T_{eg} = 400$ C ; $\dot{m}_a / \dot{m}_f = 32$; $d = 0.22$ m ; $L = 0.28$ m ; Brake radius = $R = \frac{1}{2}$ m ; $CV = 43,000$ kJ/kg ; $(C_p)_{eg} = 1.0$ kJ/(kg-K) ;

To find:- (i) ip ; (ii) bp ; and (iii) heat balance;

Solution:

$$P_{im} L A n = 3 \times 10^5 \times 0.28 \times \left(\frac{\pi}{4}\right) \times 0.22^2 \times 350$$

$$= 60,000$$

(i) ip = $\frac{60,000}{60,000}$

$$= 18.63 \text{ kW.}$$

(ii) bp = $\frac{2 \times N \times WR}{60,000} = \frac{2 \times 350 \times (65 \times 9.81) \times 0.5}{60,000}$

$$= 11.68 \text{ kW.}$$

(iii) Heat supplied in kW = $\dot{m}_f CV = (4 / 3600) \times 43,000$

$$= 47.8 \text{ kW}$$

Heat lost to cooling water = $\dot{m}_w (C_p)_w [T_{wo} - T_{wi}]$

$$= (500 / 3600) \times 4.2 \times [40 - 20]$$

$$= 11.7 \text{ kW.}$$

Heat lost in exhaust gases = $(\dot{m}_a + \dot{m}_f) (C_p)_{eg} [T_{eg} - T_{atm}]$

$$= \frac{(32 + 1) \times 4}{3600} \times 1.0 \times [400 - 20]$$



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= 13.9 kW

Heat balance sheet:

Heat Input	kW	%	Heat Expenditure	kW	%
Heat supplied by fuel	47.8	100	Heat in bp	11.68	24.4
			Heat lost to cooling Water	11.70	24.5
			Heat lost to exhaust Gases	13.90	29.1
			Unaccounted heat (by difference)	10.52	22.0
Total	47.80	100	Total	47.80	100.0



amics Notes, HIT, NDS



UNIT - 4

VAPOUR POWER CYCLES

Carnot vapour power cycle, drawbacks as a reference cycle, simple Rankine cycle; description, T-s diagram, analysis for performance. Comparison of Carnot and Rankine cycles. Effects of pressure and temperature on Rankine cycle performance. Actual vapour power cycles. Ideal and practical regenerative Rankine cycles, open and closed feed water heaters. Reheat Rankine cycle.

Vapour power cycles are used in steam power plants. In a power cycle heat energy (released by the burning of fuel) is converted into work (shaft work), in which a working fluid repeatedly performs a succession of processes. In a vapour power cycle, the working fluid is water, which undergoes a change of phase.

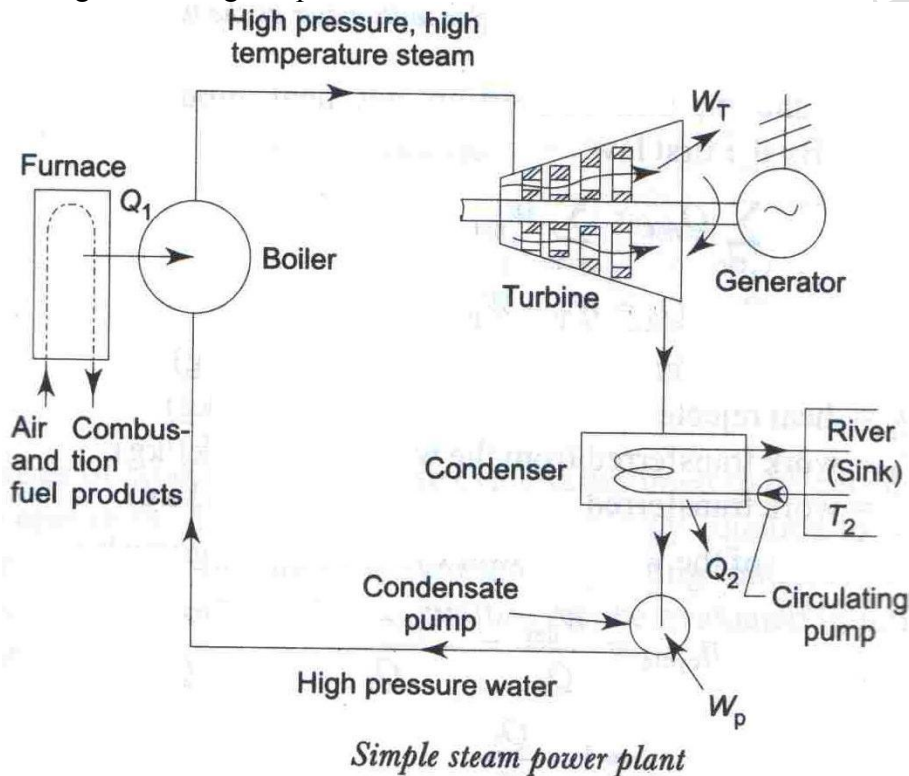


Figure shows a simple steam power plant working on the vapour power cycle. Heat is transferred to the water in the boiler (Q_H) from an external source. (Furnace, where fuel is continuously burnt) to raise steam, the high pressure high temperature steam leaving the boiler expands in the turbine to produce shaft work (W_T), the steam leaving the turbine condenses into water in the condenser (where cooling water circulates), rejecting heat (Q_L), and then the water is pumped back (W_P) to the boiler.

Since the fluid is undergoing a cyclic process, the net energy transferred as heat during the cycle must equal the net energy transfer as work from the fluid.



By the 1st law of Thermodynamics,

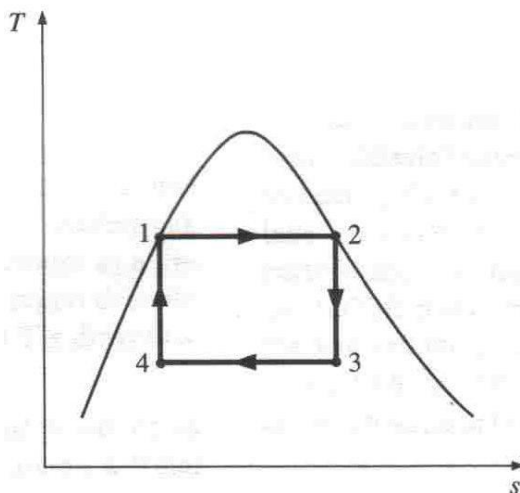
$$Q_{net\ cycle} = W_{net\ cycle}$$

Or
$$Q_H - Q_L = W_T - W_P$$

Where Q_H = heat transferred to the working fluid (kJ/kg)
 Q_L = heat rejected from the working fluid (kJ/kg)
 W_T = work transferred from the working fluid (kJ/kg)
 W_P = work transferred into the working fluid (kJ/kg)

$$\therefore \eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{W_T - W_P}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Idealized steam power cycles:



We know that the efficiency of a Carnot engine is maximum and it does not depend on the working fluid. It is, therefore, natural to examine if a steam power plant can be operated on the Carnot cycle.

Figure shows the Carnot cycle on the T-S diagram. Heat addition at constant pressure P_2 , can be achieved isothermally in the process 1-2 in a boiler. The decrease in pressure from P_2 to P_3 in the process 2-3 can also be attained through the performance of work in a steam turbine. But in order to bring back the saturated liquid water to the boiler at the state 1, the condensation process 3-4 in the condenser must be terminated at the state 4, where the working fluid is a mixture of liquid water and vapour. But it is practically impossible to attain a condensation of this kind. Difficulty is also experienced in compressing isentropically the binary mixture from state 4 to the initial state 1, where the working fluid is entirely in the liquid state. Due to these inherent practical difficulties, Carnot cycle remains an ideal one.

Rankine Cycle: The simplest way of overcoming the inherent practical difficulties of the Carnot cycle without deviating too much from it is to keep the processes 1-2 and 2-3 of the latter unchanged and to continue the process 3-4 in the condenser until all the vapour has been



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converted into liquid water. Water is then pumped into the boiler upto the pressure corresponding to the state 1 and the cycle is completed. Such a cycle is known as the Rankine cycle. This theoretical cycle is free of all the practical limitations of the Carnot cycle.

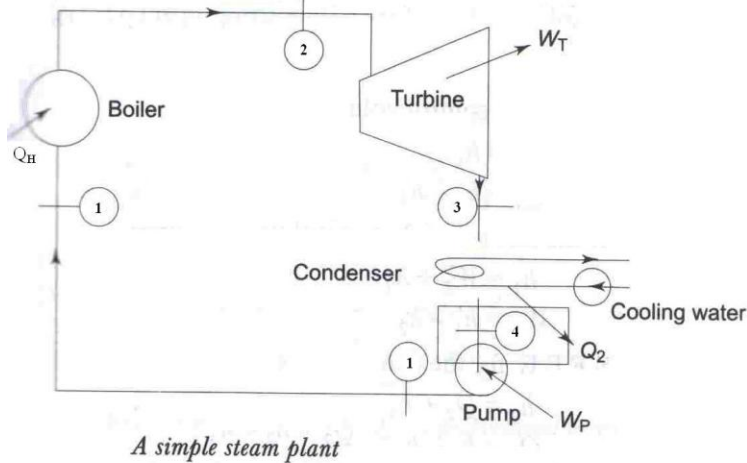


Figure (a) shows the schematic diagram for a simple steam power cycle which works on the principle of a Rankine cycle. Figure (b) represents the T-S diagram of the cycle.

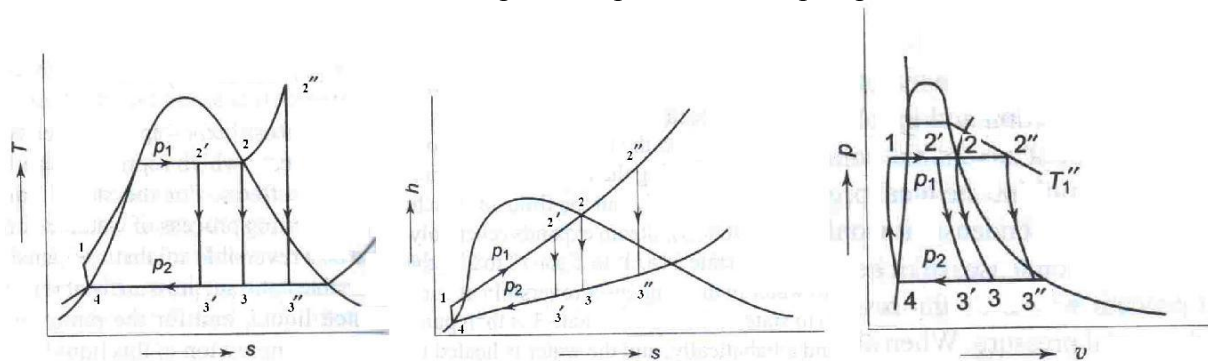
The Rankine cycle comprises the following processes.

Process 1-2: Constant pressure heat transfer process in the boiler

Process 2-3: Reversible adiabatic expansion process in the steam turbine

Process 3-4: Constant pressure heat transfer process in the condenser and

Process 4-1: Reversible adiabatic compression process in the pump.



The numbers on the plots correspond to the numbers on the schematic diagram. For any given pressure, the steam approaching the turbine may be dry saturated (state 2), wet (state 2') or superheated (state 2''), but the fluid approaching the pump is, in each case, saturated liquid (state 4). Steam expands reversibly and adiabatically in the turbine from state 2 to state 3 (or 2' to 3' or 2'' to 3''), the steam leaving the turbine condenses to water in the condenser reversibly at constant pressure from state 3 (or 3', or 3'') to state 4. Also, the water is heated in the boiler to form steam reversibly at constant pressure from state 1 to state 2 (or 2' or 2'')

Applying SFEE to each of the processes on the basis of unit mass of fluid and neglecting changes in KE & PE, the work and heat quantities can be evaluated.

For 1kg of fluid, the SFEE for the boiler as the CV, gives,



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$$h_1 + Q_H = h_2 \quad \text{i.e., } Q_H = h_2 - h_1 \quad \text{--- (1)}$$

$$\text{SFEE to turbine, } h_2 = W_T + h_3 \quad \text{i.e., } W_T = h_2 - h_3 \quad \text{--- (2)}$$

$$\text{SFEE to condenser, } h_3 = Q_L + h_4 \quad \text{i.e., } Q_L = h_3 - h_4 \quad \text{--- (3)}$$

$$\text{SFEE to pump, } h_4 + W_P = h_1 \quad \text{i.e., } W_P = h_1 - h_4 \quad \text{--- (4)}$$

$$\text{The efficiency of Rankine cycle is } \eta = \frac{W_{net}}{Q_H} = \frac{W_T - W_P}{Q_H}$$

$$\text{i.e., } \eta = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)} \quad \text{or} \quad \eta = \frac{(h_2 - h_1) - (h_3 - h_4)}{(h_2 - h_1)}$$

The pump handles liquid water which is incompressible i.e., its density or specific volume undergoes little change with an increase in pressure.

For reversible adiabatic compression, we have $Tds = dh - vdp$; since $ds = 0$
We have, $dh = vdp$

Since change in specific volume is negligible, $\Delta h = v \Delta P$

$$\text{Or } (h_1 - h_4) = v_4 (P_2 - P_3)$$

Usually the pump work is quite small compared to the turbine work and is sometimes neglected.
In that case, $h_1 = h_4$

$$\eta_{\text{rankine}} \cong \frac{(h_2 - h_3)}{(h_2 - h_1)} \cong \frac{(h_2 - h_3)}{(h_2 - h_4)}$$

The efficiency of the Rankine cycle is presented graphically in the T-S diagram

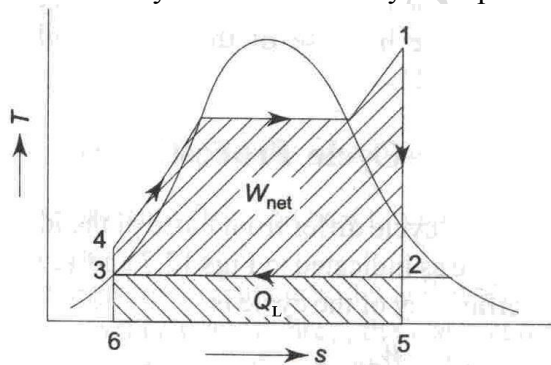


Fig. Q_H , W_{net} and Q_L are proportional to areas



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$Q_H \propto \text{area } 2-5-6-1,$

$Q_L \propto \text{area } 3-5-6-4$

$W_{\text{net}} = (Q_H - Q_L) = \text{area } 1-2-3-4 \text{ enclosed by the cycle.}$

The capacity of the steam plant is expressed in terms of steam rate defined as the rate of steam flow (kg/h) required to produce unit shaft output (1kW)

$$\therefore \text{Steam rate} = \frac{1}{W_T - W_P} \frac{\text{kg}}{\text{kJ}} \frac{1\text{kJ/s}}{1\text{kW}} = \text{Specific Steam Consumption (SSC)}$$

$$= \frac{1}{W_T - W_P} \frac{\text{kg}}{\text{kWs}} = \frac{3600}{W_T - W_P} \frac{\text{kg}}{\text{kWh}}$$

The cycle efficiency also expressed alternatively as heat rate which is the rate of heat input (Q_H) required to produce unit work output (1kW)

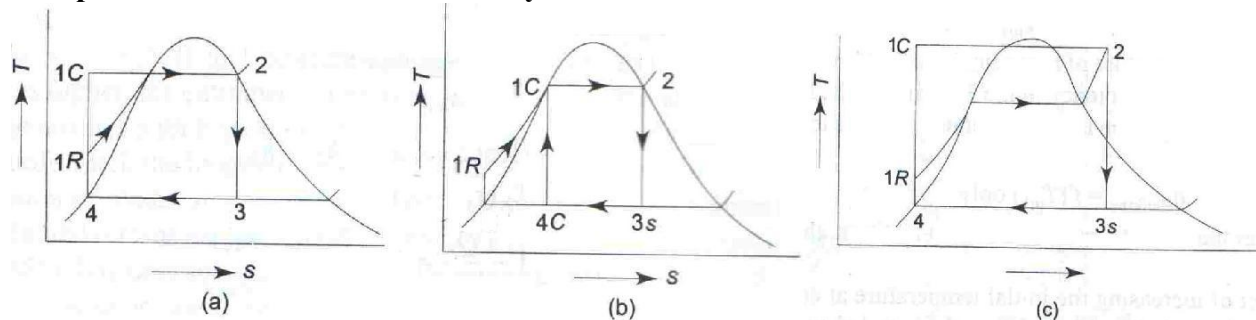
$$\text{Heat rate} = \frac{3600 Q_H}{W_T - W_P} = \frac{3600}{\eta_{\text{cycle}}} \frac{\text{kJ}}{\text{kWh}}$$

$$= \frac{3600 \delta W}{(h_2 - h_3) - (h_1 - h_4)}$$

Lastly, work ratio $r_w = \frac{\text{positive work}}{(h_2 - h_3)}$



Comparison of Rankine and Carnot cycles



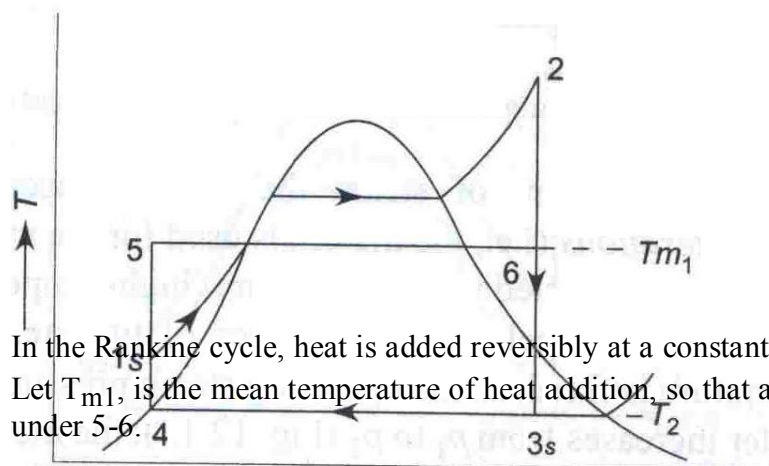
Carnot cycle has the maximum possible efficiency for the given limits of temperature. But it is not suitable in steam power plants. Figure shows the Rankine and Carnot cycles on the T-S diagram.

The reversible adiabatic expansion in the turbine, the constant temperature heat rejection in the condenser, and the Reversible adiabatic compression in the pump, are similar characteristic features of both the Rankine and Carnot cycles. But whereas the heat addition process in the Rankine cycle is reversible and at constant pressure, in the Carnot cycle it is reversible and isothermal.

In Figures (a) and (c), Q_L is the same in both the cycles, but since Q_H is more, $\eta_C > \eta_R$. The two Carnot cycles in Figure (a) and (b) have the same thermal efficiency. \therefore in Figure (b) also $\eta_C > \eta_R$.

But the Carnot cycle cannot be realized in practice because the pump work is very large. Whereas in (a) and (c) it is impossible to add heat at infinite pressures and at constant temperature from state 1C to state 2, in (b), it is difficult to control the quality at 4C, so that isentropic compression leads to a saturated liquid state.

Mean temperature of Heat addition



In the Rankine cycle, heat is added reversibly at a constant pressure, but at infinite temperatures. Let T_{m1} is the mean temperature of heat addition, so that area under 1s and 2 is equal to the area under 5-6.

$$\text{Heat added, } Q_H = h_2 - h_{1s} = T_{m1} (S_2 - S_{1s})$$

Fig. Mean temperature of heat addition
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$$\therefore T_{m1} = \text{Mean temperature of heat addition} = \frac{h_2 - h_{1s}}{s_2 - s_{1s}}$$

$$Q_L = \text{heat rejected} = h_3s - h_4 \\ = T_3 (s_2 - s_{1s})$$

$$\therefore \eta_R = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_3 (s_2 - s_{1s})}{T_{m1} (s_2 - s_{1s})}$$

$$\eta_R = 1 - \frac{T_3}{T_{m1}} \quad \text{where } T_3 = \text{temperature of heat rejection.}$$

As T_3 is lowered for a given T_{m1} , the η_R increases. But the lowest practical temperature of heat rejection is the ambient temperature T_0 .

i.e., $\eta_R = f(T_{m1})$ only.

Or higher the mean temperature of heat addition, the higher will be the cycle efficiency.

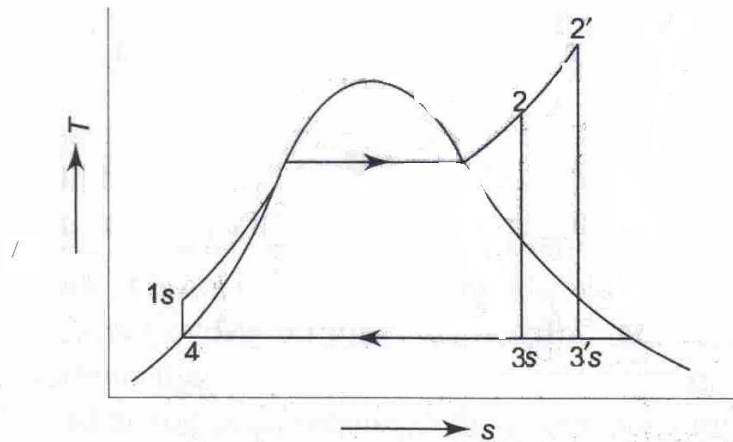


Fig. Effect of superheat on mean temperature of heat addition

The effect of increasing the initial temperature at constant pressure on cycle efficiency is shown in Figure. When the initial state changes from 2 to 2', T_{m1} between 2 and 2' is higher than T_{m1} between 1s and 2. So an increase in the superheat at constant pressure increases the mean temperature of heat addition and hence the cycle η .

But the maximum temperature of steam that can be used is fixed from metallurgical



considerations (i.e., materials used for the manufacture of the components which are subjected to high pressure, high temperature steam such as super heaters, valves, pipelines, inlet stages of turbines etc).

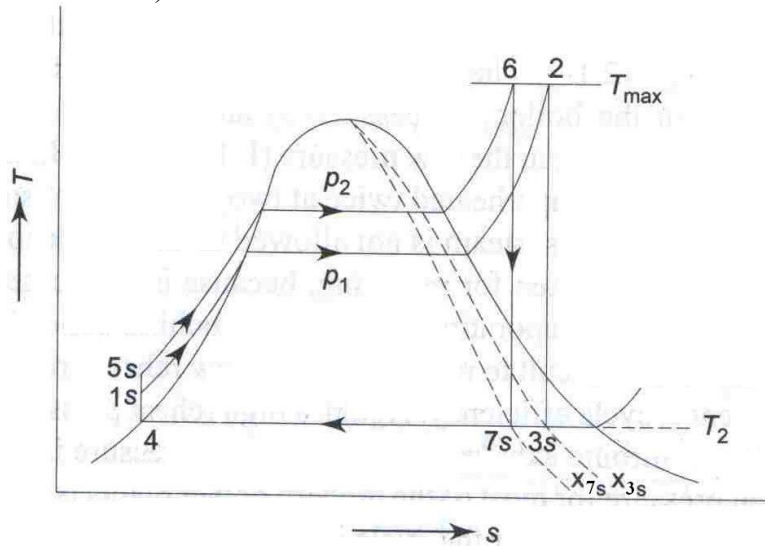


Fig. Effect of increase of pressure on Rankine cycle



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When the maximum temperature is fixed, as the operating steam pressure at which heat is added in the boiler increases from P_1 to P_2 , the mean temperature of heat addition increases (since T_{m1} between $5s$ and 6 higher than between $1s$ and 2). But when the turbine inlet pressure increases from P_1 to P_2 , the ideal expansion line shifts to the left and the moisture content at the exhaust increases ($x_{7s} < x_{3s}$)

If the moisture content of steam in the turbine is higher the entrained water particles along with the vapour coming out from the nozzles with high velocity strike the blades and erode their surfaces, as a result of which the longevity of the blades decreases. From this consideration, moisture content at the turbine exhaust is not allowed to exceed 15% or $x < 0.85$.

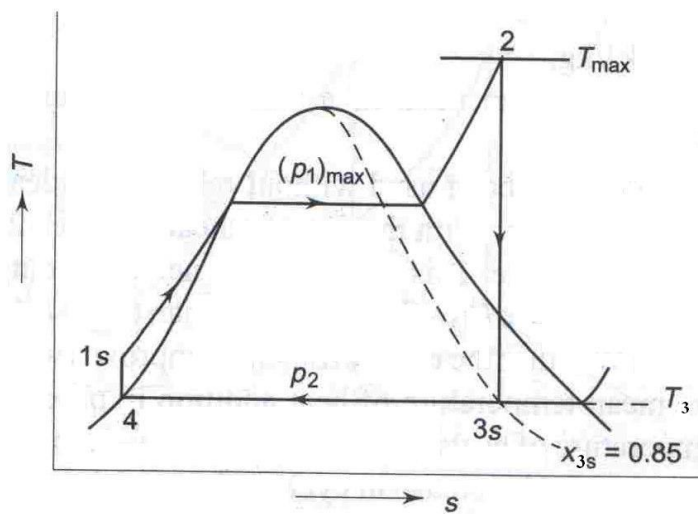


Fig. Fixing of exhaust quality, maximum temperature and maximum pressure in Rankine cycle

\therefore With the maximum steam temperature at the turbine inlet, the minimum temperature of heat rejection and the minimum quality of steam at the turbine exhaust fixed, the maximum steam pressure at the turbine inlet also gets fixed. The vertical line drawn from $3s$, fixed by T_3 and x_{3s} , intersects the T_{max} line, fixed by material, at 2 , which gives maximum steam pressure at the turbine inlet.



Effect of Boiler Pressure (Using Mollier Diagram i.e., h-s diagram)

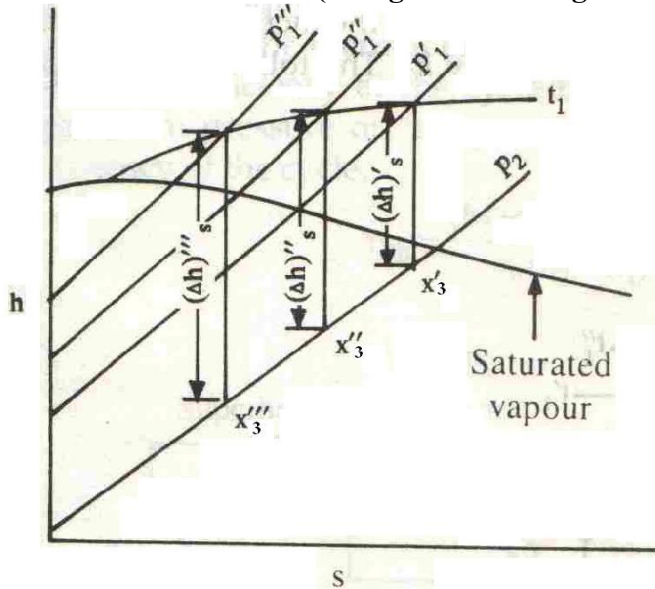


Fig. 13.5 Effect of boiler pressure

We have,

$$\eta_{th} = \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1} \text{ but } W_P \ll W_T$$

$$\therefore \eta_{th} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{(\Delta h)_S}{(h_2 - h_1)}$$

i.e., Rankine cycle η depends on h_2 , h_1 and Δh_S . From figure as $P_1''' > P_1'' > P_1'$ for the fixed maximum temperature of the steam t_1 and condenser pressure P_2 , isentropic heat drops increases with boiler pressure i.e., from the figure therefore it is evident that as boiler pressure increases, the isentropic heat drop $(\Delta h)_S$ increases, but the enthalpy of the steam entering the turbine decreases, with the result that the Rankine η increases. But quality of the steam at the exit of the turbine suffers i.e., $x_3''' < x_3'' < x_3'$, which leads to serious wear of the turbine blades.



Effect of Super Heating (Using Molliar Diagram i.e., h-s diagram)

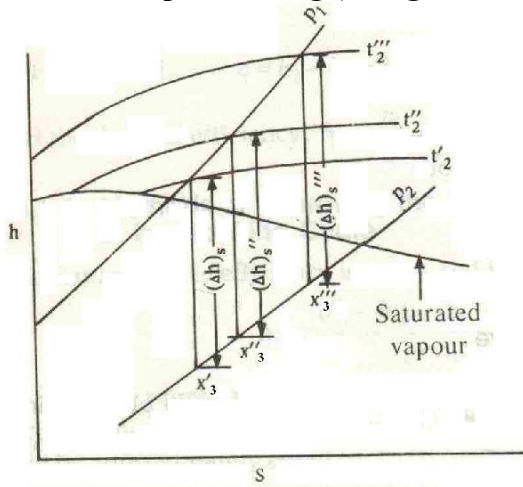
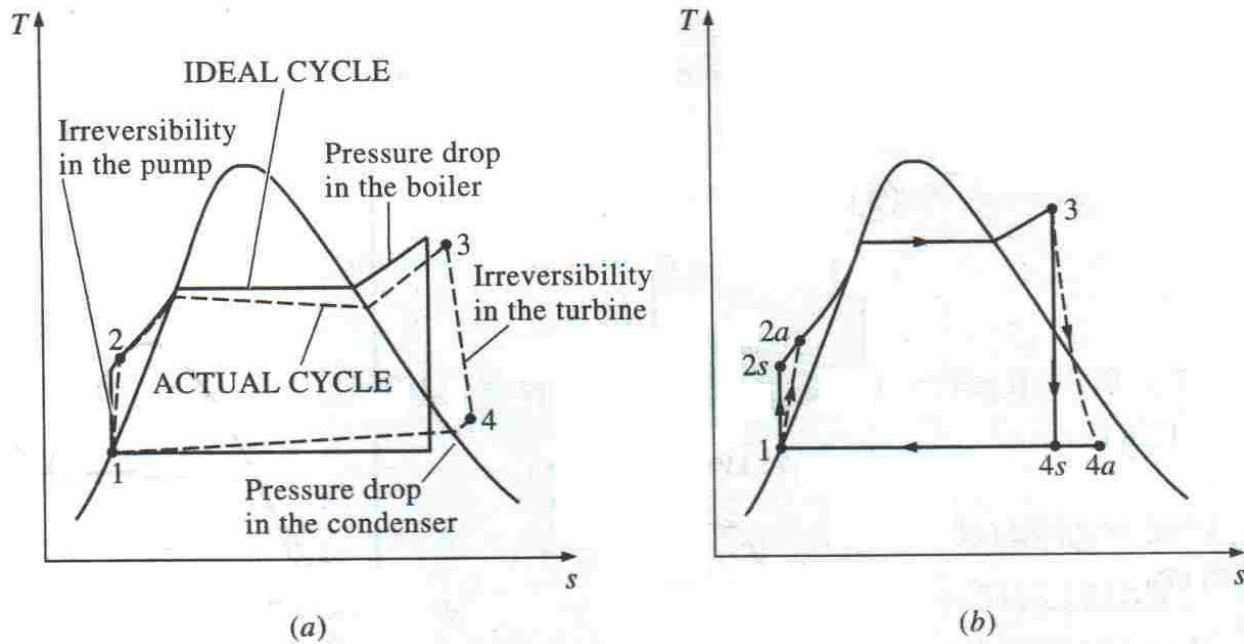


Fig. Effect of superheating

The moisture in the steam at the end of the expansion may be reduced by increasing the super heated temperature of steam t_2 . This can be seen in figure where $t_2''' > t_2'' > t_2'$, but $x_{3'} < x_{3''} < x_{3'''}$. It is, therefore, natural that to avoid erosion of the turbine blades, an increase in the boiler pressure must be accompanied by super heating at a higher temperature and since this raises the mean average temperature at which heat is transferred to the steam, the Rankine η increases.

Deviation of Actual Vapour Power cycles from Ideal cycle



The actual Vapour power cycle differs from the ideal Rankine cycle, as shown in figure, as a result of irreversibilities in various components mainly because of fluid friction and heat loss to the surroundings.

Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various
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components. As a result, steam leaves the boiler at a lower pressure. Also the pressure at the turbine inlet is lower than that at the boiler exit due to pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate these pressure drops, the water must be pumped to sufficiently higher pressure which requires the larger pump and larger work input to the pump.

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.

As a result of irreversibilities, a pump requires a greater work input, and a turbine produces a smaller work output. Under the ideal conditions, the flow through these devices are isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

$$\eta_P = \frac{W_S}{W_a} = \frac{h_{1S} - h_4}{h_1 - h_4}$$
$$\text{And } \eta_t = \frac{W_a}{W_S} = \frac{h_2 - h_3}{h_2 - h_{3S}}$$

Problems:

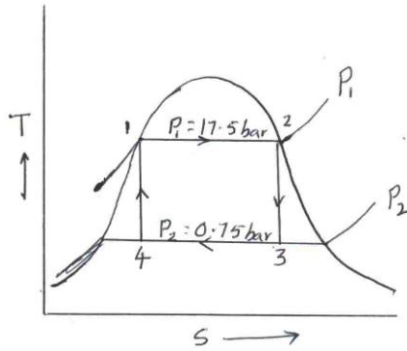
- 1. Dry saturated steam at 17.5 bar enters the turbine of a steam power plant and expands to the condenser pressure of 0.75 bar. Determine the Carnot and Rankine cycle efficiencies. Also find the work ratio of the Rankine cycle.**

Solution: $P_1 = 17.5 \text{ bar}$ $P_2 = 0.75 \text{ bar}$ $\eta_{\text{Carnot}} = ?$ $\eta_{\text{Rankine}} = ?$ a)

Carnot cycle: At pressure 17.5 bar from steam tables,



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P	t_s	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
17	204.3	871.8	1921.6	2793.4	2.3712	4.0246	6.3958
18	207.11	884.5	1910.3	2794.8	2.3976	3.9776	6.3751

For $P = 17.5$ bar, using linear interpolation



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$$\text{For } t_s, 204.3 + \frac{207.11 - 204.3}{1} \times 0.5 = 205.71^\circ \text{C}$$

$$= 478.71 \text{ K}$$

Similarly, $h_f = 878.15 \text{ kJ/kg}$ $h_{fg} = 1915.95 \text{ kJ/kg}$ $h_g = 2794.1 \text{ kJ/kg}$
 $S_f = 2.3844 \text{ kJ/kg}^\circ \text{K}$ $S_{fg} = 4.0011 \text{ kJ/kg}^\circ \text{K}$ $S_g = 6.3855 \text{ kJ/kg}^\circ \text{K}$

Also at pressure 0.75 bar from steam tables

P	t_s	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
0.8	93.51	391.7	2274.0	2665.8	1.233	6.2022	7.435
0.7	89.96	376.8	2283.3	2660.1	1.1921	6.2883	7.480

∴ For 0.75 bar, using linear interpolation,

$$t_s = 91.74^\circ \text{C} \quad h_f = 384.25 \quad h_{fg} = 2278.65 \quad h_g = 2662.95$$

$$S_f = 1.2126 \quad S_{fg} = 6.2453 \quad S_g = 7.4578$$

The Carnot cycle η , $\eta_C = \frac{T_1 - T_2}{T_1} = \frac{478.71 - 364.74}{478.71} = 0.2381$

$$\frac{1}{0} \quad \frac{1}{1}$$

Steam rate or SSC = $\delta W = W_T - W_P$

Since the expansion work is isentropic, $S_2 = S_3$

But $S_2 = S_g = 6.3855$ and $S_3 = S_{f3} + x_3 S_{fg3}$

i.e., $6.3855 = 1.2126 + x_3 (6.2453)$ ∴ $x_3 = 0.828$

∴ Enthalpy at state 3, $h_3 = h_{f3} + x_3 h_{fg3}$
 $= 384.25 + 0.828 (2278.65) = 2271.63 \text{ kJ/kg}$

∴ Turbine work or expansion work or positive work = $h_2 - h_3$
 $= 2794.1 - 2271.63 = 522.47 \text{ kJ/kg}$

Again since the compression process is isentropic i.e.,

$S_4 = S_1 = S_{f1} =$



2.3844

$$\begin{aligned} \text{Hence } 2.3844 &= S_{f4} + x_4 S_{fg4} \\ &= 1.2126 + x_4 (6.2453) \therefore x_4 = 0.188 \end{aligned}$$

\therefore Enthalpy at state 4 is $h_4 = h_{f4} + x_4 h_{fg4}$

$$\begin{aligned} &= 384.25 + 0.188 (2278.65) \\ &= 811.79 \text{ kJ/kg} \end{aligned}$$

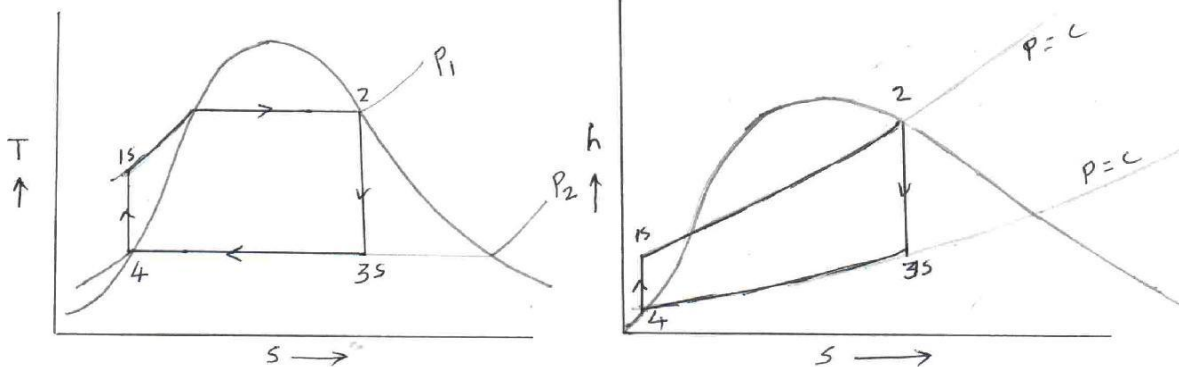
$$\begin{aligned} \therefore \text{Compression work, } &= h_1 - h_4 = 878.15 - \\ &811.79 \text{ W}_P = \\ &66.36 \text{ kJ/kg} \end{aligned}$$

$$\therefore SSC = \frac{1}{522.47 - 66.36} = 2.192 \times 10^{-3} \text{ kg/kJ}$$

$$\text{work ratio} = r = \frac{\dot{Q}_w}{w} = \frac{W - W_P}{W_T} = \frac{456.11}{522.47} = 0.873$$

+ ve work

b) Rankine cycle:



$$\eta_R = \frac{W_T - W_P}{Q_H} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

Since the change in volume of the saturated liquid water during compression from state 4 to state 1 is very small, v_4 may be taken as constant. In a steady flow process, work $W = -v dp$

$$\begin{aligned} \therefore W_P &= h_{1S} - h_4 = v_{P2} (P_1 - P_2) \\ &= 0.001037 (17.5 - 0.75) \times 10^5 \times (1/1000) \\ &= 1.737 \text{ kJ/kg} \end{aligned}$$

$$\therefore h_{1S} = 1.737 + 384.25 = 385.99 \text{ kJ/kg}$$

$$\text{Hence, turbine work} = W_T = h_2 - h_3 = 522.47 \text{ kJ/kg}$$

$$\text{Heat supplied} = Q_H = h_2 - h_{1S} = 2794.1 - 385.99 = 2408.11 \text{ kJ/kg}$$



$$\therefore \eta_R = \frac{522.47 - 1.737}{2408.11} = 0.2162$$

$$\therefore SSC = \frac{1}{522.47 - 1.737} = 19204 \times 10^{-3} \text{ kg / kJ}$$

$$\text{Work ratio, } r^w = \frac{522.47 - 1.737}{522.47} = 0.9967$$

2. If in problem (1), the turbine and the pump have each 85% efficiency, find the % reduction in the net work and cycle efficiency for Rankine cycle.

Solution: If $\eta_P = 0.85$, $\eta_T = 0.85$

$$W_P = \frac{1}{0.85} \times 0.85 \times 1.737 = 2.0435 \text{ kJ / kg}$$

$$W_T = \eta_T W_T = 0.85 (522.47) = 444.09 \text{ kJ/kg}$$

$$\therefore W_{\text{net}} = W_T - W_P = 442.06 \text{ kJ/kg}$$

$$\therefore \% \text{ reduction in work output} = \frac{520.73 - 442.06}{520.73} = 15.11\%$$

$$W_P = h_{1S} - h_4 \therefore h_{1S} = 2.0435 + 384.25 = 386.29 \text{ kJ/kg}$$

$$\therefore Q_H - h_2 - h_{1S} = 2794.1 - 386.29 = 2407.81 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{442.06}{2407.81} = 0.1836$$

$$\therefore \% \text{ reduction in cycle efficiency} = \frac{0.2162 - 0.1836}{0.2162} = 15.08\%$$

Note: Alternative method for problem 1 using h-s diagram (Mollier diagram) though the result may not be as accurate as the analytical solution. The method is as follows



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Since steam is dry saturated at state 2, locate this state at the pressure $P_2 = 17.5$ bar on the saturation line and read the enthalpy at this state. This will give the value of h_2 .

As the expansion process 2-3 is isentropic, draw a vertical line through the state 2 to meet the pressure line, $P = 0.75$ bar. The intersection of the vertical line with the pressure line will fix state 3. From the chart, find the value of h_3 .

The value of h_4 can be found from the steam tables at pressure, $P = 0.75$ bar, as $h_4 = h_{f4}$. After finding the values of h_2 , h_3 and h_4 , apply the equation used in the analytical solution for determining the Rankine cycle η and SSC.



Effect of Boiler Pressure (Using Mollier Diagram i.e., h-s diagram)

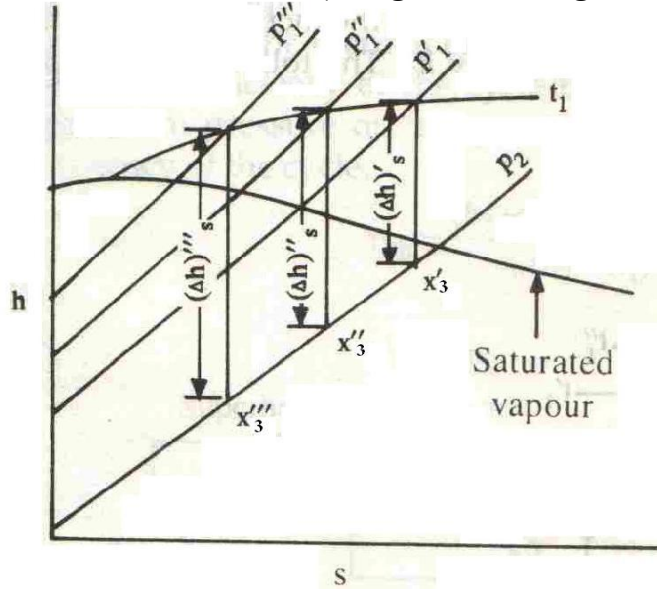


Fig. 13.5 Effect of boiler pressure

We have,

$$\eta_{th} = \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1} \text{ but } W_P \ll W_T$$

$$\therefore \eta_{th} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{(\Delta h)_S}{(h_2 - h_1)}$$

i.e., Rankine cycle η depends on h_2 , h_1 and Δh_S . From figure as $P_1''' > P_1'' > P_1'$ for the fixed maximum temperature of the steam t_1 and condenser pressure P_2 , Isentropic heat drops increases with boiler pressure i.e., from the figure therefore it is evident that as boiler pressure increases, the isentropic heat drop $(\Delta h)_S$ increases, but the enthalpy of the steam entering the turbine decreases, with the result that the Rankine η increases. But quality of the steam at the exit of the turbine suffers i.e., $x_3''' < x_3'' < x_3'$, which leads to serious wear of the turbine blades.



Effect of Super Heating (Using Molliar Diagram i.e., h-s diagram)

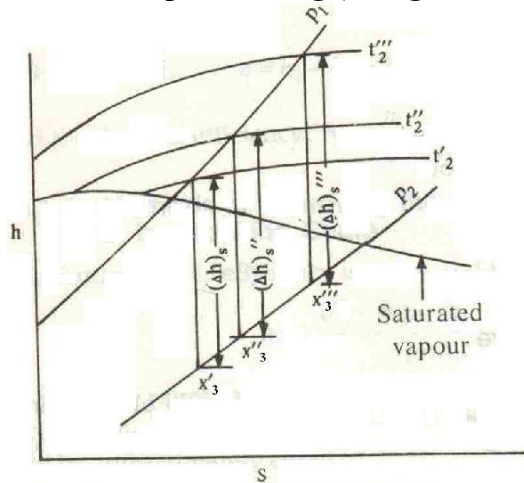
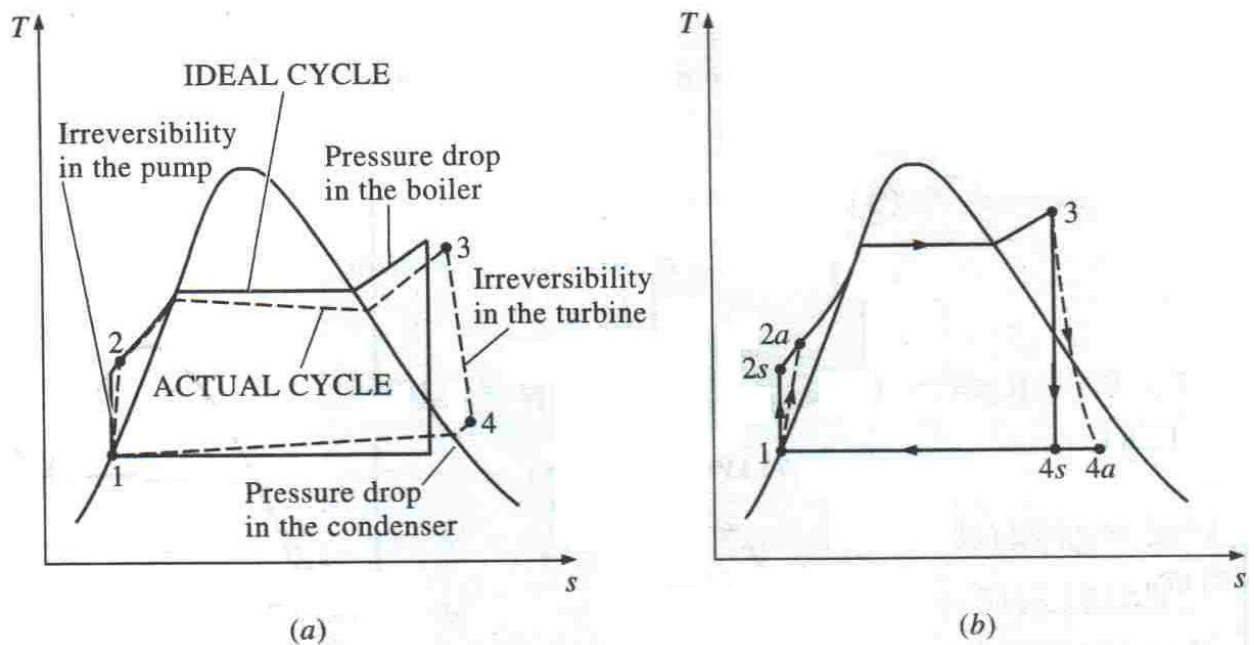


Fig. Effect of superheating

The moisture in the steam at the end of the expansion may be reduced by increasing the super heated temperature of steam t_1 . This can be seen in figure where $t_1''' > t_1'' > t_1'$, but $x_{3'} < x_{3''} < x_{3'''}$. It is, therefore, natural that to avoid erosion of the turbine blades, an increase in the boiler pressure must be accompanied by super heating at a higher temperature and since this raises the mean average temperature at which heat is transferred to the steam, the Rankine η increases.

Deviation of Actual Vapour Power cycles from Ideal cycle





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The actual Vapour power cycle differs from the ideal Rankine cycle, as shown in figure, as a result of irreversibilities in various components mainly because of fluid friction and heat loss to the surroundings.

Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various components. As a result, steam leaves the boiler at a lower pressure. Also the pressure at the turbine inlet is lower than that at the boiler exit due to pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate these pressure drops, the water must be pumped to sufficiently higher pressure which requires the larger pump and larger work input to the pump.

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.

As a result of irreversibilities, a pump requires a greater work input, and a turbine produces a smaller work output. Under the ideal conditions, the flow through these devices are isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

$$\eta_P = \frac{W_S}{W_a} = \frac{h_{1S} - h_4}{h_1 - h_4}$$
$$\text{And } \eta_t = \frac{W_a}{W_S} = \frac{h_2 - h_3}{h_2 - h_{3S}}$$

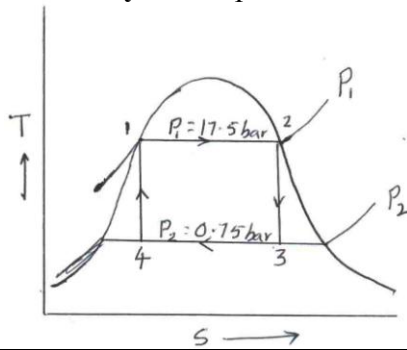


Numerical Problems:

- 1. Dry saturated steam at 17.5 bar enters the turbine of a steam power plant and expands to the condenser pressure of 0.75 bar. Determine the Carnot and Rankine cycle efficiencies. Also find the work ratio of the Rankine cycle.**

Solution: $P_1 = 17.5 \text{ bar}$ $P_2 = 0.75 \text{ bar}$ $\eta_{\text{Carnot}} = ?$ $\eta_{\text{Rankine}} = ?$ a)

Carnot cycle: At pressure 17.5 bar from steam tables,



P	t_s	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
17	204.3	871.8	1921.6	2793.4	2.3712	4.0246	6.3958
18	207.11	884.5	1910.3	2794.8	2.3976	3.9776	6.3751

For $P = 17.5 \text{ bar}$, using linear interpolation



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Also at pressure 0.75 bar from steam tables

P	t_s	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
0.8	93.51	391.7	2274.0	2665.8	1.233	6.2022	7.4352
0.7	89.96	376.8	2283.3	2660.1	1.1921	6.2883	7.4804

∴ For 0.75 bar, using linear interpolation,

$$t_s = 91.74^\circ\text{C} \quad h_f = 384.25 \quad h_{fg} = 2278.65 \quad h_g = 2662.95$$
$$S_f = 1.2126 \quad S_{fg} = 6.2453 \quad S_g = 7.4578$$

$$\text{The Carnot cycle } \eta, \eta_C = \frac{T_1 - T_2}{T_1} = \frac{478.71 - 364.74}{478.71} = 0.2381$$

$$\frac{1}{\circ} \quad \frac{1}{\circ}$$

$$\text{Steam rate or SSC} = \delta W = W_T - W_P$$

Since the expansion work is isentropic, $S_2 = S_3$

$$\text{But } S_2 = S_g = 6.3855 \text{ and } S_3 = S_{f3} + x_3 S_{fg3}$$

$$\text{i.e., } 6.3855 = 1.2126 + x_3 (6.2453) \quad \therefore x_3 = 0.828$$

$$\therefore \text{Enthalpy at state 3,} \quad h_3 = h_{f3} + x_3 h_{fg3}$$
$$= 384.25 + 0.828 (2278.65) = 2271.63 \text{ kJ/kg}$$

$$\therefore \text{Turbine work or expansion work or positive work} = h_2 - h_3$$
$$= 2794.1 - 2271.63 = 522.47 \text{ kJ/kg}$$

Again since the compression process is isentropic

$$\text{i.e., } S_4 = S_1 = S_{f1} = 2.3844$$

$$\text{Hence } 2.3844 = S_{f4} + x_4 S_{fg4}$$
$$= 1.2126 + x_4 (6.2453) \quad \therefore x_4 = 0.188$$



∴ Enthalpy at state 4 is $h_4 = h_{f4} + x_4 h_{fg4}$

$$= 384.25 + 0.188 (2278.65)$$

$$= 811.79 \text{ kJ/kg}$$

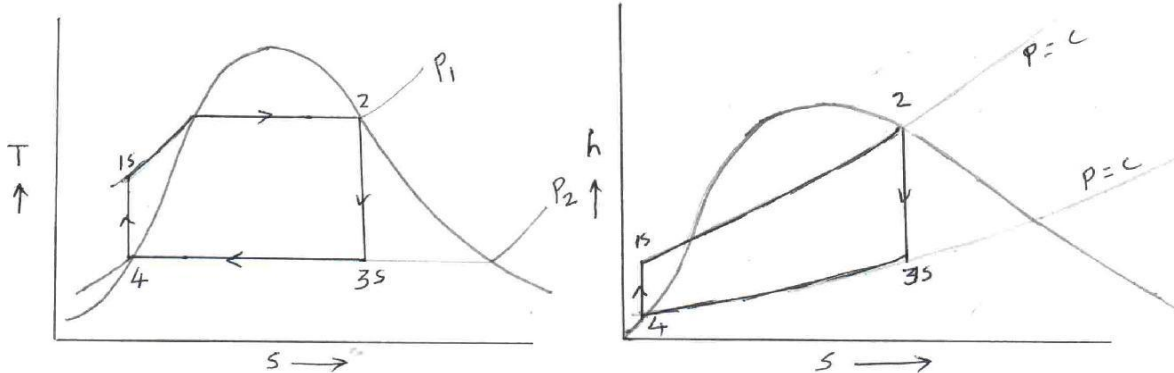
∴ Compression work, $= h_1 - h_4 = 878.15 - 811.79$

$$W_P = 66.36 \text{ kJ/kg}$$

$$\therefore SSC = \frac{1}{522.47 - 66.36} = 2.192 \times 10^{-3} \text{ kg / kJ}$$

$$\text{work ratio} = r = \frac{\oint \delta w}{\text{+ ve work}} = \frac{W - W_P}{W_T} = \frac{456.11}{522.47} = 0.873$$

b) Rankine cycle:



$$\eta_R = \frac{W_T - W_P}{Q_H} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

Since the change in volume of the saturated liquid water during compression from state 4 to state 1 is very small, v_4 may be taken as constant. In a steady flow process, work $W = -v dp$

$$\therefore W_P = h_{1s} - h_4 = v_{f4} (P_1 - P_2)$$

$$= 0.001037 (17.5 - 0.75) \times 10^5 \times (1/1000)$$

$$= 1.737 \text{ kJ/kg}$$

$$\therefore h_{1s} = 1.737 + 384.25 = 385.99 \text{ kJ/kg}$$

Hence, turbine work $= W_T = h_2 - h_3 = 522.47 \text{ kJ/kg}$

Heat supplied $= Q_H = h_2 - h_{1s} = 2794.1 - 385.99 = 2408.11 \text{ kJ/kg}$

$$\therefore \eta_R = \frac{522.47 - 1.737}{2408.11} = 0.2162$$



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$$\therefore SSC = \frac{1}{522.47 - 1.737} = 19204 \times 10^{-3} \text{ kg / kJ}$$

$$\text{Work ratio, } r^w = \frac{522.47 - 1737}{522.47} = 0.9967$$



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2. If in problem (1), the turbine and the pump have each 85% efficiency, find the % reduction in the net work and cycle efficiency for Rankine cycle.

Solution: If $\eta_P = 0.85$, $\eta_T = 0.85$

$$W_P = \frac{1}{0.85} \cdot 737 = 2.0435 \text{ kJ/kg}$$

$$W_T = \eta_T W_T = 0.85 (522.47) = 444.09 \text{ kJ/kg}$$

$$\therefore W_{\text{net}} = W_T - W_P = 442.06 \text{ kJ/kg}$$

$$\therefore \% \text{ reduction in work output} = \frac{520.73 - 442.06}{520.73} = 15.11\%$$

$$W_P = h_{1S} - h_4 \therefore h_{1S} = 2.0435 + 384.25 = 386.29 \text{ kJ/kg}$$

$$\therefore Q_H - h_2 - h_{1S} = 2794.1 - 386.29 = 2407.81 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{442.06}{2407.81} = 0.1836$$

$$\therefore \% \text{ reduction in cycle efficiency} = \frac{0.2162 - 0.1836}{0.2162} = 15.08\%$$

Note: Alternative method for problem 1 using h-s diagram (Mollier diagram) though the result may not be as accurate as the analytical solution. The method is as follows

Since steam is dry saturated at state 2, locate this state at the pressure $P_2 = 17.5$ bar on the saturation line and read the enthalpy at this state. This will give the value of h_2 .

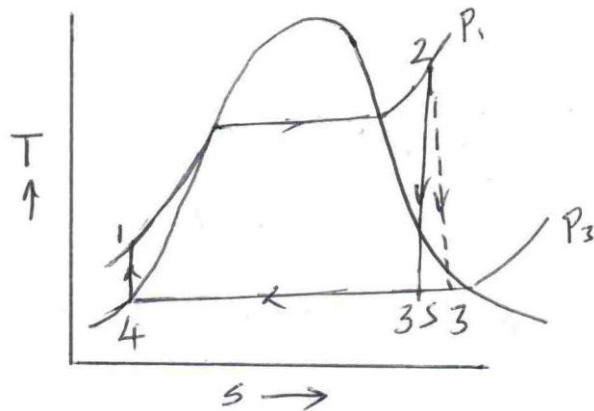
As the expansion process 2-3 is isentropic, draw a vertical line through the state 2 to meet the pressure line, $P = 0.75$ bar. The intersection of the vertical line with the pressure line will fix state 3. From the chart, find the value of h_3 .

The value of h_4 can be found from the steam tables at pressure, $P = 0.75$ bar, as $h_4 = h_{f4}$. After finding the values of h_2 , h_3 and h_4 , apply the equation used in the analytical solution for determining the Rankine cycle η and SSC.



3. Steam enters the turbine of a steam power plant, operating on Rankine cycle, at 10 bar, 300°C. The condenser pressure is 0.1 bar. Steam leaving the turbine is 90% dry. Calculate the adiabatic efficiency of the turbine and also the cycle η , neglecting pump work.

Solution:



$$P_1 = 10 \text{ bar} \quad t_2 = 300^\circ\text{C} \quad P_3 = 0.1 \text{ bar}$$

$$x_3 = 0.9 \quad \eta_t = ? \quad \eta_{\text{cycle}} = ? \quad \text{Neglect } W_p$$

From superheated steam tables,

$$\text{For } P_2 = 10 \text{ bar and } t_2 = 300^\circ\text{C}, h_2 = 3052.1 \text{ kJ/kg}, s_2 = 7.1251 \text{ kJ/kg}$$

From table A - 1, For $P_3 = 0.1 \text{ bar}$

$$t_s = 45.83^\circ\text{C} \quad h_f = 191.8 \quad h_{fg} = 2392.9$$

$$S_f = 0.6493 \quad S_{fg} = 7.5018$$

$$\begin{aligned} \text{Since } x_3 = 0.9, \quad h_3 &= h_f + x_3 h_{fg} \\ &= 191.8 + 0.9 (2392.9) \\ &= 2345.4 \text{ kJ/kg} \end{aligned}$$

Also, since process 2-3s is isentropic, $S_2 =$

$$\begin{aligned} S_{3s} \text{ i.e., } 7.1251 &= S_{fg4} + x_{3s} S_{fg3} \\ &= 0.6493 + x_{3s} (7.5018) \end{aligned}$$

$$\therefore x_{3s} = 0.863$$



$$\therefore h_{3S} = 191.8 + 0.863 (2392.9) = 2257.43 \text{ kJ/kg}$$

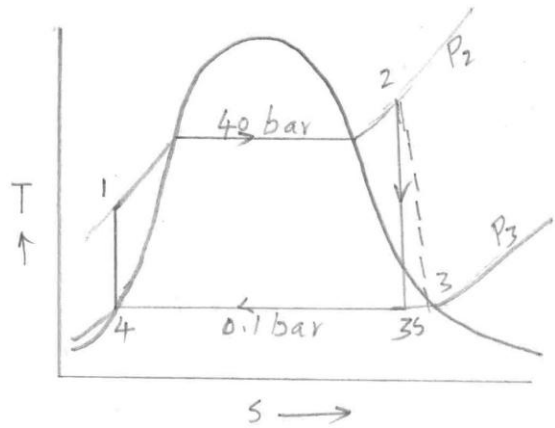
$$\therefore \text{Turbine efficiency, } \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} = \frac{3052.1 - 2345.4}{3052.1 - 2257.43} = 0.89$$

$$\eta_{\text{cycle}} = \frac{W_T}{Q_H} = \frac{h_2 - h_3}{h_2 - h_1} \quad \text{but } h_1 = 191.8 \text{ kJ/kg}$$

$$= \frac{3052.1 - 2345.4}{3052.1 - 191.8} = 0.25 \text{ i.e., } 25\%$$

4. A 40 mW steam plant working on Rankine cycle operates between boiler pressure of 4 MPa and condenser pressure of 10 KPa. The steam leaves the boiler and enters the steam turbine at 400°C. The isentropic η of the steam turbine is 85%. Determine (i) the cycle η (ii) the quality of steam from the turbine and (iii) the steam flow rate in kg per hour. Consider pump work.

Solution:



$$P_2 = 4 \text{ MPa} = 40 \text{ bar} \quad P_3 = 10 \text{ KPa} = 0.1 \text{ bar}$$

$$P = 40000 \text{ kW} \quad t_2 = 400^\circ \text{C} \quad \eta_t = 0.85 \quad \eta_{\text{cycle}} = ? \quad x_3 = ?$$

$$m = ?$$

$$h_2 = h_{g|40 \text{ bar}, 400^\circ \text{C}} = 3215.7 \text{ kJ/kg} \text{ and } s_2 = 6.7733 \text{ kJ/kg-K}$$

$$h_4 = h_{f|0.1 \text{ bar}} = 191.8 \text{ kJ/kg}$$



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Process 2-3s is isentropic i.e., $S_2 = S_{3s}$

$$6.7733 = 0.6493 + x_{3s} (7.5018)$$

$$\therefore x_{3s} = 0.816$$

$$\therefore h_{3s} = h_{f3} + x_{3s} h_{fg3} = 191.8 + 0.816 (2392.9)$$

$$= 2145.2 \text{ kJ/kg}$$

$$\text{But } \eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} \quad \text{i.e., } 0.85 = \frac{3215.7 - h_3}{3215.7 - 2145.2}$$

$$\therefore h_3 = 2305.8 \text{ kJ/kg}$$

$$\therefore W_T = h_2 - h_3 = 3215.7 - 2305.8 = 909.9 \text{ kJ/kg}$$

$$W_P = v \, dP = 0.0010102 (40 - 0.1) 10^5 / 10^2$$

$$= 4.031 \text{ kJ/kg}$$

$$= h_1 - h_4 \quad \therefore h_1 = 195.8 \text{ kJ/kg}$$

$$(i) \eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_1} = \left(\frac{909.9 - 4.031}{3215.7 - 195.8} \right) = 29.9\%$$

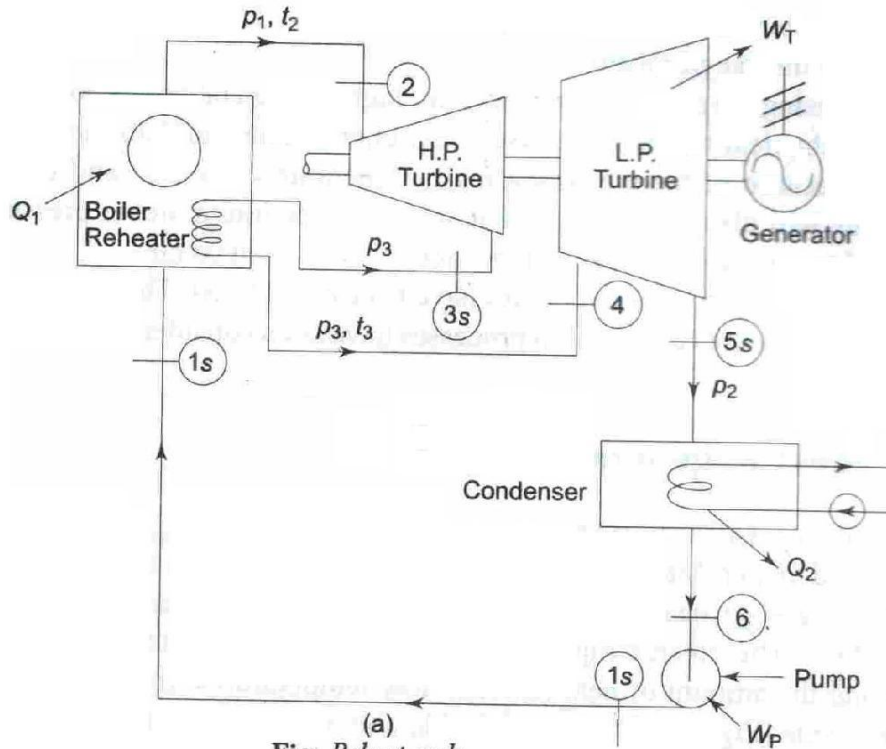
$$(ii) x_3 = ? \quad \text{we have } 2305.8 = 191.8 + x_3 (2392.9) \quad \therefore x_3 = 0.88$$

$$(iii) P = m W_{\text{net}} \quad \text{i.e., } 40000 = m (905.87)$$

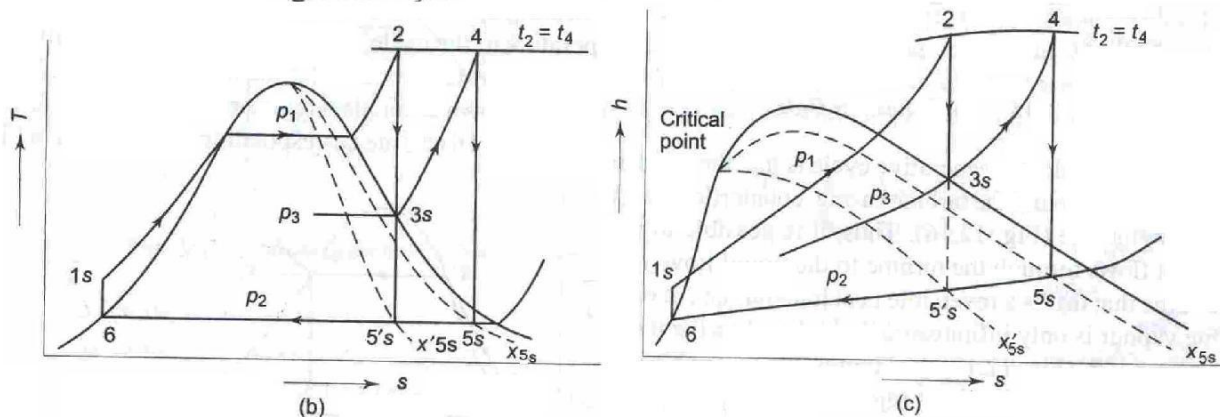
$$\therefore m = 44.2 \text{ kg/s}$$

$$= 159120 \text{ kg/hr}$$

Ideal Reheat cycle: We know that, the efficiency of the Rankine cycle could be increased by increasing steam pressure in the boiler and superheating the steam. But this increases the moisture content of the steam in the lower pressure stages in the turbine, which may lead to erosion of the turbine blade. \therefore The reheat cycle has been developed to take advantage of the increased pressure of the boiler, avoiding the excessive moisture of the steam in the low pressure stages. In the reheat cycle, steam after partial expansion in the turbine is brought back to the boiler, reheated by combustion gases and then fed back to the turbine for further expansion.



(a)
Fig. Reheat cycle



In the reheat cycle the expansion of steam from the initial state (2) to the condenser pressure is carried out in two or more steps, depending upon the number of reheats used.

In the first step, steam expands in HP turbine from state 2 to approximate the saturated vapour line (process 2-3s). The steam is then reheated (or resuperheated) at constant pressure in the boiler (or in a reheater) process 3s-4 and the remaining expansion process 4s-5 is carried out in the LP turbine.

Note: 1) To protect the reheater tubes, steam is not allowed to expand deep into the two-phase region before it is taken for reheating, because in that case the moisture particles in steam while evaporating would leave behind solid deposits in the form of scale which is difficult to remove.



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Also a low reheat pressure may bring down T_{m1} and hence cycle η . Again a high reheat pressure increases the moisture content at turbine exhaust. Thus reheat pressure is optimized. Optimum reheat pressure is about 0.2 to 0.25 of initial pressure.

We have for 1 kg of steam

$$Q_H = (h_2 - h_{1S}) + (h_4 - h_{3S}); \quad Q_L = h_{5S} - h_6$$

$$W_T = (h_2 - h_{3S}) + (h_4 - h_{5S}); \quad W_P = h_{1S} - h_6$$

$$\therefore \eta_R = \frac{W_T - W_P}{Q_H}$$

$$\text{Steam rate} = \frac{3600}{W_T - W_P} \text{ kg / kWh}$$

Since higher reheat pressure is used, W_P work is appreciable.

2) In practice, the use of reheat gives a marginal increase in cycle η , but it increases the net work output by making possible the use of higher pressures, keeping the quality of steam at turbine exhaust within a permissible limit. The quality improves from x_{51S} to x_{5S} by the use of reheat.



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Feedwater Heaters (FWH)

A practical Regeneration process in steam power plants is accomplished by extracting or bleeding, steam from the turbine at various points. This steam, which could have produced more work by expanding further in the turbine, is used to heat the feed water instead. The device where the feedwater heated by regeneration is called a Regenerator or a Feedwater Heater (FWH).

A feedwater heater is basically a heat exchanger where heat is transferred from the steam to the feedwater either by mixing the two streams (open feedwater heaters) or without mixing them (closed feedwater heaters).

Open Feedwater Heaters

An open (or direct-contact) feedwater heater is basically a mixing chamber, where the steam extracted from the turbine mixes with the feedwater exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure.

The advantages of open heater are simplicity, lower cost, and high heat transfer capacity. The disadvantage is the necessity of a pump at each heater to handle the large feedwater stream.

Closed Feedwater Heaters

In closed feedwater heater, the heat is transferred from the extracted steam to the feedwater without mixing taking place. The feedwater flows through the tubes in the heater and extracted steam condenses on the outside of the tubes in the shell. The heat released from the condensation is transferred to the feedwater through the walls of the tubes. The condensate (saturated water at the steam extraction pressure), some times called the heater-drip, then passes through a trap into the next lower pressure heater. This, to some extent, reduces the steam required by that heater. The trap passes only liquid and no vapour. The drip from the lowest pressure heater could similarly be trapped to the condenser, but this would be throwing away energy to the condenser cooling water. To avoid this waste, the drip pump feeds the drip directly into the feedwater stream.

A closed heaters system requires only a single pump for the main feedwater stream regardless of the number of heaters. The drip pump, if used is relatively small. Closed heaters are costly and may not give as high a feedwater temperature as do open heaters.

In most steam power plants, closed heaters are favoured, but atleast one open heater is used, primarily for the purpose of feedwater deaeration. The open heater in such a system is called deaerator.

Note: The higher the number of heater used, the higher will be the cycle efficiency. The number of heater is fixed up by the energy balance of the whole plant when it is found that the cost of Applied Thermodynamics, HIT, Nidasoshi



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adding another does not justify the saving in Q_H or the marginal increase in cycle efficiency. An increase in feedwater temperature may, in some cases, cause a reduction in boiler efficiency. So the number of heaters get optimized. Five feedwater heaters are often used in practice.

Characteristics of an Ideal working fluid

The maximum temperature that can be used in steam cycles consistent with the best available material is about 600°C , while the critical temperature of steam is 375°C , which necessitates large superheating and permits the addition of only an infinitesimal amount of heat at the highest temperature.

The desirable characteristics of the working fluid in a vapour power cycle to obtain best thermal η are as follows:

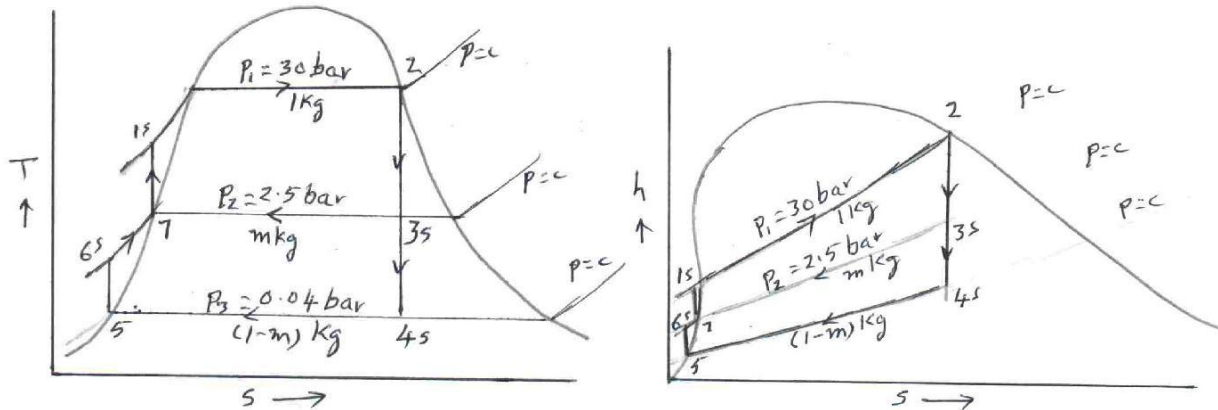
- a) The fluid should have a high critical temperature so that the saturation pressure at the maximum permissible temperature (metallurgical limit) is relatively low. It should have a large enthalpy of evaporation at that pressure.
- b) The saturation pressure at the temperature of heat rejection should be above atmosphere pressure so as to avoid the necessity of maintaining vacuum in the condenser.
- c) The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to the boiling point.
- d) The saturation vapour line of the T-S diagram should be steep, very close to the turbine expansion process so that excessive moisture does not appear during expansion.
- e) The freezing point of the fluid should be below room temperature, so that it does not get solidified while flowing through the pipe lines.
- f) The fluid should be chemically stable and should not contaminate the materials of construction at any temperature.
- g) The fluid should be nontoxic, non corrosive, not excessively viscous, and low in cost.



Numerical Problems:

1. An ideal regenerative cycle operates with dry saturated steam, the maximum and minimum pressures being 30 bar and 0.04 bar respectively. The plant is installed with a single mixing type feed water heater. The bled steam pressure is 2.5 bar. Determine (a) the mass of the bled steam, (b) the thermal η of the cycle, and (c) SSC in kg/kWh.

Solution:



$$P_1 = 30 \text{ bar} \quad P_2 = 2.5 \text{ bar} \quad P_3 = 0.04 \text{ bar}$$

From steam tables, For $P_1 = 30 \text{ bar}$, $h_2 = 2802.3 \text{ kJ/kg}$, $S_2 = 6.1838 \text{ kJ/kg}^{\circ}\text{K}$

But $S_2 = S_{3s}$ i.e., $6.1838 = 1.6072 + x_3 (5.4448)$

$$\therefore x_3 = 0.841$$

$$\therefore h_3 = 535.4 + 0.841 (2281.0) = 2452.68 \text{ kJ/kg}$$

Also $S_2 = S_{4s}$ i.e., $6.1838 = 0.4225 + x_4 (8.053)$

$$\therefore x_4 = 0.715$$

$$\therefore h_4 = 121.4 + 0.715 (2433.1) = 1862.1 \text{ kJ/kg}$$

At $P_3 = 0.04 \text{ bar}$, $h_5 = 121.4 \text{ kJ/kg}$, $v_5 = 0.001004 \text{ m}^3/\text{kg}$

$$\begin{aligned} \therefore \text{Condensate pump work} &= (h_6 - h_5) = v_5 (P_2 - P_3) \\ &= 0.001004 (2.5 - 0.04) (10^5/10^3) \\ &= 0.247 \text{ kJ/kg} \end{aligned}$$

$$\therefore h_6 = 0.247 + 121.4 = 121.65 \text{ kJ/kg}$$

Similarly, $h_1 = h_7 + v_7 (P_1 - P_2) (10^5/10^3)$



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$$= 535.4 + 0.0010676 (30 - 2.5) 10^2$$

$$= 538.34 \text{ kJ/kg}$$

a) Mass of the bled steam:

Applying the energy balance to the feed water heater

$$mh_3 + (1 - m) h_6 = 1 (h_7)$$

$$\frac{(h_7 - h_6)}{(h_3 - h_6)} = \frac{(535.4 - 121.65)}{(2452.68 - 121.65)}$$

$$\therefore m = (h_3 - h_6) = (2452.68 - 121.65) = 0.177 \text{ kg / kg of steam}$$

b) Thermal η :

$$\text{Turbine work, } W_T = 1 (h_2 - h_{3s}) + (1 - m) (h_3 - h_{4s})$$

$$= 1 (2802.3 - 2452.65) + (1 - 0.177) (2452.68 - 1862.1)$$

$$= 835.67 \text{ kJ/kg}$$

$$\text{Pump work, } W_P = (1 - m) (h_{6s} - h_5) + 1 (h_{1s} - h_7)$$

$$= (1 - 0.177) (121.65 - 121.4) + 1 (538.34 - 535.4)$$

$$= 3.146 \text{ kJ/kg}$$

$$\therefore W_{\text{net}} = W_T - W_P = 832.52 \text{ kJ/kg}$$

$$\text{Heat supplied, } Q_H = 1 (h_2 - h_{1s})$$

$$= 1 (2802.3 - 538.34)$$

$$= 2263.96 \text{ kJ/kg}$$

$$\therefore \eta_{th} = \frac{W_{\text{net}}}{Q_H} = \frac{832.52}{2263.96} = 0.368 \text{ or } 36.8\%$$

c) SSC:

$$\text{SSC} = \frac{3600}{W_{\text{net}}} = 4.324 \text{ kg / kWh}$$

2. In problem (3), also calculate the increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration)

$$\text{Solution: } T_{m1} \text{ (with regeneration)} = \frac{h_2 - h_1}{h_2 - h_6} = \frac{2263.96}{2802.3 - 121.65} = 494.68 \text{ k}$$

$$T_{m1} \text{ (without regeneration)} = \frac{h_2 - h_1}{h_2 - h_6} = \frac{2263.96}{2802.3 - 0.4225} = 465.29 \text{ k}$$

$$\therefore \text{Increase in } T_{m1} \text{ due to regeneration} = 494.68 - 465.29 = 29.39^\circ \text{K}$$

$$W_T \text{ (without regeneration)} = h_2 - h_4 = 2802.3 - 1862.1 = 940.2 \text{ kJ/kg}$$



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$$W_p \text{ (without regeneration)} = (h_1 - h_5) = v_5 (30 - 0.04) 10^2$$

$$= 0.001004 (29.96) 10^2 = 3.01 \text{ kJ/kg}$$

$$\therefore h_1 = 3.01 + 121.4 = 124.41 \text{ kJ/kg}$$

$$\therefore \eta_{th} \text{ (without regeneration)} = \frac{W_{net}}{Q_H} = \frac{(940.2 - 3.01)}{2802.3 - 124.41} = 0.349$$

$$\therefore \text{Increase in } \eta_{th} \text{ due to regeneration} = 0.368 - 0.349 = 0.018 \quad \text{i.e., 1.8\%}$$

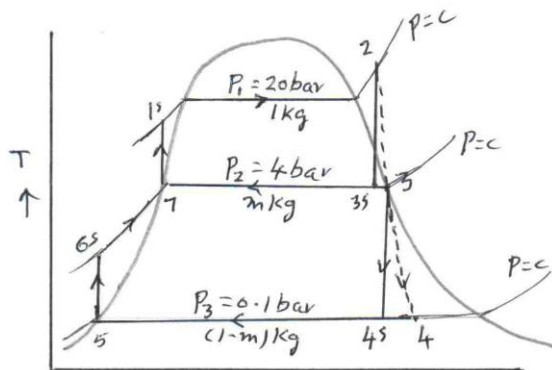
$$\text{Steam rate (without regeneration)} = 3.84 \text{ kg/kWh}$$

$$\therefore \text{Increase in steam rate due to regeneration} = 4.324 - 3.84$$

$$= 0.484 \text{ kg/kWh}$$

3. Steam at 20 bar and 300°C is supplied to a turbine in a cycle and is bled at 4 bar. The bled-steam just comes out saturated. This steam heats water in an open heater to its saturation state. The rest of the steam in the turbine expands to a condenser pressure of 0.1 bar. Assuming the turbine efficiency to be the same before and after bleeding, find:
 a) the turbine η and the steam quality at the exit of the last stage; b) the mass flow rate of bled steam 1 kg of steam flow at the turbine inlet; c) power output / (kg/s) of steam flow; and d) overall cycle η .

Solution:



$$P_1 = 20 \text{ bar} \quad t_1 = 300^\circ \text{C} \quad P_2 = 4 \text{ bar} \quad P_3 = 0.1 \text{ bar}$$

From steam tables,

$$\text{For } P_1 = 20 \text{ bar and } t_1 = 300^\circ \text{C}$$

$$v_2 = 0.12550 \quad h_2 = 3025.0 \quad S_2 = 6.7696$$

$$\text{For } P_2 = 4 \text{ bar, } h_3 = 2737.6, t_s = 143.63$$

$$h_f = 604.7, h_{fg} = 2132.9, S_f = 1.7764, S_{fg} = 5.1179, S_g = 6.8943$$

$$\text{For } P_3 = 0.1 \text{ bar, } 45.83, 191.8, 2392.9, 2584.8, 0.6493, 7.5018, 8.1511$$



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We have, $S_2 = S_{3s}$ i.e., $6.7696 = 1.7764 + x_3 (5.1179)$

$$\therefore x_3 = 0.976$$

$$\therefore h_{3s} = 604.7 + 0.976 (2132.9) = 2685.63 \text{ kJ/kg}$$

$$\therefore \eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3025 - 2737.6}{3025 - 2685.63} = 0.847$$

$S_3 = S_{4s}$ i.e., $6.8943 = 0.6493 + x_4 (7.5018)$

$$\therefore x_{4s} = 0.832$$

$$\therefore h_{4s} = 191.8 + 0.832 (2392.9) = 2183.81 \text{ kJ/kg}$$

But η^t is same before and after bleeding i.e., $\eta^t = \frac{h_3 - h_4}{h_3 - h_{4s}}$

$$\text{i.e., } 0.847 = \frac{2737.6 - h_4}{2737.6 - 2183.81}$$

$$\therefore h_4 = 2268.54 \text{ kJ/kg}$$

$$\therefore h_4 = h_{f4} + x_4 h_{fg4} \quad \therefore x_4 = 0.868$$

b) Applying energy balance to open heater, $mh_3 + (1 - m) h_{6s} = 1 (h_7)$

$$\therefore m = \frac{h_7 - h_6}{h_3 - h_6}$$

$$\text{Condensate pump work, } (h_{6s} - h_5) = v_5 (P_3 - P_2)$$

$$= 0.0010102 (3.9) 10^2 = 0.394$$

$$\text{kJ/kg } \therefore h_{6s} = 191.8 + 0.394 = 192.19 \text{ kJ/kg}$$

$$\text{Similarly, } h_{1s} = h_7 + v_7 (P_1 - P_2)$$

$$= 604.7 + 0.0010839 (16) 10^2 = 606.43 \text{ kJ/kg}$$

$$\therefore m = \frac{604.7 - 192.19}{2737.6 - 192.19} = 0.162$$

$$\text{c) Power output or } W_T = (h_2 - h_3) + (1 - m) (h_3 - h_4)$$

$$= (3025 - 2737.6) + (1 - 0.162) (2737.6 - 2268.54)$$

$$= 680.44 \text{ kJ/kg}$$

For 1kg/s of steam, $W_T = 680.44 \text{ kW}$

$$\text{d) Overall thermal efficiency, } \eta^0 = \frac{W_{net}}{Q_H}$$

$$W_P = (1 - m) (h_{6s} - h_5) + 1 (h_{1s} - h_7)$$

$$= (1 - 0.162) (192.19 - 191.8) + 1 (606.43 - 604.7)$$

$$= 2.057 \text{ kJ/kg}$$

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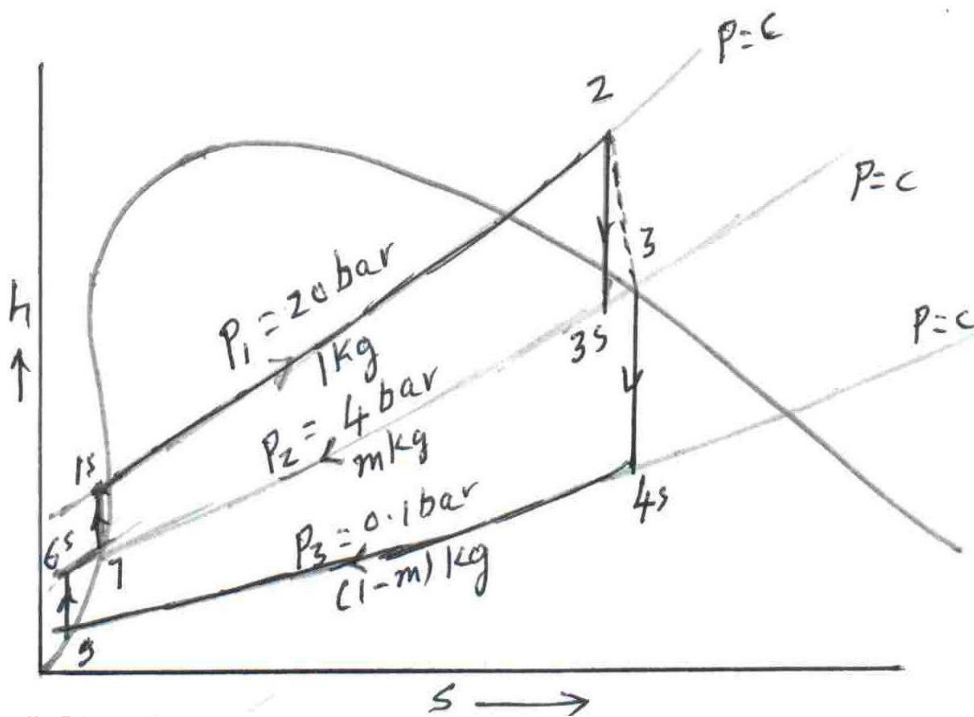


$$W_{\text{net}} = 680.44 - 2.057 = 678.38 \text{ kJ/kg}$$

$$Q_H = 1 (h_2 - h_{1s}) = (3025 - 606.43) = 2418.57 \text{ kJ/kg}$$

$$\therefore \eta_0 = \frac{678.38}{2418.57} = 0.2805$$

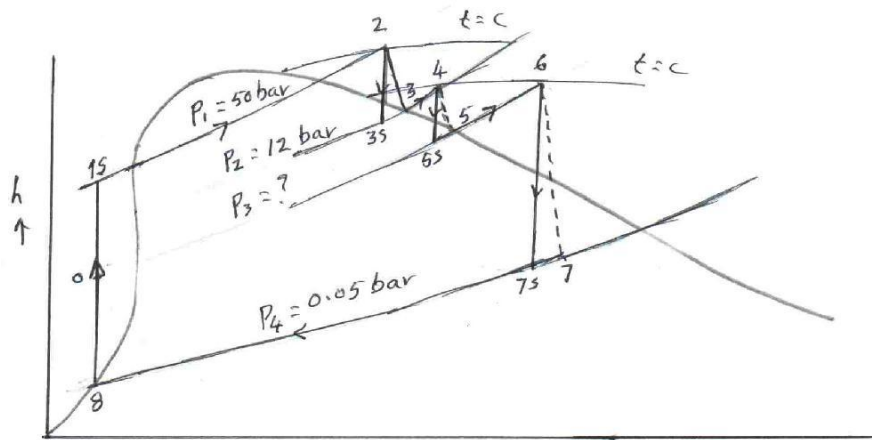
Using Moiller Diagram





4. Steam at 50 bar, 350°C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280°C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280°C . Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.

Solution:



$$P_1 = 50 \text{ bar} \quad t_2 = 350^{\circ}\text{C} \quad P_2 = 12 \text{ bar} \quad t_4 = 280^{\circ}\text{C}, \quad t_6 = 280^{\circ}\text{C}$$

$$P_3 = ? \quad P_4 = 0.05 \text{ bar}$$

From Mollier diagram

$$h_2 = 3070 \text{ kJ/kg} \quad h_{3s} = 2755 \text{ kJ/kg} \quad h_3 = 2780 \text{ kJ/kg} \quad h_4 = 3008 \text{ kJ/kg}$$

$$(a) \eta_t \text{ for HP stage} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3070 - 2780}{3070 - 2755}$$



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$$= 0.921$$

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$$\begin{aligned} h_2 - h_3 &= h_4 - h_5 \\ \text{i.e., } 3070 - 2780 &= 3008 - \\ h_5 \therefore h_5 &= 2718 \text{ kJ/kg} \end{aligned}$$

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6 \text{ bar}$,

Also from Mollier chart, $h_{5s} = 2708 \text{ kJ/kg}$, $h_6 = 3038 \text{ kJ/kg}$, $h_{7s} = 2368 \text{ kJ/kg}$

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93 \text{ kJ/kg}$$

$$\begin{aligned} \therefore \text{At a pressure } 0.05 \text{ bar, } h_7 &= h_{f7} + x_7 h_{fg7} \\ 2420.93 &= 137.8 + x_7 (2423.8) \\ \therefore x_7 &= 0.941 \end{aligned}$$

$$\begin{aligned} \text{Total power output} &= (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7) \\ &= (3070 - 2780) + (3008 - 2718) + (3038 - 2420.93) \\ &= 1197.07 \text{ kJ/kg} \end{aligned}$$

\therefore Total power output /kg of steam = 1197.07 kW For

$P_4 = 0.05 \text{ bar}$ from steam tables, $h_g = 137.8 \text{ kJ/kg}$;

$$\begin{aligned} W_P &= 0.0010052 (50 - 0.05) 10^2 = 5.021 \text{ kJ/kg} \\ &= h_g - h_{1s} \end{aligned}$$

$$\therefore h_{1s} = 142.82 \text{ kJ/kg}$$

$$\begin{aligned} \text{Heat supplied, } Q_H &= (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5) \\ &= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718) \\ &= 3475.18 \text{ kJ/kg} \end{aligned}$$

$$W_{\text{net}} = W_T - W_P = 1197.07 - 5.021 = 1192.05 \text{ kJ/kg}$$

$$\therefore \eta_{th} = \frac{W_{\text{net}}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$$\therefore T_m = 465.6 \text{ K}$$

Or

$$T_m = \frac{h_2 - h_{1s}}{\eta_{th}} = \frac{3070 - 142.82}{0.343} = 492 \text{ K}$$

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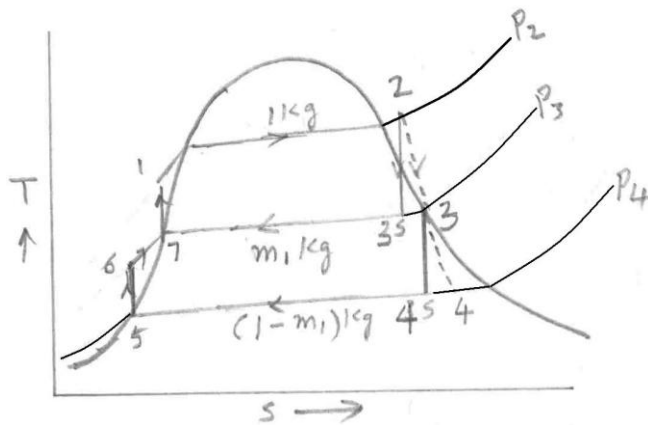


$$S_2 - S_{1s} = 6.425 - 0.4763$$

$$SSC = \frac{3600}{1192.05} = 3.02 \text{ kg / kWh}$$

5. Steam at 30 bar and 350°C is supplied to a steam turbine in a practical regenerative cycle and the steam is bled at 4 bar. The bled steam comes out as dry saturated steam and heats the feed water in an open feed water heater to its saturated liquid state. The rest of the steam in the turbine expands to condenser pressure of 0.1 bar. Assuming the turbine η to be same before and after bleeding determine (i) the turbine η , (ii) steam quality at inlet to condenser, (iii) mass flow rate of bled steam per unit mass rate at turbine inlet and (iv) the cycle η .

Solution:



$$P_2 = 30 \text{ bar} \quad t_2 = 350^\circ\text{C} \quad P_3 = 4 \text{ bar} \quad P_4 = 0.1 \text{ bar}$$

$$h_3 = h_g \text{ at } P_3 = 4 \text{ bar}, = 2737.6 \text{ kJ/kg}$$

From superheated steam tables,

$$h_2 = h_3 = h_g \Big|_{P=4\text{bar}} = 2737.6 \text{ kJ / kg}$$



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$$h_2 = h \Big|_{P_2=30\text{bar} \ \& \ t_2=350^\circ\text{C}} = 3117.5 \text{ kJ/kg} \text{ and } S_2 = 6.7471 \text{ kJ/kg-K}$$

$$h_5 = h_f \Big|_{P_5=0.1\text{bar}} = 191.8 \text{ kJ/kg}$$

$$h_7 = h_f \Big|_{P_7=4\text{bar}} = 604.7 \text{ kJ/kg}$$

Process 2-3s is isentropic, i.e., $S_2 = S_{3s}$

$$6.7471 = 1.7764 + x_{3s} (5.1179)$$

$$\therefore x_{3s} = 0.971$$

$$\therefore h_{3s} = h_{f3} + x_{3s} h_{fg3}$$

$$= 604.7 + 0.971 (2132.9)$$

$$= 2676.25 \text{ kJ/kg}$$

Process 3-4s is isentropic i.e., $S_3 = S_{4s}$ i.e.,

$$6.8943 = 0.6493 + x_{4s} (7.5018)$$

$$\therefore x_{4s} = 0.832$$



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$$\therefore h_{4S} = 191.8 + 0.832 (2392.9) = 2183.8 \text{ kJ/kg}$$

Given, η_t (before bleeding) = η_t (after bleeding)

$$\text{We have, } \eta_t \text{ (before bleeding)} = \frac{h_2 - h_3}{h_2 - h_{3S}} = \frac{3117.5 - 2737.6}{3117.5 - 2676.25} = 0.86$$

$$\therefore 0.86 = \frac{h_3 - h_4}{h_3 - h_{4S}} = \frac{2737.6 - h_4}{2737.6 - 2183.8} \quad \therefore h_4 = 2261.33 \text{ kJ/kg}$$

But $h_4 = h_{f4} + x_4 h_{fg4}$

$$2261.33 = 191.8 + x_4 (2392.9)$$

$$\therefore x_4 = 0.865$$

i.e., Dryness fraction at entry to condenser = $x_4 = \mathbf{0.865}$

iii) Let m kg of steam is bled. Applying energy balance to FWH,

$$m h_3 + (1 - m) h_6 = h_7$$

We have $W_{P1} = (h_6 - h_5)$

$$= 0.0010102 (4 - 0.1) 10^5 / 10^3$$

$$= 0.394 \text{ kJ/kg}$$

$$\therefore h_6 = 0.394 + 191.8 = 192.19$$

kJ/kg Substituting,

$$m (2737.6) + (1 - m) 192.19 = 604.7$$

$$\therefore m = 0.162 \text{ kg Also,}$$

$W_{P2} = (h_1 - h_7) = v \text{ dP}$

$$= 0.0010839 \times (30 - 4) 10^2$$

$$= 2.82 = v \text{ dP kJ/kg}$$



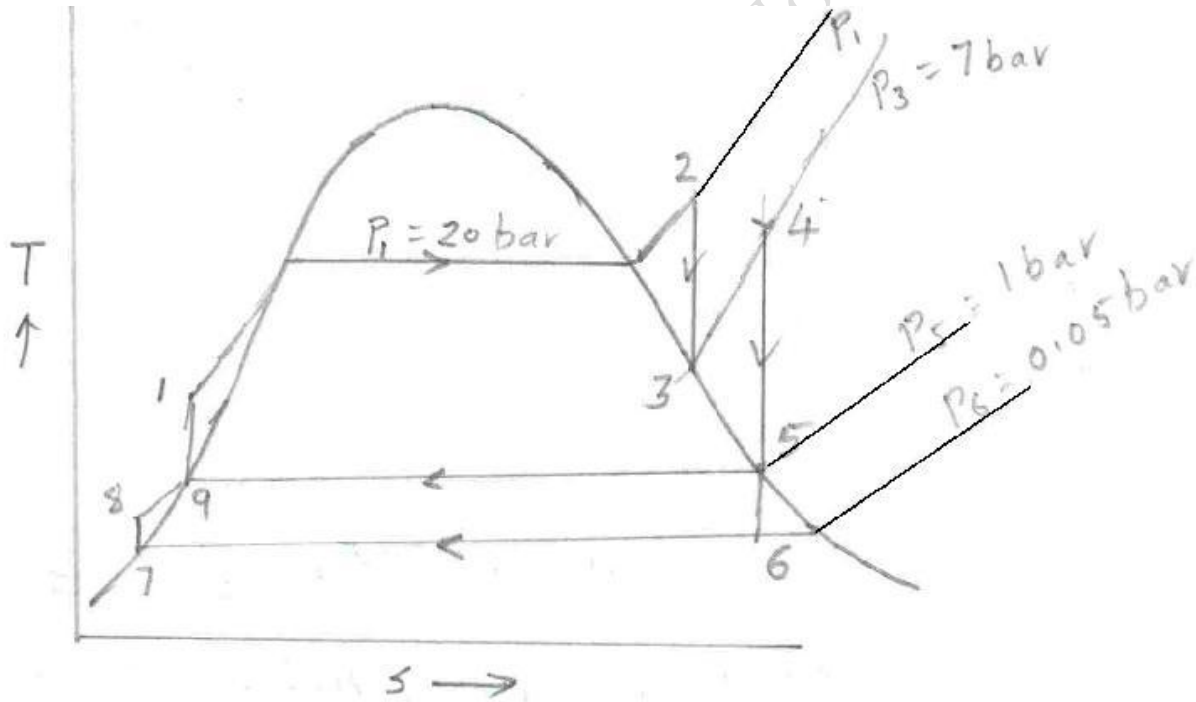
$$\therefore h_1 = 2.82 + 604.7 = 607.52 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{W_T = W_P}{Q_H} = \frac{[(h_2 - h_3) + (1-m)(h_3 - h_4)] - [(1-m)(h_6 - h_5) + (h_1 - h_2)]}{(h_2 - h_1)}$$

$$\eta_{\text{cycle}} = 0.31$$

6. In an ideal reheat regenerative cycle, the high pressure turbine receives steam at 20 bar, 300°C. After expansion to 7 bar, the steam is reheated to 300°C and expands in an intermediate pressure turbine to 1 bar. A fraction of steam is now extracted for feed water heating in an open type FWH. The remaining steam expands in a low pressure turbine to a final pressure of 0.05 bar. Determine (i) cycle thermal η , (ii) specific steam consumption, (iii) quality of steam entering condenser.

Solution:





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$$h_2 = h \Big|_{20 \text{ bar}, 300^\circ \text{C}} = 3025 \text{ kJ/kg} \text{ and } s_2 = 6.7696 \text{ kJ/kg-K}$$

Process 2-3 is isentropic

$$\text{i.e., } S_2 = S_3$$

$$6.7696 = 1.9918 + x_3 (4.7134)$$

$$\therefore x_3 = 1.014$$

i.e., state 3 can be approximated as dry saturated.

$$\therefore h_3 = h \Big|_{7 \text{ bar}, \text{dry sat.}} = 2762 \text{ kJ/kg}$$

$$\therefore h_4 = h \Big|_{7 \text{ bar}, 300^\circ \text{C}} = 3059.8 \text{ kJ/kg} \text{ and } s_4 = 7.2997 \text{ kJ/kg-K}$$

Process 4-5 is isentropic i.e., $S_4 = S_5$

$$7.2997 = 1.3027 + x_5 (6.0571)$$

$$\therefore x_5 = 0.99$$

$$\therefore h_5 = h_{f5} + x_5 h_{fg5} = 417.5 + 0.99 (2257.9) = 2652.9 \text{ kJ/kg}$$

Process 5-6 is isentropic i.e., $S_5 = S_6$

$$7.2997 = 0.4763 + x_6 (7.9197)$$

$$\therefore x_6 = 0.862$$

$$\therefore h_6 = 137.8 + 0.862 (2423.8) = 2226.1 \text{ kJ/kg}$$

$$h_7 = h_f \Big|_{0.05 \text{ bar}} = 137.8 \text{ kJ/kg}$$

Neglecting W_{p1} , $h_8 = h_7$, Also neglecting W_{p2} , $h_9 = h_1$

$$\therefore h_9 = h_f \Big|_{1 \text{ bar}} = 417.5 \text{ kJ/kg}$$



Applying energy balance to FWH

$$mh_5 + (1 - m) h_8 = h_9$$

i.e., $m(2652.9) + (1 - m) 137.8 = 417.5 \quad \therefore m = 0.111 \text{ kg/kg of steam}$

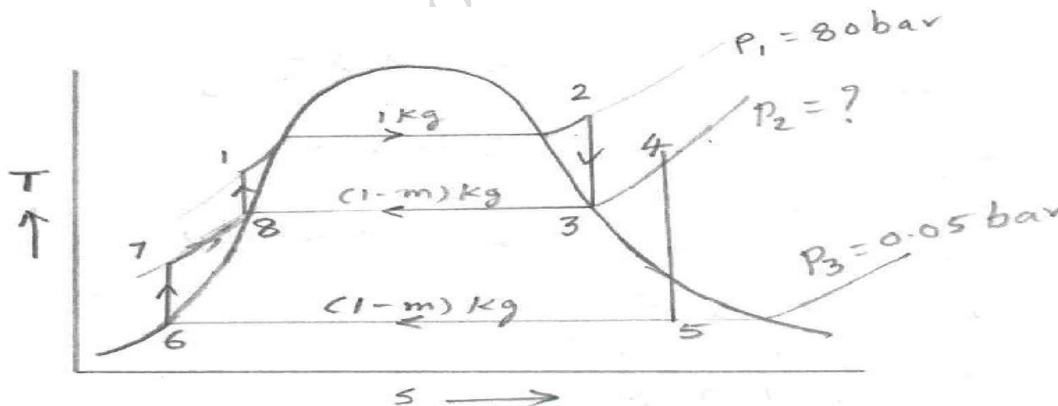
(i) $\eta_C = \frac{(h_2 - h_3) + (h_4 - h_5) + (1 - m)(h_5 - h_6)}{(h_2 - h_1) + (h_4 - h_3)} = 0.35$

(ii) $SSC = \frac{3600}{W_{net}} = 3.57 \text{ kg / kWh}$

(iii) Quality of steam entering condenser, $x_6 = 0.862$

7. The net power output of a regenerative – reheat cycle power plant is 80mW. Steam enters the high pressure turbine at 80 bar, 500 °C and expands to a pressure P_2 and emerges as dry vapour. Some of the steam goes to an open feed water heater and the balance is reheated at 400 °C at constant pressure P_2 and then expanded in the low pressure turbine to 0.05 bar. Determine (i) the reheat pressure P_2 , (ii) the mass of bled steam per kg boiler steam, (iii) the steam flow rate in HP turbine, (iv) cycle η . Neglect pump work. Sketch the relevant lines on h-s diagram. Assume expansion in the turbines as isentropic.

Solution:



$P = 80000 \text{ kW} \quad P_1 = 80 \text{ bar} \quad t_2 = 500^\circ\text{C} \quad P_2 = ? \quad t_3 = 400^\circ\text{C}$



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$$P_3 = 0.05 \text{ bar} \quad m = ? \quad m_S = ? \quad \eta_{\text{cycle}} = ?$$

$$h_2 = h \Big|_{80 \text{ bar}, 500^\circ \text{C}} = 3398.8 \text{ kJ/kg} \text{ and } s_2 = 6.7262$$

Process 2-3 is isentropic i.e., $S_2 = S_3 = 6.7262 \text{ kJ/kg-K}$

Given state 3 is dry saturated i.e., $S_3 = 6.7262 =$

From table A – 1, for dry saturated steam, at $P = 6.0 \text{ bar}$, $S_g = 6.7575$
and at $P = 7.0 \text{ bar}$, $S_g = 6.7052$

Using linear interpolation,

$$\Delta P = \left(\frac{0.0 - 1.0}{6.7052 - 6.7575} \right) \times (6.7262 - 6.7052) = 0.402 \text{ bar}$$

$$\therefore (i) P_2 = 6 + 0.402 = 6.402 \text{ bar}$$

$$\therefore h_3 = h \Big|_{P_2=6.4 \text{ bar}}$$

From table A – 1, For $P = 6 \text{ bar}$	$h_g = 2755.5$	$S_g = 6.7575$
---	----------------	----------------

For $P = 7 \text{ bar}$,	$h_g = 2762.0$	$S_g = 6.7052$
---------------------------	----------------	----------------

$$\therefore \text{For } P = 6.4 \text{ bar} \quad \frac{2762 - 2755.5}{6.7052 - 6.7575} \times (0.4) + 2755.5 = 2758.1 \text{ kJ/kg}$$

$$\therefore h_3 = 2758.1 \text{ kJ/kg}$$

$$h_4 = h \Big|_{6.4 \text{ bar}, 400^\circ \text{C}}$$

From superheated steam tables, For $P = 6.0 \text{ bar}$,	$h = 3270.6$	$s = 7.709$
--	--------------	-------------

For $P = 7.0 \text{ bar}$,	$h = 3269.0$	$s = 7.6362$
-----------------------------	--------------	--------------

$$\therefore \text{For } 6.4 \text{ bar, } h_4 = 3269.96 \text{ kJ/kg}$$

$$S_4 = 7.6798 \text{ kJ/kg-K}$$

Process 4-5 is isentropic, $S_4 = S_5$



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$$\text{i.e., } 7.6798 = 0.4763 + x_5 (7.9197)$$

$$\therefore x_5 = 0.909$$

$$\therefore h_5 = 137.8 + 0.909 (2423.8) = 2342.41 \text{ kJ/kg}$$

$$h_6 = h_f |_{0.05 \text{ bar}} = 137.8 \text{ kJ/kg}$$

$$h_7 = h_6 \text{ (since } W_{P1} \text{ is neglected)}$$

$$h_8 = h_f |_{6.4 \text{ bar}} = 681.1 \text{ kJ/kg}$$

$$h_1 = h_8 \text{ (since } W_{P2} \text{ is neglected)}$$

(ii) Applying energy balance to

$$\text{FWH, } mh_3 + (1 - m) h_7 = h_8$$

$$m (2758.1) + (1 - m) 137.8 = 681.1$$

$$\therefore m = 0.313 \text{ kg/kg of steam}$$

$$\begin{aligned} \text{(iii) } W_1 = W_{HP} &= (h_2 - h_3) = (3398.8 - 2758.1) \\ &= 640.7 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} W_2 = W_{LP} &= (1 - m) (h_4 - h_5) \\ &= (1 - 0.313) (3269.96 - 2342.41) \\ &= 637.2 \text{ kJ/kg} \end{aligned}$$

$$\therefore W_{\text{net}} = W_1 + W_2 = 1277.9 \text{ kJ/kg}$$

$$\therefore \text{Steam flow rate through HP turbine} = \frac{\text{Power}}{W_{\text{net}}} = \frac{80000}{1277.9} = 62.6 \text{ kg/s}$$



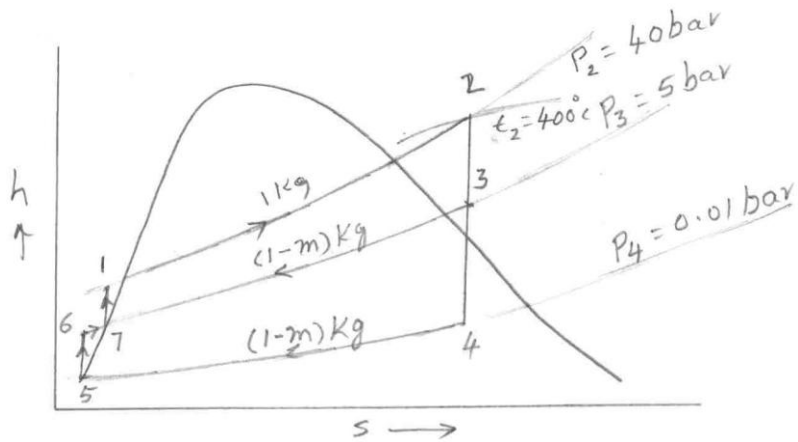
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(iv) $\eta_{\text{cycle}} = ?$ $Q_H = (h_2 - h_1) + (1 - m)(h_4 - h_3) = 3069.35 \text{ kJ/kg}$

$$\therefore \eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_H} = \frac{1277.9}{3069.35} = 0.42$$

- 8. In a single heater regenerative cycle, the steam enters the turbine at 30 bar, 400⁰ C and the exhaust pressure is 0.01 bar. The feed water heater is a direct contact type which operates at 5 bar. Find (i) thermal η and the steam rate of the cycle, (ii) the increase in mean temperature of heat addition, η and steam rate as compared to the Rankine cycle without regeneration. Pump work may be neglected.**

Solution:



$P_2 = 40 \text{ bar}$ $t_2 = 400^{\circ}\text{C}$ $P_4 = 0.01 \text{ bar}$ $P_3 = 5 \text{ bar}$

From h-s diagram,

$$h_2 = h_{\text{at } 30 \text{ bar, } 400^{\circ}\text{C}} = 3230 \text{ kJ/kg}$$

$$h_3 = 2790 \text{ kJ/kg}$$

$$h_4 = 1930 \text{ kJ/kg}$$

$$h_5 = 29.3 \text{ kJ/kg}$$

$$h_7 = 640.1 \text{ kJ/kg}$$

Since pump work may neglect, $h_6 = h_5$ & $h_1 = h_7$

(i) $\eta_{\text{cycle}} = ?$



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Let m = mass of steam bled per kg boiler steam

Applying SFEE to FWH,

$$mh_3 + (1 - m) h_6 = h_7$$

$$m(2790) + (1 - m) 29.3 = 640.1$$

$$\therefore m = 0.221 \text{ kg/kg of boiler steam}$$

$$\begin{aligned} W_T &= (h_2 - h_3) + (1 - m) (h_3 - h_4) \\ &= (3230 - 2790) + (1 - 0.221) (2790 - 1930) \\ &= 1109.73 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} Q_H &= (h_2 - h_1) = (3230 - 640.1) \\ &= 2589.9 \text{ kJ/kg} \end{aligned}$$

$$\therefore \eta_{\text{cycle}} = \frac{W_T}{Q_H} = 0.428 \quad \text{Since } W_P \text{ is neglected}$$

$$(ii) \text{ steam rate} = \frac{3600}{W_T} = 3.24 \text{ kg / kWh}$$

$$(iii) \text{ Mean temperature of heat addition, } \Delta T_m = \frac{Q_H}{s_2 - s_5}$$

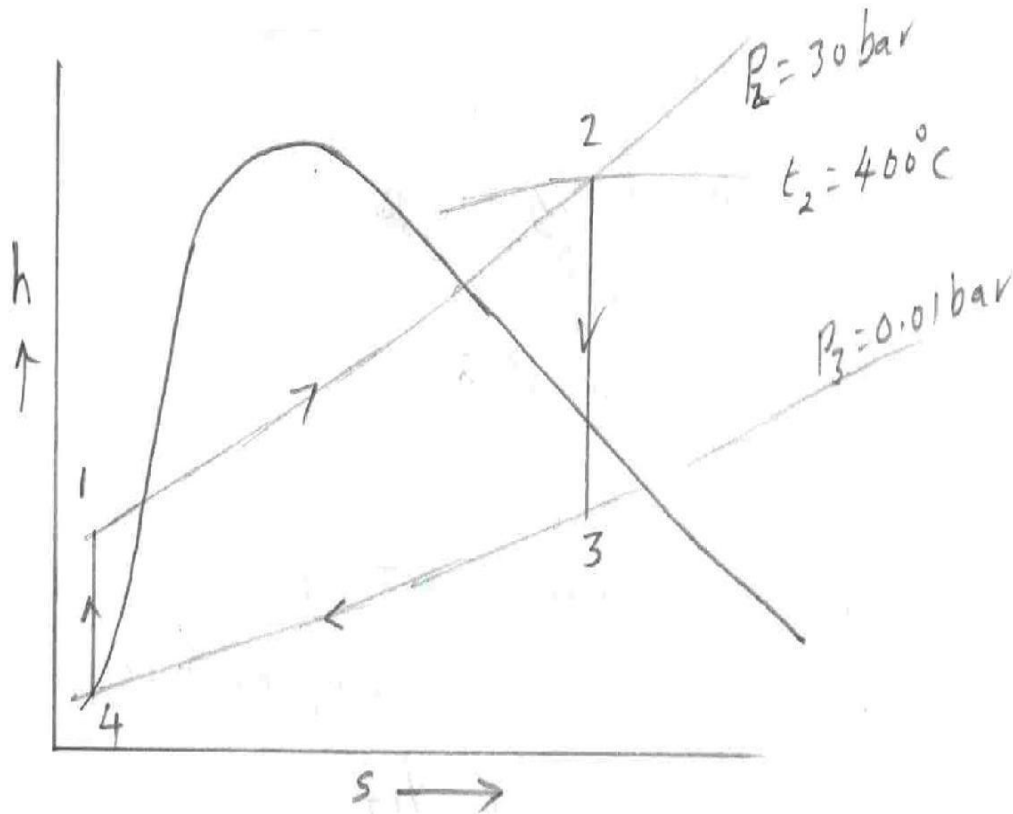
From h-s diagram, $s_2 = 6.83 \text{ kJ/kg-K}$

From steam tables, $s_5 = 0.1060 \text{ kJ/kg-K}$

$$\therefore \Delta T_m = \frac{2589.9}{(6.83 - 0.106)} = 385.2^0 \text{ K}$$



Case (ii) Rankine cycle without Regeneratio





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From h-s diagram,

$$h_2 = 3230 \text{ kJ/kg } h_3$$

$$= 1930 \text{ kJ/kg } h_4 =$$

$$29.3 \text{ kJ/kg}$$

$$h_1 = h_4$$

$$S_2 = 6.83 \text{ kJ/kg-K}$$

$$S_4 = 0.1060 \text{ kJ/kg-K}$$

$$(i) \quad \eta_{\text{cycle}} = \frac{W_T}{Q_H} = \frac{(h_2 - h_3)}{(h_2 - h_1)}$$
$$= \frac{1300}{3200.7} = 0.41$$

$$(ii) \quad \text{Steam rate} = \frac{3600}{W_T} = 2.76 \text{ kg / kWh}$$

$$(iii) \quad \text{Mean temperature of heat addition, } \Delta T = \frac{3200.7}{m(6.83 - 0.106)} = 476^0 \text{ K}$$



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Comparison	ΔT_m	η_{cycle}	Steam rate
Rankine cycle with regeneration	385.2 K	0.428	3.24 kg/kWh
Rankine cycle without regeneration	476 ⁰ K	0.41	2.76kg/kWh
\therefore Increase w.r.t Rankine cycle	- 0.19 i.e., (-19%)	0.044 (4.4%)	0.174 (17.4%)



UNIT-5

Reciprocating Compressor

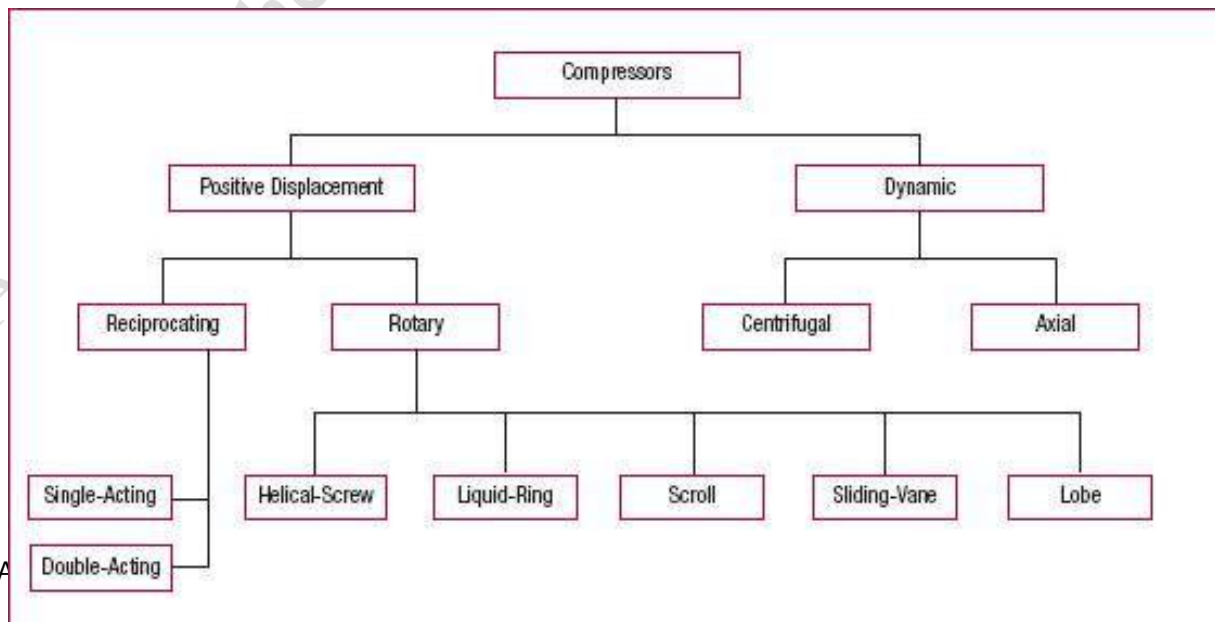
5.1 INTRODUCTION

Compressors are work absorbing devices which are used for increasing pressure of fluid at the expense of work done on fluid.

The compressors used for compressing air are called air compressors. Compressors are invariably used for all applications requiring high pressure air. Some of popular applications of compressor are, for driving pneumatic tools and air operated equipments, spray painting, compressed air engine, supercharging surface cleaning, refrigeration and air conditioning, chemical industry etc. compressors are supplied with low pressure air (or any fluid) at inlet which comes out as high pressure air (or any fluid) at outlet. Work required for increasing pressure of air is available from the prime mover driving the compressor. Generally, electric motor, internal combustion engine or steam engine, turbine etc. are used as prime movers. Compressors are similar to fans and blowers but differ in terms of pressure ratios. Fan is said to have pressure ratio up to 1.1 and blowers have pressure ratio between 1.1 to 4 while compressors have pressure ratios more than 4.

5.2 CLASSIFICATION OF COMPRESSORS

Table-5.1 Types of Compressors





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Compressors can be classified in the following different ways.

- (a) **Based on principle of operation:** Based on the principle of operation compressors can be classified as.
- (i) Positive displacement compressor.
 - (ii) Non-positive displacement compressors.

In positive displacement compressors the compression is realized by displacement of solid boundary and preventing fluid by solid boundary from flowing back in the direction of pressure gradient. Due to solid wall displacement these are capable of providing quite large pressure ratios. Positive displacement compressors can be further classified based on the type of mechanism used for compression. These can be

- (i) Reciprocating type positive displacement compressors
- (ii) Rotary type positive displacement compressors.

Reciprocating compressors generally, employ piston-cylinder arrangement where displacement of piston in cylinder causes rise in pressure. Reciprocating compressors are capable of giving large pressure ratios but the mass handling capacity is limited or small. Reciprocating compressors may also be single acting compressor or double acting compressor. Single acting compressor has one delivery stroke per revolution while in double acting there are two delivery strokes per revolution of crank shaft. Rotary compressors employing positive displacement have a rotary part whose boundary causes positive displacement of fluid and thereby compression. Rotary compressors of this type are available in the names as given below;

- (i) Roots blower
- (ii) Vane type compressors

Rotary compressors of above type are capable of running at higher speed and can handle large mass flow rate than reciprocating compressors of positive displacement type.

Non-positive displacement compressors, also called as steady flow compressors use dynamic action of solid boundary for realizing pressure rise. Here fluid is not contained in definite volume and subsequent volume reduction does not occur as in case of positive displacement compressors. Non-positive displacement compressor may be of 'axial flow type' or 'centrifugal type' depending upon type of flow in compressor.

- (b) **Based on number of stages:** Compressors may also be classified on the basis of Applied Thermodynamics, HIT, Nidasoshi



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number of stages. Generally, the number of stages depend upon the maximum delivery pressure. Compressors can be single stage or multistage. Normally maximum compression ratio of 5 is realized in single stage compressors. For compression ratio more than 5 the multistage compressors are used.

Type values of maximum delivery pressures generally available from different type of compressor are,

- (i) Single stage Compressor, for delivery pressure upto 5 bar.
- (ii) Two stage Compressor, for delivery pressure between 5 to 35 bar
- (iii) Three stage Compressor, for delivery pressure between 35 to 85 bar.
- (iv) Four stage compressor, for delivery pressure more than 85 bar

(c) **Based on Capacity of compressors** : Compressors can also be classified depending upon the capacity of Compressor or air delivered per unit time. Typical values of capacity for different compressors are given as;

- (i) Low capacity compressors, having air delivery capacity of $0.15 \text{ m}^3/\text{s}$ or less
- (ii) Medium capacity compressors, having air delivery capacity between 0.15 to $5 \text{ m}^3/\text{s}$.
- (iii) High capacity compressors, having air delivery capacity more than $5 \text{ m}^3/\text{s}$

(d) Based on highest pressure developed: Depending upon the maximum pressure available from compressor they can be classified as low pressure, medium pressure, high pressure and super high pressure compressors. Typical values of maximum pressure developed for different compressors are as under:

- (i) Low pressure compressor, having maximum pressure upto 1 bar
- (ii) Medium pressure compressor, having maximum pressure from 1 bar to 8 bar
- (iii) High pressure compressor, having maximum pressure from 8 to 10 bar
- (iv) Super high pressure compressor, having maximum pressure more than 10 bar.



5.3 Reciprocating Compressors

Reciprocating Compressor has piston cylinder arrangement as shown Fig.5.1

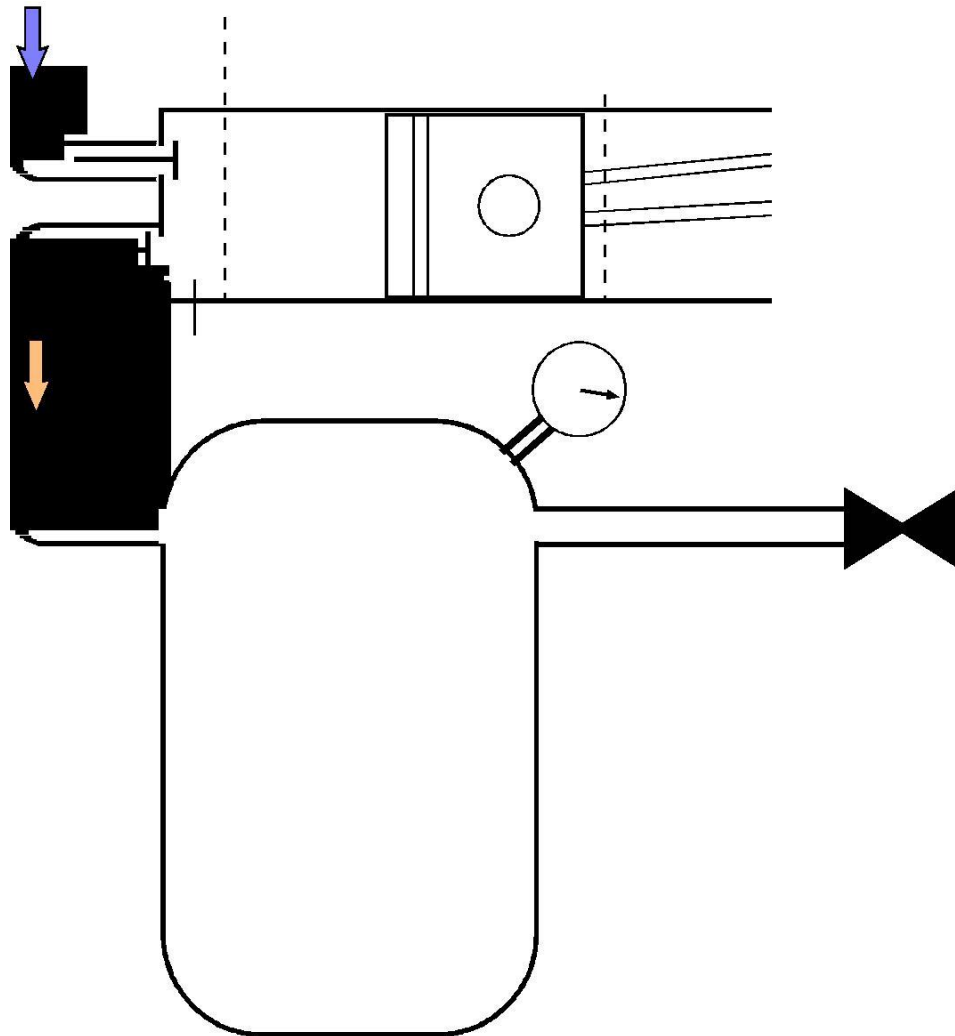


Fig.5.1 Reciprocating Compressor

Reciprocating Compressor has piston, cylinder, inlet valve, exit valve, connecting rod, crank, piston pin, crank pin and crank shaft. Inlet valve and exit valves may be of spring loaded type which get opened and closed due to pressure differential across them. Let us consider piston to be at top dead centre (TDC) and move towards bottom dead centre (BDC). Due to this piston movement from TDC to BDC suction pressure is created causing



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opening of inlet valve. With this opening of inlet valve and suction pressure the atmospheric air enters the cylinder.

Air gets into cylinder during this stroke and is subsequently compressed in next stroke with both inlet valve and exit valve closed. Both inlet valve and exit valves are of plate type and spring loaded so as to operate automatically as and when sufficient pressure difference is available to cause deflection in spring of valve plates to open them. After piston reaching BDC it reverses its motion and compresses the air inducted in previous stroke. Compression is continued till the pressure of air inside becomes sufficient to cause deflection in exit valve. At the moment when exit valve plate gets lifted the exhaust of compressed air takes place. This piston again reaches TDC from where downward piston movement is again accompanied by suction. This is how reciprocating compressor. Keeps on working as flow device. In order to counter for the heating of piston-cylinder arrangement during compression the provision of cooling the cylinder is there in the form of cooling jackets in the body. Reciprotating compressor described above has suction, compression and discharge as three prominent processes getting completed in two strokes of piston or one revolution of crank shaft.



5.4 Thermodynamic Analysis

Compression of air in compressor may be carried out following number of thermodynamic processes such as isothermal compression, polytropic compressor or adiabatic compressor. Fig.16.3 shows the thermodynamic cycle involved in compressor. Theoretical cycle is shown neglecting clearance volume but in actual cycle clearance volume can not be negligible. Clearance volume is necessary in order to prevent collision of piston with cylinder head, accommodating valve mechanism etc., Compression process is shown by process 1-2, 1-2^I, 1-2^{II} and 1-2^{III} following isothermal, polytropic and adiabatic processes.

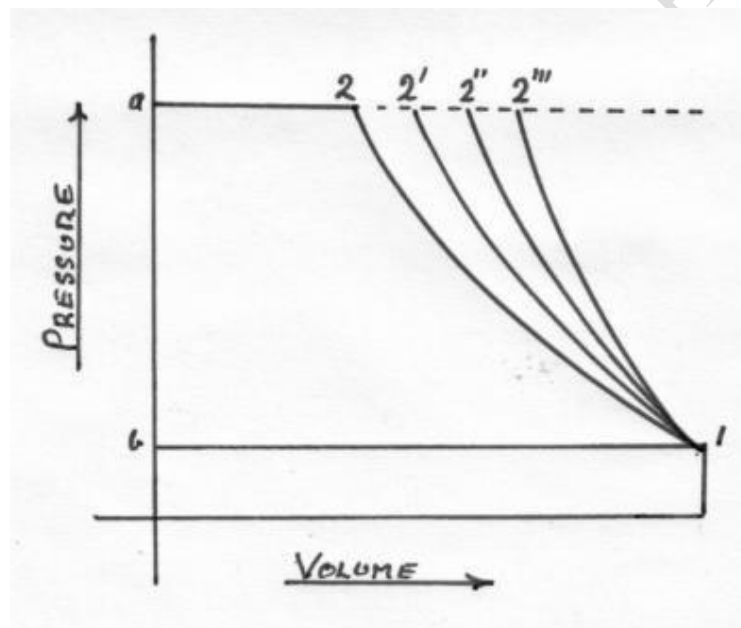


Fig.5.2 P-V diagram for Reciprocating Compressor without Clearance

On P-V diagram process 4-1 shows the suction process followed by compression during 1-2 and discharge through compressor is shown by process 2-3.

Air enters compressor at pressure p_1 and is compressed upto p_2 . Compression work requirement can be estimated from the area below the each compression process. Area on p-V diagram shows that work requirement shall be minimum with isothermal process 1-2^{''}. Work requirement is maximum with process 1-2 ie., adiabatic process. As a designer one shall be interested in a compressor having minimum compression work requirement. Therefore, ideally compression should occur isothermally for minimum work input. In Applied Thermodynamics, HIT, Nidasoshi



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practice it is not possible to have isothermal compression because constancy of temperature during compression can not be realized. Generally, compressors run at substantially high speed while isothermal compression requires compressor to run at very slow speed so that heat evolved during compression is dissipated out and temperature remains constant. Actually due to high speed running of compressor the compression process may be assumed to be near adiabatic or polytropic process following law of compression as $pV^n=C$ with of 'n' varying between 1.25 to 1.35 for air. Compression process following three processes is also shown on T-s diagram in Fig.16.4. it is thus obvious that actual compression process should be compared with isothermal compression process. A mathematical parameter called isothermal efficiency is defined for quantifying the degree of deviation of actual compression process from ideal compression process. Isothermal efficiency is defined by the ratio is isothermal work and actual indicated work in reciprocating compressor.

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Actual indicated Work}}$$



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Practically, compression process is attempted to be closed to isothermal process by air/water cooling, spraying cold water during compression process. In case of multistage compression process the compression in different stages is accompanied by intercooling in between the stages. $P_2 V_2$

Mathematically, for the compression work following polytropic process, $PV^n=C$. Assuming negligible clearance volume the cycle work done.

$W_c = \text{Area on } p\text{-}V \text{ diagram}$

$$W_c = p_2 V_2 + \frac{p_2 V_2 - p_1 V_1}{n-1} - p_1 V_1$$

$$= \frac{n}{n-1} [p_2 V_2 - p_1 V_1]$$

$$= \frac{n}{n-1} (p_1 V_1) \left[\frac{p_2 V_2}{p_1 V_1} - 1 \right]$$

$$= \frac{n}{n-1} \frac{p_2^{1/n} p_1^{(n-1)/n}}{p_1} - 1$$

$$= \frac{n}{n-1} (mR T_1) \left[\frac{p_2^{1/n}}{p_1^{(n-1)/n}} - 1 \right]$$

$$= \frac{n}{n-1} (mR) (T_2 - T_1)$$

In case of compressor having isothermal compression process, $n = 1$, i.e., $p_1 V_1 = p_2 V_2$

$$W_{iso} = p_2 V_2 + p_1 V_1 \ln r - p_1 V_1$$

$$W_{iso} = p_1 V_1 \ln r, \quad \text{where, } r = \frac{V_1}{V_2}$$

In case of compressor having adiabatic compression process,

$$\frac{\gamma}{\gamma-1} (mR T_2 - T_1)$$

Or

$$\eta_{iso} = \frac{p_1 V_1 \ln r}{\frac{n}{n-1} p_1 V_1 \left[\frac{p_2^{1/n}}{p_1^{(n-1)/n}} - 1 \right]}$$



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$n-1$ (p, V_1)

p

$$W_{adiabatic} = (m C_p) (T_2 - T_1)$$

$$W_{adiabatic} = (m) (h_2 - h_1)$$

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The isothermal efficiency of a compressor should be close to 100% which means that actual compression should occur following a process close to isothermal process. For this the mechanism be derived to maintain constant temperature during compression process.

Different arrangements which can be used are:

- (i) Faster heat dissipation from inside of compressor to outside by use of fins over cylinder. Fins facilitate quick heat transfer from air being compressed to atmosphere so that temperature rise during compression can be minimized.
- (ii) Water jacket may be provided around compressor cylinder so that heat can be picked by cooling water circulating through water jacket. Cooling water circulation around compressor regulates rise in temperature to great extent.
- (iii) The water may also be injected at the end of compression process in order to cool the air being compressed. This water injection near the end of compression process requires special arrangement in compressor and also the air gets mixed with water and needs to be separated out before being used. Water injection also contaminates the lubricant film inner surface of cylinder and may initiate corrosion etc, The water injection is not popularly used.
- (iv) In case of multistage compression in different compressors operating serially, the air leaving one compressor may be cooled upto ambient state or somewhat high temperature before being injected into subsequent compressor. This cooling of fluid being compressed between two consecutive compressors is called intercooling and is frequently used in case of multistage compressors.

Considering clearance volume: With clearance volume the cycle is represented on Fig.5.3 The work done for compression of air polytropically can be given by the area enclosed in cycle 1-2-3-4. Clearance volume in compressors varies from 1.5% to 35% depending upon type of compressor.

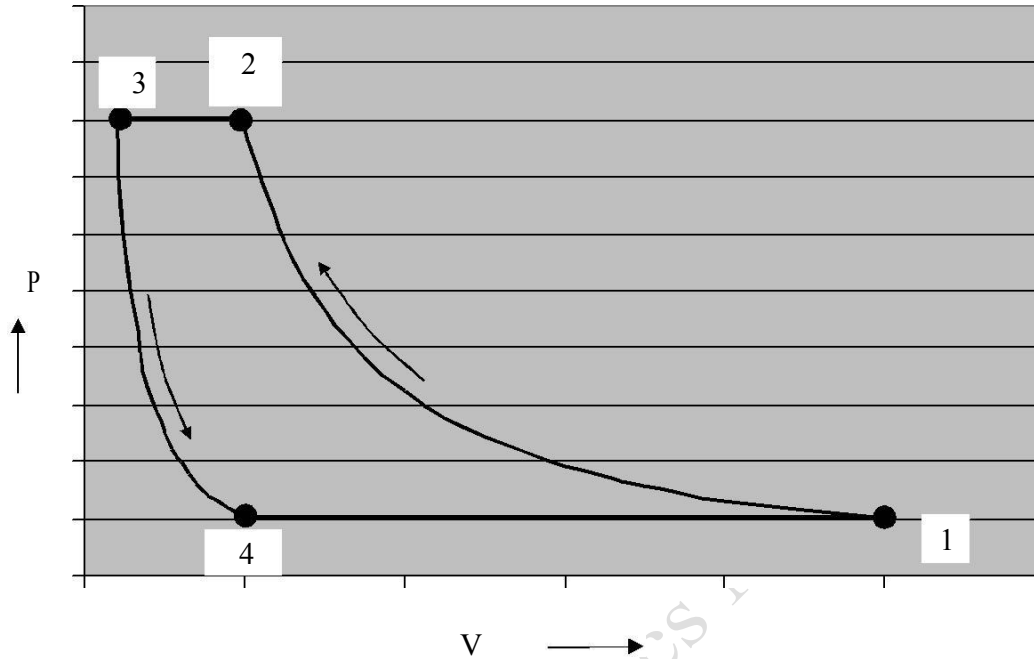


Fig.5.3 P-V diagram for Reciprocating Compressor with Clearance

$$W_{c, \text{with CV}} = \text{Area 1234}$$

$$= \frac{n}{n-1} (p_1 V_1) \frac{p_2}{p_1}^{\frac{n-1}{n}} - \frac{n}{n-1} (p_1 V_1) \frac{p_3}{p_4}^{\frac{n-1}{n}}$$

$$\text{Here } P_1 = P_4, P_2 = P_3$$

$$= \frac{n}{n-1} (p_1 V_1) \frac{p_2}{p_1}^{\frac{n-1}{n}} - \frac{n}{n-1} (p_1 V_1) \frac{p_2}{p_1}^{\frac{n-1}{n}}$$



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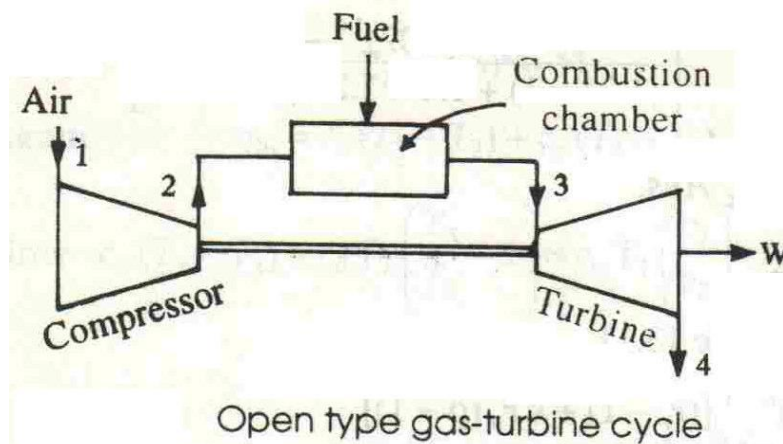
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UNIT-6 GAS TURBINES

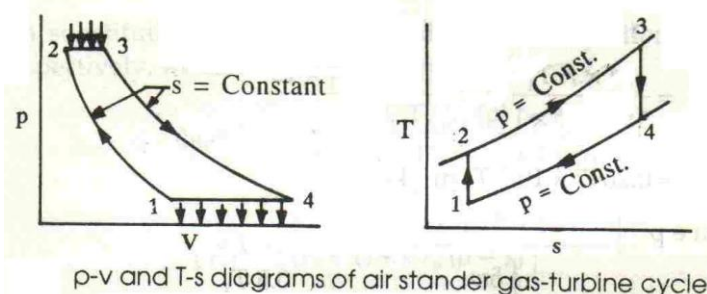
Gas Turbines and Jet Propulsion: Classification of Gas Turbines, Analysis of open cycle gasturbine cycle. Advantages and Disadvantages of closed cycle. Methods to improve thermal efficiency. Jet propulsion and Rocket propulsion.

Simple Gas Turbine Cycle



A schematic diagram of a simple gas turbine power plant is shown in figure. Air is drawn from the atmosphere into the compressor, where it is compressed reversibly and adiabatically. The relatively high pressure is then used in burning the fuel in the combustion chamber. The air fuel ratio is quite high (about 60:1) to limit the temperature of the burnt gases entering the turbine. The gases then expand isentropically in the turbine. A portion of the work obtained from the turbine is utilized to drive the compressor and the auxiliary drive, and rest of the power output is the net power of the gas turbine plant.

A gas turbine plant works using a Brayton or joule cycle. This cycle was originated by joule, a British engineer for use in a hot air reciprocating engine and later in about 1870 an American engineer George Brayton tried this cycle in a gas turbine. This cycle consists of two constant pressures and two adiabatic processes. The P-V and T-S diagrams of the cycle are as shown in figure.





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Process 1 – 2: isentropic compression in the compressor

Process 2 – 3:

constant pressure

heat addition in

the combustion

chamber Process

3 – 4: isentropic

expansion in the

turbine

Process 4 -1: constant pressure heat rejection in the atmosphere or cooling of air in the intercooler (closed cycle).

Expression of net work output:

We have net work output, $W_N = W_T - W_C$

Turbine work, $W_T = h_3 - h_4$

$= C_p (T_3 - T_4)$ since the working fluid is a perfect gas

Compressor work, $W_C = h_2 - h_1$

$= C_p (T_2 - T_1)$

$\therefore W_N = C_p (T_3 - T_4) - C_p (T_2 - T_1)$

Let $R = \frac{P_2}{P_1}$ = pressure ratio for compression

$t = T_3/T_1$ = Temperature ratio

$W_N = C_p T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} \right] - C_p T_1 + 1$

We have $\frac{T_1}{r^{r-1}} = \frac{T_2}{r^{r-1}} \quad \therefore \frac{T_2}{T_1} = \frac{P_2}{P_1} = R$



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$$\frac{T_4}{T_1} = \frac{T_3}{T_2} = R^{\frac{r-1}{r}}$$

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} = R$$

$$\therefore W_N = C_P T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} \right] = C_P T_1 \left[R^{\frac{r-1}{r}} - \frac{1}{R^{\frac{r-1}{r}}} \right] + 1$$

Expression for Thermal Efficiency:

We have thermal efficiency, $\eta_{th} = \frac{W_N}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$

Heat added, $Q_H = h_3 - h_2 = C_P (T_3 - T_2)$

Heat rejected, $Q_L = h_4 - h_1 = C_P (T_4 - T_1)$

$$\therefore \eta_{th} = 1 - \frac{C_P (T_4 - T_1)}{C_P (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now, $\frac{T_2}{T_1} = \frac{P_2}{P_1} = R^{\frac{r-1}{r}}$ & $\frac{T_3}{T_4} = \frac{P_3}{P_4} = R^{\frac{r-1}{r}}$

But as $P_2 = P_3$ & $P_1 = P_4$, it follows that $\frac{T_2}{T_1} = \frac{T_3}{T_4}$ or $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

$$\therefore \eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \text{ i.e., } \eta_{th} = 1 - \frac{1}{R^{\frac{r-1}{r}}} \text{ or } \eta_{th} = 1 - \frac{1}{R^r}$$

From the above equation, it is seen that the efficiency of the air standard gas turbine cycle increases with increase in pressure ratio (R) and the type of working fluid.

Optimum Pressure Ratio for Specific Power Output



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In a gas turbine cycle, T_1 is the temperature of the atmosphere and T_3 is the temperature of the burnt gases entering the turbine. Temperature T_3 is fixed by the metallurgical consideration of the turbine and temperature T_1 is fixed by the atmospheric condition. Between these two extreme values of temperature, there exists an optimum pressure ratio for which the work output of the turbine is maximum.

We have, the net work output of the turbine is,

$$W_N = C_p T_1 \left[t - \frac{t}{R^{\frac{\gamma-1}{\gamma}}} - R^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad \text{--- (1)}$$

The optimum pressure ratio is obtained by differentiating the net work output w.r.t. the pressure ratio and putting the derivative equal to zero i.e., $\frac{dW_N}{dR} = 0$

$$\text{Or } \frac{d}{dR} \left[t - \frac{t}{R^{\frac{\gamma-1}{\gamma}}} - R^{\frac{\gamma-1}{\gamma}} - 1 \right] = 0$$

Differentiating with respect to R we get,



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$$-t \frac{1-\gamma}{R^\gamma} - \frac{1-\gamma}{R^\gamma} = 0$$

$$\text{i.e., } -t \frac{1-\gamma}{R^\gamma} = \frac{1-\gamma}{R^\gamma}$$

$$\frac{1-\gamma}{R^\gamma} = \frac{1-\gamma}{R^\gamma}$$

$$R^\gamma = t \text{ or } R = t^{1/\gamma}$$

$$\text{or } R = t^{1/\gamma}$$

$$\text{or } (R)_{opt} = t^{1/\gamma}$$

$$\text{i.e., } R_{opt} = t^{1/\gamma}$$

Substituting this value of R in the expression for W_N , we get

$$(W_N) = C_p T_1 \left[\frac{t}{r} - \frac{t^{1-\gamma}}{r} + 1 \right]$$



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$$\begin{aligned}
 &= C_p T_1 \left[\frac{t^2}{2} - \frac{t^2}{2} + 1 \right] \\
 &= C_p T_1 \left[t^2 - t^2 + 1 \right] \\
 &= C_p T_1 \left[t^2 - 2t^2 + 1 \right] \\
 &= C_p T_1 \left[t^2 - 1 \right] \\
 (W_N)_{opt} &= C_p T_1 \left[t^2 - 1 \right]
 \end{aligned}$$

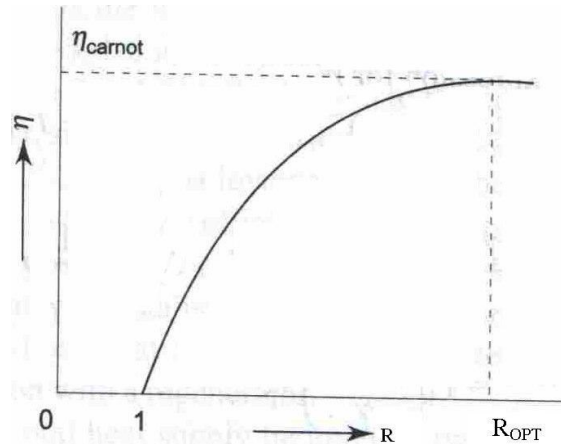
Applied Thermodynamics Notes, HIT, NDS



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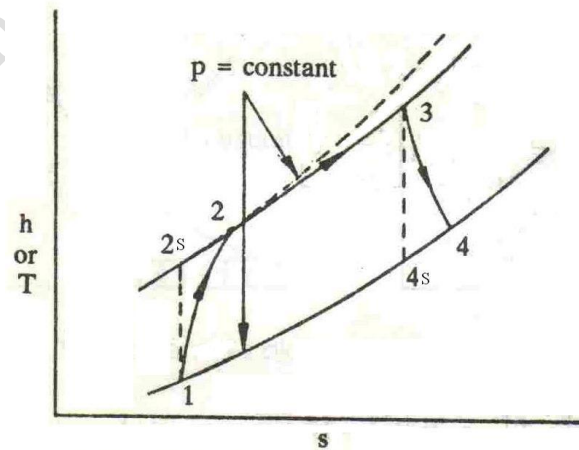
$$\eta_{th} = 1 - \frac{1}{R \frac{r-1}{r}} = 1 - \frac{1}{\frac{r}{r} \frac{r-1}{r}} = 1 - \frac{1}{t^{2(r-1)}}$$

$$\therefore (\eta_{th})_{opt} = 1 - \frac{1}{t^2}$$



Effect of pressure ratio on Brayton cycle efficiency

In an ideal gas turbine plant, the compression and expansion processes are isentropic and there is no pressure-drop in the combustion chamber. But because of irreversibilities associated in the compressor and the turbine, and the pressure-drop in the actual flow passages and combustion chamber, an actual gas turbine plant differs from ideal one. The T-S diagram of actual plant is shown in figure.



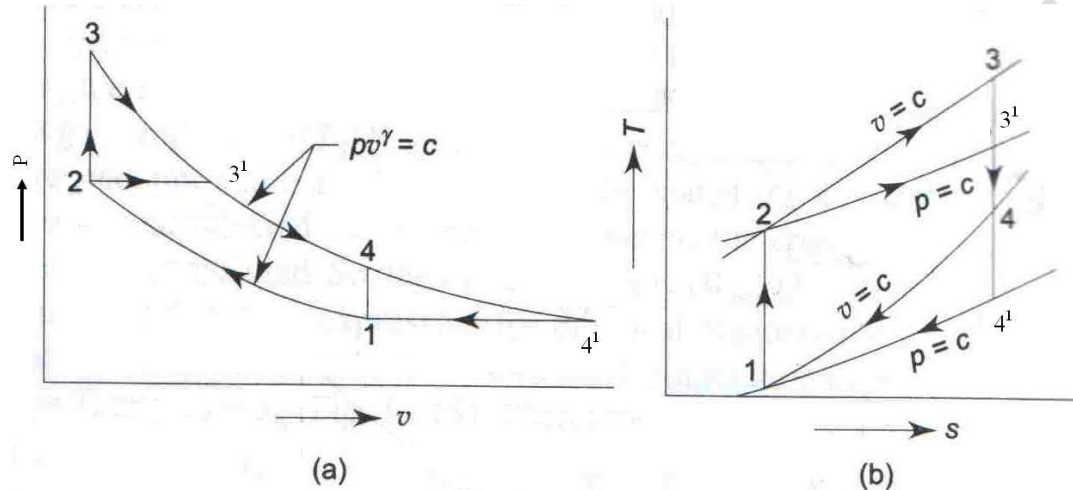
T-s diagram of the actual cycle



\therefore Compressor efficiency, $\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1}$

and the turbine efficiency, $\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$

Comparison between Brayton cycle and Otto cycle:



Comparison of Otto and Brayton cycles

1-2-3-4 Otto cycle
1-2-3¹-4¹ Brayton cycle

For same comparison ratio and work capacity, the Brayton cycle handles a larger range of volume and a smaller range of pressure and temperatures than does the Otto cycle.

In the reciprocating engine field, the Brayton cycle is not suitable. A reciprocating engine cannot efficiently handle a large volume flow of low pressure gas, for which the engine size ($\pi/4 D^2 L$) becomes large, and the friction losses also become more. So the Otto cycle is suitable in the reciprocating engine field.

In turbine plants, however, the Brayton cycle, is more suitable than the Otto cycle. An I.C. engine is exposed to the highest temperature (after the combustion of fuel) only for a short while, and it gets time to become cool in the other processes of the cycle. On the other hand, a gas turbine plant, a steady flow device, is always exposed to the highest temperature used. So to protect material, the maximum temperature of gas that can be used in a gas turbine plant cannot be as high as in I.C. engine. Also, in the steady flow machinery, it is more difficult to carryout heat transfer at constant volume than at constant pressure. Also, a gas turbine can handle a large volume flow of gas quite efficiently.



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Classification: Gas turbine are mainly divided into two group

I Constant pressure combustion gas turbine i) Open cycle, ii) Closed cycle

II Constant volume combustion gas turbine

In almost all the field open cycle gas turbine plants are used. Closed cycle plants were introduced at one stage because of their ability to burn cheap fuel.

Advantages and disadvantages of closed cycle over open cycle

Advantages of closed cycle:

- i) Higher thermal efficiency
- ii) Reduced size
- iii) No contamination
- iv) Improved heat transmission
- v) Improved part load η
- vi) Lesser fluid friction
- vii) No loss of working medium
- viii) Greater output and
- ix) Inexpensive fuel.

Disadvantages of closed cycle:

- i) Complexity
- ii) Large amount of cooling water is required. This limits its use of stationary installation or marine use
- iii) Dependent system
- iv) The wt of the system pre kW developed is high comparatively, \therefore not economical for moving vehicles
- v) Requires the use of a very large air heater.

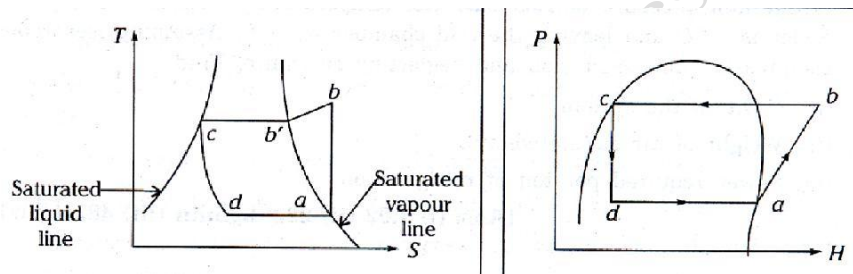
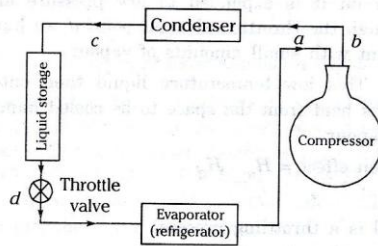


Vapour compression refrigeration system

Introduction

In vapour compression system, the refrigerants used are ammonia, carbon dioxide, freons etc. the refrigerants alternately undergoes condensation and evaporation during the cycle. When refrigerant enters the evaporator it will be in liquid state and by absorbing latent heat it become vapours. Thus the C.O.P of this system is always much higher than air refrigeration systems.

Schematic Diagram



Analysis of the cycle

The various processes are

Process ab . The vapour refrigerant entering the compressor is compressed to high pressure and temperature in a isentropic manner.

Process bc. This high pressure and high temperature vapour then enters a condenser where the temperature of the vapour first drops to saturation temperature and subsequently the vapour refrigerants condenses to liquid state.

- Process cd. This liquid refrigerant is collected in the liquid storage tank and later on it is expanded to low pressure and temperature by passing it through the throttle valve. At point d we have low temperature liquid refrigerant with small amount of vapour.
- Process da. This low temperature liquid then enters the evaporator where it absorbs heat from the space to be cooled namely the refrigerator and become vapour.

$$\text{Refrigeration effect} = H_a - H_d$$

But process c - d is a throttling process

$$H_c = H_d, \quad R.E = H_a - H_c$$



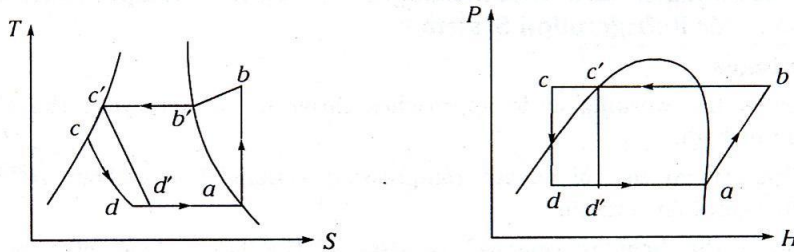
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$$\text{work done} = H_b - H_a$$

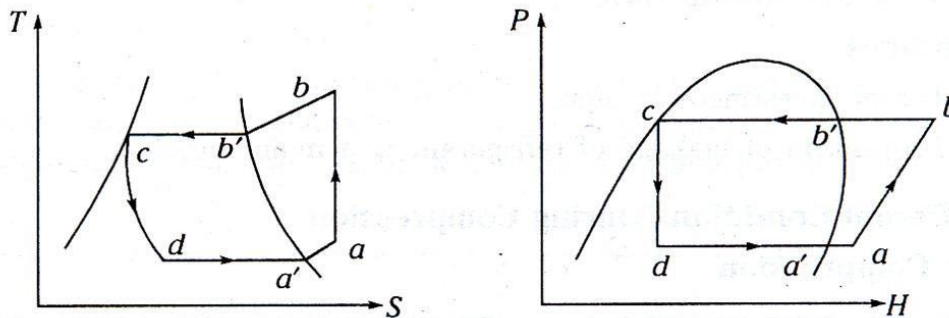
$$\frac{RE}{\text{work}} = \frac{H_a - H_c}{H_b - H_a}$$

$$C.O.P = \frac{RE}{\text{work}} = \frac{H_a - H_c}{H_b - H_a}$$

- Effect of under cooling the liquid



Effect of super heating the vapour



Advantages of Vapour compression refrigeration system over air refrigeration system

- Since the working cycle approaches closer to Carnot cycle, the C.O.P is quite high.
- Operational cost of vapour compression system is just above 1/4th of air refrigeration system.
- Since the heat removed consists of the latent heat of vapour, the amount of liquid circulated is less and as a result the size of the evaporator is smaller.
- Any desired temperature of the evaporator can be achieved just by adjusting the throttle valve.

Disadvantages of Vapour compression refrigeration system over air refrigeration system

- Initial investment is high
- Prevention of leakage of refrigerant is a major problem

Refrigerant

A refrigerant is a fluid in a refrigerating system that by its evaporating takes the heat of



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the cooling coils and gives up heat by condensing the condenser.

Identifying refrigerants by numbers

The present practice in the refrigeration industry is to identify refrigerants by numbers. The identification system of numbering has been standardized by the American society of heating, refrigerating and air conditioning engineers (ASHRAE), some refrigerants in common use are

<u>Refrigeration</u>	<u>Name and Chemical Formula</u>
R-11	Trichloromonofluoromethane CCl_3F
R-12	Dichlorodifluoromethane CCl_2F_2
R-22	Monochlorodifluoromethane CHClF_2
R-717	Ammonia NH_3
R114(R40)	Azeotropic mixture of 73.8% (R-22) and 26.2% R-152a
R-500	
R502	Azeotropic mixture of 48.8% (R-22) and 51.2% R-115
R-764	Sulphur Dioxide SO_2

Properties of Refrigerants

- **Toxicity:**
It obviously desirable that the refrigerant have little effect on people
- **Inflammability:**
Although refrigerants are entirely sealed from the atmosphere, leaks are bound to develop. If the refrigerant is inflammable and the system is located where ignition of the refrigerant may occur, a great hazard is involved.
- **Boiling Point.**
An ideal refrigerant must have low boiling temperature at atmospheric pressure
- **Freezing Point**
An ideal refrigerant must have a very low freezing point because the refrigerant should not freeze at low evaporator temperatures.
- **Evaporator and condenser pressure.**
In order to avoid the leakage of the atmosphere air and also to enable the Detection of the leakage of the refrigerant, both the Evaporator and condenser pressure should be slightly above the atmosphere pressure.
- **Chemical Stability**



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An ideal refrigerant must not decompose under operating conditions..

- **Latent heat of Evaporation.**
The Latent heat of Evaporation must be very high so that a minimum amount of refrigerant will accomplish the desired result; in other words, it increases the refrigeration effect
- **Specific Volume**
The Specific Volume of the refrigerant must be low. The lower specific volume of the refrigerant at the compressor reduces the size of the compressor.
- **Specific heat of liquid vapour.**
A good refrigerant must have low specific heat when it is in liquid state and high specific heat when it is vaporized
- **Viscosity**
The viscosity of the refrigerant in both the liquid and vapour state must be very low as improved the heat transfer and reduces the pumping pressure..
- **Corrosiveness.**
A good refrigerant should be non-corrosive to prevent the corrosion of the metallic parts of the refrigerator.
- **Coefficient of performance**
The coefficient of performance of a refrigerant must be high so that the energy spent in refrigeration will be less.
- **Odour.**
A good refrigerant must be odourless, otherwise some foodstuff such as meat, butter, etc. loses their taste
- **Lekage**
A good refrigerant must be such that any leakage can be detected by simple test.
- **Oil solvent properties.**
A good refrigerant must be not react with the lubricating oil used in the refrigerator for lubricating the parts of the compressor.
- **Cost**
The cost of the refrigerant is the major important, it will easily available and low cost.

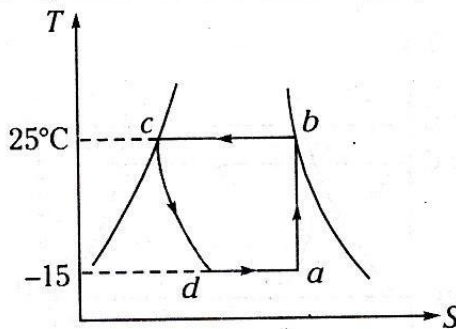


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Problem 1: 20 tons of ice is produced from water at 20°C to ice at -6°C in a day of 24 hours, when the temperature range in the compressor is from -15°C to 25°C. The condition of the vapour is dry at the end of compression. Assuming relative C.O.P as 80%, calculate the power required to drive the compressor.

Take $C_{pice} = 2.1 \text{ kJ/kg}$, Latent heat of ice = 335 k/kg

Temp °C	Liquid		Vapour	
	Enthalpy h_f	Entropy S_f	Enthalpy h_g	Entropy S_g
25	100.04	0.347	1319.2	4.4852
-15	-54.55	-2.1338	1304.99	5.0585



To find the condition of vapour at point 'a'.

Entropy at a = Entropy at b

$$S_{fa} + x_a S_{ga} = S_{gb}$$

$$-2.1338 + x_a 5.0585 - (-2.1338) = 4.4852$$

$$x_a = 0.92$$

$$H_a = h_{fa} + x_a h_{ga} = 54.55 + 0.92[1304.99 - (-54.55)]$$

$$H_a = 1196.22 \text{ kJ/kg}$$

$$H_c = h_{fc} = 100.04 \text{ kJ/kg}$$

$$H_b = h_{gb} = 1319.2 \text{ kJ/kg}$$

$$\text{Refrigeration effect R.E} = H_a - H_c$$

$$= 1196.22 - 100.04 = 1096.18 \text{ kJ/kg}$$

$$\text{work} = H_b - H_a$$

$$= 1319.2 - 1196.22 = 122.98 \text{ kJ/kg}$$



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$$C.O.P = \frac{RE}{work} = \frac{1096.18}{122.98} = 8.913$$

$$Relative\ C.O.P = 0.8$$

$$Actual\ C.O.P = 0.8 \times 8.913 = 7.13$$

$$\begin{aligned} \text{Heat extracted/kg of ice} &= C_{pw} (20 - 0) \\ &+ \text{Latent heat} + C_{pice}[0 - (-6)] \\ &= 4.187 \times 20 + 335 + 2.1 \times 6 = 431.34 \\ &= \frac{20 \times 1000}{24 \times 3600} \end{aligned}$$

$$Mass\ of\ ice\ produced/sec = 0.231\ kg / s \quad 24 \times 3600$$

$$\begin{aligned} \text{Actual heat extracted/sec} &= 431.34 \times 0.231 \\ &= 99.84\ kJ/s \end{aligned}$$

$$Actual\ C.O.P = \frac{Actual\ heat\ extracted/sec}{Actual\ work/sec}$$

$$\therefore = \frac{99.84\ Actual\ work/sec}{7.13}$$

$$Power = 14\ kW$$

Problem2: A vapour compression refrigerator working with Freon-12 has its temperature range $-10^{\circ}C$ and $30^{\circ}C$. The Vapour enters the compressor dry and under cooled by $5^{\circ}C$ in the condenser. For a capacity of 15 TOR, find: (a) C.O.P (b) mass of freon (c) Power required.

C_p for vapour = $0.56\ kJ/kgK$

C_p for liquid =

$1.003\ kJ/kgK$ Solution:

Refrigeration capacity = 15 TOR

From tables the properties of Freon 12 are

Temp $^{\circ}C$	Enthalpy			Entropy	
	hf	Hfg	hg	Sf	Sg
30	64.59	135.03	199.62	0.24	0.6853
-10	26.87	156.31	183.19	0.108	0.7019



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To find the condition of vapour at point 'b'.

Entropy at b = Entropy at a

$$s_{gb'} + C_{pV} \ln \frac{T_b}{T_{b'}} = s_{ga}$$

$$0.6853 + 0.56 \ln \frac{T_b}{303} =$$

$$0.7019 \quad T_b = 312.15K$$

$$H_b = h'_{gb} + C_{pV} (T_b - T_{b'})$$

$$= 100.62 + 0.56(312.15 - 303) =$$

$$204.74 \text{ kJ/kg} \quad H_a = h_{ga'} = 183.19 \text{ kJ/kg}$$

$$H_c = h_{fc'} - C_{PL} (T_c' - T_c)$$

$$= 64.59 - 1.003(30 - 25) = 59.575 \text{ kJ/kg}$$

$$R.E = H_a - H_c = 183.19 - 59.575 = 123.61 \text{ kJ/kg}$$

$$\text{work} = H_b - H_a = 204.74 - 183.19 = 21.55 \text{ kJ/kg}$$

$$C.O.P = \frac{RE}{\text{work}} = \frac{123.61}{21.55} = 5.73$$

$$\text{Refrigeration capacity} = 15 \text{ tons} =$$

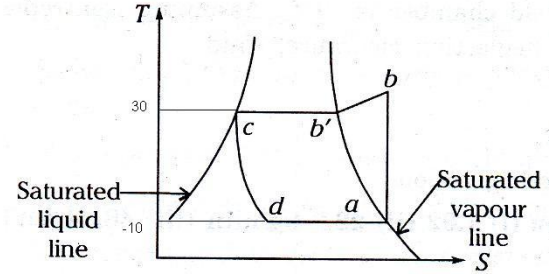
$$15 \times 3.5 = 52.5 \text{ kJ/kg}$$

$$\text{Mass of freon} = \frac{\text{Ref. capacity}}{RE}$$

$$= \frac{52.5}{123.61} = 0.424 \text{ kg/s}$$

$$\text{Power required} = \text{work/kg} \times \text{Mass of freon/s}$$

$$= 21.55 \times 0.424 = 9.152 \text{ kW}$$



Problem3: A food storage locker requires a refrigeration system of 12 tons capacity at an evaporator temperature of -8°C and a condenser temperature of 30°C . The refrigerant freon-12 is sub cooled to 25°C before entering the expansion valve and the vapour is super heated to -2°C before entering the compressor. The compression of the refrigerant is reversible adiabatic. A double action compressor with stroke equal to 1.5 times the bore is to be used operating at 900 rpm.

Determine

- COP
- Theoretical piston displacement/min
- Mass of refrigerant to be circulated/min
- Theoretical bore and stroke of the compressor.



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Take liquid specific heat of refrigerant as 1.23 kJ/kg K and the specific heat of vapour refrigerant is 0.732 kJ/kg K.

Solution:

From tables the properties of Freon 12 are

Temp °C	Enthalpy		Entropy	
	hf	hg	Sf	Sg
30	64.59	199.62	0.24	0.6853
-8	25.75	184.2	0.1142	0.7002

$$C_{pv} = 0.732 \text{ kJ/kgK}$$

$$C_{PL} = 1.235 \text{ kJ/kgK}$$

Entropy at b = entropy at a

$$S_{gb'} + C_{pv} \ln \frac{T_b}{T_{b'}} = S_{ga'} + C_{pv} \ln \frac{T_a}{T_{a'}}$$

$$0.6853 + 0.732 \ln \frac{T_b}{303} = 0.7002 + 0.733 \ln \frac{271}{265}$$

$$T_b = 317.22 \text{ K}$$

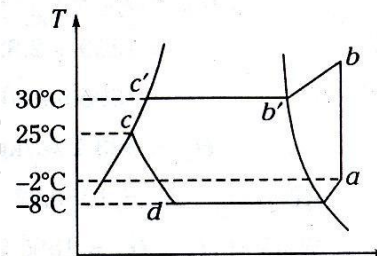
$$\begin{aligned} H_a &= h_{ga'} + C_{pv}(T_a - T_{a'}) \\ &= 184.2 + 0.732(271 - 265) \\ &= 188.59 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} H_b &= h_{gb'} - C_{pv}(T_b - T_{b'}) \\ &= 199.62 + 0.732(317.22 - 303) \\ &= 210.02 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} H_c &= h_{gc'} - C_{PL}(T_{c'} - T_c) \\ &= 64.59 - 1.235(303 - 298) = 58.41 \text{ kJ/kg} \end{aligned}$$

$$\text{R.E} = H_a - H_c = 188.59 - 58.41 = 130.181 \text{ kJ/kg}$$

$$\text{work} = H_b - H_a = 210.02 - 188.59 = 21.43 \text{ kJ/kg}$$





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$$C.O.P = \frac{RE}{work} = \frac{130.18}{21.43} = 6.07$$

$$= \frac{\text{Re.f.capacity}}{RE}$$

$$= \frac{12 \times 3}{130.18} \times 0.5 = 0.322 \text{ kg/s}$$

$$= 0.322 \times 60 = 19.35 \text{ kg/min}$$

From tables at -8°C , $V = 0.0441995 \text{ m}^3 / \text{kg}$

$$\frac{r_v}{a'} = \frac{PV}{a}$$

$$V = \frac{T \times V}{a} = \frac{271}{265} \times 0.0441995 = 0.0452$$

Theoretical piston displacement $V = \text{mass} \times V_a = 19.35 \times 0.0452 = 0.87462 \text{ m}^3 / \text{min}$

$$V = \frac{2\pi}{4} d^2 L N \quad (L = 1.5d)$$

$$0.87462 = \frac{2\pi d^2 \times 1.5d}{4} \times 900$$

$$= 0.0203 \text{ m}$$

$$d = 0.0738 \text{ m} = 7.38 \text{ cm}$$

$$L = 1.5d = 1.5 \times 7.38$$

$$= 11.08 \text{ cm}$$

Problem4:

A vapour compression refrigeration system of 5kW cooling capacity operates between -10°C and 30°C . The enthalpy of refrigerant vapour after compression is 370kJ/kg. Find the COP, refrigerating effect, mass flow rate of the refrigerant and the compressor power. The extract of the refrigerant property table is given below



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Temp	Pressure	Vf	Vg	hf	hg	Sf	Sg
°C	bar		m ³ /kg		kJ/kg		kJ/kgK
-10	226	0.7x10 ⁻³	0.08	190	345	0.95	1.5
30	7.5	0.77x10 ⁻³	0.02	220	220	1.10	1.45

Solution:

Assume the condition before compression as dry saturated vapour

$$H_c = h_{fc} = 220 \text{ kJ / kg}$$

$$H_a = h_{ga'} = 345 \text{ kJ / kg}$$

$$H_b = 370 \text{ kJ / kg (given)}$$

$$\text{R.E} = H_a - H_c = 345 - 220 = 125 \text{ kJ / kg}$$

$$\text{work} = H_b - H_a = 370 - 345 = 25 \text{ kJ / kg}$$

$$\text{C.O.P} = \frac{\text{RE}}{\text{work}} = \frac{125}{25} = 5$$

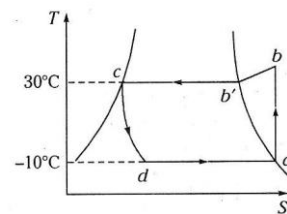
Refrigeration capacity = 5kW or kJ/s

$$\text{Mass of refrigerant} = \frac{\text{Ref. capacity}}{\text{RE}}$$

$$= \frac{5}{125} = 0.04 \text{ kg / s}$$

Compressor work = work.kg x mass of refrigerant -

$$25 \times 0.04 = 1 \text{ kW}$$



Problem5: A vapour compression refrigerator uses methyl chloride and works in the

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pressure rang of 1.19 bar and 5.67 bar. At the beginning of compression, the refrigerant is 0.96 dry and at the end of isentropic compression, its temperature is 55°C. The refrigerant liquid leaving the condenser is saturated. If the mass flow of refrigerant is 1.8kg/min, Determine

COP

The rise in temperature of cooling water if the water flow rate is 16 kg/min. the properties of methyl chloride is given below

Temp °C	Pressure bar	Enthalpy			Entropy	
		hf	Hfg	hg	Sf	Sg
30	1.19	64.59	135.03	199.62	0.24	0.6853
-10	5.67	26.87	156.31	183.19	0.108	0.7019

Take specific heat of
super heat methyl chloride
as 0.75kJ/kg K

Solution

$$x_a = 0.96$$

$$T_b = 55^\circ\text{C}$$

$$H_a = h_{fa} + x_a (h_{fga}) = h_{fa} + x_a (h_{ga} - h_{fa})$$

$$= 430.1 + 0.96(455.2 - 30.1) = 438.196 \text{ kJ / kg}$$

$$H_b = h_{gb'} - C_p (T_b - T_{b'})$$

$$= 476.5 + 0.75(55 - 25) = 499 \text{ kJ/kg}$$

$$H_c = h_{fc} = 100.5 \text{ kJ / kg}$$

$$\text{R.E} = H_a - H_c = 438.196 - 100.5 = 337.669 \text{ kJ / kg}$$

$$\text{work} = H_b - H_a = 499 - 438.196 = 60.8 \text{ kJ / kg}$$

$$\text{C.O.P} = \frac{\text{RE}}{\text{work}} = \frac{337.669}{60.8} = 5.55$$

Heat lost by the vapour in the condenser =
heat gain by cooling water



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$$m_r C_p (T_b - T_{b'}) + m_r h_{fgb'} = m_w C_{pV} \times \text{temperature rise}$$

$$1.8 \times 0.75(55 - 25) + 1.8(476.5 - 100.5)$$

$$= 16 \times 4.187 \times \text{temperature rise}$$

$$\therefore \text{Temperature rise} = 10.7^\circ\text{C}$$

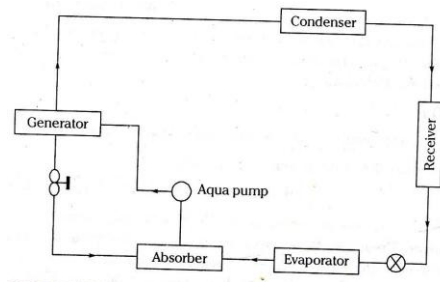
Vapour absorption refrigeration system

General

The absorption refrigeration system is a heat-operated unit which used a refrigerant that is alternately absorbed and liberated by the absorbent.

Simple Absorption system

The minimum number of primary units essential in an absorption system include an evaporation, absorber, generator and condenser.



An expansion valve, pressure reducing valve, and a pump are used in a conventional two-fluid cycle, but the pump can be eliminated by adding a gaseous third fluid. A simple absorption cycle is shown in figure

This cycle differs from a vapour compression cycle by the substitution of an absorber, generator, pumps and reducing valve for the compressor. Various combinations of fluids may be used, but that of ammonia, a strong solution that contains about as much ammonia as possible; a weak solution contains considerably less ammonia.

The weak solution containing very little ammonia is sprayed or otherwise exposed in the absorber and absorbs ammonia coming from the evaporation. Absorption of the ammonia lowers the pressure in the absorber, which in turn draws more ammonia vapour from the evaporator. Usually some forms of cooling is employed in the absorber to remove the heat of condensation and the heat of solution evolve there.

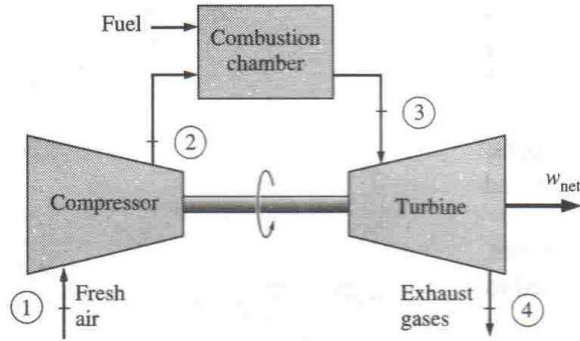
The strong solution is then pumped into a generator, which is at higher pressure and is where heat is applied the heat vapourises the ammonia driving it out of solution and into the condenser, where it is liquefied. The liquid ammonia passes on to the receiver if a separate one is used, or through the expansion valve and into the evaporator. The weak solution left in the generator after the ammonia has been driven off flows through the reducing valve back to the absorber



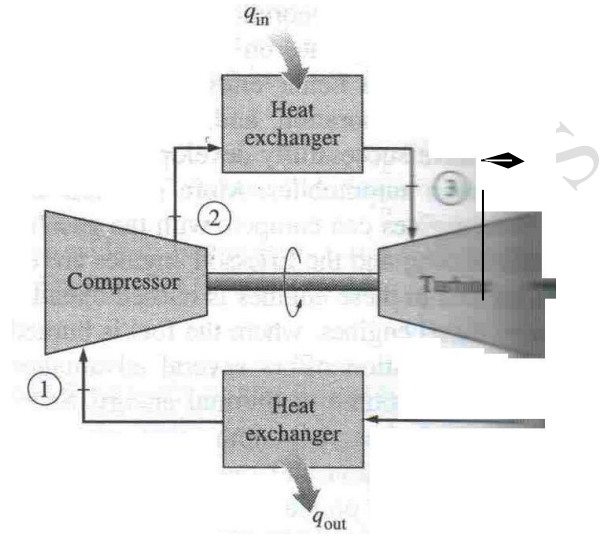
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An Open cycle and Closed cycle Gas Turbine Engines



An open-cycle, gas-turbine engine.



Methods to improve the performance of simple gas turbine plants

A closed-cycle gas-turbine engine

I Regenerative Gas Turbine Cycle: The temperature of the exhaust gases in a simple gas turbine is higher than the temperature of the air after compression process. The η of the Brayton cycle can be increased by utilizing part of the energy of the exhaust gas from the turbine in heating up the air leaving the compression in a heat exchanger called a regenerator, thereby reducing the amount of heat supplied from an external source and also the amount of heat rejected.

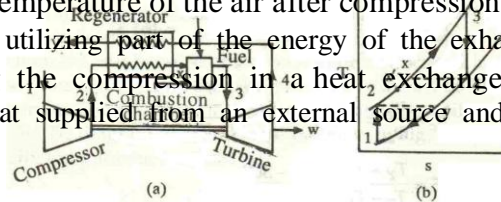


Figure shows a single stage regenerative gas turbine cycle

Air is drawn from the atmosphere into the compressor and is compressed isentropically to state 2. It is then heated at constant pressure in the regenerator to state x by the hot burnt gases from the gas turbine. Since the temperature of the air increases before it reaches the combustion chamber, less amount of fuel will be required to attain the designed turbine inlet temperature of the products of combustion. After combustion at constant pressure in the combustion chamber, the gas enters the turbine at state 3 and expands isentropically to state 4 in the turbine. It then enters the counter-flow regenerator as stated earlier, where it gives up a portion of its heat energy to the compressed air from the compressor and leaves the regenerator at state y.



4 Effectiveness of a regenerator is $\epsilon = \frac{Q_L}{Q_H}$

$$\frac{T_2 - T_1}{T_3 - T_1} \text{ when } C_p \text{ is constant.}$$

$$\epsilon = \frac{Q_L}{Q_H}$$

For an ideal regenerative gas turbine cycle,

$$Q_L = C_p(T_2 - T_1) = C_p(T_2 - T_1) \text{ and}$$

$$Q_H = C_p(T_3 - T_4) = C_p(T_3 - T_4)$$

$$\epsilon = \frac{T_2 - T_1}{T_3 - T_4}$$

Since $\frac{T_2}{T_1} = \frac{P_2}{P_1}^{1/\gamma} = R_r^{1/\gamma}$ and $\frac{T_3}{T_4} = \frac{P_3}{P_4}^{1/\gamma} = \frac{1}{R_r^{1/\gamma}}$

$$\epsilon = \frac{R_r^{1/\gamma} - 1}{R_r^{1/\gamma} - R_r^{-1/\gamma}}$$

i.e., $\epsilon = \frac{R_r^{1/\gamma} - 1}{R_r^{1/\gamma} - R_r^{-1/\gamma}}$

It is evident that the ϵ of an ideal regenerative gas turbine cycle depends not only on the pressure ratio but also on the ratio of the two extreme temperatures. For a fixed ratio of T_3/T_1 , the cycle ϵ drops with increasing pressure ratio.

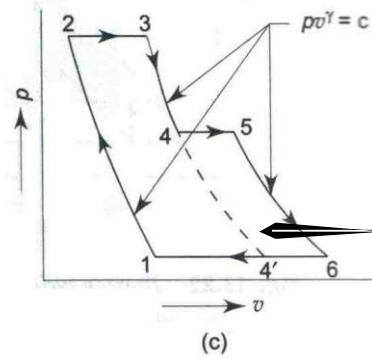
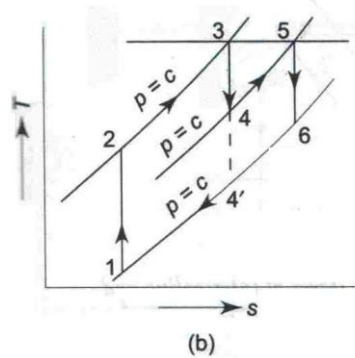
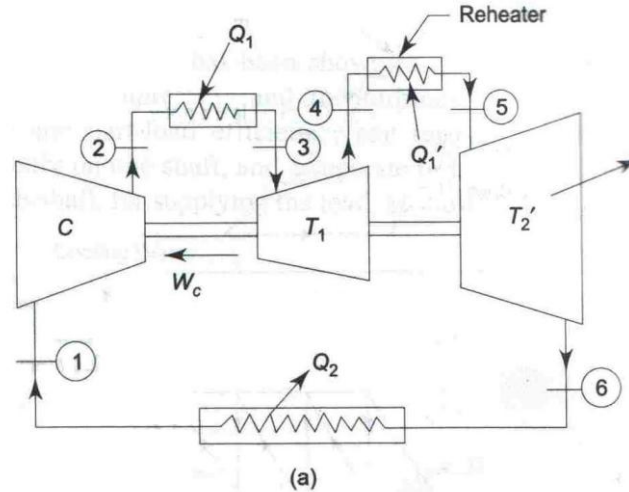
In practice the regenerator is costly, heavy and bulky, and causes pressure losses which brings about a decrease in cycle ϵ .

II Ideal regenerative cycle with inter cooling and reheat:

a) Gas turbine cycle with reheat and multistage expansion:



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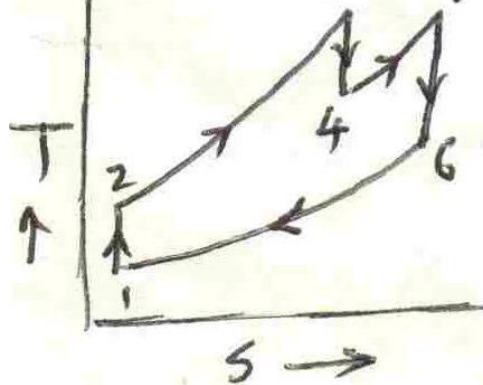


Increased work output due to reheating

Effect of reheat on Brayton cycle

Work output of the turbine is increased by multistage expansion with reheating. In the above illustration, two-stage expansion is done in low pressure (T_1) and high pressure (T_2) turbines with reheating the air in between stages.

Optimum work output for a two-stage reheat cycle:





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Assumptions: The air after one stage of expansion is reheated back to its original temperature
 i.e., $T_3 = T_5$

Let $R = \frac{P_2}{P_1}, R_1 = \frac{P_3}{P_4}$ and $R_2 = \frac{P_5}{P_6}, t = \frac{T_3}{T_1} = \text{maximum cycle temperature ratio}$

$$4 \quad R_1 R_2 = \frac{P_3}{P_4} \frac{P_5}{P_6} \quad P_5 = P_4$$

$$= \frac{P_2}{P_1} = R$$

Net work output is $W_N = C_p (T_3 - T_4) + C_p (T_5 - T_6) - C_p (T_2 - T_1)$

$$= C_p T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} + \frac{T_5}{T_1} - \frac{T_6}{T_1} - \frac{T_2}{T_1} + 1 \right]$$

we have $\frac{T_2}{T_1} = \frac{P_2}{P_1} = R$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} \frac{T_3}{T_1} = \frac{P_4}{P_3} \frac{T_3}{T_1} = \frac{t}{R_1}$$

$$\frac{T_6}{T_1} = \frac{T_6}{T_5} \frac{T_5}{T_1} = \frac{P_6}{P_5} \frac{T_5}{T_1} = \frac{t}{R_2}$$

$$4 \quad W_N = C_p T_1 \left[\frac{t}{R_1} + \frac{t}{R_2} - R + 1 \right]$$

but $R_2 = R/R_1$

$$4 \quad W_N = C_p T_1 \left[\frac{t}{R_1} + \frac{t}{R/R_1} - R + 1 \right]$$

$$= \frac{C_p T_1}{R} \left[t + tR - R^2 + R \right]$$

For given values of R, t and T_1 , W_N is maximum if $\frac{dW_N}{dR} = 0$

$$4 \quad \frac{dW_N}{dR} = C_p T_1 \left[0 + t - 2R + 1 \right] = 0$$

$$2R = t + 1$$

$$R = \frac{t + 1}{2}$$



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$$\begin{aligned}
 & \frac{dR_1}{dR_2} = 0 \\
 & + \\
 & = \\
 & \text{i.e., } \frac{dR_1}{dR_2} = 0 \quad \text{or} \quad R_1 = R_2 = R
 \end{aligned}$$

4 For maximum work output, $R_1 = R_2 = R$

Similarly if the cycle has 'N' stages of expansion, with reheating, then for maximum work output, pressure ratio for each = (compression pressure ratio)^{1/N}

Advantages: By employing multistage expansion with reheating in between the stages, the net work output from the gas turbine cycle can be increased. This is illustrated on the T-S diagram shown for a 2-stage expansion with reheating in between the stages.

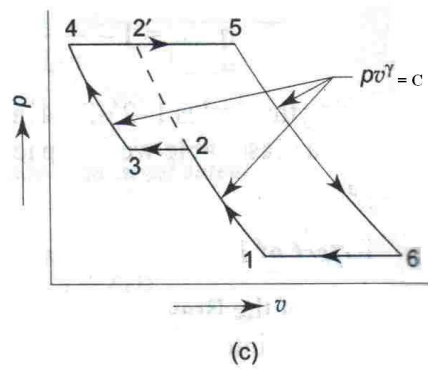
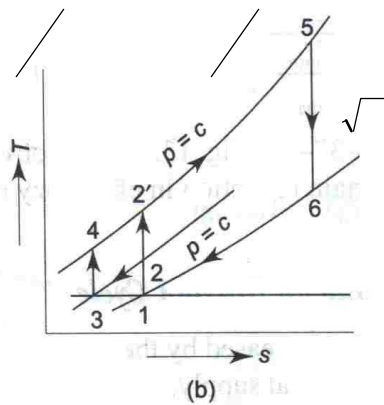
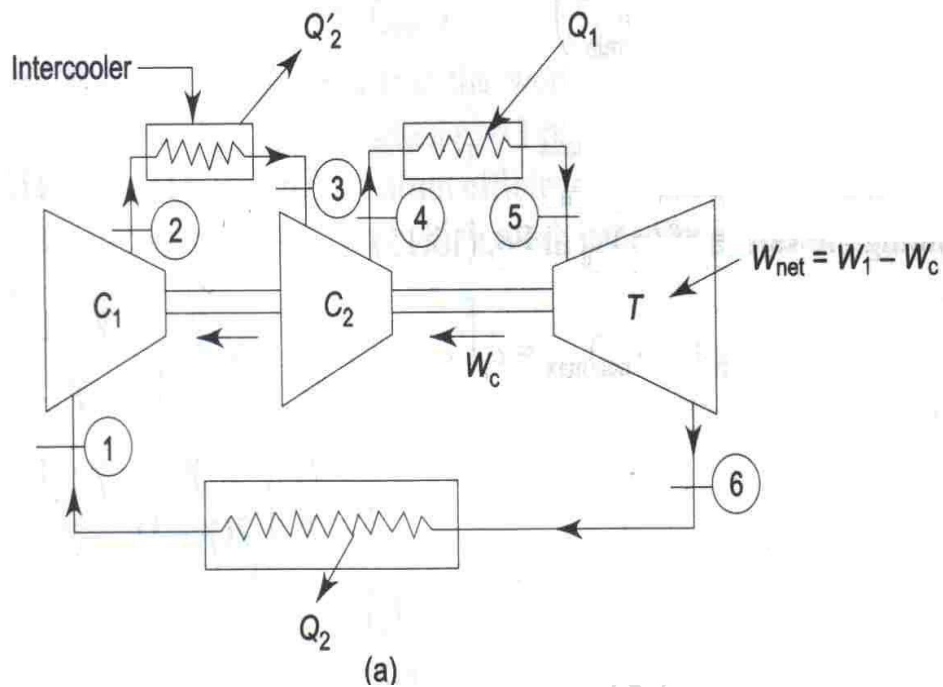
Disadvantages: But disadvantage of reheating is due to the fact that additional heat has to be supplied in order to reheat the air after each stage of expansion. This may result in a decrease in the thermal efficiency of the cycle. This is shown in figure below.

It can be seen that for a given value of t, the thermal efficiency of the reheat cycle increases with increase in R and for a given value of R, the thermal efficiency increases with increase in t. However, the thermal efficiency of a reheat cycle will be less than that of a simple cycle for a given value of R.



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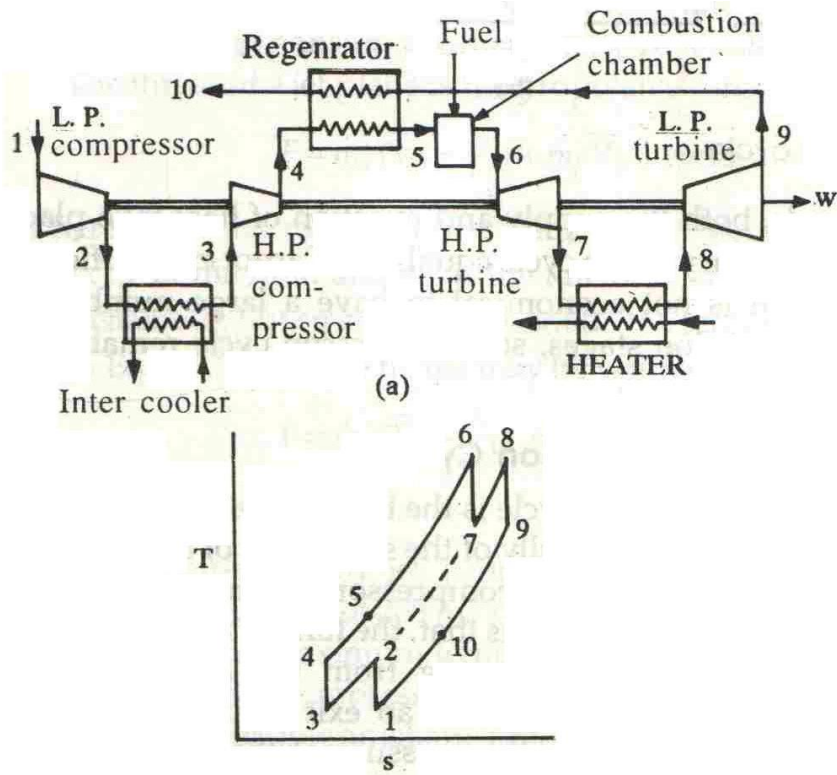
b) Gas turbine cycle with multistage compression with intercooling in between the stages:



Effect of intercooling on Brayton cycle



c) Gas turbine cycle with two stage compression two-stage expansion and regenerator.



The work output from a simple gas turbine cycle can be increased also by having multistage compression with inter cooling in between the stages. The effect of having two stage compression with inter cooling in between the stages is illustrated on the T-S diagram. It can be seen that, a higher work output has been achieved than that of simple cycle by an amount shown by the shaded area. The disadvantage of it is that more heat has to be supplied to heat the air than that is required for simple cycle. This may reduce the thermal efficiency of the cycle.

The thermal efficiency of the cycle is given by $\eta_{th} = 1 - \frac{Q_L}{Q_H}$

$$\begin{aligned}
 &= \frac{Q_H - Q_L}{Q_H} \\
 &= \frac{(h_6 - h_7) - (h_2 - h_1)}{(h_6 - h_7)} \\
 &= 1 - \frac{(h_2 - h_1)}{(h_6 - h_7)} \\
 &= 1 - \frac{(h_2 - h_1)}{(h_8 - h_7)}
 \end{aligned}$$



-27h₃ h -27h h

Deviation of Practical gas turbine cycle from ideal cycle:

- 1) The working substance will not be air throughout the cycle. Air is compressed in compressor where as the products of combustion coming out of the combustion chamber is expanded in the turbine. The value of C_p and γ will be different for expansion and heating as compared to compression process.

For compression, $C_p = 1.005 \text{ kJ/kg}^\circ\text{K}$, $\gamma = 1.4$

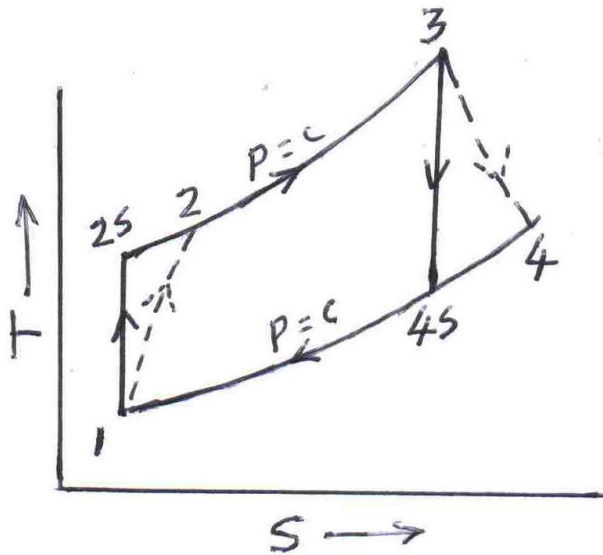
For expansion, $C_p = 1.135 \text{ kJ/kg}^\circ\text{K}$, $\gamma = 1.33$

- 2) There will be pressure loss in the piping connecting the various components of the plant. the pressure with which the products of combustion enters the turbine will be less than the pressure with which air is coming out of the compressor i.e., the pressure ratio for expansion will be less than pressure ratio for compression.

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3)



- 1-2s isentropic work of compression
- 1-2 actual work of compression
- 3-4s isentropic turbine work
- 3-4 Actual turbine work

In a practical gas turbine cycle the compression and expansion processes are not isentropic but adiabatic with certain amount of frictional losses. The friction losses are accounted for by defining a parameter called isentropic efficiency.

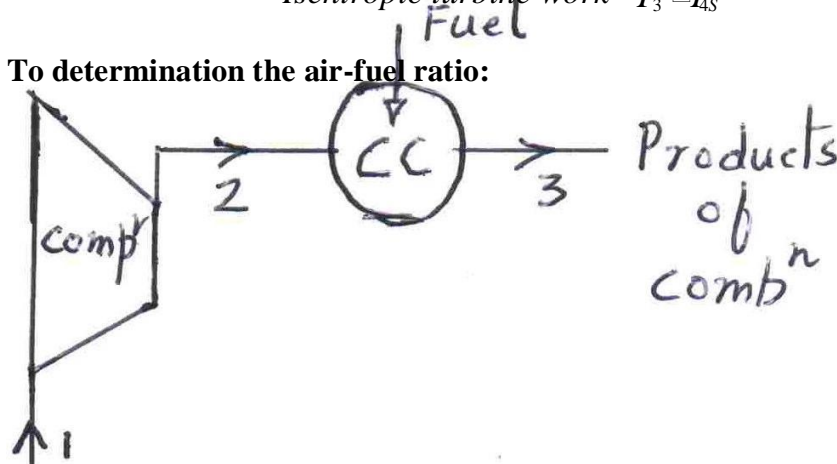
a) Isentropic efficiency of compression (η_c)

$$\eta_c = \frac{\text{Isentropic work of compression}}{\text{Actual work of compression}} = \frac{T_2 - T_1}{T_2s - T_1}$$

b) Isentropic efficiency of turbine (η_t)

$$\eta_t = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{T_3 - T_4}{T_3 - T_4s}$$

To determine the air-fuel ratio:





Let m_a = mass of air entering the combustion chamber
 m_f = mass of fuel entering the combustion chamber
 CV = Calorific value of fuel

Applying SFEE to combustion chamber, we get,

$$m_a h_2 + m_f CV = (m_a + m_f) h_3$$

by m_f $\frac{m_a}{m_f} h_2 + CV = \frac{m_a}{m_f} h_3 + h_3$

$$\frac{m_a}{m_f} h_2 + \frac{CV}{h_3} = \frac{m_a}{m_f} + 1$$

$$\frac{m_a}{m_f} (h_3 - h_2) = CV$$

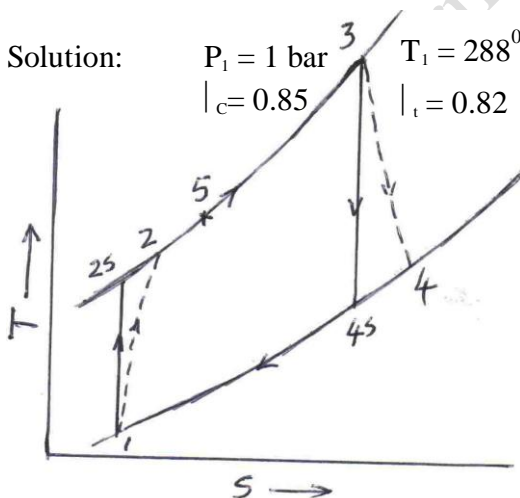
or $\frac{m_a}{m_f} = \frac{CV}{h_3 - h_2}$

Problems:

- In a G.T. installation, the air is taken in at 1 bar and 15°C and compressed to 4 bar. The isentropic efficiency of turbine and the compressor are 82% and 85% respectively. Determine (i) Compression work, (ii) Turbine work, (iii) work ratio, (iv) Theoretical Efficiency.

What would be the improvement in the th. efficiency if a regenerator with 75% effectiveness is incorporated in the cycle. Assume the maximum cycle temperature to be 825°K.

Solution: $P_1 = 1 \text{ bar}$ $T_1 = 288^\circ\text{K}$ $P_2 = 4 \text{ bar}$ $T_3 = 825^\circ\text{K}$
 $\eta_c = 0.85$ $\eta_t = 0.82$





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$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{288} = \left(\frac{4}{3}\right)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 288 \times 1.105 = 318.24 \text{ K}$$

Process 1-2 is isentropic i.e.,

$$T_2 = 288 \left(\frac{4}{3}\right)^{\frac{1.4-1}{1.4}} = 428.14 \text{ K}$$

$$T_3 = 452.87 \text{ K}$$

But $T_3 = 288 \left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$

$$T_3 = 288 \left(\frac{4}{3}\right)^{\frac{1.4-1}{1.4}} = 428.14 \text{ K}$$

Process 3-4 is isentropic

$$T_4 = 825 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = 825 \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 603.5 \text{ K}$$

But $T_3 = 428.14 \text{ K}$

$$T_3 = 428.14 \text{ K}$$

(i) Compressor work, $W_c = 1.005 (452.87 - 288) = 165.69 \text{ kJ/kg}$

(ii) Turbine work, $W_t = C_p (T_3 - T_4)$
 $= 1.005 (825 - 603.5) = 222.54 \text{ kJ/kg}$

$$\eta = \frac{\text{Net work output}}{\text{Turbine work}} = \frac{W_c}{W_t} = 0.255$$

(iv) Thermal efficiency, $\eta_{th} = \frac{W_{net}}{Q_H}$

$$\eta_{th} = \frac{165.69}{1.005 (825 - 288)} = 0.255$$

we have effectiveness $= \frac{T_3 - T_4}{T_3 - T_2} = 0.7 = \frac{T_3 - T_4}{T_3 - T_2}$



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$$\frac{452.87}{T_4} = \frac{603.57}{T_2} = 452.87$$

$$\eta_{th} = \frac{W_T - W_C}{Q_H} = \frac{56.85}{260.4} = 0.218$$

4Improvement in η_{th} due to regenerator = $\frac{0.218 - 0.152}{0.152}$
i.e., 43.6%

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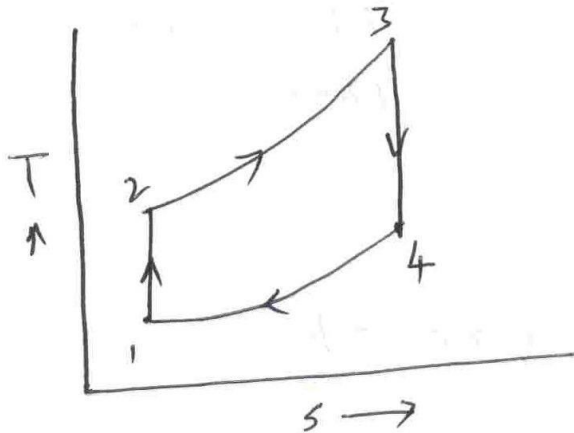
Problems:

1. In a G.T. installation, the air is taken in at 1 bar and 15°C and compressed to 4 bar. The isentropic η of turbine and the compressor are 82% and 85% respectively. Determine (i) Compression work, (ii) Turbine work, (iii) work ratio, (iv) Th. η .

What would be the improvement in the th. η if a regenerator with 75% effectiveness is incorporated in the cycle. Assume the maximum cycle temperature to be 825°K .

Solution: $P_1 = 1 \text{ bar}$ $T_1 = 288^{\circ}\text{K}$ $P_2 = 4 \text{ bar}$ $T_3 = 825^{\circ}\text{K}$
 $\eta_C = 0.85$ $\eta_t = 0.82$

Case1: Without Regeneration:



Process 1-2s is isentropic i.e., $\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$\therefore T_{2s} = 288 \left(4\right)^{\frac{0.4}{1.4}} = 428.14^{\circ}\text{K}$

But $\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1}$ i.e., $0.85 = \frac{428.14 - 288}{T_2 - 288}$ $\therefore T_2 = 452.87^{\circ}\text{K}$

Process 3-4s is isentropic $i.e., \frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$ $\therefore T_{4s} = 825 \left(\frac{1}{4}\right)^{\frac{0.4}{1.4}} = 554.96$

But $\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}}$ i.e., $0.82 = \frac{825 - T_4}{825 - 554.96}$ $\therefore T_4 = 603.57^{\circ}\text{K}$

(i) Compressor work, $W_C = C_p (T_2 - T_1)$
 $= 1.005 (452.87 - 288) = 165.69 \text{ kJ/kg}$

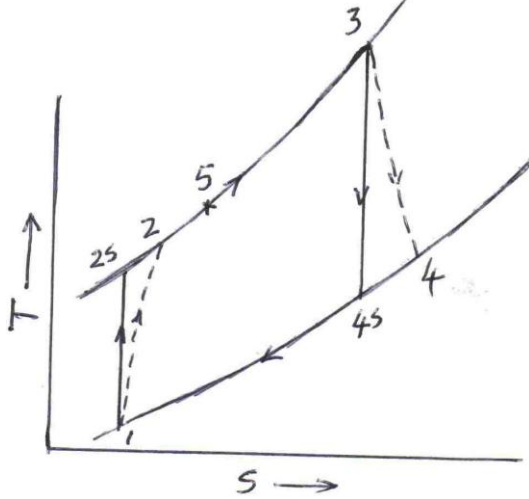
(ii) Turbine work, $W_t = C_p (T_3 - T_4)$
 $= 1.005 (825 - 603.57) = 222.54 \text{ kJ/kg}$



$$(iii) \text{ Work ratio} = W_R = \frac{\text{Network output}}{\text{Turbine work}} = \frac{W_T - W_C}{W_T} = 0.255$$

$$(iv) \text{ Thermal Efficiency } \eta, \eta_{th} = \frac{W_{net}}{Q_H} = \frac{222.54 - 165.69}{C_p (825 - 452.87)} = \frac{56.85}{373.99} = 15.2\%$$

Case2: With Regeneration:



We have effectiveness, $\epsilon = \frac{T_5 - T_2}{T_4 - T_2}$ i.e., $0.75 = \frac{T_5 - 452.87}{603.57 - 452.87}$
 $\therefore T_5 = 565.89 \text{ K}$

\therefore Heat supplied, $Q_H^1 = Q_{5-3} = C_p(T_3 - T_5)$
 $= 1.005 (825 - 565.89)$
 $= 260.4 \text{ kJ/kg}$

$\therefore \eta_{th} = \frac{W_T - W_C}{Q_H^1} = \frac{56.85}{260.4} = 0.218$

$\therefore \eta = \frac{0.218 - 0.152}{0.152}$ Improvement in η_{th} due to regenerator
 $= 0.436$ i.e., **43.6%**

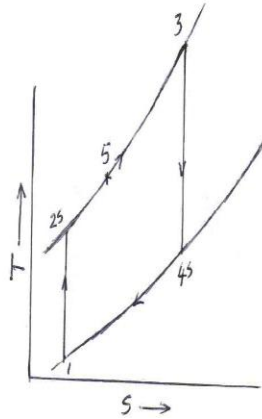
- 2. The maximum and minimum pressure and temperatures of a gas turbine are 5 bar, 1.2 bar and 1000K and 300K respectively. Assuming compression and expansion processes as isentropic, determine the η_{th} (a) when an ideal regenerator is incorporated in the plant and (b) when the effectiveness of the above regenerator is 75%.**

Solution: $P_2 = P_3 = 5 \text{ bar}$ $P_1 = P_4 = 1.2 \text{ bar}$



$$T_3 = 1000\text{K} \quad T_1 = 300\text{K}$$

3. Solve the above problem when the adiabatic efficiencies of the turbine and compressor are 90% and 85% respectively.



Process 1-2s is isentropic i.e.,

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}$$

$$\therefore T_{2s} = 300 \times \frac{1.2^{0.4}}{1.2} = 451.21 \text{ K}$$

Process 3-4s is isentropic i.e.,

$$\frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{r-1}{r}}$$

$$\therefore T_{4s} = 1000 \times \frac{1.2^{0.4}}{1.2} = 664.88 \text{ K}$$

Ideal regenerator: i.e., $T_5 = T_4$

$$\therefore \text{Heat supplied} = C_p (T_3 - T_5)$$

$$= 1.005 [1000 - 664.88] = 336.79 \text{ kJ/kg}$$

$$W_{\text{net}} = W_T - W_C = C_p (T_3 - T_4) - C_p (T_2 - T_1)$$

$$= 1.005 [1000 - 664.88 - 451.21 + 300] = 183.91$$

$$\therefore \eta_{th} = \frac{W_{\text{net}}}{Q_H} = \frac{183.91}{336.79} = 0.546 \text{ or } 54.6\%$$

Regenerator with $\epsilon = 0.75$ i.e., $0.75 = \frac{T_5 - T_2}{T_4 - T_2} = \frac{\text{actual temperature drop}}{\text{ideal temperature drop}}$

$$\text{i.e., } 0.75 = \frac{T_5 - 451.21}{(664.88 - 451.21)} \quad \therefore T_5 = 611.46 \text{ K}$$

$$\therefore \text{Heat supplied, } Q_H = C_p (T_3 - T_5)$$

$$= 1.005 (1000 - 611.46) = 390.48 \text{ kJ/kg}$$



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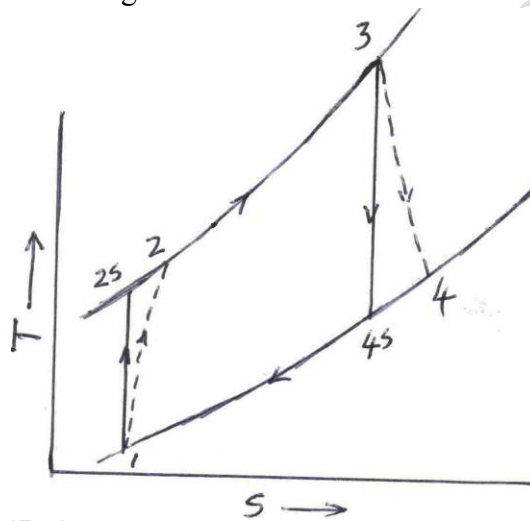
$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{185.91}{390.48} = 0.471 \text{ or } 47.1\%$$

4. A gas turbine plant uses 500kg of air/min, which enters the compressor at 1 bar, 17⁰C. The compressor delivery pressure is 4.4 bars. The product of combustion leaves the combustion chamber at 650⁰ C and is then expanded in the turbine to 1 bar. Assuming isentropic efficiency of compressor to be 75% and that of the turbine to be 85%, calculate (i) mass of the fuel required /min, of the CV of fuel is 39000KJ/Kg. (ii) net power output (iii) Overall thermal efficiency of the plant. Assume C_p=1.13KJ/Kg-K, γ=1.33 for both heating and expansion.

Solution: $m_a = 500 \text{ kg / min} = 8.33 \text{ kg / sec}$

$P_1 = 1 \text{ bar}$	$T_1 = 290 \text{ K}$	
$P_2 = 4.4 \text{ bar}$	$T_3 = 923 \text{ K}$	
$m_f = ?$	$W_N = ?$	$\eta_{th} = ?$

$\eta_c = 0.75$ $\eta_t = 0.85$
 Calorific Value = 39000 kJ/kg



Process 1-2s is isentropic compression i.e., $P_1 V_1^\gamma = P_2 V_2^\gamma$ or $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ or $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

$$\therefore T_{2s} = \frac{P_2}{P_1}^{\frac{\gamma-1}{\gamma}} = 290(4.4)^{\frac{0.4}{1.33}} = 443.02 \text{ K}$$

But $\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$ i.e., $0.75 = \frac{443.02 - 290}{T_2 - 290}$ $\therefore T_2 = 494.03 \text{ K}$

$$\frac{T_{4s}}{T_3} = \frac{P_4}{P_3}^{\frac{\gamma-1}{\gamma}} \quad T_{4s} = 923 \frac{1}{4.4}^{\frac{0.32}{1.33}} = 639.18 \text{ K}$$

But $\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}}$ i.e., $0.85 = \frac{923 - T_4}{923 - 639.18}$ $\therefore T_4 = 681.76 \text{ K}$

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(i) m_f

$$m_f C_p (T_3 - T_2) = m_f 1.13(923 - 494.03)$$

$$\therefore m_f = \mathbf{6.21 \text{ kg/min}}$$

(ii) $W_N = ?$

$$\begin{aligned} \text{Compressor work, } W_C &= C_p (T_2 - T_1) \\ &= 1.005 (494.03 - 290) \\ &= 205.05 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Turbine work, } W_T &= C_p (T_3 - T_4) \\ &= 1.13 (923 - 681.76) \\ &= 272.6 \text{ kJ/kg} \end{aligned}$$

$$\therefore W_N = W_T - W_C = 67.55 \text{ kJ/kg}$$

$$\therefore \text{Net work output per minute} = (m_a + m_f) W_N$$

$$\begin{aligned} \therefore \text{Power output} &= \frac{34194.49}{60} = 569.91 \text{ kW} \\ &= (500 + 6.21) (67.55) = 34194.49 \text{ kJ/min} \end{aligned}$$

(iii) $\eta_{th} = ?$

$$\begin{aligned} \text{Heat supplied, } Q_H &= C_p (T_3 - T_2) \\ &= 1.33 (923 - 494.03) \\ &= 570.53 \text{ kJ/kg} \end{aligned}$$

$$\therefore \eta_{th} = \frac{W_N}{Q_H} = \frac{67.55}{570.53} = \mathbf{0.118 \text{ or } 11.8\%}$$



5. A G.T. cycle having 2 stage compression with intercooling in between stages and 2 stages of expansion with reheating in between the stages has an overall pressure ratio of 8. The maximum cycle temperature is 1400°K and the compressor inlet conditions are 1 bar and 27°C . The compressors have η_s of 80% and turbines have η_s of 85%. Assuming that the air is cooled back to its original temperature after the first stage compression and gas is reheated back to its original temperature after 1st stage of expansion, determine (i) the net work output (ii) the cycle η_{th} .

Solution: $\frac{P_4}{P_1} = 8 \text{ bar}$ $T_5 = 1400^{\circ}\text{K}$ $T_1 = 300^{\circ}\text{K}$, $P_1 = 1 \text{ bar}$

$$\eta_{C1} = 0.8 = \eta_{C2}, \quad \eta_{t1} = \eta_{t2} = 0.85,$$

$$T_3 = T_1 \quad T_7 = T_5$$

For maximum work output, $\frac{P_2}{P_1} = \frac{P_4}{P_3} = \frac{P_5}{P_6} = \frac{P_7}{P_8} = \frac{P_4}{P_1} = \frac{P_5}{P_8} = \bar{8}$

\therefore Intermediate Pressure, $P_2 = P_3 = P_6 = P_7 = 2.83 \text{ bar}$

$$\frac{P_2}{P_1} = 2.83$$

$$= 300 (2.83)^{0.286} = 403.95^{\circ}\text{K}$$

But $\eta_{c1} = 0.8 = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{403.95 - 300}{T_2 - 300} \therefore T_2 = 429.9^{\circ}\text{K}$

Since $T_3 = T_1$ and $\frac{P_4}{P_3} = \frac{P_2}{P_1}$

We have $T_{4s} = T_{2s} = 403.95^{\circ}\text{K}$

Also since $\eta_{C1} = \eta_{C2}$, $T_4 = T_2 = 429.9^{\circ}\text{K}$



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$$\begin{aligned}\therefore \text{Compressor work, } W_C &= C_p (T_2 - T_1) + C_p (T_4 - T_3) \\ &= 2 C_p (T_2 - T_1) \\ &= 2 (1.005) (429.9 - 300) \\ &= 261.19 \text{ kJ/kg}\end{aligned}$$

For process 5 – 6,

$$\frac{T_{6s}}{T_5} = \frac{P_6}{P_5}^{\frac{\gamma-1}{\gamma}} \quad \therefore T_{6s} = \frac{1400}{2.83} 1^{0.286} = 1039.72 \text{ K}$$

$$\text{But } \eta_{i1} = \frac{T_5 - T_6}{T_5 - T_{6s}} \quad \text{i.e., } 0.85 = \frac{1400 - T_6}{1400 - 1039.72} \quad \therefore T_6 = 1093.76 \text{ K}$$

Since $T_7 = T_5$ and $\frac{P_5}{P_6} = \frac{P_7}{P_8}$, then $T_8 = T_6$

Since $\eta_{t1} = \eta_{t2}$, $T_6 = T_8 = 1093.76 \text{ K}$

$$\begin{aligned}\therefore \text{Turbine work, } W_t &= C_p (T_5 - T_6) + C_p (T_7 - T_8) \\ &= 2 C_p (T_5 - T_6) \\ &= 2 (1.005) (1400 - 1093.76) \\ &= 615.54 \text{ kJ/kg}\end{aligned}$$

$$\therefore W_N = W_T - W_C = 354.35 \text{ kJ/kg}$$

$\eta_{th} = ?$

$$\begin{aligned}\text{Heat Supplied, } Q_H &= C_p (T_5 - T_4) + C_p (T_7 - T_6) \\ &= 1.005 (1400 - 429.9 + 1400 - 1093.76) \\ &= 1282.72 \text{ kJ/kg}\end{aligned}$$

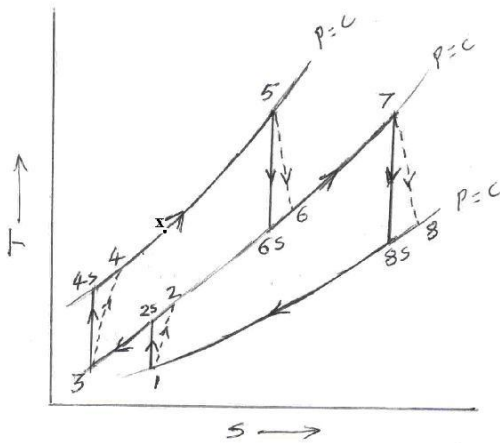
$$\therefore \eta_{th} = \frac{354.35}{1282.72} = 0.276 \text{ or } 27.6\%$$



6. A two stage gas turbine cycle receives air at 100 kPa and 15⁰ C. The lower stage has a pressure ratio of 3, while that for the upper stage is 4 for the compressor as well as the turbine. The temperature rise of the air compressed in the lower stage is reduced by 80% by intercooling. Also, a regenerator of 78% effectiveness is used. The upper temperature limit of the cycle is 1100⁰ C. The turbine and the compressor η_s are 86%. Calculate the mass flow rate required to produce 6000kW.

Solution: $P_1 = 1 \text{ bar}$ $T_1 = 288^0 \text{ K}$ $\frac{P_2}{P_1} = 3, \frac{P_4}{P_3} = 4$ $\eta_{IC} = 0.8$
 $= \eta_{reg} = 0.78, T_5 = 1373^0 \text{ K},$ $\eta_{C1} = \eta_{C2} = \eta_{t1} = \eta_{t2} = 0.86$

$m = ?$ if $P = 6000 \text{ kW}$



Process 1-2s is isentropic compression

$$\therefore \frac{T_{2s}}{T_1} = \frac{P_2}{P_1}^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_{2s} = 288 (3)^{0.286} = 410.75^0 \text{ K}$$

But $\eta_{C1} = \frac{T_{2s} - T_1}{T_2 - T_1}$ i.e., $0.86 = \frac{410.75 - 288}{T_2 - 288}$ $\therefore T_2 = 430.73^0 \text{ K}$

Also, $\eta_{IC} = \frac{T_2 - T_3}{T_2 - T_1}$ i.e., $0.8 = \frac{430.73 - T_3}{430.73 - 288}$ $\therefore T_3 = 316.54^0 \text{ K}$

Process 3-4s is 2nd

stage isentropic compression

$$\therefore \frac{T_{4s}}{T_3} = \frac{P_4}{P_3}^{\frac{\gamma-1}{\gamma}}$$



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$$\therefore T_{4s} = 316.54 (4)^{0.286} = 470.57^{\circ}K$$

$$\text{But } \eta_{c2} = \frac{T_{4s} - T_3}{T_4 - T_3} \quad \text{i.e., } 0.86 = \frac{470.57 - 316.54}{T_4 - 316.54} \quad \therefore T_4 = 495.64^{\circ}K$$

Process 5-6s is 1st stage isentropic expansion $\therefore \frac{T_{6s}}{T_5} = \left(\frac{P_6}{P_5}\right)^{\frac{\gamma-1}{\gamma}}$

$$\therefore T_{6s} = 1373 \frac{1}{4}^{0.286} = 923.59^{\circ}K$$

$$\text{But } \eta_{t1} = \frac{T_5 - T_6}{T_5 - T_{6s}} \quad \text{i.e., } 0.86 = \frac{1373 - T_6}{1373 - 923.59} \quad \therefore T_6 = 986.51^{\circ}K$$

Process 6-7 is reheating, assume $T_7 = T_5 = 1373^{\circ}K$

Process 7-8s is 2nd stage isentropic expansion i.e., $\frac{T_{8s}}{T_7} = \left(\frac{P_8}{P_7}\right)^{\frac{\gamma-1}{\gamma}}$

$$\therefore T_{8s} = 1373 \frac{1}{3}^{0.286} = 1002.79^{\circ}K$$

$$\text{But } \eta_{t2} = \frac{T_7 - T_8}{T_7 - T_{8s}} \quad \text{i.e., } 0.86 = \frac{1373 - T_8}{1373 - 1002.79} \quad \therefore T_8 = 1054.63^{\circ}K$$

Regenerator is used to utilize the temperature of exhaust gases $\text{i.e., } \epsilon = \frac{T_x - T_4}{T_8 - T_4}$

$$\text{i.e., } 0.78 = \frac{T_x - 495.64}{1054.63 - 495.64} \quad \therefore T_x = 931.65^{\circ}K$$



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We have, Compressor work: $W_C = C_p (T_2 - T_1) + C_p (T_4 - T_3)$
 $= 1.005 (430.73 - 288 + 495.64 - 316.54)$
 $= 323.44 \text{ kJ/kg}$

Also, Turbine work : $W_T = C_p (T_5 - T_6) + C_p (T_7 - T_8)$
 $= 1.005 (1373 - 986.51 + 1373 - 1054.63)$
 $= 708.38 \text{ kJ/kg}$

\therefore Net work output, $W_N = W_T - W_C$
 $= 384.95 \text{ kJ/kg}$

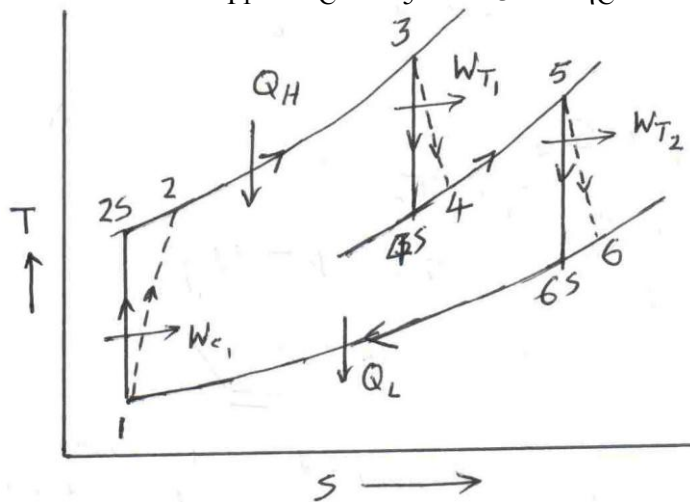
But, power produced, $P = m W_N$
 i.e., $6000 \times 1000 = m \times 384.95 \times 1000$
 $\therefore m = 15.59 \text{ kg/sec}$

We have, heat supplied, $Q_H = C_p (T_5 - T_x) + C_p (T_7 - T_6)$
 $= 1.005 (1373 - 931.65 + 1373 - 986.51)$
 $= 831.98 \text{ kJ/kg}$

$\therefore \eta_{th} = \frac{W}{Q_H} = 0.463 \text{ or } 46.3\%$

9. In a reheat gas turbine cycle, comprising one compressor and two turbine, air is compressed from 1 bar, 27°C to 6 bar. The highest temperature in the cycle is 900°C. The expansion in the 1st stage turbine is such that the work from it just equals the work required by the compressor. Air is reheated between the two stages of expansion to 850°C. Assume that the isentropic η_s of the compressor, the 1st stage and the 2nd stage turbines are 85% each and that the working fluid is air and calculate the cycle η .

Solution: $P_1 = 1 \text{ bar}$ $T_1 = 300\text{K}$ $P_2 = 6 \text{ bar}$ $T_3 = 1173\text{K}$
 $W_{T1} = W_C$ $T_5 = 1123\text{K}$ $\eta_C = 0.85$ $\eta_{t1} = \eta_{t2} = 0.85$



We have process 1-2s is isentropic i.e.,

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$



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$$\therefore T_{2S} = 300 \frac{6^{0.4}}{1} = 500.5$$

But $\eta_c = \frac{T_1 - T_{2S}}{T_2 - T_1}$ i.e., $0.85 = \frac{500.5 - 300}{T_2 - 300}$

\therefore Compressor work, $W_C = C_p (T_2 - T_1)$
 $= 1.005 (536 - 300)$

From data, $W_{T1} = W_C = 237 \text{ kJ/kg}$
 $= C_p (T_3 - T_4)$

But $\eta_{t1} = \frac{T_3 - T_4}{T_3 - T_{4S}}$ i.e., $0.85 = \frac{1173 - 93}{1173 - T_{4S}}$

Process 3-4 is isentropic i.e.,

$$\frac{P_4}{P_3} = \left(\frac{T_4}{T_3} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore P_4 = 6 \frac{895^{1.4}}{1173} = 2$$

From T-S diagram, intermediate pressure, $P_4 = P_5 = 2.328 \text{ bar}$

Process 5-6s is isentropic in the 2nd stage turbine

i.e., $\frac{T_6s}{T_5} = \left(\frac{P_6}{P_5} \right)^{\frac{\gamma-1}{\gamma}} = 1123 \frac{1}{2.328}^{0.4} = 882K$

But $\eta_{t2} = \frac{T_5 - T_6}{T_5 - T_{6S}}$ i.e., $0.85 = \frac{1123 - T_6}{1123 - 882}$ $\therefore T_6 = 918K$

$\therefore W_{T2} = C_p (T_5 - T_6)$
 $= 1.005 (1123 - 918) = 206 \text{ kJ/kg}$

\therefore Net work output $= W_T - W_C$
 $= (W_{T1} + W_{T2}) - W_C = 206 \text{ kJ/kg}$

Net heat transfer or heat supplied, $Q = Q_H + Q_R$
 $= C_p (T_3 - T_2) + C_p (T_5 - T_4)$
 $= 640 + 187 = 827 \text{ kJ/kg}$

\therefore Cycle efficiency, $\eta_{cycle} = \frac{W_{net}}{Q_{net}} = \frac{206}{827} = 25\%$

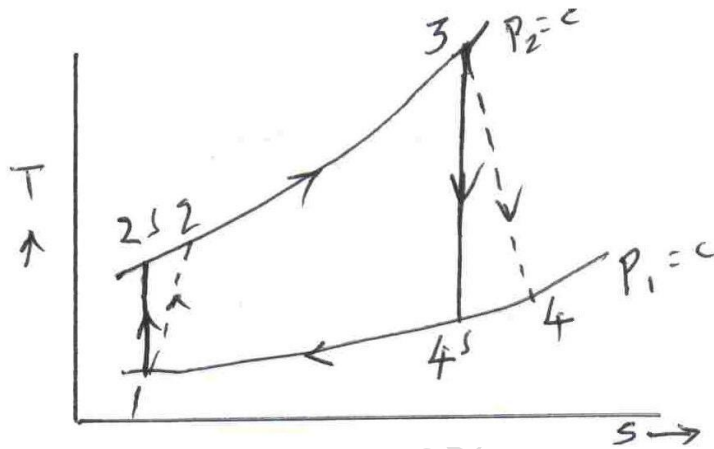


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Further, $W_{net} = (W_T - W_C) = (m_a + m_f) C_{Pg} (T_3 - T_4) - m_a C_{Pa} (T_2 - T_1)$

12. In a simple gas turbine unit, the isentropic discharge temperature of air flowing out of compressor is 195°C, while the actual discharge temperature is 240°C. Conditions of air at the beginning of compression are 1 bar and 17°C. If the air-fuel ratio is 75 and net power output from the unit is 650kW. Compute (i) isentropic η of the compressor and the turbine and (ii) overall η . Calorific value of the fuel used is 46110 kJ/kg and the unit consumes 312 kg/hr of fuel. Assume for gases $C_p = 1.09$ kJ/kg-K and $\gamma = 1.32$ and for air $C_p = 1.005$ kJ/kg-K and $\gamma = 1.4$.

Solution: $T_{2S} = 195 + 273 = 468$ K $T_2 = 240 + 273 = 513$ K $T_1 = 290$ K $P_1 = 1$ bar
 $A/F = 75$, Power output = $W_{net} = W_T - W_C = 650$ kW $\eta_C = ?$ $\eta_T = ?$ $\eta_{cycle} = ?$
 $CV = 46110$ kJ/kg $m_f = 312$ kg/hr = 0.0867 kg/s $C_{Pg} = 1.09$ kJ/kg-K, $\gamma_g = 1.30$
 $C_{Pa} = 1.005$ kJ/kg-K, $\gamma_a = 1.4$



We have, Compressor Efficiency, $\eta_C = \frac{T_{2S} - T_1}{T_2 - T_1}$ i.e., $\frac{468 - 290}{513 - 290} = 0.79$

Also, $m_a = \frac{A}{F} m_f$
 $= 75 (0.0867) = 6.503$ kg/s

$\frac{T_2}{T_1} = \frac{\gamma}{\gamma-1} = \frac{468}{290} = 1.614$ $\frac{\gamma}{\gamma-1} = \frac{1.4}{0.4} = 3.5$
 $\therefore T_2 = 290 \times 3.5 = 1015$ K

Applying SFEE to the constant pressure heating process 2-3,

$m_f CV = (m_a + m_f) C_{Pg} (T_3 - T_2)$
 $0.0867 (46110) = (6.503 + 0.0867) 1.09 (T_3 - 513)$ $\therefore T_3 = 1069.6$ K

Also, $T_{4S} = T_3$



$$\frac{P_4}{P_3} = \left(\frac{T_4}{T_3} \right)^{\frac{\gamma_g - 1}{\gamma_g}} = \left(\frac{1069.6 - T_4}{1069.6} \right)^{\frac{1.32 - 1}{1.32}}$$

$$\therefore T_{4S} = 712.6K$$

$$\text{i.e., } 650 = (6.503 + 0.0867) 1.09 (1069.6 - T_4) - 6.503 (1.005) (513 - 290) \therefore T_4 = 776K$$

$$\text{Now, Turbine Efficiency, } \eta_T = \frac{T_3 - T_4}{T_3 - T_{4S}} = \frac{1069.6 - 776}{1069.6 - 712.6} = 0.822$$

$$\text{And, } \eta_{cycle} = \frac{W_{net}}{m_f CV} = \frac{650}{0.0867(46110)} = 0.163$$

Or

$$\eta_{cycle} = \frac{W_{net}}{Q_H}$$

$$= \frac{650}{(m_a + m_f) C_{Pg} (T_3 - T_2)} = \frac{650}{3997.9} = 0.162$$



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UNIT-7

REFRIGERATION

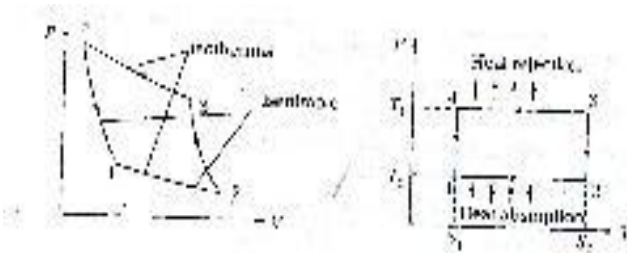
Definition

Refrigeration is the process of providing and maintaining temperature of the system below that of the surrounding atmosphere.

Carnot Cycle

The reversed Carnot cycle can be considered in refrigeration system.

$$C.O.P = \frac{T_2}{T_2 - T_1} \quad \text{where } T_2 < T_1$$



Unit of Refrigeration

The common unit used in the field of refrigeration is known as Ton of refrigeration.

A ton of refrigeration is defined as the quantity of heat required to be removed to produce one ton (1000kg) of ice within 24 hours when the initial condition of water is 0°C

$$\text{Ton of refrigeration} = \frac{1000 \times 335}{24 \times 3600} = 3.5 \text{ kJ/s}$$

Consider a refrigerator of T tons capacity,

Refrigeration capacity = 3.5 kJ/s

Heat removed from

refrigerator = Refrigeration effect = R.E. kJ/s

Power of the compressor = work/kg of refrigerant x mass flow rate

Air Refrigeration system working on Bell-coleman cycle



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In air refrigeration system, air is used as the refrigerant which always remains in the gaseous phase. The heat removed consists only of sensible heat and as a result, the coefficient of performance (C.O.P) is low.

The various processes are:

Process 1-2:

The air leaving the evaporator enters a compressor. Where it is compressed isentropically to higher pressure and temperature.

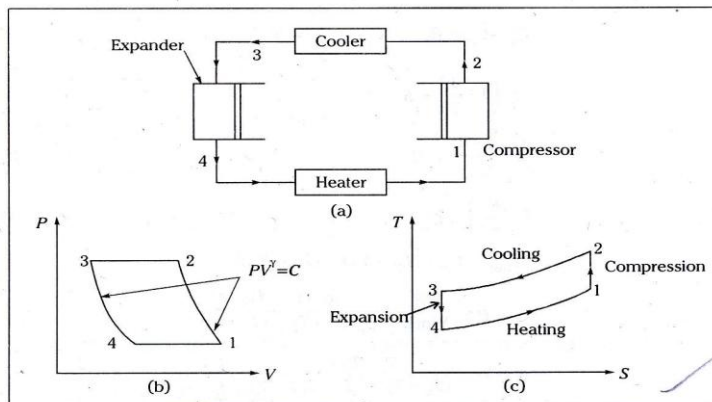
Process 2-3:

This high pressure, high temperature air, then enters a cooler where it is cooled at constant pressure to a low temperature.

Process 3-4: This high pressure, low temperature air is then expanded in an expander to lower pressure and temperature in a isentropic manner. At point 4, the temperature of the air will be lowest.

Process 4-1: This low temperature air is then passed through the heater coils where it absorbs heat from the space to be cooled namely the refrigerator and the air gets heated back to the initial temperature, but in the process, it cools the refrigerator. And the cycle repeats.

Air refrigeration system



Expression C.O.P when compression and expansion are Isentropic

Refrigeration Effect = Heat removed from the refrigerator

$$= C_p (T_1 - T_4) \text{ kJ / kg}$$

$$\text{Work input} = W_C - W_E = \gamma \frac{P_2 V_2 - P_1 V_1}{\gamma - 1} - \gamma \frac{P_3 V_3 - P_4 V_4}{\gamma - 1}$$

$$\text{Work input} = W_C - W_E = \frac{\gamma}{\gamma - 1} [P_2 V_2 - P_1 V_1 - (P_3 V_3 - P_4 V_4)]$$

$$= \frac{\gamma R}{\gamma - 1} [(T_2 - T_1) - (T_3 - T_4)]$$



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But $C_p = \frac{\gamma R}{\gamma - 1}$

$$W_{net} = C_p [(T_2 - T_1) - (T_3 - T_4)]$$

Process 1-2 is isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{-----(2)}$$

Process 3-4 is isentropic

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{-----(3)}$$

From (2) and (3)

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$C.O.P = \frac{Work}{Q_{in}} = \frac{C_p(T_1 - T_4)}{C_p[(T_2 - T_1) - (T_3 - T_4)]}$$

$$C.O.P = \frac{(T_1 - T_4)}{[(T_2 - T_1) - (T_3 - T_4)]} = \frac{1}{\frac{T_2 - T_3}{T_1 - T_4} - 1} \quad \text{---(1)}$$

$$\frac{T_2}{T_3} = \frac{T_1}{T_4}$$

$$\frac{T_2}{T_3} - 1 = \frac{T_1}{T_4} - 1$$

$$\frac{T_2 - T_3}{T_3} = \frac{T_1 - T_4}{T_4}$$

$$\frac{T_2 - T_3}{T_3} = \frac{T_1 - T_4}{T_4} \quad \text{----- (4)}$$

From (1) and (4)

$$C.O.P = \frac{1}{\frac{T_2 - T_3}{T_4} - 1}$$

$$C.O.P = \frac{T_4}{T_3 - T_4}$$



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For Polytropic process

Net work

$$W_{\text{net}} = \frac{n}{n-1} \frac{\gamma-1}{\gamma} c_p [(T_2 - T_1) - (T_3 - T_4)]$$

$$COP = \frac{T_4}{(T_3 - T_4) \frac{n}{n-1} \frac{\gamma-1}{\gamma}}$$

Advantages of air refrigeration system

1. Air is cheap, easily available.
2. It is not flammable.
3. For a given capacity, weight of air refrigeration system is less compared to other system and hence it is widely used for aircraft cooling.

Disadvantages

1. Since heat removed by air consists only of sensible heat, weight of air required is high.
2. C.O.P of the system is low compared to other systems.



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Problem 1

A cold storage is to be maintained at -5°C (268k) while the surroundings are at 35°C . The heat leakage from the surroundings into the cold storage is estimated to be 29kW . The actual C.O.P of the refrigeration plant is one third of an ideal plant working between the same temperatures. Find the power required to drive the plant. (VTU Jan 2007)

Solution : -

$$T_1 = 35^{\circ}\text{C} = 308\text{k} \quad T_2 = 5^{\circ}\text{C} = 268\text{k}$$

C.O.P of the ideal plant is nothing but

C.O.P based on carnot cycle.

$$\begin{aligned} \therefore \text{C.O.P ideal} &= \frac{T_2}{T_1 - T_2} \\ &= \frac{268}{308 - 268} = 6.7 \end{aligned}$$

$$\begin{aligned} \text{Actual C.O.P} &= \frac{1}{3} \text{ ideal C.O.P} \\ &= \frac{1}{3} \times 6.7 = 2.233 \end{aligned}$$

Q_2 = The heat removed from low temperature reservoir (cold storage) must be equal to heat leakage from surroundings to the cold storage (which is 29kW)

$$Q_2 = 29\text{kW}$$

$$\text{Actual C.O.P} = \frac{Q_2}{W}$$

$$W = \frac{Q_2}{\text{Actual C.O.P}} = \frac{29}{2.233}$$

$$\text{Power required} = 12.98 \text{ kW}$$

Problem 2



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A refrigeration machine of 6 tones capacity working on Bell coleman cycle has an upper limit pressure of 5.2 bar. The pressure and temperature at the start of the compression are 1 bar and 18°C respectively. The cooled compressed air enters the expander at 41°C. assuming both expansion and compression to be adiabatic with an index of 1.4.

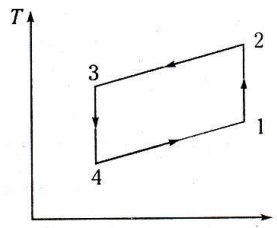
Calculate:-

- (i) Co-efficient of performance.
- (ii) Quantity of air circulated per minute.
- (iii) Piston displacement of compressor and expander
- (iv) Bore of compression and expansion cylinder when the unit runs at 240 rpm and is double acting with stroke length =200 mm
- (v) Power required to drive the unit

Solution : -

$$T_1 = 18^\circ\text{C} \quad P_1 = 1\text{bar}$$

$$T_3 = 41^\circ\text{C} \quad P_2 = 5.2\text{bar}$$



$$\begin{aligned} \text{Work input} &= C_p [(T_2 - T_1) - (T_3 - T_4)] \\ &= 1.005 [(466 - 291) - (314 - 196)] = 57\text{kJ/kg} \end{aligned}$$

$$C.O.P = \frac{\text{Refrigeration effect}}{\text{Work input}}$$

$$= \frac{95.42}{57} = 1.6757$$

$$\text{Refrigeration capacity} = 6 \text{ tons} = 6 \times 3.5 = 21\text{kJ/s}$$

$$\text{Mass of air/sec} = \frac{\text{Refrigeration capacity}}{C.O.P}$$

$$= \frac{21}{95.42} = 0.22\text{kg/s}$$

$$\text{Power required} = \text{workdone/kg of air} \times \text{Mass of air/sec} = 57 \times 0.22 = 12.54\text{kW}$$

$$\text{Mass of air/min} = 0.22 \times 60 = 13.2\text{kg/min}$$



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$$V_1 = \frac{mRT_1}{P_1} = \frac{13.2 \times 0.287 \times 291}{1 \times 10^4} = 11 \text{ m}^3 / \text{min}$$

Piston displacement of compressor $V = 11 \text{ m}^3 / \text{min}$

$$V_4 = \frac{mRT_4}{P_4} = \frac{13.2 \times 0.287 \times 196}{1 \times 10^4} = 7.42 \text{ m}^3 / \text{min}$$

Piston displacement of expander $V = 7.42 \text{ m}^3 / \text{min}$

But
$$V_1 = 2 \frac{\pi}{4} d_1^2 L N$$
$$11 = 2 \frac{\pi}{4} d_1^2 \times 0.2 \times 240$$

$d_1 = \text{diameter of compressor cylinder} = 0.38 \text{ m} = 38 \text{ cm}$

$$V_4 = 2 \frac{\pi}{4} d_2^2 L N$$

$$7.42 = 2 \frac{\pi}{4} d_1^2 \times 0.2 \times 240$$

$d_1 = \text{diameter of expander cylinder} = 0.313 \text{ m} = 31.3 \text{ cm}$



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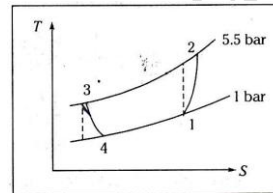
Problem3 An air refrigerator system operating on Bell Coleman cycle, takes in air from cold room at 268 K and compresses it from 1 bar to 5.5 bar the index of compression being 1.25. the compressed air is cooled to 300 K. the ambient temperature is 20°C. Air expands in expander where the index of expansion is 1.35.

Calculate:

- i) **C.O.P of the system**
- ii) **Quantity of air circulated per minute for production of 1500 kg of ice per day at 0°C from water at 20°C.**
- iii) **Capacity of the plant.**

Solution

$$T_2 = \frac{P_2}{P_1}^{\frac{\gamma-1}{\gamma}} = 268(5.5)^{\frac{1.25-1}{1.25}} = 376.8\text{K}$$



$$T_4 = \frac{P_4}{P_3}^{\frac{\gamma-1}{\gamma}} = 300 \frac{1}{5.5}^{\frac{1.35-1}{1.35}} = 192.83\text{K}$$

$$W_C = \frac{n}{n-1} \frac{\gamma-1}{\gamma} C_p (T_2 - T_1) = \frac{1.25}{1.25-1} \frac{1.4-1}{1.4} 1.005(376.8 - 268) = 156.2\text{kJ/kg}$$

$$W_E = \frac{n}{n-1} \frac{\gamma-1}{\gamma} C_p (T_3 - T_4) = \frac{1.35}{1.35-1} \frac{1.4-1}{1.4} 1.005(300 - 192.83) = 118.69\text{kJ/kg}$$

$$\text{Network} = W_C - W_E = 156.2 - 118.69 = 37.5\text{kJ/kg}$$

$$R.E = C_p (T_1 - T_4) = 1.005(268 - 192.83) = 75.54\text{kJ/s}$$

$$C.O.P = \frac{R.E}{\text{Network}} = \frac{75.54}{37.5} = 2$$



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$$\begin{aligned} \text{Heat extracted/kg of ice} &= C_{pw} (20 - 0) + L \\ &= 4.187(20) + 335 = 418.74 \text{ kJ/kg} \\ &\quad \underline{1500} \end{aligned}$$

$$\text{Mass of ice produced/sec} = \frac{24 \times 3600}{1500} = 0.0173 \text{ kg / s}$$

$$\begin{aligned} \text{Actual heat extracted/sec} &= 418.74 \times 0.0173 \\ \text{or Refrigeration capacity} &= 7.26 \text{ kJ/s} = \frac{1.20}{3.5} = 2.02 \text{ tons} \end{aligned}$$

$$\begin{aligned} \text{Mass flow rate} &= \frac{\text{Refrigeration Capacity}}{\text{Refrigeration effect}} = \frac{7.26}{75.54} \\ &= 0.096 \text{ kg / s} \end{aligned}$$

Problem 4

An air refrigeration system is to be designed according to the following specifications

Pressure of air at compressor inlet = 101 kPa

Pressure of work at compressor outlet = 404 kPa

Pressure loss in the inter cooler = 12 kPa

Pressure loss in the cold chamber = 3 kPa

Temperature of air at compressor inlet = 7°C

Temperature of air at turbine inlet = 27°C

Isentropic efficiency of compressor = 85%

Isentropic efficiency of turbine = 85%

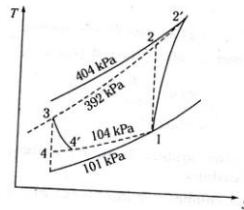
Determine

- i) C.O.P of cycle
- ii) Power required to produce 1 ton of refrigeration
- iii) Mass flow rate of air required for 1 ton of refrigeration

Solution : -

$$T_1 = -7^\circ\text{C} \quad P_1 = 101 \text{ kPa}$$

$$T_3 = 27^\circ\text{C} \quad \eta_T = 0.85; \eta_C = 0.85$$



Process 1-2 is isentropic, Hence T_2

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\begin{aligned} &= 266 \frac{404}{101}^{\frac{1.4-1}{1.4}} = 395.4 \text{ K} \end{aligned}$$



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$$\eta_c = \frac{T_2 - T_1}{T_2' - T_1} \text{ or } T_2' = \frac{T_2 - T_1}{\eta_c} + T_1$$

$$T_2' = \frac{395.4 - 266}{0.88} + 266$$

$$T_2' = 418.2 \text{ K}$$

$$P_4 - P_1 = 0.03 P_1 \quad \therefore P_4 = 1.03 P_1 = 1.03 \times 101 = 104 \text{ kPa}$$

$$P_2 - P_3 = 0.03 P_2 \quad \therefore P_3 = 0.97 P_2 = 0.97 \times 404 = 392 \text{ kPa}$$

$$T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300 \left(\frac{104}{392} \right)^{\frac{1.4-1}{1.4}} = 202.3 \text{ K}$$

$$\eta_E = \frac{T_3 - T_4}{T_3 - T_4'} \quad \therefore T_4' = T_3 - \eta_T (T_3 - T_4)$$

$$T_4' = 300 - 0.85 \times [300 - 202.3] = 216.53 \text{ K Re}$$

$$\text{refrigeration effect/kg of air} = C_p (T_1 - T_4')$$

$$= 1.005 \times [266 - 216.53] = 50.47 \text{ kJ/kg}$$

$$\text{Compressor work/kg of air} = C_p (T_2' - T_1)$$

$$= 1.005 \times [418.2 - 266] = 152.96 \text{ kJ/kg}$$

$$\text{Turbine work/kg of air } W_T = C_p (T_3 - T_4')$$

$$= 1.005 \times [300 - 216.53] = 84.9 \text{ kJ/kg}$$

$$\text{Net work Input/kg of air } W_{\text{net}} = W_C - W_T$$

$$W_{\text{net}} = 152.96 - 80.9 = 72.06 \text{ kJ/kg}$$

$$C.O.P = \frac{RE}{Work} = \frac{46.73}{72.06} = 0.73$$

Power required per tons of refrigeration

$$= \frac{\text{Refrigeration capacity}}{C.O.P}$$

$$\text{Refrigeration capacity} = 1 \text{ ton} = 3.5 \text{ kJ/s}$$

$$\text{Mass of air} = \frac{\text{Refrigeration capacity}}{\text{capacity RE}}$$



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$$= \frac{3.5}{50.47} = 0.075 \text{ kg / s}$$

$$\text{Power} = W_{net} \times \text{mass of air / sec} = 72.06 \times 0.075 = 5.42 \text{ kW}$$

Applied Thermodynamics Notes, HIT, NDS



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Problem 1

Moist air at 30°C, 1.01325 bar has a relative humidity of 80%. Determine without using the psychrometry chart

- 1) Partial pressures of water vapour and air
- 2) Specific humidity
- 3) Specific Volume and
- 4) Dew point temperature (V.T.U. July 2004)

Solution : At 30°C from table $p_{vs} = 4.2461 \text{ kPa}$

$$\phi = \frac{p_v}{p_{vs}}$$

$$p_v = 0.8 \times 4.2461 = 3.397 \text{ kPa}$$

$$\omega = \frac{0.622 p_v}{p - p_v} = 0.622 \times \frac{3.397}{101.325 - 3.397}$$
$$= 0.213 \text{ kg/kg of dry air.}$$

Corresponding to $P_v = 3.397 \text{ kPa}$ from tables, we get dew point temperature = 28.9°C

Problem 2:

Atmospheric air at 101.325 kPa has 30°C DBT and 15°C DPT. Without using the psychrometric chart, using the property values from the table, Calculate

1. Partial pressure of air and water vapour
2. Specific humidity
3. Relative humidity
4. Vapour density and
5. Enthalpy of moist air

Solution :

$$p = 101.325 \text{ kPa} =$$
$$1.01325 \text{ bar DBT} = 30^\circ\text{C},$$
$$\text{DPT} = 15^\circ\text{C}$$

From table

$$\text{Corresponding to DBT} = 30^\circ\text{C}, \text{ we have } p_{vs} = 0.042461 \text{ bar}$$

$$\text{Corresponding to DPT} = 15^\circ\text{C}, \text{ we have } p_v = 0.017051 \text{ bar}$$

$$\text{Partial pressure of air} = p - p_v = 1.01325 - 0.017051$$
$$= 0.984274 \text{ bar}$$



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$$\text{Specific humidity} = \frac{0.622 p_v}{p_a} = \frac{0.622 \times 0.017051}{0.984274}$$

$$= 0.01077 \text{ kg of dry air}$$

$$\text{Relative humidity} = \frac{p_v}{p_{vs}} = \frac{0.017051}{0.042461} = 0.4015$$

$$= 40.15\%$$

$$\text{Enthalpy} = 1.005 t_{db} + \omega(2500 + 1088 t_{db})$$

$$= 1.005 \times 30 + 0.010775(2500 + 1.88 \times 30)$$

$$= 57.69 \text{ kJ/kg of dry air}$$

$$\text{Specific volume of dry air, } v_a = \frac{RT}{P}$$

$$= \frac{0.2872 \times 303}{0.98425 \times 100} = 0.874 \text{ m}^3 / \text{kg}$$

$$\text{Vapour density } \rho_w = \frac{\omega}{v_a} = \frac{0.010775}{0.847} = 0.12 \text{ kg / m}^3$$

Problem 3:

Air at 30°C DBT and 25°C WBT is heated to 40°C. if the air is 300 m³/min, find the amount of heat added/min and RH and WBT of air. Take air pressure to be 1 bar

Solution:

At 25°C WBT from tables $p_{wsbt} = 0.03166 \text{ bar}$

$$\therefore p_v = (P_{VS})_{wbt} - \frac{(p - p_{vsbt})(t_{db} - t_{wb})}{1547 - 1.44 t_{wb}}$$

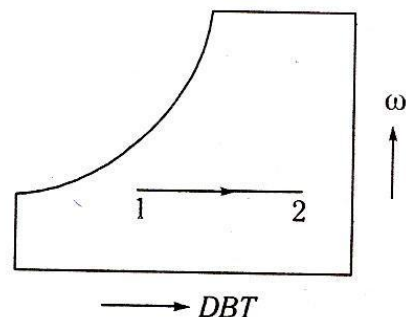
$$= 0.03166 - \frac{(1 - 0.03166)(30 - 25)}{1547 - 1.44 \times 25}$$

$$= 0.0284 \text{ bar}$$

$$\omega_1 = 0.622 \frac{p_v}{p - p_v}$$

$$= \frac{0.622 \times 0.0284}{1 - 0.0284}$$

$$= 0.0179 \text{ kJ / kg of dry air}$$





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At 40°C DBT

$$P_{VS} = 0.07375 \text{ bar}$$

During sensible heating ω and p_v remain

constant $p_v = 0.0284 \text{ bar}$

$$RH = \phi = \frac{p_v}{p_{vs}} = \frac{0.0284}{0.07375}$$
$$= 0.385 = 38.5\%$$

$$H_2 = 1.005 \times 40 + 0.0179(2500 + 1.88 \times 40)$$
$$= 86.29 \text{ kJ/kg of dry air}$$

$$\text{Weight of } 300 \text{ m}^3 / \text{min of air} = \frac{(p - p_v)V}{RT}$$
$$= \frac{(1 - 0.0284) \times 300 \times 10^3}{0.287 \times 303} = 335.18 \text{ kg / min}$$

$$\therefore \text{Heat added/min} = 335.18(86.29 - 76) =$$

$$3449 \text{ kJ/min From chart WBT} = 27.2^\circ\text{C}$$

Problem 4:

One stream of air at 5.5 m³/min at 15°C and 60% RH flows into another stream of air at 35 m³/min at 25°C and 70% RH, calculate for the mixture

1) Dry bulb temperature, 2) Wet bulb temperature 3) Specific Humidity and 4) Enthalpy

Solution:

For air at 15°C and 60% RH, $V = 5.5 \text{ m}^3/\text{min}$

$$\therefore p_{vs} = 0.017051 \text{ bar}$$

$$RH = \phi = \frac{p_v}{p_{vs}}$$

$$\therefore p_v = 0.6 \times 0.017051 = 0.01023 \text{ bar}$$

$$\text{Mass of air} = \frac{(p - p_v)V}{RT} = \frac{(1.01325 - 0.01023) \times 10^3 \times 5.5}{0.287 \times 288}$$

$$m_1 = 6.672 \text{ kg / min}$$

$$\omega = \frac{0.622 p_v}{(p - p_v)} = \frac{0.622 \times 0.01023}{(1.01325 - 0.01023)}$$
$$= 0.006343 \text{ kg / kg of dry air}$$



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$$\begin{aligned} H_1 &= 1.005t_{db} + \omega_1(2500 + 1.88t_{db}) \\ &= 1.008 \times 18 + 0.006343(2500 + 1.88 \times 15) \\ &= 34.12 \text{ kJ/kg of dry air} \end{aligned}$$

For air at 25°C and 70% RH, $V = 35 \text{ m}^3 / \text{min}$ $P_{vs} = 0.03169 \text{ bar}$

$$\phi = RH = \frac{P_v P_{vs}}{P}$$

$$\begin{aligned} p_v &= 0.03169 \times 0.7 = 0.02218 \text{ bar} \quad H_1 = 1.005t_{db} + \omega_1(2500 + 1.88t_{db}) \\ &= 1.008 \times 18 + 0.006343(2500 + 1.88 \times 15) \\ &= 34.12 \text{ kJ/kg of dry air} \end{aligned}$$

For air at 25°C and 70% RH, $V = 35 \text{ m}^3 / \text{min}$ $P_{vs} = 0.03169 \text{ bar}$

$$\phi = RH = \frac{p_v}{p_{vs}}$$

$$p_v = 0.03169 \times 0.7 = 0.02218 \text{ bar}$$

$$\text{Mass of air} = \frac{(1.01325 - 0.02218 \times 10^2 \times 35)}{0.287 \times 298}$$

$$m_2 = 40.55 \text{ kg} \cdot \text{min}$$

$$\omega_2 = \frac{0.622 \times 0.02218}{(1.01325 - 0.02218)} = 0.01392 \text{ kg / kg of dry air}$$

$$H_2 = (1.005 \times 25) + 0.01392(2500 + 1.88 \times 25) \quad H_2 = 60.59 \text{ kJ / kg of dry air}$$

Mass of dry air/Unit mass of moist air

$$m_{a1} = \frac{m_1}{1 + \omega_1} = \frac{6.672}{1 + 0.006343} = 6.6299$$

$$\text{Since } m_{a2} = \frac{m_2}{1 + \omega_2} = \frac{40.55}{1 + 0.01392} = 39.993$$

Then enthalpy of the mixed air,

$$\begin{aligned} H_{\text{mix}} &= \frac{m_{a1} H_1 + m_{a2} H_2}{m_1 + m_2} \\ &= \frac{6.6299(34.12) + 39.993(60.56)}{6.672 + 40.55} \\ &= 55.96 \text{ kJ/kg of dry air} \end{aligned}$$

Specific Humidity of the mixed air,

$$\begin{aligned} \omega_{\text{mix}} &= \frac{m_{a1}(\omega_1) + m_{a2}(\omega_2)}{m_1 + m_2} \\ &= \frac{(6.6299 \times 0.006343) + (39.993 \times 0.01932)}{6.672 + 40.55} \end{aligned}$$



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$$6.672 + 40.55 \\ = 0.01268 \text{ kg/kg of dry air}$$

$$\text{But } H_{\text{mix}} = 1.005t_{db} + \omega_{\text{mix}}(2500 + 1.88t_{db})$$

$$55.96 = 1.005xt_{db} + 0.01234(2500 + 1.88t_{db})$$

$$t_{db} = 24.42^\circ\text{C}$$

DBT of the mixture = 24.42°C

From chart WBT = 19°C

RH = 67%

Problem 5:

An air conditioning system is designed under the following conditions

Outdoor conditions: 30°C DBT , 75% RH

Required indoor conditions: 22°C DBT , 70% RH

RH Amount of Free air circulated $3.33 \text{ m}^3/\text{s}$

Coil dew point temperature $\text{DPT} = 14^\circ$

The required condition is achieved first by cooling and dehumidification and then by heating. Estimate

- 1) The capacity of the cooling coil in tons of refrigeration
- 2) Capacity of the heating coil in kW
- 3) The amount of water vapour removed in kg/hr

Solution:

Locate point 'a' 30°C DBT , 75%RH out door

condition Locate point 'd' 22°C DBT , 70%RH

required condition Locate point 'b' 14°C DPT , coil

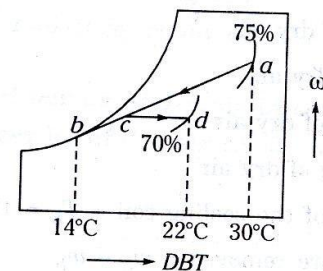
surface temperature Join ab

at d, draw a horizontal line to cut

the line ab at point c.

ac \rightarrow cooling and

dehumidification cd \rightarrow heating





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From chart

$$H_a = 83 \text{ kJ / kg of air}$$

$$H_b = 40 \text{ kJ / kg of air}$$

$$H_d = 53 \text{ kJ / kg of air}$$

$$H_c = 48 \text{ kJ / kg of air}$$

$$W_a = 0.0202 \text{ kg / kg of dry air}$$

$$W_c = W_d = 0.0118 \text{ kg / kg of dry air}$$

$$V = 0.88 \text{ m}^3 / \text{kg}$$

$$\text{Mass of air} = \frac{V}{V_a} = \frac{3.33}{0.88} = 3.78 \text{ kg / s}$$

$$\text{Capacity of cooling coil} = \frac{m_a (H_a - H_c)}{3.5}$$

$$= \frac{3.78(83 - 48)}{3.5} = 37.84 \text{ tons of refrigeration}$$

$$\text{Capacity of heating coil} = m_a (H_d - H_c) = 3.78(53 - 48) = 18.92 \text{ kW}$$

$$\begin{aligned} \text{Amount of water vapour removed} &= m_a (\omega_a - \omega_d) 3600 \\ &= 3.78(0.0202 - 0.0118) 3600 \\ &= 114.3 \text{ kg/hr} \end{aligned}$$

Problem 6:

A summer air conditioning system for hot and humid weather (DBT=32°C and 70% RH)

Consists in passing the atmosphere air over a cooling coil where the air is cooled and dehumidified. The air leaving the cooling coil is saturated at the coil temperature. It is then sensibly heated to the required comfort condition of 24°C and 50%RH by passing it over an electric heater then delivered to the room.

Sketch the flow diagram of the arrangement and represent the process undergone by the air on a skeleton psychometric chart and determine

- 1) The temperature of the cooling coil
- 2) The amount of moisture removed per kg of dry air in the cooling coil.
- 3) The heat removed per kg of dry air in the cooling coil and
- 4) The heat added per kg of dry air in the heating coil

From chart



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$H_a = 86 \text{ kJ / kg}$ of
 air $H_b = 38 \text{ kJ / kg}$
 of air $H_c = 48.5 \text{ kJ /}$
 kg of air
 $\omega_a = 0.021 \text{ kg / kg}$ of dry
 air $\omega_b = 0.0092 \text{ kg / kg}$ of
 dry air

The temperature of the cooling

$$\text{coil} = T_b = 13^\circ\text{C}$$

Amount of moisture removed = $\omega_a - \omega_b$
 $= 0.021 - 0.0092 = 0.0108 \text{ kg/kg}$ of dry

air Heat removed = $H_a - H_b = 86 - 38$

$= 48 \text{ kJ/kg}$ of dry air Heat added

$= H_c - H_b = 48.5 - 38 = 10.5$

kJ/kg of dry air

Locate point 'a' 32°C , 70%RH out door condition

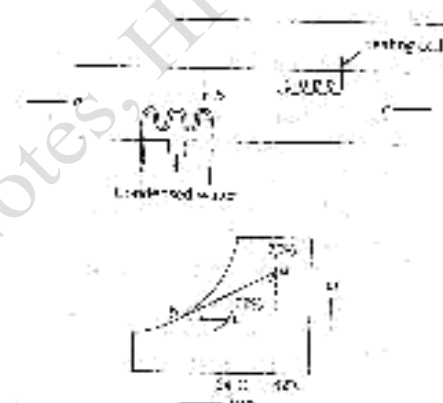
Locate point 'c' 24°C DBT, 50%RH required condition

At c draw a horizontal line to cut the saturation
line at point '

b' Join ab

ab \rightarrow cooling and

dehumidification bc \rightarrow heating



Problem 7

It is required to design an air conditioning plant for an office room with the following conditions.

Outdoor conditions: 14°C DBT, 10°C WB

Required conditions: 20°C DBT, 60% RH

Amount of air circulated

$0.3 \text{ m}^3/\text{min}/\text{person}$ Starting capacity of the
office = 60

The required condition is achieved first by heating and then by adiabatic humidifying. Determine the following.

Heating capacity of the coil in kW and the surface temperature required, if the by



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pass factor of the coil is 0.4
Capacity of the humidifier.

Locate point 'a' 14°C , and 10°CWBT (out door condition) Locate point 'c' 20°C DBT, 60%RH required condition At a draw a horizontal line
At 'c' draw a constant enthalpy line to cut the horizontal line at point ' b'

Join ab

ab → heating

bc → *adiabatic* humidification

From chart

$$H_a = 30 \text{ kJ / kg of air}$$

$$H_b = H_c = 43 \text{ kJ / kg of air}$$

$$\omega_a = \omega_b = 0.006 \text{ kg / kg of dry air}$$

$$\omega_c = 0.00875 \text{ kg/kg of dry air}$$

$$\text{Specific volume } V_{sa} = 0.8175 \text{ m}^3 / \text{kg}$$

$$\text{Volume of air supplied} = V = \frac{0.5 \times 0.00}{60} = 0.3 \text{ m}^3 / \text{sec}$$

$$\text{Weight of air supplied } m_a = \frac{V}{V_a} = \frac{0.3}{0.8175} = 0.3669 \text{ kg/sec}$$

$$\text{Capacity of the heating coil} = m_a (H_b - H_a) = 0.3669(43 - 30) =$$

$$4.77 \text{ kW From chart } T_b = 26.5^\circ \text{C}$$

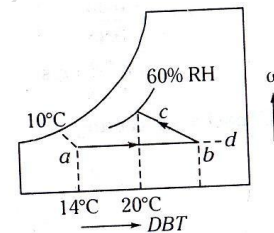
Let coil surface temperature be T_d

$$\text{By passing factor} = \frac{T_d - T_b}{T_d - T_a} = 0.4 T_d - 5.6 = T_d - 26 \text{ kJ}$$

$$0.4 = \frac{T_d - 26.5}{T_d - 1.4}$$

$$T_d = 34.8^\circ \text{C}$$

$$\text{Capacity of the humidifier} = m_a (\omega_c - \omega_b) \times 3600 = 0.3669(0.00875 - 0.006) \times 3600$$





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$$= 3.63 \text{ kg/hour}$$

Problem 8

An air conditioned system is to be designed for a hall of 200 seating capacity when the following conditions are given:

Atmospheric condition = 30°C DBT and 50% RH

Indoor condition = 22°C DBT and 60% RH

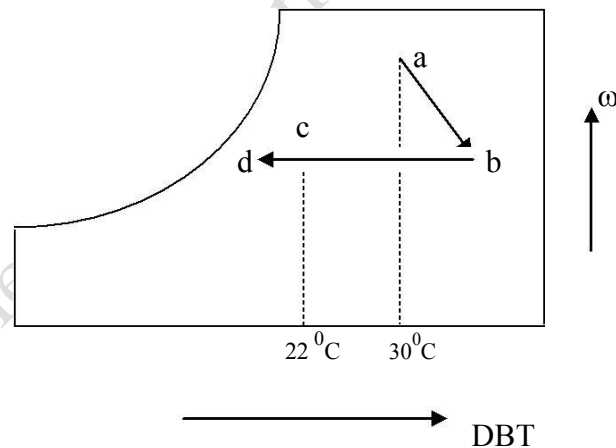
Volume of air required = $0.4 \text{ m}^3/\text{min}/\text{person}$

The required condition is achieved first by chemical dehumidification and after that by sensible cooling.

Find the following .

- DBT of the air leaving the dehumidifier.
- The quantity of water vapour removed in the dehumidifier per hour.
- The capacity of cooling coil in tons of refrigeration.
- Surface temperature of the coil if the by pass factor of the coil is 0.25.

Solution:



Locate point 'a', 30°C DBT, 50% RH, the atmospheric condition.

Locate point 'c', 22°C DBT, 60% RH, the required indoor condition.

“Since chemical dehumidification process follows constant enthalpy line”
at a draw a line parallel to constant enthalpy line.

At 'c' draw a constant ω line to cut the previous line at point b.

a) DBT of air leaving the dehumidifier $T_b = 40.5^{\circ}\text{C}$

From chart

$$H_b = H_a = 65 \text{ kJ/kg}, \quad \omega_a = 0.013 \text{ kg/kg of dry air}$$

$$H_c = 45 \text{ kJ/kg}, \quad \omega_b = 0.009 \text{ kg/kg of dry air}$$



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$$V_{sa} = 0.875 \text{ m}^3/\text{min}$$

$$\text{Volume of air} = 200 \times 0.4 = 80 \text{ m}^3/\text{min}$$

$$W_a = \text{Weight of air} = V/V_{sa} = 80/0.875 = 91.42 \text{ kg/min}$$

$$\begin{aligned} \text{b) Quantity of water vapour removed/hour} &= W_a(\omega_a - \omega_b)60 \\ &= 91.42(0.13 - 0.009)60 = 21.94 \text{ kg/hr} \end{aligned}$$

$$\begin{aligned} \text{c) Capacity of cooling coil} &= W_a(H_a - H_b) / (60 \times 3.5) = 91.42(65 - 45) / (60 \times 3.5) \\ &= 8.7 \text{ tons} \end{aligned}$$

$$\text{d) By pass factor} = (T_c - T_d) / (T_b - T_d) = 0.25$$

$$T_d = \text{Temperature of cooling coil} = 15.83^\circ\text{C}$$

Problem 9

An air conditioned system is to be designed for a cinema hall of 1000 seating capacity when the following conditions are given:

Outdoor condition = 11°C DBT and 70% RH Required

indoor condition = 20°C DBT and 60% RH

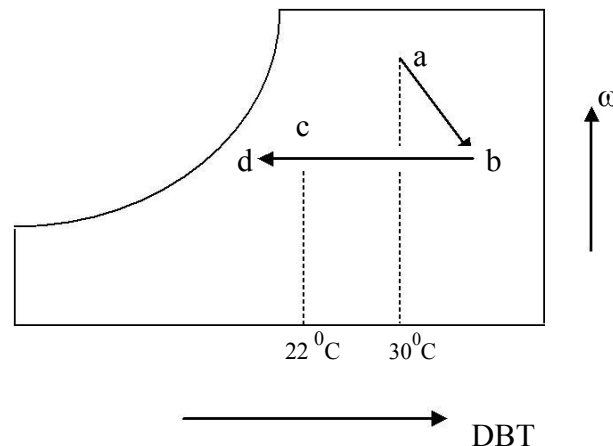
Amount of air required = $0.3 \text{ m}^3/\text{min}/\text{person}$

The required condition is achieved first by heating, then by humidifying and finally by heating. The condition of air coming out of the humidifier is 75% RH.

Find the following .

- a) Heating capacity of the first heater in kW and condition of the air coming out of the first heater in kW and condition of the air

Solution:





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Locate point 'a', 30⁰C DBT, 50% RH, the atmospheric condition.

Locate point 'c', 22⁰C DBT, 60% RH, the required indoor condition.

“Since chemical dehumidification process follows constant enthalpy line”
at a draw a line parallel to constant enthalpy line.

At 'c' draw a constant ω line to cut the previous line at point

b. e) DBT of air leaving the dehumidifier $T_b = 40.5^0\text{C}$

From chart

$$H_b = H_a = 65\text{kJ/kg}, \omega_a = 0.013\text{ kg/kg of dry air}$$

$$H_c = 45\text{ kJ/kg}, \omega_b = 0.009\text{ kg/kg of dry air}$$

$$V_{sa} = 0.875\text{ m}^3/\text{min}$$

$$\text{Volume of air} = 200 \times 0.4 = 80\text{ m}^3/\text{min}$$

$$W_a = \text{Weight of air} = V/V_{sa} = 80/0.875 = 91.42\text{ kg/min}$$

$$\begin{aligned} \text{f) Quantity of water vapour removed/hour} &= W_a(\omega_a - \omega_b)60 \\ &= 91.42(0.013 - 0.009)60 = 21.94\text{ kg/hr} \end{aligned}$$

$$\begin{aligned} \text{g) Capacity of cooling coil} &= W_a(H_a - H_b) / (60 \times 3.5) = 91.42(65 - 45) / (60 \times 3.5) \\ &= 8.7\text{ tons} \end{aligned}$$

$$\text{h) By pass factor} = (T_c - T_d) / (T_b - T_d) = 0.25$$

$$T_d = \text{Temperature of cooling coil} = 15.83^0\text{C}$$



UNIT-8 PSYCHOMETRY

Problem 1

Moist air at 30°C, 1.01325 bar has a relative humidity of 80%. Determine without using the psychrometry chart

- 1) Partial pressures of water vapour and air
- 2) Specific humidity
- 3) Specific Volume and
- 4) Dew point temperature (V.T.U. July 2004)

Solution : At 30°C from table $p_{vs} = 4.2461 \text{ kPa}$

$$\phi = \frac{p_v}{p_{vs}}$$

$$p_v = 0.8 \times 4.2461 = 3.397 \text{ kPa}$$

$$\omega = \frac{0.622 p_v}{p - p_v} = 0.622 \times \frac{3.397}{101.325 - 3.397}$$

$$= 0.213 \text{ kg/kg of dry air.}$$

Corresponding to $p_v = 3.397 \text{ kPa}$ from tables, we get dew point temperature = 28.9°C

Problem 2:

Atmospheric air at 101.325 kPa has 30°C DBT and 15°C DPT. Without using the psychrometric chart, using the property values from the table, Calculate

1. Partial pressure of air and water vapour
2. Specific humidity
3. Relative humidity
4. Vapour density and
5. Enthalpy of moist air

Solution :

$$p = 101.325 \text{ kPa} = 1.01325 \text{ bar} \quad \text{DBT} = 30^\circ\text{C},$$

$$\text{DPT} = 15^\circ\text{C}$$

From table

$$\text{Corresponding to DBT} = 30^\circ\text{C}, \text{ we have } p_{vs} = 0.042461 \text{ bar}$$

$$\text{Corresponding to DPT} = 15^\circ\text{C}, \text{ we have } p_v = 0.017051 \text{ bar}$$

$$\text{pressure of air} = p - p_v = 1.01325 - 0.017051$$

$$= 0.984274 \text{ bar}$$



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$$\begin{aligned} \text{Specific humidity} &= \frac{p}{0.622} \frac{v}{p_a} = \frac{0.622 \times 0.0170}{0.984274} \\ &= 0.01077 \text{ kJ/kg of dry air} \end{aligned}$$

$$\begin{aligned} \text{Relative humidity} &= \frac{p}{p_{vs}} = \frac{0.017051}{0.042461} \\ &= 40.15\% \end{aligned}$$

$$\begin{aligned} \text{Enthalpy} &= 1.005t_{db} + \omega(2500 + 1088t_{db}) \\ &= 1.005 \times 30 + 0.010775(2500 + 1.88 \times 30) \\ &= 57.69 \text{ kJ/kg of dry air} \end{aligned}$$

$$\begin{aligned} \text{Specific volume of dry air, } v_a &= \frac{RT}{P} \\ &= \frac{0.2872 \times 303}{0.98425 \times 100} = 0.874 \text{ m}^3 / \text{kg} \\ \text{Vapour density } \rho_w &= \frac{\omega}{v_a} = \frac{0.010775}{0.847} = 0.12 \text{ kg / m}^3 \end{aligned}$$

Problem 3:

Air at 30°C DBT and 25°C WBT is heated to 40°C. if the air is 300 m³/min, find the amount of heat added/min and RH and WBT of air. Take air pressure to be 1 bar

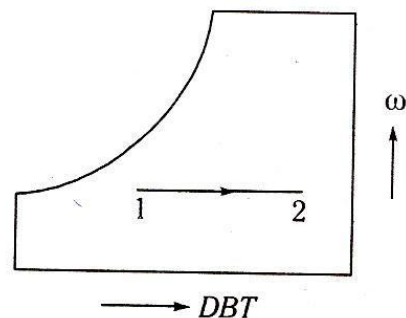
Solution:

At 25°C WBT from tables $p_{wswb} = 0.03166 \text{ bar}$

$$\therefore p_v = (p_{vs})_{wb} - \frac{(p - p_{vswb})(t_{db} - t_{wb})}{1547 - 1.44t_{wb}}$$

$$\begin{aligned} &= 0.03166 - \frac{(1 - 0.03166)(30 - 25)}{1547 - 1.44 \times 25} \\ &= 0.0284 \text{ bar} \end{aligned}$$

$$\begin{aligned} \omega_1 &= 0.622 \frac{p}{p - p_v} \\ &= 0.622 \frac{0.0284}{1 - 0.0284} \end{aligned}$$





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At 40°C DBT

$$P_{VS} = 0.07375 \text{ bar}$$

During sensible heating ω and p_v remain

constant $p_v = 0.0284 \text{ bar}$

$$RH = \phi = \frac{p_v}{p_{vs}} = \frac{0.0284}{0.07375}$$

$$= 0.385 = 38.5\%$$

$$H_2 = 1.005 \times 40 + 0.0179(2500 + 1.88 \times 40)$$

$$= 86.29 \text{ kJ/kg of dry air}$$

$$\text{Weight of } 300 \text{ m}^3 / \text{min of air} = \frac{(p - p_v)V}{RT}$$

$$= \frac{(1 - 0.0284) \times 300 \times 10^2}{0.287 \times 303} = 335.18 \text{ kg / min}$$

$$\therefore \text{Heat added/min} = 335.18(86.29 - 76) =$$

$$3449 \text{ kJ/min From chart WBT} = 27.2^\circ\text{C}$$

Problem 4:

One stream of air at 5.5 m³/min at 15°C and 60% RH flows into another stream of air at 35 m³/min at 25°C and 70% RH, calculate for the mixture

1) Dry bulb temperature, 2) Wet bulb temperature 3) Specific Humidity and 4) Enthalpy

Solution:

For air at 15°C and 60% RH, $V = 5.5 \text{ m}^3/\text{min}$

$$\therefore p_{vs} = 0.017051 \text{ bar}$$

$$RH = \phi = \frac{p_v}{p_{vs}}$$

$$\therefore p_v = 0.6 \times 0.017051 = 0.01023 \text{ bar}$$

$$\text{Mass of air} = \frac{(p - p_v)V}{RT} = \frac{(1.01325 - 0.01023) \times 10^2 \times 5.5}{0.287 \times 288}$$

$$m_1 = 6.672 \text{ kg / min}$$

$$\omega = \frac{0.622 p_v}{(p - p_v)} = \frac{0.622 \times 0.01023}{(1.01325 - 0.01023)}$$

$$= 0.006343 \text{ kg / kg of dry air}$$



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$$\begin{aligned} H_1 &= 1.005t_{db} + \omega_1(2500 + 1.88t_{db}) \\ &= 1.008 \times 18 + 0.006343(2500 + 1.88 \times 15) \\ &= 34.12 \text{ J/kg of dry air} \end{aligned}$$

For air at 25°C and 70% RH, $V = 35 \text{ m}^3 / \text{min}$ $P_{vs} = 0.03169 \text{ bar}$

$$\phi = RH = \frac{P_v P_{vs}}$$

$$P_v = 0.03169 \times 0.7 = 0.02218 \text{ bar} \quad H_1 = 1.005t_{db} + \omega_1(2500 + 1.88t_{db})$$

$$\begin{aligned} &= 1.008 \times 18 + 0.006343(2500 + 1.88 \times 15) \\ &= 34.12 \text{ J/kg of dry air} \end{aligned}$$

For air at 25°C and 70% RH, $V = 35 \text{ m}^3 / \text{min}$ $P_{vs} = 0.03169 \text{ bar}$

$$\phi = RH = \frac{P_v}{P_{vs}}$$

$$P_v = 0.03169 \times 0.7 = 0.02218 \text{ bar}$$

$$\text{Mass of air} = \frac{(1.01325 - 0.02218 \times 10^2) \times 35}{0.287 \times 298}$$

$$m_2 = 40.55 \text{ kg} \cdot \text{min}$$

$$\omega_2 = \frac{0.622 \times 0.02218}{(1.01325 - 0.02218)} = 0.01392 \text{ kg / kg of dry air}$$

$$H_2 = (1.005 \times 25) + 0.01392(2500 + 1.88 \times 25) \quad H_2 = 60.59 \text{ kJ / kg of dry air}$$

Mass of dry air/Unit mass of moist air

$$m_{a1} = \frac{m_1}{1 + \omega_1} = \frac{6.672}{1 + 0.006343} = 6.6299$$

$$\text{Since } m_{a2} = \frac{m_2}{1 + \omega_2} = \frac{40.55}{1 + 0.01392} = 39.993$$

Then enthalpy of the mixed air,

$$H_{\text{mix}} = \frac{m_{a1} H_1 + m_{a2} H_2}{m_1 + m_2}$$

$$\begin{aligned} &= \frac{6.6299(34.12) + 39.993(60.56)}{6.672 + 40.55} \\ &= 55.96 \text{ kJ/kg of dry air} \end{aligned}$$

Specific Humidity of the mixed air,

$$\omega_{\text{mix}} = \frac{m_{a1}(\omega_1) + m_{a2}(\omega_2)}{m_1 + m_2}$$

$$= \frac{(6.6299 \times 0.006343) + (39.993 \times 0.01392)}{6.672 + 40.55}$$



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$$6.672 + 40.55 \\ = 0.01268 \text{ kg/kg of dry air}$$

$$\text{But } H_{\text{mix}} = 1.005 t_{db} + \omega_{\text{mix}} (2500 + 1.88 t_{db})$$

$$55.96 = 1.005 x t_{db} + 0.01234 (2500 + 1.88 t_{db})$$

$$t_{db} = 24.42^\circ\text{C}$$

DBT of the mixture = 24.42°C

From chart WBT = 19°C

RH = 67%

Problem 5:

An air conditioning system is designed under the following conditions

Outdoor conditions: 30°C DBT , 75% RH

Required indoor conditions: 22°C DBT , 70% RH

RH Amount of Free air circulated $3.33 \text{ m}^3/\text{s}$

Coil dew point temperature $\text{DPT} = 14^\circ$

The required condition is achieved first by cooling and dehumidification and then by heating. Estimate

- 1) The capacity of the cooling coil in tons of refrigeration
- 2) Capacity of the heating coil in kW
- 3) The amount of water vapour removed in kg/hr

Solution:

Locate point 'a' 30°C DBT , 75% RH out door

condition Locate point 'd' 22°C DBT , 70% RH

required condition Locate point 'b' 14°C DPT , coil

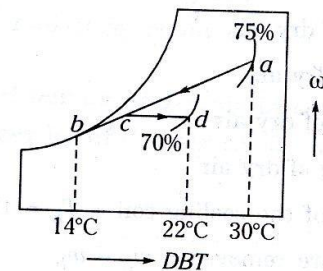
surface temperature Join ab

at d, draw a horizontal line to cut

the line ab at point c.

ac \rightarrow cooling and

dehumidification cd \rightarrow heating





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From chart

$$H_a = 83 \text{ kJ / kg of air}$$

$$H_b = 40 \text{ kJ / kg of air}$$

$$H_d = 53 \text{ kJ / kg of air}$$

$$H_c = 48 \text{ kJ / kg of air}$$

$$W_a = 0.0202 \text{ kg / kg of dry air}$$

$$W_c = W_d = 0.0118 \text{ kg / kg of dry air}$$

$$V = 0.88 \text{ m}^3 / \text{kg}$$

$$\text{Mass of air} = \frac{V}{V_{sa}} = \frac{3.33}{0.88} = 3.78 \text{ kg / s}$$

$$\text{Capacity of cooling coil} = \frac{m_a (H_a - H_c)}{3.5}$$

$$= \frac{3.78(83 - 48)}{3.5} = 37.84 \text{ tons of refrigeration}$$

$$\text{Capacity of heating coil} = m_a (H_d - H_c) = 3.78(53 - 48) = 18.92 \text{ kW}$$

$$\begin{aligned} \text{Amount of water vapour removed} &= m_a (\omega_a - \omega_d) 3600 \\ &= 3.78(0.0202 - 0.0118) 3600 \\ &= 114.3 \text{ kg/hr} \end{aligned}$$

Problem 6:

A summer air conditioning system for hot and humid weather (DBT=32°C and 70% RH)

Consists in passing the atmosphere air over a cooling coil where the air is cooled and dehumidified. The air leaving the cooling coil is saturated at the coil temperature. It is then sensibly heated to the required comfort condition of 24°C and 50%RH by passing it over an electric heater then delivered to the room.

Sketch the flow diagram of the arrangement and represent the process undergone by the air on a skeleton psychrometric chart and determine

- 1) The temperature of the cooling coil
- 2) The amount of moisture removed per kg of dry air in the cooling coil.
- 3) The heat removed per kg of dry air in the cooling coil and
- 4) The heat added per kg of dry air in the heating coil

From chart



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$$H_a = 86 \text{ kJ / kg of}$$

$$\text{air } H_b = 38 \text{ kJ / kg}$$

$$\text{of air } H_c = 48.5 \text{ kJ}$$

$$/ \text{ kg of air}$$

$$\omega_a = 0.021 \text{ kg / kg of dry}$$

$$\text{air } \omega_b = 0.0092 \text{ kg / kg of}$$

$$\text{dry air}$$

The temperature of the cooling

$$\text{coil} = T_b = 13^\circ \text{C}$$

Amount of moisture removed = $\omega_a -$

ω_b

$$= 0.021 - 0.0092 = 0.0108 \text{ kg/kg of dry}$$

air Heat removed = $H_a - H_b = 86 - 38$

= 48 kJ/kg of dry air Heat added

$$= H_c - H_b = 48.5 - 38 = 10.5$$

kJ/kg of dry air

Locate point 'a' 32°C , 70%RH out door condition

Locate point 'c' 24°C DBT, 50%RH required condition

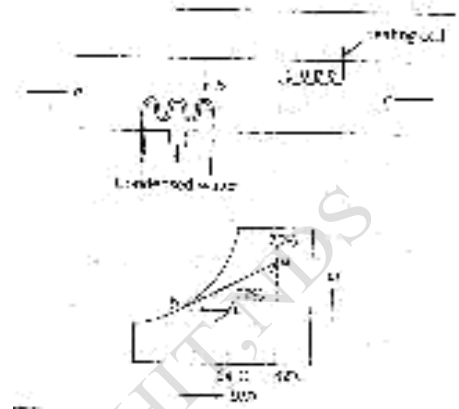
At c draw a horizontal line to cut the saturation

line at point '

b' Join ab

ab → cooling and

dehumidification bc → heating



Problem 7

It is required to design an air conditioning plant for an office room with the following conditions.

Outdoor conditions: 14°C DBT,

10°C WBWBT Required conditions:

20°C DBT, 60% RH Amount of air

circulated $0.3 \text{ m}^3/\text{min}/\text{person}$ Starting

capacity of the office = 60



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The required condition is achieved first by heating and then by adiabatic humidifying. Determine the following.

Heating capacity of the coil in kW and the surface temperature required, if the by pass factor of the coil is 0.4

Capacity of the humidifier.

Locate point 'a' 14°C , and 10°CWBT (out door condition) Locate point 'c' 20°C DBT, 60%RH required condition At a draw a horizontal line
At 'c' draw a constant enthalpy line to cut the horizontal line at point ' b'

Join ab

ab → heating

bc → *adiabatic* humidification

From chart

$$H_a = 30 \text{ kJ / kg of air}$$

$$H_b = H_c = 43 \text{ kJ / kg of air}$$

$$\omega_a = \omega_b = 0.006 \text{ kg / kg of dry air}$$

$$\omega_c = 0.00875 \text{ kg/kg of dry air}$$

$$\text{Specific volume } V_{sa} = 0.8175 \text{ m}^3 / \text{kg}$$

$$\text{Volume of air supplied} = V = \frac{0.3 \times 60}{60} = 0.3 \text{ m}^3 / \text{sec}$$

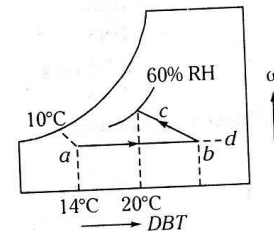
$$\text{Weight of air supplied } m_a = \frac{V}{V_a} = \frac{0.3}{0.8175} = 0.3669 \text{ kg/sec}$$

$$\begin{aligned} \text{Capacity of the heating coil} &= m_a (H_b - H_a) \\ &= 0.3669(43 - 30) = \end{aligned}$$

$$4.77 \text{ kW From chart } T_b = 26.5^\circ \text{C}$$

Let coil surface temperature be T_d

$$\begin{aligned} \text{By passing factor} &= \frac{T_d - T_b}{T_d - T_a} = 0.4 T_d - 5.6 = T_d - \\ & \frac{26 \text{ kJ } T_d - T_a}{T_d - 26.5} \\ 0.4 &= \frac{26.5 - T_d}{T_d - 26.5} \end{aligned}$$





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$$T_d - 1.4$$

$$T_d = 34.8^\circ\text{C}$$

$$\begin{aligned}\text{Capacity of the humidifier} &= m_a (\omega_c - \omega_b) \times 3600 \\ &= 0.3669(0.00875 - 0.006) \times 3600 \\ &= 3.63 \text{ kg/hour}\end{aligned}$$

Problem 8

An air conditioned system is to be designed for a hall of 200 seating capacity when the following conditions are given:

Atmospheric condition = 30°C DBT and 50%

RH Indoor condition = 22°C DBT and 60% RH

Volume of air required = $0.4 \text{ m}^3/\text{min}/\text{person}$

The required condition is achieved first by chemical dehumidification and after that by sensible cooling.

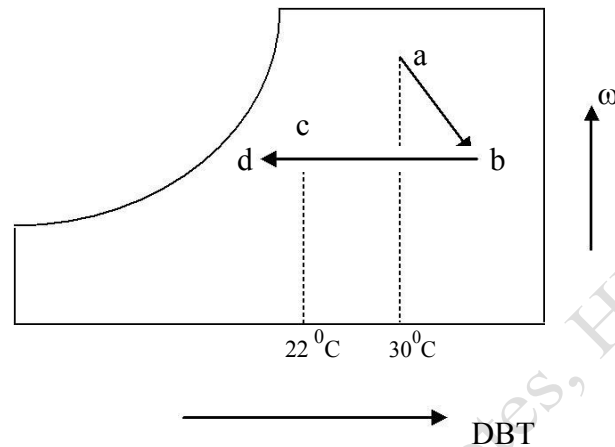
Find the following .

- DBT of the air leaving the dehumidifier.
- The quantity of water vapour removed in the dehumidifier per hour.
- The capacity of cooling coil in tons of refrigeration.
- Surface temperature of the coil if the by pass factor of the coil is 0.25.



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Solution:



Locate point 'a', 30⁰C DBT, 50% RH, the atmospheric condition.
Locate point 'c', 22⁰C DBT, 60% RH, the required indoor condition.
“Since chemical dehumidification process follows constant enthalpy line”
at a draw a line parallel to constant enthalpy line.

At 'c' draw a constant ω line to cut the previous line at point b.

a) DBT of air leaving the dehumidifier $T_b = 40.5^0\text{C}$

From chart

$$H_b = H_a = 65 \text{ kJ/kg}, \quad \omega_a = 0.013 \text{ kg/kg of dry air}$$

$$H_c = 45 \text{ kJ/kg}, \quad \omega_b = 0.009 \text{ kg/kg of dry air}$$

$$V_{sa} = 0.875 \text{ m}^3/\text{min}$$

$$\text{Volume of air} = 200 \times 0.4 = 80 \text{ m}^3/\text{min}$$

$$W_a = \text{Weight of air} = V/V_{sa} = 80/0.875 = 91.42 \text{ kg/min}$$

$$\begin{aligned} \text{b) Quantity of water vapour removed/hour} &= W_a(\omega_a - \omega_b)60 \\ &= 91.42(0.013 - 0.009)60 = 21.94 \text{ kg/hr} \end{aligned}$$

$$\begin{aligned} \text{c) Capacity of cooling coil} &= W_a(H_a - H_b) / (60 \times 3.5) = 91.42(65 - 45) / (60 \times 3.5) \\ &= 8.7 \text{ tons} \end{aligned}$$

$$\text{d) By pass factor} = (T_c - T_d) / (T_b - T_d) = 0.25$$

$$T_d = \text{Temperature of cooling coil} = 15.83^0\text{C}$$

Problem 9

An air conditioned system is to be designed for a cinema hall of 1000 seating capacity when the following conditions are given:



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Outdoor condition = 11°C DBT and 70% RH Required
indoor condition = 20°C DBT and 60% RH

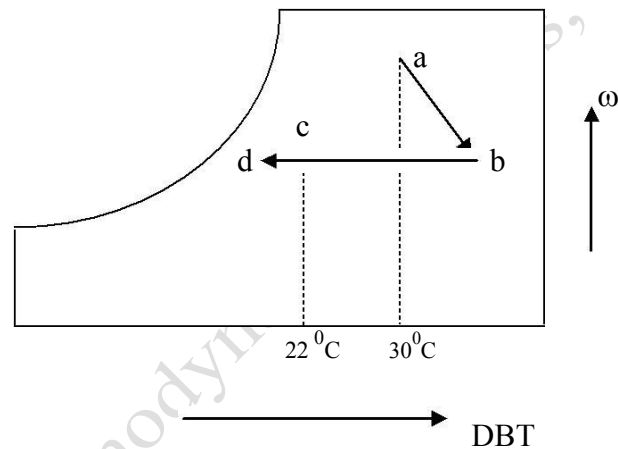
Amount of air required = $0.3\text{m}^3/\text{min}/\text{person}$

The required condition is achieved first by heating, then by humidifying and finally by heating. The condition of air coming out of the humidifier is 75% RH.

Find the following .

- a) Heating capacity of the first heater in kW and condition of the air coming out of the first heater in kW and condition of the air

Solution:



Locate point 'a', 30°C DBT, 50% RH, the atmospheric condition.

Locate point 'c', 22°C DBT, 60% RH, the required indoor condition.

“Since chemical dehumidification process follows constant enthalpy line”
at a draw a line parallel to constant enthalpy line.

At 'c' draw a constant ω line to cut the previous line at point

- b. e) DBT of air leaving the dehumidifier $T_b = 40.5^{\circ}\text{C}$

From chart

$$H_b = H_a = 65\text{kJ/kg}, \omega_a = 0.013\text{ kg/kg of dry air}$$

$$H_c = 45\text{ kJ/kg}, \omega_b = 0.009\text{ kg/kg of dry air}$$

$$V_{sa} = 0.875\text{ m}^3/\text{min}$$

$$\text{Volume of air} = 200 \times 0.4 = 80\text{ m}^3/\text{min}$$

$$W_a = \text{Weight of air} = V/V_{sa} = 80/0.875 = 91.42\text{ kg/min}$$

- f) Quantity of water vapour removed/hour = $W_a(\omega_a - \omega_b)60$
 $= 91.42(0.013 - 0.009)60 = 21.94\text{ kg/hr}$

- g) Capacity of cooling coil = $W_a(H_a - H_b) / (60 \times 3.5) = 91.42(65 - 45) / (60 \times 3.5)$



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= 8.7 tons

h) By pass factor = $(T_c - T_d) / (T_b - T_d) = 0.25$

T_d = Temperature of cooling coil = 15.83°C

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