

Department of Electronics & Communication Engg.

Course : Network Security

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CRYPTOGRAPHY AND NETWORK SECURITY

Unit-03 Public-Key Cryptography and RSA

Private-Key Cryptography

- traditional private/secret/single key cryptography uses one key
- shared by both sender and receiver
- if this key is disclosed communications are compromised
- also is **symmetric**, parties are equal
- hence does not protect sender from receiver forging a message & claiming is sent by sender

Public-Key Cryptography

- probably most significant advance in the 3000 year history of cryptography
- uses two keys a public & a private key
- asymmetric since parties are not equal
- uses clever application of number theoretic concepts to function
- complements rather than replaces private key crypto

Why Public-Key Cryptography?

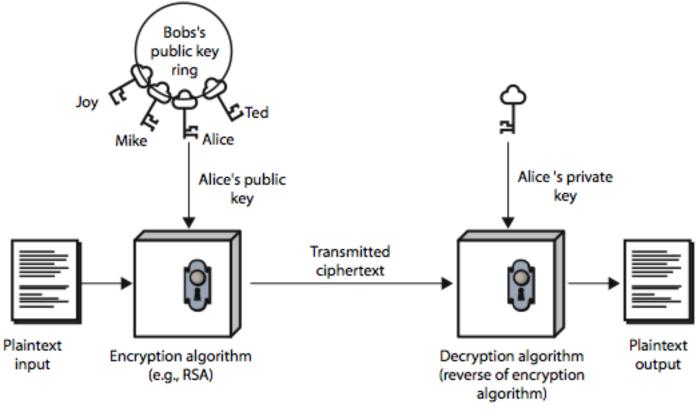
- developed to address two key issues:
 - key distribution how to have secure communications in general without having to trust a KDC with your key
 - digital signatures how to verify a message comes intact from the claimed sender
- public invention due to Diffie & Hellman at Stanford University in 1976

known earlier in classified community

Public-Key Cryptography

- **public-key/two-key/asymmetric** cryptography involves the use of **two** keys:
 - a public-key, which may be known by anybody, and can be used to encrypt messages, and verify signatures
 - a private-key, known only to the recipient, used to decrypt messages, and sign (create) signatures
- is asymmetric because
 - those who encrypt messages or verify signatures cannot decrypt messages or create signatures

Public-Key Cryptography

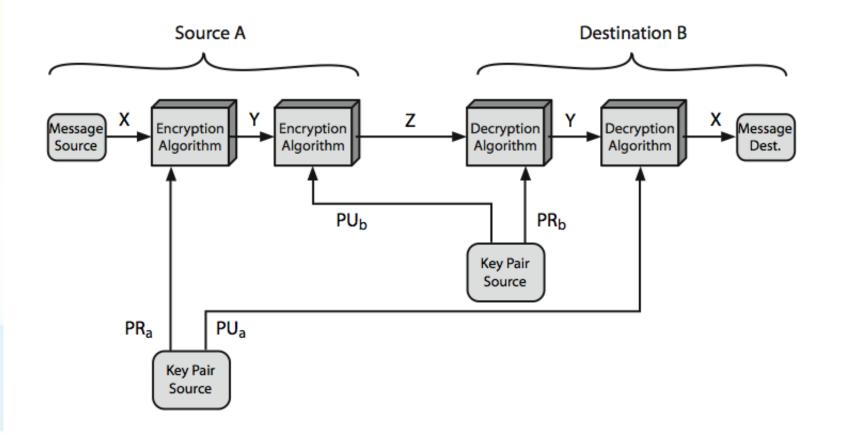


(a) Encryption

Public-Key Characteristics

- Public-Key algorithms rely on two keys where:
 - it is computationally infeasible to find decryption key knowing only algorithm & encryption key
 - it is computationally easy to en/decrypt messages when the relevant (en/decrypt) key is known
 - either of the two related keys can be used for encryption, with the other used for decryption (for some algorithms)

Public-Key Cryptosystems



Public-Key Applications

- can classify uses into 3 categories:
 - encryption/decryption (provide secrecy)
 - digital signatures (provide authentication)
 - key exchange (of session keys)
- some algorithms are suitable for all uses, others are specific to one

Security of Public Key Schemes

- like private key schemes brute force exhaustive search attack is always theoretically possible
- but keys used are too large (>512bits)
- security relies on a large enough difference in difficulty between easy (en/decrypt) and hard (cryptanalyse) problems
- more generally the hard problem is known, but is made hard enough to be impractical to break
- requires the use of very large numbers
- hence is slow compared to private key schemes

RSA

- by Rivest, Shamir & Adleman of MIT in 1977
- best known & widely used public-key scheme
- based on exponentiation in a finite (Galois) field over integers modulo a prime
- uses large integers (e.g., 1024 bits)
- security due to cost of factoring large numbers

RSA Key Setup

- each user generates a public/private key pair by:
- selecting two large primes at random p,q
- computing their system modulus n=p.q
 -define ø(n)=(p-1)(q-1)
- selecting at random the encryption key e
 - where 1<e<ø(n), gcd(e,ø(n))=1
- solve following equation to find decryption key d
 e.d=1 mod ø(n) and 0≤d≤n
- publish their public encryption key: PU={e,n}
- keep secret private decryption key: PR={d,n}

RSA Use

- to encrypt a message M the sender:
 - obtains public key of recipient PU={e,n}
 - computes: $C = M^e \mod n$, where $0 \le M \le n$
- to decrypt the ciphertext C the owner:
 - uses their private key PR={d,n}
 - computes: $M = C^d \mod n$
- note that the message M must be smaller than the modulus n (block if needed)

Why RSA Works

- because of Euler's Theorem:
 - $-a^{\phi(n)} \mod n = 1$ where gcd(a,n) = 1
- in RSA have:
 - n=p.q
 - $\phi(n) = (p-1)(q-1)$
 - carefully chose e & d to be inverses mod ø(n)
 - hence e.d=1+k.ø(n) for some k
- hence :
 - $C^{d} = M^{e.d} = M^{1+k.\emptyset(n)} = M^{1}.(M^{\emptyset(n)})^{k}$
 - $= M^{1}.(1)^{k} = M^{1} = M \mod n$

RSA Example - Key Setup

- 1. Select primes: p=17 & q=11
- 2. Compute $n = pq = 17 \ge 11 = 187$
- 3. Compute $\phi(n) = (p-1)(q-1) = 16 \ge 10 = 160$
- 4. Select e: gcd(e,160)=1; choose *e*=7
- 5. Determine d: *de*=1 mod 160 and *d* < 160 Value is d=23 since 23x7=161= 10x160+1
- 6. Publish public key PU={7,187}
- 7. Keep secret private key PR={23,187}

RSA Example - En/Decryption

- sample RSA encryption/decryption is:
- given message M = 88
- encryption:
 C = 88⁷ mod 187 = 11
- decryption:
 M = 11²³ mod 187 = 88

Efficient Encryption

- encryption uses exponentiation to power e
- hence if e small, this will be faster
 - often choose e=65537 (2¹⁶-1)
 - also see choices of e=3 or e=17
- but if e too small (eg e=3) can attack
 - using Chinese remainder theorem & 3 messages with different modulii

Efficient Decryption

- decryption uses exponentiation to power d

 this is likely large, insecure if not
- can use the Chinese Remainder Theorem (CRT) to compute mod p & q separately. then combine to get desired answer

approx 4 times faster than doing directly

 only owner of private key who knows values of p & q can use this technique

RSA Key Generation

- users of RSA must:
 - determine two primes at random p, q
 - select either e or d and compute the other
- primes p,q must not be easily derived from modulus n=p.q
 - means must be sufficiently large
 - typically guess and use probabilistic test
- exponents e, d are inverses, so use Inverse algorithm to compute the other

RSA Security

- possible approaches to attacking RSA are:
 - brute force key search (infeasible given size of numbers)
 - mathematical attacks (based on difficulty of computing ø(n), by factoring modulus n)
 - chosen ciphertext attacks (given properties of RSA)

Factoring Problem

- mathematical approach takes 3 forms:
 - factor n=p.q, hence compute ø(n) and then d
 - determine ø(n) directly and compute d
 - find d directly
- currently assume 1024-2048 bit RSA is secure
 - ensure p, q of similar size and matching other constraints

Timing Attacks

- developed by Paul Kocher in mid-1990's
- exploit timing variations in operations
 - eg. multiplying by small vs large number
 - or IF's varying which instructions executed
- infer operand size based on time taken
- RSA exploits time taken in exponentiation
- countermeasures
 - use constant exponentiation time
 - add random delays

Chosen Ciphertext Attacks

RSA is vulnerable to a Chosen Ciphertext Attack (CCA)

 attackers chooses ciphertexts & gets decrypted plaintext back

-choose ciphertext to exploit properties of RSA to provide info to help cryptanalysis

Summary

- have considered:
 - principles of public-key cryptography
 - RSA algorithm, implementation, security

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Unit-03(Continued...) Other Public Key Cryptosystems

Other Public Key Cryptosystems

Amongst the tribes of Central Australia every man, woman, and child has a secret or sacred name which is bestowed by the older men upon him or her soon after birth, and which is known to none but the fully initiated members of the group. This secret name is never mentioned except upon the most solemn occasions; to utter it in the hearing of men of another group would be a most serious breach of tribal custom. When mentioned at all, the name is spoken only in a whisper, and not until the most elaborate precautions have been taken that it shall be heard by no one but members of the group. The native thinks that a stranger knowing his secret name would have special power to work him ill by means of magic.

-The Golden Bough, Sir James George Frazer

Diffie-Hellman Key Exchange

- > first public-key type scheme proposed
- by Diffie & Hellman in 1976 along with the exposition of public key concepts
 - note: now know that Williamson (UK CESG) secretly proposed the concept in 1970
- is a practical method for public exchange of a secret key
- > used in a number of commercial products

Diffie-Hellman Key Exchange

> a public-key distribution scheme

- cannot be used to exchange an arbitrary message
- rather it can establish a common key
- known only to the two participants
- value of key depends on the participants (and their private and public key information)
- based on exponentiation in a finite (Galois) field (modulo a prime or a polynomial) - easy
- security relies on the difficulty of computing discrete logarithms (similar to factoring) hard

Diffie-Hellman Setup

> all users agree on global parameters: large prime integer or polynomial q a being a primitive root mod q > each user (eg. A) generates their key • chooses a secret key (number): $x_A < q$ compute their **public key**: $y_A = a^{x_A} \mod q$ each user makes public that key y_A

Diffie-Hellman Key Exchange

- shared session key for users A & B is K_{AB}:
 - $K_{AB} = a^{x_{A}, x_{B}} \mod q$
 - $= y_A^{x_B} \mod q$ (which **B** can compute)
 - $= y_B^{x_A} \mod q$ (which A can compute)
- K_{AB} is used as session key in private-key encryption scheme between Alice and Bob
- if Alice and Bob subsequently communicate, they will have the same key as before, unless they choose new public-keys
- attacker needs an x, must solve discrete log

Diffie-Hellman Example

- > users Alice & Bob who wish to swap keys:
- > agree on prime q=353 and a=3
- > select random secret keys:
 - A chooses x_A=97, B chooses x_B=233
- > compute respective public keys:
 - $y_A = 3^{97} \mod 353 = 40$ (Alice) • $y_B = 3^{233} \mod 353 = 248$ (Bob)
- compute shared session key as:
 - $K_{AB} = y_B^{x_A} \mod 353 = 248^{97} = 160$ (Alice) $K_{AB} = y_A^{x_B} \mod 353 = 40^{233} = 160$ (Bob)

Key Exchange Protocols

- users could create random private/public D-H keys each time they communicate
- users could create a known private/public D-H key and publish in a directory, then consulted and used to securely communicate with them
- both of these are vulnerable to a meet-inthe-Middle Attack
- > authentication of the keys is needed

Man-in-the-Middle Attack

- 1. Darth prepares by creating two private / public keys
- 2. Alice transmits her public key to Bob
- 3. Darth intercepts this and transmits his first public key to Bob. Darth also calculates a shared key with Alice
- 4. Bob receives the public key and calculates the shared key (with Darth instead of Alice)
- 5. Bob transmits his public key to Alice
- 6. Darth intercepts this and transmits his second public key to Alice. Darth calculates a shared key with Bob
- 7. Alice receives the key and calculates the shared key (with Darth instead of Bob)
- Darth can then intercept, decrypt, re-encrypt, forward all messages between Alice & Bob

ElGamal Cryptography

> public-key cryptosystem related to D-H > so uses exponentiation in a finite (Galois) > with security based difficulty of computing discrete logarithms, as in D-H > each user (eg. A) generates their key chooses a secret key (number): $1 < x_A < q-1$ compute their **public key**: $y_A = a^{x_A} \mod q$

ElGamal Message Exchange

Bob encrypt a message to send to A computing

- represent message M in range 0 <= M <= q-1</p>
 - longer messages must be sent as blocks
- chose random integer k with 1 <= k <= q-1</p>
- compute one-time key K = y_A^k mod q
- encrypt M as a pair of integers (C₁,C₂) where
 - $C_1 = a^k \mod q$; $C_2 = KM \mod q$
- A then recovers message by
 - recovering key K as K = C₁^{xA} mod q
 computing M as M = C₂ K⁻¹ mod q

a unique k must be used each time otherwise result is insecure

ElGamal Example

- > use field GF(19) q=19 and a=10
- Alice computes her key:
 - A chooses $x_A = 5$ & computes $y_A = 10^5 \mod 19 = 3$
- > Bob send message m=17 as (11,5) by
 - chosing random k=6
 - computing $K = y_A^k \mod q = 3^6 \mod 19 = 7$ • computing $C_1 = a^k \mod q = 10^6 \mod 19 = 11;$
 - computing $C_1 = a^{\kappa} \mod q = 10^{\circ} \mod 19 = 11$ $C_2 = KM \mod q = 7.17 \mod 19 = 5$

> Alice recovers original message by computing:

- recover K = $C_1^{xA} \mod q = 11^5 \mod 19 = 7$
- compute inverse K⁻¹ = 7⁻¹ = 11

recover $M = C_2 K^{-1} \mod q = 5.11 \mod 19 = 17$

Elliptic Curve Cryptography

- majority of public-key crypto (RSA, D-H) use either integer or polynomial arithmetic with very large numbers/polynomials
- > imposes a significant load in storing and processing keys and messages
- > an alternative is to use elliptic curves
- offers same security with smaller bit sizes
- > newer, but not as well analysed

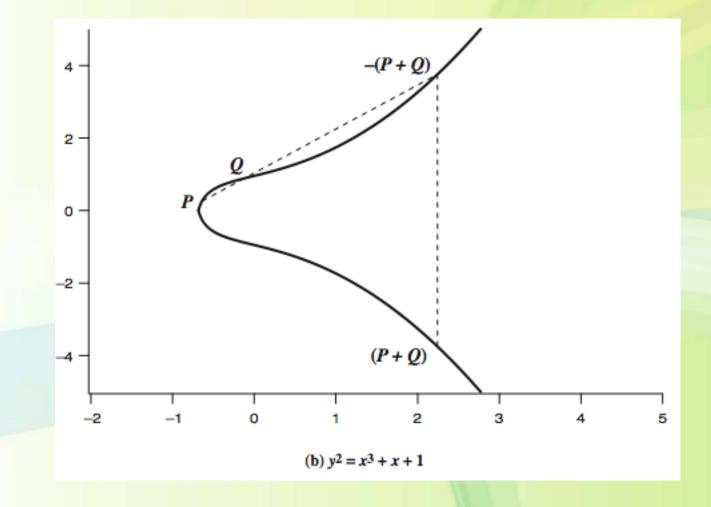
Real Elliptic Curves

- > an elliptic curve is defined by an equation in two variables x & y, with coefficients
- > consider a cubic elliptic curve of form

 $y^2 = x^3 + ax + b$

- where x,y,a,b are all real numbers
- also define zero point O
- consider set of points E(a,b) that satisfy
- have addition operation for elliptic curve geometrically sum of P+Q is reflection of the intersection R

Real Elliptic Curve Example



Finite Elliptic Curves

 Elliptic curve cryptography uses curves whose variables & coefficients are finite
 have two families commonly used:

- prime curves E_p(a,b) defined over Z_p
 - use integers modulo a prime
 - best in software
- binary curves E_{2m}(a,b) defined over GF(2ⁿ)
 - use polynomials with binary coefficients
 - best in hardware

Elliptic Curve Cryptography

- ECC addition is analog of modulo multiply
- ECC repeated addition is analog of modulo exponentiation
- > need "hard" problem equiv to discrete log
 - Q=kP, where Q,P belong to a prime curve
 - is "easy" to compute Q given k,P
 - but "hard" to find k given Q,P
 - known as the elliptic curve logarithm problem
- Certicom example: E₂₃(9,17)

ECC Diffie-Hellman

> can do key exchange analogous to D-H > users select a suitable curve $E_a(a,b)$ > select base point $G=(x_1,y_1)$ with large order n s.t. nG=0 > A & B select private keys $n_A < n$, $n_B < n$ > compute public keys: $P_A = n_A G$, $P_B = n_B G$ > compute shared key: K=n_AP_B, K=n_BP_A same since K=n_An_BG > attacker would need to find k, hard

ECC Encryption/Decryption

- several alternatives, will consider simplest
- must first encode any message M as a point on the elliptic curve P_m
- select suitable curve & point G as in D-H
- each user chooses private key n_A<n</p>
- and computes public key P_A=n_AG
- to encrypt P_m : C_m={kG, P_m+kP_b}, k random
- > decrypt C_m compute:
 - $P_{m}+kP_{b}-n_{B}(kG) = P_{m}+k(n_{B}G)-n_{B}(kG) = P_{m}$

ECC Security

- > relies on elliptic curve logarithm problem
 > fastest method is "Pollard rho method"
 > compared to factoring, can use much smaller key sizes than with RSA etc
 > for equivalent key lengths computations are roughly equivalent
- > hence for similar security ECC offers significant computational advantages

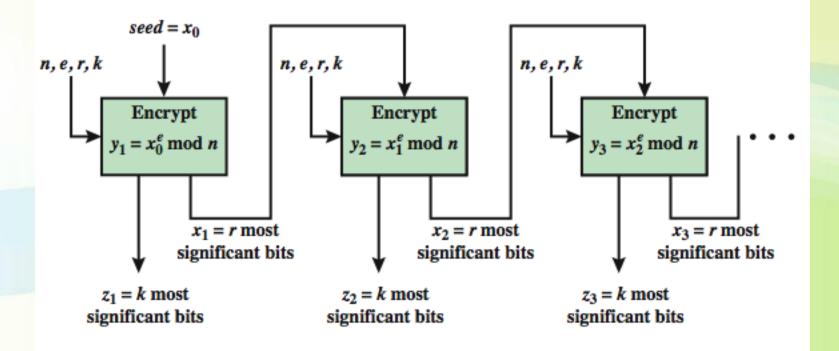
Comparable Key Sizes for Equivalent Security RSA/DSA Symmetric ECC-based scheme scheme (modulus size in (key size in bits) (size of *n* in bits) bits)

Pseudorandom Number Generation (PRNG) based on Asymmetric Ciphers

- > asymmetric encryption algorithm produce apparently random output
- > hence can be used to build a pseudorandom number generator (PRNG)
- > much slower than symmetric algorithms
- hence only use to generate a short pseudorandom bit sequence (eg. key)

PRNG based on RSA

have Micali-Schnorr PRNG using RSA in ANSI X9.82 and ISO 18031



PRNG based on ECC

> dual elliptic curve PRNG NIST SP 800-9, ANSI X9.82 and ISO 18031 > some controversy on security /inefficiency > algorithm for i = 1 to k do set $s_i = x(s_{i-1}P)$ set $r_i = lsb_{240} (x(s_i Q))$ end for return r_1, \ldots, r_k > only use if just have ECC

Summary

- have considered:
 - Diffie-Hellman key exchange
 - ElGamal cryptography
 - Elliptic Curve cryptography
 - Pseudorandom Number Generation (PRNG)
 based on Asymmetric Ciphers (RSA & ECC)

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Unit-03 Key Management in Public key cryptosystems

Key Management

- public-key encryption helps address key distribution problems
- have two aspects of this:
 - distribution of public keys
 - use of public-key encryption to distribute secret keys

Distribution of Public Keys

- can be considered as using one of:
 - public announcement
 - publicly available directory
 - public-key authority
 - public-key certificates

Public Announcement

- users distribute public keys to recipients or broadcast to community at large
 - E.g., append PGP keys to email messages or post to news groups or email list
- major weakness is forgery
 - anyone can create a key claiming to be someone else and broadcast it
 - until forgery is discovered can masquerade as claimed user

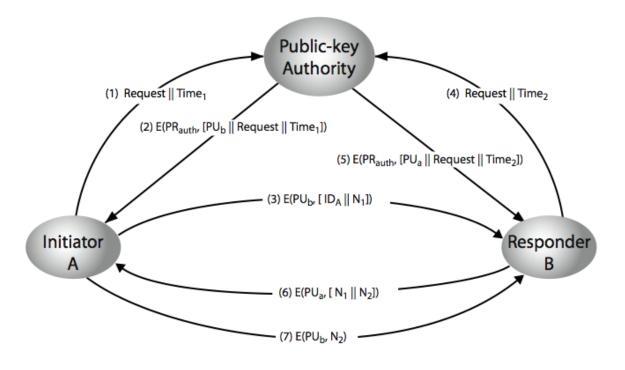
Publicly Available Directory

- can obtain greater security by registering keys with a public directory
- directory must be trusted with properties:
 contains {name,public-key} entries
 - participants register securely with directory
 - participants can replace key at any time
 - directory is periodically published
 - directory can be accessed electronically
- still vulnerable to tampering or forgery

Public-Key Authority

- improve security by tightening control over distribution of keys from directory
- requires users to know public key for the directory
- then users interact with directory to obtain any desired public key securely
 - does require real-time access to directory when keys are needed

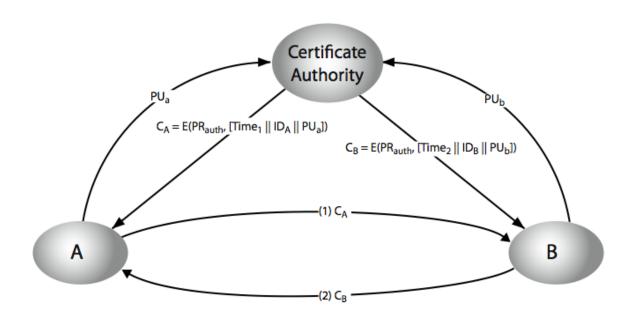
Public-Key Authority



Public-Key Certificates

- certificates allow key exchange without real-time access to public-key authority
- a certificate binds identity to public key
 -contains other info such as period of validity,
 rights of use, etc.
- with all contents signed by a trusted Public-Key or Certificate Authority (CA)
- can be verified by anyone who knows the public-key authorities public-key

Public-Key Certificates



Public-Key Distribution of Secret Keys

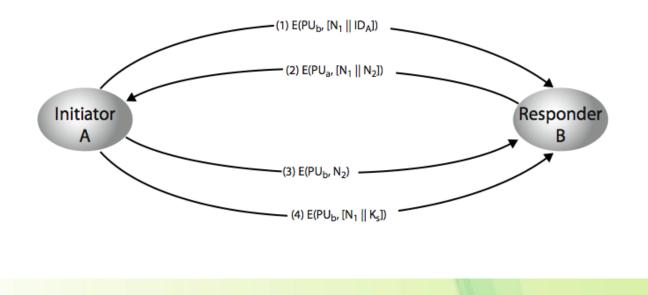
- use previous methods to obtain public-key
- can use for secrecy or authentication
- but public-key algorithms are slow
- so usually want to use private-key encryption to protect message contents
- hence need a session key
- have several alternatives for negotiating a suitable session key

Simple Secret Key Distribution

- proposed by Merkle in 1979
 - A generates a new temporary public key pair
 - A sends the public key and its identity to B
 - B generates a session key K and sends it to A encrypted using the supplied public key
 - A decrypts the session key and both use
- problem is that an opponent can intercept and impersonate both halves of protocol

Public-Key Distribution of Secret Keys

- A & B securely exchange public-keys:
 - -Insures both
- (i) confidentiality and (ii) authentication.



Hybrid Key Distribution

- use of private-key KDC
- shares secret master key with each user
- distributes session key using master key
- public-key used to distribute master keys
 - Three-level hierarchy.
- rationale
 - Performance (public key cryptography is used the least frequently.)

Summary

- have considered:
 - distribution of public keys
 - public-key distribution of secret keys

CRYPTOGRAPHY AND NETWORK SECURITY

Unit-03(Continued...) Message Authentication and Hash Functions

Message Authentication and Hash Functions

- At cats' green on the Sunday he took the message from the inside of the pillar and added Peter Moran's name to the two names already printed there in the "Brontosaur" code. The message now read: "Leviathan to Dragon: Martin Hillman, Trevor Allan, Peter Moran: observe and tail." What was the good of it John hardly knew. He felt better, he felt that at last he had made an attack on Peter Moran instead of waiting passively and effecting no retaliation. Besides, what was the use of being in possession of the key to the codes if he never took advantage of it?
- — Talking to Strange Men, Ruth Rendell

Message Authentication

- message authentication is concerned with:
 - protecting the integrity of a message
 - validating identity of originator
 - non-repudiation of origin (dispute resolution)
- will consider the security requirements
- then three alternative functions used:
 - message encryption
 - message authentication code (MAC)
 - hash function

Security Requirements

- disclosure
- traffic analysis
- masquerade
- content modification
- sequence modification
- timing modification
- source repudiation
- destination repudiation

Message Encryption

- message encryption by itself also provides a measure of authentication
- if symmetric encryption is used then:
 - receiver know sender must have created it
 - since only sender and receiver now key used
 - know content cannot of been altered
 - if message has suitable structure, redundancy or a checksum to detect any changes

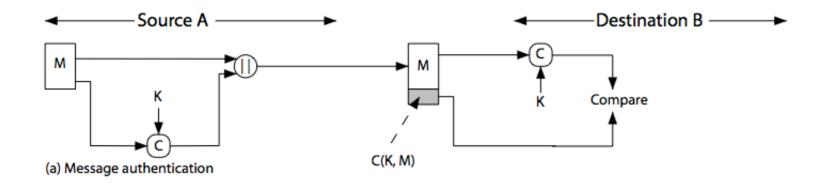
Message Encryption

- if public-key encryption is used:
 - encryption provides no confidence of sender
 - since anyone potentially knows public-key
 - however if
 - sender signs message using their private-key
 - then encrypts with recipients public key
 - have both secrecy and authentication
 - again need to recognize corrupted messages
 - but at cost of two public-key uses on message

Message Authentication Code (MAC)

- generated by an algorithm that creates a small fixed-sized block
 - depending on both message and some key
 - like encryption though need not be reversible
- appended to message as a signature
- receiver performs same computation on message and checks it matches the MAC
- provides assurance that message is unaltered and comes from sender

Message Authentication Code



Message Authentication Codes

- as shown the MAC provides authentication
- can also use encryption for secrecy
 - generally use separate keys for each
 - can compute MAC either before or after encryption
 - is generally regarded as better done before
- why use a MAC?
 - sometimes only authentication is needed
 - sometimes need authentication to persist longer than the encryption (eg. archival use)
- note that a MAC is not a digital signature

MAC Properties

- a MAC is a cryptographic checksum MAC = C_K(M)
 - condenses a variable-length message M
 - using a secret key K
 - to a fixed-sized authenticator
- is a many-to-one function
 - potentially many messages have same MAC
 - but finding these needs to be very difficult

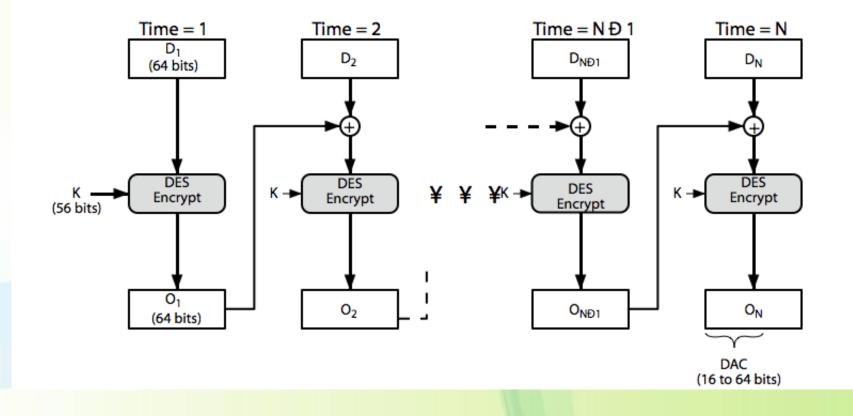
Requirements for MACs

- taking into account the types of attacks
- need the MAC to satisfy the following:
 - 1. knowing a message and MAC, is infeasible to find another message with same MAC
 - 2. MACs should be uniformly distributed
 - 3. MAC should depend equally on all bits of the message

Using Symmetric Ciphers for MACs

- can use any block cipher chaining mode and use final block as a MAC
- Data Authentication Algorithm (DAA) is a widely used MAC based on DES-CBC
 - using IV=0 and zero-pad of final block
 - encrypt message using DES in CBC mode
 - and send just the final block as the MAC
 - or the leftmost M bits (16≤M≤64) of final block
- but final MAC is now too small for security

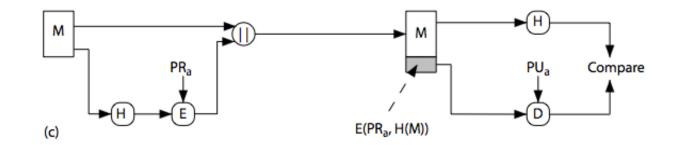
Data Authentication Algorithm



Hash Functions

- condenses arbitrary message to fixed size
 h = H(M)
- usually assume that the hash function is public and not keyed
 - cf. MAC which is keyed
- hash used to detect changes to message
- can use in various ways with message
- most often to create a digital signature

Hash Functions & Digital Signatures



Requirements for Hash Functions

- 1. can be applied to any sized message M
- 2. produces fixed-length output h
- 3. is easy to compute h=H(M) for any message M
- 4. given h is infeasible to find x s.t. H(x)=h
 - one-way property
- 5. given x is infeasible to find y s.t. H(y)=H(x)
 - weak collision resistance
- 6. is infeasible to find any x,y s.t. H(y)=H(x)
 - strong collision resistance

Simple Hash Functions

- are several proposals for simple functions
- based on XOR of message blocks
- not secure since can manipulate any message and either not change hash or change hash also
- need a stronger cryptographic function (next chapter)

Birthday Attacks

- might think a 64-bit hash is secure
- but by Birthday Paradox is not
- birthday attack works thus:
 - opponent generates $2^{m/2}$ variations of a valid message all with essentially the same meaning
 - opponent also generates 2^{m/2} variations of a desired fraudulent message
 - two sets of messages are compared to find pair with same hash (probability > 0.5 by birthday paradox)
 - have user sign the valid message, then substitute the forgery which will have a valid signature
- conclusion is that need to use larger MAC/hash

Block Ciphers as Hash Functions

- can use block ciphers as hash functions
 - using H₀=0 and zero-pad of final block
 - compute: $H_i = E_{M_i} [H_{i-1}]$
 - and use final block as the hash value
 - similar to CBC but without a key
- resulting hash is too small (64-bit)
 - both due to direct birthday attack
 - and to "meet-in-the-middle" attack
- other variants also susceptible to attack

Hash Functions & MAC Security

- like block ciphers have:
- brute-force attacks exploiting
 - strong collision resistance hash have cost $2^{m/2}$
 - have proposal for h/w MD5 cracker
 - 128-bit hash looks vulnerable, 160-bits better
 - MACs with known message-MAC pairs
 - can either attack keyspace (cf key search) or MAC
 - at least 128-bit MAC is needed for security

Hash Functions & MAC Security

- cryptanalytic attacks exploit structure
 - like block ciphers want brute-force attacks to be the best alternative
- have a number of analytic attacks on iterated hash functions
 - $CV_i = f[CV_{i-1}, M_i]; H(M) = CV_N$
 - typically focus on collisions in function f
 - like block ciphers is often composed of rounds
 - attacks exploit properties of round functions

Summary

- have considered:
 - message authentication using
 - message encryption
 - MACs
 - hash functions
 - general approach & security

