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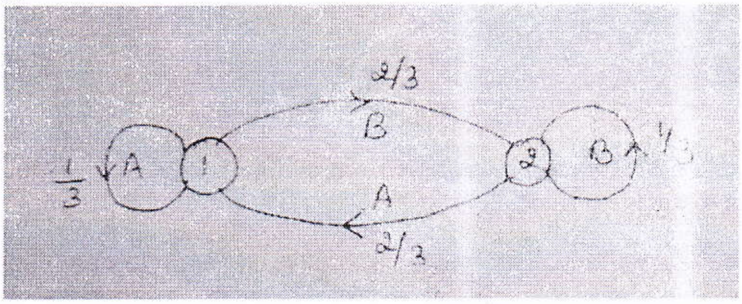
ECE Dept.
Exam.
Internal Assessment
Odd Sem(2019-20)

FIRST INTERNAL ASSESSMENT

Sem.: V Semester | **Sub: Information Theory and Coding** | **Sub. Code: 17EC54**

Date: 16/09/2019 | **Time: 3.00PM to 4.00PM** | **Max. Marks:30**

Note: Answer two full questions, draw sketches wherever necessary.

Q. No		Discription of Question	Marks	CO	RBT Level
1	a	Show that $H(S))^n = nH(S)$. Where n is the extension number.	7	CO304.1	L2
	b	A source emits one of the four probable messages m1, m2, m3, and m4 with probabilities 1/2, 1/8, 1/8, and 1/4 respectively. Find the entropy of the source.	8	CO304.1	L3
OR					
2	a	State and prove the extremal property of entropy with an example	7	CO304.1	L2
	b	The state diagram of the Markov source is as shown in the Figure below:  i) Find the stationary distribution ii) Find the entropy of each state and hence the entropy of the source. iii) Find the entropy of the adjoint source and verify that $H(S)<H(S')$	8	CO304.1	L3
3	a	Show that (a)) 1Nat=1.443bits (b) 1Hartley=2.303Nats	7	CO304.1	L2
	b	The output of an information source consists of 128 symbols 16 of which occur with probability of 1/32 and remaining occurs with probability of 1/224. The source emits 1000 smbols/sec assuming that symbols are chosen independently. Find the average information of the source.	8	CO304.1	L3
OR					
4	a	State and prove the theorem $H(S)) \leq \sum_{k=1}^{n} p_k \log\left(\frac{1}{q_k}\right)$	7	CO304.1	L2
	b	A binary source is emitting an independent sequence of 0s and 1s with probabilities p and 1-p respectively. Plot the entropy of the source versus 'p'. Indicate for what condition entropy reaches maximum.	8	CO304.1	L2

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Sem : V		Subject : Information Theory & Coding — Sub Code : 17EC54	Date : 16/09/2019		
Q. No.	Bit	Description	Marks	CO's	RBT Level
1	(a)	proof of the entropy property: $H(S)^n = n \cdot H(S)$ where n is the integer & indicates extension number. Let us consider a binary source 'S' emitting symbols S_1 & S_2 with probabilities P_1 & P_2 such that $P_1 + P_2 = 1$. Then the second extension of this binary source is: S_1S_1, S_1S_2, S_2S_1, and S_2S_2 with probabilities P_1P_1, $P_1 \cdot P_2$, P_2P_1, and P_2P_2 respectively. And, $P_1 \cdot P_1 = P_1^2$, $P_2 \cdot P_2 = P_2^2$, $P_1P_2 = P_2P_1$ Also, $P_1^2 + 2P_1P_2 + P_2^2 = 1$. $\because (P_1+P_2)^2 = 1 \Rightarrow 1^2 = 1$. By definition $H(S) = \sum_{i=1}^{2} P_i \log \frac{1}{P_i} = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2}$ The entropy of 2^{nd} extension is given by $H(S)^2 = \sum_{j=1}^{4} P_j \log \frac{1}{P_j}$ $= P_1^2 \log \frac{1}{P_1^2} + P_1P_2 \log \frac{1}{P_1P_2} + P_2P_1 \log \frac{1}{P_2P_1} + P_2^2 \log \frac{1}{P_2^2}$ $= 2P_1^2 \log \frac{1}{P_1} + 2P_1P_2 \log \frac{1}{P_1} + 2P_1P_2 \log \frac{1}{P_2} + 2P_2^2 \log \frac{1}{P_2}$ $= 2P_1(P_1+P_2) \log \frac{1}{P_1} + 2P_2(P_1+P_2) \log \frac{1}{P_2}$ $= 2P_1 \log \frac{1}{P_1} + 2P_2 \log \frac{1}{P_2} = 2 \cdot H(S)$ Similarly it can be extended to n^{th} extension $\Rightarrow H(S)^n = n \cdot H(S)$ //	07	CO304.1	L2

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Sem : V	Subject : Information Theory & Coding	Sub Code : 17EC54	Date : 16/09/2019.		
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
1	(b)	Given: Source $S = \{m_1, m_2, m_3, m_4\}$ & $P = \{1/2, 1/8, 1/8, 1/4\}$ $H(S) = ?$ We Know that $H(S) = \sum_{i=1}^{q} P_i \log_2 \frac{1}{P_i}$ where q is no. of source symbols. $\therefore H(S) = \sum_{i=1}^{4} P_i \log_2 \frac{1}{P_i}$ $= \frac{1}{2} \log \frac{1}{1/2} + \frac{1}{8} \log \frac{1}{1/8} + \frac{1}{8} \log \frac{1}{1/8} + \frac{1}{4} \log \frac{1}{1/4}$ $= \frac{1}{2} \times \log_2 2 + 2 \times \frac{1}{8} \log_2 8 + \frac{1}{4} \log_2 4$ $= \frac{1}{2} + \frac{1}{4} \times 3 + \frac{1}{4} \times 2$ $= \frac{1}{2} + \frac{3}{4} + \frac{1}{2} = 1.75$ bits/symb. $\because \log_2 8 = \log_2 2^3 = 3$ & $\log_2 4 = \log_2 2^2 = 2$ $\log_2 2 = 1$.	08	CO304.1	L3
2	(a)	Extremal property of Entropy: This is an important property of an entropy from which lower and upper bounds can be calculated. The entropy of source 'S' is given by $H(S) = \sum_{i=1}^{q} P_i \log \frac{1}{P_i}$ & $\sum_{i=1}^{q} P_i = 1$. Where q is the total number of source symbols.	07	CO304.1	L2

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2	(a)	cont'd...			

Lower bound: It is obvious from the property of entropy is that it can not be negative

$\therefore H(S)_{min} = 0.$

For $P=0$, $H(S) = P\log\frac{1}{P} = \underline{0}$ & For $P=1$, $H(S) = 1\log\frac{1}{1} = 1\times 0 = \underline{0}$

Also, $0 \leq P \leq 1.$

Upper bound:

Let $\log q - H(S) = \left[\sum_{i=1}^{q} P_i\right]\log q - \sum_{i=1}^{q} P_i \log\frac{1}{P_i} \rightarrow (1)$ $\quad \because \sum_{i=1}^{q} P_i = 1.$

$= \sum_{i=1}^{q} P_i\left(\log q - \log\frac{1}{P_i}\right)$ $\quad \because \log\left(\frac{m}{n}\right) = \log m - \log n$

$= \sum_{i=1}^{q} P_i \log qP_i$

$= \sum_{i=1}^{q} P_i \frac{\log_e qP_i}{\log_e^2}$ $\quad \log_2^m = \frac{\log_e^m}{\log_e^2}$

$\therefore \log q - H(S) = \log_2^e \sum_{i=1}^{q} P_i \log qP_i \rightarrow (2)$

Using log inequality:

Let $y = x-1$ &

$y = \ln x \Rightarrow \ln x \leq x-1$

or $\ln\frac{1}{x} \geq 1-x$

And, let $x = \frac{1}{qP_i}$

$\Rightarrow \ln qP_i \geq 1 - \frac{1}{qP_i}$

$\Rightarrow \sum_{i=1}^{q} P_i \ln qP_i \geq \sum_{i=1}^{q} P_i\left(1 - \frac{1}{qP_i}\right) \rightarrow (3)$ $\quad \because$ multiplying both sides by $\sum_{i=1}^{q} P_i \; (=1)$

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Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
2	(a)	cont'd... Multiplying ③ by $\log_2^e$ both sides we get $\log_2^e \sum_{i=1}^{q} P_i \ln qP_i \geq \log_2^e \left[\sum_{i=1}^{q} P_i - \sum_{i=1}^{q} \frac{1}{q}\right] \to (4)$ $\because x = 1/qP_i$ $\therefore$ RHS of ④ is always zero equality only for $P_i = 1/q$. $(\forall i)$ i.e., $\log q - H(S) \geq 0$ or $H(S) \leq \log_2 q$ $\Rightarrow H(S)_{max} = \log_2 q$ Example: let $S = \{S_1, S_2, S_3, S_4\}$ $P = \{1/4, 1/4, 1/4, 1/4\}$ $\quad \underline{\underline{q=4}}$ $\therefore H(S) = 4 \cdot \frac{1}{4} \log_2 \frac{1}{1/4} = 2$ bits/symb. or $H(S) = \log_2 q = \log_2 4 = \underline{\underline{2}}$ bits/symb.			
2	(b) (i)	From the state diagram: $P(1) = P(A) = \frac{1}{3}P(A) + \frac{2}{3}P(B) \to (1)$ $\quad P(1) \Rightarrow P(A)$ $P(2) = P(B) = \frac{2}{3}P(A) + \frac{1}{3}P(B) \to (2)$ $\quad P(2) \Rightarrow P(B)$ From (1) $\quad \frac{2}{3}P(B) = P(A) - \frac{1}{3}P(A)$ $= \frac{2}{3}P(A)$ $\Rightarrow P(B) = P(A) \to (3)$ And, we know that $P(A) + P(B) = 1$ $\Rightarrow P(A) + P(A) = 1 \Rightarrow P(A) = 1/2 = P(B)$ //	08	CO304.1	L3

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Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
2	(b)	Cont'd....			
		The entropy of state 1 or A is given by			
	(ii)	$i=1 \atop =A$ $H_1 = H_A = \sum_{j=A}^{B} P_{ij} \log_2 \frac{1}{P_{ij}}$			
		$= P_{AA} \log \frac{1}{P_{AA}} + P_{AB} \log \frac{1}{P_{AB}}$			
		$= \frac{1}{3} \log_2 \frac{1}{1/3} + \frac{2}{3} \log_2 \frac{1}{2/3}$			
		$= \frac{1}{3} \log_2 3 + \frac{2}{3} \log_2 1.5$			
		$= 0.5283 + 0.3899$			
		$= \underline{\underline{0.9182}}$ bits/symbol			
		$i=2 \atop =B$ $H_2 = H_B = \sum_{j=A}^{B} P_{ij} \log_2 \frac{1}{P_{ij}}$			
		$= P_{BA} \log_2 \frac{1}{P_{BA}} + P_{BB} \log_2 \frac{1}{P_{BB}}$			
		$= \frac{2}{3} \log_2 \frac{1}{2/3} + \frac{1}{3} \log \frac{1}{1/3}$			
		$= 0.9182$ bits/symbol			
		$\therefore H(S) = \sum_{i=A}^{B} P_i H_i = P(A) \cdot H(A) + P(B) H(B)$			
		$= \frac{1}{2} \times 0.9182 + \frac{1}{2} \times 0.9182$			
		$= 0.9182$ bits/symbol			
		Entropy of adjoint source: $H(S') \Rightarrow \sum_{i=1}^{9} P_i \log \frac{1}{P_i}$			

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Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
2	(b)	cont'd...			
	(iii)	$H(S') = \frac{1}{2}\log\frac{1}{1/2} + \frac{1}{2}\log\frac{1}{1/2} = 1.$ $\therefore H(S) < H(S')$			
3	(a)	(i) 1 Nat = 1.443 bits proof: $I = \log_e \frac{1}{p}$ nats $\longrightarrow (1)$ $\therefore 1\text{ Nat} = I / \log_e\frac{1}{p} = \frac{\log_2 1/p}{\log_e 1/p} = \frac{-\log_2 p}{-\log_e p}$ $1\text{Nat} = \frac{\frac{1}{\log_p 2}}{1/\log_p e} = \frac{\log_p e}{\log_p 2} = \frac{\log_e e}{\cancel{\log_e p}} \Big/ \frac{\log_e 2}{\cancel{\log_e p}}$ $\therefore 1\text{ Nat} = \frac{1}{\log_e 2} = 1.443$ bits $\{\log_e e = 1$ ii) 1 Hartley = 2.303 Nats proof: $I = \log_{10} 1/p$ Hartleys — (1) $\therefore 1\text{ Hartley} = \frac{I}{\log_{10}\frac{1}{p}} = \frac{\log_e 1/p}{\log_{10} 1/p} = \frac{-\log_e p}{-\log_{10} p}$ $= \frac{\frac{1}{\log_p e}}{1/\log_p 10} = \frac{\log_p 10}{\log_p e} = \frac{\frac{\log_e 10}{\cancel{\log_e p}}}{\frac{\log_e e}{\cancel{\log_e p}}} = \log_e 10$ $\Rightarrow$ 1 Hartley = 2.303 Nats //	07	CO304.1	L2

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3	(b)	Given: Total no of Symbols = 128 $16/128$ occur with prob. $1/32$ & Remaining $112/128$ occur with prob. $1/224$ $r_s = 1000$ symbols/sec $H = ?$ $\therefore H = 16 \times \frac{1}{32} \log_2 \frac{1}{1/32} + 112 \times \frac{1}{224} \log \frac{1}{1/224}$ $= \frac{1}{2} \times 5 + \frac{1}{2} \log_2 224 = 2.5 + 3.9036$ $H(S) = 6.40$ bits/symbol.	08	CO304.1	L3
4	(a)	Suppose S_1 & S_2 are two ZMSs with probabilities are $P_1, P_2, P_3 \cdots P_n$ for source S_1 and $q_1, q_2, q_3 \cdots q_n$ for source S_2. Show that the entropy of source S_1 $H(S_1) \leq \sum_{k=1}^{n} P_k \log \frac{1}{q_k}$ proof: Given S_1 & S_2 are two zero memory sources $\therefore H(S_1) = \sum_{k=1}^{n} P_k \log \frac{1}{P_k} \rightarrow (1)$ & $\sum_{k=1}^{n} P_k = 1$ (Total prob. theorem) III'ly $H(S_2) = \sum_{k=1}^{n} q_k \log \frac{1}{q_k}$ —(2) & $\sum_{k=1}^{n} q_k = 1.$	07	CO304.1	L2

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Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
4	(a)	Cont'd.... Consider $H(S_1) - \sum_{k=1}^{n} P_k \log \frac{1}{q_k} = \sum_{k=1}^{n} P_k \log \frac{1}{P_k} - \sum_{k=1}^{n} P_k \log \frac{1}{q_k}$ $= \sum_{k=1}^{n} P_k \left(\log \frac{1}{P_k} - \log \frac{1}{q_k}\right) = \sum_{k=1}^{n} P_k \cdot \log_2 \left(\frac{q_k}{P_k}\right)$ $= \sum_{k=1}^{n} P_k \left[\frac{\log_e (q_k/P_k)}{\log_e 2}\right] = \log_2^e \sum_{k=1}^{n} P_k \ln\left(\frac{q_k}{P_k}\right)$ We know the log inequality $\ln \frac{1}{x} \geq 1 - x$ or $-\ln x \geq 1 - x$ Removing -ve sign $\ln x \leq x - 1$ let $x = \frac{q_k}{P_k}$ $\therefore \ln\left(\frac{q_k}{P_k}\right) \leq \frac{q_k}{P_k} - 1$ Multiplying by P_k taking summation for $\forall k (k = 1, 2, .. n)$ & then multiplying by $\log_2 e$ on both sides we get $\log_2^e \sum_{k=1}^{n} P_k \ln\left(\frac{q_k}{P_k}\right) \leq \log_2^e \sum_{k=1}^{n} P_k \left(\frac{q_k}{P_k} - 1\right)$ $= H(S_1) - \sum_{k=1}^{n} P_k \ln\left(\frac{q_k}{P_k}\right) \leq \log_2^e \underbrace{\sum_{k=1}^{n} (q_k - P_k)}_{=0}$ $\therefore H(S_1) \leq \sum_{k=1}^{n} P_k \log \frac{1}{q_k}$ //			

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4	(b)	(see below)	08	CO304.1	L2

Given Let $P(0)$ & $P(1)$ be the probabilities of emitting 0s & 1s respectively

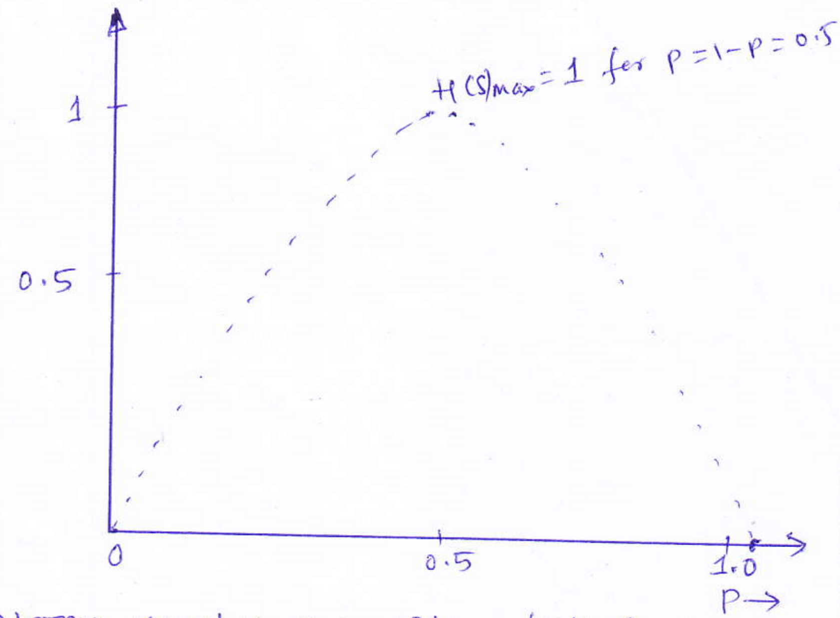
Then $P(0) = P$ & $P(1) = 1-P$

$\Rightarrow P(0) + P(1) = P + 1 - P = 1.$

$\therefore$ Entropy $= H(S) = \sum_{i=1}^{n} P_i \log \frac{1}{P_i} = \sum_{i=1}^{2} P_i \log \frac{1}{P_i}$

$H(S) = P \log_2 \frac{1}{P} + (1-P) \log \left(\frac{1}{1-P}\right)$

P	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
H(S)	0	0.469	0.722	0.881	0.971	1	0.971	0.881	0.722	0.469	0



Entropy reaches max when both symbols are equiprobable i.e., $P = 1 - P = 0.5$

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