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ECE Dept.
Exam. 12
Internal Assessment
2019-20(Odd Sem)

FIRST INTERNAL ASSESSMENT

Sem: V | **Sub: DSP** | **Sub. Code: 17EC52**
Date: 15/09/2019 | **Time: 3PM to 4PM** | **Max. Marks:30**

Note: Answer two full questions, draw sketches wherever necessary.

Q. No		Description of Question	Marks	CO	RBT LEVEL
1	a	Prove that the sampling of Fourier transform of a sequence x(n) results in N-point DFT.	8	CO302.1	L1, L2
	b	Compute DFT of the given sequence using linearity property. x(n)= coshan, n=0,1,2..N-1.	7	CO302.1	L1, L2
OR					
2	a	Compute the 8-point DFT of the sequence x(n) given bellow. x(n)=(1,1,1,1,0,0,0,0)	8	CO302.1	L1, L2
	b	Derive the relationship between DFT and Z-transform. Find the Z-transform of sequence x(n)=[0.5, 0, 0.5, 0] using Z-transform result find its DFT.	7	CO302.1	L1, L2
3	a	Prove the Circular time shift and frequency shift properties.	8	CO302.1	L1, L2
	b	Let x(n) be a finite length sequence with X(K) ={0, 1+j, 1, 1-j} using the properties of DFT find DFT of the following sequence i) $x_1(n) = e^{j(\pi/2)n}.x(n)$ ii) $x_2(n)=\cos\{(\pi/2)n\}$. iii) $x((n-1))_4$	7	CO302.1	L1, L2
OR					
4	a	Find the IDFT of the sequence X(K)={5, 0, 1-j, 0, 1, 0, 1-j, 0}.	8	CO302.1	L1, L2
	b	Find the N-point DFT of following sequences i) x(n)= an ii) $x(n)=a^n$	7	CO302.1	L1, L2

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IA -I SCHEME OF EVALUATION

Sem : V	Subject : Digital Signal Processing	Sub Code : 17EC52	Date : 15/09/2019		
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
1)	a)	The Fourier transform of the signal is given by equation $X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} \rightarrow (1)$ Here $x(n)$ is the discrete time signal & Ω is the frequency. Here Ω is the continuous function varies from 0 to 2π. Eventhough $x(n)$ is discrete, its spectrum $X(\Omega)$ is continuous. Such a continuous function cannot be evaluated on digital processor. To overcome the problem on digital processing, the spectrum $X(\Omega)$ is sampled uniformly. WKT $X(\Omega)$ is periodic with period of 2π. Hence samples taken from 0 to 2π are only important for our processing.	2M.	CO302.1	L1, L2
		Let 'N' samples are taken from 0 to 2π. Hence spacing between successive samples will be $\frac{2\pi}{N}$. Therefore in eq (1) put $\Omega = \frac{2\pi N}{K}$ $\therefore X\left(\frac{2\pi}{N}K\right) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\frac{2\pi}{N}Kn} \rightarrow (2)$ Here $K = 0$ to $N-1$. Thus K is an index for samples.	1M		

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IA - I SCHEME OF EVALUATION

Sem : V	Subject : DSP	Sub Code : 17EC52	Date : 15/09/2019		
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
		The equation (2) can be written as $X\left(\frac{2\pi}{N}k\right) = \cdots + \sum_{n=-N}^{-1} x(n) e^{-j\frac{2\pi}{N}kn} + \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} + \sum_{n=N}^{2N-1} x(n) e^{-j\frac{2\pi}{N}kn} + \cdots$ The above individual summations can also be represented as $X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n) e^{-j\frac{2\pi}{N}kn}$ Now let us change index n by $n-lN$ $\therefore X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN) e^{-j\frac{2\pi}{N}k(n-lN)}$ $\therefore X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN) e^{-j\frac{2\pi}{N}kn} e^{j2\pi kl}$ Here $e^{j2\pi kl} = 1$ always. Since k & l are both integers. $\boxed{\therefore X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi}{N}kn}}$ where $x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$ $\therefore x_p(n) = \cdots + x(n+2N) + x(n+N) + x(n) + x(n-N) + \cdots$ This means that $x_p(n)$ is periodic with period of N samples of $x(n)$	5M	CO302.1	L1, L2

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Sem : V		Subject : DSP	Sub Code : 17EC52	Date : 15/09/2019	
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
1)	b)	$x(n) = \cosh an$; $n = 0$ to $N-1$. The N-point DFT of sequence $x(n)$ is given by $X(K) = DFT\{x(n)\} = DFT\{\cosh an\}$ $= DFT\{\frac{1}{2}e^{an} + \frac{1}{2}e^{-an}\}$ Applying linearity property we get $\therefore X(K) = \frac{1}{2}DFT\{e^{an}\} + \frac{1}{2}DFT\{e^{-an}\}$	2M		
		$\therefore X(K) = \frac{1}{2}\sum_{n=0}^{N-1} e^{an} W_N^{kn} + \frac{1}{2}\sum_{n=0}^{N-1} e^{-an} W_N^{kn}$ $= \frac{1}{2}\sum_{n=0}^{N-1}(e^{a}W_N^{k})^n + \frac{1}{2}\sum_{n=0}^{N-1}(e^{-a}W_N^{k})^n$ $= \frac{1}{2}\left[\frac{e^{aN} \cdot W_N^{KN} - 1}{e^{a}W_N^{K} - 1}\right] + \frac{1}{2}\left[\frac{e^{-aN} W_N^{KN} - 1}{e^{-a}W_N^{K} - 1}\right]$ $= \frac{1}{2}\left[\frac{(e^{aN} - 1)(e^{-a}W_N^{K} - 1) + (e^{a}W_N^{K} - 1) \times (e^{-aN} - 1)}{(e^{a}W_N^{K} - 1)(e^{-a} \cdot W_N^{K} - 1)}\right]$ $\therefore X(K) = \frac{1 - \cosh Na + W_N^{K}[\cosh(N-1)a - \cosh a]}{1 - 2W_N^{K}\cosh a + W_N^{K}}$	5M	CO302.1	L1, L2

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IA -I SCHEME OF EVALUATION

Sem : V	Subject : DSP	Sub Code : 17EC52	Date : 15/09/2019		
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
2)	a)	$x(n) = \{1,1,1,1,0,0,0,0\}$ 8-point DFT of the above sequence is given by $\therefore X(K) = \sum_{n=0}^{N-1} x(n) W_N^{Kn}$ $\underline{N=8}$ $X(K) = \sum_{n=0}^{7} x(n) W_8^{Kn}$ $\therefore X(K) = 1 + W_8^{K} + W_8^{2K} + W_8^{3K}$ $K=0;$ $X(0) = 1+1+1+1 = 4$ $K=1;$ $X(1) = 1 + W_8^1 + W_8^2 + W_8^3 = 1 - j2.414$ $K=2;$ $X(2) = 1 + W_8^2 + W_8^4 + W_8^6 = 0$ $K=3;$ $X(3) = 1 + W_8^3 + W_8^6 + W_8^9 = 1 - j0.414$ $K=4;$ $X(4) = 1 + W_8^4 + W_8^8 + W_8^{12}$ WKT $W_8^0 = W_8^8$, $W_8^4 = W_8^{12}$ $X(4) = 0.$ $K=5;$ $X(5) = 1 + W_8^5 + W_8^{10} + W_8^{15}$ $= 1 + W_8^5 + W_8^2 + W_8^7 = 1 + j0.414$ $K=6;$ $X(6) = 1 + W_8^6 + W_8^4 + W_8^2 = 0$ $K=7;$ $X(7) = 1 + W_8^7 + W_8^6 + W_8^5$ $= 1 + j2.414$	$1 \times 8 = 8$	CO302.1	L1, L2

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IA -I SCHEME OF EVALUATION

Sem : V		Subject : DSP	Sub Code : 17EC52	Date : 15/09/2019	
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
2)	b)	Relationship between DFT & Z-transform WKT Z-Transform of sequence $x(n)$ is given as $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ where $z = e^{j\omega}$ Let us sample $X(z)$ at 'N' equally spaced points. then $z_k = e^{j\frac{2\pi k}{N}}$ $k=0$ to $N-1$ It means $\omega = \frac{2\pi}{N}k$ $\therefore X\left(\frac{2\pi}{N}k\right) = X(z)\Big\vert_{z=e^{j\frac{2\pi k}{N}}}$ $= \sum_{n=-\infty}^{\infty} x(n) z^{-n}\Big\vert_{z=e^{j\frac{2\pi k}{N}}}$ $\therefore X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$ $\therefore X(z)\Big\vert_{z=e^{j\frac{2\pi k}{N}}} = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}}$ $\boxed{\therefore X(k) = X(z)\Big\vert_{z_k=e^{j\frac{2\pi k}{N}}}}$ This means if Z-transform is evaluated on unit circle at evenly spaced points only, then it becomes DFT	3M	CO302.1	L1, L2

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Sem : V	Subject : DSP	Sub Code : 17EC52	Date : 15/09/2019

Q. No.	Bit	Description	Marks	CO's	RBT Level
		$x(n) = \{0.5, 0, 0.5, 0\}$ Z-transform of above sequence $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ $= 0.5 + 0z^{-1} + 0.5z^{-2} + 0.z^{-3}$ $= 0.5 + 0.5z^{-2}$ The Z-transform & DFT are related as $X(k) = X(z)\Big/_{z = e^{j\frac{2\pi}{N}k}}$ Here $N=4$ $\therefore X(k) = 0.5 + 0.5e^{-j\frac{2\pi}{4}k \times 2}$ $X(k) = 0.5 + 0.5(-1)^k$ $\therefore k=0;\ X(0) = 0.5+0.5 = 1$ $k=1;\ X(1) = 0.5-0.5 = 0$ $k=2;\ X(2) = 0.5+0.5 = 1$ $k=3;\ X(3) = 0.5-0.5 = 0$ $\therefore X(k) = \{1, 0, 1, 0\}$	4M	CO3021	L1, L2

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Sem : V	Subject : DSP	Sub Code : 17EC52	Date : 15/09/2019		
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
3)	a)	Circular time shift Property If $x(n) \xleftrightarrow[N]{DFT} X(K)$ then this Property states that $x((n-m))_N \xleftrightarrow[N]{DFT} W_N^{Km} X(K)$ Proof:- WKT IDFT eq is given by $x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) W_N^{-Kn}$ Replace n by n-m $x(n-m) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) W_N^{-K(n-m)}$ $= \frac{1}{N}\sum_{K=0}^{N-1} X(K) W_N^{-Kn} W_N^{Km}$ $\therefore x((n-m))_N = \frac{1}{N}\sum_{K=0}^{N-1} X(K) W_N^{Km} W_N^{-Kn}$ $\therefore$ By above equation we can write the circular time shift property as $DFT\{x((n-m))_N\} = W_N^{Km} X(K)$ Or $IDFT\{W_N^{Km} X(K)\} = x((n-m))_N$	4M	CO302.1	L1, L2

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Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
		Circular frequency Shift Property This Property States that. if $x(n) \xleftrightarrow[N]{DFT} X(k)$ then $DFT\{W_N^{-ln} x(n)\} = X((k-l))_N$ Proof :- WKT DFT equation is given by $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$ Replace k by k-l $\therefore X((k-l))_N = \sum_{n=0}^{N-1} x(n) W_N^{(k-l)n}$ $\therefore X((k-l))_N = \sum_{n=0}^{N-1} x(n) W_N^{kn} W_N^{-ln}$ $\therefore X((k-l))_N = \sum_{n=0}^{N-1} \{x(n) W_N^{-ln}\} W_N^{kn}$ From the above equation Circular frequency Shift Property States that $DFT\{x(n) W_N^{-ln}\} = X((k-l))_N$ Or $IDFT\{X((k-l))_N\} = x(n) W_N^{-ln}$	4M	CO302.1	L1, L2

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Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
3)	b)	$x(k) = \{0, 1+j, 1, 1-j\}$ i) $x_1(n) = e^{j(\pi/2)n} \cdot x(n)$ $x_1(n) = e^{j\frac{2\pi}{4}n} \cdot x(n)$ $x_1(n) = W_4^{-n} \cdot x(n)$ using frequency shift property. $DFT\{W_N^{-ln} x(n)\} = X((k-l))_N$	2M		
		$X_1(k) = X((k-1))_4$ $\therefore X_1(k) = \{1-j, 0, 1+j, 1\}$ ii) $x_2(n) = \cos\{\frac{\pi}{2}n\} = \frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}$ $\therefore x_2(n) = \frac{1}{2}[W_4^{-n} + W_4^{n}]$ By using circular frequency shift $\therefore X_2(k) = \frac{1}{2}[X((k-1))_4 + X((k+1))_4]$ $X_2(k) = \frac{1}{2}[\{1-j, 0, 1+j, 1\} + \{1+j, 1, 1-j, 0\}]$ $X_2(k) = \frac{1}{2}[2, 1, 2, 1] = [1, 1/2, 1, 1/2]$	2M	CO302.1	L1, L2
		iii) $x((n-1))_4$ By using time shift property Let $y(k) = W_4^{k} X(k)$ $k=0;\ Y(0) = 0;\quad k=1;\ Y(1) = W_4^1 X(1) = (-j)(1+j) = 1+j = 1-j$ $k=2;\ Y(2) = W_4^2 X(2) = (-1)1 = -1\quad k=3;\ Y(3) = W_4^3 X(3) = (j)(1-j) = 1+j$ $\therefore Y(k) = \{0, 1-j, -1, 1+j\}$	3M		

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Sem : V	Subject : DSP	Sub Code : 17EC52	Date : 15/09/2019		
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
4)	a)	$x(k) = \{5, 0, 1-j, 0, 1, 0, 1-j, 0\}$ WKT IDFT equation $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(k) W_N^{-kn}$ $N=8$ $\therefore x(n) = \frac{1}{8}\sum_{k=0}^{7} x(k) W_8^{-kn}$ $n=0$; $x(0) = \frac{1}{8}[5+0+1-j+0+1+0+1-j+0]$ $x(0) = \frac{1}{8}\times 8 = 1$ $n=1$; $x(1) = \frac{1}{8}[5+(1-j)(j)+1(-1)+(1+j)(-j)]$ $= \frac{1}{8}\times 6 = 0.75$ $n=2$; $x(2) = \frac{1}{8}[5+(1-j)(-1)+1(1)+(1+j)(-1)]$ $= \frac{1}{8}\times 4 = 0.5$ $n=3$; $x(3) = \frac{1}{8}[5+(1-j)(-j)+1(-1)+(1+j)(j)]$ $= \frac{1}{8}\times 2 = 0.25$ $n=4$; $x(4) = \frac{1}{8}[5+(1-j)(1)+1(1)+(1+j)(1)]$ $= \frac{1}{8}\times 8 = 1$ $n=5$; $x(5) = \frac{1}{8}[5+(1-j)(j)+1(1)+(1+j)(-j)]$ $= \frac{1}{8}\times 6 = 0.75$ $n=6$; $x(6) = \frac{1}{8}[5+(1-j)(-1)+1(1)+(1+j)(-1)]$ $= \frac{1}{8}\times 4 = 0.5$ $n=7$; $x(7) = \frac{1}{8}[5+(1-j)(-j)+1(-1)+(1+j)(j)]$ $= \frac{1}{8}\times 2 = 0.25$ $\therefore x(n) = \{1, 0.75, 0.5, 0.25, 1, 0.75, 0.5, 0.25\}$	1×8= 8M	CO302.1	L1, L2

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IA -I SCHEME OF EVALUATION

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Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
4)	b)	i) $x(n) = an$ $X(k) = DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n) W_N^{kn}$ $X(k) = \sum_{n=0}^{N-1} an W_N^{kn} = a\sum_{n=0}^{N-1} n W_N^{kn}$ WKT $\sum_{n=0}^{N-1} b^n = \frac{b^N - 1}{b-1}$; $b \neq 1$ Differentiating above equation with respect to b $\sum_{n=0}^{N-1} n b^{n-1} = \frac{(b-1)(Nb^{N-1}) - (b^N - 1)}{(b-1)^2}$ $\sum_{n=0}^{N-1} n b^n = \frac{b(Nb^N - Nb^{N-1} - b^N + 1)}{(b-1)^2}$ $= \frac{b(b^N(N-1) - Nb^{N-1} + 1)}{(b-1)^2}$ Letting $b = W_N^k$ in above expression $\sum_{n=0}^{N-1} n W_N^{kn} = \frac{W_N^k[W_N^{kN}(N-1) - N W_N^{k(N-1)} + 1]}{[W_N^k - 1]^2}$ $\therefore \sum_{n=0}^{N-1} n W_N^{kn} = \frac{W_N^k[N - 1 - N W_N^{-k} + 1]}{[W_N^k - 1]^2}$ $= \frac{N[W_N^k - 1]}{[W_N^k - 1]^2} = \frac{N}{[W_N^k - 1]}$ $\therefore X(k) = \frac{aN}{[W_N^k - 1]}$; $k \neq 0$ $k=0$; $X(0) = a\sum_{n=0}^{N-1} n = \frac{aN(N-1)}{2}$	5M	CO302.1	L1, L2

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Sem : V		Subject : DSP	Sub Code : 17EC52	Date : 15/09/2019	
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT Level**
		$\therefore X(k) = \begin{cases} \frac{aN(N-1)}{2} ; & k=0 \\ \frac{aN}{W_N^k - 1} ; & k \neq 0. \end{cases}$			
		ii) $x(n) = a^n$			
		$\therefore X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} = \sum_{n=0}^{N-1} a^n W_N^{kn}$			
		$\therefore X(k) = \sum_{n=0}^{N-1} (a W_N^k)^n = \frac{a^N \cancel{W_N^{kN}}^{=1} - 1}{a W_N^k - 1}$	2M	CO302.1	L1,L2
		$\therefore X(k) = \frac{a^N - 1}{a W_N^k - 1} ; 0 \leq k \leq N-1.$			

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