Department of Electronics & Communication Engg.

Course : Signals and Systems   Engg-15EC44.   Sem.: 4th (2017-18, Even)

Course Coordinator:

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Convergence

- Existence of *z-transform*: exists only if $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$ converges
- Necessary condition: absolute summability of $x[n]z^{-n}$, since $|x[n]z^{-n}| = |x[n]r^{-n}|$, the condition is

$$\sum_{n=-\infty}^{\infty} |x[n]r^{-n}| < \infty$$

- The range r for which the condition is satisfied is called the *range of convergence* (ROC) of the *z-transform*
- ROC is very important in analyzing the system stability and behavior
- We may get identical *z-transform* for two different signals and only ROC differentiates the two signals
- The *z-transform* exists for signals that do not have DTFT.
- existence of DTFT: absolute summability of $x[n]$
- by limiting restricted values for r we can ensure that $x[n]r^{-n}$ is absolutely summable even though $x[n]$ is not
- Consider an example: the DTFT of $x[n] = \alpha^n u[n]$ does not exists for $|\alpha| > 1$
- If $r > \alpha$, then r^{-n} decays faster than $x[n]$ grows
- Signal $x[n]r^{-n}$ is absolutely summable and *z-transform* exists

- ROC is related to characteristics of $x[n]$
- ROC can be identified from $X(z)$ and limited knowledge of $x[n]$
- The relationship between ROC and characteristics of the $x[n]$ is used to find inverse z-transform

Property 1

ROC can not contain any poles

- ROC is the set of all z for which z-transform converges
- $X(z)$ must be finite for all z
- If p is a pole, then $|H(p)| = \infty$ and z-transform does not converge at the pole
- Pole can not lie in the ROC

Property 2

The ROC for a finite duration signal includes entire z-plane except $z = 0$ or/and $z = \infty$

- Let $x[n]$ be nonzero on the interval $n_1 \leq n \leq n_2$. The z-transform is

$$X(z) = \sum_{n=n_1}^{n_2} x[n]z^{-n}$$

The ROC for a finite duration signal includes entire z-plane except $z = 0$ or/and $z = \infty$

Properties of Z-transform

- Linearity
- Time reversal
- Time shift
- Multiplication by α^n
- Convolution
- Differentiation in the z-domain

The z-transform

- The *z-transform* of any signal $x[n]$ is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

- The *inverse z-transform* of $X(z)$ is

$$x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

Inverse Z transform:

Three different methods are:

1.Partial fraction method

2.Power series method

Long division method

Queries …?