Department of Electronics & Communication Engg.

Course : Signals and Systems Engg-15EC44. Sem.: 4th (2017-18, Even)

Course Coordinator:

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Finding Unit Impulse Response and step response S(n)

Stability and Causality

Definition: A system is stable if and only if every bounded input produces a bounded output. A bounded input/output is a signal for which for all values of *t.*

Stability for LTI Systems

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

– *if and only if* holds in either direction

- A theorem which applies to LTI systems states that a system(LTI system) is stable if and only of

Causality

Definition: A system is causal if and only if the output at the present time does not depend upon future values of the input.

Causal for LTI Systems

$$h(t) = 0 \quad \text{for } t < 0$$

- A theorem which applies to LTI systems is

Example: Step Response from $h(t) = e^{-at}u(t)$

- Knowing the impulse response of a system we can find the respond to a step input by just integrating the output, since $u(n)$ at the input is obtained by integrating $\delta(t)$
- Thus we can write that

$$y(t) = u(t)*h(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

$$= \int_{-\infty}^{t} e^{-a\tau}u(\tau)d\tau = \int_{0}^{t} e^{-a\tau}d\tau$$

$$= \left.\frac{e^{-a\tau}}{-a}\right|_{0}^{t} = \frac{1}{a}[1 - e^{-at}]u(t)$$

- This result is consistent with earlier analysis

Example: LTI with $h(t) = e^{-at}u(t)$

- For stability

$$\int_{-\infty}^{\infty} \left| e^{-at}u(t) \right| dt = \int_{0}^{\infty} e^{-at} dt$$

$$= \left. \frac{e^{-at}}{-a} \right|_{0}^{\infty} = \frac{1}{a}, a > 0$$

- We must have $a > 0$ for stability
- Note that $a = 0$ result in $h(t) = u(n)$, which is an integrator system, hence an integrator system is not stable

The Fourier series of a periodic continuous-time signal

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$$

Eq. (3.32) is referred to as the ***Synthesis equation, and Eq. (3.33) is referred to as analysis***
equation. The set of coefficient { } k a are often called the Fourier series coefficients of the
spectral coefficients of *x(t) .*
The coefficient 0 *a is the* ***dc or constant component and is given with k = 0 , that is***

$$a_0 = \frac{1}{T}\int_T x(t)dt ,$$

Properties of the Continuous-Time Fourier Series

Let $x(t)$ and $y(t)$ denote two periodic signals with period T and which have Fourier series coefficients denoted by a_k and b_k, that is

$$x(t) \xleftrightarrow{FS} a_k \text{ and } y(t) \xleftrightarrow{FS} b_k,$$

then we have

$$z(t) = Ax(t) + By(t) \xleftrightarrow{FS} c_k = Aa_k + Bb_k. \tag{3.48}$$

3.5.2 Time Shifting

When a time shift to a periodic signal $x(t)$, the period T of the signal is preserved.

If $x(t) \xleftrightarrow{FS} a_k$, then we have

$$x(t - t_0) \xleftrightarrow{FS} e^{-jk\omega_0 t} a_k. \tag{3.49}$$

The magnitudes of its Fourier series coefficients remain unchanged.

3.4.3 Time Reversal

If $x(t) \overset{FS}{\longleftrightarrow} a_k$, then

$$x(-t) \overset{FS}{\longleftrightarrow} a_{-k}. \quad (3.50)$$

Time reversal applied to a continuous-time signal results in a time reversal of the corresponding sequence of Fourier series coefficients.

If $x(t)$ is even, that is $x(t) = x(-t)$, the Fourier series coefficients are also even, $a_{-k} = a_k$. Similarly, if $x(t)$ is odd, that is $x(-t) = -x(t)$, the Fourier series coefficients are also odd, $a_{-k} = -a_k$.

3.5.4 Time Scaling

If $x(t)$ has the Fourier series representation $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$, then the Fourier series representation of the time-scaled signal $x(\alpha t)$ is

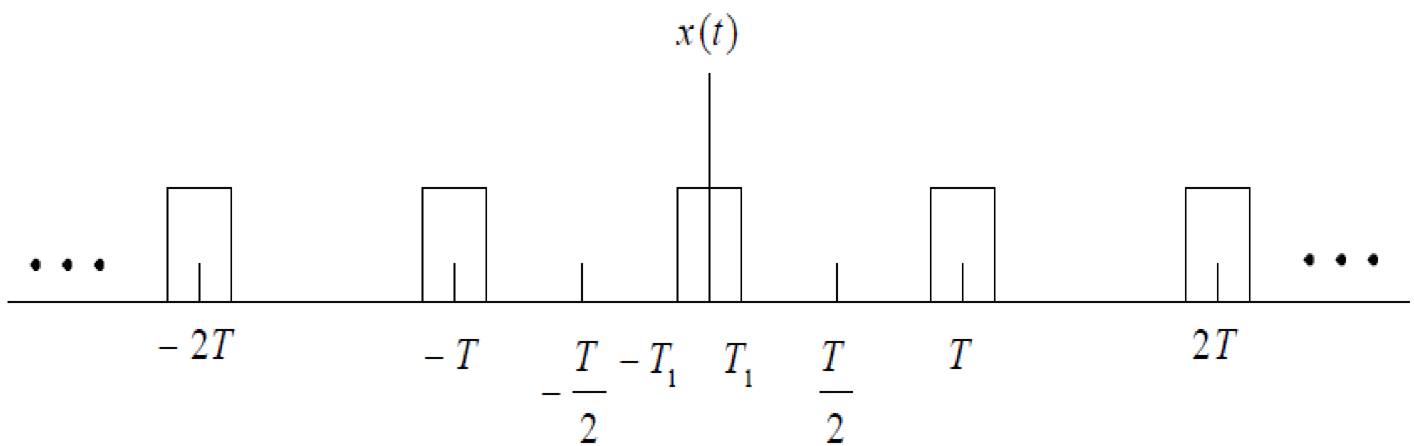
Parseval's Relation for Continuous-Time periodic Signals is

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2 ,$$

Example: The periodic square wave, sketched in the figure below and define over one period is

$$x(t)=\begin{cases}1, & |t|<T_1 \\ 0, & T_1<|t|<T/2\end{cases}, \qquad (3.35)$$

The signal has a fundamental period T and fundamental frequency $\omega_0 = 2\pi / T$.



Example: consider the signal $x(t) = \sin \omega_0 t$.

$$\sin \omega_0 t = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t} .$$

Comparing the right-hand sides of this equation and Eq. (3.32), we have

$$a_1 = \frac{1}{2j}, \qquad a_{-1} = -\frac{1}{2j}$$

$$a_k = 0, \qquad k \neq +1 \ or \ -1$$

To determine the Fourier series coefficients for $x(t)$, we use Eq. (3.33). Because of the symmetry of $x(t)$ about $t = 0$, we choose $-T/2 \leq t \leq T/2$ as the interval over which the integration is performed, although any other interval of length T is valid the thus lead to the same result.

For $k = 0$,

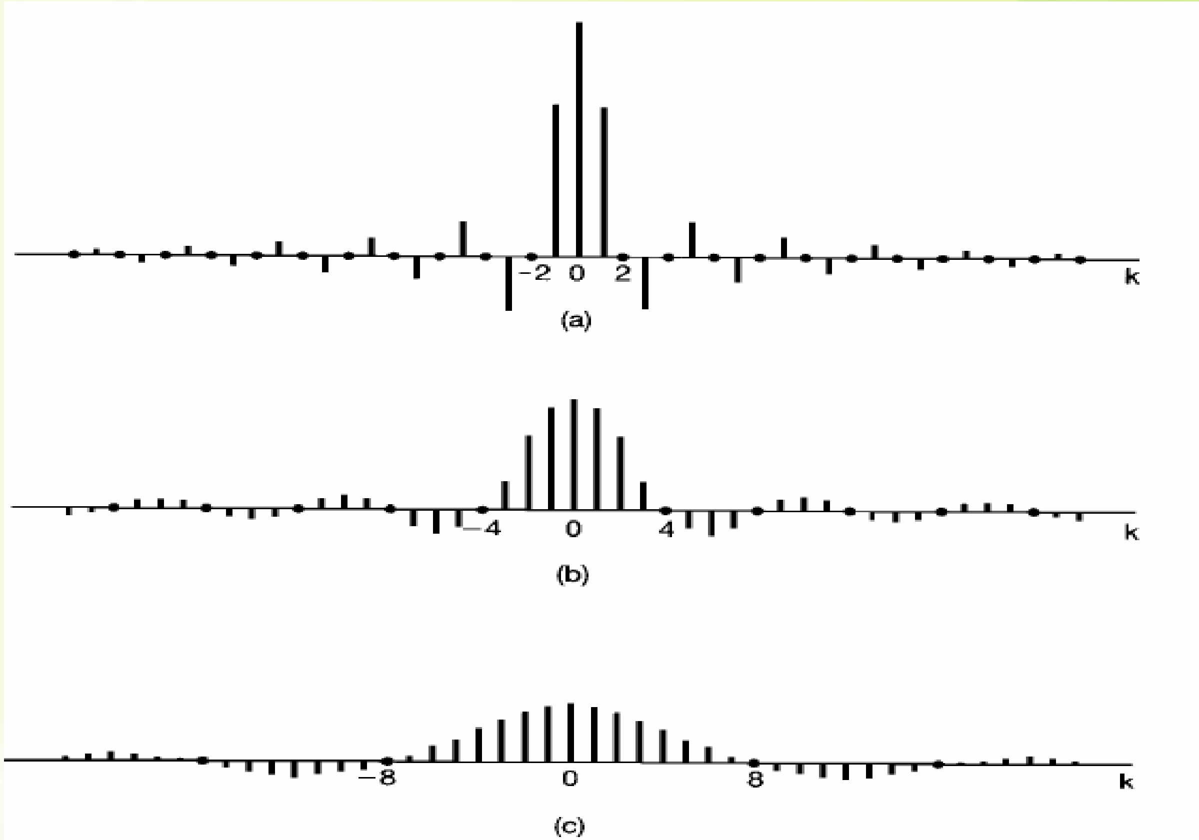
$$a_0 = \frac{1}{T}\int_{-T_1}^{T_1} x(t)dt = \frac{1}{T}\int_{-T_1}^{T_1} dt = \frac{2T_1}{T}, \tag{3.36}$$

For $k \neq 0$, we obtain

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\,dt = -\frac{1}{jk\omega_0 T} e^{-jk\omega_0 t}\Big|_{-T_1}^{T_1}$$

$$= \frac{2}{k\omega_0 T}\left[\frac{e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}}{2j}\right]$$

$$= \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T} = \frac{\sin(k\omega_0 T_1)}{k\pi}$$



The above figure is a bar graph of the Fourier series coefficients for a fixed T_1 and several values of T. For this example, the coefficients are real, so they can be depicted with a single graph. For complex coefficients, two graphs corresponding to the real and imaginary parts or amplitude and phase of each coefficient, would be required.

Queries …?