CBCS Scheme

15EC44

USN 2HN16EC406

Fourth Semester B.E. Degree Examination, June/July 2017

Signals and Systems

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Sketch the even and odd part of the signals shown in Fig. Q1(a) and (b). **(08 Marks)**

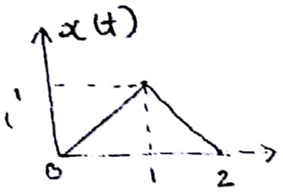

Fig. Q1(a)

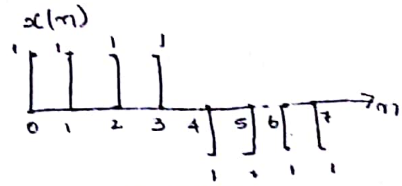

Fig. Q1(b)

b. Determine whether the following signal is periodic or not if periodic find the fundamental period. $x(n) = \cos\left(\frac{n\pi}{5}\right)\sin\left(\frac{n\pi}{3}\right)$ **(03 Marks)**

c. Express x(t) in terms g(t) if x(t) and g(t) are shown in Fig. Q1(c). **(05 Marks)**

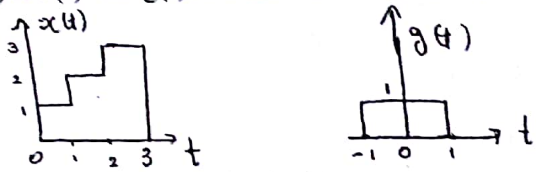

Fig. Q1(c)

OR

2 a. Determine whether the following systems are memory less, causal, time invariant, linear and stable. i) $y(n) = n\,x(n)$ ii) $y(t) = x(t/2)$. **(08 Marks)**

b. For the signal x(t) and y(t) shown in Fig. Q2(b) sketch the following signals.
i) $x(t+1)\cdot y(t-2)$ ii) $x(t)\cdot y(t-1)$ **(08 Marks)**

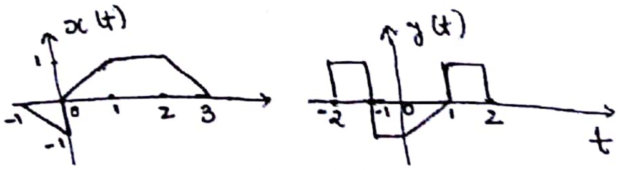

Fig. Q2(b)

Module-2

3 a. Prove the following :

i) $x(n)*[h_1(n)+h_2(n)] = x(n)*h_1(n)+x(n)*h_2(n)$

ii) $x(n)*u(n) = \sum_{k=-\infty}^{n} x(k)$. **(08 Marks)**

b. Compute the convolution sum of $x(n) = u(n) - u(n-8)$ and $h(n) = u(n) - u(n-5)$. **(08 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

4 a. State and prove the associative, integral and commutative properties of convolution. **(08 Marks)**

b. Compute the convolution integral of $x(t) = u(t) - u(t-2)$ and $h(t) = e^{-t}\,u(t)$. **(08 Marks)**

Module-3

5 a. A system consists of several subsystems connected as shown in Fig. Q5(a). Find the operator H relating x(t) to y(t) for the following sub system operators. **(04 Marks)**

$H_1 : y_1(t) = x_1(t)\,x_1(t-1)$

$H_2 : y_2(t) = |x_2(t)|$

$H_3 : y_3(t) = 1+2x_3(t)$

$H_4 : y_4(t) = \cos(x_4(t))$

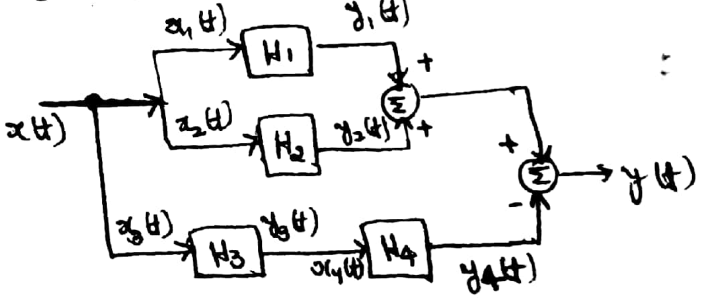

Fig. Q5(a)

b. Determine whether the following systems defined by their impulse responses are causal, memory less and stable.

i) $h(t) = e^{-2t}\,u(t-1)$ ii) $h(n) = 2u[n] - 2u(n-5)$ **(06 Marks)**

c. Evaluate the step response for the LTI systems represented by the following impulse responses. i) $h(t) = u(t+1) - u(t-1)$ ii) $h(n) = \left(\frac{1}{2}\right)^n u(n)$. **(06 Marks)**

OR

6 a. State the following properties of CTFS. i) Time shift ii) Differentiation in time domain iii) Linearity iv) Convolution v) Frequency shift vi) Scaling. **(06 Marks)**

b. Determine the DTFS coefficients for the signal shown in Fig.Q6 (b) and also plot $|x(k)|$ and $\arg\{x(k)\}$. **(10 Marks)**

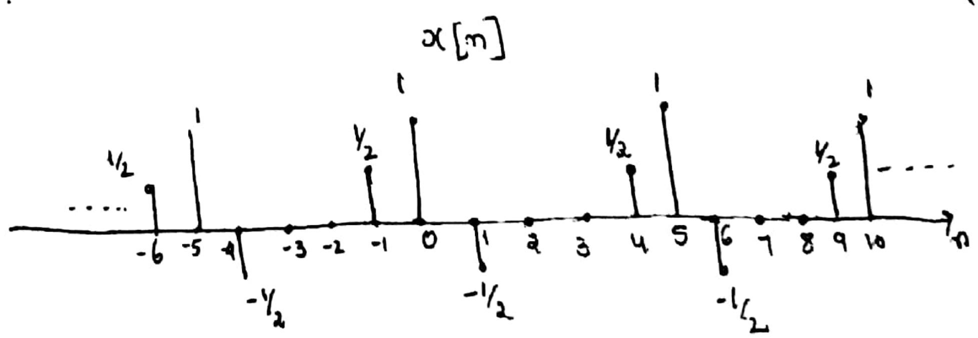

Fig. Q6(b)

Module-4

7 a. State and prove the following properties :

i) $y(t) = h(t) * x(t) \xleftrightarrow{FT} y(jw) = x(jw)H(jw)$

ii) $\frac{d}{dt}x(t) \xleftrightarrow{FT} jw\,x(jw)$ **(06 Marks)**

b. Find DTFT of the following signals.

i) $x(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$ ii) $x(n) = \left(\frac{3}{4}\right)^n u[n]$. **(10 Marks)**

OR

8 a. Specify the Nyquist rate for the following signals
i) $x_1(t) = \sin(200\pi t)$ ii) $x_2(t) = \sin(200\pi t) + \cos(400\pi t)$. **(04 Marks)**

b. Use partial fraction expansion to determine the time domain signals corresponding to the following FTs.

i) $x(jw) = \dfrac{-jw}{(jw)^2 + 3jw + 2}$

ii) $x(jw) = \dfrac{jw}{(jw+2)^2}$ **(08 Marks)**

c. Find FT of the signal $x(t) = e^{-2t} u(t-3)$. **(04 Marks)**

Module-5

9 a. Explain properties of ROC with example. **(06 Marks)**

b. Determine the z-transform of the following signals

i) $x(n) = \left(\frac{1}{4}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)$

ii) $x(n) = n\left(\frac{1}{2}\right)^n u(n)$ **(10 Marks)**

OR

10 a. Find the time domain signals corresponding to the following z-transforms.

$$x(z) = \frac{\left(\frac{1}{4}\right)z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$ with ROC $\frac{1}{4} < |z| < \frac{1}{2}$. **(06 Marks)**

b. Determine the transfer function and the impulse response for the causal LTI system described by the difference equation

$$y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y(n-2) = -x(n) + 2x(n-1)$$ **(10 Marks)**

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Fourth Semester B.E Degree Examination, ~~Dec~~ June-2017

Signals and Systems

Module 1.

1. a. Sketch the even and odd part of the signals shown in fig (a) & (b). (08M)

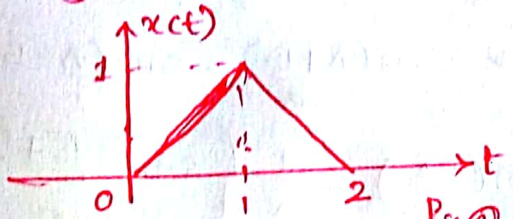


Fig (a).

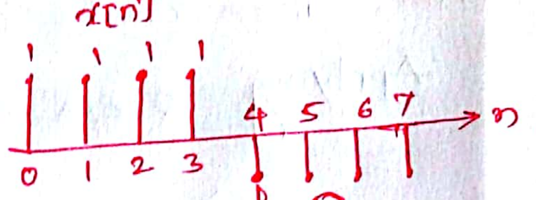
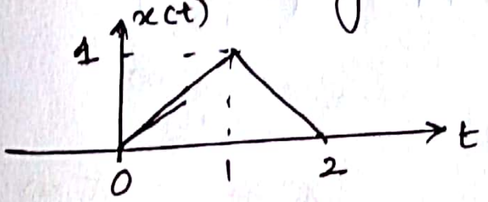
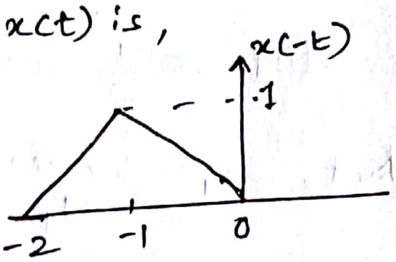


fig (b).

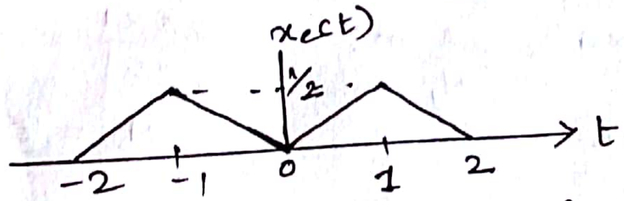
Ans: (a). The given signal $x(t)$ is,



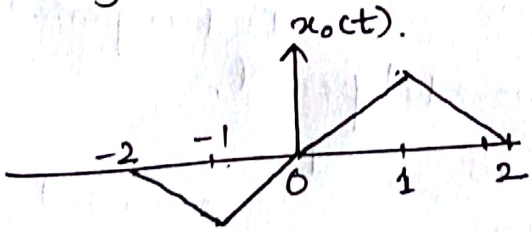
$\Rightarrow$



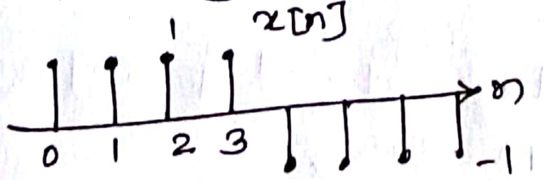
Even signal $x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$.



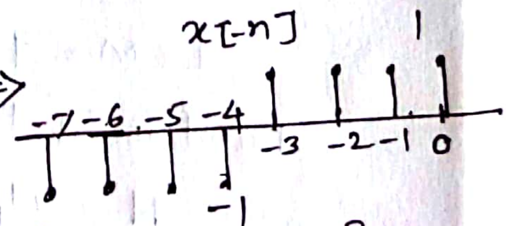
odd signal is $x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$.



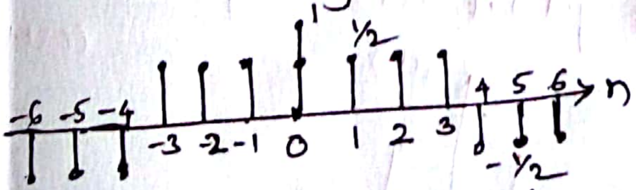
(b). Given signal is



$\Rightarrow$

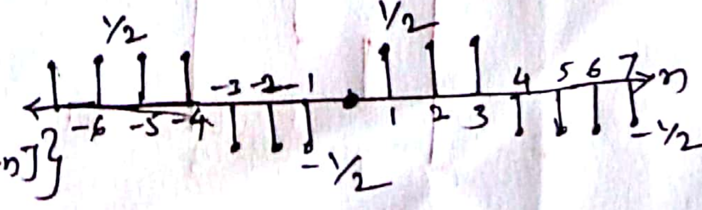


The even signal is $x_e[n] = \frac{1}{2}\{x[n] + x[-n]\}$



& odd signal

$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\}$



1. b. Determine whether the following signal is periodic or not? if periodic find the fundamental period. (03M).

$$x[n] = \cos\left(\frac{n\pi}{5}\right)\sin\left(\frac{n\pi}{3}\right)$$

Ans: Given $x[n] = \cos\left(\frac{n\pi}{5}\right) \cdot \sin\left(\frac{n\pi}{3}\right)$.

Applying $2\cos A \sin B \rightarrow \sin(A+B) - \sin(A-B)$.

$$\Rightarrow \cos\frac{n\pi}{5} \cdot \sin\left(\frac{n\pi}{3}\right) = \frac{1}{2}\left\{\sin\left(\frac{n\pi}{5}+\frac{n\pi}{3}\right) - \sin\left(\frac{n\pi}{5}-\frac{n\pi}{3}\right)\right\}$$

$$x[n] = \frac{1}{2}\sin\left(\frac{8n\pi}{15}\right) + \sin\left(\frac{2n\pi}{15}\right)$$

Let, $\Omega_1 = \frac{2\pi m}{N_1}$ $\quad\quad$ $\Omega_2 = \frac{2\pi m}{N_2}$

$\Rightarrow \frac{8\pi}{15} = \frac{2\pi m}{N_1}$ $\quad\quad$ $\frac{2\pi}{15} = \frac{2\pi m}{N_2}$

$\Rightarrow \frac{2\pi \times 4}{15} = \frac{2\pi m}{N_1}$. $\quad\quad$ $\frac{2\pi \times 1}{15} = \frac{2\pi m}{N_2}$

$N_1 = 15$ $\quad\quad$ $N_2 = 15$.

Step 1: $\frac{N_1}{N_2} = \frac{15}{15} = 1$ rational no. hence the signal $x[n]$ is periodic

To find it's fundamental period,

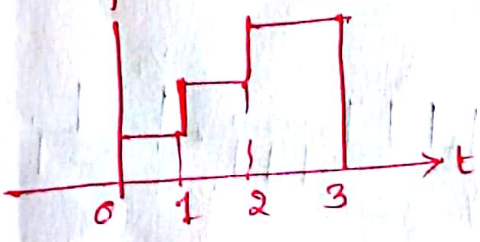
$\frac{N_1}{N_2} \rightarrow \frac{15}{15} = 1$ $\quad$ no. need of gcd.

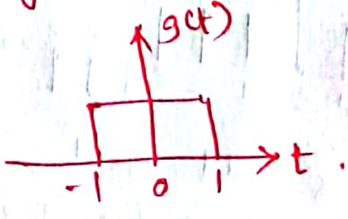
LCM = 1.

∴ Fundamental time interval of $x[n]$ is,

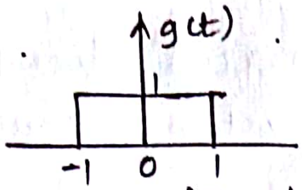
$$N = N_1(1) = 15$$

1. c. Express $x(t)$ in terms of $g(t)$ if $x(t)$ and $g(t)$ are shown in fig. (05M)



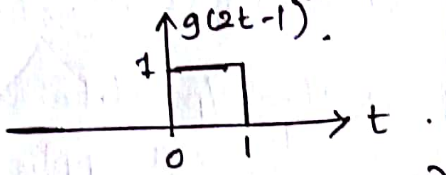


c. Ans. Given.

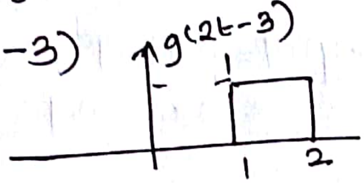


When we operate shifting and scaling on g(t)

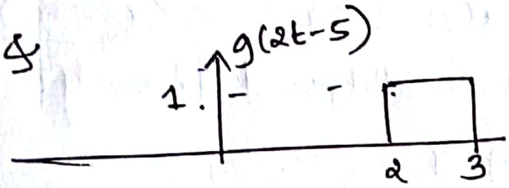
g(2t−1) the signal becomes.



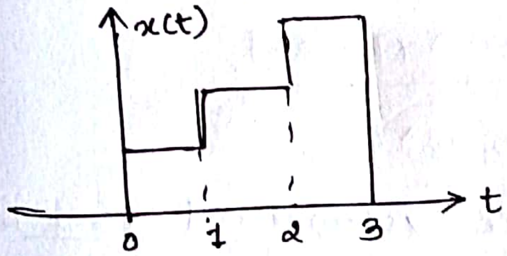
When g(2t−3)



&



To obtain



We need

$$x(t) = g(2t-1) + 2g(2t-3) + 3g(2t-5)$$

Q.2.a. Determine whether the following systems are memoryless, Causal, time invariant, linear and stable (08M)

i> $y(n) = n\,x(n)$ ii> $y(t) = x(t/2)$

Ans: The system is $y(n) = n\,x[n]$.

$x[n] \longrightarrow \boxed{H\{\cdot\}} \longrightarrow y[n] = H\{x[n]\}$

system function is multiplication by n. to the applied i/p.

i> Memoryless: when y[n] depends only on present value of x[n]. As it doesn't depend on past or future values of the i/p it is memoryless.

$y[n] = n\,x[n]$

↑ present value of i/p

ii) Causal : As $y[n] = n x[n]$ doesn't depend on future values the system is causal //

iii) Time invariant : system is said to be time invariant, if $y[n-n_0] = H\{x(n-n_0)\}$.

Now

$H\{x(n-n_0)\}$ is the o/p obtained when $x[n-n_0]$ is applied to the system.

$$x[n] \to \boxed{n} \to y[n] = n x[n]$$

$x[n-n_0]$ $\quad = n x[n-n_0]$

& it simply multiplies the i/p by $\cancel{x}$ n.

$$\therefore H\{x(n-n_0)\} = n\,x(n-n_0).$$

& the o/p $y(n-n_0)$ is obtained from $y[n]$.

$$y[n] = n x[n].$$

put $n = n-n_0$ in above eqn.

$$y[n-n_0] = (n-n_0)\,x[n-n_0].$$

& $y[n-n_0] \neq H\{x(n-n_0)\}$.

hence system is not time invariant.

iv) Linear : System is Linear if it satisfies the superposition principle

$$H\{a x_1(n) + b x_2(n) \ldots\} = a y_1(n) + b y_2(n) + \ldots$$

Now, $H\{a x(n) + b x_2(n)\} = ?$

$$\underbrace{\{a x_1(n) + b x_2(n)\}}_{z(n)} \to \boxed{n} \to n\,\underbrace{\{a x_1(n) + b x_2(n)\}}_{z(n)}.$$

$$= a n x_1(n) + b n x_2(n).$$

&

$$a y_1(n) + b y_2(n) = a x_1(n) n + b n x_2(n).$$

As $H\{a x_1(n) + b x_2(n)\} = a y_1(n) + b y_2(n)$

System is linear //

ii) $y(t) = x(t/2)$. $\Rightarrow$ $x(t) \rightarrow \boxed{T\{\cdot\}} \rightarrow y(t) = x(t/2)$
time scaling.

i) <u>memoryless</u> : As the system is $y(t) = x(t/2)$

When $t = 2$; $y(t) = x(1)$ previous value.

& When $t = -1$; $y(t) = x(-1/2)$ future value.

$\therefore$ The system depends on past and future values of the i/p when $t \neq 0$. Hence system is <u>not memoryless</u>.

ii) <u>Causal</u> : When $t < 0$; the o/p of the system depends on future values of the i/p.

Hence system is <u>non Causal</u> for $t < 0$.

But when $t \geq 0$

The o/p of the system depends on present/past values of the i/p hence <u>Causal</u>.

Causal for $t \geq 0$
non causal for $t < 0$.

iii) <u>Time invariant</u> : When $T\{x(t-t_0)\} = y(t-t_0)$.
system will be time invariant.

Now. $T\{x(t-t_0)\}$ is

$$x(t-t_0) \rightarrow \boxed{T\{\cdot\}} \rightarrow x(t/2 - t_0).$$

& $y(t-t_0) = ?$

$$y(t) = x(t/2)$$

$t = t - t_0$

$$y(t-t_0) = x\left(\frac{t-t_0}{2}\right).$$

as $x(t/2 - t_0) \neq x\left(\frac{(t-t_0)}{2}\right)$

System is not time invariant

ix) Linear : System $y(t) = x(t/2)$ is linear

$T\{ax_1(t)+bx_2(t)\} = ay_1(t) + by_2(t)$

$T\{ax_1(t)+bx_2(t)\} = ?$

$\{a x_1(t) + b x_2(t)\} \longrightarrow \boxed{T\{\cdot\}} \longrightarrow a x_1(t/2) + b x_2(t/2)$.

& $ay_1(t) + by_2(t) = a x_1(t/2) + b x_2(t/2)$

as $T\{ax_1(t)+bx_2(t)\} = ay_1(t)+by_2(t)$ //

System is linear.

Q2 b. For the signal $x(t)$ and $y(t)$ shown in Fig. $Q_2(b)$, sketch following signals. (08M)

i) $x(t+1)y(t-2)$ ii) $x(t).y(t-1)$.

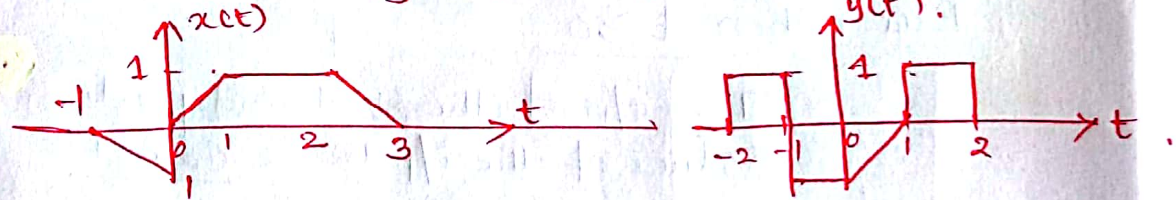
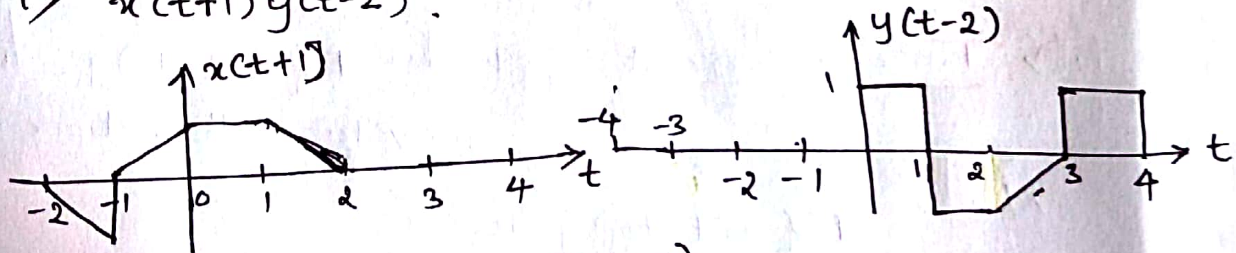
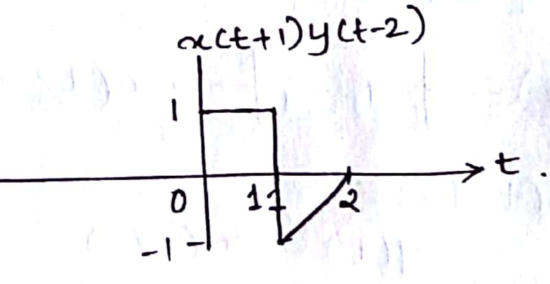


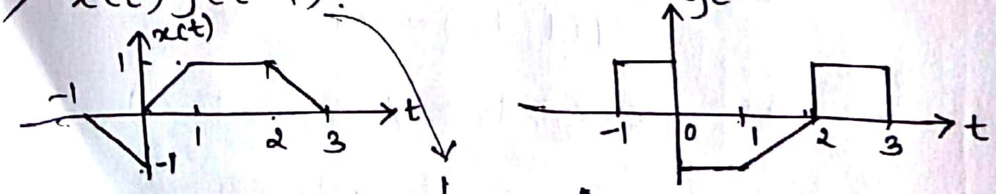
Fig. $Q_2(b)$

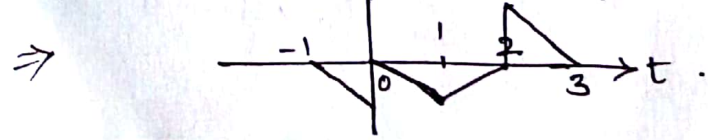
Ans! i) $x(t+1)y(t-2)$.





ii) $x(t)y(t-1)$.





3.a. Prove the following:

i) $x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

ii) $x(n) * u(n) = \sum_{k=-\infty}^{n} x[k]$. (08M)

Ans: i) $x[n] * [h_1(n) * h_2[n]] = x(n) * h_1(n) + x(n) * h_2(n)$.

$x(n) * h_1(n) + x(n) * h_2[n]$. — ②

w.k.t. $x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$\therefore \sum_{k=-\infty}^{\infty} x(k) h_1(n-k) + \sum_{k=-\infty}^{\infty} x(k) h_2(n-k)$

$x(k) \left\{ \sum_{k=-\infty}^{\infty} h_1(n-k) + h_2(n-k) \right\}$

let $h(n-k) = h_1(n-k) + h_2(n-k)$.

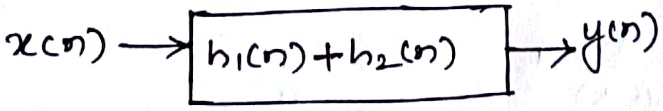
$\Rightarrow \quad x(k) \sum_{k=-\infty}^{\infty} h(n-k)$

$= \quad x(n) * h(n)$.

$= \quad x(n) * [h_1(n) + h_2(n)]$. as $h(n-k) = h_1(n-k) + h_2(n-k)$
$h(n) = h_1(n) + h_2(n)$

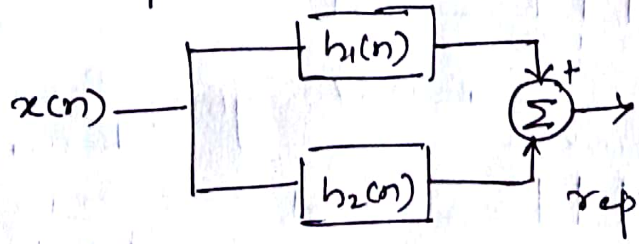
$= x(n) * [h_1(n) + h_2(n)]$.

$\Rightarrow \quad x(n) * h_1(n) + x(n) * h_2(n)$



represents $x(n) * \{h_1(n) + h_2(n)\}$

&



represents $x(n) * h_1(n) + x(n) * h_2(n)$

ii) $x(n) * u(n) = \sum_{k=-\infty}^{\infty} x(k)\, u(n-k)$

we know that, $u(n-k) = 1 \; ; \; n-k \geq 0$
$= 0 \quad n-k < 0$.

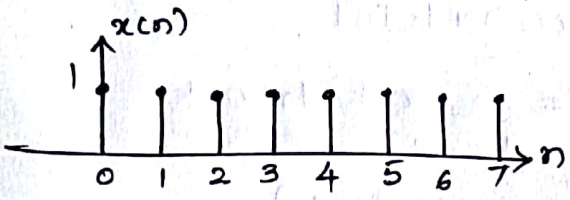
$n-k \geq 0$
$n \geq k$

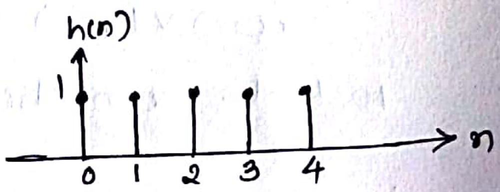
$$x(n) * u(n) = \sum_{k=-\infty}^{n} x[k] \qquad n \geq k$$

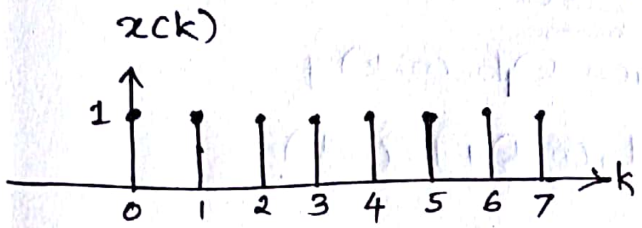
$$= 0 \qquad n < k.$$

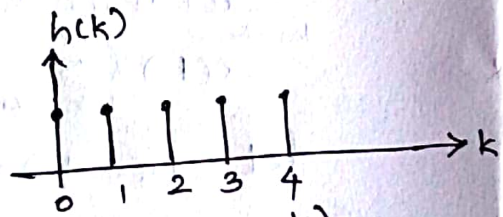
b. Compute the convolution sum of $x(n) = u(n) - u(n-8)$ & $h(n) = u(n) - u(n-5)$ (08M)

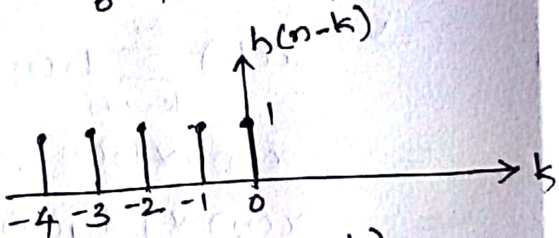
Ans: $x(n) = u(n) - u(n-8)$ $\qquad$ $h(n) = u(n) - u(n-5)$





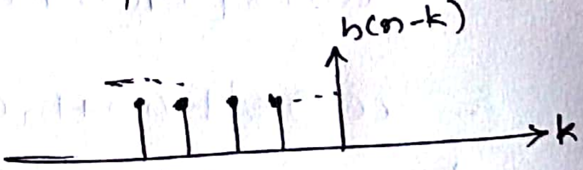






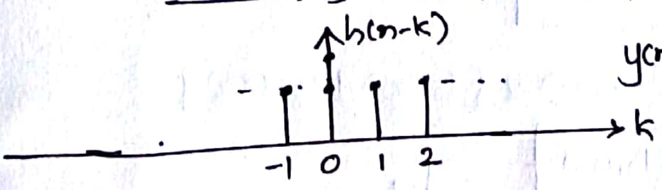
$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

When $n < 0$



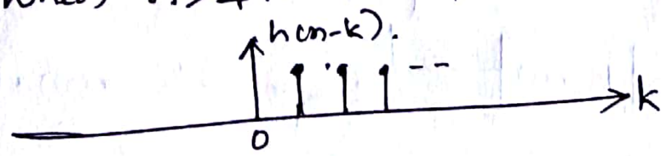
$$\Rightarrow \quad y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) = 0.$$

When $n > 0$; & $n \leq 4$ $\qquad 0 \leq n \leq 4$



$$y(n) = \sum_{k=0}^{n} x(k) h(n-k) = \sum_{k=0}^{n} 1 \cdot 1 = n.$$

When $n > 4$. $\qquad 4 \leq n \leq 7$

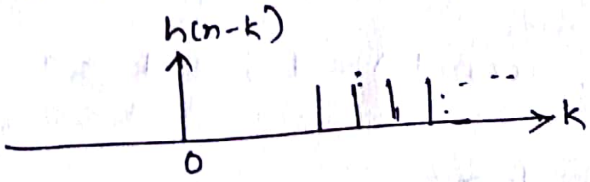


$$y(n) = \sum_{k=n-4}^{n} x(k) h(n-k)$$

$$= \sum_{k=n-4}^{n} 1 = n - n + 4 + 1$$

$$= 5$$

When $n > 7$ $\qquad 7 \leq n \leq 11$



$$y(n) = \sum_{k=n-4}^{7} x(k) h(n-k)$$

$$= \sum_{k=n-4}^{7} 1 = 7 - n + 4 + 1$$

$$= -n + 4 + 1$$

$$= 12 - n$$

4.
Q. State and prove the associative, integral and commutative properties of Convolution (08M)

Ans: Associative property

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t).$$

$$\{x(t) * h_1(t)\} * h_2(t) = x(t) * \{h_1(t) * h_2(t)\}.$$

L.H.S $\rightarrow$ $\{x(t) * h_1(t)\} * h_2(t)$

Let $x(t) * h_1(t) = z(t)$

Now $y(t) = \int_{-\infty}^{\infty} z(\tau) h_2(t-\tau)\, d\tau$

$$z(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\eta) h_1(\tau-\eta) h_2(t-\tau)\, d\tau\, d\eta$$

$$= \int_{-\infty}^{\infty} x(\eta) \int_{-\infty}^{\infty} h_1(\tau-\eta) h_2(t-\tau)\, d\tau\, d\eta$$

put $m = \tau - \eta$
$dm = d\tau$

$$= \int_{-\infty}^{\infty} x(\eta) \int_{-\infty}^{\infty} h_1(m) h_2(t-m-\eta)\, dm\, d\eta.$$

$$= \int_{-\infty}^{\infty} x(\eta) h_1(t-\eta) * h_2(t-\eta)\, d\eta.$$

$$= \int_{-\infty}^{\infty} x(\eta) h(t-\eta)\, d\eta \quad (\because h(t-\eta) = h_1(t-\eta) * h_2(t-\eta)).$$

$$= x(t) * h(t)$$

$$= x(t) * \{h_1(t) * h_2(t)\}.$$

& Commutative property $x(t) * h(t) = h(t) * x(t)$.

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau.$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t).$$

Put $m = t - \tau$
$dm = -d\tau$

$$= -\int_{+\infty}^{\infty} x(t-m) h(m)\, dm$$

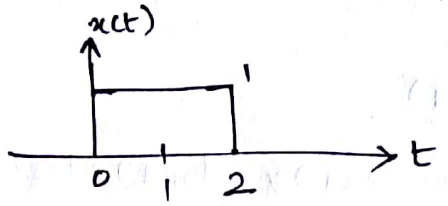
$$= \int_{-\infty}^{\infty} h(m) x(t-m)\, dm = h(t) * x(t)$$

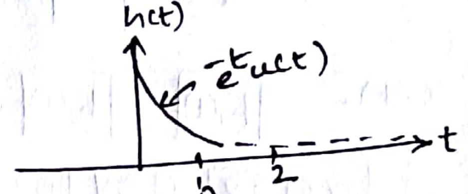
$$= h(t) * x(t).$$

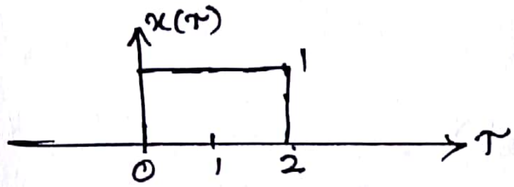
4 b. Compute the Convolution integral of $x(t) = u(t) - u(t-2)$ and $h(t) = e^{-t}u(t)$ (08M).

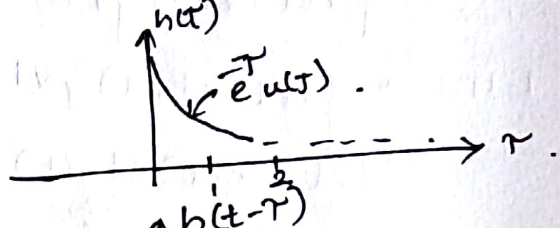
Ans:

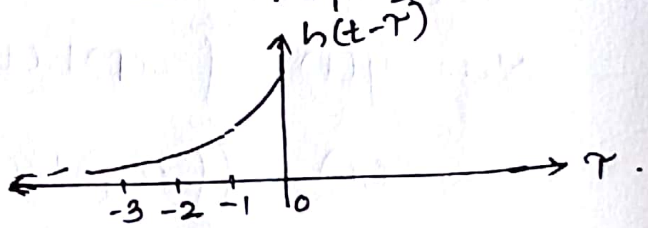
$x(t) = u(t) - u(t-2)$ & $h(t) = e^{-t}u(t)$.











$y(t) = 0$; $t < 0$. $\quad y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$

when $t \geq 0$ $y(t) \neq 0$.

$0 \leq t \leq 2$ $\quad y(t) = \int_0^t x(\tau)h(t-\tau)\,d\tau = \int_0^t 1 \cdot e^{-(t-\tau)}\,d\tau = \int_0^t e^{-(t-\tau)}\,d\tau$

$= e^{-t}\left\{ e^{\tau}\Big|_0^t \right\} = e^{-t}(e^{t} - e^{0}) = 1 - e^{-t}$ //.

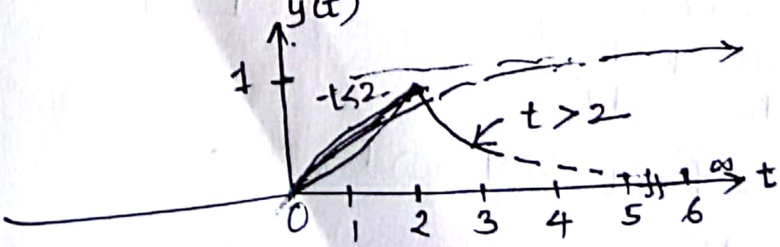
& $t > 2$. $\quad y(t) = \int_0^2 x(\tau)h(t-\tau)\,d\tau = \int_0^2 1 \cdot e^{-(t-\tau)}\,d\tau$

$= \int_0^2 e^{-t} \cdot e^{\tau}\,d\tau = e^{-t} \cdot e^{\tau}\Big|_0^2 = e^{-t} \cdot (e^{2} - e^{0})$

$= e^{-t}(e^{2} - 1)$ //.

The o/p $y(t)$ is,

$$y(t) = \begin{cases} 0 \,; & t < 0 \\ 1 - e^{-t} \,; & 0 \leq t \leq 2 \\ e^{-t}(e^{2} - e^{0}) \,; & t > 2 \end{cases}$$



Module - 3

5a. A system consists of several subsystems connected as shown in fig Q5(a). Find the operation H relating x(t) to y(t) for the following sub system operators. (04M)

$H_1 : y_1(t) = x_1(t)\, x_1(t-1)$

$H_2 : y_2(t) = |x_2(t)|$

$H_3 : y_3(t) = 1 + 2x_3(t)$

$H_4 : y_4(t) = \cos x_4(t)$

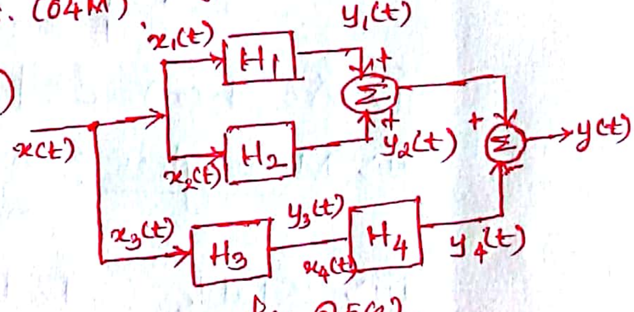


fig. Q5(a).

Ans: For the given fig. Q5(a).

The o/p $y(t) = y_2(t) - y_4(t)$.

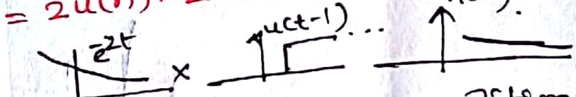
$= y_1(t) + y_2(t) - \cos x_4(t)$

$= x_1(t)\, x_1(t-1) + |x_2(t)| - \cos y_3(t)$

$= x_1(t)\, x_1(t-1) + |x_2(t)| - \cos(1 + 2x_3(t))$ //

b. Determine whether the following systems defined by their impulse responses are causal, memoryless & stable. (06M)

i) $h(t) = e^{-2t} u(t-1)$ ii) $h(n) = 2u(n) - 2u(n-5)$



Ans: i) Given $h(t) = e^{-2t} u(t-1)$

Causal: As $h(t) = 0$; for $t < 0$ satisfies the condition for causality, hence the given system is causal //

Memoryless: System is memoryless ~~as~~ when $h(t) = 0$; for $t \neq 0$.

But given system $h(t) \neq 0$ for $t \neq 0$, hence the given system is not memoryless or it ~~possess~~ has memory.

Stable: The system is said to be stable if,

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau = \int_{-\infty}^{\infty} e^{-2\tau} u(t-1)\, d\tau < \infty.$$

$$\int_{1}^{\infty} e^{-2\tau}\, d\tau = \left.\frac{e^{-2\tau}}{-2}\right|_{\tau=1}^{\infty} = \frac{e^{-0} - e^{-2}}{-2} < \infty$$

$$= \frac{e^{-2} - 1}{2} < \infty$$

hence the system is stable.

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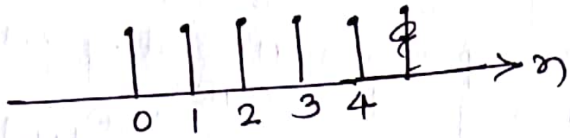
ii) $h(n) = 2u(n) - 2u(n-5)$



i) As. $h(n) = 0$; for $n < 0$ system is Causal.

ii) **Memoryless**: As $h(n) \neq 0$ for $n \neq 0$; system has **Memory**. not memoryless.

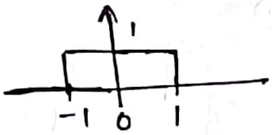
iii) **Stable**: To be stable

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty.$$

$$\sum_{k=0}^{4} |h(k)| = 1+1+1+1+1 = 5 < \infty.$$

c. Evaluate the step response for the LTI systems represented by the following o/p responses i) $h(t) = u(t+1) - u(t-1)$ (06M)
ii) $h(n) = (1/2)^n u(n)$

Ans: i) $h(t) = u(t+1) - u(t-1)$



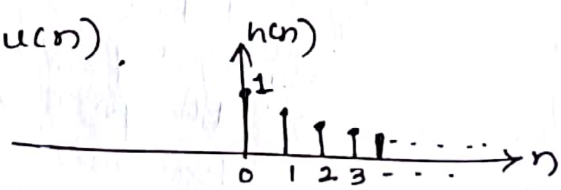
step response is given by.

$$s(t) = \int_{-\infty}^{\infty} h(\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, d\tau$$

$$= \int_{-1}^{1} d\tau = \tau \Big|_{-1}^{1} = 1+1 = 2$$

Step response $s(t) = 2$

ii) $h(n) = (1/2)^n u(n)$



$$s(n) = \sum_{k=-\infty}^{\infty} h(k)$$

$$= \sum_{k=-\infty}^{n} (1/2)^n = \sum_{k=0}^{n} (1/2)^n = \frac{1-(1/2)^{n+1}}{1-(1/2)} = 2(1-(1/2)^{n+1})$$

$$= \cancel{\tfrac{1}{2} - (1/2)^{n+2}} \qquad = 2 - (1/2)^n$$

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Module-3

6.
a. State the following properties of CTFS
 i) Time shift   ii) Differentiation in time domain
 iii) Linearity   iv) Convolution   v) Frequency shift
 vi) Scaling.   (06M).

Ans. CTFS properties.

i) Time shift : $x(t-t_0) \xleftrightarrow{FS;\omega_0} e^{-jk\omega_0 t_0} X(k)$

ii) Diff^n in time domain : $\frac{d}{dt}x(t) \xleftrightarrow{FS;\omega_0} jk\omega_0 X(k)$

iii) Linearity : $ax(t)+by(t) \xleftrightarrow{FS,\omega_0} aX(k)+bY(k)$.

iv) Convolution : $x(t)*y(t) \xleftrightarrow{''} TX(k)Y(k)$

v) Freq. shift : $e^{jk_0\omega_0 t}x(t) \xleftrightarrow{''} X(k-k_0)$

vi) Scaling : $x(at) \longleftrightarrow X(k)$

b. Determine the DTFS coefficients for the signal shown in fig Q6(b) and also plot $|X(k)|$ and $\arg\{X(k)\}$.   (10M).

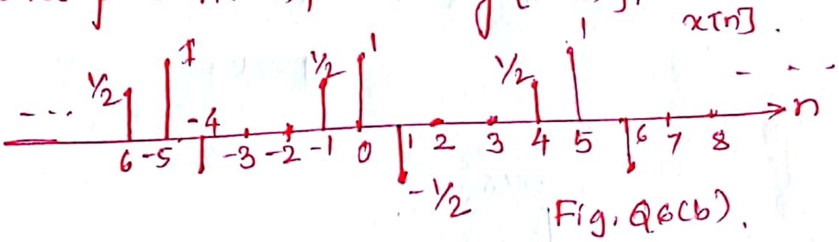


Fig. Q6(b).

Ans: For the given signal $N=5$, $\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{5}$.

$$X(k) = \frac{1}{N}\sum_{k=0}^{4} x(n)e^{-jk\omega_0 n}$$

$$= \frac{1}{5}\sum_{k=0}^{4} x(n)e^{-jk\frac{2\pi}{5}n}$$

$$X(k) = \frac{1}{5}\left\{x(0)e^{-jk\frac{2\pi}{5}\times 0} + x(1)e^{-jk\frac{2\pi}{5}\cdot 1} + x(2)e^{-jk\frac{2\pi}{5}\times 2} + x(3)e^{-jk\frac{2\pi}{5}\times 3} + x(4)e^{-jk\frac{2\pi}{5}\times 4}\right\}$$

$$= \frac{1}{5}\left\{1 + -\frac{1}{2}e^{-j\frac{2\pi}{5}} + 0 + 0 + \frac{1}{2}e^{-jk\frac{8\pi}{5}}\right\}$$

$$= \frac{1}{5}\left\{1 + \frac{1}{2}e^{-jk(2\pi-\frac{2\pi}{5})} - \frac{1}{2}e^{-j\frac{2\pi k}{5}}\right\}$$

$$= \frac{1}{5}\left\{1 + \frac{1}{2}\left\{e^{+jk2\pi/5} - e^{-jk2\pi/5}\right\}\right\}$$

$$= \frac{1}{5}\left\{1 + j\sin\frac{2\pi k}{5}\right\}$$

$$X(k) = \frac{1}{5}\left\{1 + j\sin\left(\frac{2\pi k}{5}\right)\right\}.$$

$k=0;$ $\quad X(0) = \frac{1}{5} \Rightarrow |X(0)| = 0.2 \;;\; \arg\{X(0)\} = 0.$

$k=1$ $\quad X(1) = \frac{1}{5}\left\{1 + j\sin\frac{2\pi}{5}\right\} = 0.275\angle 43.53^\circ.$

$= 0.276\angle 0.76$ rad.

$k=2$ $\quad X(2) = \frac{1}{5}\left\{1 + j\sin\frac{4\pi}{5}\right\} = \frac{1}{5}\{1 + j0.587\}$

$0.232\angle -0.53$

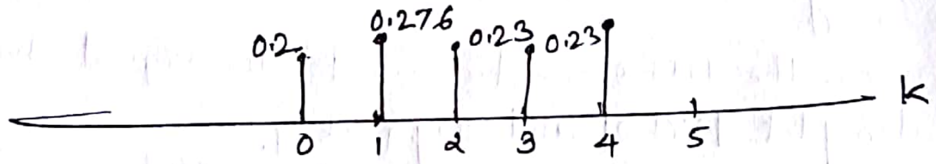
$k=3$ $\quad X(3) = \frac{1}{5}\left\{1 + j\sin\frac{6\pi}{5}\right\} = \frac{0.2 + j(-0.587)}{5}$

$= 0.2 - j0.117 = 0.231\angle -30.3 = 0.231\angle -0.5$

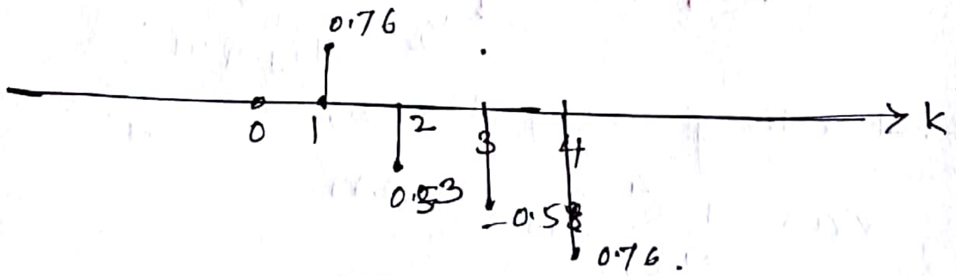
$k=4$ $\quad X(4) = \frac{1}{5}\left\{1 + j\sin\frac{8\pi}{5}\right\} = 0.2 + j(-0.19).$

$= \frac{1}{5}\{0.2 - j0.19\} = 0.276\angle -0.76$ rad.

$|X(k)|$



$\measuredangle X(k)$



7.a. State and prove the following properties: (06M)

Ans: i) $y(t) = h(t) * x(t) \xleftrightarrow{FT} Y(j\omega) = X(j\omega)H(j\omega)$

ii) $\frac{d\,x(t)}{dt} \xleftrightarrow{FT} j\omega X(j\omega)$

i) $$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \{h(t) * x(t)\} e^{-j\omega t}\, dt.$$

$$= \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \right\} e^{-j\omega t}\, dt$$

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Changing the order of the integration

$$Y(j\omega) = \int_{-\infty}^{\infty} h(\tau) \left\{ \int_{-\infty}^{\infty} x(t-\tau) e^{-j\omega t} dt \right\} d\tau$$

put $t-\tau = a$;

$dt = da$.

$$Y(j\omega) = \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} x(a) e^{-j\omega(a+\tau)} da\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \int_{-\infty}^{\infty} x(a) e^{-j\omega a}$$

$$Y(j\omega) = \underline{H(j\omega) X(j\omega)} \;//$$ by the definition.

ii) $\frac{d}{dt} x(t) \xleftrightarrow{FT} j\omega X(j\omega)$.

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

differentiating both the sides w.r.t t we get,

$$\frac{d\,x(t)}{dt} = \frac{d}{dt} \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \right\}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega X(j\omega) e^{j\omega t} d\omega$$

$$\frac{d}{dt} x(t) = j\omega \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \right\}$$

$$= j\omega X(j\omega) \;//$$

7.b. Find DTFT of the following signals. (10M)

i) $x(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$ ii) $x(n) = \left(\frac{3}{4}\right)^n u(n)$

Ans: DTFT of the given signals is,

i) $x(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

$$= \sum_{n=-2}^{2} x(n) e^{-j\Omega n}$$

$$= x(-2) e^{j2\Omega} + x(-1) e^{j\Omega} + 3 + 2e^{-j1\Omega} + x(2) e^{-j\Omega \times 2}$$

$$= e^{j2\Omega} + 2e^{j\Omega} + 3 + 2e^{-j1\Omega} + e^{-j2\Omega}$$

$$= 3 + 4\cos\Omega + 2\cos 2\Omega \;//$$

ii) $x(n) = (3/4)^n u(n)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} = \sum_{n=0}^{\infty} (3/4)^n (e^{-j\Omega})^n$$

$$= \frac{1}{1-3/4\, e^{-j\Omega}} = \cancel{\frac{4}{4-3} = 4}$$

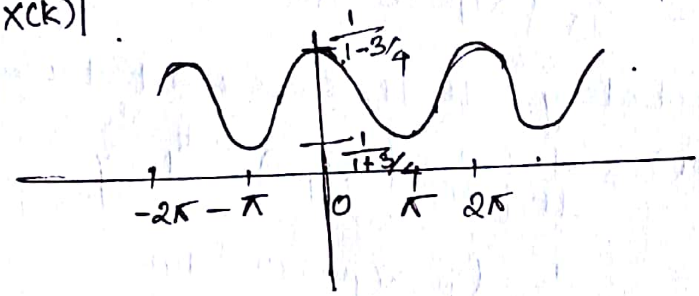
$$= \frac{1}{1-3/4\, e^{-j\Omega}} = \frac{1}{1-3/4\{\cos\Omega - j\sin\Omega\}}$$

$$|X(e^{j\Omega})| = \frac{1}{\{(1-3/4\cos\Omega)^2 + (3/4\sin\Omega)^2\}^{1/2}}$$

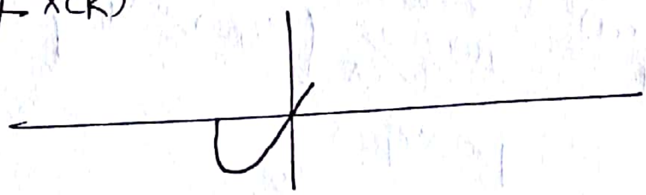
$$\measuredangle(X(e^{j\Omega})) = -\tan^{-1}\left\{\frac{3/4\sin\Omega}{1-3/4\cos\Omega}\right\}$$

$$= -\tan^{-1}\left\{\frac{3/4\sin\Omega}{1-3/4\cos\Omega}\right\}$$

$|X(k)|$



$\measuredangle X(k)$



8a. Specify the Nyquist rate for the following signals (04M)

i) $x_1(t) = \sin(200\pi t)$ ii) $x_2(t) = \sin(200\pi t) + \cos 400\pi t$.

Ans: i) $x_1(t) = \sin 200\pi t$.

Comparing with $\sin 2\pi f t$.

$\Rightarrow f = 100\,Hz$, $\therefore$ Nyquist rate $f_s = 2f = 200\,Hz$

ii) $x_2(t) = \sin 200\pi t + \cos 400\pi t$

Among the given signal $f_1 = 100\,Hz$ & $f_2 = 200\,Hz$.

$\Rightarrow$ Nyquist rate $= 2 \times$ highest freq. of i/p signal.

$= 2 \times 200\,Hz = 400\,Hz$

8. b. Use partial fraction expansion to determine the time domain signals corresponding to the following FTs. (08M)

i) $X(j\omega) = \dfrac{-j\omega}{(j\omega)^2 + 3j\omega + 2}$

ii) $X(j\omega) = \dfrac{j\omega}{(j\omega+2)^2}$

Ans i) $X(j\omega) = \dfrac{-j\omega}{(j\omega)^2 + 3j\omega + 2}$

As DR power is more than NR. directly we can apply partial fractions. we get,

$$X(j\omega) = \frac{-j\omega}{(j\omega+1)(j\omega+2)}$$

$$\frac{-j\omega}{(j\omega+1)(j\omega+2)} = \frac{A}{(j\omega+1)} + \frac{B}{(j\omega+2)}$$

Solving for A & B,

$$= \frac{1}{(j\omega+1)} - \frac{2}{(j\omega+2)}$$

$$x(t) = e^{-t}u(t) - 2e^{-2t}u(t).$$

$$x(t) = \underline{(e^{-t} - 2e^{-2t})u(t)}$$ //

ii) $X(j\omega) = \dfrac{j\omega}{(j\omega+2)^2}$

w.k.t. FT of $e^{-2t}u(t) \xrightarrow{FT} \dfrac{1}{2+j\omega}$

by freq. differentiation property we get,

$$-jte^{-2t}u(t) \xrightarrow{FT} \frac{d}{d\omega}\left\{\frac{1}{2+j\omega}\right\}$$

$$-jte^{-2t}u(t) \xrightarrow{FT} \frac{(2+j\omega)0 - 1(j)}{(2+j\omega)^2} = \frac{-j}{(2+j\omega)^2}$$

$$\Rightarrow te^{-2t}u(t) \xrightarrow{FT} \frac{1}{(2+j\omega)^2}$$

by time differentiation property,

$$\frac{d}{dt}\{te^{-2t}u(t)\} = j\omega \frac{1}{(2+j\omega)^2}$$

$$\therefore (te^{-2t}(-2) + e^{-2t})u(t) \Rightarrow x(t) = (1-2t)e^{-2t}u(t)$$ //

8 c. Find the FT of the signal $x(t) = e^{-2t} u(t-3)$ (04M)

Ans: $x(t) = e^{-2t} u(t-3)$

$$X(j\omega) = \int_0^\infty \cancel{e^{-at}} x(t) e^{-j\omega t} dt$$

$$= \int_0^\infty e^{-2t} u(t-3) e^{-j\omega t} dt$$

$$= \int_3^\infty e^{-2t} e^{-j\omega t} dt = \int_3^\infty e^{-(2+j\omega)t} dt$$

~~put x~~ $$X(j\omega) = \frac{e^{-(2+j\omega)t}}{-(+2+j\omega)}\Bigg|_{t=3}^{\infty} = \frac{e^{-\infty} - e^{-(2+j\omega)3}}{-(2+j\omega)}$$

$$X(j\omega) = \frac{e^{-(6+j3\omega)}}{(2+j\omega)}$$

$$|X(j\omega)| = \frac{e^{-6}}{\sqrt{4+\omega^2}}$$

& $\angle X(j\omega) = -3\omega - \tan^{-1}(\omega/2)$.

9. Q. Explain properties of ROC with example. (06M)

Ans: Properties of Region of Convergence.

1. The ROC of $X(z)$ consist of a Ring in the z-plane centered about the origin.
2. The ROC doesn't contain any poles
3. If $x(n)$ is of finite duration, the the ROC is the entire z-plane except possibly $|z|=0$ & or $|z|=\infty$.
4. If $x(n)$ is right sided sequence, then the ROC is the ~~entire~~ z-plane outside the outermost pole. ie
5. If $x(n)$ is left sided sequence, the ROC is the region in z-plane inside the innermost pole.
6. If $x(n)$ is two sided sequence, then ROC is concentric ring in the z-plane.
7. If $x(z)$ is rational, then it's ROC is bounded by poles or extends to ∞.

9 b. Determine the z-transform of the following signals (10 M)

i) $x(n) = (1/4)^n u(n) - (1/2)^n u(-n-1)$

ii) $x(n) = n(1/2)^n u(n)$

Ans z-transform of

i) $x(n) = (1/4)^n u(n) - (1/2)^n u(-n-1)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \{ (1/4)^n u(n) - (1/2)^n u(-n-1) \} z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (1/4)^n u(n) z^{-n} - \sum_{n=-\infty}^{\infty} (1/2)^n u(-n-1) z^{-n}$$

$$= \sum_{n=0}^{\infty} (1/4)^n z^{-n} - \sum_{n=-\infty}^{-1} (1/2)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (1/4 z^{-1})^n - \sum_{n=1}^{\infty} ((1/2 z^{-1})^{-1})^n = \sum_{n=0}^{\infty} (1/4 z^{-1})^n - \sum_{n=1}^{\infty} ((1/2)^{-1} z)^n$$

$$= \sum_{n=0}^{\infty} (1/4 z^{-1})^n - \sum_{n=1}^{\infty} (2z) = \frac{1}{1 - \frac{1}{4} z^{-1}} - \frac{2z}{1-2z}$$

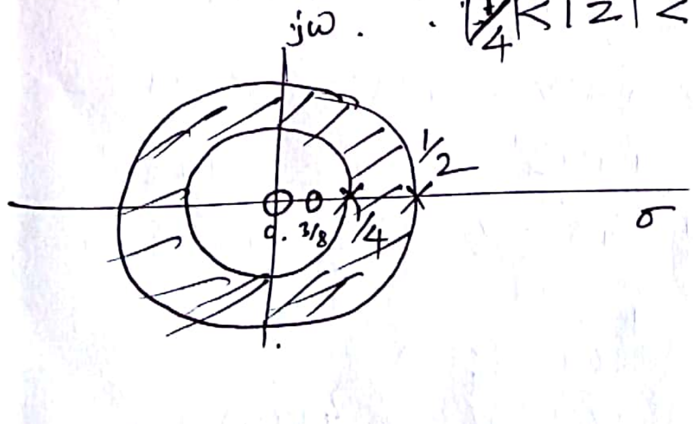
$$= \frac{z}{z - 1/4} + \frac{z}{(z - 1/2)} = \frac{z^2 - 1/2 z + z^2 - 1/4 z}{(z-1/2)(z-1/4)}$$

$$= \frac{2z^2 - 3/4 z}{(z-1/2)(z-1/4)} = \frac{z(2z - 3/4)}{(z-1/2)(z-1/4)}$$

if ROC. $\left|\frac{1}{4} z^{-1}\right| < 1$ & $|2z| < 1$

$\left|\frac{1}{4}\right| < |z|$ $\quad |z| < |1/2|$.

$\left|\frac{1}{4}\right| < |z| < |1/2|$.



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ii) $x(n) = n(\frac{1}{2})^n u(n)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum n x(n) z^{-n}.$$

using diff^n in z-domain property.

$$x(n) = n x(n) \longrightarrow -z\frac{d}{dz}\{X(z)\}.$$

$$x(n) = (\tfrac{1}{2})^n u(n). \xrightarrow{\text{Z Transform}} \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z}{z-\frac{1}{2}}$$

$$\Rightarrow \quad -z\frac{d}{dz}\left\{\frac{z}{z-\frac{1}{2}}\right\} = -z\left\{\frac{\cancel{z}-\frac{1}{2}-\cancel{z}}{(z-\frac{1}{2})^2}\right\} = \frac{\frac{1}{2}z}{(z-\frac{1}{2})^2}$$

10 a. Find the time domain signals corresponding to the following z-transforms. 06M

$$X(z) = \frac{\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} \quad \text{with ROC } \tfrac{1}{4}<|z|<\tfrac{1}{2}.$$

Ans: $X(z) = \dfrac{\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$

$$= \frac{A}{(1-\frac{1}{2}z^{-1})} + \frac{B}{(1-\frac{1}{4}z^{-1})} = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{-1}{1-\frac{1}{4}z^{-1}}$$

$$= \underbrace{\frac{1}{1-\frac{1}{2}z^{-1}}}_{\text{LH}} - \frac{1}{1-\frac{1}{4}z^{-1}} \leftarrow \text{RH},$$

for $\frac{1}{4}<|z|<\frac{1}{2}$

$$= -(\tfrac{1}{2})^n u(-n-1) - (\tfrac{1}{4})^n u(n) \ //.$$

b. Determine the T.F and the impulse response for the causal LTI system described by the difference eq^n. (10M)

$$y(n) - \tfrac{1}{4}y(n-1) - \tfrac{3}{8}y(n-2) = -x(n) + 2x(n-1)$$

Ans: $y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y(n-2) = -x(n) + 2x(n-1)$

$\downarrow$ z-transform

$$Y(z) - \tfrac{1}{4}z^{-1}Y(z) - \tfrac{3}{8}z^{-2}Y(z) = -X(z) + 2z^{-1}X(z).$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-1+2z^{-1}}{1-\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2}} = \frac{-1+2z^{-1}}{(1+\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})}$$

$$H(z) = \frac{-2}{(1+\frac{1}{2}z^{-1})} + \frac{1}{(1-\frac{3}{4}z^{-1})}$$

$$\Rightarrow h(n) = \left\{-2(-\tfrac{1}{2})^n + (\tfrac{3}{4})^n\right\} u(n) \ //.$$