Dr. V.B.K
Angle mod^n

Module 2



Basic Defn :

Let $\theta_i(t)$ be the angle of a modulated sinusoidal carrier at time 't'.

Then $s(t) = A_c \cos(\theta_i(t)) \rightarrow (1)$

$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} \rightarrow (2)$ instantaneous freq

$\theta_i(t) = 2\pi f_c t + \phi_c$ $\phi_c$ is the value when $t=0$

1. Phase mod^n: $\theta_i(t) = 2\pi f_c t + k_p m(t)$

$\Rightarrow s(t) = A_c \cos(2\pi f_c t + k_p m(t))$

2. Freq mod^n $f_i(t) = f_c + k_f m(t)$ — (3)

Where $k_f \rightarrow$ freq sensitivity $\rightarrow$ Hz/volt

Assuming $m(t)$ is voltage waveform.

Integrating (3) w.r.t. 't' & multiplying the result by $2\pi$, we get

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau).d\tau.$$

$\therefore$ $s(t) = A_c \cos\theta_i(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau).d\tau\right)$ is FM.

Properties of Angle modulated waves:

1. Constancy of Txd power $P_{av} = \frac{1}{2}A_c^2$ $R = 1\Omega$
2. Nonlinearity of the modulation process

$m(t) = m_1(t) + m_2(t)$

$s(t) = A_c \cos[2\pi f_c t + k_p(m_1(t) + m_2(t))]$

$s_1(t) = A_c \cos[2\pi f_c t + k_p m_1(t)]$ and

$s_2(t) = A_c \cos(2\pi f_c t + k_p m_2(t))$

$s(t) \neq s_1(t) + s_2(t)$. Similar result holds for FM.

3. Irregularity of zero crossings (nonlinear char. of mod^n)

Infn resides in zero crossings also provided $f_c \gg f_{m,max}$

4. Visualization Difficulty of message waveform can't be distinguished like AM (Envelope)

5. Tradeoff of increased transmission BW for improved noise performance.

FM.

$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

1) Narrow Band FM: $\beta < 1$ rad. 2) Wideband FM $\beta > 1$ rad

$s(t) = A_c \cos 2\pi f_c t \cdot \cos(\beta \sin(2\pi f_m t)) - A_c \sin(2\pi f_c t) \cdot \sin(\beta \sin 2\pi f_m t)$

$\Rightarrow$ Assuming that $\beta$ is small compared to 1 rad

Then $\cos(\beta \sin(2\pi f_m t)] \cong 1$ & $\sin(\beta \sin(2\pi f_m t)] \cong \beta \sin 2\pi f_m t$

$\therefore s(t) \cong A_c \cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \cdot \sin 2\pi f_m t$.

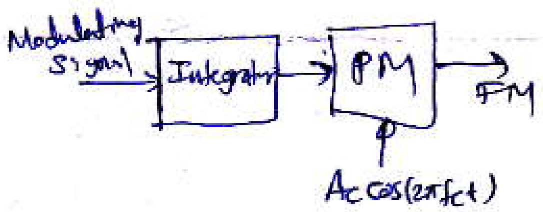
on simplication

$s(t) \cong A_c \cos 2\pi f_c t + \frac{\beta A_c}{2}\left[\cos(2\pi(f_c+f_m)t) - \cos(2\pi(f_c-f_m)t)\right]$ $\rightarrow$ FM (NB)

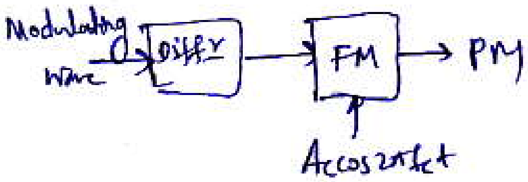
$s(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\left[\cos(2\pi(f_c+f_m)t) + \cos(2\pi(f_c-f_m)t)\right]$ $\rightarrow$ AM.

$\Rightarrow$ Phase noise: The unintensional PM is called phase noise.

Narrow Band FM Gen^y

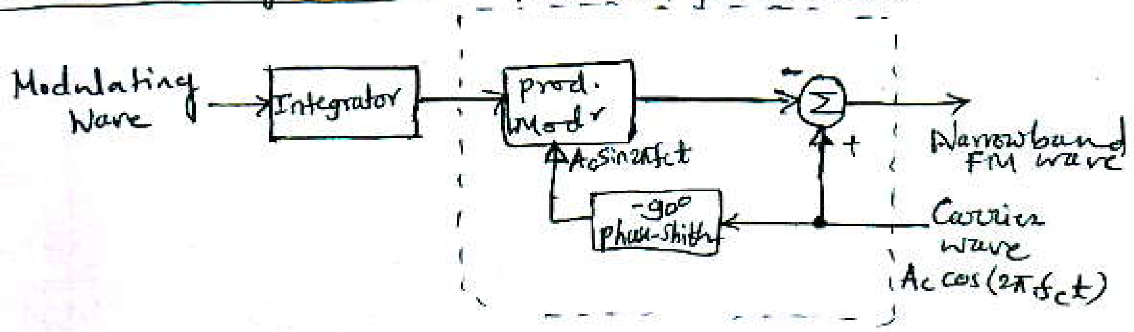



OR




* Introduced by oscillators in bandpass commu^n & has no. of causes
* Some causes are deterministic such as those created by changes by changes in oscillator temp, supply voltage, physical vibration magnetic field, humidity, or o/p load load & impedance.
* The phase noise due to those sources may be minimized by good desing
* Other sources are catogorised as: random which can be controlled but not eliminated by appropriate circuitary such as PLL

## Narrow Band FM Generation:




## Wideband Frequency Modulation:

Rewriting : $s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

→ Assuming fc is large enough compared to BW of FM & & representing complex baseband representation of a modulated signal:

$$s(t) = Re(A_c \exp(j2\pi f_c t + j\beta \sin 2\pi f_m t) - ① \quad \} = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

$$= Re(\tilde{s}(t) \exp(j2\pi f_c t)) - ② \qquad e^{j\theta} = \cos\theta + j\sin\theta$$

where $\tilde{s}(t) = A_c \exp(j\beta \sin(2\pi f_m t)) - ③ \qquad e^x . e^y = e^{x+y}$

$$\tilde{s}(t) = A_c \exp(j\beta \sin(2\pi f_m(t + k/f_m)) - ④$$

Eqn (3) is the complex envelop of the FM wave $s(t)$

The complex envelop $\tilde{s}(t)$ is a periodic function of time with a fundamental freq equal to the modulation freq $f_m$

$\boxed{t \to t + k/f_m \text{ for some integer } k}$

$$\Rightarrow \tilde{s}(t) = A_c \exp(j\beta \sin(2\pi f_m t + 2k\pi))$$

$$= A_c \exp(j\beta \sin 2\pi f_m t)$$

# SAHYADRI
COLLEGE OF ENGINEERING & MANAGEMENT
MANGALURU

UGC 2(f) Recognition & NAAC Accredited with "A" Grade, ISO 9001-2008 recertified

## Dr. Karisiddappa
Hon'ble Vice Chancellor
Visvesvaraya Technological University
Belagavi

**Academic Qualifications**
- B.E. (Civil Engineering) in 1981 from University of Mysore securing 7th Rank.
- M.Tech. (Structural Engineering) in 1986 from IIT, Madras.
- Ph.D. in 1994 from IIT Roorkee (Formerly University of Roorkee, Roorkee).

**Positions held**
- Dr. Karisiddappa is the present Vice Chancellor of Visvesvaraya Technological University, Belagavi, a leading University in Asia and the only technical university in the State.
- He joined as Lecturer at Adichunchanagiri Institute of Technology, Chikmagalur in 1982.
- He served as Assistant Engineer in PWD for very short period in 1982.
- He served the Malnad College of Engineering, Hassan, from 1983 to 2010 for about 27 years in the capacity of Lecturer, Assistant Professor, Professor, Head of the Department, Vice Principal and Dean (Academic Affairs).
- Prior to his appointment as Vice Chancellor of VTU, he served as the Principal of Government Engineering College, Hassan from 2010 to 2016.
- He has served VTU in various capacities. He was the member of Board of Studies, Board of Examiners and also member of various Local Inquiry Committee of VTU.

**Other Academic Activities:**
- He has teaching and research experience of about 34 years in the field of Structural Engineering.
- He has carried out Research Projects sanctioned by AICTE, KSCST and DRDO and completed various Structural Engineering Consultancy Projects.
- He has published 47 Research Papers in refereed International and National Journals, in the area of Concrete Technology, Finite Element Analysis, Neural Networks, Structural Stability and Soil Structure Interaction.
- He has organized, attended, and delivered Lectures in various International and National Conferences.
- He has guided over 34 M.Tech candidates and about 40 batches of B.E. students for their Dissertation. He has also guided 5 candidates for their Ph.D. and 1 candidate for M.Sc.(Engg.) by Research.
- Under his administration the Government Engineering College, Hassan, is recognized as one of the best engineering colleges in the State of Karnataka, implementing good practices in teaching, learning and research to impart quality education.

**He is a member of many professional bodies like**
- Fellow Institution of Engineers (F.I.E.)
- Life Member of Indian Society for Technical Education (M.I.S.T.E.)
- Fellow Indian Association of Structural Engineers (F.I.A.Struct)
- Life Member of Indian Society for Wind Engineering (M.I.S.W.E.)
- Member of Indian Geotechnical Society (M.I.G.S.)
- International Society for Soil Mechanics & Geotechnical Engineering (I.S.S.M.G.E.)

**He is member of the Governing Councils of various organisations such as**
- Karnataka Examination Authority(KEA)
- Dept. of IT, BT and ST, Bengaluru
- Water Land Management Institute, Dharwad
- Rajiv Gandhi Institute of Steel Technology, Bengaluru
- Govt. Tool Room & Training Centre, Bengaluru Also, he is the member of the Executive Council of Karnataka State Higher Education Council and Karnataka State Council for Science & Technology.

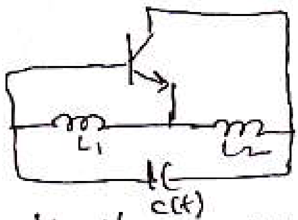
**His Vision as Vice Chancellor of VTU**
- To transform the University from a teaching centric University into a teaching, research, consultancy and knowledge centric University.
- To create exciting and supportive learning environment that transforms the engineering students and inspire them to make a real difference to the society.
- To be the leading pioneers in the field of technical education and research.
- To train the young faculty in terms of curriculum delivery, teaching methodology and to design appropriate need based curriculum through Academic Staff College.

Dr. Karisiddappa, Vice Chancellor - VTU, Belagavi visits Sahyadri

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{1}{2} A_c^2 \quad \because \sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1.$$

$$BW_{FM} \simeq 2\Delta f + 2f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right) \quad \text{Carson's rule}$$

## Gen^n of FM Signals:




Hartley Oscillator

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}}$$

$$C(t) = C_0 + \Delta C \cos 2\pi f_m t$$

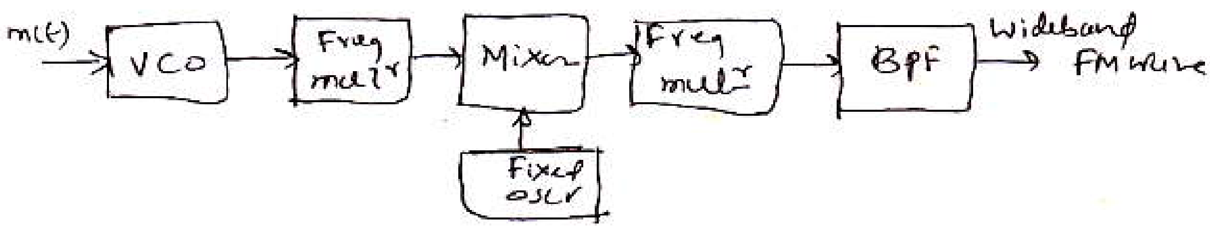
$$f_i(t) = f_0\left(1 + \frac{\Delta C}{C_0}\cos(2\pi f_m t)\right)^{-1/2}$$

where $f_0 = 1 / 2\pi\sqrt{C_0(L_1+L_2)}$

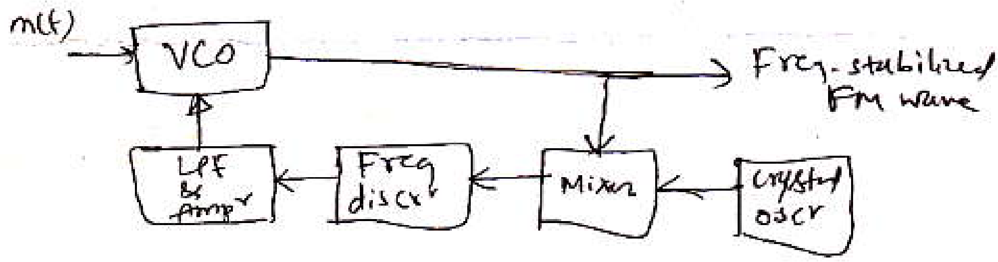
$$f_i(t) \simeq f_0\left(1 - \frac{\Delta C}{2C_0}\cos 2\pi f_m t\right) \quad \text{Let } \frac{\Delta C}{2C_0} = -\frac{\Delta f}{f_0}$$

Without feedback

$$\Rightarrow \quad f_i(t) = f_0 + \Delta f \cos 2\pi f_m t$$



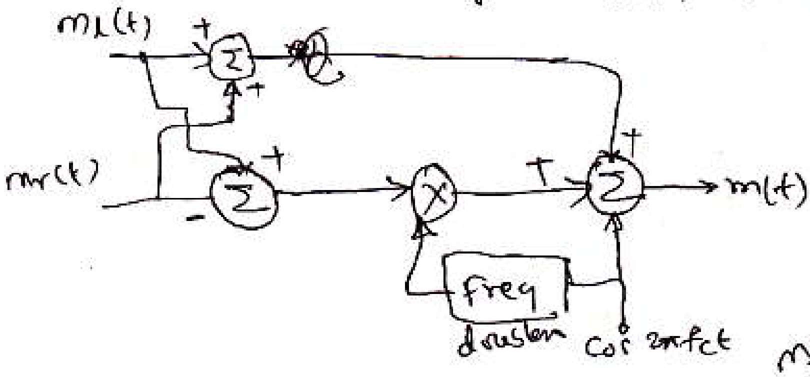

With feedback



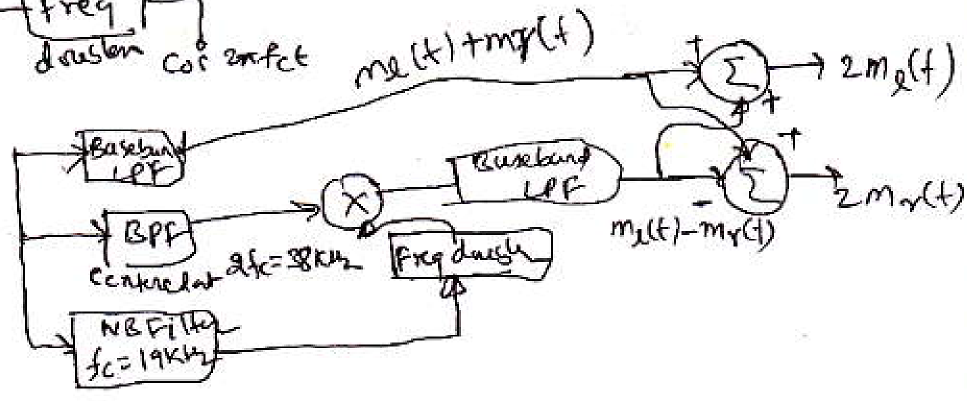

## FM stereo multiplexing

Two requirements

1. The transmission has to operate within the allocated FM broadcast channels
2. It has to be compatible with monophonic radio receivers

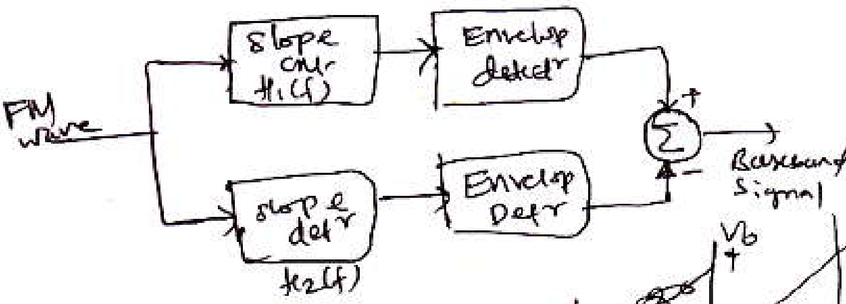


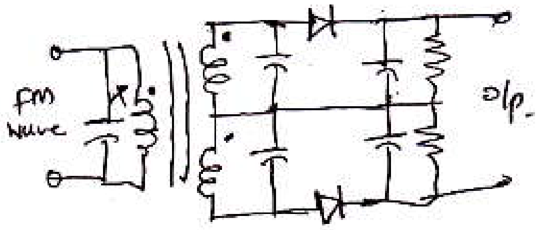

Mux

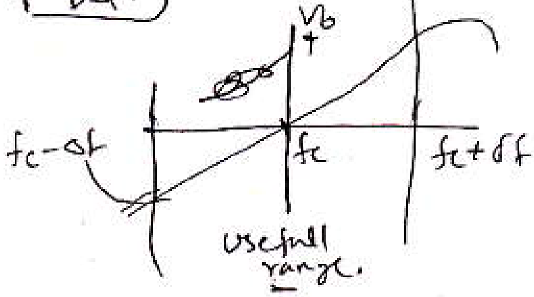


## FM Demodulators:



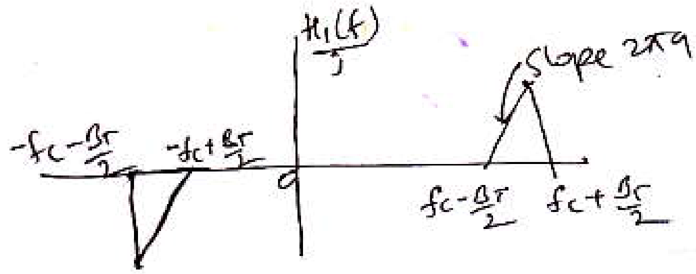



1) Slope-det$^r$ (single)
2) Balanced
3) phase discriminator
4) Ratio detect

$$H_1(f) = \begin{cases} j2\pi a\left(f - f_c + \frac{B_T}{2}\right), & f_c - \frac{B_T}{2} \le f \le f_c + \frac{B_T}{2} \\ j2\pi a\left(f + f_c - \frac{B_T}{2}\right) & -f_c - \frac{B_T}{2} \le f \le -f_c + \frac{B_T}{2} \\ 0 & \text{elsewhere} \end{cases}$$




$$H_2(f) = \tilde{H}_1(-f)$$

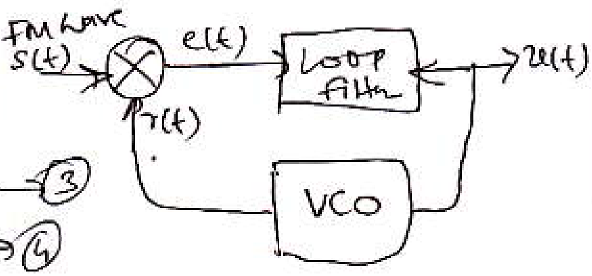
**PLL:**

$$s(t) = A_c \sin(2\pi f_c t + \phi_1(t)) \rightarrow (1)$$

$$\phi_1(t) = 2\pi K_f \int_0^t m(\tau)\,d\tau \rightarrow (2)$$

VCO o/p $\quad r(t) = A_v \cos(2\pi f_c t + \phi_2(t)) \rightarrow (3)$

$K_f \rightarrow$ freq sensitivity $\quad \phi_2(t) = 2\pi K_v \int_0^t v(t)\,dt \rightarrow (4)$




where $K_v \rightarrow$ freq sensitivity of the VCO (Hz/volt)

⇒ The object of PLL loop is to generate VCO o/p $r(t)$ that has same phase angle (except 90° phase diff in degrees) as the FM i/p signal

Models of PLL: Non linear & Linear Model. is on multiplying $s(t)$ & $r(t)$

1. A high freq component, represented by the double freq term
$$K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$$
2. A low freq component represented by the difference freq term
$$K_m A_c A_v \sin(\phi_1(t) - \phi_2(t))$$
where $K_m$ is the multiplier gain, measured in volt$^{-1}$

~~where φ~~ $e(t) = K_m A_c A_v \sin(\phi_e t)$ → (5)

where $\phi_e(t)$ is the phase error defined by

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$= \phi_1(t) - 2\pi k_v \int_0^t v(\tau)\,d\tau \rightarrow (6)$$

The loop filter operates on the input $e(t)$ to produce an o/p $v(t)$ defined by the convolution integral

$$v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau \rightarrow (7)$$ where $h(t)$ is the impulse response of loop filter

Using (4) to (6) to relate $\phi_e(t)$ & $\phi_1(t)$, we obtain following nonlinear integro-differential eqn as the descriptor

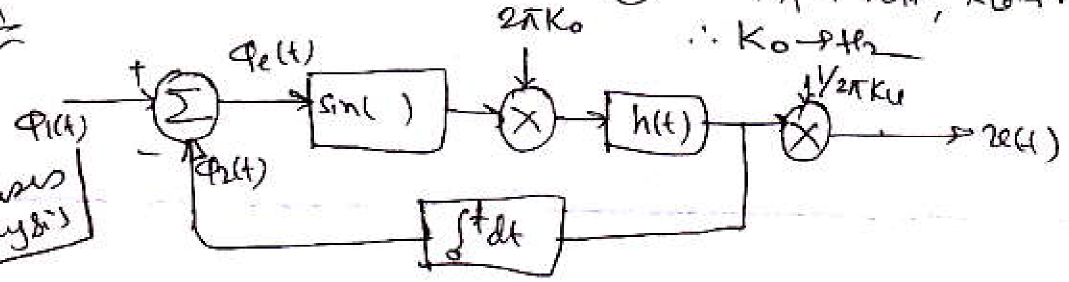
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_0 \int_{-\infty}^{\infty} \sin \phi_e(\tau)\, h(t-\tau)\, d\tau \rightarrow (8)$$

where $K_0$ is loop gain parameter defined by

$$K_0 = K_m K_v A_c A_v \rightarrow (9)$$

$A_c$ & $A_v$ → volts
$K_m$ → volt$^{-1}$, $K_v$ → ~~volts/Hz~~ Hz/volt
∴ $K_0$ → Hz

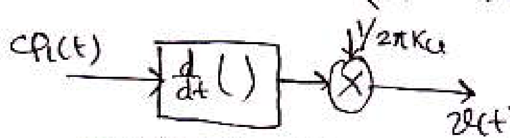
**NM model**
B. diagram



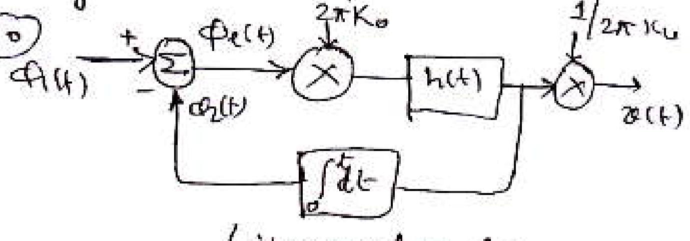

Drawback: greatly increases the analysis

**Linear model**: When the phase error $\phi_e(t)$ is zero, the PLL is said to be in phase-lock. When $\phi_e(t)$ is at all times small compared with one radian, we may use the approximation

$$\sin(\phi_e(t)) \cong \phi_e(t) \rightarrow (10)$$




Simplified model when loop gain is >> 1




Linearized model

$$\frac{d\phi_e(t)}{dt} + 2\pi K_0 \int_{-\infty}^{\infty} \phi_e(\tau)\, h(t-\tau)\, d\tau = \frac{d\phi_1(t)}{dt} \quad (11)$$

Transforming eqn (11) in to freq domain & solving for $\phi_e(t)$, the F.T of $\phi_e(t)$ in terms of $\Phi_1(f)$, the F.T of $\phi_1(t)$ we get—

$$\Phi_e(f) = \frac{1}{1+L(f)} \Phi_1(f) \quad (12)$$ where $$L(f) = K_0 \frac{H(f)}{jf} \quad (13)$$

where $H(f)$ is transfer fn of loop filter & $L(f)$ is called open loop transfer fn of PLL.

⟹ When $L(f) >> 1$, $\phi_e(f) \cong 0$ i.e, VCO phase or PLL phase = FM signal phase

$\therefore \quad V(f) = \frac{K_o}{K_v} H(f) \cdot \Phi_e(f)$ (14) <u>From diagram of Linear model</u>

rewriting $V(f) = \frac{jf}{K_v} L(f) \cdot \Phi_e(f)$ (15), using (13)

substituting (12) in (15)

$$V(f) = \frac{(jf/K_v)\, L(f)}{1+L(f)} \Phi_1(f)$$

again $|L(f)| >> 1$ for the freq band of interest we may approximate

$$V(f) \cong \frac{jf}{K_v} \Phi_1(f)$$

$$\Rightarrow v(t) \cong \frac{1}{2\pi K_v} \frac{d\Phi_1(t)}{dt}$$

$\Rightarrow$ $L(f)$ is very large for all frequencies of interest

$\therefore$ PLL acts as a differentiator with o/p scaled by $\frac{1}{2\pi K_v}$

Also $\Phi_1(t) \propto m(t)$ <u>as in FM</u>

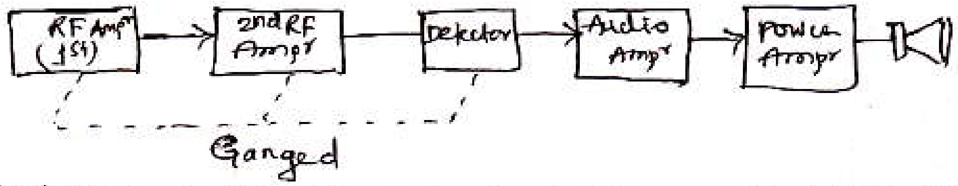
$$\therefore v(t) \cong \frac{K_f}{K_v} m(t)$$

o/p is (scaled) original info

IES 2018-CS

# SuperHeterodyne Receivers: (SHR)

⇒ Tuned radio-frequency receivers: Simple, Freq fixed, high sensitivity and used in only fixed applications (special)

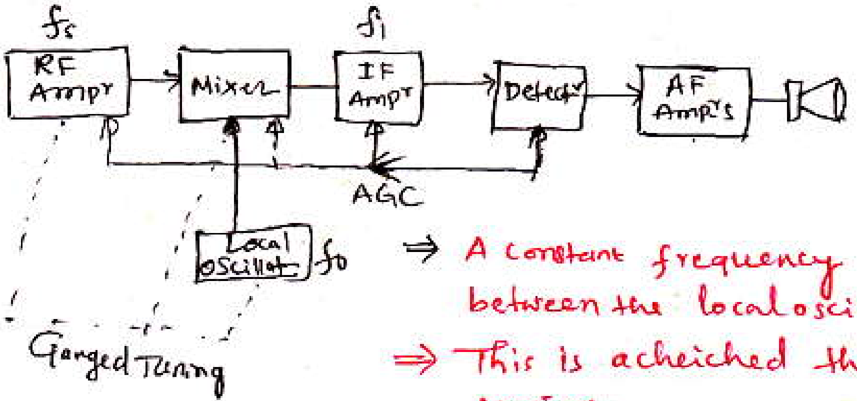



Drawbacks: i) Variation in bandwidth over the tuning range

ii) Difficulties at high frequencies i.e., instability associated with high gain being acheived at one frequency by a multistage ampr.

iii) poor selectivity (poor adjacent-freq rejection)

⇒ To overcome, SHRs are used:




AGC: Automatic Gain Control

⇒ A constant frequency difference is maintained between the local oscillator and the RF circuits.

⇒ This is acheiched through Ganged capacitive tuning.

⇒ A simplified form of the SHR is also in existence, in which the mixer output is infact audio. Such a direct conversion receiver has been used by amatures with good results.

Advantages: i) stable, ii) Good selectivity, Fidelity iii) fixed bandwidth etc

Applications: AM, FM communications, SSB, TV & Radar receivers etc.

Advantages of RF Ampr: Greater gain, improved image frequency rejection, improved S/N ratio, improved rejection of unwanted / adjacent signal (Good selectivity), Better coupling betn Antenna & Receiver, prevention of spurious frequencies from entering the mixer & heterodyning there to produce an interfering frequency equal to the IF from the desired signal And, prevention of reradiation of the local oscillator through the antenna of the receiver.

Sensitivity: Ability of the receiver to produce (standard) o/p (typically atleast 50mW) & S/N ≥ 20dB for the weakest signals, Expressed in µV.

⇒ Sensitivity is not fixed, varies over the band of frequencies.

⇒ Receiver should not be too sensitive.

**Selectivity:** Ability of the receiver to reject unwanted signals.

**Fidelity:** Ability to produce weakest audio signals uniformally over the audiorange.

⇒ Local oscillator frequency is always greater than signal frequency.

⇒ Armstrong & Hartley oscillators are used upto 36 MHz (SW)

⇒ >36MHz, Colpitts, clapp or ultra audion oscillators are used.

⇒ IF for MW band (540-1650 KHz) is 455KHz   'IF' → Intermediate freq

Choice of If:

1) If the 'IF' is too high, poor selectivity

SB → Sideband

2) If 'IF' is too high, tracking difficulties

3) As 'IF' is lowered, image frequency rejection becomes poorer

4) A very low 'IF' can make selectivity too sharp, cutting off the SBs ⇒ Q must be low when fi is low.

5) If 'IF' is very low, freq stability of LO must be high.

6) 'IF' must not fall in the receivers' tuning range. ⇒ Instability will occur and heterodyne whistles will be heard, making it impossible to tune to the frequency band immediately adjacent to the IF.

Frequencies used:

1) MW: 540K–1650KHz
SW: 6 – 18MHz
European long-wave band (150K–350K)
} IF: 438K to 465K
Most popular: 455KHz

2) AM, SSB and other receivers for SW or VHF: First option 1.6M to 2.3MHz
Second option: 30MHz.
⇒ Some receivers have two or more IF frequencies.

3) FM radios (88MHz–108MHz) use IF: 10.7MHz

4) TV receivers in the VHF band 54 to 223 MHz & in the UHF band (470–940MHz) use an IF between 26 & 46 MHz with approximately 36MHz & 46MHz are two most popular ones.

5) Microwave and Radar Receivers: Operating on frequencies in the 1-to-10-GHz range use 30, 60 and 70 GHz IF.

⇒ Communication receivers and some high-quality domestic AM receivers have more than one IF – generally two, but sometimes even more. When receiver has two ~~or more~~ different IFs is called double conversion receiver.

⇒ In suchase high IF comes first e.g., 36 MHz & 12 MHz etc.

⇒ If not image frequency will not be sufficiently rejected.

Examples:

1) When a SHR is tuned to 555 KHz, its local oscillator provides the mixer with an input at 1010 KHz. What is the image freq? The antenna of this receiver is connected to the mixer via a tuned circuit whose loaded Q is 40. What will be the rejection ratio for the calculated image frequency

soln $f_s = 555\,KHz$ ; $Q = 40$;

$f_o = 1010\,KHz \Rightarrow f_i = f_o - f_i = 1010K - 555\,KHz = 455\,KHz$

$\therefore f_{si} = f_s + 2f_i = f_o + f_i = 1010 + 455 = 1465\,KHz$

Image frequency rejection ratio $(\alpha) = \sqrt{1+Q^2\rho^2}$

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1465}{555} - \frac{555}{1465} = 2.639 - 0.3788 = 2.26$$

$$\therefore \alpha = \sqrt{1+40^2 \times 2.26^2} = 90.41$$

2) Find $\alpha$ for $f_{s_1} = 1200\,KHz$, $f_{s_2} = 20\,MHz$ ; given IF $= 450\,KHz$ & $Q = 65$

soln a) $f_{si} = 1200K + 2\times 450 = 2100\,KHz$

$$\therefore \rho = \frac{2100}{1200} - \frac{1200}{2100} = 1.75 - 0.571 = 1.178$$

$$\therefore \alpha = \sqrt{1+65^2 \times 1.178^2} = 76.57.$$

b) $f_{si} = 20\,MHz + 2\times 450K = 20.9\,MHz$

$$\therefore \rho = \frac{20.9}{20} - \frac{20}{20.9} = 1.045 - 0.9569 = 0.088$$

$\Rightarrow \alpha = 5.810.$

eg. GATE-2016-Set 1: A superheterodyne receiver (SHR) operates in the frequency range of 58MHz – 68MHz. It is required that the image frequencies fall outside the range. The intermediate frequency $f_{IF}$ & L.O. $f_{LO}$ are chosen such that $f_{IF} \leq f_{LO}$. The minimum required $f_I$.

soln:

$f_o - f_s = f_I$ Also $f_I \leq f_o$ & $f_o > f_s$ $f_{si} = f_s + 2f_i$

$\Rightarrow$ $\Rightarrow 58 + 2 \times 5 MHz \Rightarrow 68 MHz$ min or (lower end)

$f_o - 58$ MHz (1)

$f_o - 68$ MHz (2)

on solving $\Rightarrow$ 5MHz.

$\Rightarrow$ $[(1)-(2)]/2$ to cross the range

eg GATE-2016-Set 1: The amplitude of a sinusoidal carrier is modulated by a single sinusoid to obtain the amplitude modulated signal: $s(t) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos 2000\pi t$. The value of modulation index is

soln N.K.T

$$s(t) = V_c\left[1 + \frac{V_m}{V_c}\cos w_m t\right]\cos w_c t$$

$$= 20\left\{1 + \frac{V_m}{V_c}\cos(200\pi)t\right]\cos 1800\pi t$$

$$= V_c(1 + m_a \cos 200\pi t)\cos 1800\pi t$$

$$\Rightarrow \frac{V_c m_a}{2} = 5 \qquad m_a = 0.5 //$$

or

$$V_c \cos w_c t + \frac{V_c m_a}{2}\cos(w_c - w_m)t - \frac{V_c m_a}{2}\cos(w_c + w_m)t$$

on comparing $\frac{V_c m_a}{2} = 5 \Rightarrow m_a = 0.5 //$