

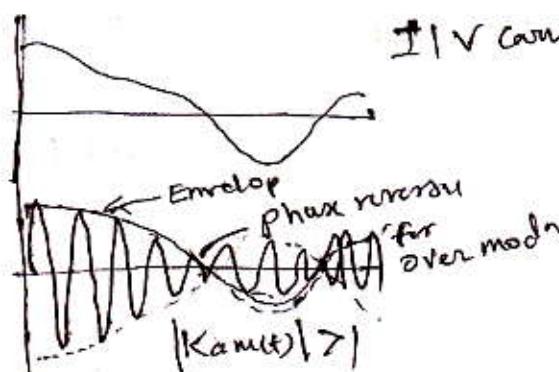
Amplitude Modulation:

Let  $c(t) = A_c \cos(2\pi f_c t)$  ① → Sinusoidal carrier wave.

$s(t) = A_c(1 + K_a m(t)) \cos 2\pi f_c t \rightarrow$  AM Wave → ②

$m(t) \rightarrow$  base band signal  $K_a \rightarrow$  Amplitude sensitivity

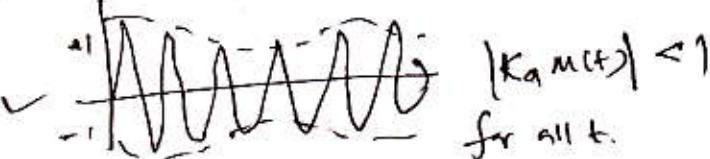
$\Rightarrow$   $\sqrt{V}$  carrier (unmodulated)



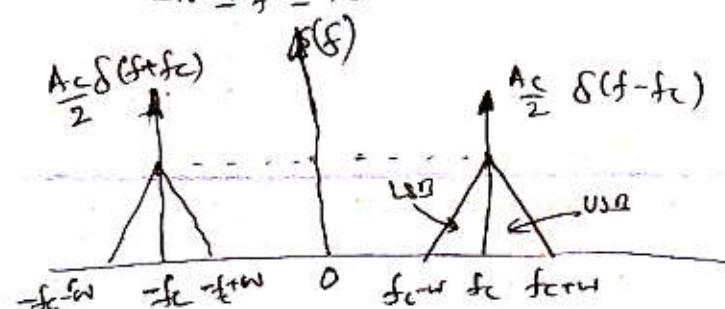
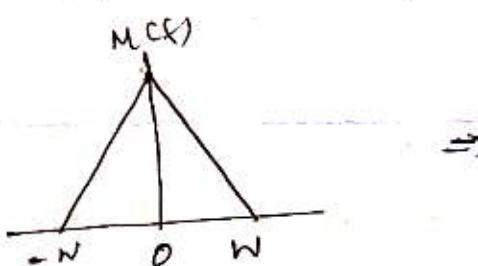
F.T. of ②  $s(t)$

$$S(f) = \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)]$$

$$+ \frac{K_a A_c}{2} [M(f-f_c) + M(f+f_c)] \rightarrow ③$$



$|K_a m(t)| < 1$   $m(t)$  is band limited to the interval  $-W \leq f \leq W$



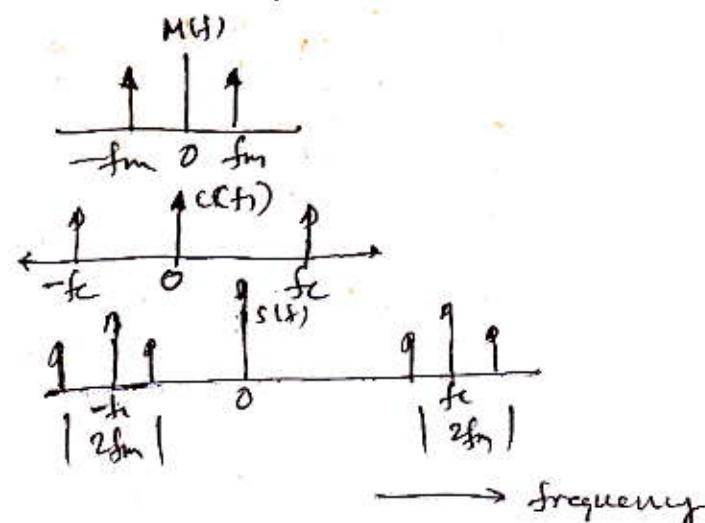
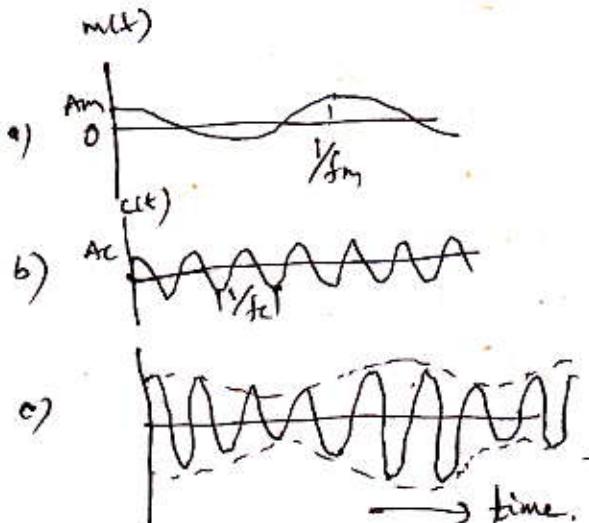
Example: Single tone Modulation:  $m(t) = A_m \cos(2\pi f_m t)$

$$s(t) = A_c (1 + \mu \cos(2\pi f_m t)) \cos(2\pi f_c t)$$

$\mu = K_a A_m \rightarrow$  modulation factor or modulation index

$$\text{eg } \mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$$



$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} M A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} M A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of  $s(t)$  is therefore

$$\begin{aligned} s(f) &= \frac{1}{2} A_c [\delta(f-f_c) + \delta(f+f_c)] \\ &\quad + \frac{1}{4} M A_c [\delta(f-f_c-f_m) + \delta(f+f_c+f_m)] \\ &\quad + \frac{1}{4} M A_c [\delta(f-f_c+f_m) + \delta(f+f_c-f_m)] \end{aligned} \quad \text{--- (1)}$$

Thus the spectrum of an AM wave, for the special case of sinusoidal modulation, consists of delta functions at  $\pm f_c$ ,  $f_c \pm f_m$ , and  $-f_c \mp f_m$ .

$\Rightarrow s(t)$  is current or voltage wave.

$\Rightarrow$  Average power delivered to a  $1\Omega$  resistor by  $s(t)$  is composed of three components:

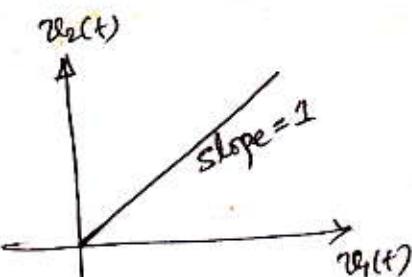
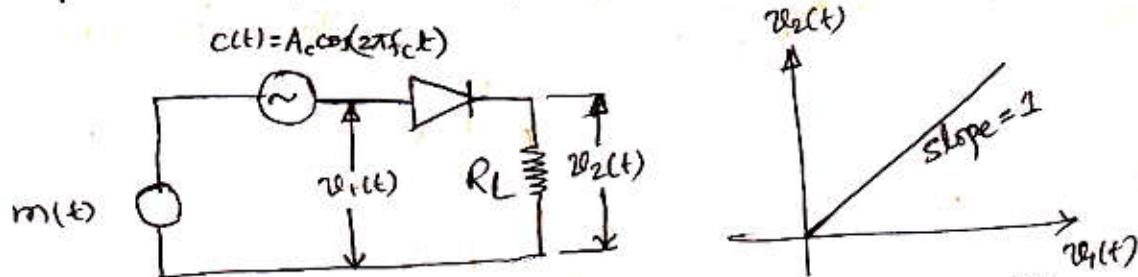
- Carrier power =  $\frac{1}{2} A_c^2$   $\rightarrow$  (5)
- USB power =  $\frac{1}{8} M^2 A_c^2$   $\rightarrow$  (6) Note: When  $R \neq 1\Omega$ , has to be divided as per actual value.
- LSB power =  $\frac{1}{8} M^2 A_c^2$   $\rightarrow$  (7)

Switching Modulator: is an AM generator

$\Rightarrow$  It is assumed that the carrier wave  $c(t)$  applied to the diode is large in amplitude, so that it swings right across the characteristic curve of the diode.

$\Rightarrow$  Assumed that diode acts as ideal switch, i.e., presents zero impedance when it is forward biased.

$\Rightarrow$  Transfer characteristics of the diode-load resistor combination by a piecewise-linear characteristics.



$$v_2(t) = A_c \cos(2\pi f_c t) + m(t) \quad \rightarrow (8)$$

where  $|m(t)| \ll A_c$ , the resulting load voltage  $v_2(t)$  is

$$v_2(t) \approx \begin{cases} v_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases} \quad \rightarrow (9)$$

Since diode passes only positive half cycles of carrier,  $v_L(t)$  may be written as

$$v_L(t) \geq [A_c \cos 2\pi f_c t + m(t)] g_{T_0}(t) \quad \text{--- (10)}$$

Where  $g_{T_0}(t)$  is a periodic pulse train of duty cycle equal to one-half, and period  $T_0 = 1/f_c$ . Representing this by its Fourier series

$$g_{T_0}(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t (2n-1)] \quad \text{--- (11)}$$

Substituting (11) in (10), we find that the load voltage  $v_L(t)$  consists of the sum of two components:

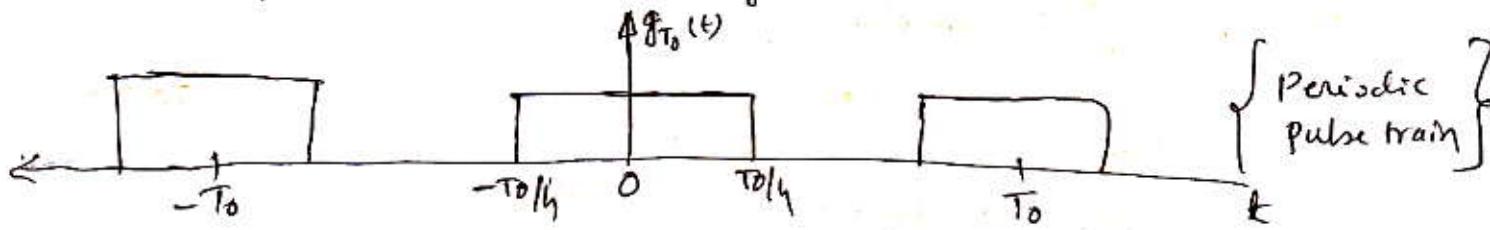
### 1. The component

$$\frac{A_c}{2} \left( 1 + \frac{1}{\pi A_c} m(t) \right) \cos(2\pi f_c t)$$

which is the desired AM wave with amplitude sensitivity  $K_a = 4/\pi A_c$

⇒ The switch modulator is therefore made more sensitive by reducing the carrier amplitude  $A_c$ ; however, it must be maintained large enough to make the diode act like an ideal switch.

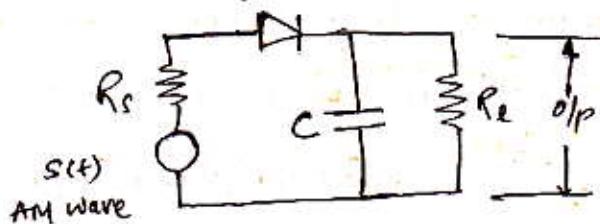
### 2. Unwanted Components, the spectrum of which contains delta functions at $0, \pm 2f_c, \pm 4f_c$ and so on, and which occupy freq intervals of width $2W$ centered at $0, \pm 3f_c, \pm 5f_c$ , and so on, where $W$ is the message bandwidth.



⇒ Unwanted terms are removed from the load voltage  $v_L(t)$  by means of a bandpass filter with mid frequency  $f_c$  and bandwidth  $2W$ , provided that  $f_c > 2W$ .

⇒ This later condition ensures that the frequency separations between the desired AM wave and the unwanted components are large enough for the band-pass filter to suppress the unwanted components.

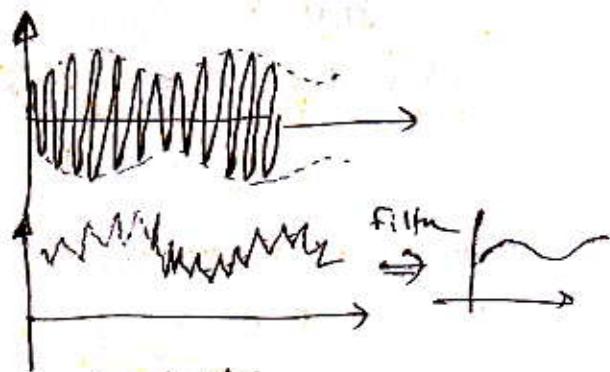
Envelope Detector: Is a demodulator (recovery circuit) which extracts original modulating signal  
 ⇒ Modulating signal lies in the envelope, hence envelope detector.



$$(r_f + R_s)C \ll \frac{1}{f_c}$$

$$\frac{1}{f_c} \ll R_L C \ll \frac{1}{W}$$

W → message bandwidth.



### Virtues, Limitations, and Modifications of Amplitude Modulation

→ AM is wasteful of power

⇒ AM is " " × BW.

∴ Power saving:

m2

$$P_t = P_c \left(1 + \frac{m^2}{2}\right) = 1.5 P_c \text{ for } m=1$$

(100%)

a)  $P_{SB} = P_c \frac{m^2}{4} = P_c / 4 = 0.25 P_c \text{ only one SB}$

$$\% \text{ Saving} = \frac{1.5 P_c - 0.25 P_c}{1.5 P_c} = \frac{1.25}{1.5} = 0.833 \Rightarrow 83.3\%$$

b)  $P_t = P_c \left(1 + \frac{0.5^2}{2}\right) = 1.125 P_c$

(50%)

$$P_{SB} = P_c \cdot \frac{0.5^2}{4} = 0.0625 P_c$$

$$\% \text{ Saving} = \frac{1.125 - 0.0625}{1.125} = \frac{1.0625}{1.125} = 0.944 \Rightarrow 94.4\%$$