Department of Electronics & Communication Engg.

Course : Control Systems **Sem.: 4^{th} (2017-18)**

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Module 3

ROOT LOCUS TECHNIQUE-INTRODUCTION

Possibility of unstable operation is inherent in all feedback control systems because of the very nature of the feedback itself.

An unstable system cannot perform the control task required of it.

While analyzing a given system the very first investigation that needs to be made is

----whether the system stable.determination of stability of system ,is necessary but not sufficient ,for a stable system with low damping is still undesirable.

In an analysis problem one must, therefore proceed to determine not only the absolute stability of a system but also its relative stability.

Relative stability is directly related to the location of the closed –loop poles of a system.

Consider now a design problem in which the designer is required to achieve the desired performance for a system by adjusting the location of its closed loop poles in the s –plane by varying one or more system parameters.

Rouths's criterion obviously does not help much in such problems.

For determining the location of the closed poles :
one may resort to the classical techniques of factoring the characteristic polynomial & determining its roots,since the closed loop poles are the roots of the characteristic equation.

This technique is very laborious when the degree of the characteristic polynomial is three or higher.

Repeated calculations are required as a system parameter is varied for adjustments.

A simple technique known as the ROOT LOCUS TECHNIQUE,for finding the roots of the characteristic equation .

Introduced by W.R.Evans is extensively used in control engineering practice.

Technique provides a graphical method of ploting the locus of the roots in the s-plane as a given system parameter is varied over the complete range of values.

Roots corresponding to a particular value of the system parameter can then be located on the locus or the value of the parameter for a desired root location can be determined from the locus.

Root locus is a powerful technique as it brings into focus the complete dynamic response of the system.

Graphical technique an approximate root locus sketch can be made quickly & the designer can easily visualize the effects of varying various system parameters on root locations.

The root locus also provides a measure of sensitivity of roots to the variation in the parameter being considered. .

Basic Concept of Root Locus

The characteristic equation of a closed loop system is given as
1+G(s)H(s) =0

For Root locus ,gain 'K' is assumed to be a variable parameter & is part of forward path of the closed loop system.

Consider the system shown G(s)=KG'(s) K—Gain of the amplifier in forward path or system gain.

The characteristic equation becomes 1+G(s)H(s)=0 i.e 1+KG'(s)H(s)=0. K—variable parameter.

Closed loop poles i.e roots of above equation are now dependent on values of 'K'.

If now Gain'K' is varied from -∞ to +∞ ,for each separate value of 'K' we will get separate set of locations of the roots of the characteristic equation.If all such locations are joined ,resulting locus is called Root Locus.

If all such locations are joined, resulting locus is called Root locus as,the locus of the closed loop poles obtained when system gain ‘K’ is varied from -∞ to + ∞ is called Root Locus.

K varied from 0 to + ∞ plot is called Direct Root Locus.

K varied from - ∞ to + 0 plot is called Inverse Root Locus.

Angle & Magnitude Condition :

The characteristic equation for a general closed loop system is

$1+G(s)H(s)=0$

i.e $G(s)H(s)=-1$

As s - plane is complex we can write above equation as,

$G(s)H(s)= -1+j0$.

All s-values can be expressed as ‘$\sigma+j\omega$’ i.e G(s) H(s) term is also complex one.

So for any value of ‘s’ if it has to be on the root locus ,it must satisfy above equation.

Construction Rules of Root Loci of 1+G(s)H(s)=0 ,G(s)H(s) is known in pole zero form with n—number of open loop poles,m—number of open loop zeros.

Rule 1: The root locus is symmetrical about the real axis.

Rule 2: As K increases from zero to infinity,each branch of the root locus originates from an open loop pole with k=0 & terminates either on an open loop zero or on infinity with k=∞.the number of branches terminating on infinity equals the number of open loop poles minus zeros.Branch direction always remains from open loop poles towards open loop zeros.

Rule 3: A point on the real axis lies on the locus if the number of open loop poles plus zeros on the real axis to the right of this points is odd.

Rule 4: The (n-m) branches of the root locus which tend to infinity ,do so along straight line asymptotes whose angles are given by

ϕ_A = (2q+1)180deg/n-m; q=0.1.2,…..,(n-m-1).

Rule 5: The asymptotes cross the real axis ata point known as centroid,determined by the relationship:(sum of real parts of poles-sum of real parts of zeros)/(number of poles-number of zeros).

Rule 6: The breakaway points (points at which multiple roots of the characteristic equation occur) of the root locus are the solutions of dK/ds=0.

Breakaway points on the real axis:
Method 1: An analytical approach:requires the determination of the roots of the equation dK/ds=0 to evaluate the breakaway points.

Method 2: Graphical approach: more practical method for determining the breakaway points.

Rule 7:The angle of departure from an open loop pole is given by

$\Phi p = \pm 180$ deg$(2q+1) + \Phi$;$q=0,1,2$ Φ is net angle contribution ,at this pole ,of all other open loop poles & zeros.

Angle of arrival at an open loop zeros is given by

$\Phi z = \pm 180$ deg$(2q+1) - \Phi$; $q=0,1,2$ Φ is net angle contribution ,at this zero,of all other open loop poles & zeros.

Rule 8: The intersection of root locus branches with the imaginary axis can be determined by use of Routh Criterion.

Rule 9: The open loop gain K in pole zero form at any point So on the root locus is given by

K= Product of Phasor lengths* from So to open loop poles
      Product of phasor lengths* from So to open loop zeros.

General Predictions about existence of breakaway points.

If there are adjacently placed poles/zero on the real axis & the real axis between them is a part of the root locus then there exists minimum one breakaway point in between adjacently placed poles/zeros.

General steps to solve the problem on Root Locus:

1. Get the general information about number of open loop poles,zeros,number of branches etc from G(s)H(s).
2. Draw the pole-zero plot.Identify sections of real axis for the existence of the root locus.And predict minimum number of breakaway points by using general predictions.
3. Calculate angles of asymptotes.
4. Determine the centroid.Sketch a separate sketch for step 3 & 4.

Calculate the breakaway points .If breakaway points are complex conjugaates ,then use angle condition to check them for their validity as a breakaway points.

6. Calculate the intersection points of root locus with the imaginary axis.

7.Calculate the angles of departures or arrivals if applicable.

8.Combine steps 1 to 7 & draw the final sketch on the root locus.

9.Predict the stability & performance of the given system by using Root Locus.

NOTE:

Complex conjugate roots should not be considered while counting the poles & zeros to the right side .

To decide valid breakaway points: substitute breakaway point value to the equation of K to get value of K.If K is +ve that breakaway point is valid for root locus.

If no complex poles,then angle of departure is not required.

Module-3

Stability Analysis

Stability in a system: Implies that small changes in the system i/p, in initial conditions or in system parameters do not result in large changes in system o/p.

--Is a very important characteristic of the transient performance of a system.

--Every working system is designed to be stable --almost.

Within the boundaries of parameter variations permitted by stability considerations,--then seek to improve system performance.

A linear time invariant system is stable if the following two notions of system stability is satisfied.

i) When the system is excited by a bounded input,the o/p is bounded.

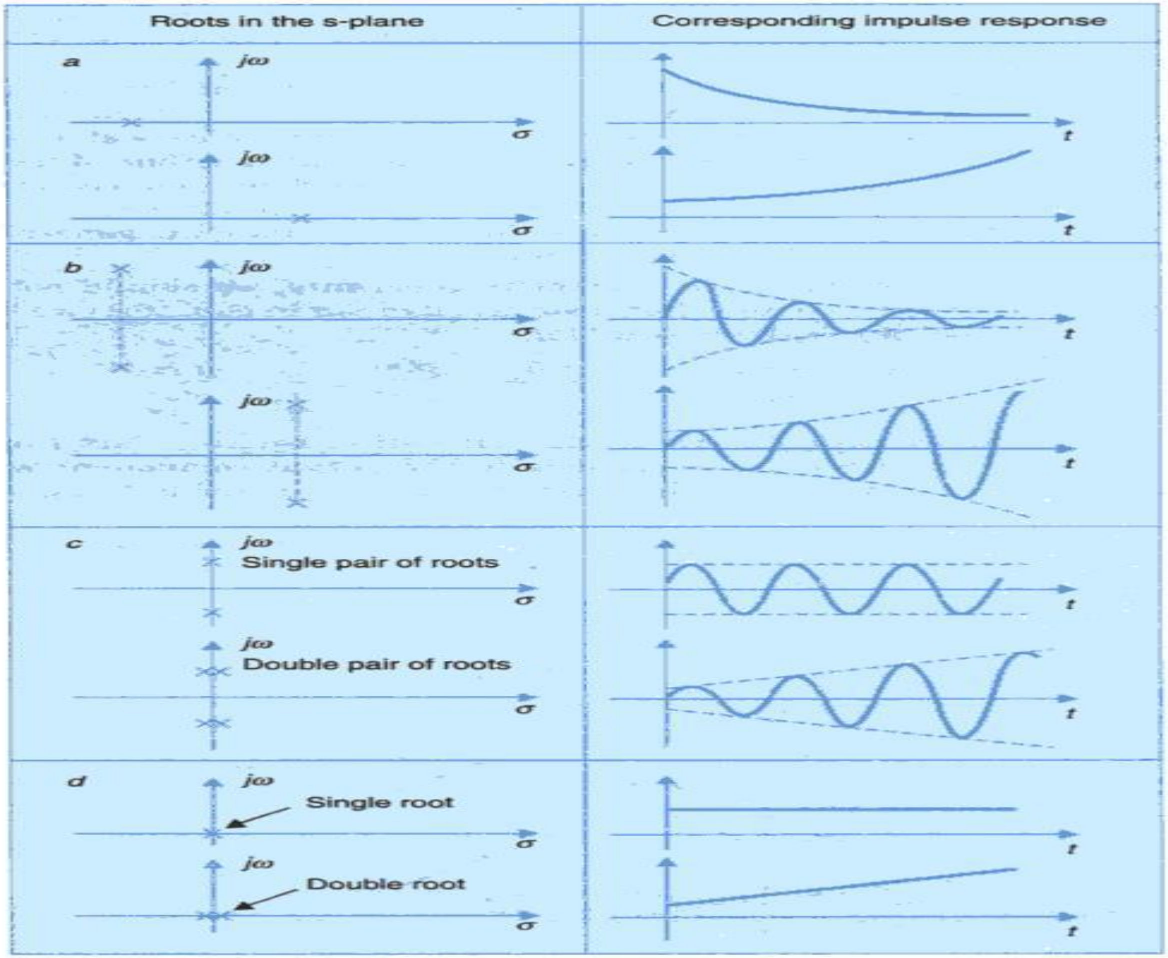


Fig. 6.1. Response terms contributed by various types of roots.

The response to initial conditions is not evident from the model of eqn. (6.1) since the transfer function of a system is derived with the assumption of zero initial conditions. However,

ii) In the absence of the i/p,o/p tends towards zero irrespective of initial conditions. Stability concept is known as Asymptotic stability.

Concern of notion:

II nd notion concerns a free system relative to its transient behaviour.

I st notion concerns a system under the influence of an i/p .

If a system is subjected to an unbounded input & produces an unbounded response ,nothing can be said about its stability.

--But if it is subjected to a bounded i/p & produces an unbounded response ,it is by defn unstable.

---The o/p of an unstable system may increase to a certain extent & then system may break down or become nonlinear after the o/p exceeds a certain magnitude,--linear mathematical model no longer applies.

The two notions of stability defined above are essentially equivalent in

linear time invariant systems.

------simple & powerful tools are available to determine the stability of such systems.

-------For nonlinear systems ,b'coz of possible existence of multiple equilibrium states & other anomalies ,concept of stability is difficult ,even to define.

-------Free stable nonlinear system,-no guarantee that o/p will be bounded whenever i/p is bounded.

If o/p is bounded for a particular bounded i/p ,it may not be bounded for other bounded i/p's.

Many of the important results obtained thus far concern the stability of the nonlinear system in the sense of second notion above,when the system has no i/p.

Observations made from the table

All the roots which have non zero real parts [case I,ii,iii & iv],contribute response terms with a multiplying factor of eσi.
If s < 0 (i.e,the roots have negative real parts),response terms vanish as t ∞ & if σ >0(i.e the roots have positive real parts),the response terms increase without bound.

Roots on the jw axis with multiplicity two or higher[case vi & viii] also contribute terms which increase without bound as t ∞.

Single root at origin [case (vii)] or non –multiple root pairs [case v] on the jw axis contribute response terms which are constant amplitude or constant amplitude oscillation.

Observations made from the table

GENERAL CONCLUSIONS REGARDING SYSTEM STABILITY FROM THE OBSERVATIONS:

1.If all the roots of the characteristic equation have negative real parts ,then the impulse response is bounded & eventually decreases to zero. therefore,∫ 0 to ∞ g(τ)dτ is finite & the system is bounded-input,bounded –output stable.

2.If any root of the characteristic equation has a +ve real part,g(t) is unbounded & ∫ 0 to ∞ g(τ) dτ is infinite.the system is therefore unstable.

3) If the characteristic equation has repeated roots on the jw axis ,g(t) is unbounded & ∫ 0 to ∞ g(τ) dτ is infinite. The system is therefore unstable.

4) If one or more nonrepeated roots of the characteristic equation are on the jw axis,then g(t) is bounded but ∫ 0 to ∞ g(τ) dτ is infinite.The system is therefore unstable.

Module-4

BODE PLOT:

Logarithmic Plot which consists of two graphs:
One giving the logarithm of | G(jw) | & other phase angle of G(jw).

Both plotted against frequency in logarithmic scale.

Magnitude Plot:Magnitude expressed in logarithmic values against logarithmic values of frequency.

Phase angle plot:Phase angle in degrees against logarithmic values of frequency.

Magnitude Plot: Find 20log | G(jw) | --unit as decibel.

Steps to sketch bode plot:

i) Express given G(s)H(s) into time constant form.

ii) Draw a line of 20 Log K dB.

iii) Draw a line of appropriate slope representing poles or zeros at the origin passing through intersection point of $\omega=1$ & 0 db.

iv) Shift this intersection point on 20 Log K line & draw parallel line to the line drawn in step 3.This is addition of constant K &
& number of poles or zeros at the origin.

v) Change the slope of this line at various corner frequencies by appropriate value i.e depending upon which factor is occuring at corner frequency.
For a simple pole,slope must be changed by -20db/decade,for a simple zero by +20 db/decade.
Change the slope of line obtained in step 5 by respective value & draw line with resultant slope.continue this line till it intersects next corner frequency line.change the slope & continue.
Apply necessary correction for quadratic factor.

vi) Prepare the phase angle table & obtain the table of ω & resultant phase angle φR by actual calculation.Plot these points & draw smooth curve obtaining necessary phase angle plot.

To Remember: At every corner frequency slope of resultant line must change.

Frequency Response Specifications

Basic objective of control system design----Performance Specifications.

Specifications are the constraints or limitations put on the mathematical functions describing the system characteristics.

--Bandwidth: Defined as the range of frequencies over which the system will respond satisfactorily.Range of frequencies in which magnitude response is almost flat in nature.

--Cutoff frequency: Frequency at which magnitude of the closed loop response is 3db down from its zero frequency value.

-----Cutoff rate: slope of the resultant magnitude curve near the cutoff frequency is called cutoff rate.

-----Resonant peak: Max value of magnitude of the closed loop frequency response.Measure of relative stability of the system.

----Resonant frequency: Frequency at which resonant peak occurs in the closed loop frequency response.

Gain cross over frequency: (ωgc):Frequency at which magnitude of $G(j\omega)H(j\omega)$ is unity.

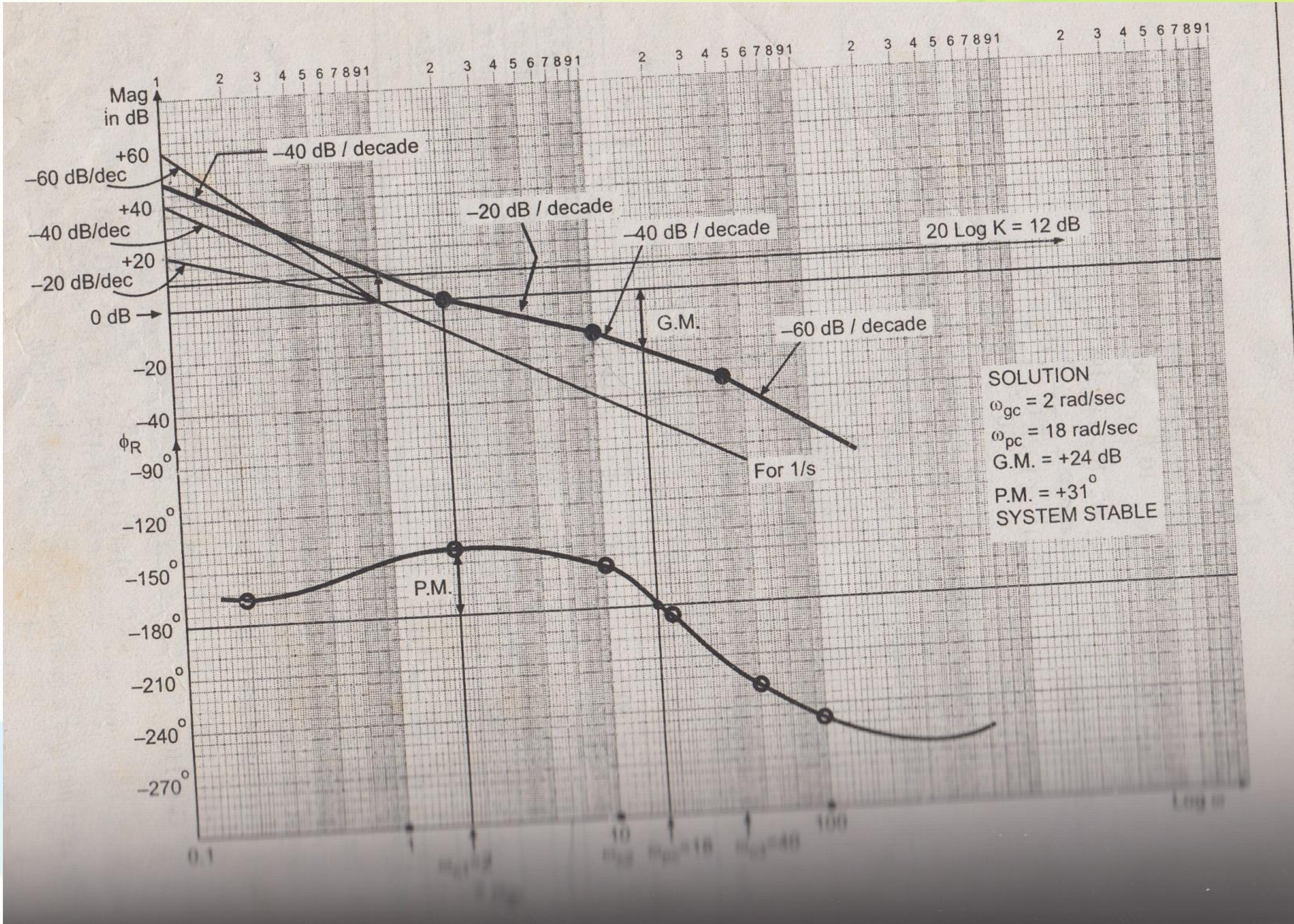
Frequency at which magnitude of $G(j\omega)H(j\omega)$ is 0 db is ωgc.

Phase cross over frequency:(ωpc) : Frequency at which phase angle of $G(j\omega)H(j\omega)$ is -180 deg .

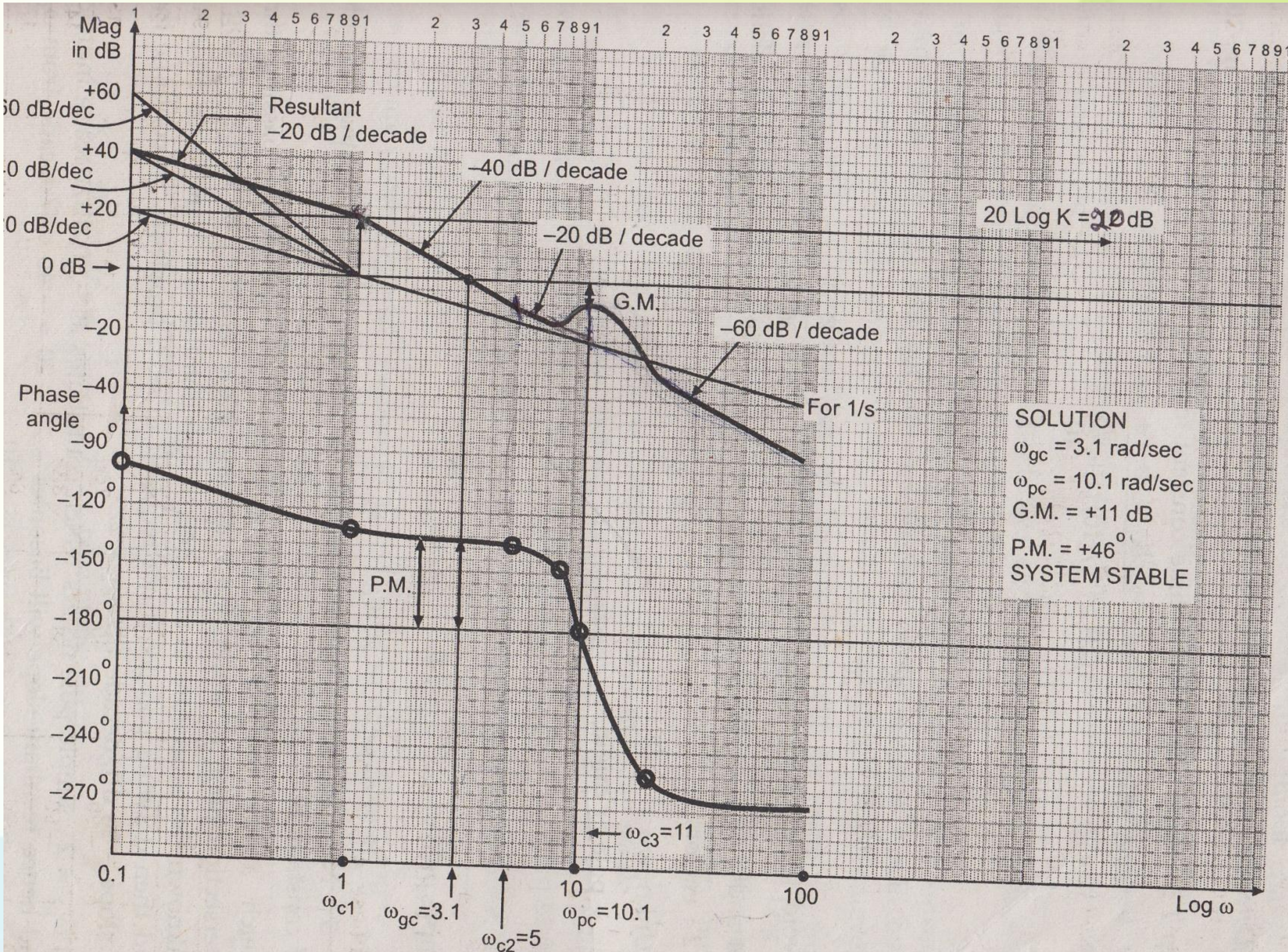
Gain Margin: Gain margin is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge of instability.

+ve gain margin---system is stable.

-ve gain margin---system is unstable.



Mag in dB
+60
–60 dB/dec
+40
–40 dB/dec
+20
–20 dB/dec
0 dB →
–20
–40
ϕ_R
–90°
–120°
–150°
–180°
–210°
–240°
–270°
–40 dB / decade
–20 dB / decade
–40 dB / decade
20 Log K = 12 dB
G.M.
–60 dB / decade
For 1/s
P.M.
SOLUTION
ω_{gc} = 2 rad/sec
ω_{pc} = 18 rad/sec
G.M. = +24 dB
P.M. = +31°
SYSTEM STABLE
0.1
1
10
100



Mag in dB
+60
0 dB/dec
+40
0 dB/dec
+20
0 dB/dec
0 dB
−20
−40
Resultant −20 dB / decade
−40 dB / decade
−20 dB / decade
20 Log K = 20 dB
G.M.
−60 dB / decade
For 1/s
Phase angle
−90°
−120°
−150°
−180°
−210°
−240°
−270°
P.M.
ω_c3=11
0.1
1
ω_c1
ω_gc=3.1
ω_c2=5
10
ω_pc=10.1
100
Log ω
SOLUTION
ω_gc = 3.1 rad/sec
ω_pc = 10.1 rad/sec
G.M. = +11 dB
P.M. = +46°
SYSTEM STABLE

i.e. any = 0° out

wherever necessary.

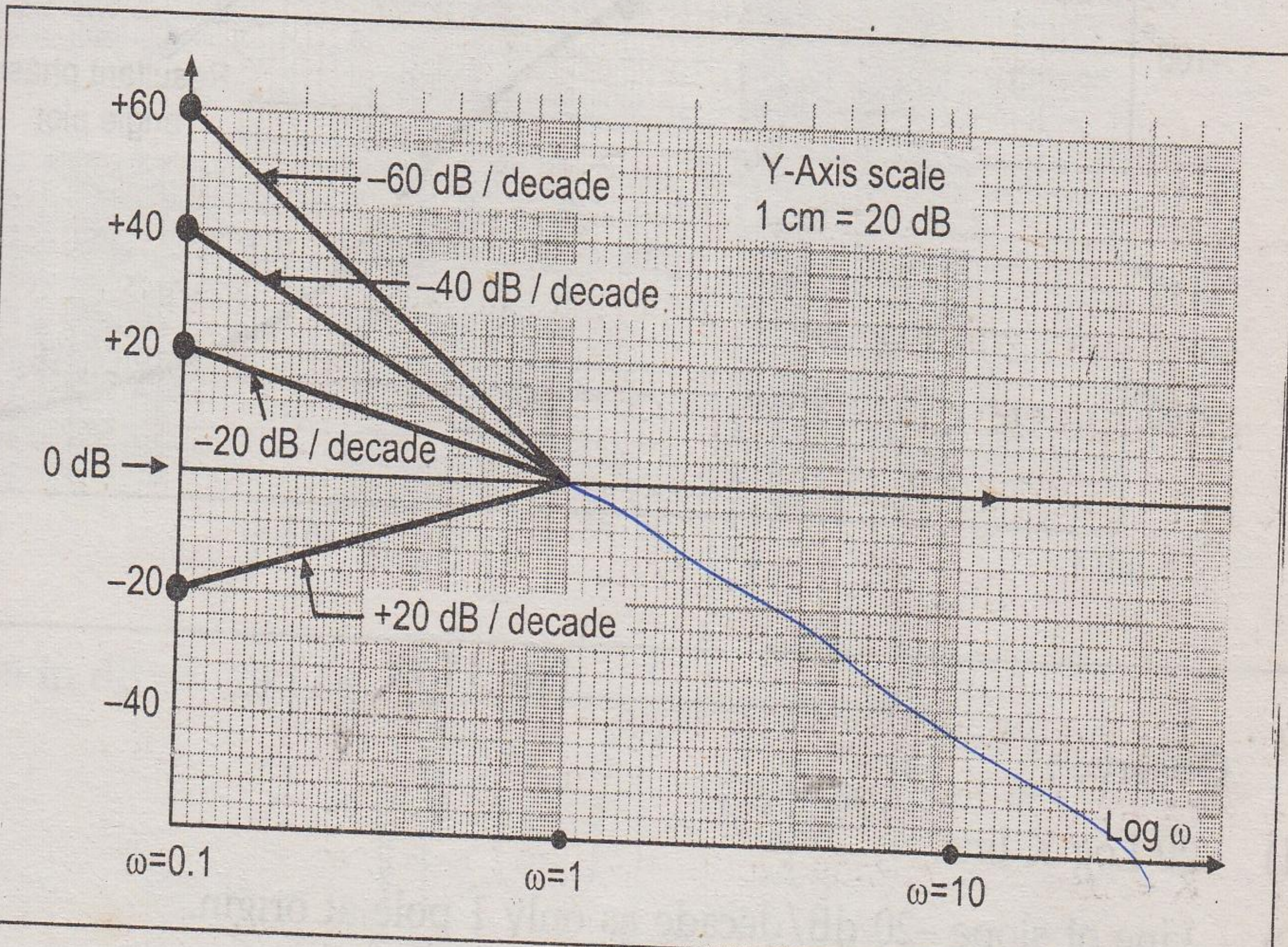


Fig. 11.15

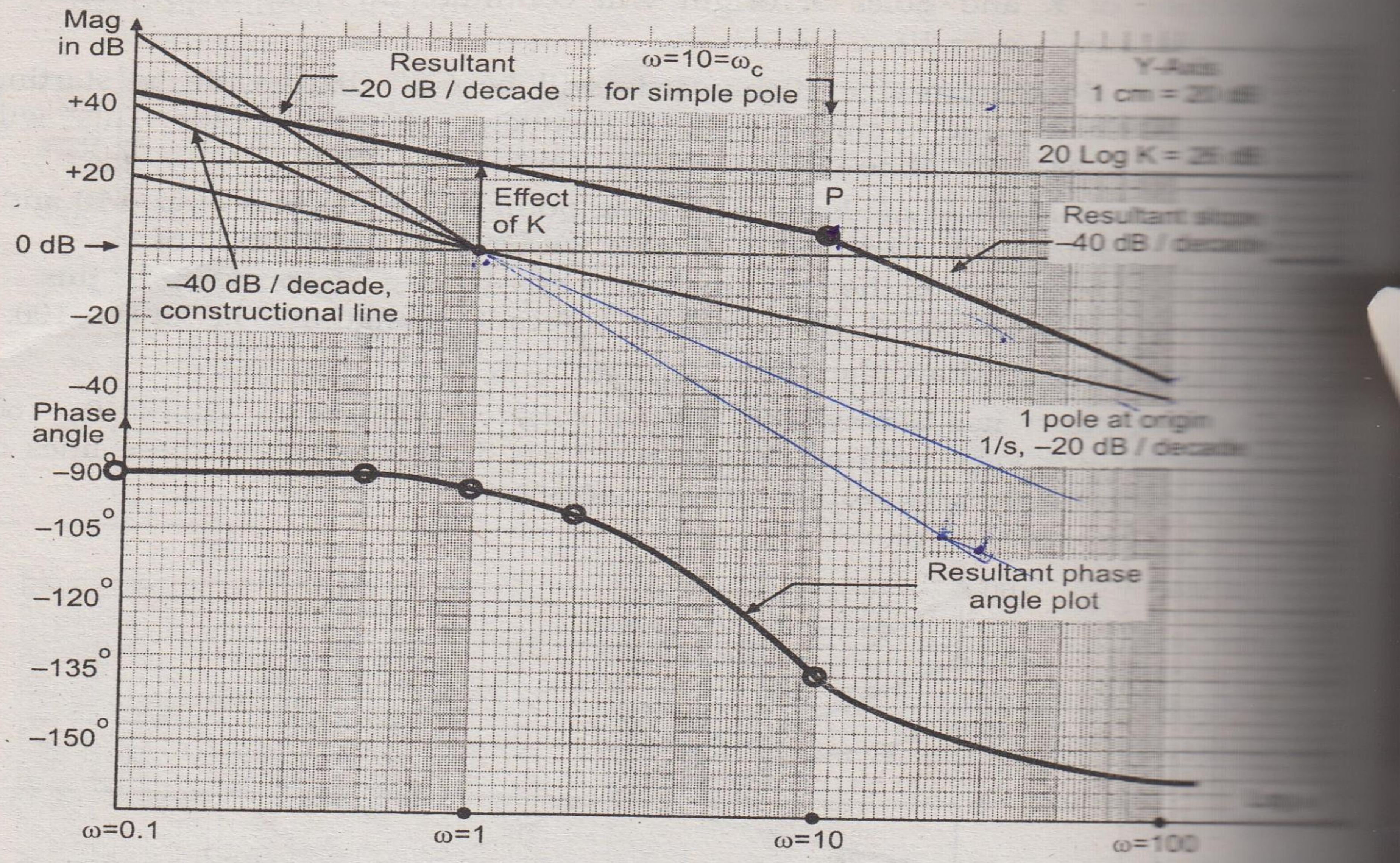
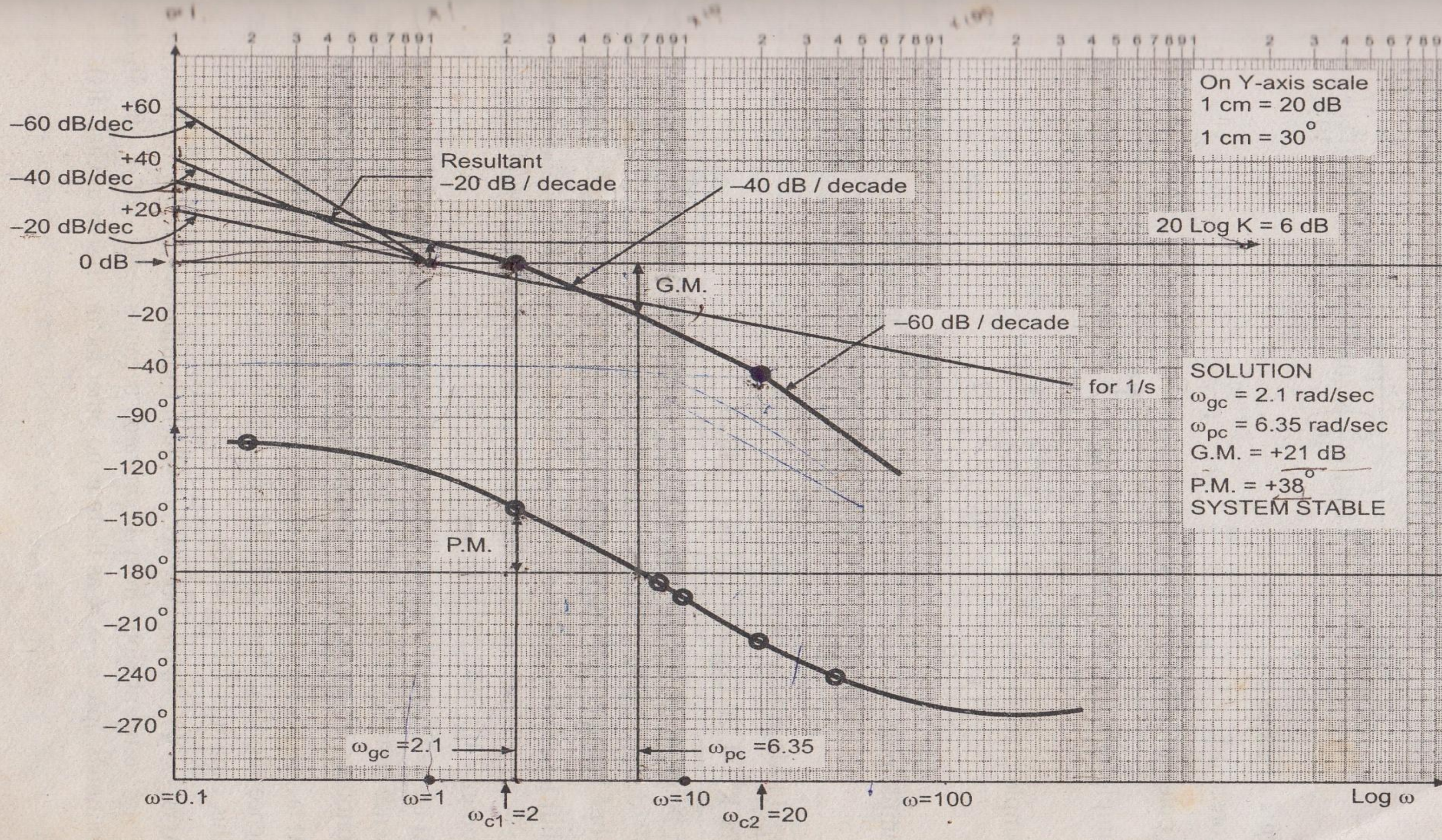


Fig. 11.16

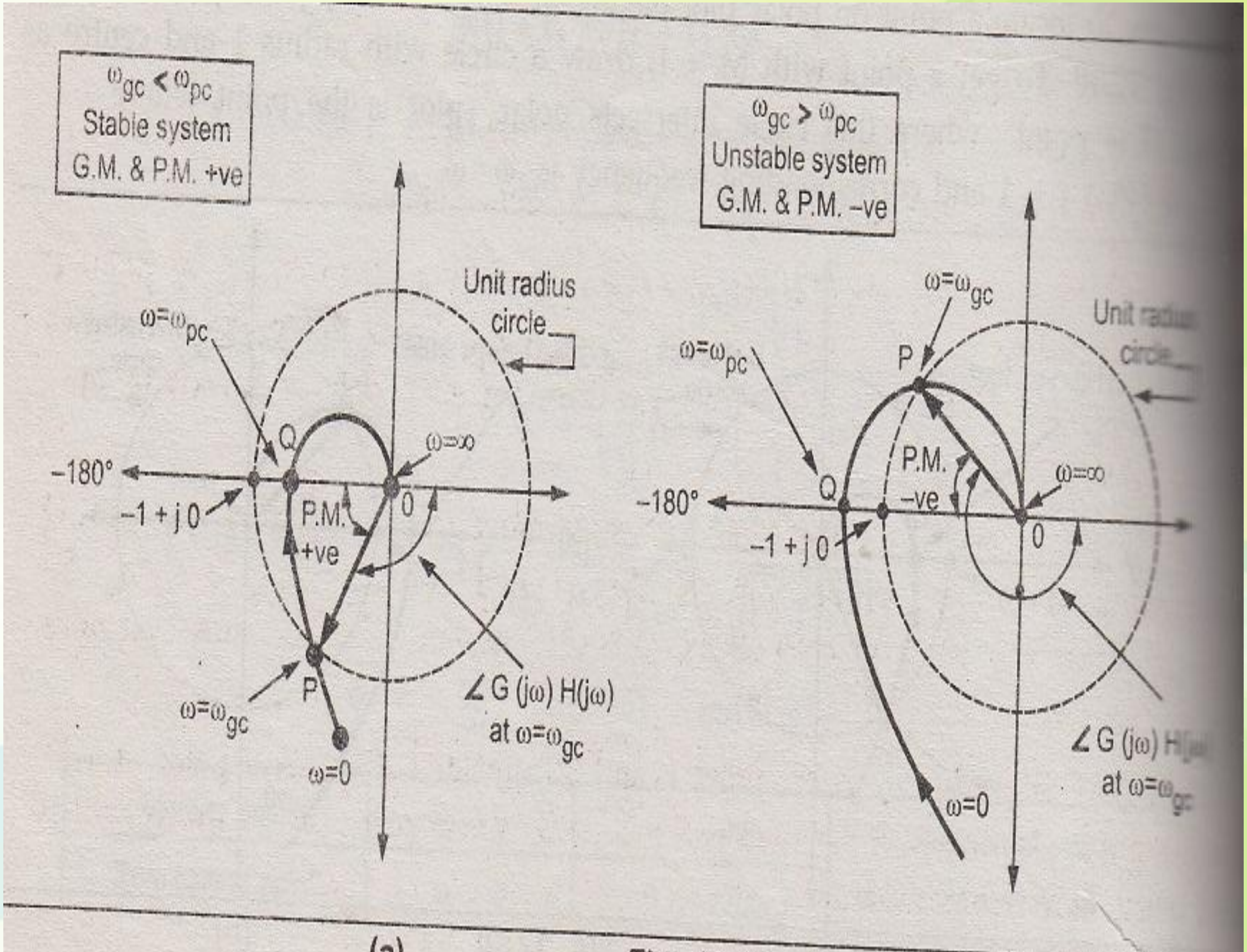
Observe :

1) 20 Log K line



+60
–60 dB/dec
+40
–40 dB/dec
+20
–20 dB/dec
0 dB
–20
–40
–90°
–120°
–150°
–180°
–210°
–240°
–270°
Resultant
–20 dB / decade
–40 dB / decade
–60 dB / decade
G.M.
P.M.
for 1/s
20 Log K = 6 dB
On Y-axis scale
1 cm = 20 dB
1 cm = 30°
SOLUTION
ωgc = 2.1 rad/sec
ωpc = 6.35 rad/sec
G.M. = +21 dB
P.M. = +38°
SYSTEM STABLE
ωgc =2.1
ωpc =6.35
ω=0.1
ω=1
ωc1 =2
ω=10
ωc2 =20
ω=100
Log ω

Fig.11.27



ωgc < ωpc
Stable system
G.M. & P.M. +ve
Unit radius circle
ω=ωpc
Q
ω=∞
−180°
−1 + j 0
0
P.M. +ve
P
ω=ωgc
ω=0
∠G (jω) H(jω) at ω=ωgc
(a)
ωgc > ωpc
Unstable system
G.M. & P.M. −ve
ω=ωgc
P
ω=ωpc
Q
P.M. −ve
ω=∞
−180°
−1 + j 0
0
ω=0
(b)

Fig. 12.11

Module 5

Digital Control Systems

- In control systems dealt so far—signal at every point in the system is a continuous function of time.
- Controller elements are such that the controller produces continuous time signals from continuous time input signals.—Analog controller.
- As complexity of a control system increases—arises severe demands of flexibility ,adaptability & optimally & even demands to account for economic control function complexity.
- Constructing a complex control function may even become technically infeasible,if one is restricted to use only analog elements.

- Use of a digital computer as a compensator (controller) device has grown during the past two decades as the price of digital computers has reduced & their reliability has improved drastically.

- Powerful but inexpensive computers called microcomputers –16 bit or 32 bit word have readily available.

- Computers can be equipped with sufficiently large memories to handle a large amount of data in a complex control process.

- Speed has increased & today systems with speed as high as 300MHz are readily available.

- Besides these general purpose computers, many specific control systems use embedded microcontroller (8 or 16 bit depending upon complexity of the built in chip)as their heart.

- A digital controller in which either a special purpose computer or a general purpose computer forms the heart ,is therefore choice for complex control systems.

- A general purpose computer if used lends itself to time-shared use for other control functions in the plant or process.

- Digital controllers used in digital control systems have the inherent characteristics that they accept data as short duration pulses and produce a similar kind of output as control signal.

- Typical system with digital controller diagram

A sampler & ADC is needed at the computer input. The sampler converts the continuous time error signal into a sequence of pulses which are then expressed in numerical code.

Numerically coded o/p data of digital computer are decoded into continuous time signal by DAC & hold ckt.

This continuous time signal then controls the plant.This overall system is hybrid in which the signal is in sampled form in the digital controller & in continuous form in rest of system.

System of this kind is referred to as a sampled data control system.

- Even in relatively simple control schemes, sampling may be warranted from other considerations.
- Sampling is a necessity wherever a high degree of accuracy is a prerequisite. Is the case in most automated machine tools.
- Eg—if it is required to move the table of a drilling machine within an accuracy of 0.01mm over a distance of 1 m.resolution of 1 in 100,000 is needed which is impossible to measure using an analog type,o/p transducer say a potentiometer.
- Sampled data technique is most appropriate for control systems requiring long distances data transmission.

- Circumstances that lead to the use of sampled data control systems are summarized below

- 1.For using digital computer(or MP) as part of the control loop.
- 2. For time sharing of control components.

- 3.Whenever a transmission channel forms part of the control loop.
- Whenever the o/p of a control component is essentially in discrete form

Sampling implies that the signal at the o/p end of the sampler is available in form of short duration pulses each followed by a skip period when no signal is available so that the control system essentially operates open loop during the skip period.

- Uniform periodic sampling is illustrated in fig.

- If the sampling rate is too low,significant information contained in the i/p signal may be missed in the o/p.

- Minimum sampling rate has a definite relationship with the highest significant signal frequency.

- Assuming sample width (time) as fixed,other forms of sampling are:

- Multi-order sampling: A particular sampling pattern is repeated periodically.

- Multiple rate sampling : Two simultaneous sampling operations with different time periods are carried out on the signal to produce the sampled o/p.

- Random Sampling: Sampling instants are random with a particular kind of distribution.

  The mathematical model of a sampled –data control system is essentially in the form of difference equations.

  Analysis & design of sampled data systems with linear elements may be effectively carried out by use of z transform.

State Variable Analysis & Design:

several methods of analysis & design of feedback systems ----root locus & frequency response methods.

Methods require---physical system be modelled in the form of a transfer function.

Transfer function ---simple & powerful analysis,design techniques ,suffer from certain drawbacks.---TF defined under zero initial conditions.

---limitation of TF---

TF model is only applicable to linear time invariant systems—SISO systems—cumbersome in MIMO systems.

It reveals only the system o/p for a given i/p---provides no information regarding internal state of system.

----situations: o/p of a system is stable & some of the elements may have tendency to exceed their specified ratings.

Sometimes necessary & advantages to provide a feedback .

Proportional to some internal variables of system.

- Classsical methods based on TF model are trial & error procedures. Difficult to visualize& organize

.

- More general mathematical representation of a system which along with the o/p, yields information about the state of the system

  variables at some predetermined points along the flow of signals.
- Approach---state variable approach---direct time domain approach ----basis for modern control theory & system optimization.

- Powerful technique for the analysis & design of linear & nonlinear time invariant or time varying MIMO systems.

- Laplace transform is needed for continuous time & z transform is needed for discrete time systems.

- Note: state variable approach can completely replace classical approach is incorrect.

- B’coz classical approaches provide the control engineer with a deep physical insight into the system & greatly aid the preliminary system design .

- A complex system is approximated by a more manageable model.

Queries …?