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ECE Dept.
NA
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Department of Electronics & Communication Engg.

Course :Network Analysis-15EC34 **Sem.: 3rd (2016-17)**

Course Coordinator:

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MODULE I

Basic Concepts

- Practical sources, Source transformations, Network reduction using Star – Delta transformation, Loop and node analysis With linearly dependent and independent sources for DC and AC networks, Concepts of super node and super mesh

Network: Any interconnection of network or circuit elements (R, L, C, Voltage and Current sources).

- Circuit: Interconnection of network or circuit elements in such a way that a closed path is formed and an electric current flows in it.
- Active Circuit elements deliver the energy to the network (Voltage and Current sources)
- Passive Circuit elements absorb the energy from the network (R, L and C).
- Active elements:
- Ideal Voltage Source is that energy source whose terminal voltage remains constant regardless of the value of the terminal current that flows. Fig.1a shows the representation of Ideal voltage source and Fig.1b, it’s V-I characteristics.

Ideal voltage source representation & V-I characteristics

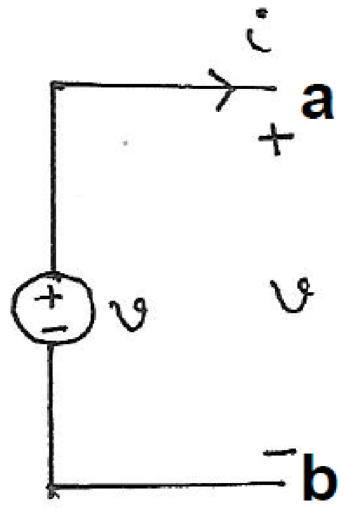


Fig.1a: Ideal Voltage source Representation

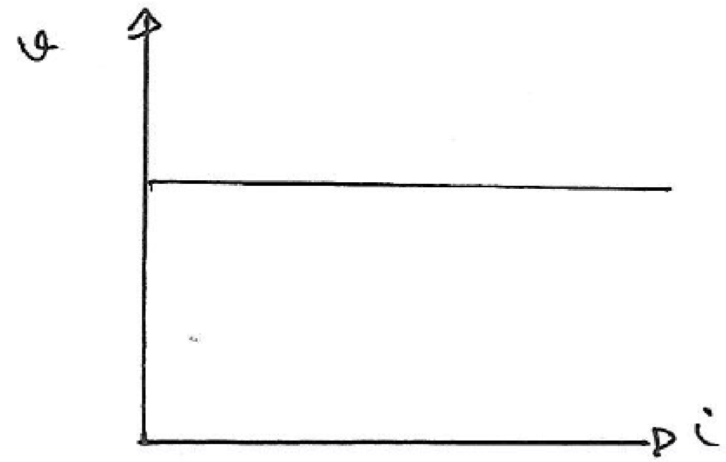


Fig. 1b: V-I characteristics

Practical Voltage source:

- It is that energy source whose terminal voltage decreases with the increase in the current that flows through it.
- The practical voltage source is represented by an ideal voltage source and a series resistance called internal resistance.
- It is because of this resistance there will be potential drop within the source and with the increase in terminal current or load current, the drop across resistor increases, thus reducing the terminal voltage.

practical voltage source and it's V-I characteristics.

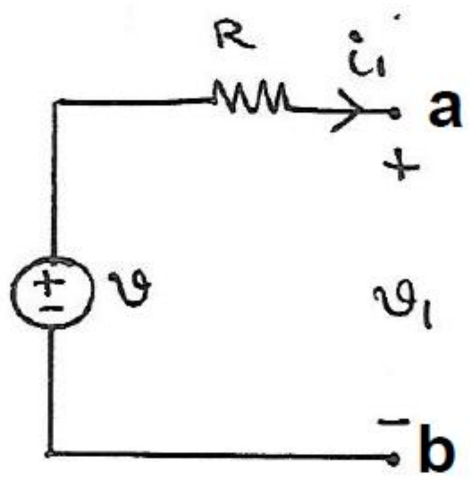


Fig. 2a: Practical Current Source

Fig. 2b: V-I characteristics

- Dependent or Controlled Sources: These are the sources whose voltage/current depends on voltage or current that appears at some other location of the network. We may observe 4 types of dependent sources.
- i) Voltage Controlled Voltage Source (VCVS)
- ii) Voltage Controlled Current Source (VCCS)
- iii) Current Controlled Voltage Source (CCVS)
- iv) Current Controlled Current Source (CCCS)

Fig.3a, 3b, 3c and 3d represent **VCVS,VCCS,CCVS,CCCS** .

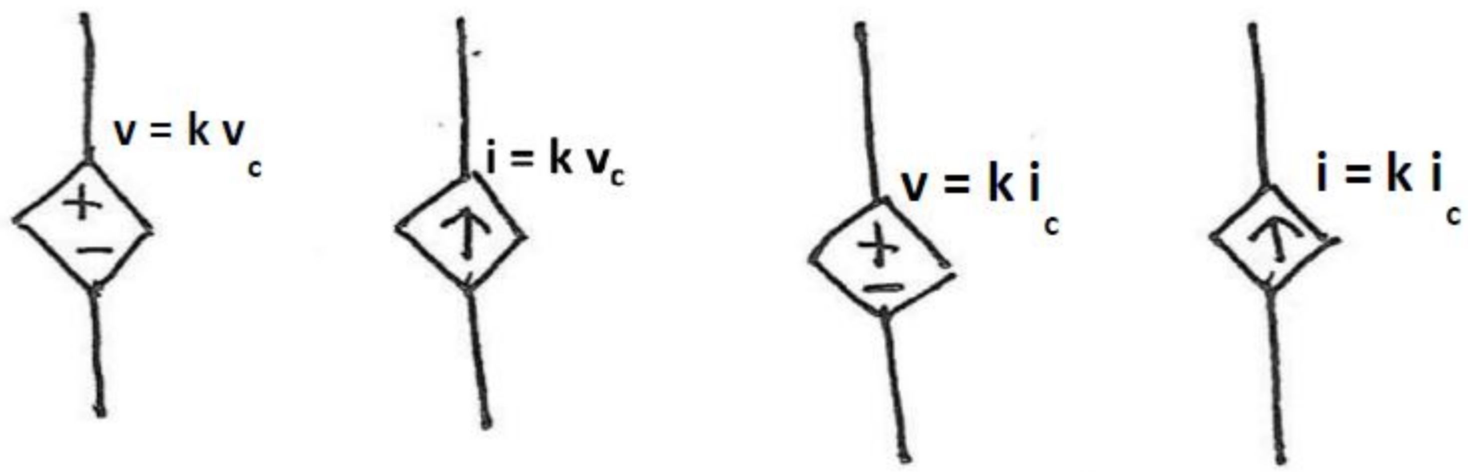
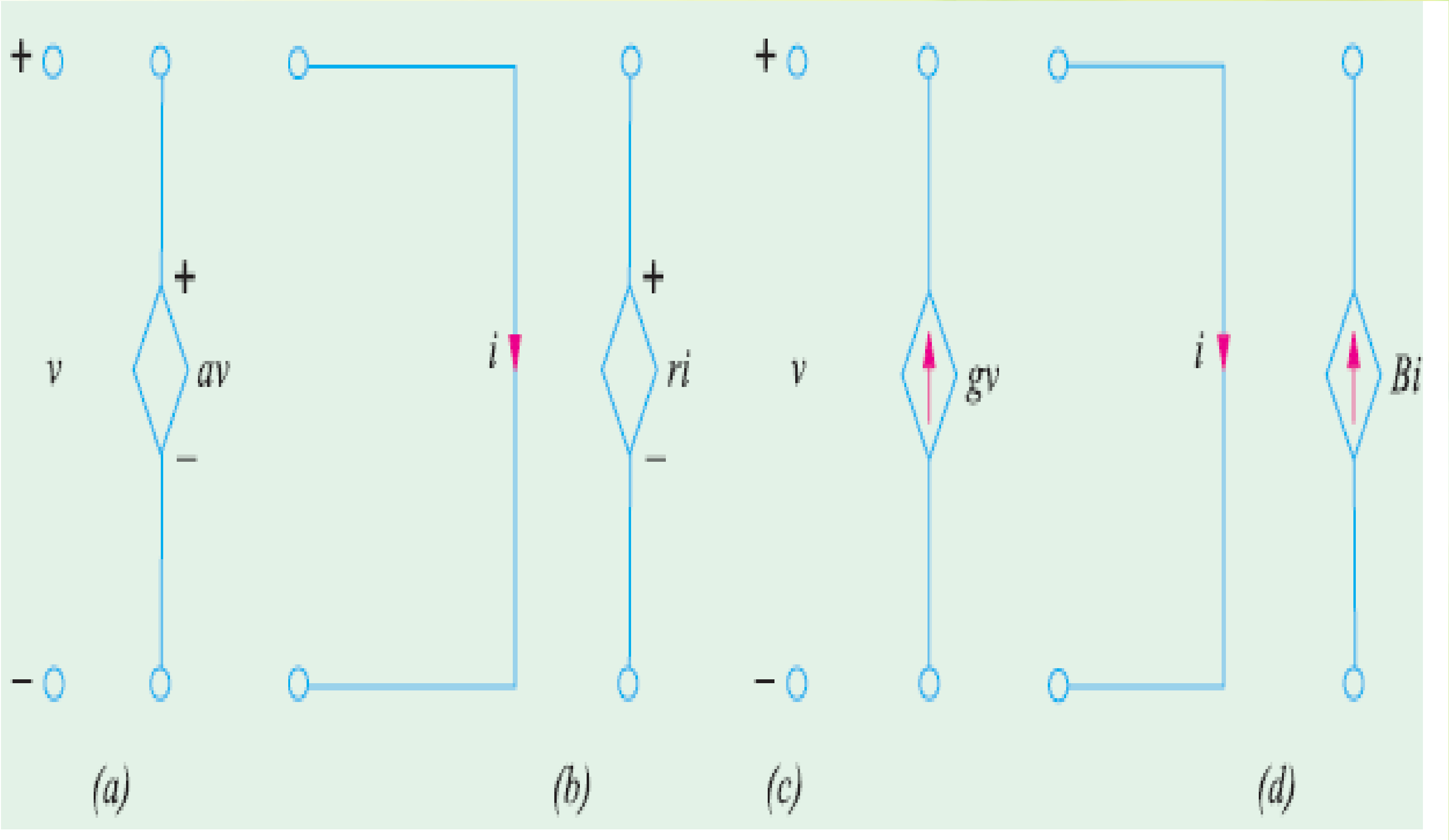


Fig. 3 a) VCVS b) VCCS c) CCVS d) CCCS

Dependent voltage and current sources



Fundamental Laws

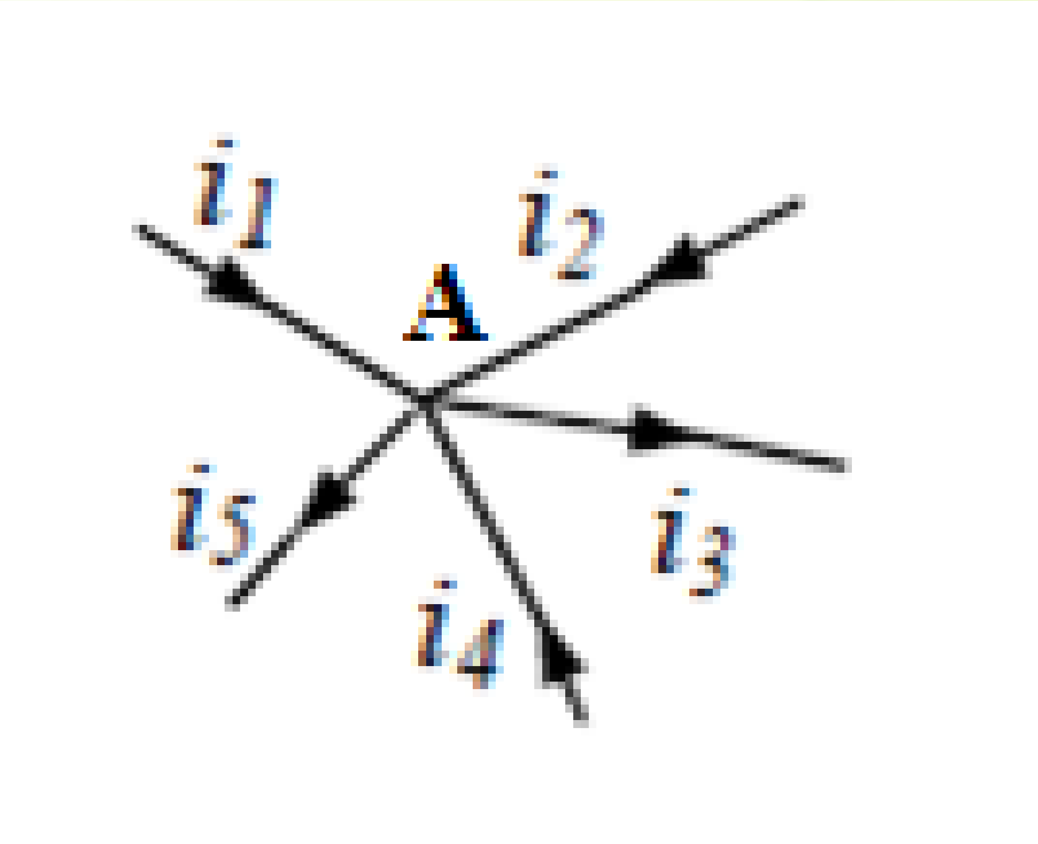
- The fundamental laws that govern electric circuits are Ohm's law and Kirchoff's laws.
- *Ohm's Law*
- Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through it.
- $v \propto i$, $v = R \cdot i$ where R is the proportionality constant.
- A *short circuit in a circuit element is when the resistance (and any other impedance) of the element approaches zero. [The term impedance is similar to resistance but is used in alternating current theory for other components]*
- An *open circuit in a circuit element is when the resistance (and any other impedance) of the element approaches infinity.*
- In addition to Ohm's law we need the Kirchoff's voltage law and the Kirchoff's current law to analyse circuits.

Kirchoff’s Current Law

- Kirchoff’s Current Law states that the algebraic sum of the currents entering a node is zero. It simply means that the total current leaving a junction is equal to the current entering that junction.
- $\Sigma i = 0$
- Consider the case of a few conductors meeting at a point A as in Fig.1.4. Some conductors have currents leading to point A, whereas some have currents leading away from point A.

Assuming the incoming currents to be positive and the outgoing currents negative, we have

$i1 + i2 - i3 + i4 - i5 = 0$

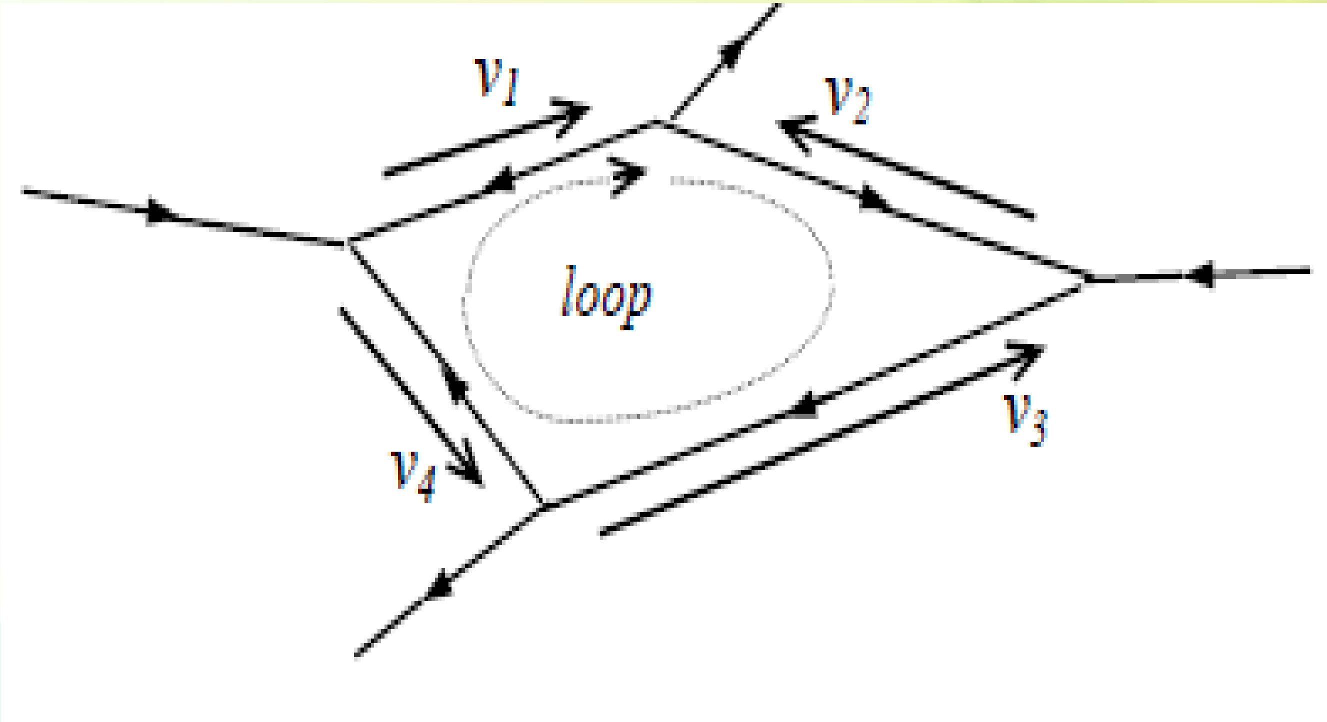


Kirchoff’s Voltage Law

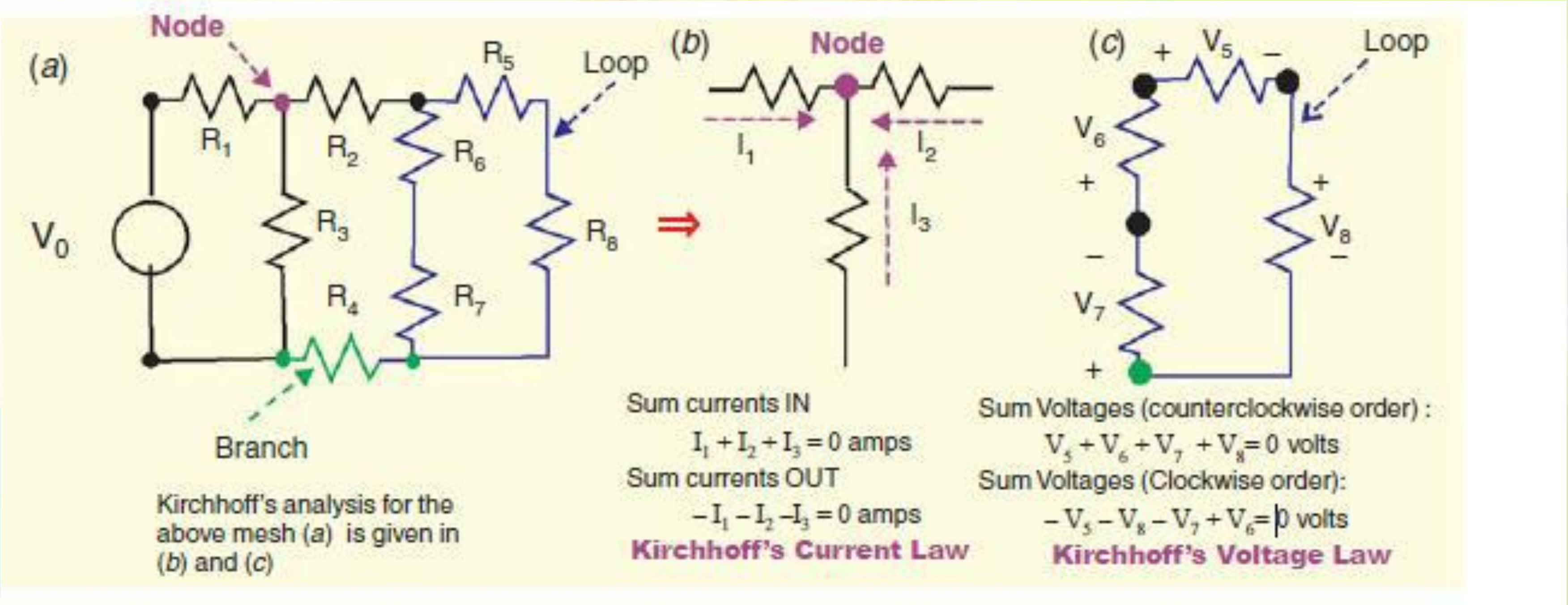
Kirchoff’s Voltage Law states that the algebraic sum of all voltages around a closed path (or loop) is zero.

- $\Sigma v = 0$
- In other words, *... round a mesh*
- Consider a circuit as shown in Fig.1.5, we have
- $-v1 + v2 + v3 + v4 = 0$
- depending on the convention, you may also write
- $v1 - v2 - v3 - v4 = 0$

Kirchoff's Voltage Law



Kirchoff's analysis circuit



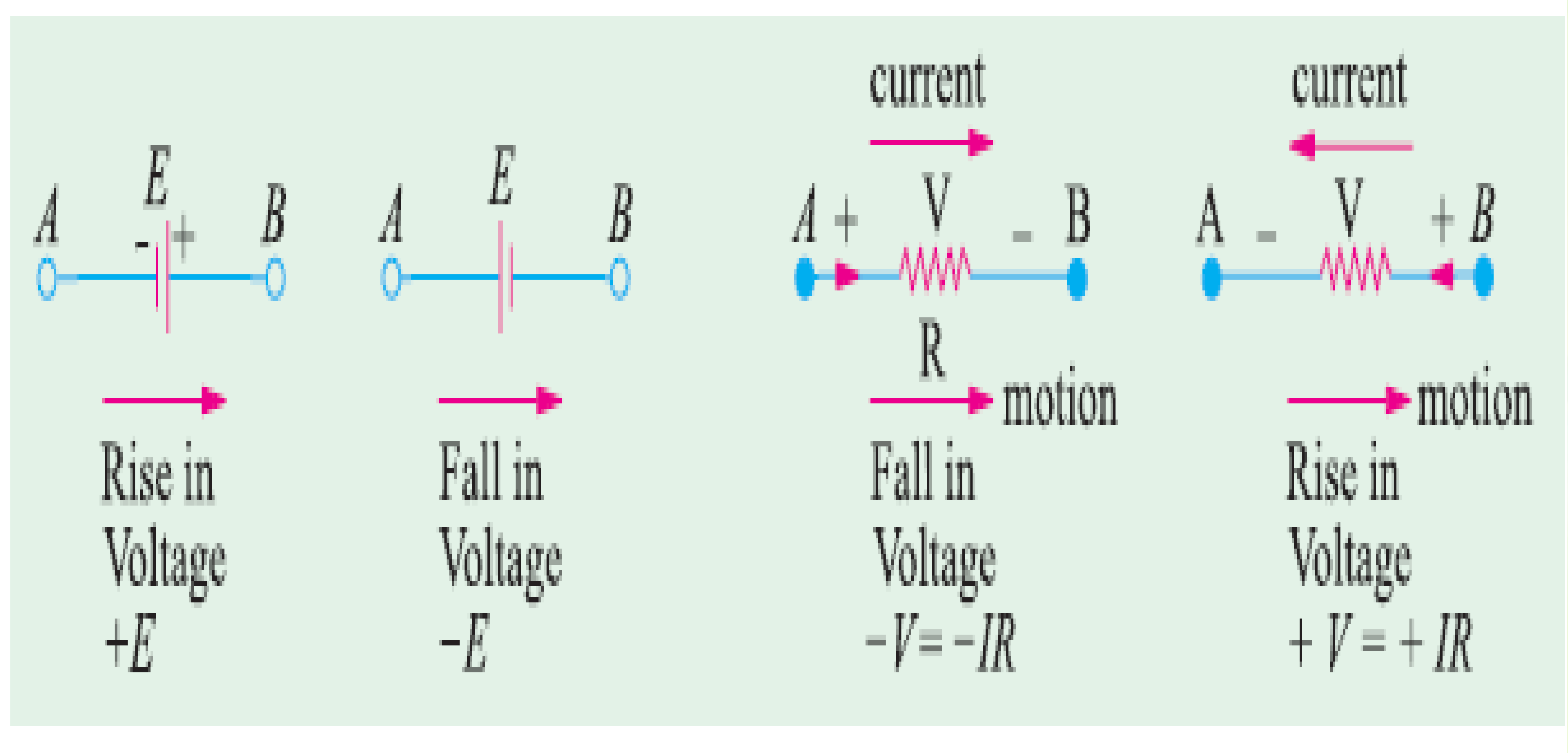
Determination of Voltage sign

In applying Kirchhoff’s laws to specific problems, for example, the circuit shown in Fig, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs. Following sign conventions is suggested

Sign of Battery E.M.F.

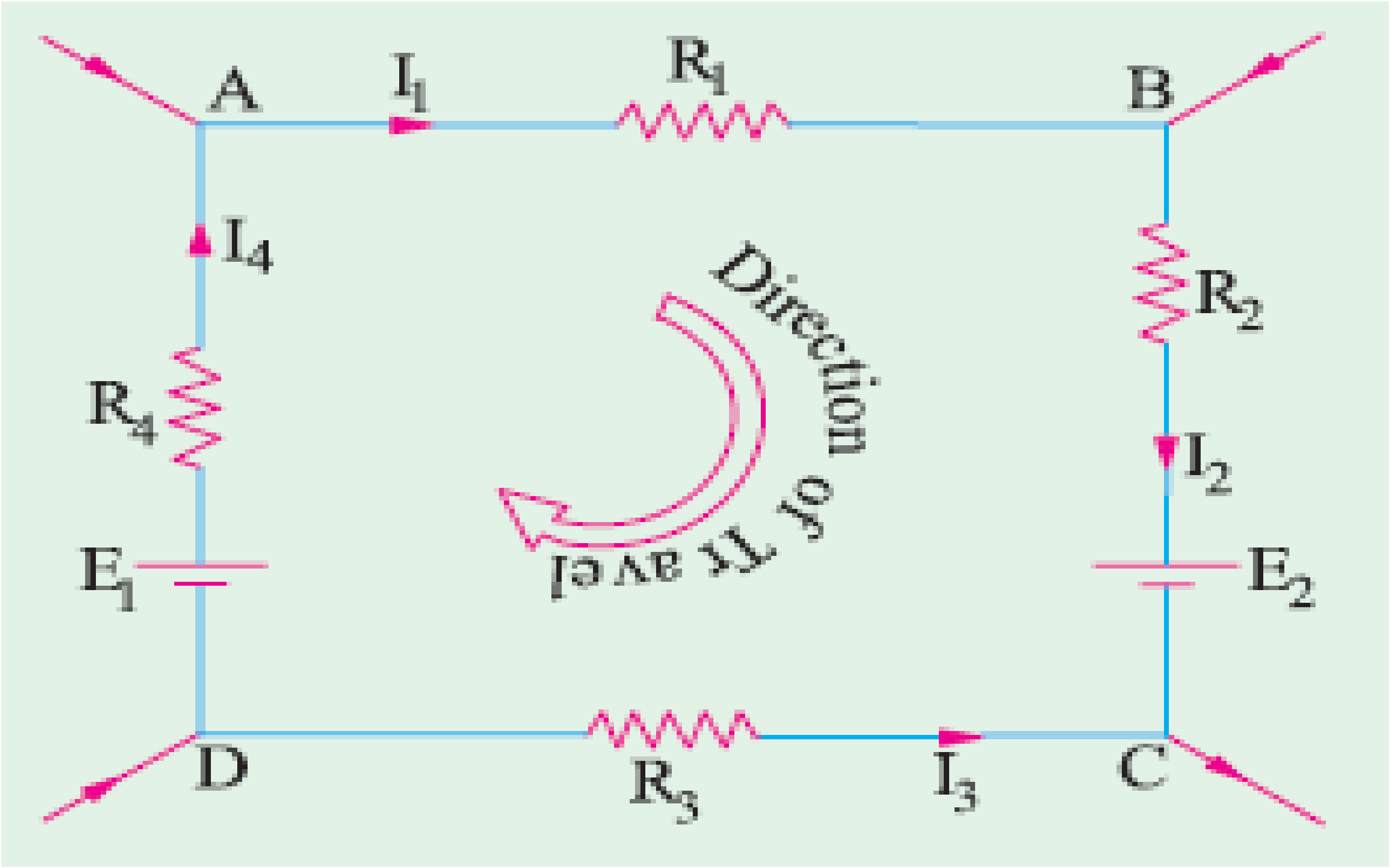
- A rise in voltage should be given a +ve sign and a fall in voltage a –ve sign. Keeping this in mind, it is clear that as we go from the –ve terminal of a battery to its +ve terminal as shown in Fig.there is a rise in potential, hence this voltage should be given a +ve sign.
- On the other hand, if we go from the +ve terminal of a battery to its -ve terminal) there is a fall in potential, hence this voltage should be preceeded by a -ve sign. It is important to note that the sign of the battery e.m.f is independent of the direction of the current through that branch.

Voltage Sign



Sign of IR Drop

- Now, take the case of a resistor for Fig. If we go through a resistor in the same direction as of the current, then there is a fall in potential because current flows from a higher to lower potential.
- Hence this voltage fall should be taken –ve. However, if we go in a direction opposite of the current, then there is a rise in voltage.
- Hence this voltage rise should be given a +ve.
- Consider the closed path ABCDA in Fig, as we travel around the mesh in clockwise direction, using KVL we get.

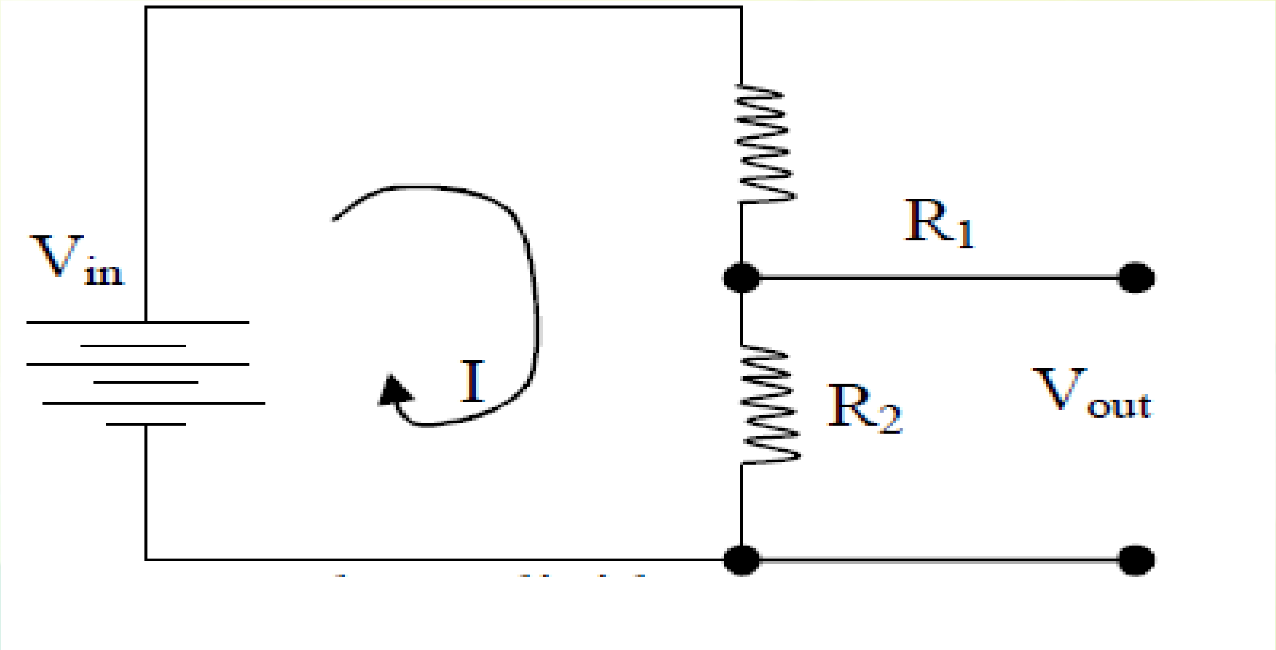


A
I1
R1
B
I4
R2
R4
I2
E1
E2
Direction of Tr avel
D
R3
I3
C

Assumed Direction of Current

- The direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of the current is not actual direction, then on solving the question, this current will be found to have a minus sign.
- If the answer is positive, then assumed direction is same as actual direction.

Voltage Divider Circuit



Ohm's law gives $V_{out} = IR_2$ …… (1)

and we know that $I = \frac{Vin}{Re} = \frac{V_{in}}{R_1 + R_2}$……. (2)

Substituting eqn (2) into eqn (1) gives eqn (3)

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2} \text{---------} (3)$$

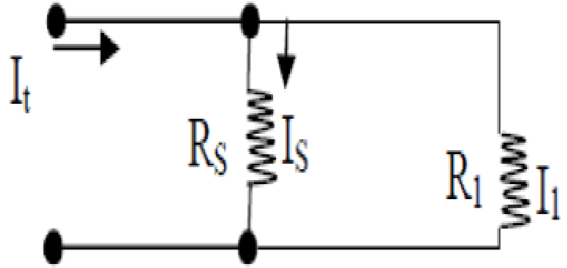
In general, if there are n resistors in series, the voltage across resistor Rx is given by

$$V_X = V_{in} \times \frac{R_X}{R_1 + R_2 + R_3 \ldots + R_n}$$

Current Divider

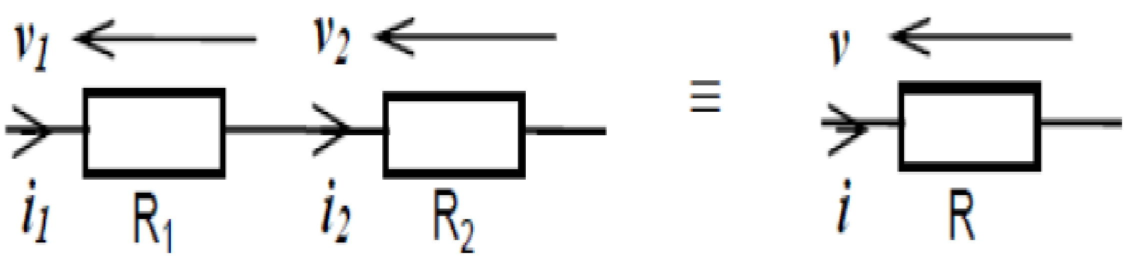
The two-resistor circuits shown in the circuit Fig.1.10 is a current divider circuit. The current through R_1 is given by,

$$I_1 = I_t \times \frac{R_S}{R_1 + R_S}$$



Series Circuits

1.6.1 Series Circuits

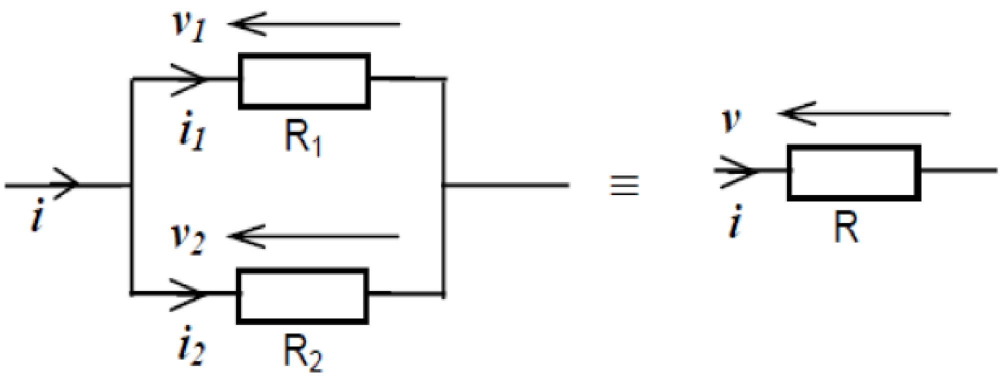


When elements are connected in series, from Kirchoff's current law, $i_1 = i_2 = i$ and from Kirchoff's Voltage Law, $v_1 + v_2 = v$. Also from Ohm's Law,

$v_1 = R_1 i_1$, $v_2 = R_2 i_2$, $v = R I$

$\therefore R_1 i + R_2 i = R i$, or $R = R_1 + R_2$

Parallel Circuits



When elements are connected in parallel, from Kirchoff's current law, $i_1 + i_2 = i$ and from Kirchoff's Voltage Law, $v_1 = v_2 = v$. Also from Ohm's Law, $v_1 = R_1 i_1$, $v_2 = R_2 i_2$, $v = R i$

$$\therefore \quad \frac{v}{R_1}+\frac{v}{R_2}=\frac{v}{R} \quad or \quad \frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \quad or \quad R=\frac{R_1R_2}{R_1+R_2}$$

$$Also, \ \frac{i_1}{i_2}=\frac{\frac{v_1}{R_1}}{\frac{v_2}{R_2}}=\frac{R_2\,v}{R_1\,v}=\frac{R_2}{R_1} \quad and \ \frac{i_1}{i}=\frac{R_2}{R_1+R_2}, \ \frac{i_2}{i}=\frac{R_1}{R_1+R_2}$$ ....current division rule

Problems on KVL & KCL

1. What is the voltage V_S across the open switch in the circuit shown in Fig. Q1?

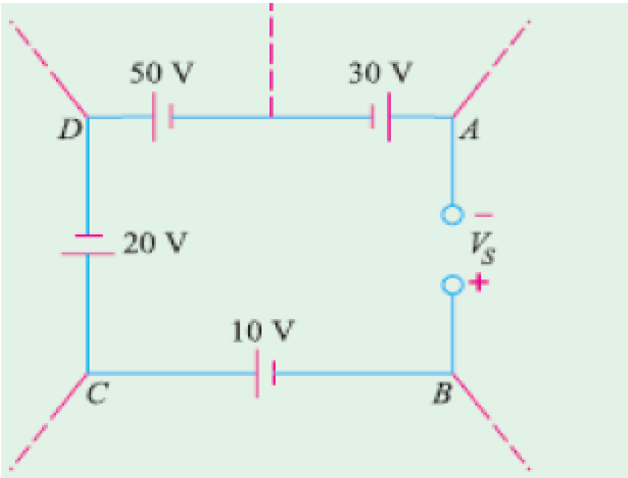


Fig. Q1

Solution

Solution:

We will apply KVL to find V_S. Starting from point A in the clockwise direction

$V_S + 10 - 20 - 50 + 30 = 0 \quad \therefore V_S = 30V$

2. Find the unknown voltage V_1 in the circuit of Fig. Q2.

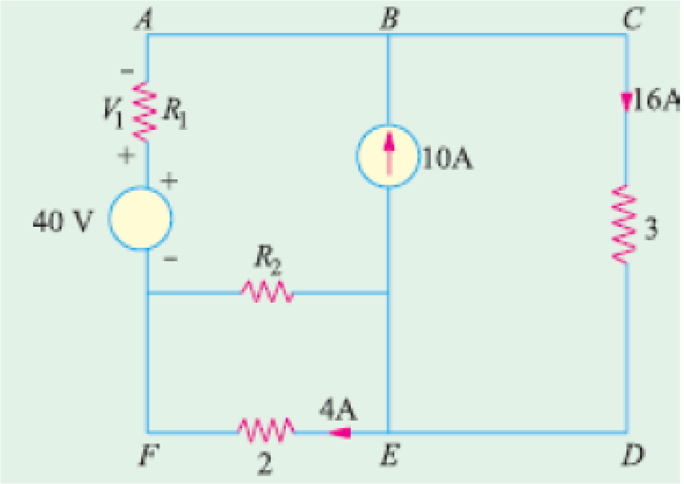


Fig. Q2

Solution:

Taking the outer closed loop ABCDEFA and applying KVL to it, we get

$$(-16 \times 3) - (4 \times 2) + 40 - V_1 = 0$$

$$\therefore V_1 = -16V$$

Problem 2

3.For the circuit shown in Fig. Q3, find V_{CE} and V_{AG}.

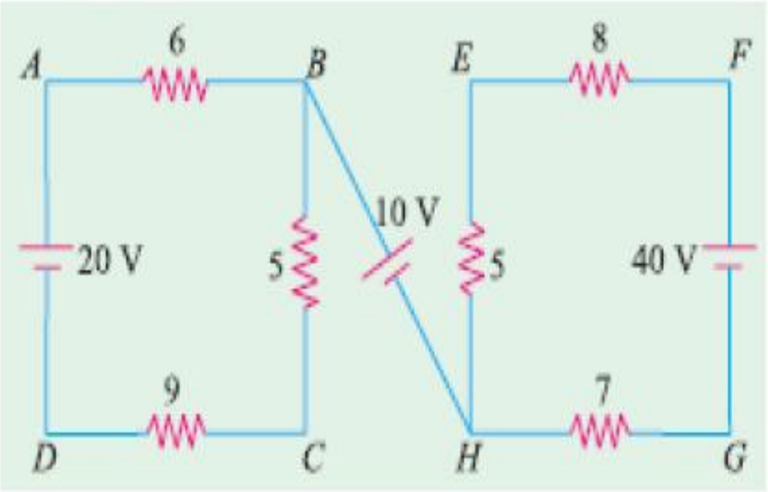


Fig. Q3

Solution 2

Solution:

Consider the two battery circuits of Fig Q3 separately. Current in the 20V battery circuit ABCD is $\frac{20}{6+5+9} = 1A$.

Similarly, current in the 40V battery circuit EFGH is $\frac{40}{5+8+7} = 2A$

For finding V_{CE}, we will find the algebraic sum of the voltage drops from point E to C via H and B.

$$\therefore V_{CE} = (-5 \times 2) + 10 - (5 \times 1) = -5V$$

The –ve sign shows that the point C is negative with respect to point E.

For finding V_{AG}, we will find the algebraic sum of the voltage drops from point E to C via H and B.

$$V_{AG} = (7 \times 2) + 10 + (6 \times 1) = 30V$$

4.Using Kirchhoff's Current Law and Ohm's law, find the magnitude and polarity of voltage V in Fig. Q4

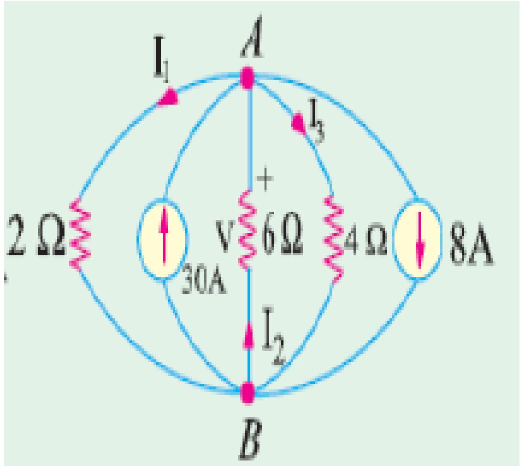


Fig. Q4

Solution:

Applying KCL to node A, we have $I_1 - I_2 + I_3 = 22$ ---(i)

Applying Ohm's law, we have

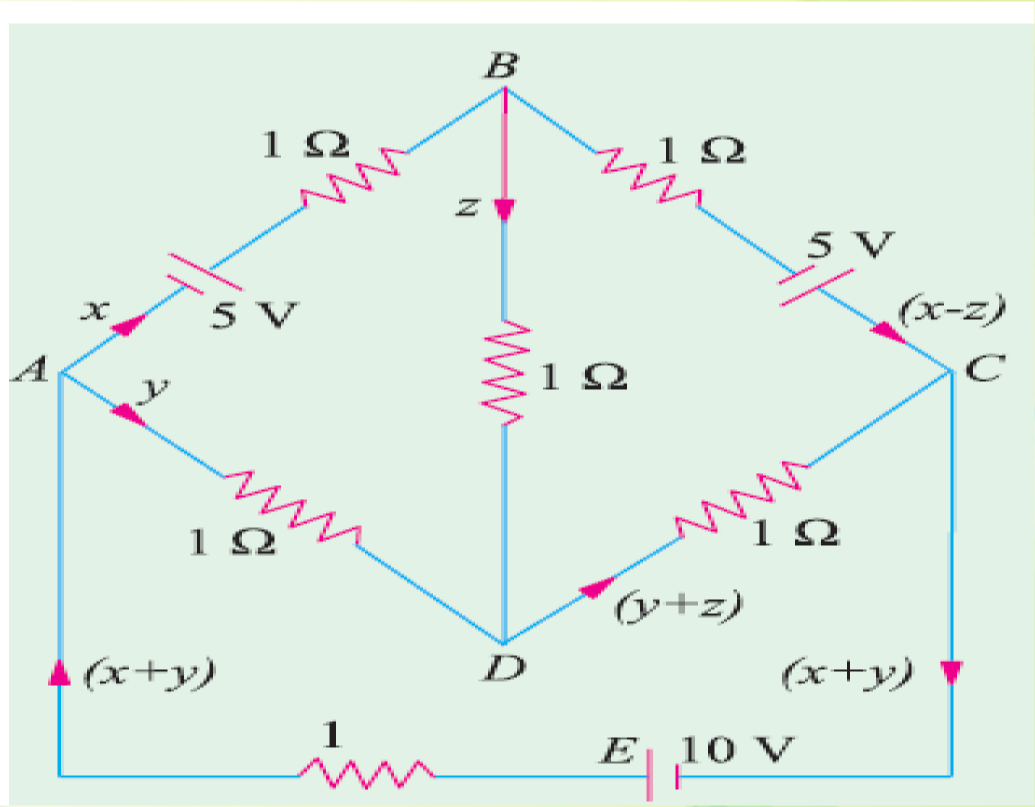
$I_1 = V/2$, $I_3 = V/4$, $I_2 = -V/6$

Substituting these values in eqn (i) ,we get V=24V

$I_1 = 12A, I_2 = -4A, I_3 = 6A$

The negative sign of I2 indicates that actual direction of its flow is opposite to that of shown in Fig. Q4.

5.Determine the branch currents in the network of Fig. Q.5.



Solution:

Apply KCL to the closed circuit ABDA, we get

$5 - x - z + y = 0 \quad or \quad x - y + z = 5$

Similarly, circuit BCDB GIVES

$-(x - z) + 5 + (y + z) + z = 0$

or $\quad x - y - 3z = 5$

From circuit ADCEA,we get

$-y - (y + z) + 10 - (x + y) = 0$

or $\quad x + 3y + z = 10$

On solving we get $z = 0, x = 6.25A$ and $y = 1.24A$

Current in branch AB = current in branch $BC = 6.25A$

Current in branch $BD = 0$; current in branch AD = current in branch $DC = 1.25A$; current in branch $CEA = 7.5A$.

Self Assessment

1.Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistors of the circuit shown in Fig. Q.6. Also determine the potential difference across the external resistors.

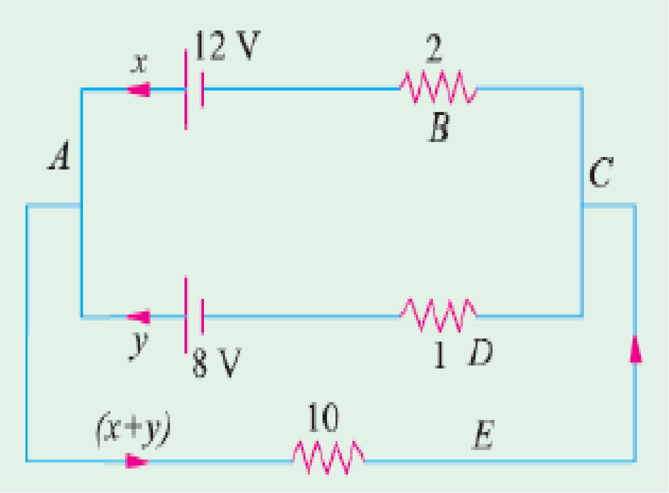
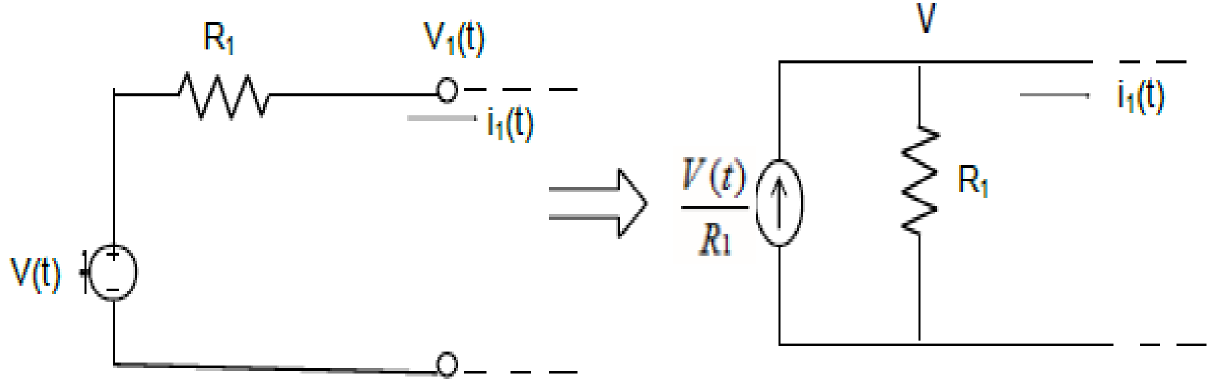


Fig. Q. 6

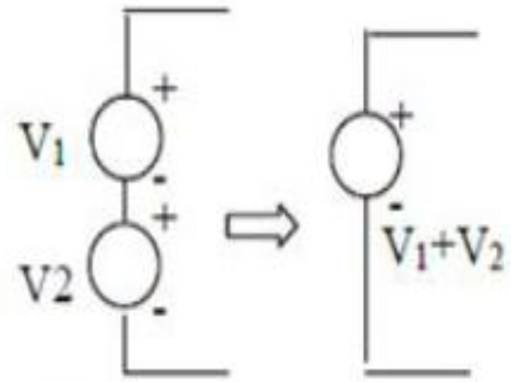
Source Transformation

In network analysis it may be required to transform a practical voltage source into its equivalent practical current source and vice versa which are depicted in Fig.1.11. These are as explained follows.

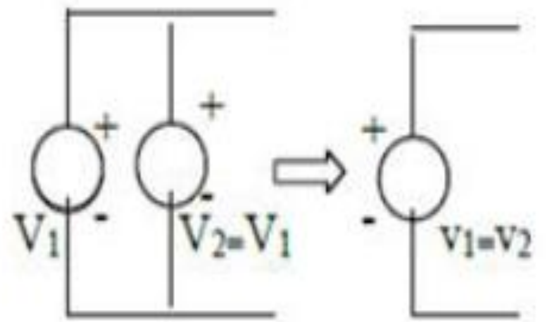


Applying KVL,

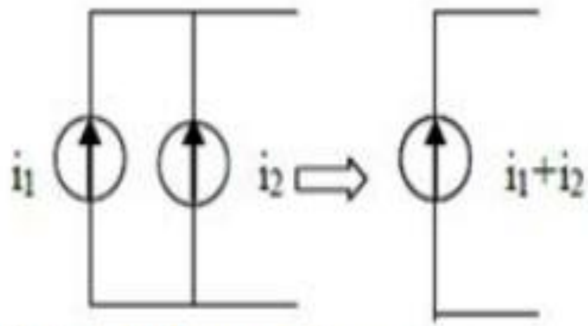
$$V(t) - i_1(t).R_1 - V_1(t) = 0 \quad \text{or} \quad i_1(t) = \frac{V(t)}{R_1} - \frac{V_1(t)}{R_1}$$



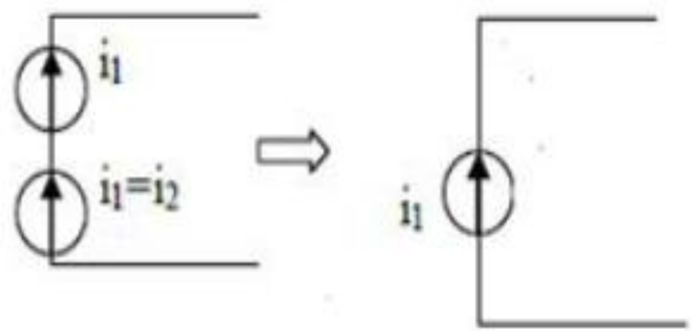
(i)Series voltage sources



(ii) Parallel voltage sources(ideal)



(iii) Parallel current sources



(iv) Series current sources(ideal)

Source Transformation

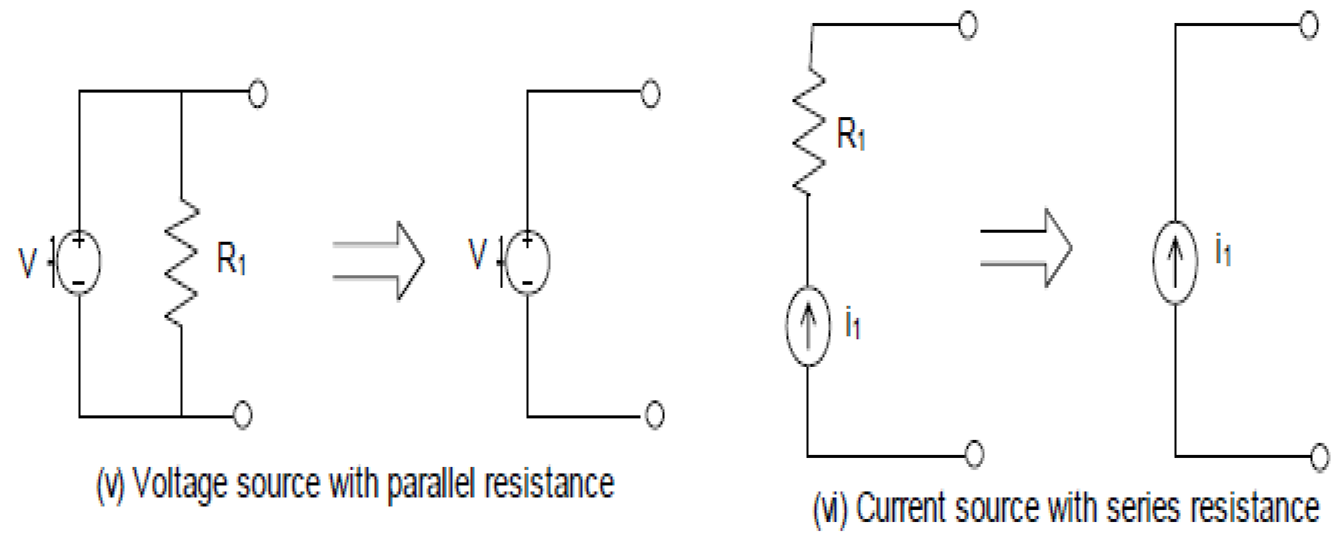


Fig.1.11: Source Transformation

Self Assessment

1.Using successive source transformation, simplify the network shown in Fig. Q8 betweenX &Y.

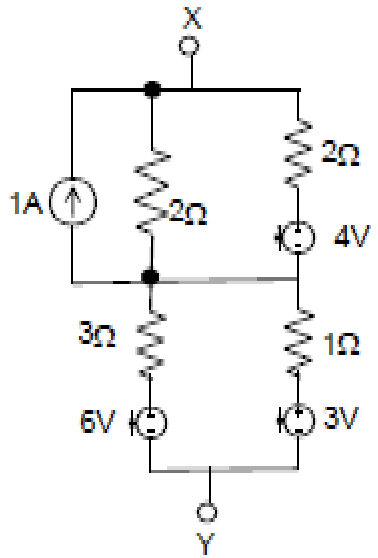
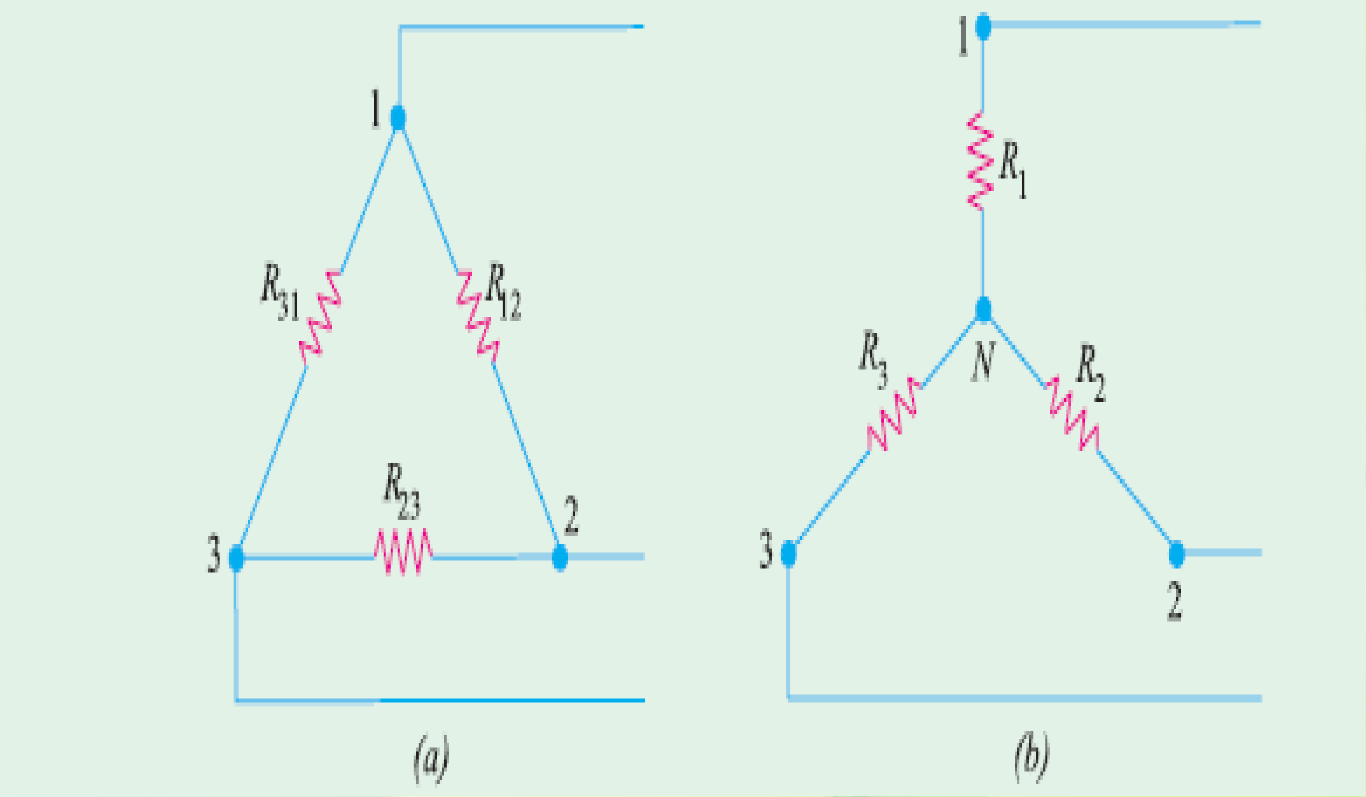


Fig.Q.8

Delta/Star Transformation

- In solving networks by the applications of Kirchhoff's laws, one sometimes experiences great difficulty due to a large number of simultaneous equations that have to be solved.
- However such complicated network can be simplified by successively replacing delta meshes by equivalent star system and vice versa.
- A delta connected network of three resistances (or impedances) R12, R23, and R31 can be transformed into a star connected network of three resistances (or impedances) R1, R2, and R3 as shown in Fig.using following transformations

Source Transformation



$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} \quad ; \quad R_2 = \frac{R_{23}R_{12}}{R_{12} + R_{23} + R_{31}} \; and \; R_3 = \frac{R_{31}R_{23}}{R_{12} + R_{23} + R_{31}}$$

Star/Delta transformation

- This transformation can be easily done by the following equations

$$R_{12} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_3}\ ;\ R_{23} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_1}\ \text{and}\ R_{31} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2}$$

- The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistances.

1. Calculate the equivalent resistance between the terminals A and B in the network shown in Fig. Q.9.

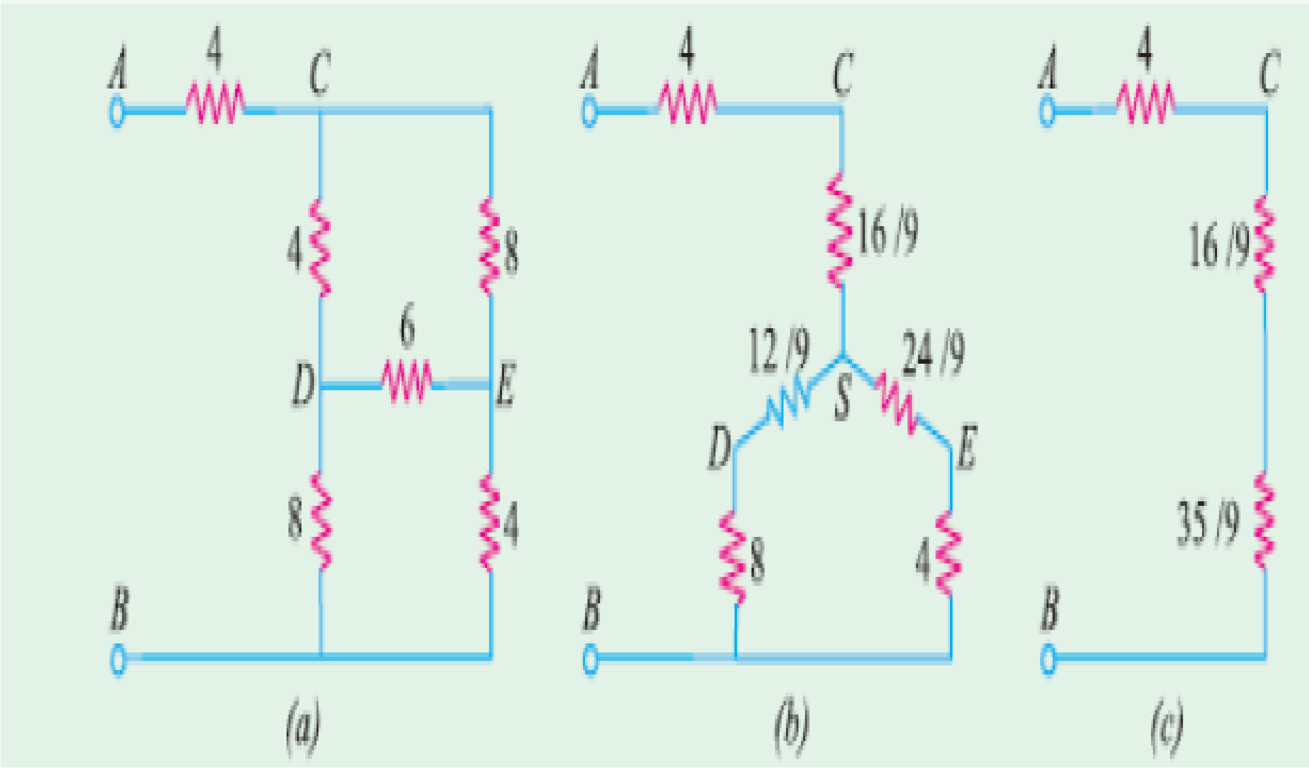


Fig. Q.9

Solution

Solution:

R_{CS}= 16/9Ω, R_{ES}=24/9Ω and R_{DS}=12/9Ω

R_{AB}= 4+(16/9)+(35/9)=87/9Ω

Self Assessment

1. Calculate the current flowing through the 10Ω resistor of Fig. Q.10.

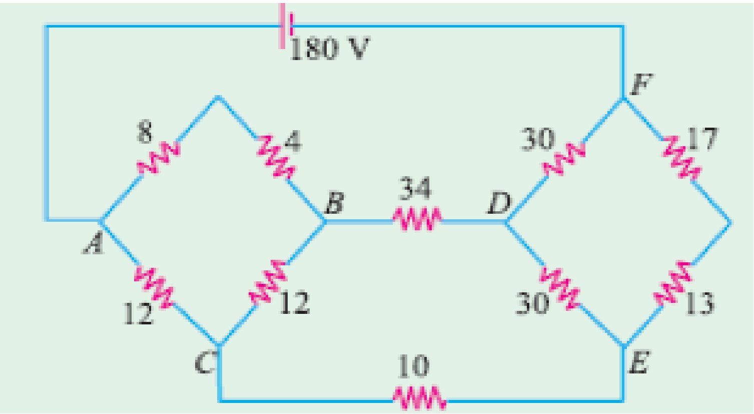


Fig. Q.10

2. A network of resistances is formed as shown in Fig.Q.11. Compute the network resistance measured between (i) A and B (ii) B and C (iii) C and A.

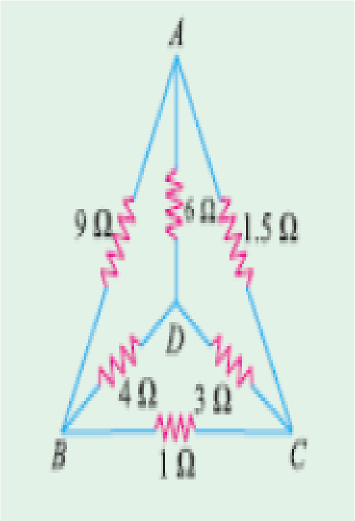


Fig. Q.11

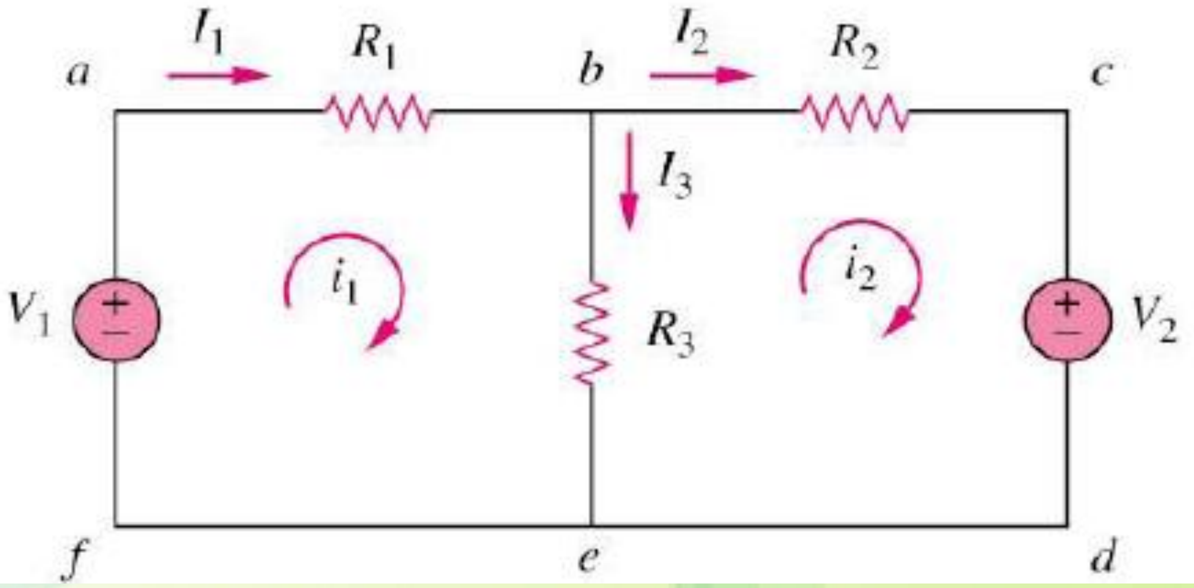
Introduction to Nodal and Mesh Analysis

- When we want to analyse a given network, we try to pick the minimum number of variables and the corresponding number of equations to keep the calculations to a minimum.

- Thus we would normally work with either currents only or voltages only. This can be achieved using these two analyses.

Mesh or Loop Analysis

Circuit with independent voltage sources



- Using KVL for the circuit as shown in Fig, at loops 1 and 2, we form KVL equations using the current and components in the loops in terms of the loop currents. Important thing to look at it is the subtraction of the opposing loop current in the shared section of the loop.

- Equations:
- $R1 \cdot i1 + (i1 – i2) \cdot R3 = V1$
- $R2 \cdot i2 + R3 \cdot (i2 – i1) = -V2$
- i..e. $(R1 + R3) \cdot i1 - i2 \cdot R3 = V1$
- $- R3 \cdot i1 + (R2 + R3) \cdot i2 = -V2$

$I_1 = i_1; I_2 = i_2; I_3 = (i_1 - i_2)$

Formalization: Network equations by inspection

$$\begin{pmatrix} (R_1 + R_3) & -R_3 \\ -R_3 & (R_2 + R_3) \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ -V_2 \end{pmatrix}$$

Impedance matrix

Excitation

Mesh currents

Use determinants and Cramer's rule for solving network equations through manipulation of their co-efficients.

Problem

1.Determine the current supplied by each battery in the circuit shown in Fig. Q.12.

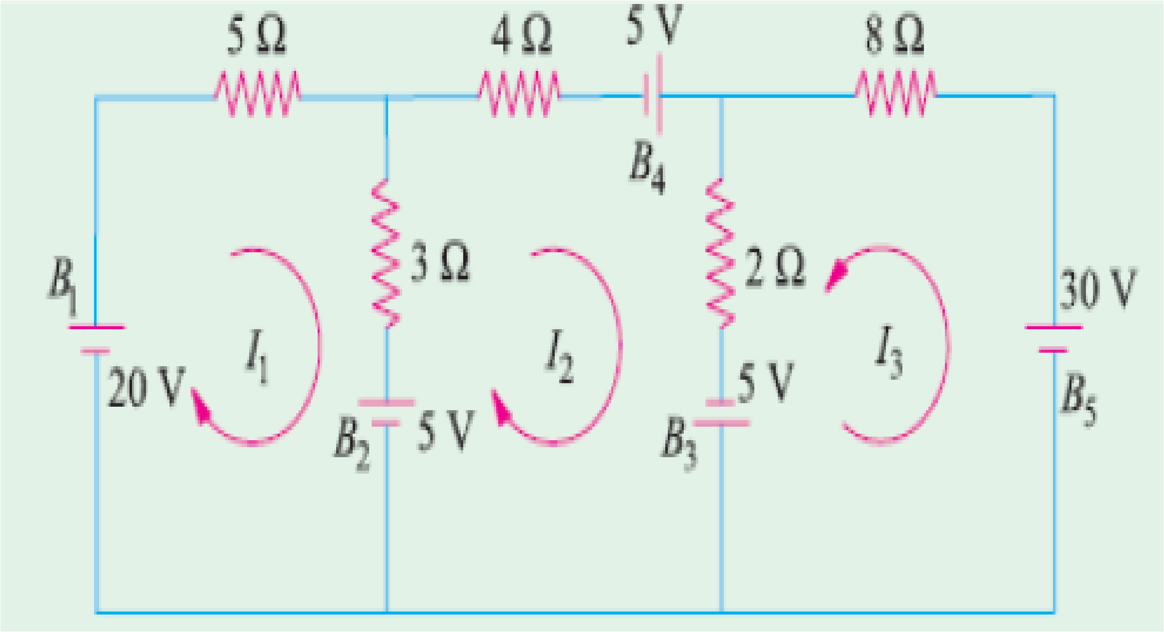


Fig. Q.12

Solution:

For loop 1 we get

$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$ or $8I_1 - 3I_2 = 15$

For loop 2 we have

$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$ or $3I_1 - 9I_2 + 2I_3 = -15$

Similarly, for loop 3, we get

$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$ or $2I_2 - 10I_3 = 35$

On solving, we get I_1 = 765/299 A, I_2 = 542/299 A and I_3 = -1875/598 A

So current supplied by each battery is

B_1=765/299 A

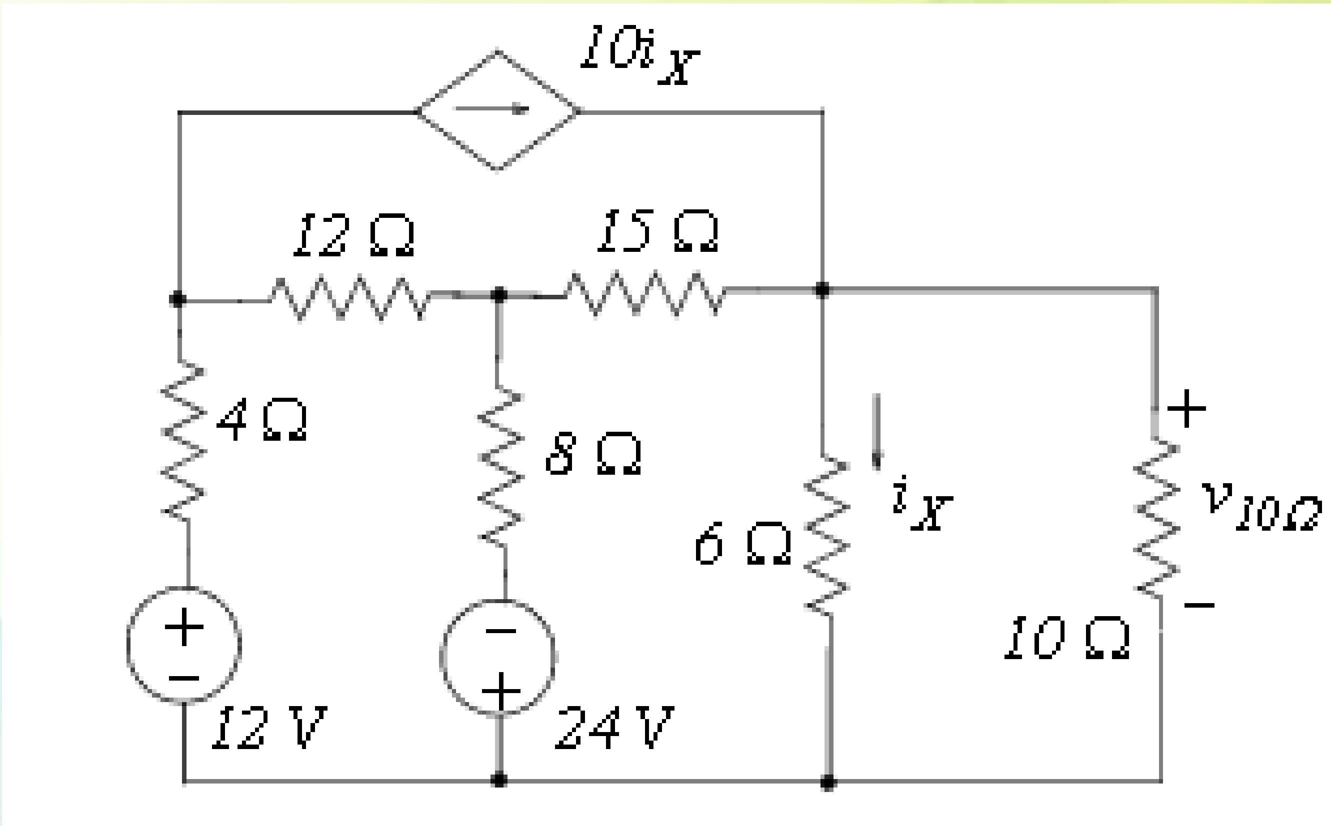
$B_3 = I_2+I_3$ = 2965/598 A

B_5 = 1875/598 A

$B_2 = I_1-I_2$=220/299 A

$B_4 = I_2$=545/299 A

Use mesh analysis to compute the voltage V10Ω in Fig. Q.13.



Solution

Solution:

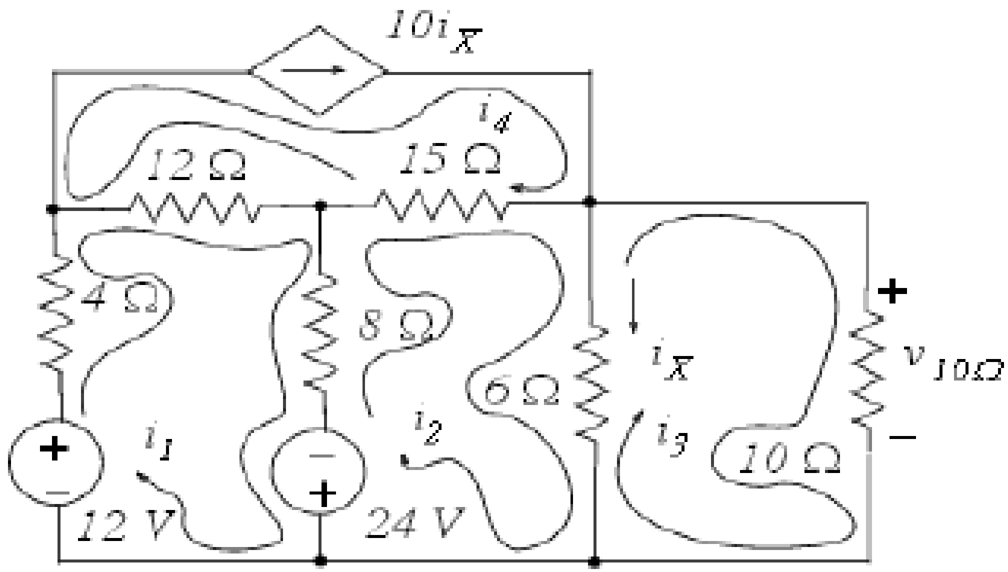


Fig. Q13.(a)

- On applying KVL to Fig. Q13.(a) , We have
- Mesh 1: $24i_1-8i_2-12i_4-24-12=0$ or $6i_1-2i_2-3i_4=9$
- Mesh 2: $-8i_1+29i_2-6i_3-15i_4=-24$
- Mesh 3: $-6i_2+16i_3=0$ or $-3i_2+8i_3=0$
- Mesh 4: $i_4=10i_x=10(i_2-i_3)$ or $10i_2-10i_3-i_4=0$

- On solving, we get
- i_1=1.94A i_2=0.13A i_3=0.05A i_4=0.79A
- Now, we find $V_{10\Omega}$ by ohm's law, that is,
- $V_{10\Omega} = 10i_3 = 10 * 0.05 = 0.5V$

Using mesh analysis, find Io for the circuit shown in Fig. Q14.

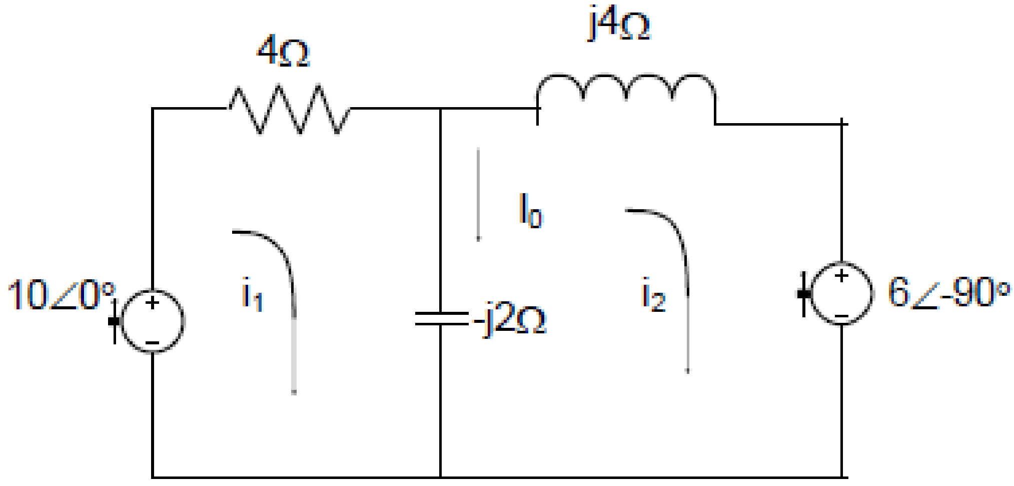


Fig. Q.14

- Solution:
- On applying KVL, we have
- Mesh 1: 10□0□ □-4i1 + j2(i1-i2) = 0 or (2-j)i1 + ji2 = 5
- Mesh 2: -j4i2 + j2(i2-i1) - 6/-90° = 0 or –j2i1 + (-j4+j2)i2 = 6□-90□ □
- I0 = (i1 - i2)
- On solving, we get
- i1= 2+j0.5
- i2=1-j0.5
- I0=1+j=1.414/45°

Nodal Analysis

The node-equation method is based directly on KCL. In nodal analysis, basically we work with a set of node voltages. It provides a general procedure for analyzing circuits using node voltages as the circuit variables.

Example

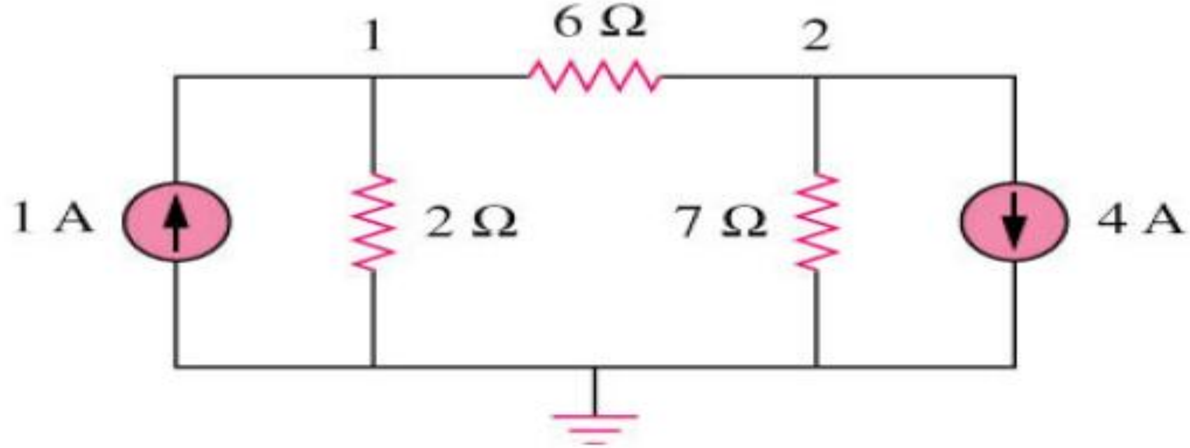


Fig.1.14: Nodal analysis for independent current sources

On applying KCL to the circuit shown in Fig.1.14, we get

At node 1

$1A = V_1/2 + (V_1-V_2)/6$ or $0.66V_1-0.166V_2 = 1A$

At node 2

$(V_1-V_2)/6 = V_2/7+4A$ or $0.166V_1 - 0.309V_2 = 4A$

On solving, we get

$V_1 = -2.01V$ and $V_2 = -14.02V$

1. Using nodal analysis, find the node voltages V_1 and V_2 in Fig. Q.15

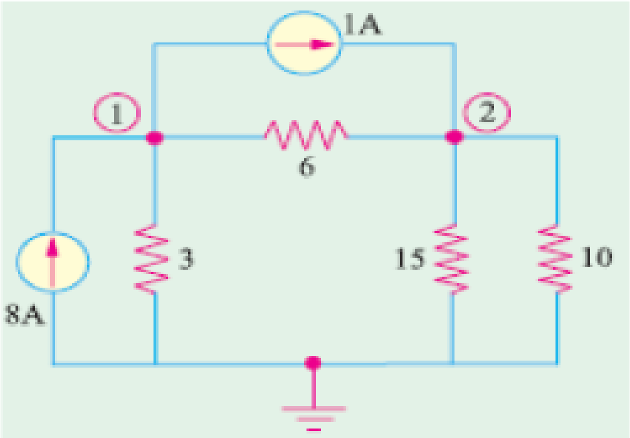


Fig. Q.15

Solution:

Applying KCL to node 1, we get
$8 - 1 - V_1/3 - (V_1 - V_2)/6 = 0$ or $3V_1 - V_2 = 42$

Similarly, applying KCL to node 2, we get
$1 + (V_1 - V_2)/6 - V_2/15 - V_2/10 = 0$ or $V_1 - 2V_2 = -6$

Solving for V_1 and V_2, we get
$V_1 = 18V$ and $V_2 = 12V$

2. Use nodal analysis to determine the value of current i in the network of Fig. Q.16

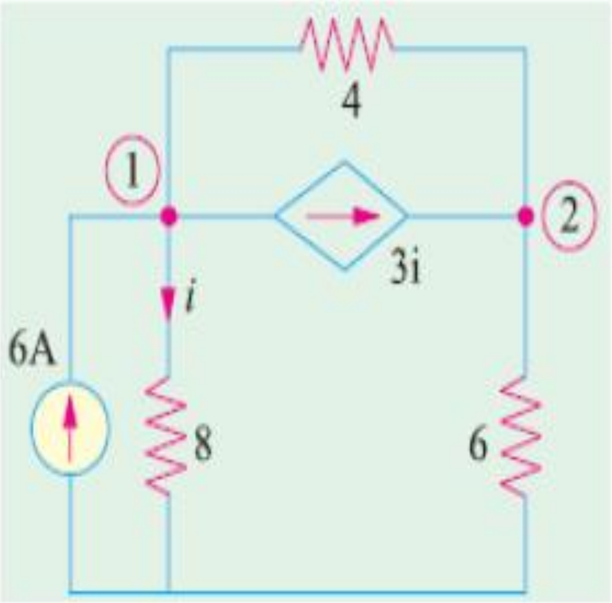


Fig. Q.16

Solution:

Applying KCL to node 1, we get

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3i$$

As seen, $i = \frac{V_1}{8}$. Hence, the above equation becomes

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3\frac{V_1}{8} \quad \text{or} \quad 3V_1 - V_2 = 24$$

Similarly, applying KCL to node 2, we get

$$\frac{V_1 - V_2}{4} + 3i = \frac{V_2}{6} \quad \text{or} \quad \frac{V_1 - V_2}{4} + 3\frac{V_1}{8} = \frac{V_2}{6} \quad \text{or} \quad 3V_1 = 2V_2$$

From the above two equations, we get

$V_1 = 16V$ $\quad \therefore i = 16/8 = 2A$

Magnetically coupled circuits:

FARADAY'S LAW

- The physical or experimental law governing the principle of magnetic induction.
- "The electromotive force (EMF) induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit."
- The EMF which is shown in Fig. 1.15 can either be produced by changing B (induced EMF) or by changing the area, e.g., by moving the wire (motional EMF).

Inductance EMF

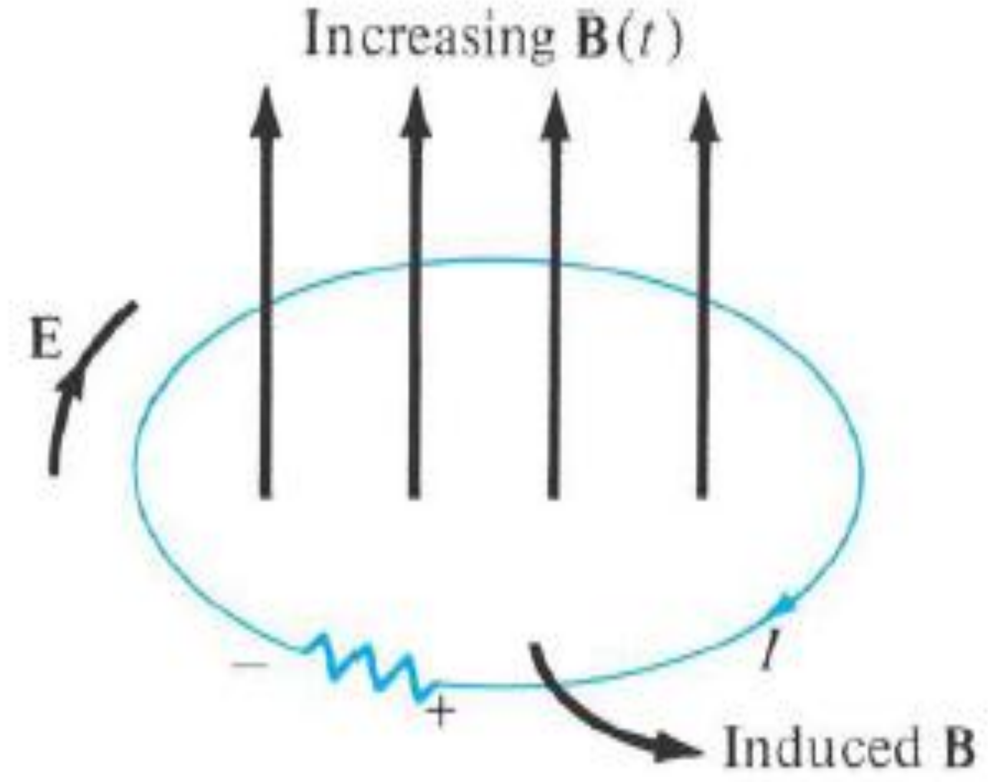
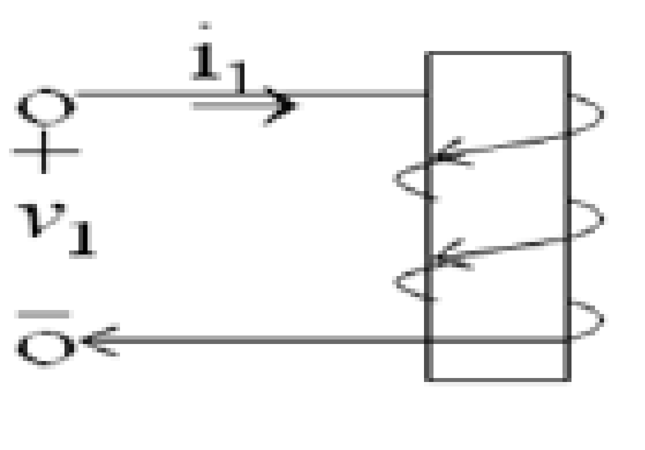
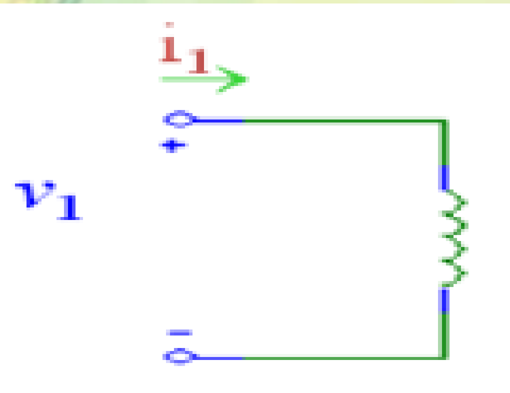


Fig. 1.15: The induced EMF

Self Inductance

According to Faraday's law, the voltage induced in a coil is proportional to the number of turns N and the time rate of change of the magnetic flux φ.





Mutual Inductance

- Two coils in a close proximity are linked together by the magnetic flux produced by current in one coil, thereby inducing voltage in the other.
- It is the ability of one inductor to induce a voltage across a neighbouring inductor, measured in henrys (H).

Queries …?