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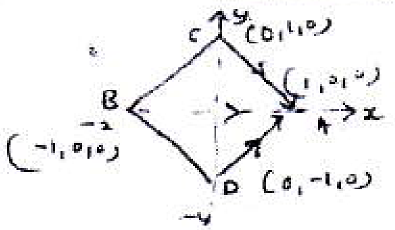
Pr. Dattath

Signature of Scrutinizer

Scheme & Solutions

Subject Title: Engineering Electromagnetics **Subject Code:** 15EC36

Question Number	Solution	Marks Allocated
1 a)	 $\overline{F_B} = \frac{Q^2}{4\pi\epsilon_0 r_B^2}\overline{a_{rB}}$ ①; $r_B = 2$	fig-01m (01)
	$\overline{F_C} = \frac{Q^2}{4\pi\epsilon_0 r_C^2}\overline{a_{rC}} = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\overline{a_x} - \overline{a_y}}{2\sqrt{2}}\right]$	2
	$\overline{F_D} = \frac{Q^2}{4\pi\epsilon_0 r_D^2}\overline{a_{rD}} = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\overline{a_x} + \overline{a_y}}{2\sqrt{2}}\right]$	2
	$\overline{F} = \overline{F_B} + \overline{F_C} + \overline{F_D} = 21.5\times10^{-6}\,\overline{a_x}\ N$	2
b)	Electric field intensity is the vector force on unit positive test charge $\overline{E} = \frac{F_t}{q_t}$ V/m	2
	Electric flux Density $\overline{D} = \frac{\psi}{S}$ C/m² The number of flux lines which pass unit area of a surface held normal to the direction of the flux lines.	2
c)	$\overline{E} = \frac{\rho_L}{2\pi\epsilon_0 R}\overline{a_R}$; $\overline{R} = \sqrt{(-2-0)^2 + (2-0)^2 + (8-8)^2}$; $R = \sqrt{8}$; $\overline{a_R} = \frac{\overline{R}}{R} = \frac{-2\overline{a_x} + 2\overline{a_y}}{\sqrt{8}}$	2
	$\overline{E} = \frac{4\times10^{-9}}{2\pi\times8.854\times10^{-12}\sqrt{8}}\left[\frac{-2\overline{a_x} + 2\overline{a_y}}{\sqrt{8}}\right]$	2
	$= -180\vec{a}_x + 180\vec{a}_y$ V/m	

Approved

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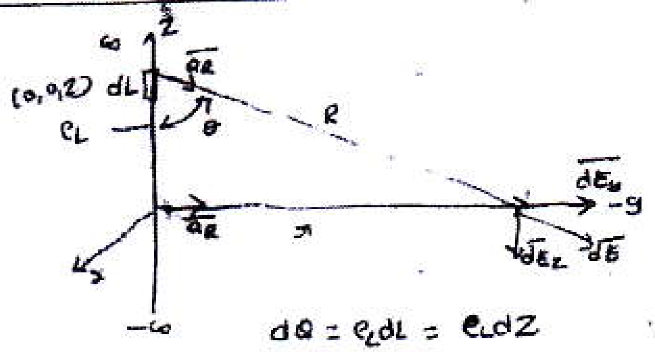
2a)



1

$dQ = \rho_L dL = \rho_L dz$ 1

$\overline{R} = r\overline{a_y} - z\overline{a_z}$, $|\overline{R}| = \sqrt{r^2+z^2}$

$$\overline{dE} = \frac{dQ}{4\pi\epsilon_0 R^2}\overline{a_R} = \frac{\rho_L dz}{4\pi\epsilon_0(\sqrt{r^2+z^2})^2}\left[\frac{r\overline{a_y} - z\overline{a_z}}{\sqrt{r^2+z^2}}\right]$$ 2

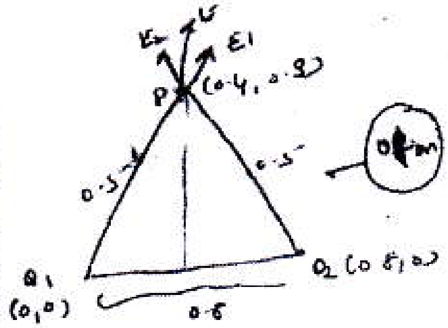
$$d\overline{E} = \frac{\rho_L dz}{4\pi\epsilon_0(\sqrt{r^2+z^2})^2}\frac{r\overline{a_y}}{\sqrt{r^2+z^2}}$$ 2

Finding E. After integration

$$\overline{E} = \frac{\rho_L}{2\pi\epsilon_0 r}\overline{a_y} \text{ V/m}$$ 2

8M

(b)



0.5m

$$\overline{E_1} = \frac{Q_1}{4\pi\epsilon(R_{1P})^2}\overline{a_{1P}}$$ 1m

$$\overline{a_{1P}} = \frac{0.4\overline{a_x} + 0.3\overline{a_y}}{0.5}$$ 1m

$$\overline{E_1} = \frac{2\times10^{-9}}{4\pi\times 8.854\times10^{-12}\times 5(0.5)^2}\left[\frac{0.4\overline{a_x}+0.3\overline{a_y}}{0.5}\right]$$ 02

$$\overline{E_1} = 28.76[0.4\overline{a_x} + 0.3\overline{a_y}]$$
$$= 11.50\overline{a_x} + 8.62\overline{a_y} \text{ V/m}$$

Similarly
$$\overline{E_2} = \frac{5\times10^{-9}}{4\pi\times5\times8.854\times10^{-12}(0.5)^2}\frac{(-0.4\overline{a_x}+0.3\overline{a_y})}{0.5}$$ 02m
$$= -28.76\overline{a_x} + 21.5\overline{a_y}$$

$$\overline{E} = \overline{E_1} + \overline{E_2} = [-17.26\overline{a_x} + 30.17\overline{a_y}]$$ 02m

Subject Title: Engineering Electromagnetics — Subject code: 10EC36

Question Number	Solution	Marks allotted

3 a)

$\vec{D} = 2xy\vec{a_x} + x^2\vec{a_y}\ C/m^2 \qquad \oint \vec{D}\cdot\vec{ds} = \int_v (\vec{\nabla}\cdot\vec{D})dv$ — **2**

$$\oint_s \vec{D}\cdot\vec{ds} = \int_0^3\int_0^2 \vec{D}\cdot(-dydz\,\vec{a_x})_{x=0} + \int_0^3\int_0^2 \vec{D}\cdot(dydz\,\vec{a_x})_{x=1}$$

$$+ \int_0^3\int_0^1 \vec{D}\cdot(-dxdz\,\vec{a_y})_{y=0} + \int_0^3\int_0^1 \vec{D}\cdot dxdz\,\vec{a_y}\,_{y=2}$$

$$= -\int_0^3\int_0^2 D_x\,dydz\,_{x=0} + \int_0^3\int_0^2 D_x\,dydz\,_{x=1} - \int_0^3\int_0^1 D_y\,dxdz\,_{y=0}$$

$$+ \int_0^3\int_0^1 D_y\,dxdz\,_{y=2}$$ — **3**

D_x at $x=0$ = 0 and D_y at $y=0$ = D_y at $y=2$ needs to be evaluated

$$\oint_s \vec{D}\cdot\vec{ds} = \int_0^3\int_0^2 D_x\,dydz\,_{x=1} = \int_0^3\int_0^2 2y\,dydz$$

$$= \int_0^3 4dz = 12 \quad -(1)$$

$$\vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}(2xy) + \frac{\partial}{\partial y}(x^2) = 2y$$

volume integral is: — **3**

$$\int_v (\vec{\nabla}\cdot\vec{D})dv = \int_0^3\int_0^2\int_0^1 2y\,dxdydz = \int_0^3\int_0^2 2y\,dydz$$

$$= \int_0^3 4dz = 12 \quad -(2)$$

from (1) and (2) divergence theorem is verified.

Question Number	Solution	Marks Allocated
3a)	~~given $\vec{J} = 2ay \hat{a}_x$ ... $1m^2$~~ ~~$\oint \vec{J}\cdot d\vec{s} = \int (\nabla\cdot J) dv$~~ ~~$\oint \vec{J}\cdot d\vec{s} = 1\alpha$~~ ~~$\int (\nabla\cdot J) dv = 1\alpha$~~ ~~LHS = RHS.~~	
b)	Equation of Continuity. The current through closed surface is $I = \oint_s J\cdot ds$ —(1) outward flow must be balanced by decrease of positive charge $I = \oint_s J\cdot ds = -\frac{dQ_i}{dt}$ —(2) Apply Divergence theorem $\int_s J\cdot ds = \iiint (\nabla\cdot J) dv$ —(3) $\int_{vol} (\nabla\cdot J) dv = -\frac{d}{dt}\int_{vol} \rho_v dv$ —(4) $\int_{v} (\nabla\cdot J) dv = -\int_{vol} \frac{\partial \rho_v}{\partial t} dv$ —(5) $(\nabla\cdot J) dv = -\frac{\partial \rho_v}{\partial t} dv$ —(6) $\nabla\cdot J = -\frac{\partial \rho_v}{\partial t}$ —(7)	1 1 2 2
c)	$I = \int_s \vec{J}\cdot\vec{ds} = \int_0^{2\pi}\int_2^{2.8} 10e^3 z\, d\phi\, dz \times 10^{-3} = 3.26A$	2M
4a)	Gauss law in point form: The divergence of electric displacement density in a medium at a point is equal to the charge/unit volume at the same point By gauss law $\oint \vec{D}\cdot \vec{ds} = \Delta Q$ $\therefore \frac{\oint \vec{D}\cdot\vec{ds}}{\Delta v} = \frac{\Delta Q}{\Delta v}$ $\Delta v \to 0$ $\frac{\oint \vec{D}\cdot ds}{\Delta v} = \lim_{\Delta v\to 0}\left(\frac{\Delta Q}{\Delta v}\right)$ $\nabla\cdot D = \rho_v$ or $\vec{\nabla}\cdot\vec{D} = \rho_v$	2 2 1

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Question Number	Solution	Marks Allocated

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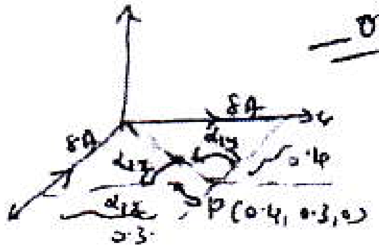
5 a) Uniqueness theorem: Under the given boundary condition, Laplace's equation has one and only one solution. The same holds good for Poisson's equation also. — 2

$V_1 = V_2$ proof — — — — 6

b)



— 02 marks

Consider current along x axis for which $\alpha_{1x} = -90°$

$\alpha_{2x} = \tan^{-1}\frac{0.4}{0.3} = 53.12°$

ρ = radial distance measured from x axis = 0.3

$\bar{H} = \frac{I}{4\pi\rho}[\sin\alpha_2 - \sin\alpha_1]\,\bar{a_\phi}$

$\bar{H_x} = \frac{8}{4\pi\rho}[\sin\alpha_{2x} - \sin\alpha_{1x}]\,\bar{a_\phi}$

$= 3.8197\,\bar{a_\phi}$

unit vector a_ϕ referred to x axis is $-\bar{a_z}$

$\bar{H_x} = -3.8197\,\bar{a_z}$ A/m — 02 marks

Similarly on the y axis $\alpha_{1y} = -\tan^{-1}\left(\frac{0.3}{0.4}\right)$

$\alpha_{1y} = -36.86°$ $\alpha_{2y} = 90°$ where $\rho = 0.4$

$\bar{H_y} = \frac{8}{4\pi\times0.4}\left[\sin 90° - \sin(-36.86)\right]$

$= 2.54\,\bar{a_\phi}$

unit vector $\bar{a_\phi}$ referred to y axis is $-\bar{a_z}$

$\bar{H_y} = -2.54\,\bar{a_z}$ A/m

$\bar{H} = \bar{H_x} + \bar{H_y} = -6.36\,\bar{a_z}$ A/m — 4

6(a) $V = 6\rho\phi z$

for P $x = 0.5,\ y = 1.5,\ z = 1$

in cylindrical system $\phi = \tan^{-1}(y/x)$

$= 71.56$

$r = \sqrt{x^2+y^2}$ (2)

P(1.58, $\phi = 71.56, z = 1$) $= 1.5811$

ϕ in radians $= \frac{\pi \times 71.56}{180} = 1.249$ rad

$V = 6 \times 1.5811 \times 1.249 \times 1 = 11.84\,V$ (1)

$E = -\nabla V$

$= -\left(\frac{\partial V}{\partial r}\bar{a_r} + \frac{1}{r}\frac{\partial V}{\partial \phi}a_\phi + \frac{\partial V}{\partial z}\bar{a_z}\right)$ (1)

$= -6\phi z\,\bar{a_r} - 6z\,\bar{a_\phi} - 6r\phi\,\bar{a_z}$ (1)

$D = \epsilon_0 \vec{E}$ (1)

$\rho_v = \nabla \cdot D = \frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

$= \frac{1}{r}\times(-6\phi z\epsilon_0) + \frac{1}{r}\times 0 + 0$ (1)

$= \frac{-6\phi z\epsilon_0}{r}$

$\rho_v = \frac{-6 \times 1.249 \times 1 \times 8.854\times10^{-12}}{1.5811}$ (1)

$= -41.96\,pC/m^3$

(b) Scalar magnetic potential

V_m is scalar magnetic potential.

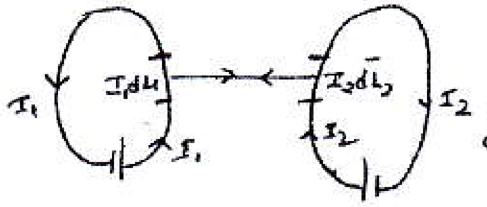
$\nabla \times \nabla V_m = 0$

$H = -\nabla V_m$ (4)

$\nabla(-\bar{H}) = 0$ $\quad \nabla \times H = 0$

But $\nabla \times \bar{H} = J$ $\quad$ ie $J = 0$

Scalar magnetic potential V_m can be defined for source free region where current density is

Question Number	Solution	Marks Allocated
	Vector magnetic potential	
	It is denoted by $\overline{A}$	
	$\nabla\cdot(\nabla\times\overline{A})=0$, $\nabla\cdot B=0$	
	$\overline{B}=\nabla\times\overline{A}$.	
	w.k.t $\nabla\times H=J$, $\nabla\times\frac{B}{\mu_0}=J$, $\nabla\times\overline{B}=\mu_0\overline{J}$	
	$\nabla\times\nabla\times\overline{A}=\mu_0\overline{J}$	(4)
	using vector identity $\nabla(\nabla\cdot\overline{A})-\nabla^2A=\mu_0 J$	
	$J=\frac{1}{\mu_0}(\nabla\times\nabla\times\overline{A})=\frac{1}{\mu_0}\left[\nabla(\nabla\cdot\overline{A})-\nabla^2\overline{A}\right]$	
7(a)	Force between Differential current Elements	
		
	$d(dF_1)=I_1 dL_1\times d\overline{B_2}$	
	$\overline{dB_2}=\mu_0\, d\overline{H_2}$	
	$=\mu_0\left[\frac{I_2\, dL_2\, a_{R21}}{4\pi R_{21}^2}\right]$	
	– 01 marks	
	$d[dF_1]=\frac{\mu_0 I\,\overline{dL_1}\times(I_2\, dL_2\times a_{R21})}{4\pi R_{21}^2}$	- 2 marks
	$F_1=\frac{\mu_0 I_1 I_2}{4\pi}\oint_{L1}\oint_{L2}\frac{\overline{dL_1}\times(\overline{dL_2}\times\overline{a_{R21}})}{R_{21}^2}$	3.
	$\overline{F_2}=\frac{\mu_0 I_1 I_2}{4\pi}\oint_{L2}\oint_{L1}\frac{dL_2\times(\overline{dL_1}\times\overline{a_{R12}})}{R_{12}^2}$	
(b)	(i) $\mu=1.8\times10^{-5}$ & $H=120$ A/m	
	$M=(\mu_r-1)H=\left(\frac{\mu}{\mu_0}-1\right)H=1599$ A/M	(2)
	(ii) $M=(8.3\times10^{28})(4.5\times10^{-27})$	(2)
	$=373.5$ A/M	

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(4b) For movement of charge from (2,0,0) to (0,0,0) only x coordinate value is changing from (2 to 0)

$$dW_x = -Q\left[E_x \vec{a_x} \cdot dx \vec{a_x}\right]$$

$$\vec{E} = 2x\vec{a_x} - 4y\vec{a_y} \quad , \quad \vec{E_x} = 2x\vec{a_x}$$

$$dW_x = -4x\,dx$$ (2M)

movement from (0,0,0) to (0,2,0)

$$dW_y = -Q\left[E_y \vec{a_y} \cdot dy \vec{a_y}\right]$$

$$\vec{E_y} = -4y\vec{a_y}$$

$$dW_y = -Q\left[-4y\vec{a_y} \cdot dy\vec{a_y}\right]$$

$$= 8y\,dy$$ (2M)

$dW_z = 0$ (no z component)

$$W = \int dW_x + \int dW_y + \int dW_z$$

$$W = -\int_2^0 4x\,dx + \int_0^2 8y\,dy + 0 = \underline{24\,J}$$ (1)

(c)

$$V = \frac{60\sin\theta}{r^2}$$

$$\vec{E} = -\left[\frac{\partial V}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{a_\phi}\right]$$ 1

$$\vec{E} = -\left[\frac{-120\sin\theta}{r^3}\vec{a_r} + \frac{60\cos\theta}{r^3}\vec{a_\theta} + 0\right]$$ 3

$$\vec{D} = \epsilon_0\vec{E} = \frac{60\epsilon_0}{r^3}\left[2\sin\theta\,\vec{a_r} + \cos\theta\,\vec{a_\theta}\right]$$ 1

$$\vec{D} = \cancel{+\,...\times 10^{-10}\,\vec{a_r} + 2.95\times 10^{-11}\,\vec{a_\theta}}$$ 1

$\boxed{OR}$

$$\vec{D} = 34.86\,\vec{a_r} - 9.835\,\vec{a_\theta}\ \text{pC/m}^2$$

Question Number	Solution	Marks Allocated
	$\bar{F} = ILB\sin\theta$	
(b)	The normal component $\overline{B_{N1}} = (\overline{B_1}\cdot\overline{a_{N12}})\overline{a_{N12}}$	
	$z=0$, there is medium 2 and above $z=0$ there is medium 1. The field vector travels from medium 1 to medium 2	(2)
	$\overline{a_{N12}} = -\overline{a_z}$	
	$\overline{B_{N1}} = [(2\overline{a_x} - 3\overline{a_y} + \overline{a_z})\cdot(-\overline{a_z})]\cdot(-\overline{a_z})$	(1)
	$= \overline{a_z}\ mT$	
	Tangential component of $\overline{B_1}$ is	
	$\overline{B_{tan1}} = \overline{B_1} - \overline{B_{N1}}$	
	$= (2\overline{a_x} - 3\overline{a_y} + \overline{a_z} - \overline{a_z})\times 10^{-3}$	
	$= 2\overline{a_x} - 3\overline{a_y}\ mT$	(1)
	using B.C $\overline{B_{N2}} = \overline{B_{N1}} = \overline{a_z}\ mT$	
	$\overline{H_{tan1}} - \overline{H_{tan2}} = \overline{a_{N12}}\times\bar{K}$,	
	$\overline{H_{tan1}} = \dfrac{B_{tan1}}{\mu_0\mu_{r1}}$	(1)
	$= 500\overline{a_x} - 750\overline{a_y}\ A/m$	
	putting value of $\overline{H_{tan1}}$ in above expression,	(1)
	$[(500\overline{a_x} - 750\overline{a_y}) - \overline{H_{tan2}}] = (-\overline{a_z})\times(100\overline{a_x})$	
	$\overline{H_{tan2}} = 500\overline{a_x} - 650\overline{a_y}\ A/m$	
	$\overline{B_{tan2}} = \mu_2 H_{tan2} = (6\times10^{-6})(500\overline{a_x} - 650\overline{a_y})$	(1)
	$\overline{B_{tan2}} = (3\overline{a_x} - 3.9\overline{a_y})\ mT$	
	$B_2 = \overline{B_{tan2}} + \overline{B_{N2}}$	(1)
	$= (3\overline{a_x} - 3.9\overline{a_y} + \overline{a_z})\ mT$	

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(iii) $B = 300 \times 10^{-6}\, T \quad \chi_m = 15$

$B = \mu_0 \mu_r H$

$\chi_m = M/H$ or $H = \frac{M}{\chi_m}$

$B = \frac{\mu_0 \mu_r M}{\chi_m}$ (2)

$M = \frac{(300 \times 10^{-6}) \times 15}{4\pi \times 10^{-7} \times (15+1)} = 224\ A/m$

(c) $\bar{F} = I \bar{dL} \times \bar{B}$

$I = 10A, \ \bar{dL} = 4\bar{a_y} \quad \bar{B} = 0.05 \bar{a_z}$ (2)

$\bar{F} = [10(4\bar{a_y}) \times 0.05 \bar{a_z})]$ (2)

$= 2(-\bar{a_x})\, N$

8 a) Consider a differential element of length dL of the conductor. If ρ_v is the charge/unit volume in the conductor then the charge dQ in the differential element is ← 02 mark

$dQ =$ Charge density × volume

$dQ = \rho_v dv$

$dv = A \cdot dL$ (A is the area of conductor) (2)

$dQ = \rho_v A\, dL$ ——— 03M

I be the current flowing in the conductor

$\bar{dF} = dQ(\vec{v} \times \vec{B}) = dL \rho_v A (\vec{v} \times \vec{B})$

$\bar{dF} = \rho_v v A (\bar{dL} \times \vec{B})$

But $\rho_v v A = I$ current flow

WKT $\bar{dF} = I(\bar{dL} \times \vec{B})$

$\bar{dF} = I\, dL\, B \sin\theta$ ——— 03 mark

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10 a) Statement: The energy dissipated in a given volume free of any sources is equal to the sum of, the rate at which the decrease in electric and magnetic energies stored in the volume takes place, and the rate at which energy in flow occurs through its surface (2)

$$-\oint_{area} (E \times H)\, ds = \int_{vol} J \cdot E\, dv + \frac{d}{dt}\int_{vol} \frac{1}{2} D \cdot E\, dv + \frac{d}{dt}\int_{vol} \frac{1}{2} B \cdot H\, dv$$

Proof. — (6)

(b) using Ampere circuital law (1)

$$\nabla \times H = \frac{\partial D}{\partial t} = J_D$$

$$\nabla \times \bar{H} = \begin{vmatrix} \bar{a_x} & \bar{a_y} & \bar{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 10^6 \cos(377t + 1.25\times10^{-6} z) & 0 \end{vmatrix}$$ (1)

(1)

$$= 1.25 \sin(377t + 1.2566\times10^{-6} z)\, \bar{a_x}$$

$J_D' = \frac{\partial D}{\partial t} = \nabla \times \bar{H}$ for free space (1)

$$J_D = 1.2566 \sin(377t + 1.25\times10^{-6} z)\, \bar{a_x} \text{ A/m}^2$$

Amplitude is 1.2566 A/m² (1)

(c) $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi f (\mu_0 \mu_r)\sigma}}$ (1)

$\sqrt{\sigma} = 5.0329$ ℧/m, $\sigma = 25.3302$ ℧/m (2)

9(a) $\nabla \times H = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}\epsilon E = \epsilon\frac{\partial E}{\partial t}$

LHS $\nabla \times \overline{H} = \begin{vmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & y+2\times10^6 t \end{vmatrix}$ (4)

$= \underline{\overline{a_x}}$

RHS $= \epsilon\frac{\partial}{\partial t}E = \epsilon\frac{\partial}{\partial t}[20y - kt]\overline{a_x}$

$= \epsilon\frac{\partial}{\partial t}(-kt)\overline{a_x}$

$= -k\epsilon\overline{a_x}$

LHS = RHS (4)

$-k\epsilon\,\overline{a_x} = \overline{a_x}$

$k = -1/\epsilon = -\frac{1}{4\times10^{-9}} = \underline{-2.5\times10^8}$

(b) ratio $\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2\pi f\epsilon_0\epsilon_r} = \underline{1.12}$ (2)

So it is a conductor

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi f\mu\sigma}{2}} = 2513$ (2)

$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ = 35.5\angle 45^\circ$ (1)

$v = \frac{\omega}{\beta} = \frac{2\pi\times16\times10^{6}}{2513} = 4\times10^7$ m/s (1)

$\delta = \frac{1}{\sqrt{\pi f\mu_0\mu_r\sigma}} = \cancel{0.0398m}$ (2)

0.3978mm