Module 1 (Dec-2017/Jan 2018)

Q: 1.a. State and explain Coulomb's law in vector form (05M).

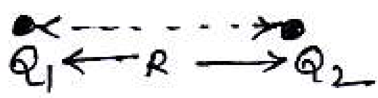
Ans: Coulomb's law states that the force of attraction or repulsion between the two point charges Q_1 & Q_2 is,

i> directly proportional to the product of charges Q_1 & Q_2.

ii> Inversely proportional to the square of the distance between the charges &

iii> Acts along the line joining the two charges.

Consider the two point charges Q_1 and Q_2 as shown below separated by the distance R.



$$F \propto \frac{Q_1 Q_2}{R^2}$$

$$F = \frac{k Q_1 Q_2}{R^2}$$

Where $k = \frac{1}{4\pi\varepsilon}$ - Constant of proportionality

Here ε - permittivity of the medium in which charges are placed.

$$\varepsilon = \varepsilon_r \varepsilon_0$$

ε_r - relative permittivity of the media.

& ε_0 - permittivity of free space

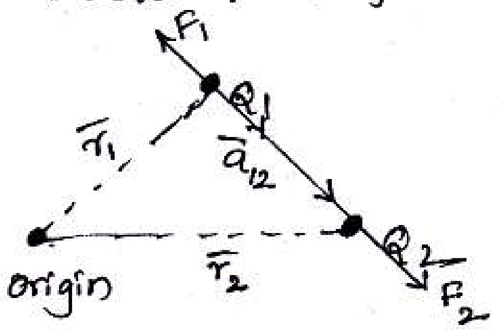
$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}.$$

$$\frac{1}{4\pi\varepsilon_0} = 8.98 \times 10^{9} \text{ F/m}.$$

When both charges are of same polarity force between them is repulsive and when they are of opposite polarity force is attractive.

To learn the vector force between them, Let's consider the arrangement as in next fig.

Vector form of Coulomb's law.



The charges Q_1 and Q_2 are located at the places $\bar{r}_1$ and $\bar{r}_2$ distance away from origin.

Let's find the force exerted at Q_2 charge due to the Q_1 charge. Here Force is directive and acts along the line joining Q_1 to Q_2, given by $\bar{F}_{12}$ or $\bar{F}_2$

$$\bar{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\bar{a}_{12} \qquad (\because R_{12} = |R|$$

$$\bar{a}_{12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|}$$

$$F_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 |R_{12}|^2}\frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|} \text{ N.}$$

If we find the force at Q_1 due to Q_2 than,

$$\bar{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\bar{a}_{21}$$

$$\bar{a}_{21} = \frac{\bar{R}_{21}}{|\bar{R}_{21}|} = \frac{\bar{r}_1 - \bar{r}_2}{|\bar{r}_1 - \bar{r}_2|}$$

$$\& \quad \bar{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 |R_{21}|^2}\frac{\bar{r}_1 - \bar{r}_2}{|\bar{r}_1 - \bar{r}_2|} \text{ N.}$$

$$\& \quad \bar{F}_2 = -\bar{F}_1$$

which indicates the direction of the force opposite when we change the calculating charge.

Q:1, b. Find the electric field $\bar{E}$ at the origin, if the following charge distributions are present in free space.

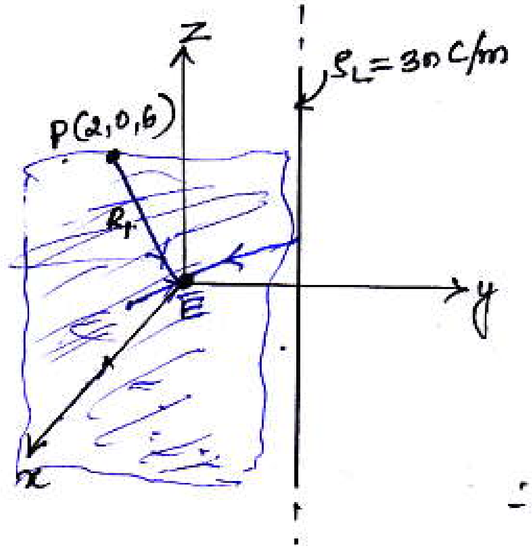
i) point charge 12nC at P(2, 0, 6)

ii) Uniform line charge of linear charge density 3nC/m at $x = 2, y = 3$

Sol^n : $Q = 12\,nC$ at $P(2,0,6)$

Line charge along $x=2, y=3$ point means lies along z-axis, & $\rho_L = 3\,nC/m$.

Sheet charge at $x = 2$, $\rho_s = 0.2\,nC/m^2$



Total $\bar{E}$ at point $(0,0,0)$ i.e. origin is,

$$\bar{E} = \bar{E}_Q + \bar{E}_l + \bar{E}_s$$

$\bar{E}$ due to point charge is,

$$E_Q = \frac{Q}{4\pi\varepsilon_0 R_1^2}\bar{a}_{r_1}$$

R_1 is directed from point P to O

$\therefore\ \bar{R}_1 = O - P$ ie. $(0,0,0) - (2,0,6)$

$$\bar{R}_1 = -2\bar{a}_x - 6\bar{a}_z$$

$$|\bar{R}_1| = \sqrt{4+36} = \sqrt{40}$$

$$\bar{a}_r = \frac{-2\bar{a}_x - 6\bar{a}_z}{\sqrt{40}}$$

$$\bar{E}_Q = \frac{12\times10^{-9}\times 8.98\times10^{9}}{(40)^{3/2}}\cdot\{-2\bar{a}_x - 6\bar{a}_z\}$$

$$= -0.8526\bar{a}_x - 2.555\bar{a}_z\ V/m.$$

$\bar{E}$ due to line charge is,

$$\bar{E}_l = \frac{\rho_L}{2\pi\varepsilon_0 r_{l0}}\bar{a}_r$$

$\bar{r}_{l0} = 0 -$ line charge point $(2,3,z)$

$$\bar{r}_{l0} = -2\bar{a}_x - 3\bar{a}_y$$

As line charge is along the z-axis, z component will be absent in the field.

$$|\bar{r}_{l0}| = \sqrt{4+9} = \sqrt{13}$$

$$\bar{a}_r = \frac{-2\bar{a}_x - 3\bar{a}_y}{\sqrt{13}}$$

$$\therefore\ \bar{E}_l = \frac{3\times10^{-9}\{-2\bar{a}_x - 3\bar{a}_y\}}{2\times\pi\times8.854\times10^{-12}\times\sqrt{13}\sqrt{13}} = -8.29\bar{a}_x - 12.44\bar{a}_y\ V/m$$

$\bar{E}$ due surface charge is,

$$\bar{E}_s = \frac{\rho_s}{2\varepsilon_0}(-\bar{a}_x) \qquad (\because \text{ as vector runs from } z=2 \text{ to origin, } -x \text{ direction}$$

$$\bar{E}_s = \frac{0.2\times10^{-9}}{2\times8.854\times10^{-12}}(-\bar{a}_x)$$

$$= -11.294\bar{a}_x \text{ V/m}.$$

Now the total $\bar{E} = \bar{E}_Q + \bar{E}_l + \bar{E}_s$ is,

$$\bar{E} = -20.44\bar{a}_x - 12.44\bar{a}_y - 2.555\bar{a}_z \text{ V/m}.$$

Q:1c. Define volume charge density. Also find the total charge within each of the indicated volumes

i) $0\le r\le 0.1,\ 0\le\phi\le\pi,\ 2\le z\le 4;\ \rho_v = r^2z^2\sin 0.6\phi$

ii) Universe: $\rho_v = \frac{e^{-2r}}{r^2}$ (05M)

Ans: sol^n. Volume charge density: When the charge is distributed in three dimensions, we refer it as volume charge. It can be in the form of cube/ rectangle/ cylinder or sphere. The charge can be positive or negative.

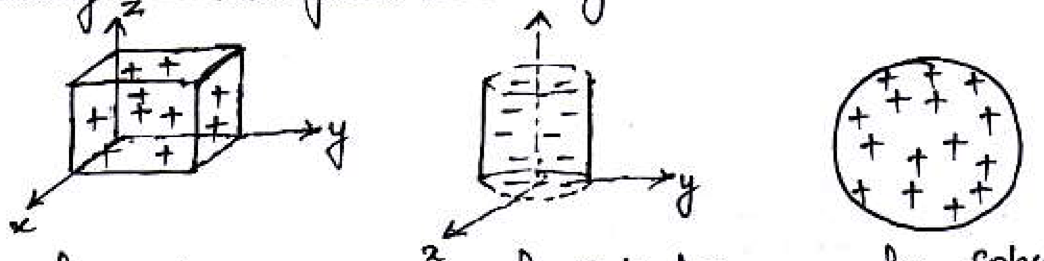


fig. cube   fig. Cylinder   fig. sphere

Volume charge density is represented by ρ_v. The total charge is Q, and volume is V, then

$$\rho_v = \frac{\text{total charge in coulomb}}{\text{Total volume in cubic meters}} \ (\text{C/m}^3).$$

In-terms of differential elements it is dV. and the charge enclosed by the differential element is dQ.

$$dQ = \rho_v\, dV$$

$$\& \quad Q = \int_{\text{Volume}} dQ = \int_{\text{Vol}} \rho_v\, dV.$$

i) $\rho_v = r^2 z^2 \sin(0.6\phi)$ belongs to cylindrical system.

$$\therefore Q = \int_{vol} \rho_v \, dv$$

$$= \int_{r=0}^{0.1} \int_{\phi=0}^{\pi} \int_{z=2}^{4} r^2 z^2 \sin(0.6\phi) . r dr d\phi dz$$

$$= \int_{r=0}^{0.1} \int_{\phi=0}^{\pi} \int_{z=2}^{4} r^3 dr . \sin(0.6\phi) d\phi . z^2 dz$$

$$= \left.\frac{r^4}{4}\right|_{r=0}^{0.1} \left.\frac{-\cos 0.6\phi}{0.6}\right|_{\phi=0}^{\pi} \left.\frac{z^3}{3}\right|_{z=2}^{4}$$

$$Q = \underline{1.018 mC}.$$

$\therefore$ The charge enclosed by the region is $Q = 1.018 mC$.

ii) Universe; $\rho_v = \frac{e^{-2r}}{r^2}$

Universe is in spherical shape; having it's r from 0 to ∞ and θ & ϕ over their complete range.

$$\therefore Q = \int_{Vol} \rho_v \, dv$$

$$= \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{e^{-2r}}{r^2} . r^2 \sin\theta d\theta dr d\phi$$

$$= \int_{r=0}^{\infty} \frac{e^{-2r}}{\cancel{r^2}} \cancel{r^2} dr . \int_{\theta=0}^{\pi} d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \left.\frac{e^{-2r}}{-2}\right|_{r=0}^{\infty} . 4\pi = \frac{e^{-\infty} - e^{0}}{-2} . 4\pi$$

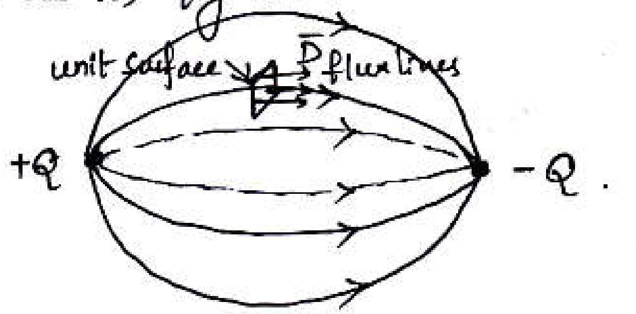
$$= 2\pi$$

$$= \underline{6.28 C}$$

$\therefore$ Total charge enclosed by the sphere (universe of $\rho_v = \frac{e^{-2r}}{r^2}$ //

Q:2 a. Define Electric flux and flux density (04M).

Ans: Consider two charges as in fig. below, As field exists in the form of electric lines of force. Consider a unit surface area as in fig. held normal to the direction of flux lines



Total number of electric lines of force around the charge is flux. represented by ψ. & $\psi = Q$ in Coulomb.

"The net flux passing normal through the unit surface area is called the electric flux density" represented by $\bar{D}$.

$$\bar{D} = \frac{\psi}{S}$$

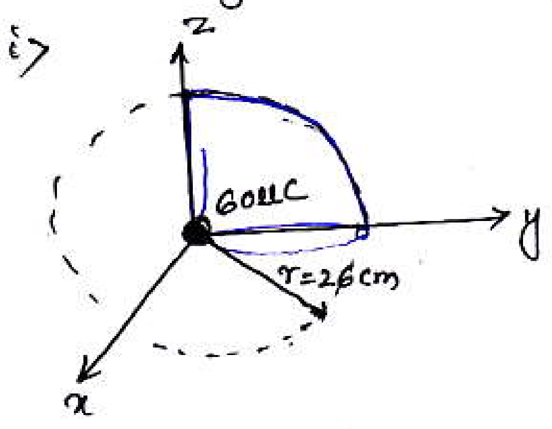
Where ψ - total flux &
S - total surface area of the sphere

$\bar{D}$ is measured in C/m^2

Q:2 b. Given the 60µC point charge located at the origin, find the total electric flux passing through:
i) The portion of the sphere $r = 26$cm bounded by $0 \leq \theta \leq \pi/2$ and $0 \leq \phi \leq \pi/2$
ii) The closed surface defined by $r = 26$cm & $z = \pm 26$ cm.
iii) The plane $z = 26$cm. (07M)

Ans: The charge of 60µC is at origin

i)



The sphere of radius $r = 26$cm will enclose the entire charge of 60µC when it is complete.

total area through which flux passes when sphere is complete is $4\pi r^2$

$$S_{total} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$= 4\pi r^2$$

The portion $r = 26\,cm, 0 \le \theta \le \pi/2$ & $0 \le \phi \le \pi/2$

$$S = \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{\pi/2} r^2 \sin\theta \, d\theta \, d\phi$$

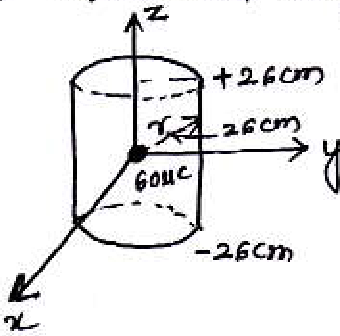
$$= r^2 \left. -\cos\theta \right|_{\theta=0}^{\pi/2} \pi/2 = \frac{\pi r^2}{2}$$

$\therefore$ The charge enclosed is, $\frac{S}{S_{total}} \times$ charge value

$$= \frac{\pi r^2/2}{4\pi r^2} \times 60\mu C = \underline{7.5\mu C}.$$

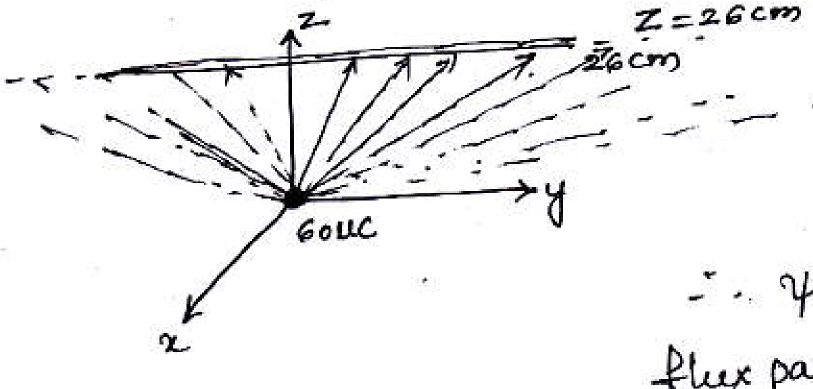
The flux passing through or charge enclosed by $1/8$ of the portion of the sphere is $\underline{7.5\mu C}$

ii) The closed surface defined by $r = 26\,cm$ and $z = \pm 26\,cm$.



As the cylinder is complete having it's radius as $r = 26\,cm$, $z = \pm 26\,cm$ and by default ϕ is from 0 to 2π.
Since it is the closed surface and encloses the point charge $-60\mu C$ comfortably. As per the gauss law the charge enclosed is $\psi = Q = 60\mu C$.

iii) The plane $z = 26\,cm$

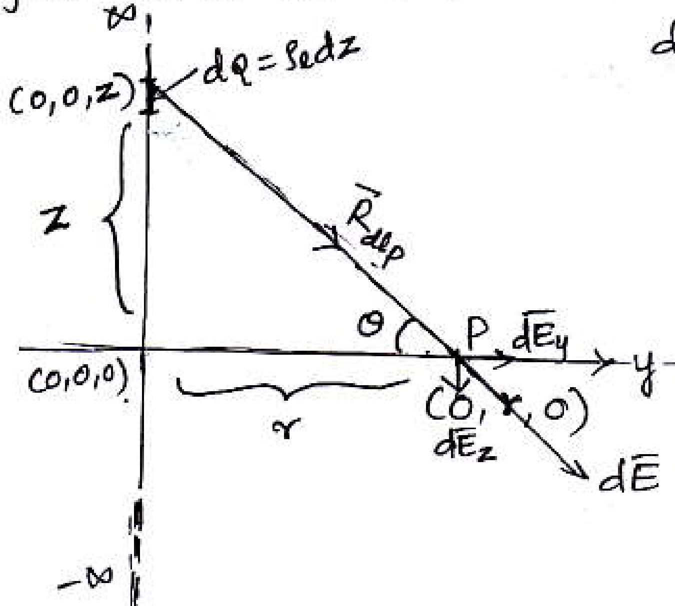


As the surface is infinitesimal almost half the flux lines of the charge pass through the $z = 26\,cm$ surface

$\therefore \psi = \frac{Q}{2} = 30\mu C$. amount of flux passes through the surface.

Q:2 c. Derive the expression for $\bar{E}$ due to infinite line of charge with ρ_L C/m (05M).

Ans: Electric field Intensity due an infinite line of ρ_L C/m charge density. Let the line charge lies along z-axis ranging from $-\infty$ to ∞. Consider a differential element of length dl of charge value dQ



$$dQ = \rho_l dl = \rho_l dz$$

at a distance z from origin. Let's find the $\bar{E}$ at point $P(0, r, 0)$ along the y-axis. The line joining the element dl ~~mat~~ and P makes an angle θ with y-axis.

The vector length $\bar{R}_{dlp} = P - dl = r\bar{a}_y - z\bar{a}_z$

$$|\bar{R}_{dlp}| = \sqrt{r^2 + z^2}$$

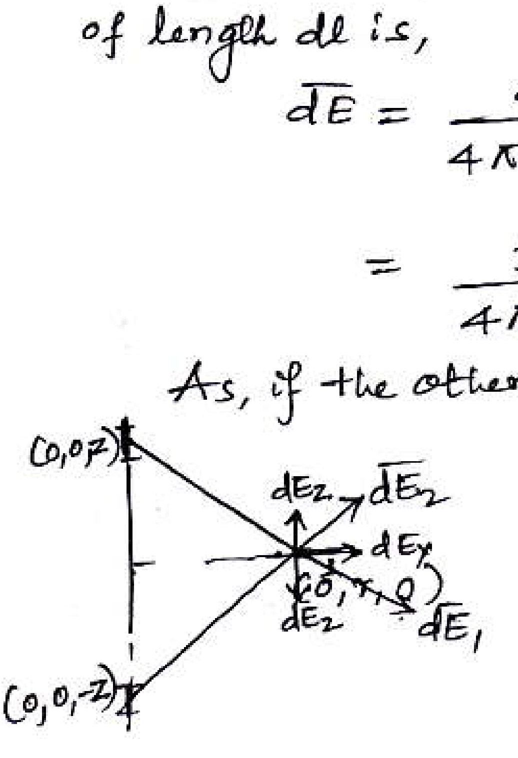
$$\bar{a}_{r_{dlp}} = \frac{r\bar{a}_y - z\bar{a}_z}{\sqrt{r^2 + z^2}}$$

The electric field at point p due to the considered element of length dl is,

$$\bar{dE} = \frac{dQ}{4\pi\epsilon_0 |\bar{R}_{dlp}|^2} \bar{a}_{rdlp}$$

$$= \frac{\rho_l dz}{4\pi\epsilon_0 (r^2 + z^2)^{3/2}} (r\bar{a}_y - z\bar{a}_z)$$

As, if the other element of dl is considered at $(0, 0, -z)$, the electric field intensity $d\bar{E}_2$ will have it's z component opposite to that of $d\bar{E}_1$ whereas y components add together. Therefore neglecting the z components as they cancel out.



Now, the $\bar{E}$ due to entire line charge is,

$$\bar{E} = \int_{-\infty}^{\infty} d\bar{E} = \int_{-\infty}^{\infty} \frac{\rho_l \, dz \, r \, \bar{a}_y}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}.$$

let $z = r\tan\theta$

$dz = r\sec^2\theta \, d\theta.$

when $z = -\infty$, $\theta = -\pi/2$ & $z = \infty$, $\theta = \pi/2$

$$\bar{E} = \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_l \, r^2 \sec^2\theta \, d\theta \, \bar{a}_y}{4\pi\varepsilon_0 (r^2 + r^2\tan^2\theta)^{3/2}}$$

$$= \frac{\rho_l}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{r^2\sec^2\theta \, d\theta}{r^3(1+\tan^2\theta)^{3/2}} \bar{a}_y$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \bar{a}_y$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 r} \sin\theta \Big|_{-\pi/2}^{\pi/2} \bar{a}_y$$

$$= \frac{\rho_l}{4\pi\varepsilon_0 r} 2 \, \bar{a}_y$$

$$\bar{E} = \frac{\rho_l}{2\pi\varepsilon_0 r} \bar{a}_y \text{ V/m}.$$

The electric field intensity due to an infinite line charge will not have it's Component along the axis in which line charge is located.