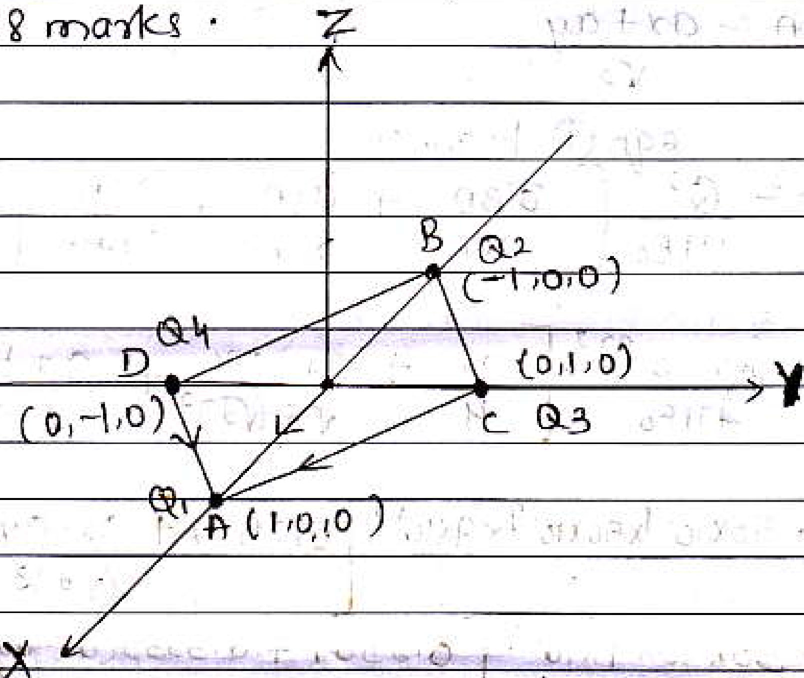
Engg Electromagnetics.

Dec 2016 | Jan 2017.

1) a) point charges of 50 nC each are located at A (1,0,0), B (−1,0,0), C (0,1,0) & D (0,−1,0) in free space. Find the total force on the charge at A – 8 marks.

Sol^n:-



Let Q_1, Q_2, Q_3 & Q_4 are located at A, B, C, D respectively.

$$Q_1 = Q_2 = Q_3 = Q_4 = 50nC = Q.$$

$$\bar{F}_A = \bar{F}_{BA} + \bar{F}_{CA} + \bar{F}_{DA}$$

$$F_A = \frac{Q_1 Q_2}{4\pi\epsilon_0 r_{BA}^2}\bar{a}_{BA} + \frac{Q_1 Q_3}{4\pi\epsilon_0 r_{CA}^2}\bar{a}_{CA} + \frac{Q_1 Q_4}{4\pi\epsilon_0 r_{DA}^2}\bar{a}_{DA} \quad \text{— (1)}$$

$r_{BA} = A - B \quad (1,0,0) - (-1,0,0)$

$r_{BA} = 2a_x$

$|r_{BA}| = \sqrt{2^2}$

$|r_{BA}| = 2$

$a_{BA} = \frac{2a_x}{2} = \bar{a}_x$

$r_{CA} = A - C \quad (1,0,0) - (0,1,0)$

$r_{CA} = \bar{a}_x - \bar{a}_y$

$|r_{CA}| = \sqrt{1^2 + (-1)^2}$

$|r_{CA}| = \sqrt{2}$

$a_{CA} = \frac{\bar{a}_x - \bar{a}_y}{\sqrt{2}}.$

$r_{DA} = A - D \quad (1,0,0) - (0,-1,0)$

$r_{DA} = a_x + a_y$.

$|r_{DA}| = \sqrt{1^2+1^2}$

$= \sqrt{2}$

$$a_{DA} = \frac{a_x + a_y}{\sqrt{2}}$$

eqn (1) becomes

$$\bar{F}_A = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\bar{a}_{BA}}{r_{BA}^2} + \frac{\bar{a}_{CA}}{r_{CA}^2} + \frac{\bar{a}_{DA}}{r_{DA}^2}\right]$$

$$\bar{F}_A = \frac{(50\times10^{-9})^2}{4\pi\epsilon_0}\left[\frac{a_x}{4} + \frac{a_x - a_y}{\sqrt{2}(\sqrt{2})^2} + \frac{a_x + a_y}{2(\sqrt{2})^2}\right]$$

$$= 50\times10^{-9}\times50\times10^{-9}\times9\times10^{+9}\left[0.25a_x + \frac{a_x - a_y}{(\sqrt{2})^3} + \frac{a_x + a_y}{(\sqrt{2})^3}\right]$$

$$= 50\times50\times9\times10^{-9}\left[0.25a_x + 0.3535(a_x - a_y) + 0.3535(a_x + a_y)\right]$$

$$= 22500\times10^{-9}\left[0.25a_x + 0.3535a_x - 0.3535a_y + 0.3535a_x + 0.3535a_y\right]$$

$$= 22500\times10^{-9}\left[0.957\bar{a}_x\right]$$

$\bar{F}_A = 21.532\times10^{-6}\,N\ \bar{a}_x$

$F = 21.53\,\mu N.\ \bar{a}_x$

b) Define electric field intensity & electric flux density

→ 1) Electric field intensity :-

The region where a perticular charge exerts a force on any other charge located in that region is called electric field of that charge.

The force exerted per unit charge is called electric field intensity.

2) Electric flux density :-

The net flux passing normal through the unit surface area is called the electric flux density

$$D = \frac{\psi}{S}$$

Where ψ - total flux

S - total surface area of sphere

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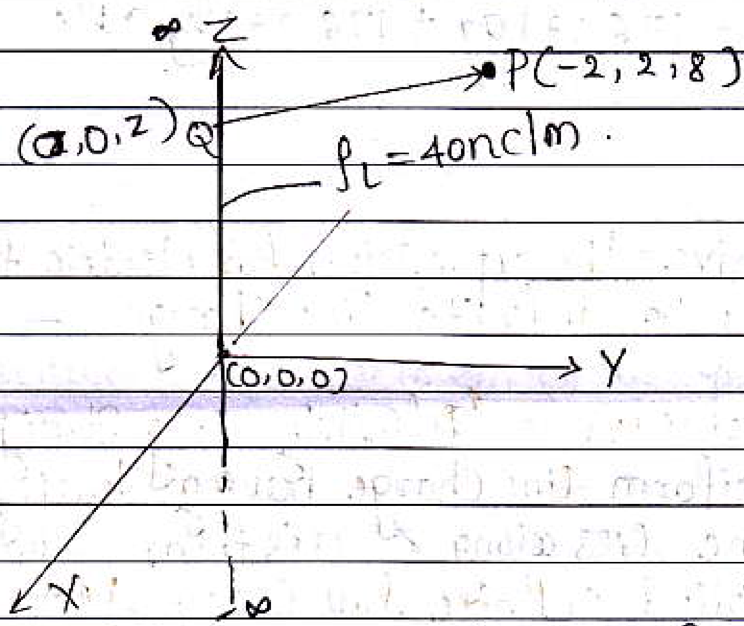
c) A uniform line charge of infinite length with $\rho_L = 40 nC/m$ lies along z-axis. Find $\bar{E}$ at $(-2,2,8)$ in air — 4 marks.



→ Let us consider a point $P(-2,2,8)$.

$\bar{E}$ at $(-2,2,8)$

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r_{QP}} a_{QP} \quad \text{—} (1)$$

$$r_{QP} = P - Q \quad (-2,2,8) - (0,0,2)$$

$$r_{QP} = -2a_x + 2a_y$$

$$|r_{QP}| = \sqrt{2^2 + 2^2}$$

$$= \sqrt{4+4}$$

$$|r_{QP}| = \sqrt{8}$$

$$a_{QP} = \frac{-2a_x + 2a_y}{\sqrt{8}}$$

$$\bar{E} = \frac{40 \times 10^{-9}}{2 \times 3.142 \times 8.854 \times 10^{-12} \times \sqrt{8}} \cdot \frac{-2a_x + 2a_y}{\sqrt{8}}$$

$$= \frac{40 \times 10^{-9}}{2 \times 3.142 \times 8.854 \times 10^{-12} \times 8} (-2a_x + 2a_y)$$

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$$= \frac{40\times10^{-9}}{4.45108\times10^{-10}}(-2a_x+2a_y)$$

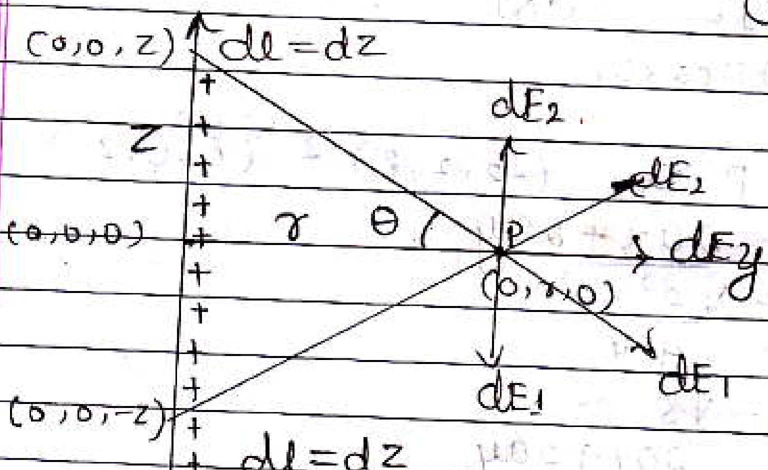
$$= 89.86(-2a_x+2a_y)$$

$$\bar{E} = -176.731a_x + 176.731a_y \text{ N/C}.$$

2)

a) Derive the expression for electoic field intensity due to infinite line Charge — 08 marks.

→ Consider a infinitely long straight line carrying uniform line charge having density ρ_L C/m. Let the line lies along z-axis from $-\infty$ to $+\infty$ and hence called infinite line charge. Let point p is on y axis at which electric field intensity is to be determined. The distance of point p from the origin is 'r' as shown in below figure.



$dQ = \rho_L dz$.

Consider a small differential length dl carrying a charge dQ, along the line as shown in the fig above. It is along z axis hence $dl = dz$.

The coordinate of dQ are (0,0,z) while the coordinates of p(0,r,0). Hence the distance vector $\bar{R}$ can be written as

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$\bar{R}_{dlp} = \bar{r}_p - \bar{r}_{dl} = r a_y - z a_z$ — ①

$|R_{dlp}| = \sqrt{r^2+z^2}$ — ①

$a_{dlp} = \frac{r a_y - z a_z}{\sqrt{r^2+z^2}}$ — ②

The electric field due to Small element dl is given by at point P

$$dE_p = \frac{dQ}{4\pi\epsilon_0 R_{dlp}^2} a_{dlp} \quad ③$$

Substitute eqn ① & ② in eqn ③ we get

$$dE_p = \frac{dQ}{4\pi\epsilon_0 (\sqrt{r^2+z^2})^2} \frac{(r\bar{a}_y - z\bar{a}_z)}{\sqrt{r^2+z^2}}$$

$$dE_p = \frac{dQ\,(r\bar{a}_y - z\bar{a}_z)}{4\pi\epsilon_0 (r^2+z^2)\sqrt{r^2+z^2}}$$

$$dE_p = \frac{\rho_L\, dz}{4\pi\epsilon_0 (r^2+z^2)^{3/2}} (r a_y - z a_z)$$

$\bar{E}$ due to Entire line charge is

$$\bar{E}_{p1} = \int dE_p = \int_{-\infty}^{\infty} \frac{\rho_L\, dz}{4\pi\epsilon_0 (r^2+z^2)^{3/2}} (r\bar{a}_y - z\bar{a}_z) \quad ④$$

$$\bar{E}_{p2} = \int dE_p = \int_{-\infty}^{\infty} \frac{\rho_L\, dz}{4\pi\epsilon_0 (r^2+z^2)^{3/2}} (r a_y + z a_z) \quad ⑤$$

$$E_p = \bar{E}_{p1} + \bar{E}_{p2}$$

$$E_p = \int_{-\infty}^{\infty} \frac{\rho_L\, dz}{4\pi\epsilon_0 (r^2+z^2)^{3/2}} r a_y$$

W.K.T $\tan\theta = \frac{z}{r}$ let $z = r\tan\theta$

$$z = r\tan\theta$$
$$dz = dr\sec^2\theta\, d\theta$$

$$\bar{E_p} = \int_{\theta=-\pi/2}^{\theta=\pi/2} \frac{\rho_L\, r\sec^2\theta\, d\theta}{4\pi\epsilon_0 (r^2+r^2\tan^2\theta)^{3/2}}\, r\,\bar{a_y}$$

when $z = -\infty$, $\theta = -\pi/2$
$z = \infty$, $\theta = \pi/2$

$$\bar{E_p} = \int_{-\pi/2}^{\pi/2} \frac{\rho_L\, r^2\sec^2\theta\, d\theta}{4\pi\epsilon_0\, r^3 (1+\tan^2\theta)^{3/2}}\,\bar{a_y}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\rho_L\sec^2\theta\, d\theta}{4\pi\epsilon_0\, r\sec^3\theta}\,\bar{a_y}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\rho_L}{4\pi\epsilon_0\, r\sec\theta}\, d\theta\,\bar{a_y}$$

$$= \frac{\rho_L}{4\pi\epsilon_0 r}\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta\,\bar{a_y}$$

$$= \frac{\rho_L}{4\pi\epsilon_0 r}\Big[\sin\theta\Big]_{\theta=-\pi/2}^{\theta=\pi/2}\,\bar{a_y}$$

$$= \frac{\rho_L}{4\pi\epsilon_0 r}(1+1)\,\bar{a_y}$$

$$\boxed{E_p = \frac{\rho_L}{2\pi\epsilon_0 r}\,\bar{a_y}}$$

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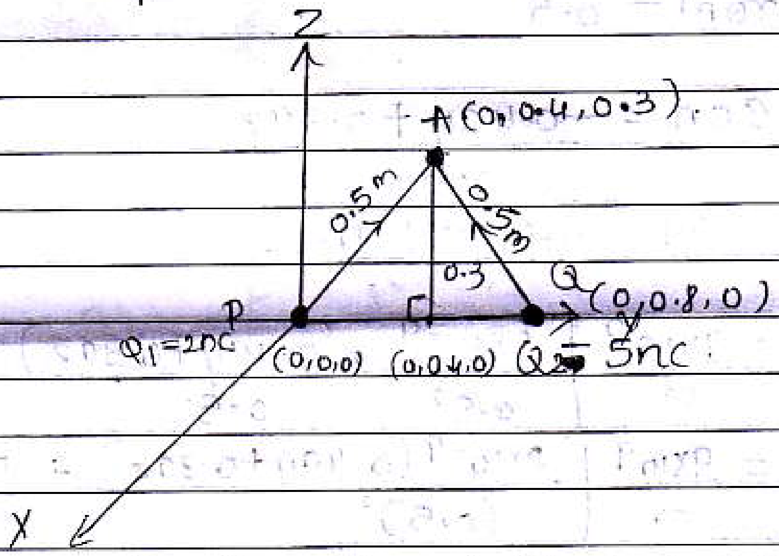
b) Two particles having charges 2nC and 5nC are spaced 80 cm apart. Determine the electric field intensity at point 'A' situated at a distance of 0.5 m from each of the particles. Assume dielectric constant of 5.



Let us consider the charge $Q_1 = 2nC$ & $Q_2 = 5nC$ are placed at A & B respectively 0.8 m apart from each other.

The electric field intensity at a point A which is 0.5m from the both charges is as follows.

$$E = E_{PA} + E_{QA}$$

$$= \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{Q_1}{r_{PA}^2} \bar{a}_{PA} + \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r_{QA}^2} \bar{a}_{QA}$$

$$E = \frac{9\times10^9}{\epsilon_r} \left[\frac{Q_1}{r_{PA}^2} \bar{a}_{PA} + \frac{Q_2}{r_{QA}^2} \bar{a}_{QA} \right] \quad -(1)$$

Where

$r_{PA} = A - P \quad (0, 0.4, 0.3) - (0, 0, 0)$

$r_{PA} = 0.4 a_y + 0.3 a_z$

$|r_{PA}| = \sqrt{(0.4)^2 + (0.3)^2}$

$= \sqrt{(0.25)}$

$|r_{PA}| = 0.5$

$$a_{PA} = \frac{0.4 a_y + 0.3 a_z}{0.5}$$

$r_{QA} = A - Q \quad (0, 0.4, 0.3) - (0, 0.8, 0)$

$r_{QA} = -0.4\, a_y + 0.3\, a_z$

$|r_{QA}| = \sqrt{(-0.4)^2 + (0.3)^2}$

$= \sqrt{(0.25)}$

$|r_{QA}| = 0.5$

$a_{QA} = \frac{-0.4\, a_y + 0.3\, a_z}{0.5}$

Substitute all these values in eqn (1).

$$\bar{E} = \frac{9\times10^9}{\varepsilon_r}\left[\frac{2\times10^{-9}}{0.5^2} \times \frac{(0.4 a_y + 0.3 a_z)}{0.5} + \frac{5\times10^{-9}}{0.5^2} \times \frac{-0.4 a_y + 0.3 a_z}{0.5}\right]$$

$$= \frac{9\times10^9}{\varepsilon_r}\left[\frac{2\times10^{-9}(0.4 a_y + 0.3 a_z)}{(0.5)^3} + \frac{5\times10^{-9}\times(-0.4 a_y + 0.3 a_z)}{(0.5)^3}\right]$$

$$= \frac{9\times10^9}{\varepsilon_r}\left[1.6\times10^{-8}(0.4 a_y + 0.3 a_z) + 4\times10^{-8}(-0.4 a_y + 0.3 a_z)\right]$$

$$= \frac{9\times10^9}{\varepsilon_r}\left[(6.4\times10^{-9}) a_y + (4.8\times10^{-9}) a_z + (-1.6\times10^{-8}) a_y + 1.2\times10^{-8}\, a_z\right]$$

$$\cancel{= 9\times10^9\left[(6.4 - 1.6)\times10^{-9}\, a_y + (4.8 + 1.2)\right.}$$

$$= \frac{9\times10^9}{\varepsilon_r}\left[\left(6.4\times10^{-9} + (-1.6\times10^{-8})\right) a_y + (4.8\times10^{-9} + 1.2\times10^{-8})\, a_z\right]$$

$$= \frac{9\times10^9}{\varepsilon_r}\left[-9.6\times10^{-9}\, a_y + 1.68\times10^{-8}\, a_z\right]$$

$$\bar{E} = \frac{1}{\varepsilon_r}(-86.4\, a_y + 151.2\, a_z)$$

where $\varepsilon_r = 5$

$$= \frac{1}{5}(-86.4\, a_y + 151.2\, a_z)$$

$$\bar{E} = -17.28\, a_y + 30.24\, a_z \quad \text{V/m}$$