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ECE Dept.
EE
III Sem
2018-19

Department of Electronics & Communication Engg.

Course : Engineering Electromagnetics -17EC36. **Sem.: 3rd (2018-19)**

Course Coordinator:

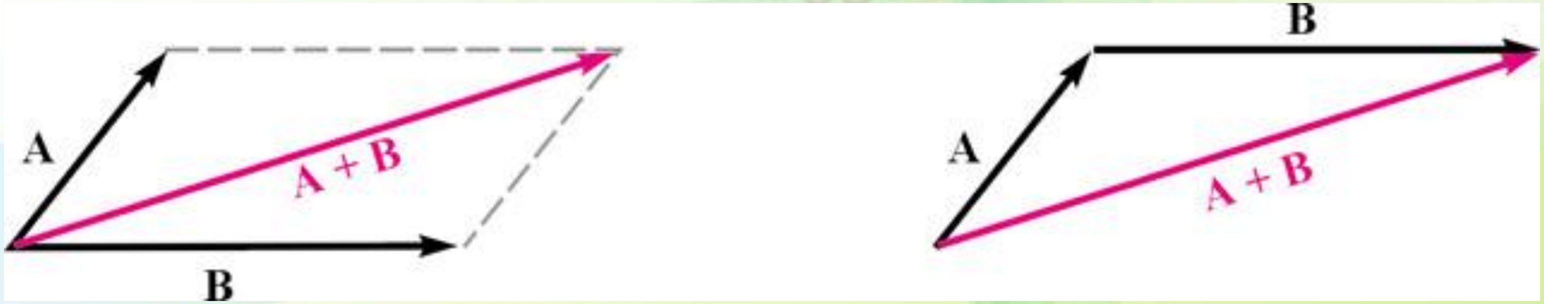
Prof. S. S. KAMATE

Vector Analysis

Scalars and Vectors

Scalar Fields (temperature)
Vector Fields (gravitational, magnetic)

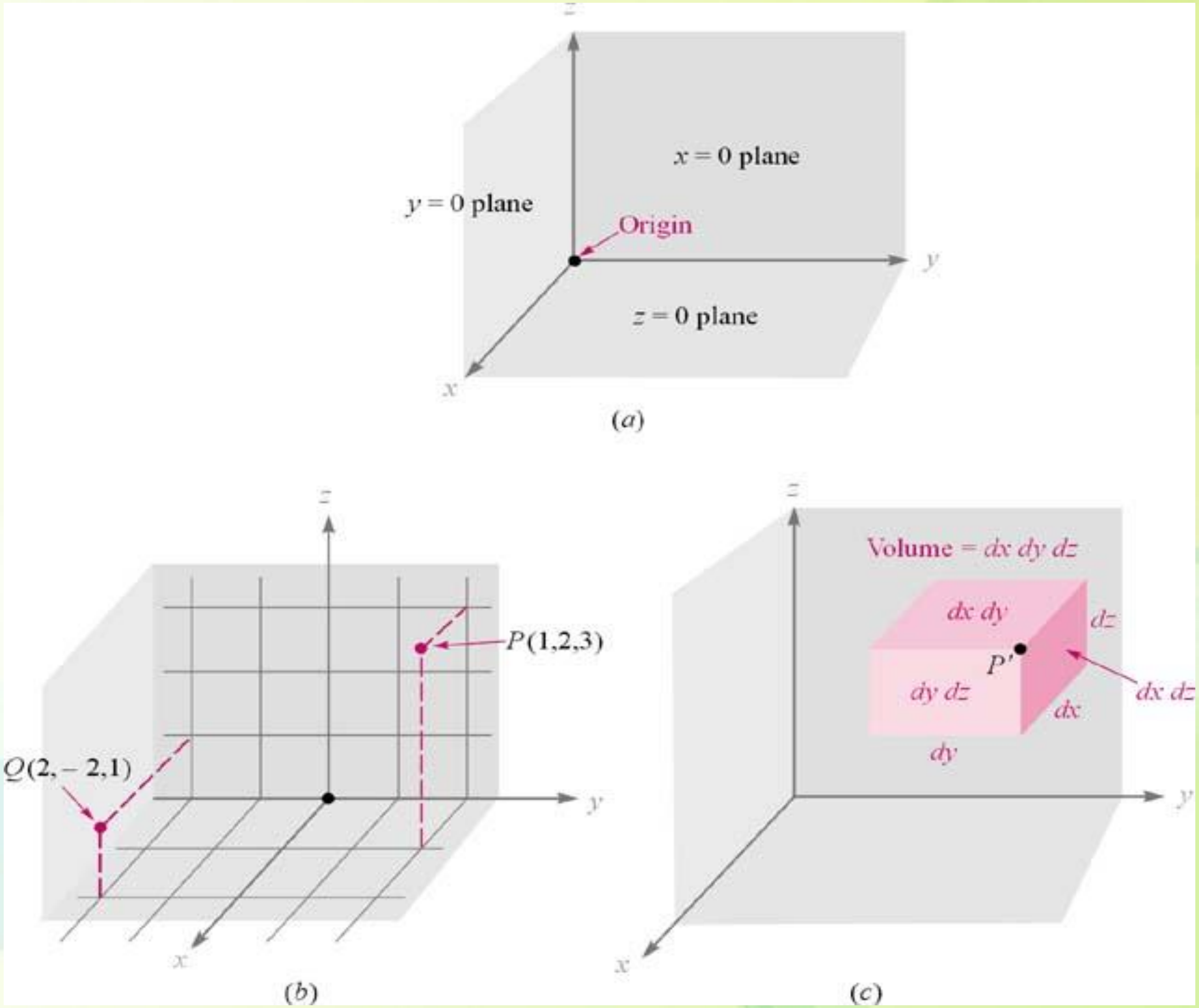
Vector Algebra



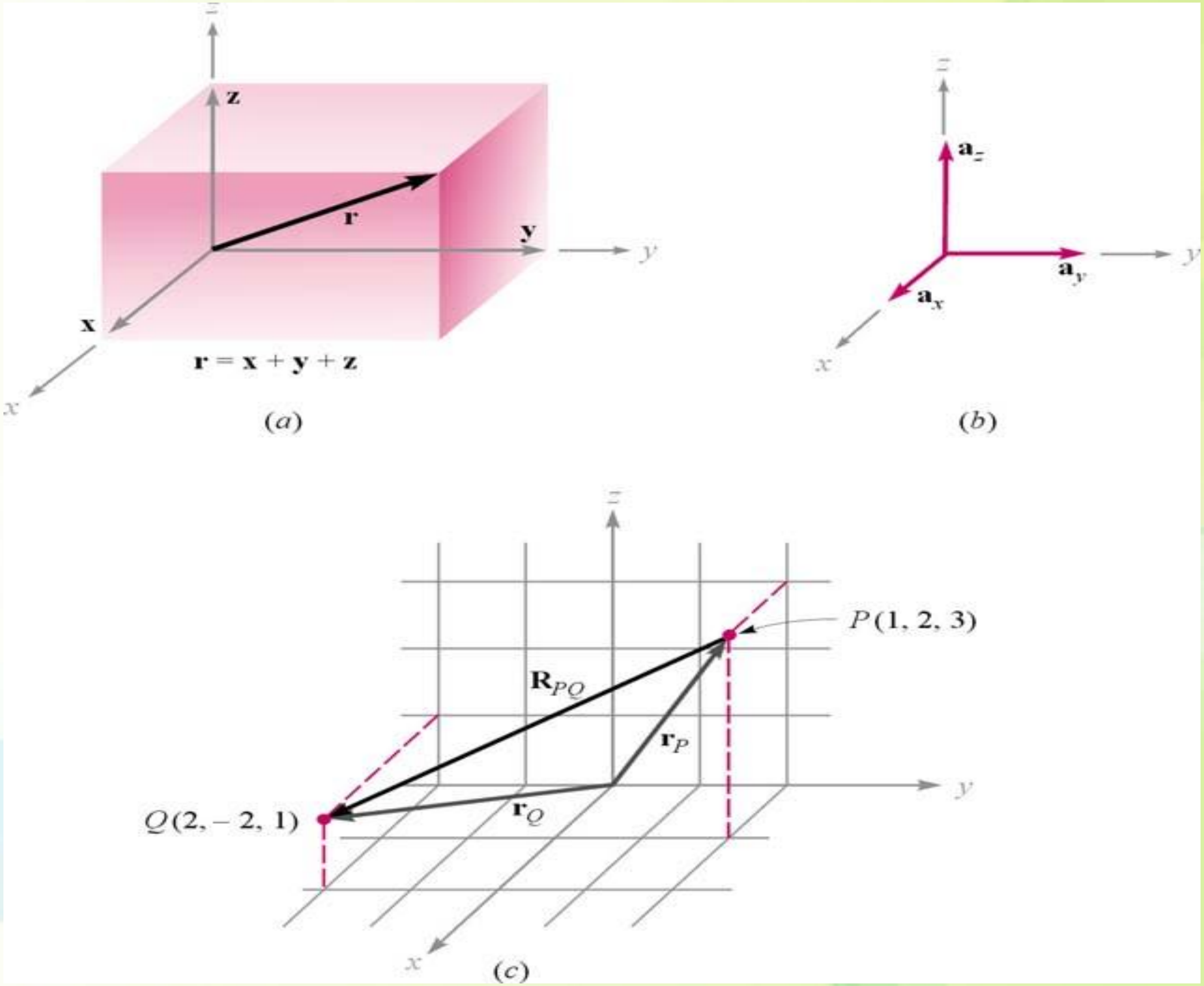
There are three co-ordinate systems

1. Cartesian or rectangular co-ordinate system
2. Cylindrical co-ordinate System
3. Spherical co-ordinate System

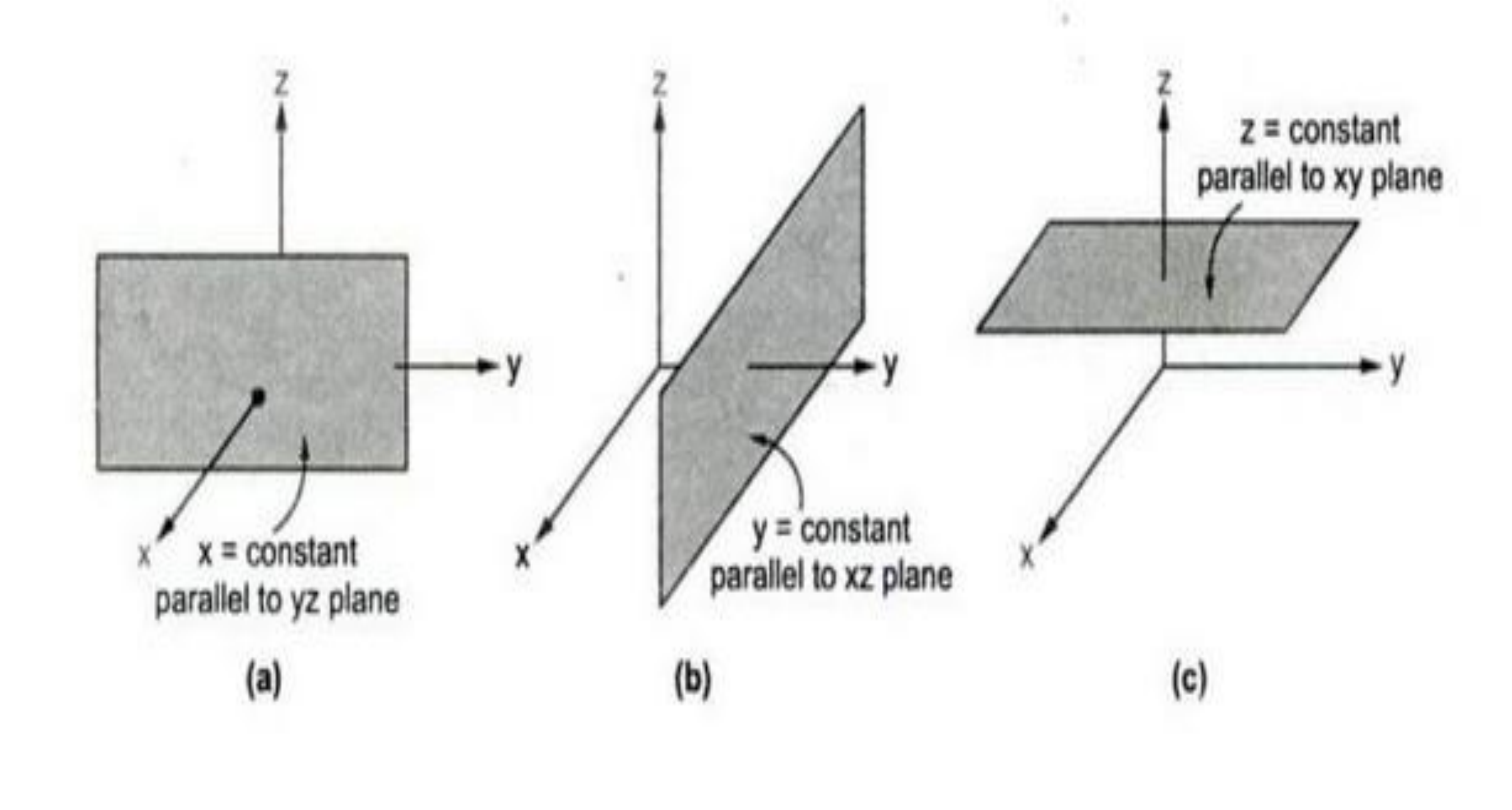
The Cartesian Coordinate System : vertices are x,y,z



Vector Components and Unit Vectors



Constant planes



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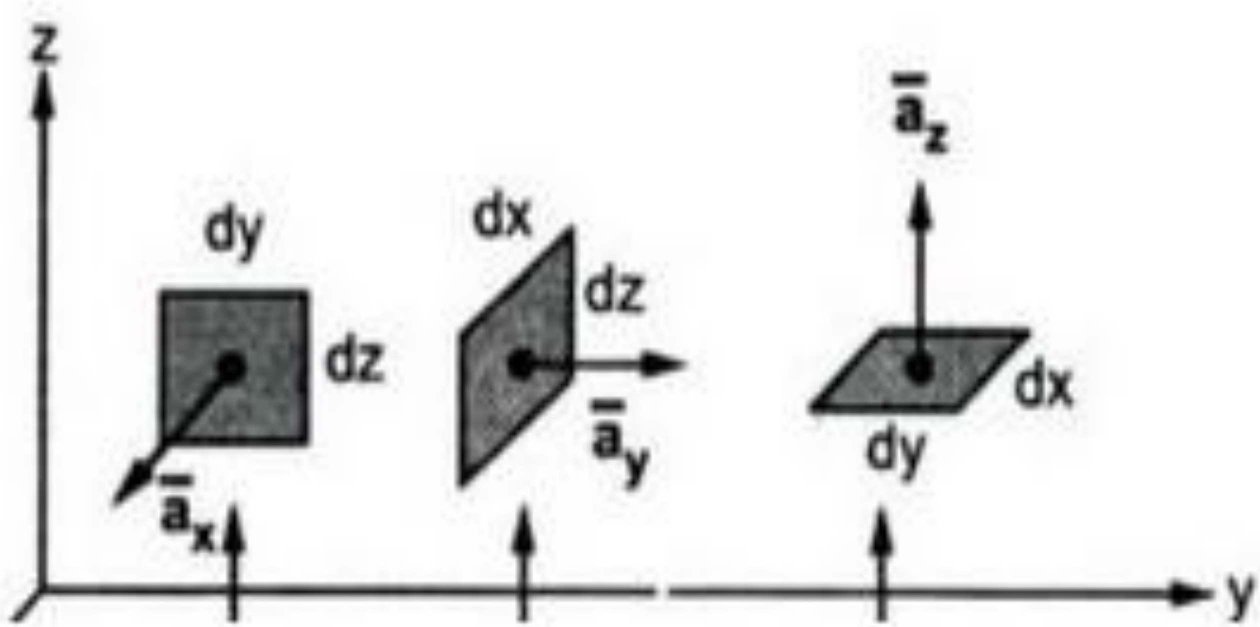
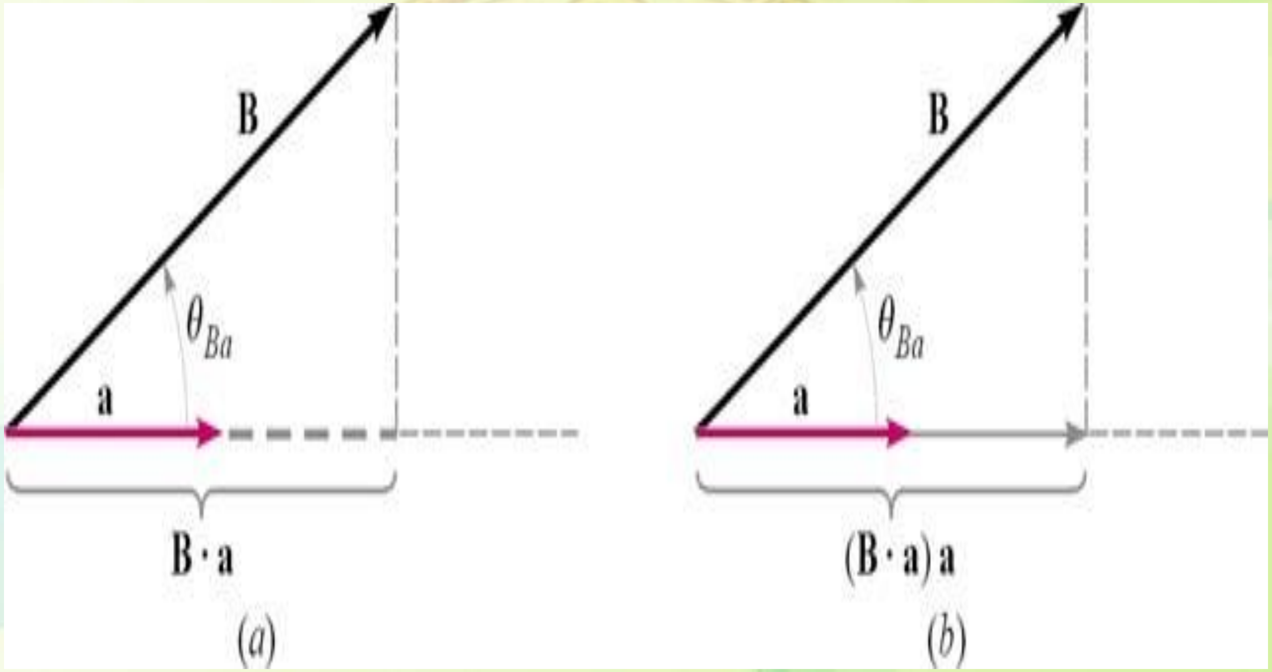


Distance vector - dl
Differential surfaces -
dv =

The Dot product

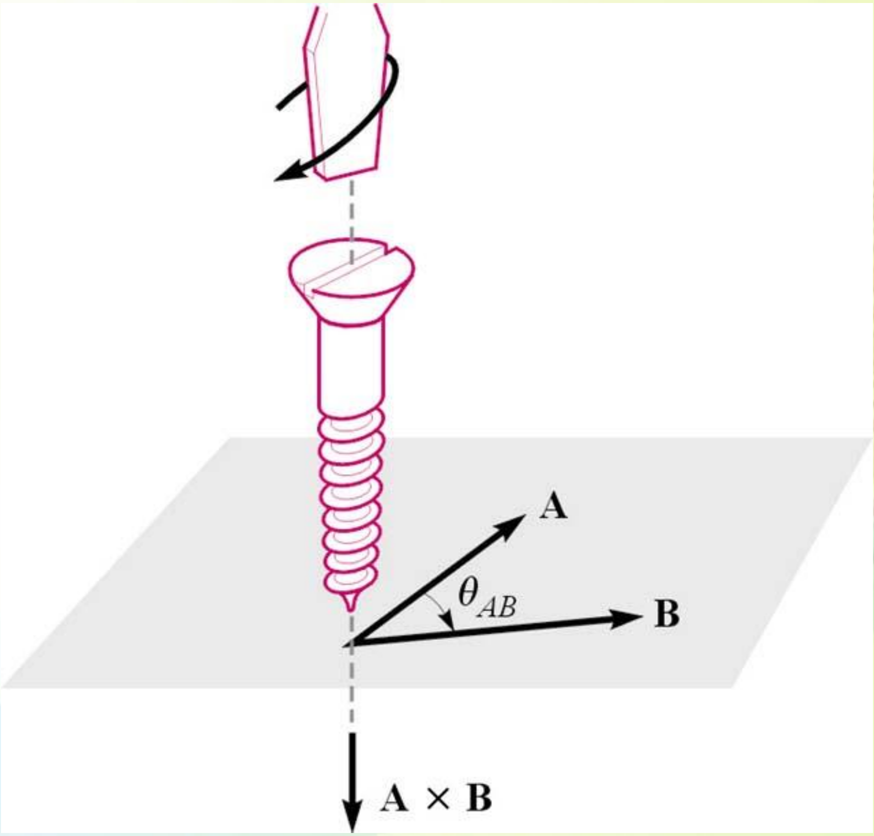
$$A.B = |A||B| \cos \theta_{AB}$$

B in the direction of A
You need to normalize a before the dot product.



The Cross Product

$$A \times B = a_N|A||B| \sin \theta_{AB}$$



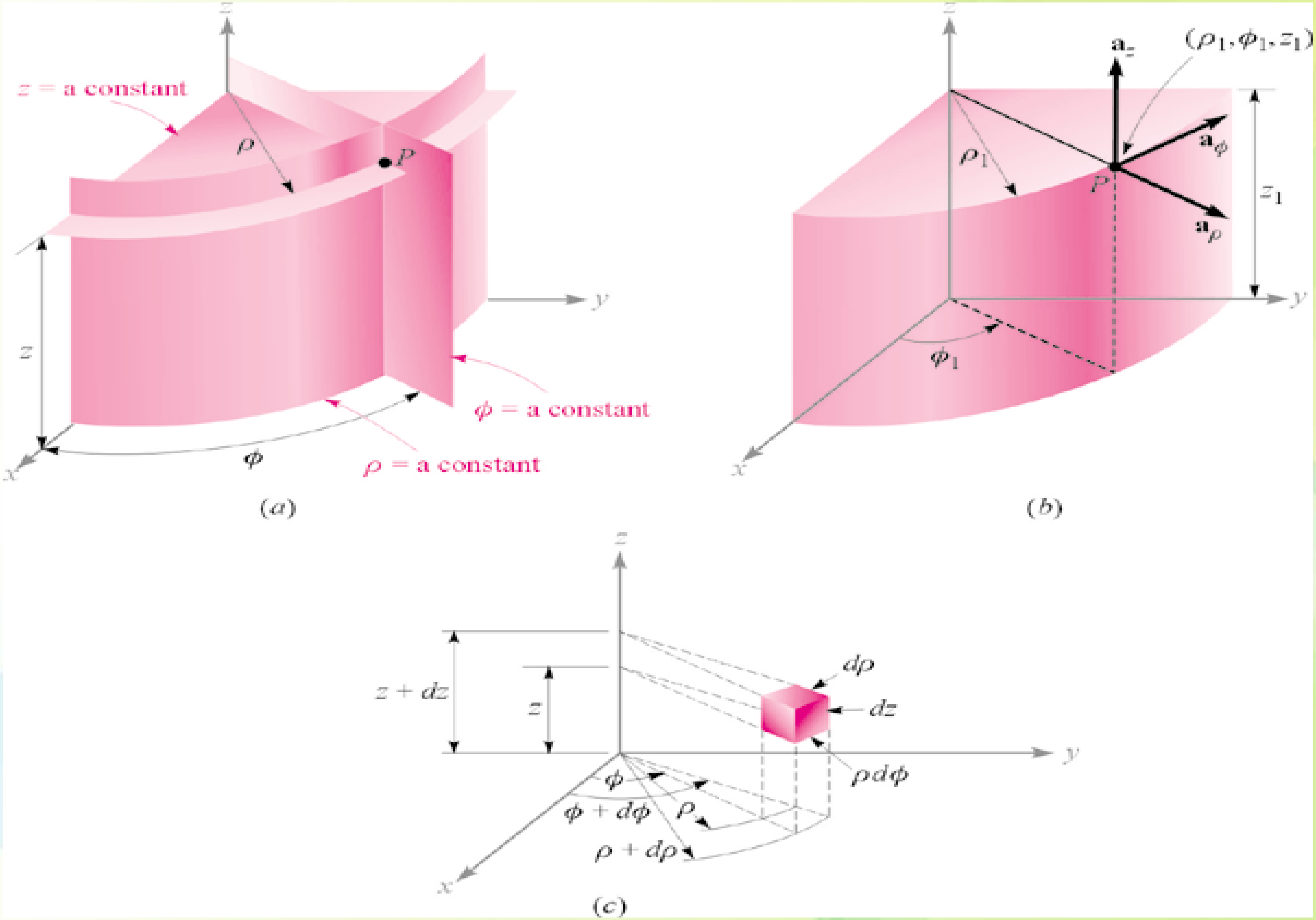
$$A \times B = \begin{pmatrix} ax & ay & az \\ Ax & Ay & Az \\ Bx & By & Bz \end{pmatrix}$$

Example

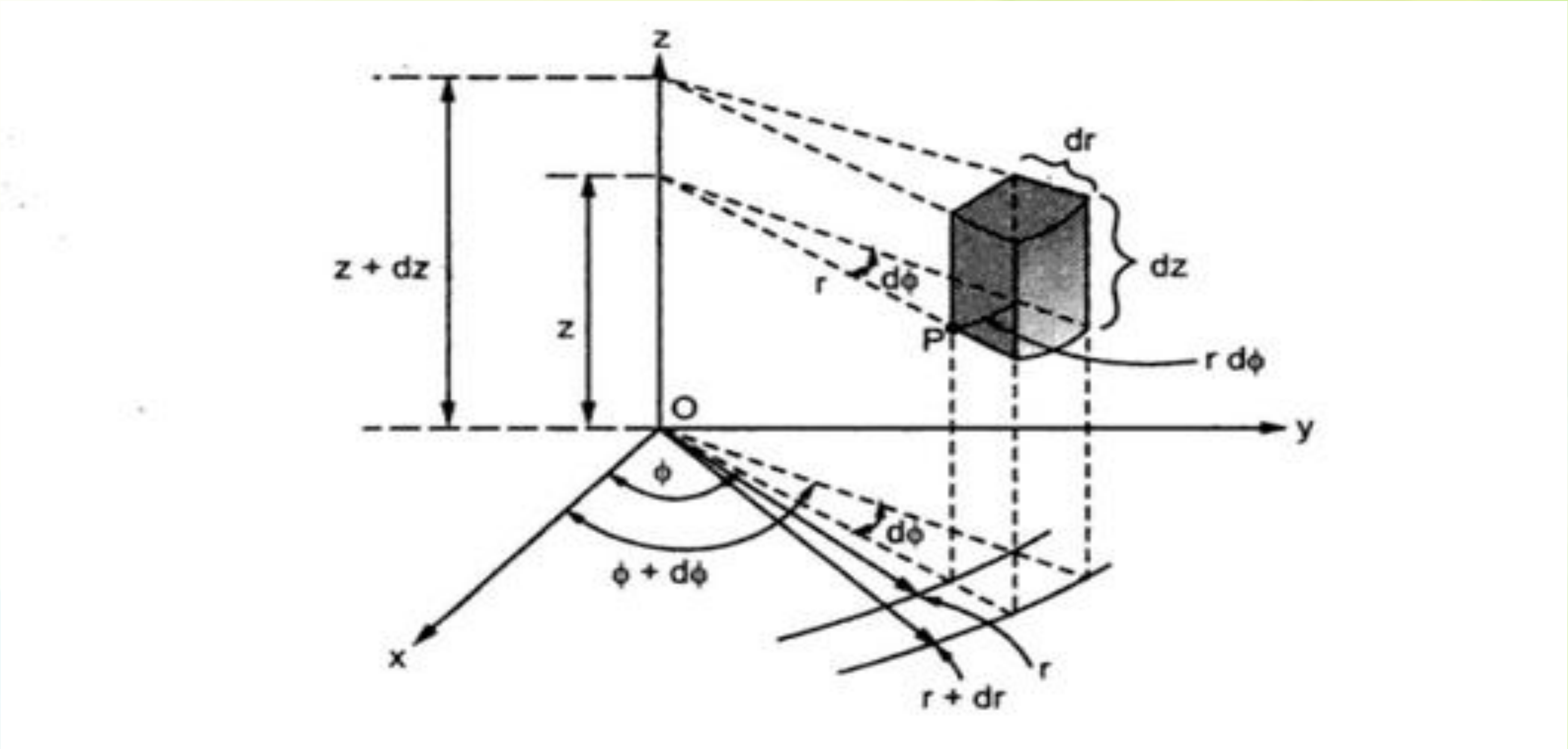
$$A := \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \qquad B := \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix}$$

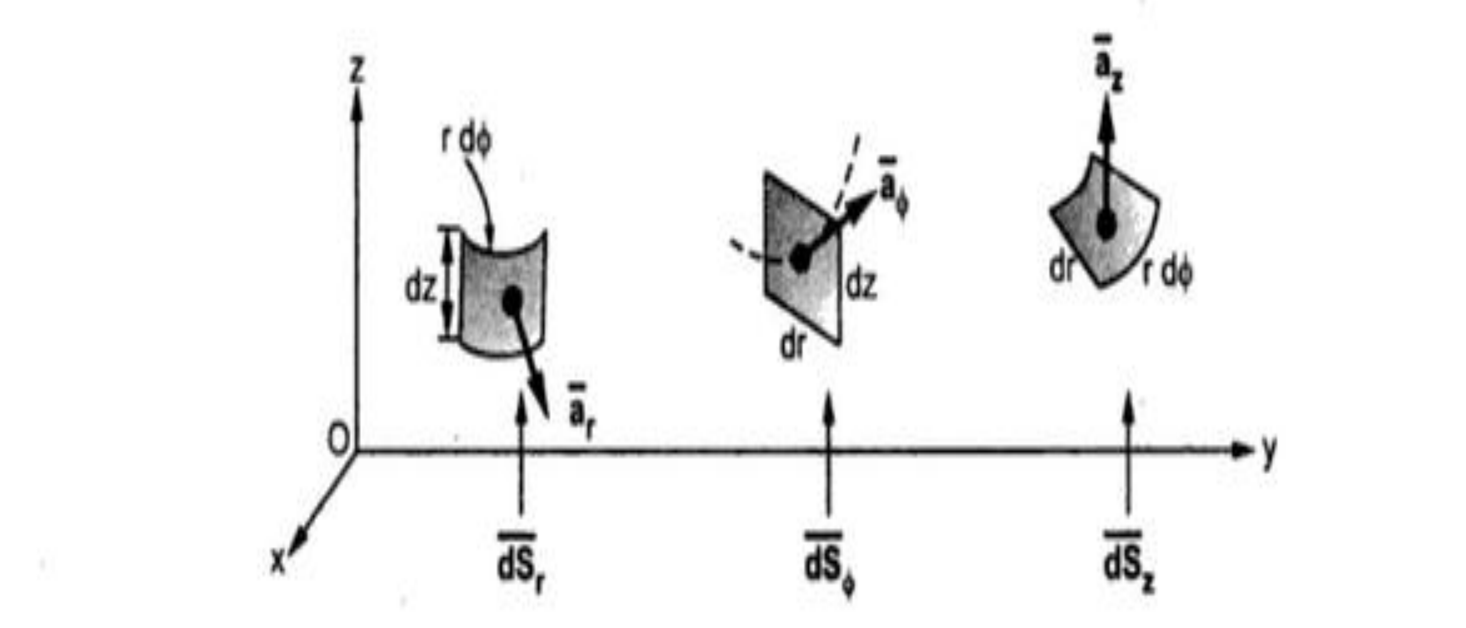
$$A \times B = \begin{pmatrix} -13 \\ -14 \\ -16 \end{pmatrix}$$

Circular Cylindrical Coordinate System



Differential volume





$\overline{dS}_r$ = Differential vector surface area normal to r direction

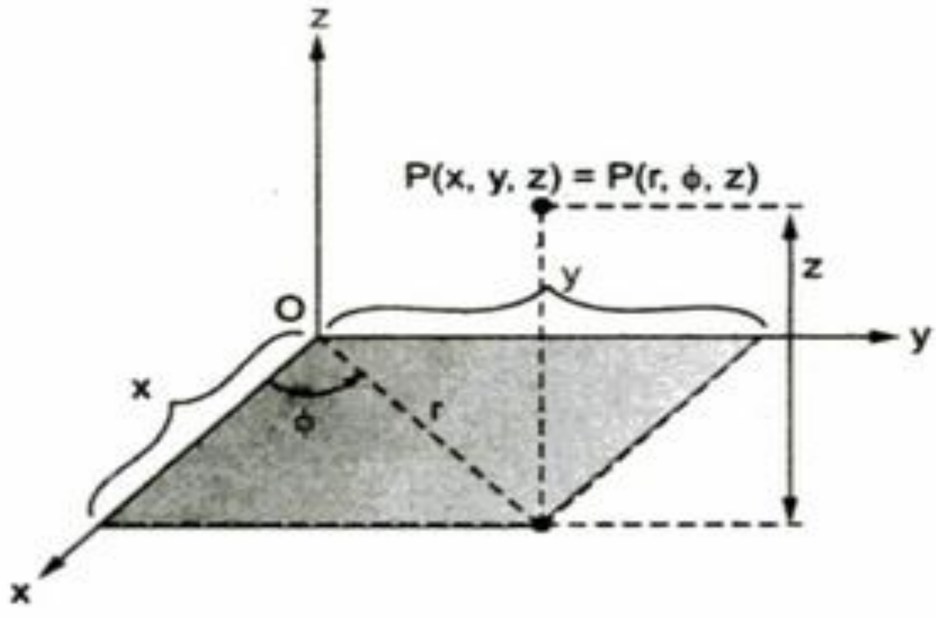
$= r\,d\phi\,dz\,\bar{a}_r$

$\overline{dS}_\phi$ = Differential vector surface area normal to ϕ direction

$= dr\,dz\,\bar{a}_\phi$

$\overline{dS}_z$ = Differential vector surface area normal to z direction

$= r\,dr\,d\phi\,\bar{a}_z$

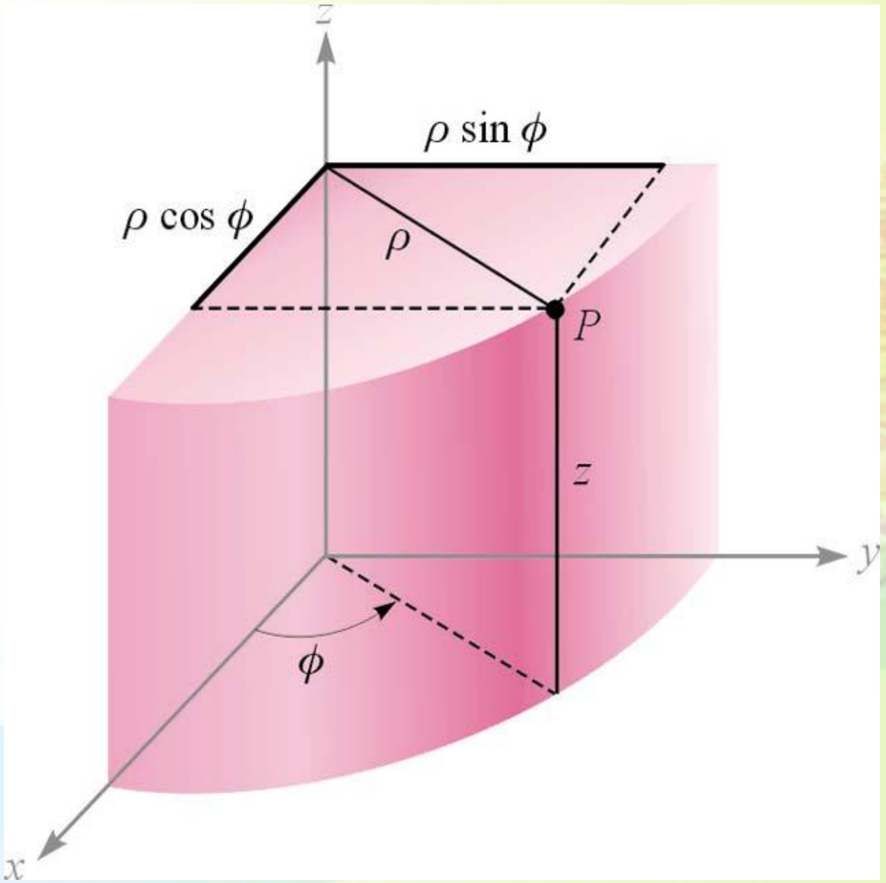


$$\boxed{x = r\cos\phi, \quad y = r\sin\phi \text{ and } \quad z = z}$$

It can be seen that, r can be expressed interms of x and y as,

$$r = \sqrt{x^2 + y^2}$$

Circular Cylindrical Coordinate System



$$x = \rho \cdot \cos(\phi)$$

$$y = \rho \cdot \sin(\phi)$$

$$z = z$$

$$\rho = \sqrt{x^2 + y^2} \qquad \rho \geq 0$$

$$\phi = \operatorname{atan}\left(\frac{y}{x}\right)$$

$$z = z$$

Dot Product

$$A = Ax \cdot ax + Ay \cdot ay + Az \cdot az$$

$$A = A\rho \cdot a\rho + A\phi a\phi + Az \cdot az$$

$$A\rho = A \cdot a\rho \qquad A\phi = A \cdot a\phi \qquad Az = Az$$

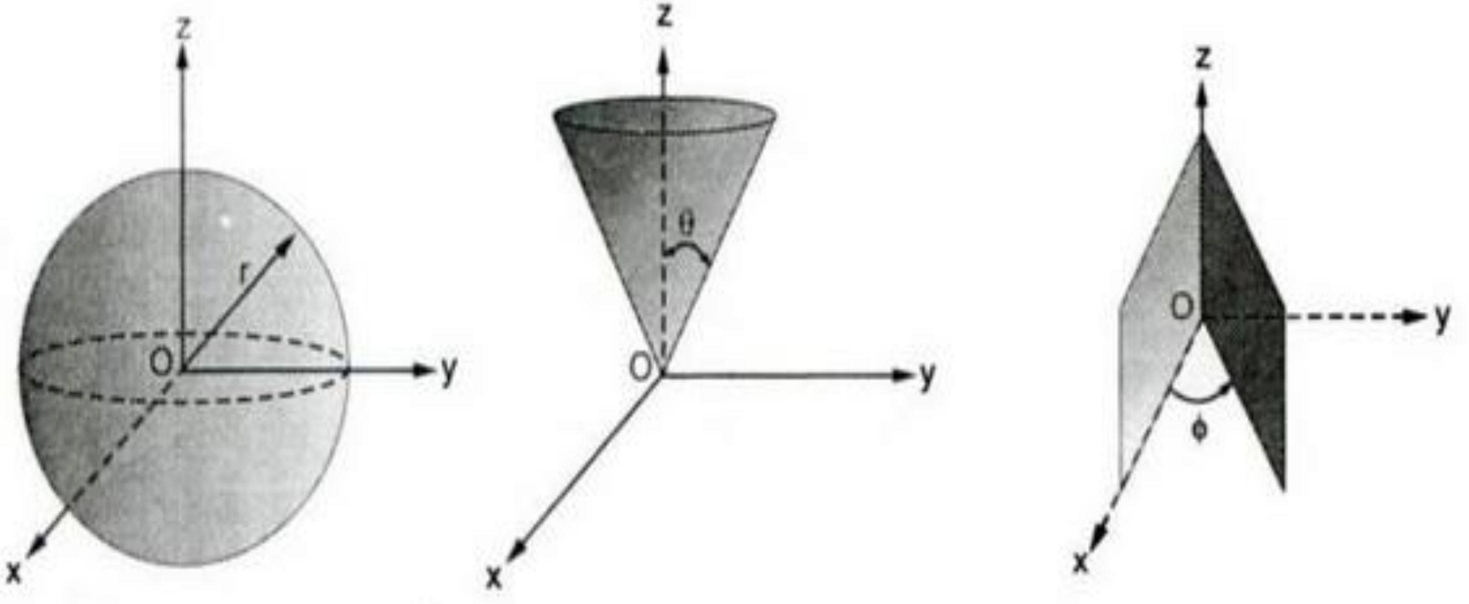
$$A\rho = (Ax \cdot ax + Ay \cdot ay + Az \cdot az) \cdot a\rho = Ax \cdot ax \cdot a\rho + Ay \cdot ay \cdot a\rho$$

$$A\phi = (Ax \cdot ax + Ay \cdot ay + Az \cdot az) \cdot a\phi = Ax \cdot ax \cdot a\phi + Ay \cdot ay \cdot a\phi$$

$$Az = (Ax \cdot ax + Ay \cdot ay + Az \cdot az) \cdot az = Az \cdot az \cdot az = Az$$

$$az \cdot a\rho = az \cdot \phi = 0$$

Spherical co-ordinate system

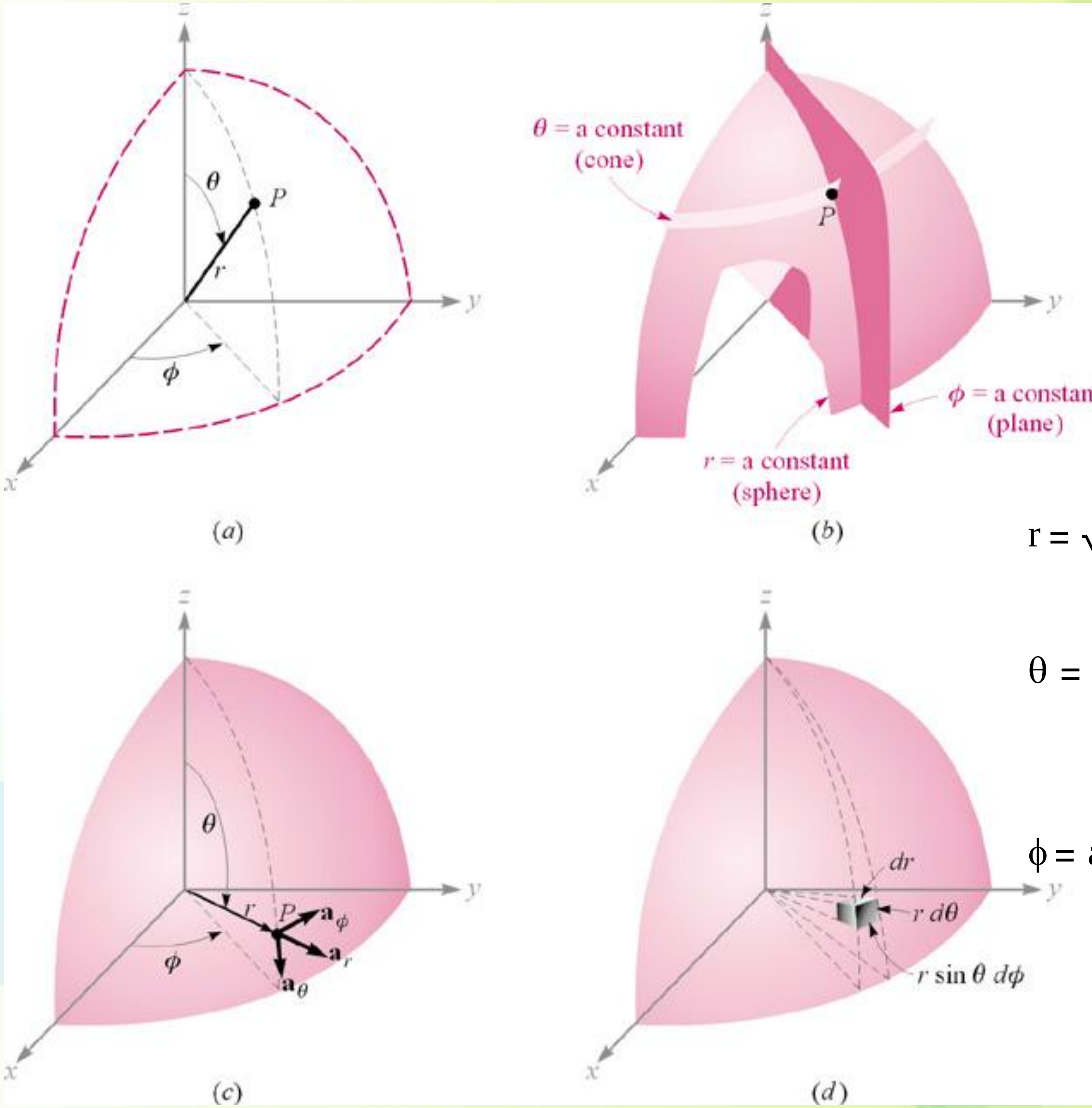


(a) Sphere of radius r with centre as origin

(b) Right circular cone with apex at origin

(c) Half plane perpendicular to xy plane

The Spherical Coordinate System



$$x = r\cdot\sin(\theta)\cdot\cos(\phi)$$

$$y = r\cdot\sin(\theta\cdot\sin(\phi))$$

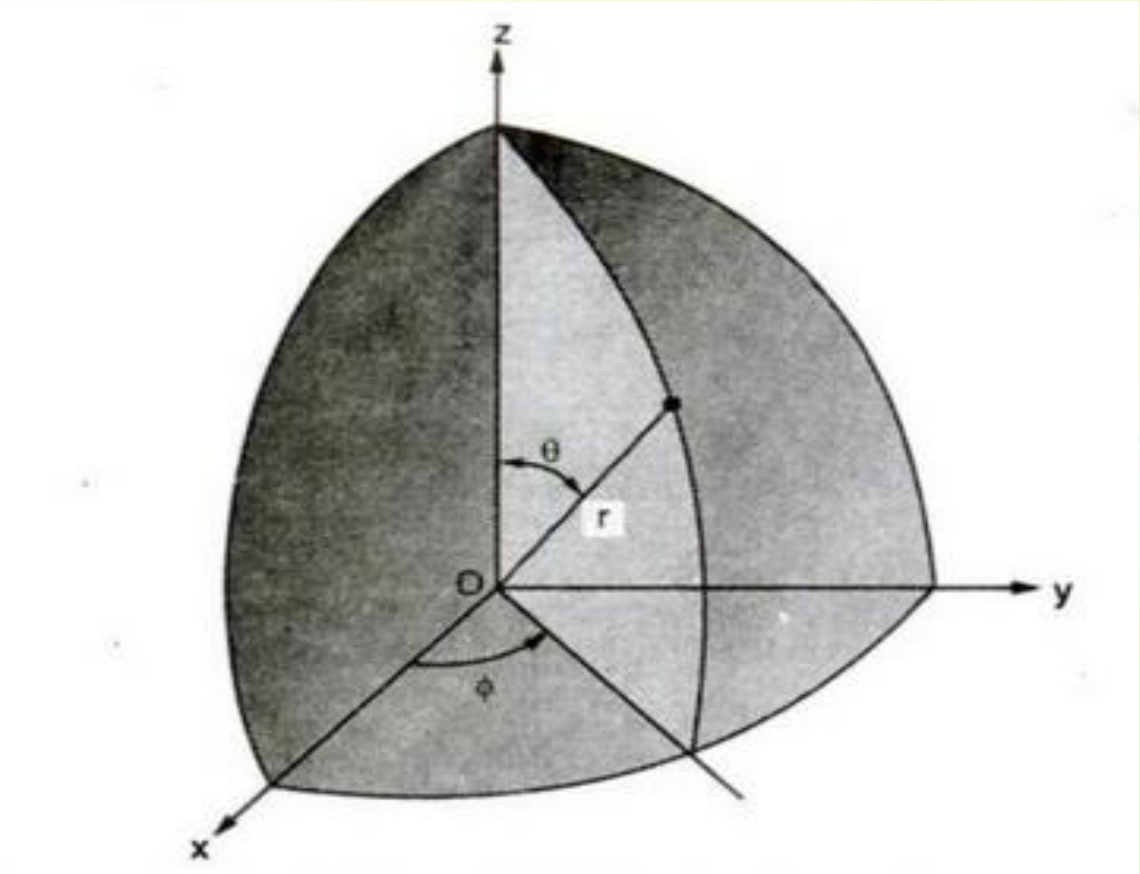
$$z = r\cdot\cos(\theta)$$

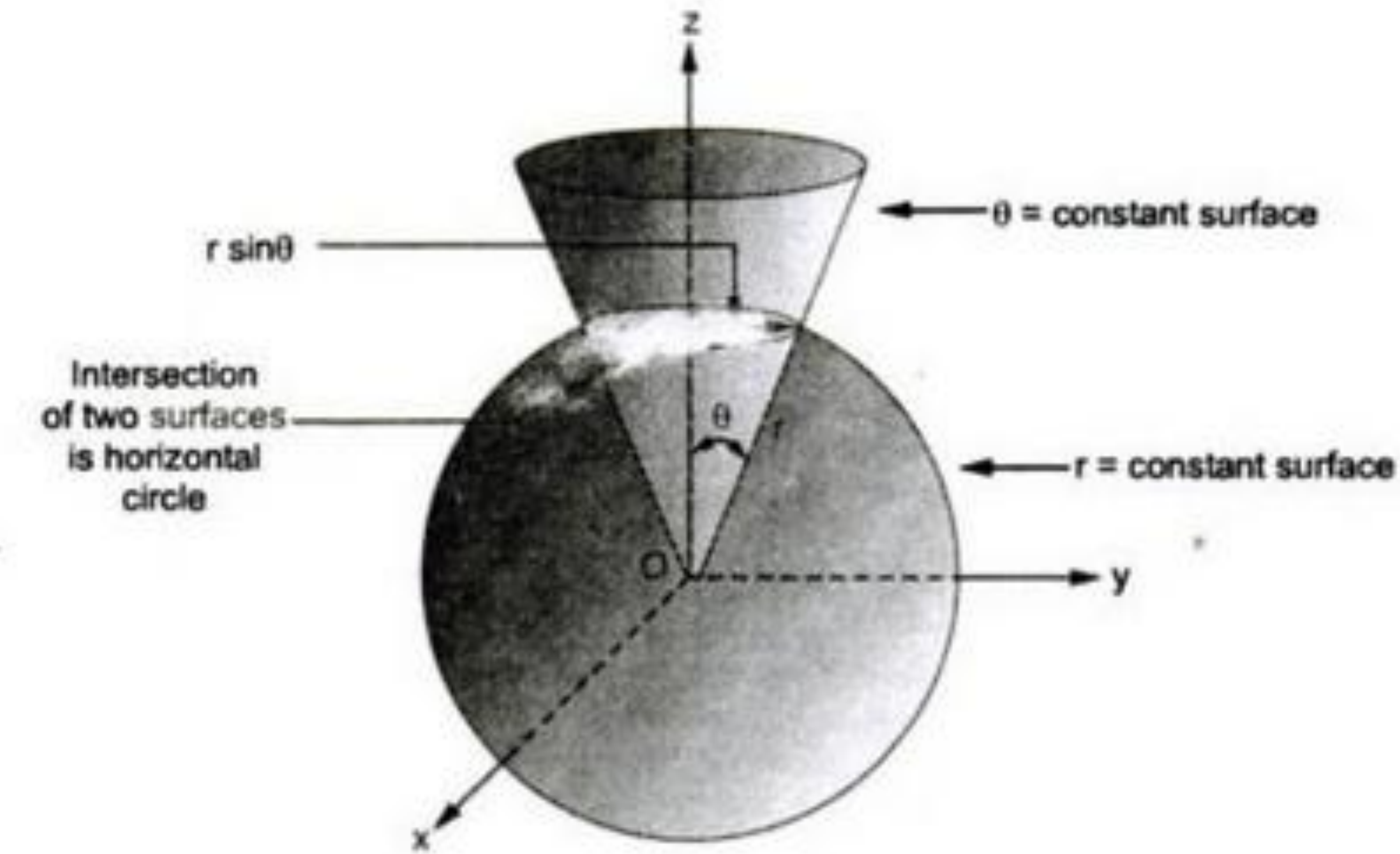
$$r = \sqrt{x^2 + y^2 + z^2} \qquad r \ge 0$$

$$\theta = \mathrm{acos}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right) \qquad 0 \le \theta \le 180$$

$$\phi = \mathrm{atan}\left(\frac{y}{x}\right)$$

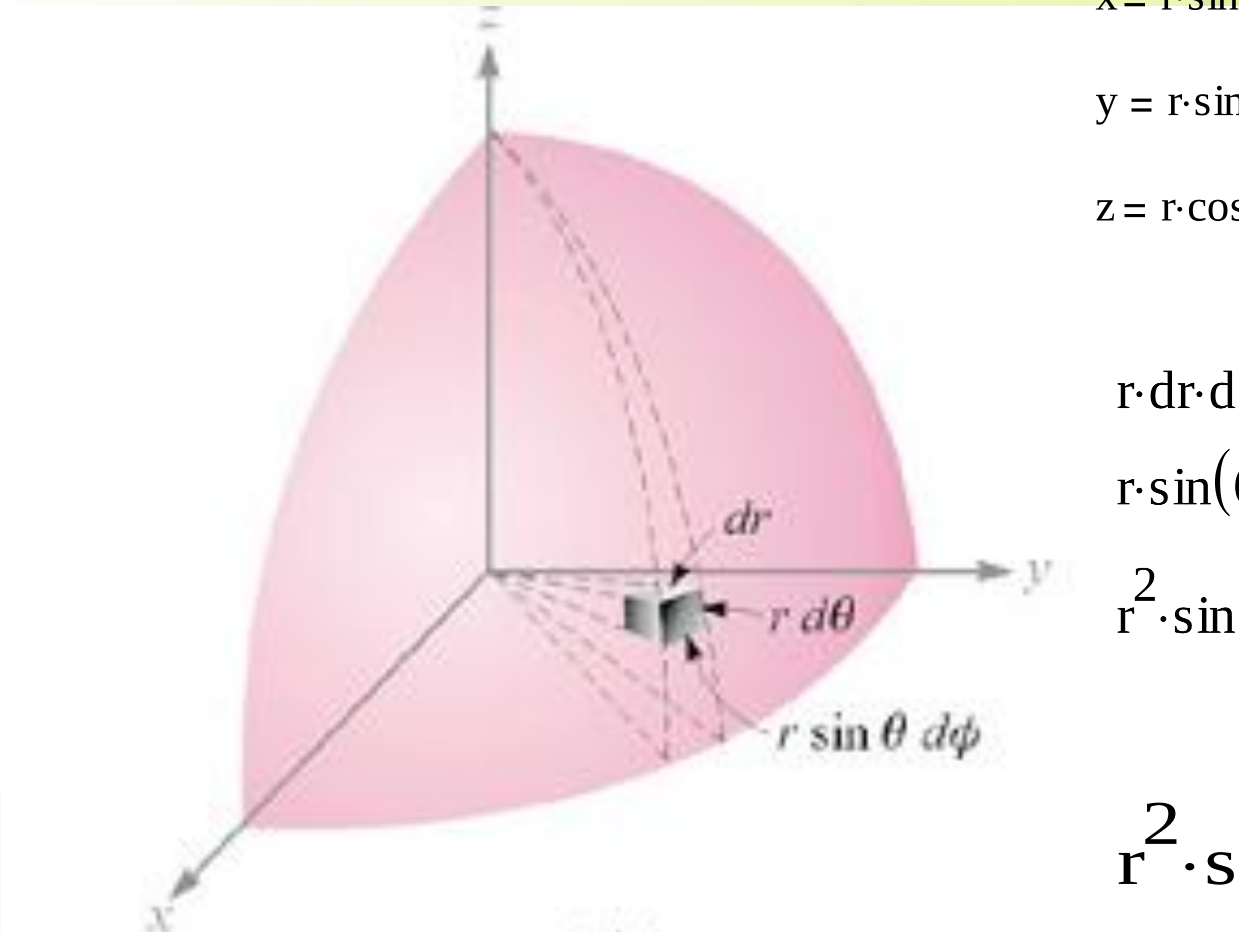
Sperical Co-ordinate System





z
θ = constant surface
r sinθ
Intersection
of two surfaces
is horizontal
circle
θ
r
r = constant surface
O
y
x

The Spherical Coordinate System



(d)

$$x = r \cdot \sin(\theta) \cdot \cos(\phi)$$

$$y = r \cdot \sin(\theta \cdot \sin(\phi))$$

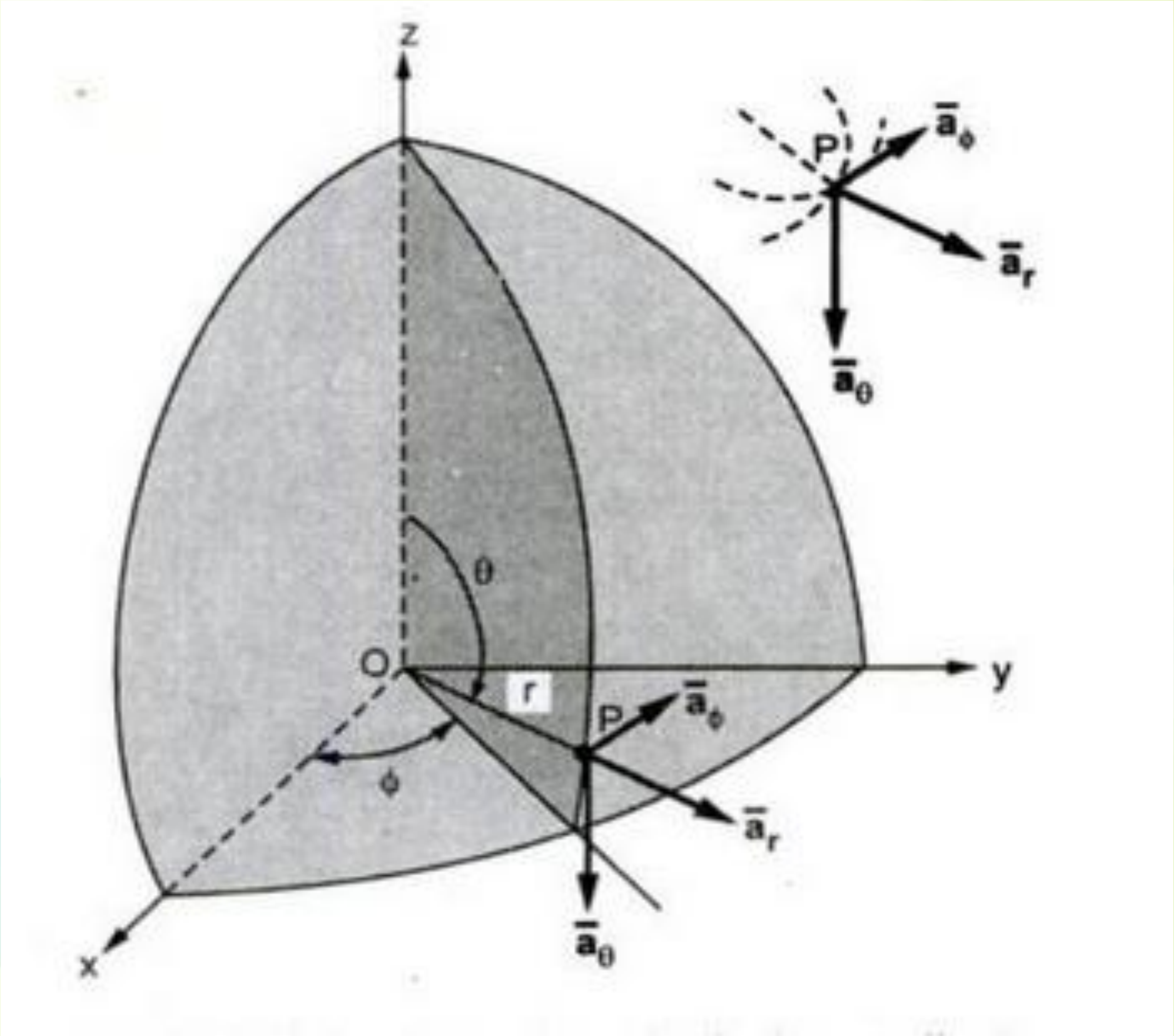
$$z = r \cdot \cos(\theta)$$

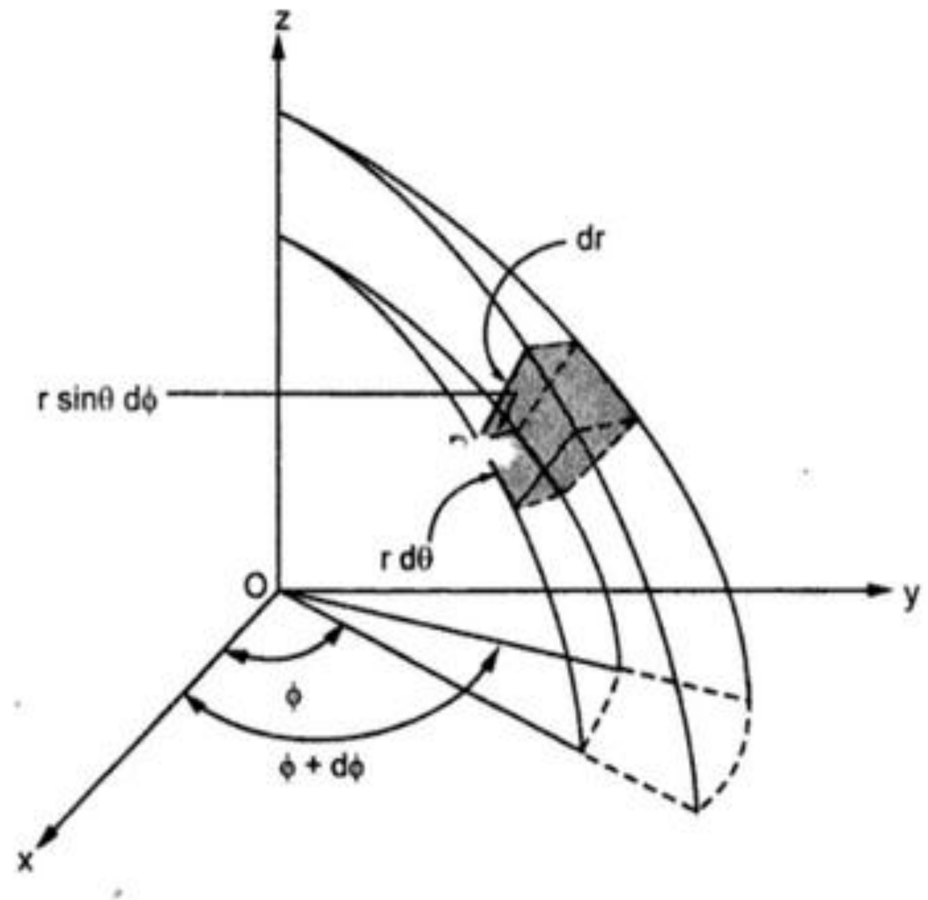
$$r \cdot dr \cdot d\theta$$

$$r \cdot \sin(\theta) \cdot dr \cdot d\phi$$

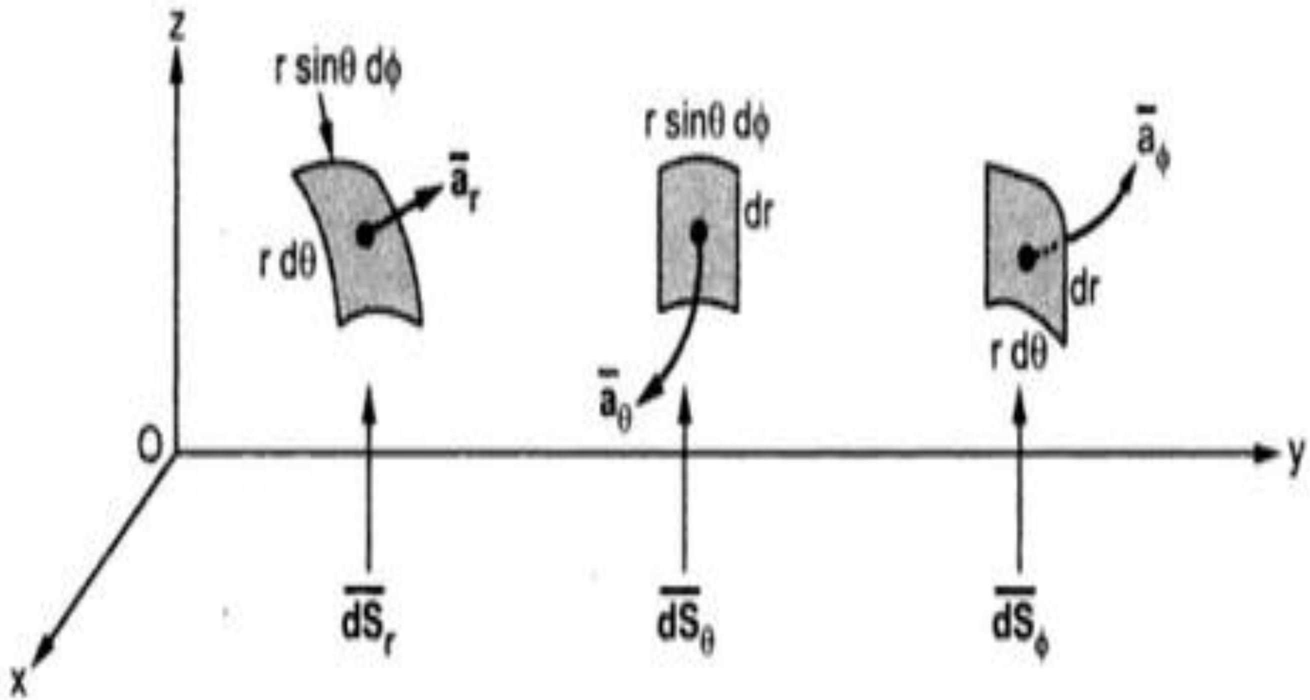
$$r^2 \cdot \sin(\theta) \cdot d\theta \cdot d\phi$$

$$r^2 \cdot \sin(\theta) \cdot dr \cdot d\theta \cdot d\phi$$


z
P
$\bar{a}_\phi$
$\bar{a}_r$
$\bar{a}_\theta$
θ
O
r
y
P
$\bar{a}_\phi$
φ
$\bar{a}_r$
x
$\bar{a}_\theta$



z
dr
r sinθ dφ
r dθ
O
y
φ
φ + dφ
x



$\overline{dS_r}$ = Differential vector surface area normal to r direction

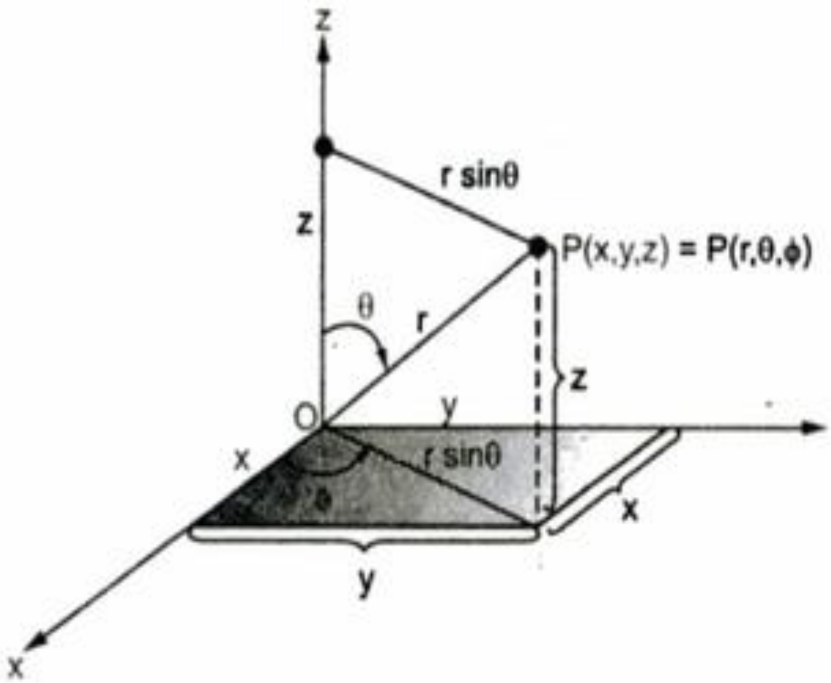
$= r^2 \sin\theta \, d\theta \, d\phi$

$\overline{dS_\theta}$ = Differential vector surface area normal to θ direction

$= r \sin\theta \, dr \, d\phi$

$\overline{dS_\phi}$ = Differential vector surface area normal to ϕ direction

$= r \, dr \, d\theta$



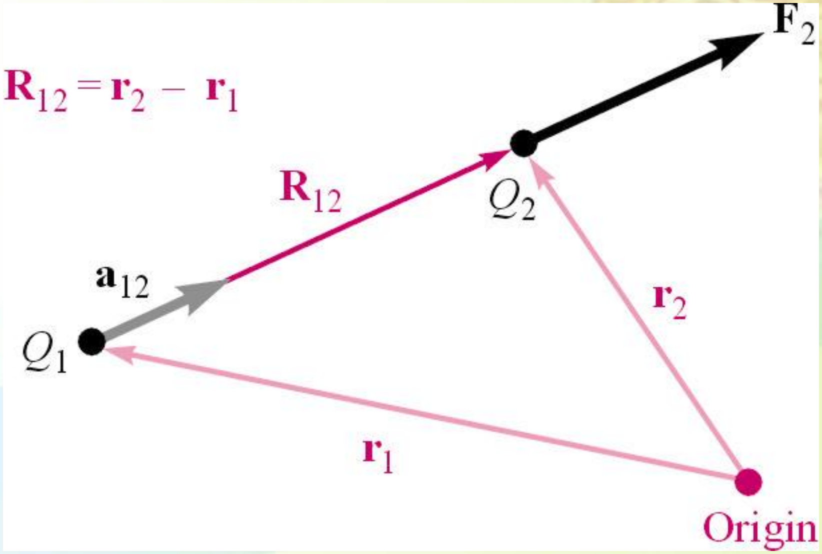
$$x = r\sin\theta\cos\phi \quad \text{and} \quad y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$\boxed{x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi \ \text{ and } \ z = r\cos\theta}$$

The Experimental Law of Coulomb

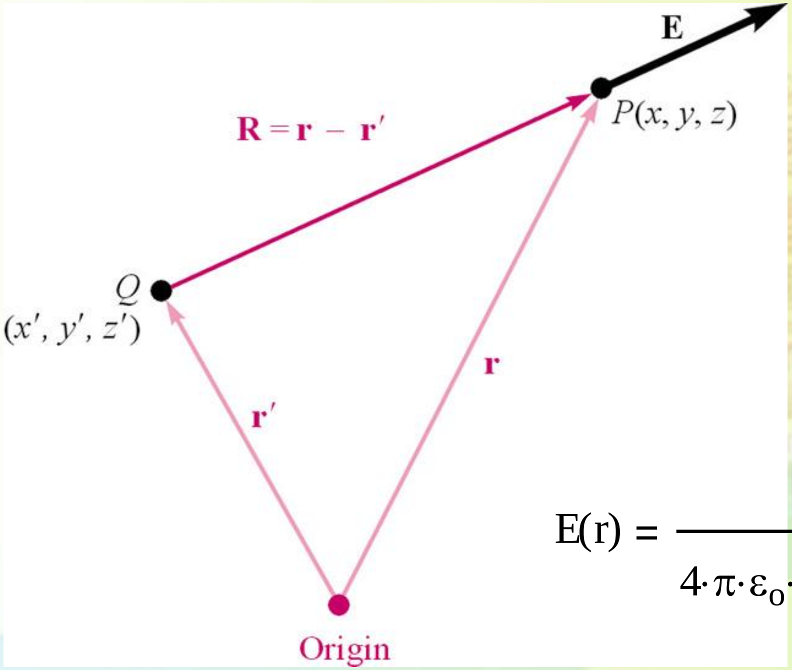
$$F = k \cdot \frac{Q1 \cdot Q2}{R^2} \qquad k = \frac{1}{4 \cdot \pi \cdot \varepsilon_0} \qquad \varepsilon_0 = 8.854 \cdot 10^{-12} = \frac{1}{36 \cdot \pi} \cdot 10^{-9} \quad \frac{F}{m} \qquad \longrightarrow \qquad F = \frac{Q1 \cdot Q2}{4 \cdot \pi \cdot \varepsilon_0 \cdot R^2}$$



$$F = \frac{Q1 \cdot Q2}{4 \cdot \pi \cdot \varepsilon_0 \cdot R^2} \cdot a_{12} \qquad a_{12} = \frac{r2 - r1}{\left| r2 - r1 \right|}$$

Electric Field Intensity

$$F_t = \frac{Q1 \cdot Qt}{4 \cdot \pi \cdot \varepsilon_0 \cdot (R_{1t})^2} \cdot a_{1t}$$



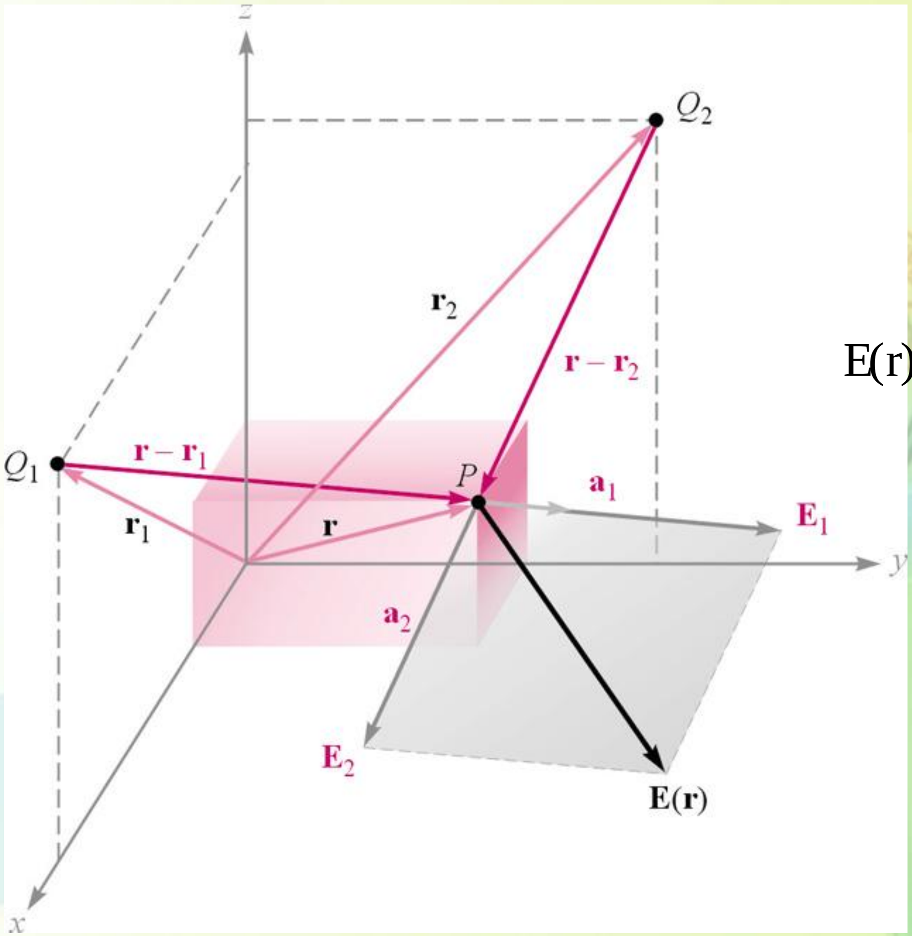
$$\frac{F_t}{Qt} = \frac{Q1}{4 \cdot \pi \cdot \varepsilon_0 \cdot (R_{1t})^2} \cdot a_{1t}$$

$$E = \frac{F_t}{Qt}$$

$$E(r) = \frac{Q}{4 \cdot \pi \cdot \varepsilon_o \cdot (|r - r1|)^2} \cdot \frac{r - r1}{|r - r1|}$$

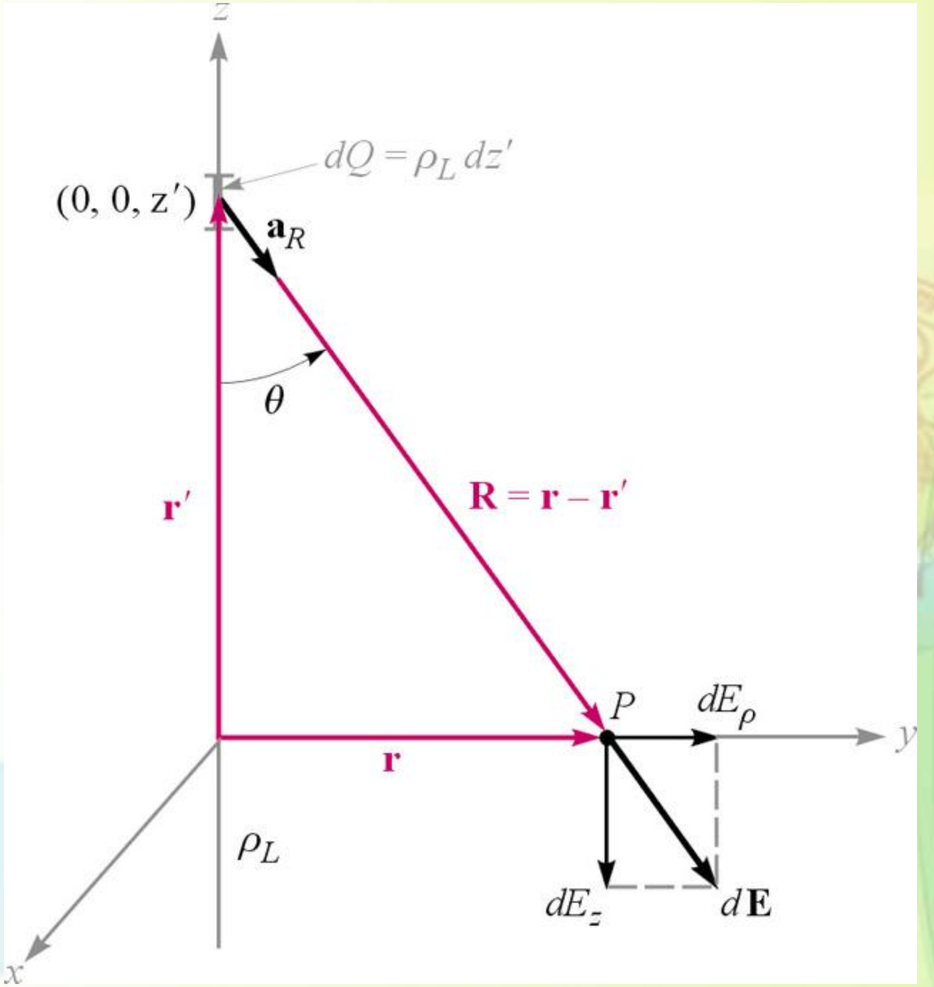
$$E(r) = \frac{Q \cdot \left[(x - x1) \cdot a_x + (y - y1) \cdot a_y + (z - z1) \cdot a_z \right]}{4 \cdot \pi \cdot \varepsilon_o \cdot \left[(x - x1)^2 + (y - y1)^2 + (z - z1)^2 \right]^{\frac{3}{2}}}$$

Electric Field Intensity



$$E(r) = \sum_{m=1}^{n} \frac{Qm}{4\cdot\pi\cdot\varepsilon_0\cdot\left(\left|r - r_m\right|\right)^2}\cdot a_m$$

Field of a Line Charge



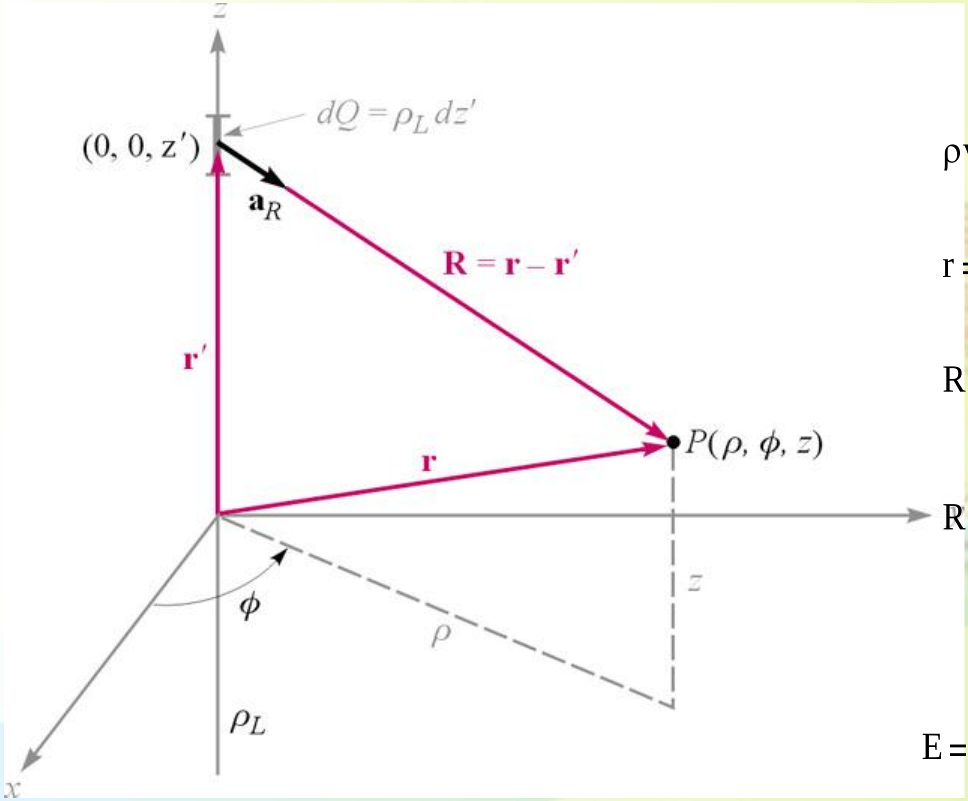
$$E\rho = \int_{-\Omega}^{\Omega} \frac{\rho L \cdot \rho}{4 \cdot \pi \cdot \varepsilon_0 \cdot \left(\rho^2 + z^2\right)^{\frac{3}{2}}} \, dz$$

$$E\rho = \frac{\rho L}{2 \cdot \pi \cdot \varepsilon_0 \cdot \rho}$$

$$E = \frac{\rho L}{2 \cdot \pi \cdot \varepsilon_0 \cdot \rho} \cdot a_\rho$$

Field of a Line Charge (neglect symmetry)

$$E = \int\int\int \frac{\rho vq}{4\cdot\pi\cdot\varepsilon 0}\cdot\frac{(r - r1)}{(|r - r1|)^3}\,dx1\,dy1\,dz1$$



$$\rho v1 = \rho L\cdot dz1$$

$$r = \rho\cdot a\rho + z\cdot az \qquad r1 = z1\cdot az$$

$$R = r - r1 = \rho\cdot a\rho + (z - z1)\cdot az$$

$$R = \sqrt{\rho^2 + (z - z1)^2} \qquad aR = \frac{\rho\cdot a\rho + (z - z1)\cdot az}{\sqrt{\rho^2 + (z - z1)^2}}$$

$$E = \int_{-\Omega}^{\Omega} \frac{(\rho L\cdot dz1)\cdot\left[\rho\cdot a\rho + (z - z1)\cdot az\right]}{\left[4\cdot\pi\cdot\varepsilon 0\left[\rho^2 + (z - z1)^2\right]\right]^{\frac{3}{2}}}\,dz1$$

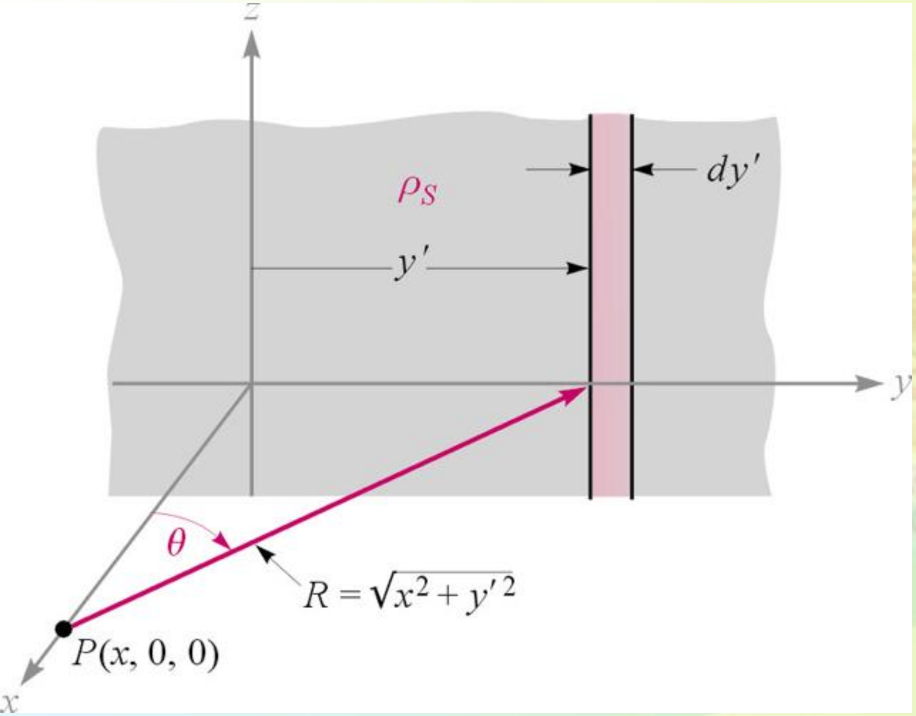
Field of a Line Charge (neglect symmetry)

$$E = \frac{\rho L}{(4\cdot\pi\cdot\varepsilon 0)}\cdot\left[a\rho\cdot\int_{-\Omega}^{\Omega}\frac{(\rho\cdot dz1)}{\left[\left[\rho^2+(z-z1)^2\right]\right]^{\frac{3}{2}}}\,dz1 + az\cdot\int_{-\Omega}^{\Omega}\frac{((z-z1))}{\left[\left[\rho^2+(z-z1)^2\right]\right]^{\frac{3}{2}}}\,dz1\right]$$

$$E = \frac{\rho L}{(4\cdot\pi\cdot\varepsilon 0)}\cdot\left[a\rho\cdot\rho\cdot\frac{1}{\rho^2}\cdot\overset{-\Omega\ \text{to}\ \Omega}{\frac{-(z-z1)}{\sqrt{\rho^2+(z-z1)^2}}} + az\cdot\overset{-\Omega\ \text{to}\ \Omega}{\frac{1}{\sqrt{\rho^2+(z-z1)^2}}}\right]$$

$$E = \frac{\rho L}{(4\cdot\pi\cdot\varepsilon 0)}\cdot\left(a\rho\cdot\frac{2}{\rho} + az\cdot 0\right) = \frac{\rho L}{(2\cdot\pi\cdot\varepsilon 0)\cdot\rho}\cdot a\rho$$

Field of a Sheet of Charge



$$R = \sqrt{x^2 + y^2}$$

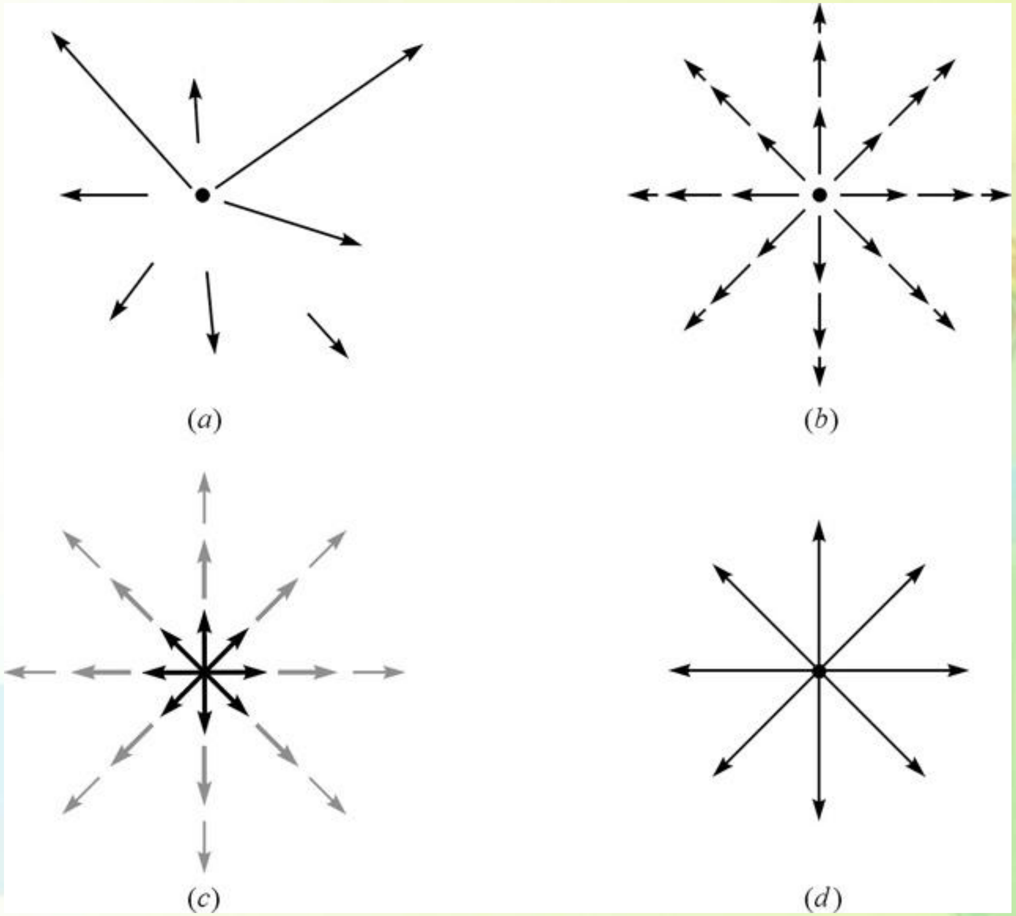
$$dEx = \rho s \cdot \frac{dy1}{2 \cdot \pi \cdot \varepsilon 0 \cdot \sqrt{x^2 + y^2}} \cdot \cos(\theta) = \frac{\rho s}{(2 \cdot \pi \cdot \varepsilon 0)} \cdot \frac{x \cdot dy1}{x^2 + y^2}$$

$$Ex = \frac{\rho s}{(2 \cdot \pi \cdot \varepsilon 0)} \cdot \int_{-\Omega}^{\Omega} \frac{x}{x^2 + y^2}\, dy1 = \frac{\rho s}{(2 \cdot \pi \cdot \varepsilon 0)} \cdot \operatorname{atan}\left(\frac{y1}{x}\right) \quad -\Omega \text{ to } \Omega$$

$$Ex = \frac{\rho s}{2 \cdot \varepsilon 0} \qquad E = \frac{\rho s}{2 \cdot \varepsilon 0} \cdot aN$$

This is a very interesting result. The field is constant in magnitude and direction. It is as strong a million miles away from the sheet as it is right of the surface.

Streamlines and Sketches of Fields

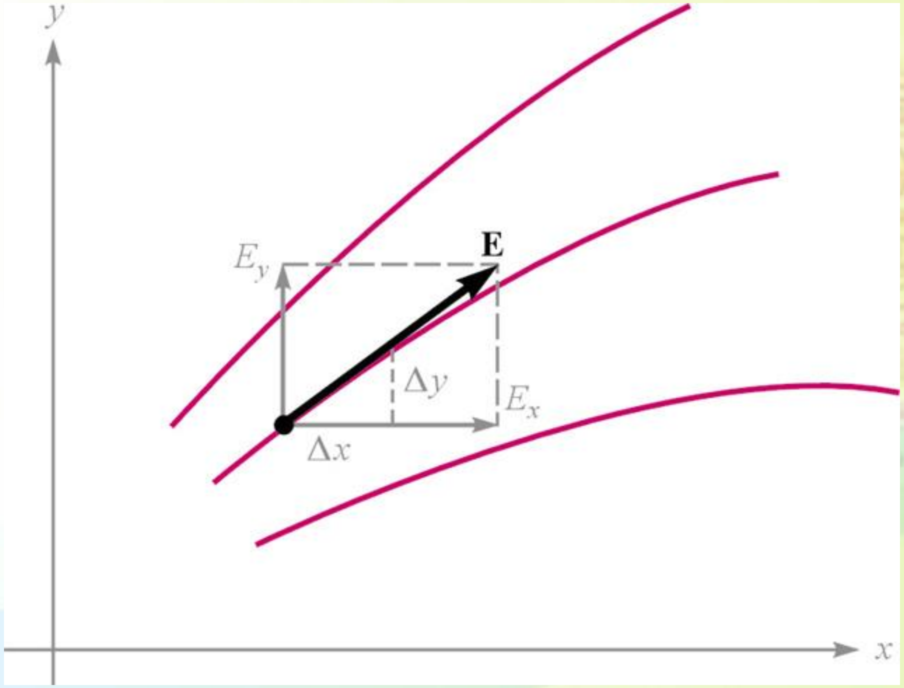


Cross-sectional view of the line charge.

Lengths proportional to the magnitudes of E and pointing in the direction of E

Streamlines and Sketches of Fields

$$\frac{Ey}{Ex} = \frac{dy}{dz}$$



$$E = \frac{1}{\rho} \cdot a\rho \qquad E = \frac{x}{x^2 + y^2} \cdot ax + \frac{y}{x^2 + y^2} \cdot ay$$

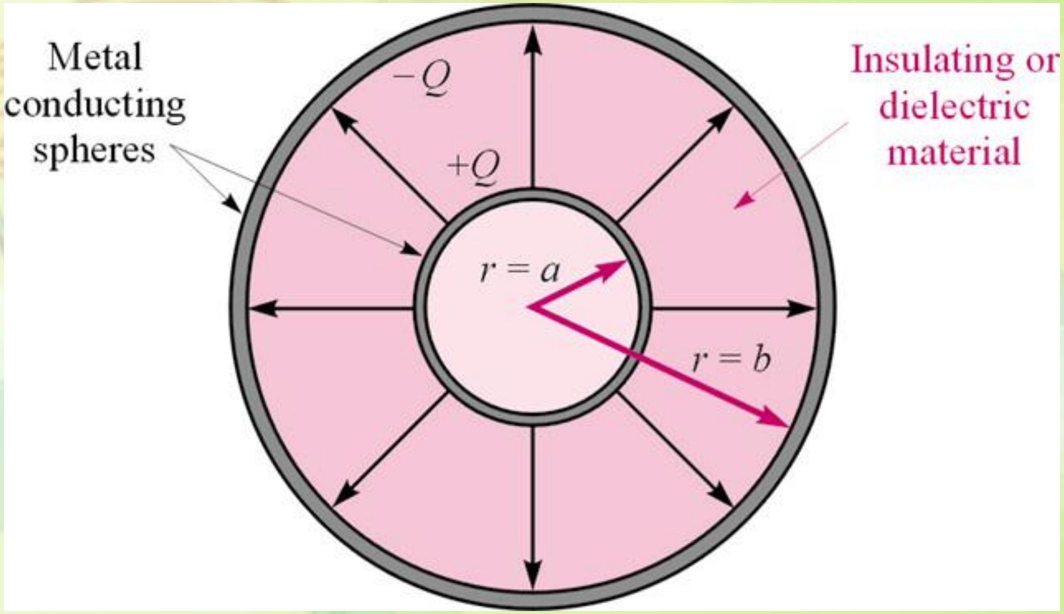
$$\frac{dy}{dx} = \frac{Ey}{Ex} = \frac{y}{x} \qquad \frac{dy}{y} = \frac{dx}{x}$$

$$\ln(y) = \ln(x) + C1 \qquad \ln(y) = \ln(x) + \ln(c)$$

$$y = C \cdot x$$

3.1 Electric Flux Density

- Faraday's Experiment



$\text{Flux} = \Psi,$ same units as Q

Ψ is responsible for creating $-Q$ on outer sphere

Electric Flux Density, D

- Units: C/m^2
- Magnitude: Number of flux lines (coulombs) crossing a surface normal to the lines divided by the surface area.
- Direction: Direction of flux lines (same direction as **E**).
- For a point charge: $\mathbf{D} = \dfrac{Q}{4\pi r^2}\,\mathbf{a}_r$
- For a general charge distribution,

$$\mathbf{D} = \epsilon_0\,\mathbf{E} = \int_{\text{vol}} \frac{\rho_v\,dv}{4\pi R^2}\,\mathbf{a}_r$$

D3.1

Given a 60-uC point charge located at the origin, find the total electric flux passing through:

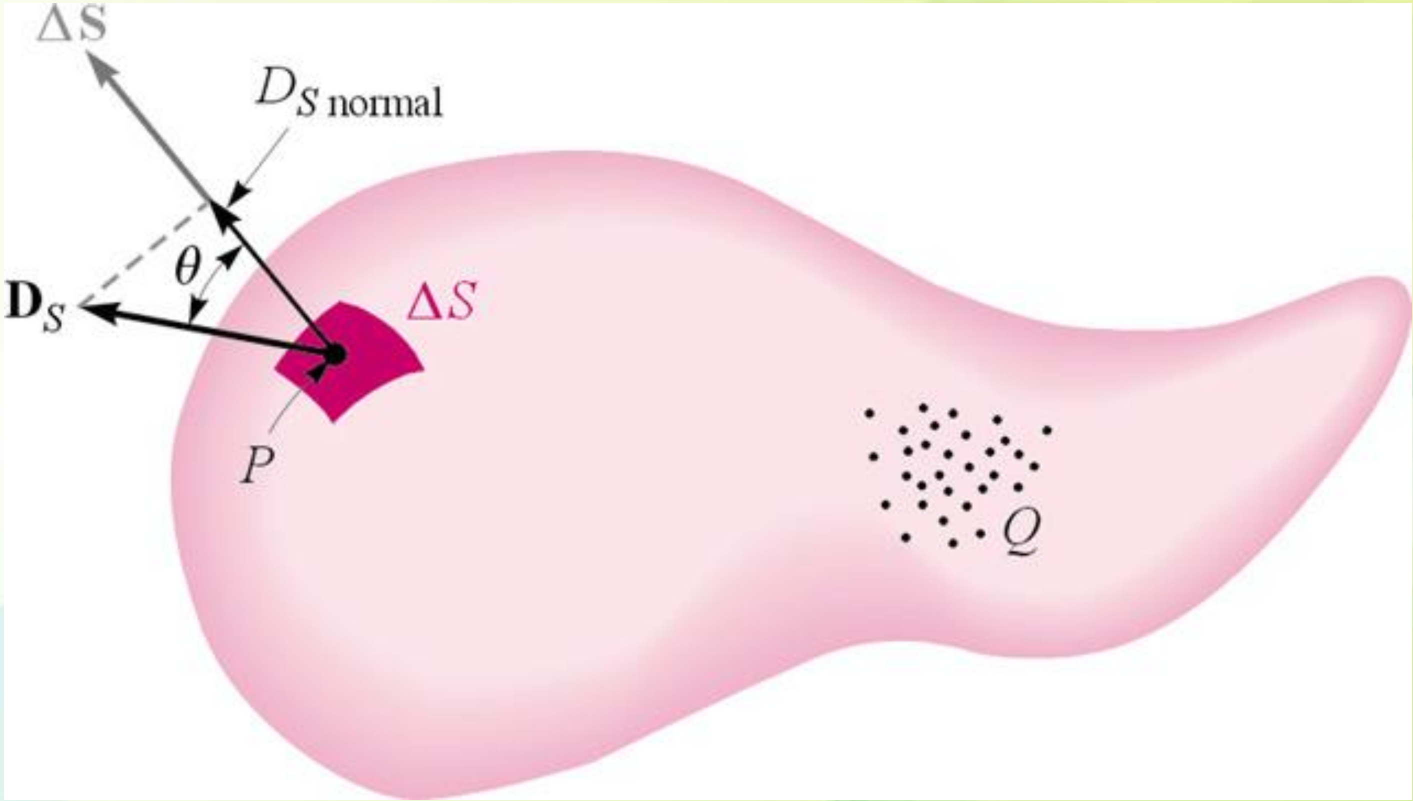
(a) That portion of the sphere r = 26 cm bounded by $0 < \theta < \pi/2$ and $0 < \phi < \pi/2$

Gauss's Law

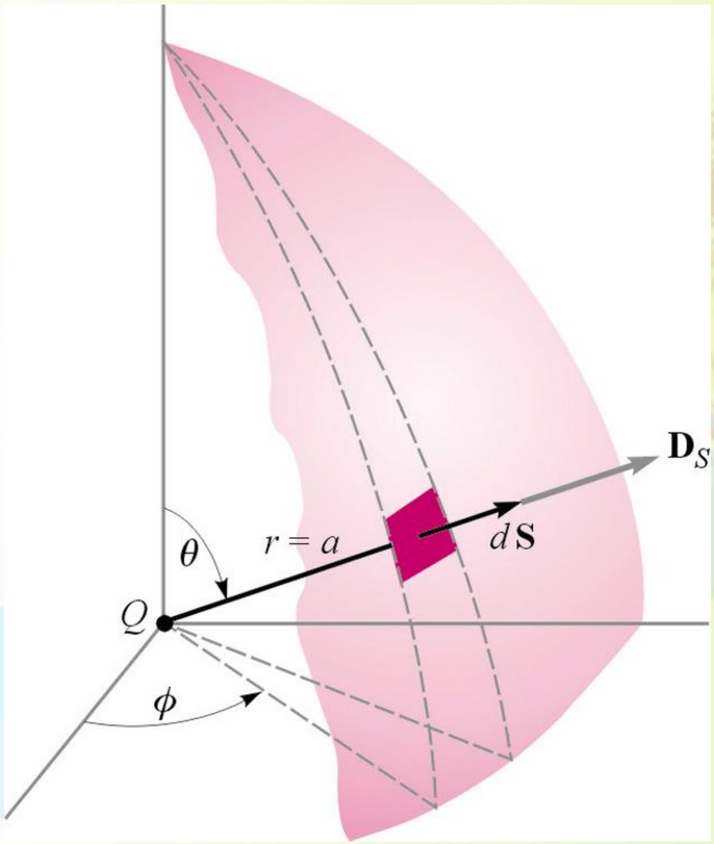
- "The electric flux passing through any closed surface is equal to the total charge enclosed by that surface."

$$\Psi = \oint_S \mathbf{D}_S \cdot d\mathbf{S} = \text{charge enclosed} = Q$$

- The integration is performed over a *closed* surface, i.e. *gaussian surface.*



- We can check Gauss's law with a point charge example.



$$\int_0^{2\pi}\int_0^{\pi}\frac{q}{4\pi a^2}\,a^2\,\mathrm{Sin}[\theta]\,d\theta\,d\phi$$

q

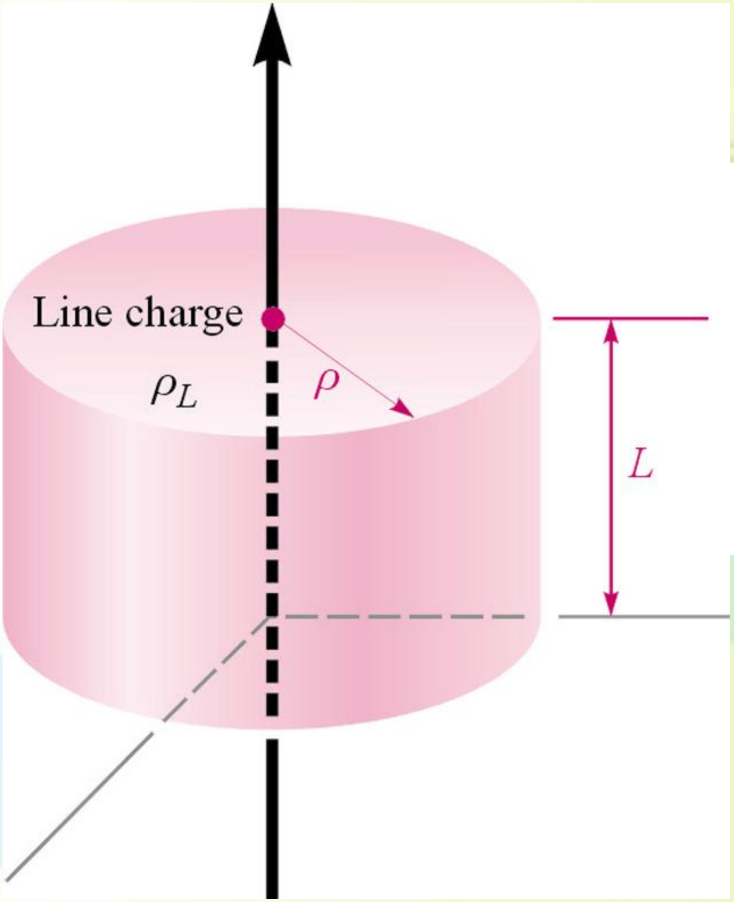
Symmetrical Charge Distributions

- Gauss's law is useful under two conditions.

1. $\mathbf{D}_S$ is everywhere either normal or tangential to the closed surface, so that $\mathbf{D}_S \cdot d\mathbf{S}$ becomes either $D_S\, dS$ or zero, respectively.

1. On that portion of the closed surface for which $\mathbf{D}_S \cdot dS$ is not zero, D_S = constant.

Gauss's law simplifies the task of finding **D** near an infinite line charge.

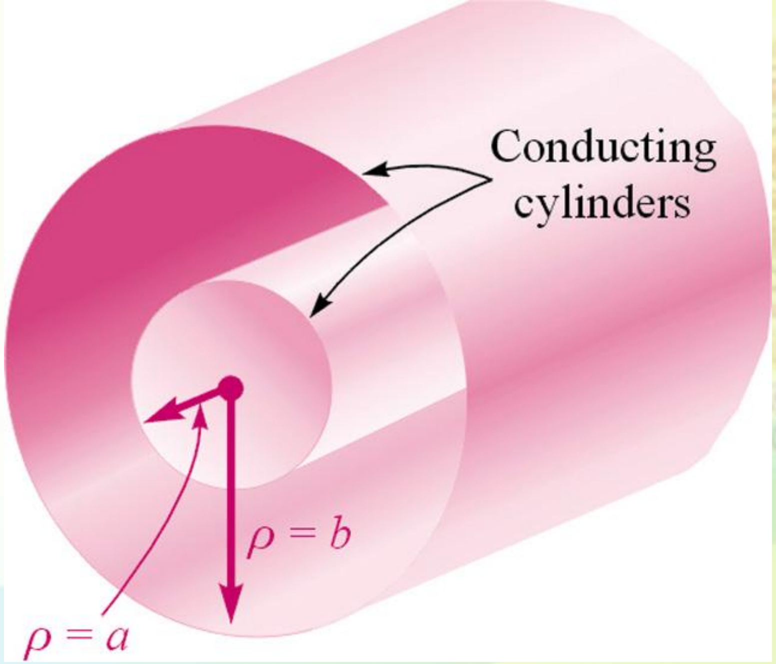


$$Q = \oint_{\text{cyl}} \mathbf{D}_S \cdot d\mathbf{S} = D_S \int_{\text{sides}} dS + 0 \int_{\text{top}} dS + 0 \int_{\text{bottom}} dS$$

$$= D_S \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \rho \, d\phi \, dz = D_S \, 2\pi\rho L$$

$$D_S = D_\rho = \frac{Q}{2\pi\rho L} = \frac{\rho_L}{2\pi\rho}$$

Infinite coaxial cable:

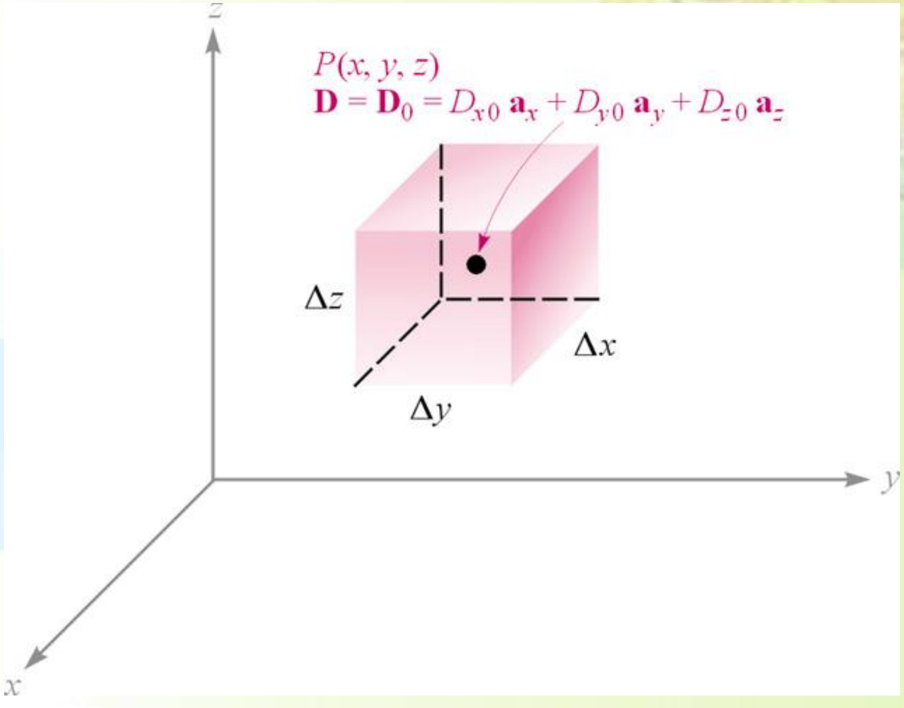


$$D = \frac{\rho_L}{2\pi\rho}\,\mathbf{a}_\rho \quad (a < \rho < b)$$

$$D = 0 \quad (\rho > b)$$

Differential Volume Element

- If we take a small enough closed surface, then **D** is almost constant over the surface.



$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_{\text{front}} + \int_{\text{back}} + \int_{\text{left}} + \int_{\text{right}} + \int_{\text{top}} + \int_{\text{bottom}}$$

$$\int_{\text{front}} \doteq \left(D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y \Delta z$$

$$\int_{\text{back}} \doteq \left(-D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y \Delta z$$

$$\int_{\text{front}} + \int_{\text{back}} \doteq \frac{\partial D_x}{\partial x}\Delta x \Delta y \Delta z$$

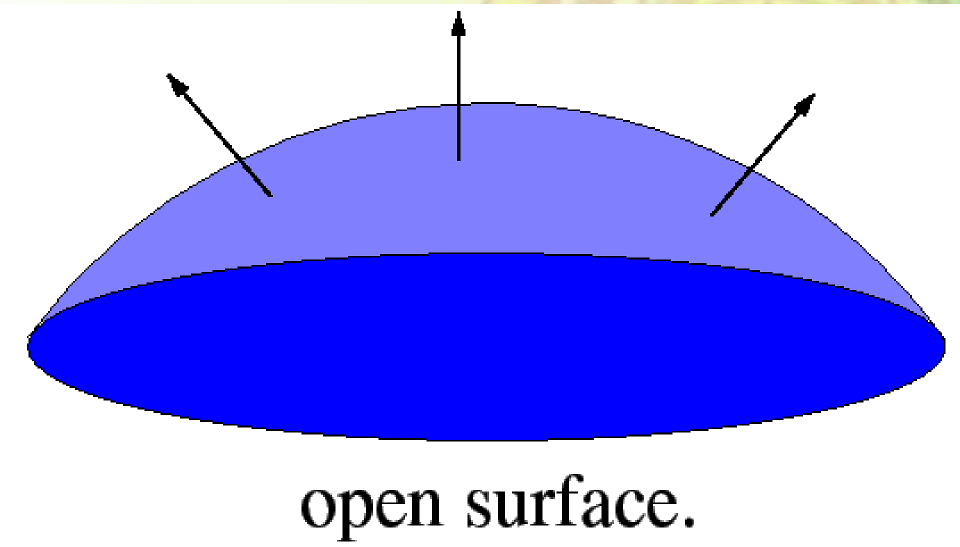
$$\vdots$$

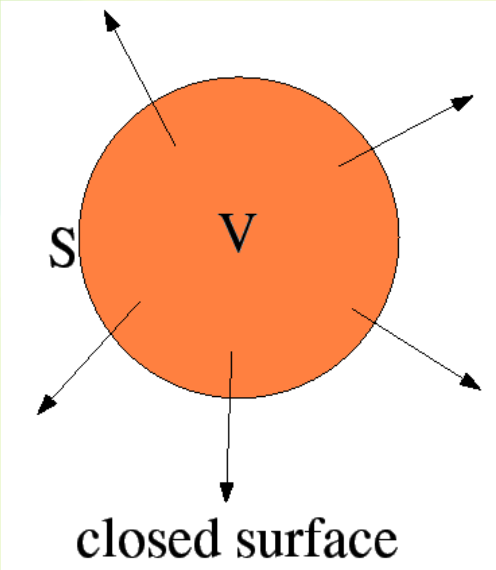
$$\oint_S \mathbf{D} \cdot d\mathbf{S} \doteq \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta x \Delta y \Delta z$$

Charge enclosed in volume $\Delta v \doteq \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \times$ volume Δv

Divergence

Divergence is the outflow of flux from a small closed surface area (per unit volume) as volume shrinks to zero.





-Water leaving a bathtub

-Closed surface (water itself) is essentially incompressible

-Net outflow is zero

-Air leaving a punctured tire

-Divergence is positive, as closed surface (tire) exhibits net outflow

Mathematical definition of divergence

$$\operatorname{div}(\mathbf{D}) = \lim_{\Delta v \to 0} \int \frac{\mathbf{D}}{\Delta v} \, d\mathbf{S}$$

Surface integral as the volume element (Δv) approaches zero

D is the vector flux density

$$\boxed{\operatorname{div}(\mathbf{D}) = \left(\frac{\delta D_x}{\delta x} + \frac{\delta D_y}{\delta y} + \frac{\delta D_z}{\delta z} \right)}$$

- *Cartesian*

Divergence in Other Coordinate Systems

Cylindrical

$$\mathrm{div}(\mathbf{D}) = \frac{1}{\rho}\cdot\frac{\delta}{\delta\rho}\left(\rho\cdot D_{\rho}\right) + \frac{1}{\rho}\cdot\frac{\delta D_{\phi}}{\delta\phi} + \frac{\delta D_{z}}{\delta z}$$

Spherical

$$\mathrm{div}(\mathbf{D}) = \frac{1}{r^{2}}\cdot\frac{\delta\left(D_{r}\cdot r^{2}\right)}{\delta r} + \frac{1}{r\cdot\sin(\theta)}\cdot\frac{\delta\left(D_{\theta}\cdot\sin(\theta)\right)}{\delta\theta} + \frac{1}{r\cdot\sin(\theta)}\cdot\frac{\delta D_{\phi}}{\delta\phi}$$

4.1 Energy to move a point charge through a Field

- Force on Q due to an electric field

$$F_E = QE$$

- Differential work done by an external source moving Q

$$dW = -QE \cdot dL$$

- Work required to move a charge a finite distance

$$W = -Q \int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L}$$

Line Integral

- Work expression without using vectors

  EL is the component of E in the dL direction

$$W = -Q \cdot \int_{\text{initial}}^{\text{final}} E_L \, dL$$

$$d\mathbf{L} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z \qquad \text{(cartesian)}$$

$$d\mathbf{L} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z \qquad \text{(cylindrical)}$$

$$d\mathbf{L} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi \qquad \text{(spherical)}$$

- Uniform electric field density

$$W = -QE \cdot L_{BA}$$

Potential

- Measure potential difference between a point and something which has zero potential “ground”

$$V_{AB} = V_A - V_B$$

Potential Field of a Point Charge

- Let V=0 at infinity

$$V = \frac{Q}{4\pi\varepsilon_0 \cdot r}$$

- Equipotential surface:
  - A surface composed of all points having the same potential

Potential due to *n* point charges

Continue adding charges

$$V(r) = \frac{Q1}{4\cdot\pi\cdot\varepsilon_0\cdot|r - r1|} + \frac{Q2}{4\cdot\pi\cdot\varepsilon_0\cdot|r - r2|} + ... + \frac{Qn}{4\cdot\pi\cdot\varepsilon_0\cdot|r - r_n|}$$

$$V(r) = \sum_{m=1}^{n} \frac{Qm}{4\cdot\pi\cdot\varepsilon_0\cdot|r - r_m|}$$

Potential as point charges become infinite

Volume of charge

$$V(r) = \int \frac{\rho_v(r_{prime})}{4 \cdot \pi \cdot \varepsilon_0 \cdot |r - r_{prime}|} \, dv_{prime}$$

Line of charge

$$V(r) = \int \frac{\rho_L(r_{prime})}{4 \cdot \pi \cdot \varepsilon_0 \cdot |r - r_{prime}|} \, dL_{prime}$$

Surface of charge

$$V(r) = \int \frac{\rho_S(r_{prime})}{4 \cdot \pi \cdot \varepsilon_0 \cdot |r - r_{prime}|} \, dS_{prime}$$

Potential as point charges become infinite

Volume of charge

$$V(r) = \int \frac{\rho_v(r_{prime})}{4\cdot\pi\cdot\varepsilon_0\cdot\left|r - r_{prime}\right|}\, dv_{prime}$$

Line of charge

$$V(r) = \int \frac{\rho_L(r_{prime})}{4\cdot\pi\cdot\varepsilon_0\cdot\left|r - r_{prime}\right|}\, dL_{prime}$$

Surface of charge

$$V(r) = \int \frac{\rho_S(r_{prime})}{4\cdot\pi\cdot\varepsilon_0\cdot\left|r - r_{prime}\right|}\, dS_{prime}$$

Gradients in different coordinate systems

The following equations are found on page 104 and inside the back cover of the text:

$$\mathrm{grad}\,V = \frac{\delta V}{\delta x}\cdot a_x + \frac{\delta V}{\delta y}\cdot a_y + \frac{\delta V}{\delta z}\cdot a_z$$

Cartesian

$$\mathrm{grad}\,V = \frac{\delta V}{\delta \rho}\cdot a_\rho + \frac{1}{\rho}\cdot\frac{\delta V}{\delta \phi}\cdot a_\phi + \frac{\delta V}{\delta z}\cdot a_z$$

Cylindrical

$$\mathrm{grad}\,V = \frac{\delta V}{\delta r}\cdot a_r + \frac{1}{r}\cdot\frac{\delta V}{\delta \theta}\cdot a_\theta + \frac{1}{r\cdot\sin(\theta)}\cdot\frac{\delta V}{\delta \phi}\cdot a_\phi$$

Spherical

Chapter 7 – Poisson's and Laplace Equations

A useful approach to the calculation of electric potentials
Relates potential to the charge density.
The electric field is related to the charge density by the divergence relationship

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field
ρ = charge density
ε_0 = permittivity

The electric field is related to the electric potential by a gradient relationship

$$E = -\nabla V$$

Therefore the potential is related to the charge density by Poisson's equation

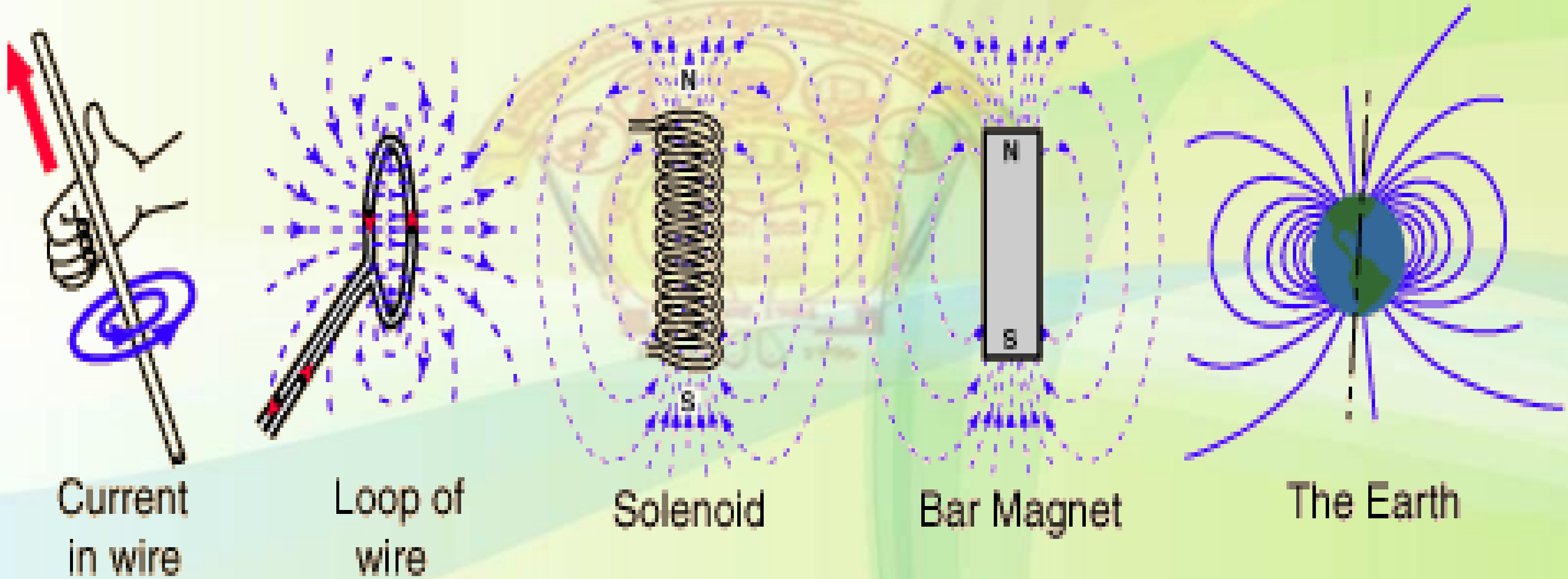
$$\nabla \cdot \nabla V = \nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

In a charge-free region of space, this becomes Laplace's equation

$$\nabla^2 V = 0$$

Magnetic Field Sources

Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.



Magnetic Field Sources

Maxwell's equations

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} - \underline{K}_c - \underline{K}_i$$

$$\nabla \times \underline{H} = \frac{\partial \underline{D}}{\partial t} + \underline{J}_c + \underline{J}_i$$

$$\nabla \cdot \underline{D} = q_{ev}$$

$$\nabla \cdot \underline{B} = q_{mv}$$

Maxwell's equations for TVF

Differential form	Controlling principle	Integral form	
$\nabla \times \vec{H} = \dot{\vec{D}} + \vec{J}$	Ampere's Circuital Law	$\oint \vec{H} \cdot d\vec{L} = \int \dot{\vec{D}} + \vec{J} \cdot d\vec{S}$	(I)
$\nabla \times \vec{E} = -\dot{\vec{B}}$	**Potential around a closed path is zero**	$\oint \vec{E} \cdot d\vec{L} = -\int \dot{\vec{B}} \cdot d\vec{S}$	(II)
$\nabla \bullet \vec{D} = \rho$	Gauss's Law	$\oint \vec{D} \cdot d\vec{S} = \int \rho \, dv$	(III)
$\nabla \bullet \vec{B}$	Non-existence of isolated magnetic poles	$\oint \vec{B} \cdot d\vec{S} = 0$	(IV)

Queries ...?