The Steady State Magnetic Field

The Concept of Field (Physical Basis ?)

Why do Forces Must Exist?

Magnetic Field – Requires Current Distribution

Effect on other Currents – next chapter

Free-space Conditions

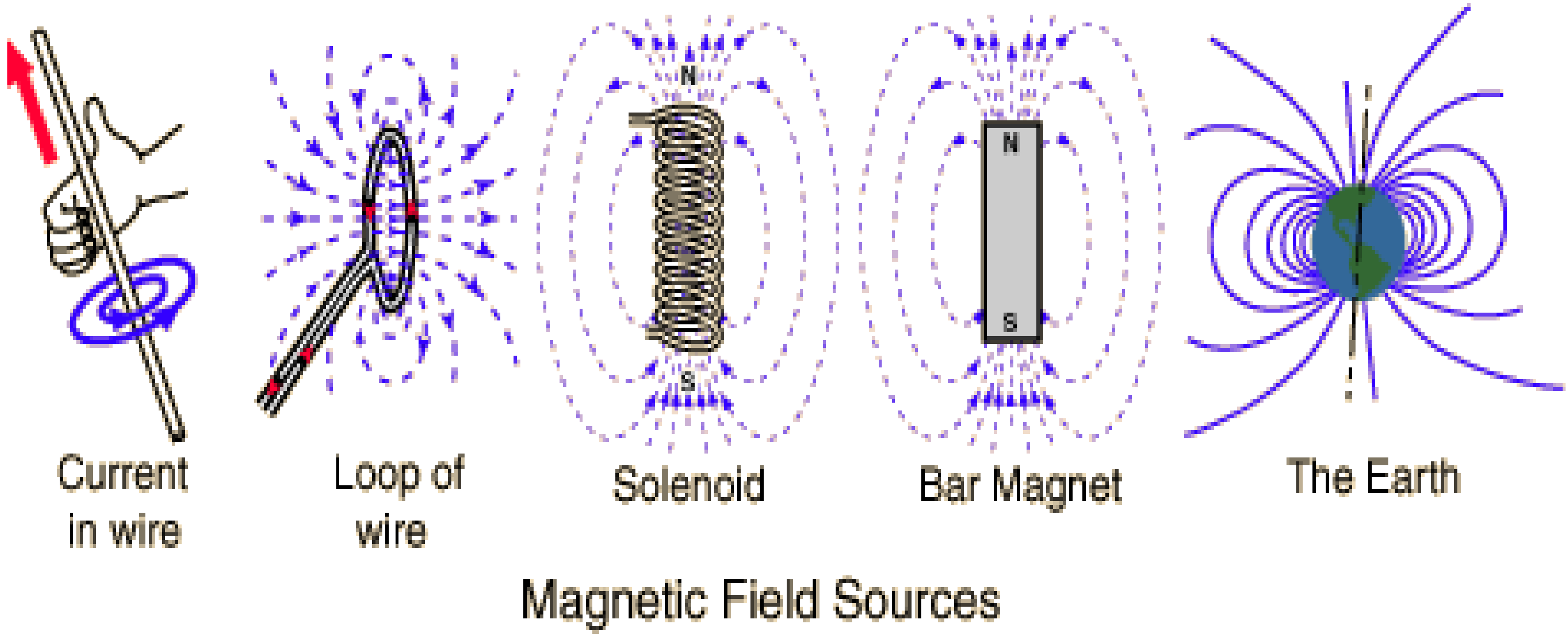
Magnetic Field - Relation to its source – more complicated

Accept Laws on “faith alone” – later proof (difficult)

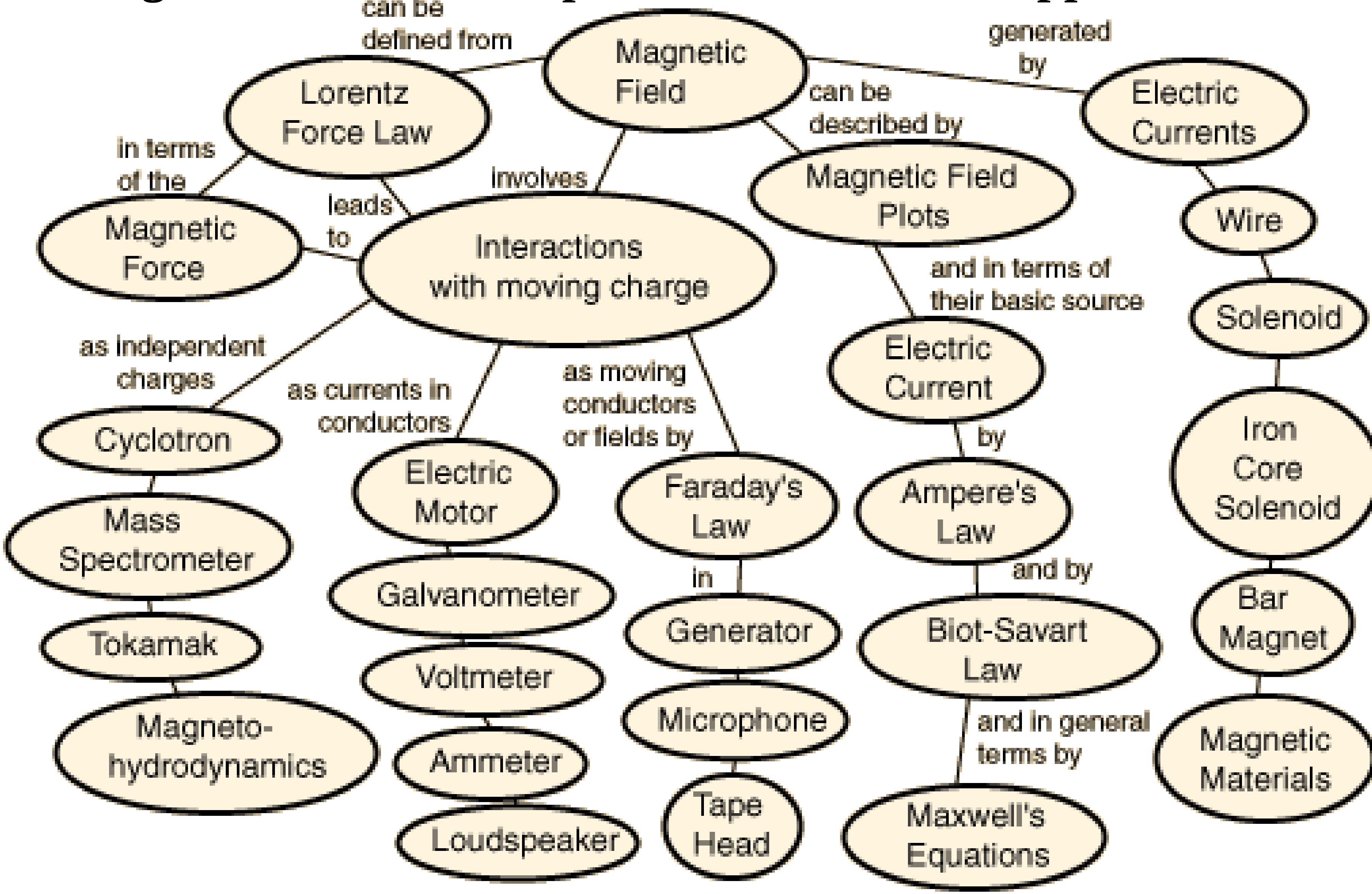
Do we need faith also after the proof?

Magnetic Field Sources

Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.

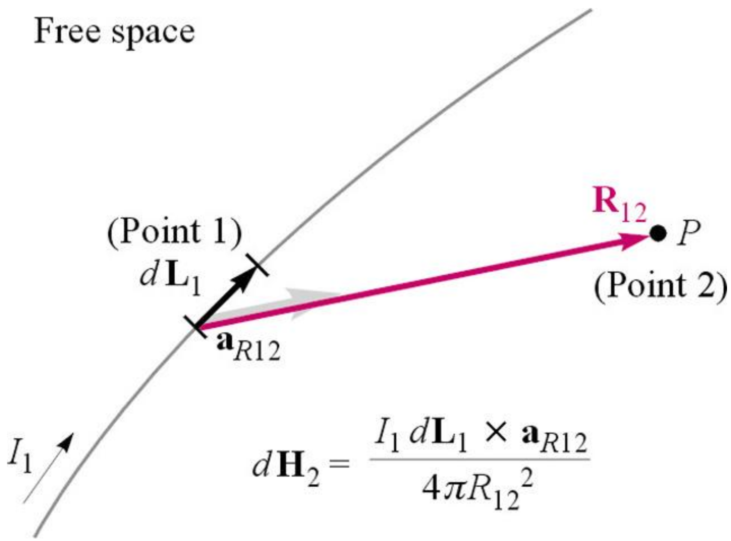


Magnetic Field – Concepts, Interactions and Applications



From GSU Webpage

Biot-Savart Law



At any point P the magnitude of the magnetic field intensity produced by a differential element is proportional to the product of the current, the magnitude of the differential length, and the sine of the angle lying between the filament and a line connecting the filament to the point P at which the field is desired; also, the magnitude of the field is inversely proportional to the square of the distance from the filament to the point P. The constant of proportionality is $1/4\pi$

$$dH = \frac{IdL \times a_R}{4 \cdot \pi \cdot R^2} = \frac{IdL \times \vec{R}}{4 \cdot \pi \cdot R^3}$$

Magnetic Field Intensity A/

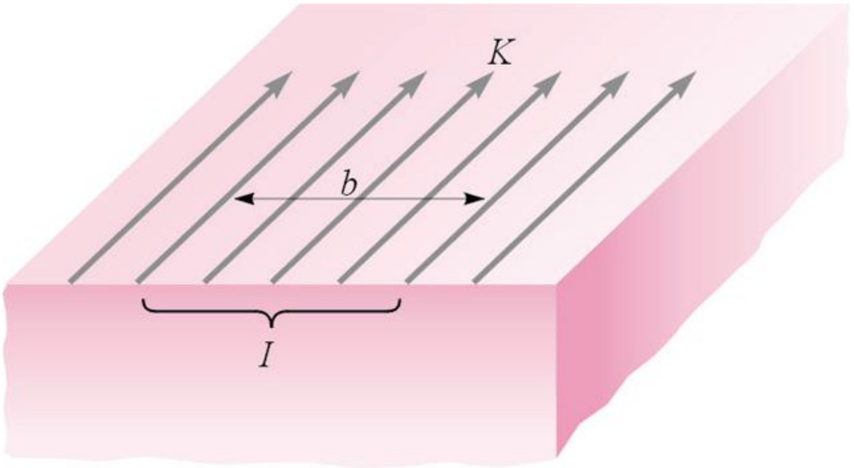
$$H = \oint \frac{I}{4 \cdot \pi \cdot R^2} dL \times a_R$$

Verified experimentally

Biot-Savart = Ampere's law for the current element.

Biot-Savart Law

B-S Law expressed in terms of distributed sources



The total current I within a transverse Width b, in which there is a uniform surface current density K, is Kb.

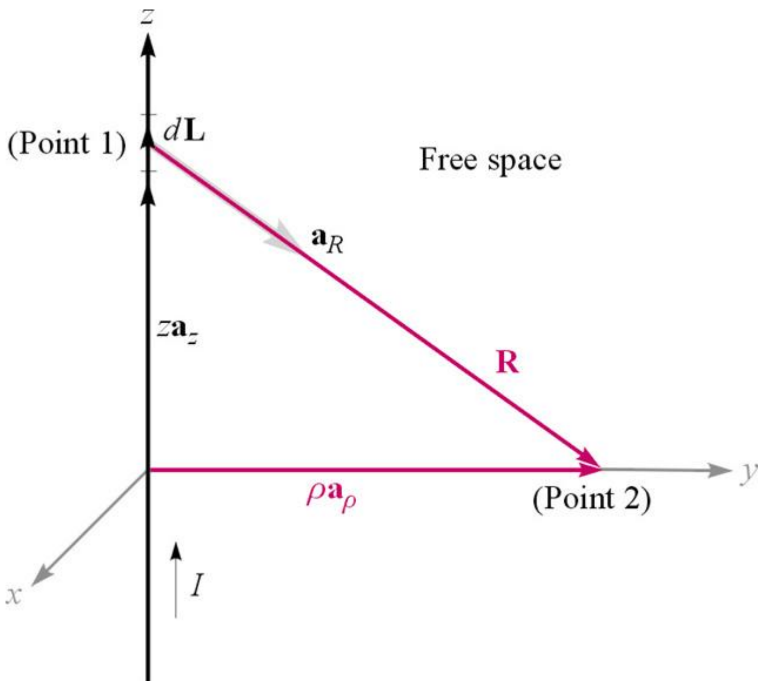
$$I = \int K\,dN$$

For a non-uniform surface current density, integration is necessary.

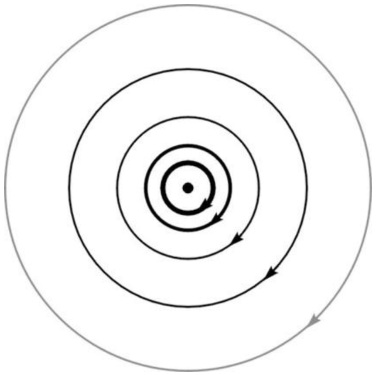
Alternate Forms

$$H = \int \frac{K\_x}{4 \cdot \pi \cdot R^2}\,dS \cdot a_R \qquad H = \int \frac{J\_x}{4 \cdot \pi \cdot R^2}\,dv \cdot a_R$$

Biot-Savart Law



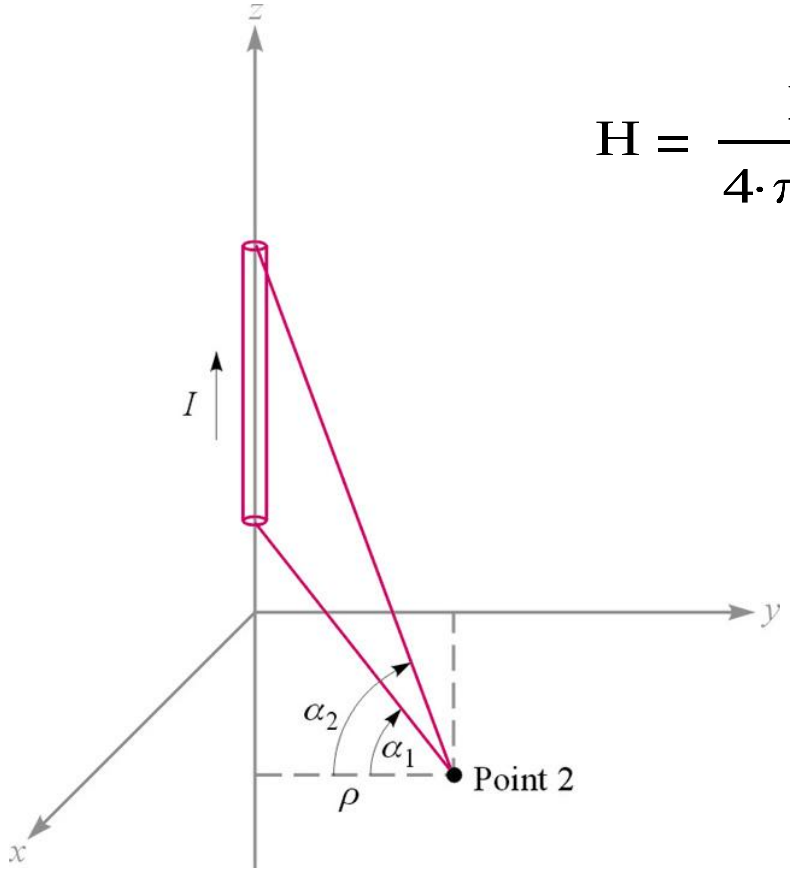
The magnitude of the field is not a function of phi or z and it varies inversely proportional as the distance from the filament.
The direction is of the magnetic field intensity vector is circumferential.



$$H_2 = \int_{-\infty}^{\infty} \frac{I}{4\cdot\pi\cdot\left(\rho^2 + z1^2\right)^{\frac{3}{2}}}\, dz1\cdot a_z \times \left(\rho\cdot a_\rho - z1\cdot a_z\right) = \frac{I}{4\cdot\pi}\cdot\int_{-\infty}^{\infty} \frac{\rho}{\left(\rho^2 + z1^2\right)^{\frac{3}{2}}}\, dz1\cdot a_\phi$$

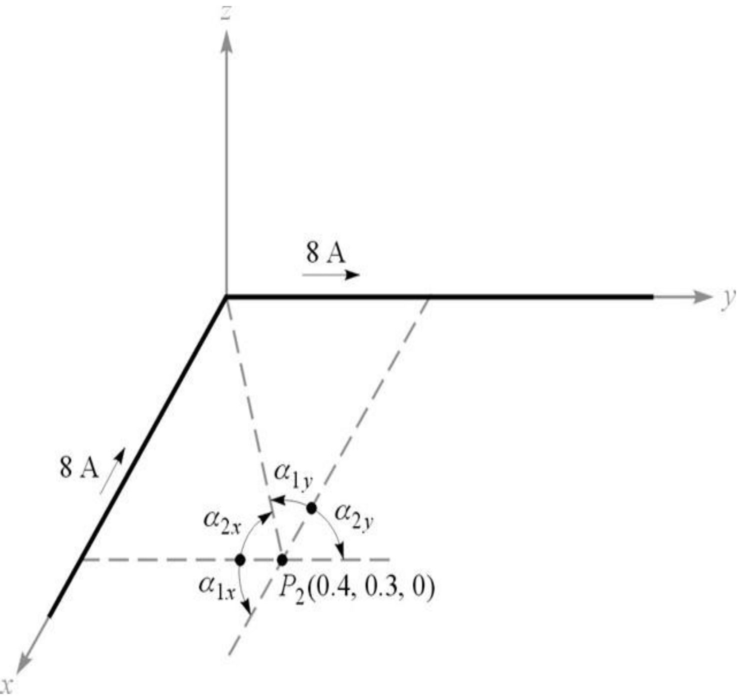
$$H_2 = \frac{I}{2\cdot\pi\cdot\rho}\cdot a_\phi$$

Biot-Savart Law



$$H = \frac{I}{4 \cdot \pi \cdot \rho} \cdot \left(\sin\left(\alpha_2\right) - \sin\left(\alpha_1\right)\right) \cdot a_{\phi}$$

Example 8.1



$$\alpha 1x := -90 \cdot \frac{\pi}{180} \qquad \alpha 2x := \operatorname{atan}\left(\frac{0.4}{0.3}\right)$$

$$\alpha 1y := -\operatorname{atan}\left(\frac{0.3}{0.4}\right) \qquad \alpha 2y := 90 \cdot \frac{\pi}{180}$$

$$H2x = \frac{8}{4 \cdot \pi \cdot (0.3)} \cdot \left(\sin\left(53.1 \cdot \frac{\pi}{180}\right) + 1\right) \cdot a_{\phi}$$

$$H2x := \frac{8}{4 \cdot \pi \cdot (0.3)} \cdot \left(\sin\left(53.1 \cdot \frac{\pi}{180}\right) + 1\right) \qquad H2x = 3.819$$

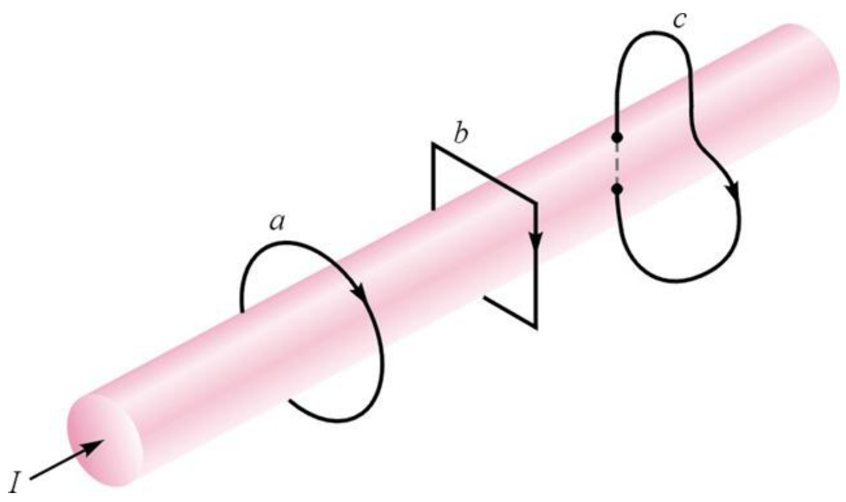
a_{ϕ} must be refered to the x axis - which becomes $-a_z$

$$H2y = \frac{8}{4 \cdot \pi \cdot (0.4)} \cdot \left(1 + \sin\left(36.9 \cdot \frac{\pi}{180}\right)\right) \cdot (-a_z)$$

$$H2y := \frac{8}{4 \cdot \pi \cdot (0.4)} \cdot \left(1 + \sin\left(36.9 \cdot \frac{\pi}{180}\right)\right) \qquad H2y = 2.547$$

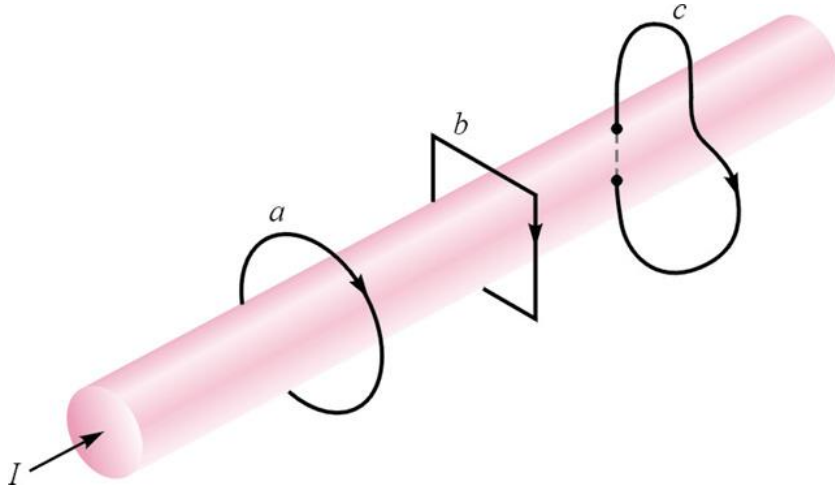
$$H2 := -H2x - H2y \qquad H2 = -6.366 \qquad a_z$$

Ampere's Circuital Law



The magnetic field in space around an electric current is proportional to the electric current which serves as its source, just as the electric field in space is proportional to the charge which serves as its source.

Ampere’s Circuital Law

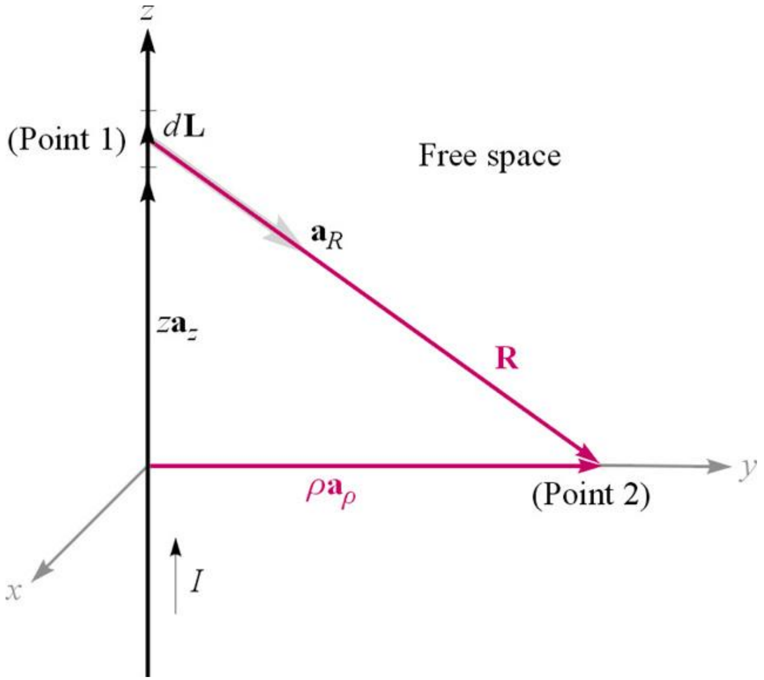


$\oint$ H_dot_ dL = I

Ampere’s Circuital Law states that the line integral of H about any closed path is exactly equal to the direct current enclosed by the path.

We define positive current as flowing in the direction of the advance of a right-handed screw turned in the direction in which the closed path is traversed.

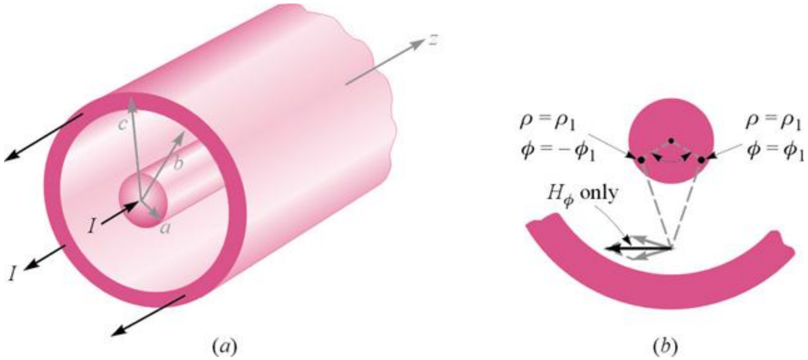
Ampere's Circuital Law - Example



$$\oint H\_dot\_\, dL = \int_0^{2\cdot\pi} H_\phi \cdot \rho \, d\phi = H_\phi \cdot \rho \cdot \int_0^{2\cdot\pi} 1\, d\phi = I$$

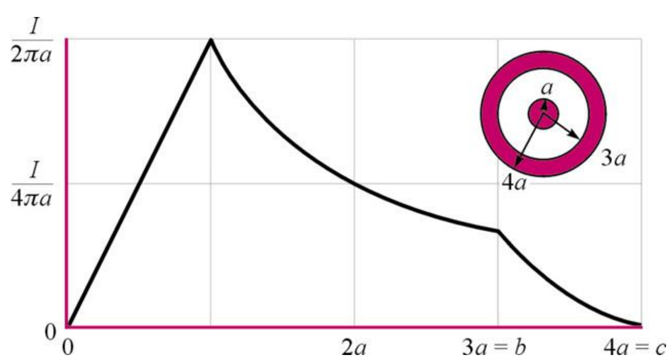
$$H_\phi = \frac{I}{2\cdot\pi\cdot\rho}$$

Ampere's Circuital Law - Example



$$H_\phi = \frac{I}{2\cdot\pi\cdot\rho} \qquad a < \rho < b$$

$$H_\phi = \frac{I\cdot\rho}{2\cdot\pi\cdot a^2} \qquad \rho < a \qquad\qquad H_\phi = 0 \qquad \rho > c$$

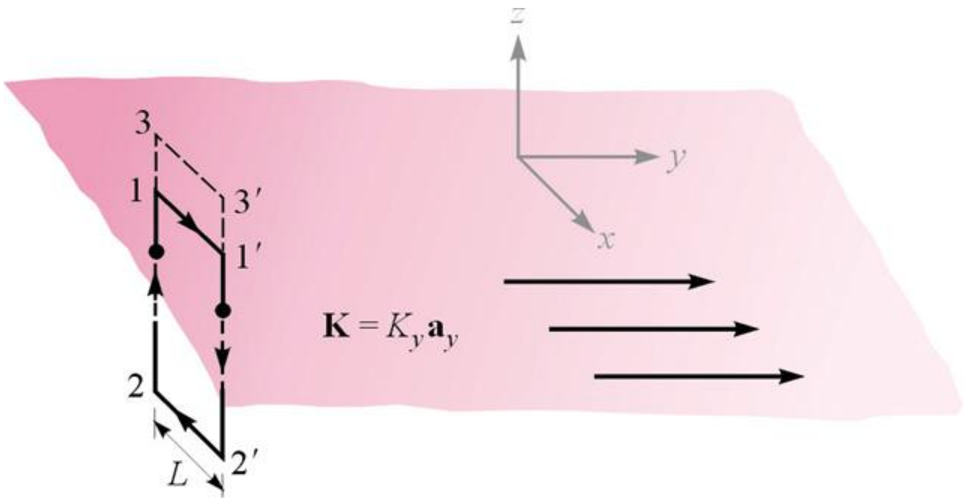


$$2\cdot\pi\cdot\rho\cdot H_\phi = I - I\cdot\left(\frac{\rho^2 - b^2}{c^2 - b^2}\right)$$

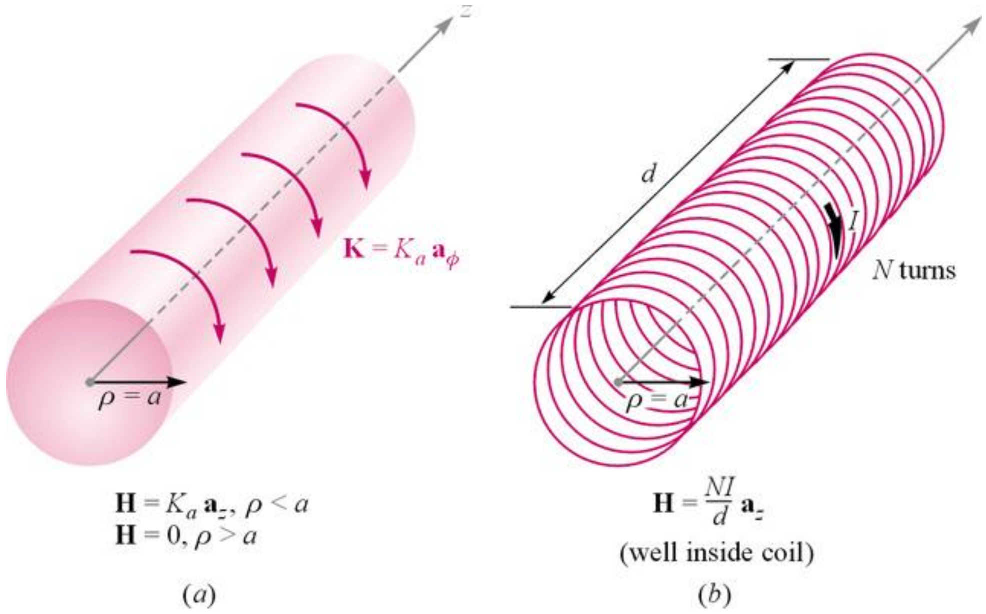
$$H_\phi = \frac{I}{2\cdot\pi}\cdot\frac{\rho^2 - b^2}{c^2 - b^2}$$

$$H_\phi(\rho) := \frac{I}{2\cdot\pi\cdot\rho}\cdot\frac{c^2 - \rho^2}{c^2 - b^2}$$

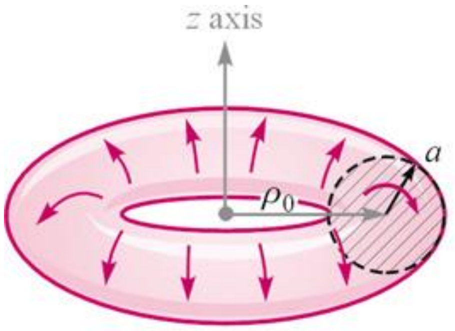
Ampere’s Circuital Law - Example



Ampere’s Circuital Law - Example



Ampere's Circuital Law - Example

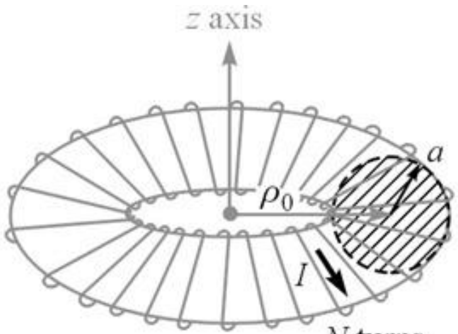


$\mathbf{K} = K_a\,\mathbf{a}_z$ at $\rho = \rho_0 - a,\ z = 0$

$\mathbf{H} = K_a \dfrac{\rho_0 - a}{\rho}\,\mathbf{a}_\phi$ (inside toroid)

$\mathbf{H} = 0$ (outside)

(*a*)



$\mathbf{H} = \dfrac{NI}{2\pi\rho}\,\mathbf{a}_\phi$ (well inside toroid)

(*b*)

CURL

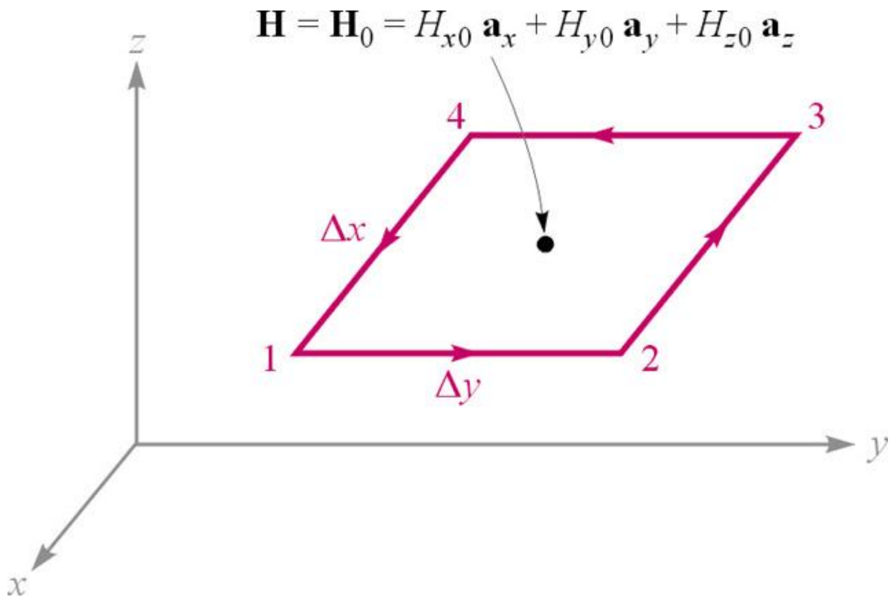
The curl of a vector function is the vector product of the del operator with a vector function

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\nabla \times E = \begin{vmatrix} \frac{\mathbf{1_r}}{\mathbf{r}} & 1_{\boldsymbol{\theta}} & \frac{\mathbf{k}}{\mathbf{r}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ E_{\mathbf{r}} & \mathbf{r}E_{\boldsymbol{\theta}} & E_{\mathbf{z}} \end{vmatrix}$$

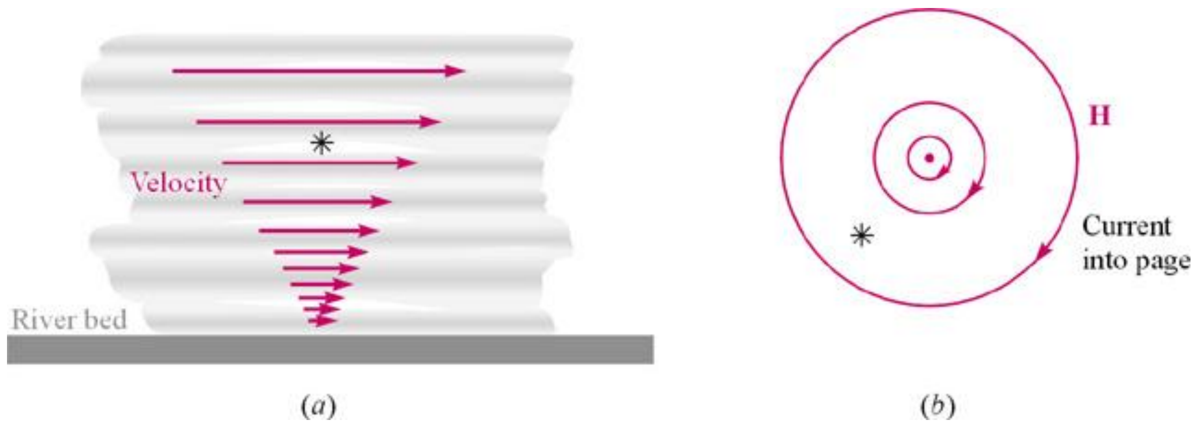
$$\nabla \times E = \begin{vmatrix} \frac{1_{\mathbf{r}}}{r^2 \sin\theta} & \frac{1_{\boldsymbol{\theta}}}{r \sin\theta} & \frac{1_{\boldsymbol{\phi}}}{\mathbf{r}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ E_{\mathbf{r}} & \mathbf{r}E_{\boldsymbol{\theta}} & \mathbf{r \sin \boldsymbol{\theta}} E_{\boldsymbol{\phi}} \end{vmatrix}$$

CURL



$$(\text{curl\_H})\mathbf{a}_N = \lim_{\Delta S_N \to 0} \frac{\oint \text{H\_dot\_ dL}}{\Delta S_N}$$

CURL



$$\text{curl\_H} = \nabla \times H$$

$$\nabla \times H = J$$

Ampere's Circuital Law
Second Equation of Maxwell

$$\nabla \times E = 0$$

Third Equation

CURL

Illustration of Curl Calculation

$$\mathrm{CurlH} = \left(\frac{d}{dy}H_z - \frac{d}{dz}H_y\right)\cdot a_x + \left(\frac{d}{dz}H_x - \frac{d}{dx}H_z\right)\cdot a_y + \left(\frac{d}{dx}H_y - \frac{d}{dy}H_x\right)\cdot a_z$$

$$\mathrm{CurlH} = \begin{pmatrix} a_x & a_y & a_z \\ \frac{d}{dx}\blacksquare & \frac{d}{dy}\blacksquare & \frac{d}{dz}\blacksquare \\ H_x & H_y & H_z \end{pmatrix}$$

CURL

Example 1

In a certain conducting region, H is defined by:

$H1x(x,y,x) := y \cdot x \cdot \left(x^2 + y^2\right)$ $\quad H1y(x,y,z) := -y^2 \cdot x \cdot z$ $\quad H1z(x,y,z) := 4 \cdot x^2 \cdot y^2$

Determine J at: $x := 5$ $\quad y := 2$ $\quad z := -3$

$DelXHx := \frac{d}{dy}H1z(x,y,z) - \frac{d}{dz}H1y(x,y,z)$ $\quad DelXHx = 420$ $\quad \vec{ax}$

$DelXHy := \frac{d}{dz}H1x(x,y,z) - \frac{d}{dx}H1z(x,y,z)$ $\quad DelXHy = -98$ $\quad \vec{ay}$

$DelXHz := \frac{d}{dx}H1y(x,y,z) - \frac{d}{dy}H1x(x,y,z)$ $\quad DelXHz = 75$ $\quad \vec{az}$

CURL

Example 2

$H2x(x,y,x) := 0$ $\quad$ $H2y(x,y,z) := x^2 \cdot z$ $\quad$ $H2z(x,y,z) := -y^2 \cdot x$

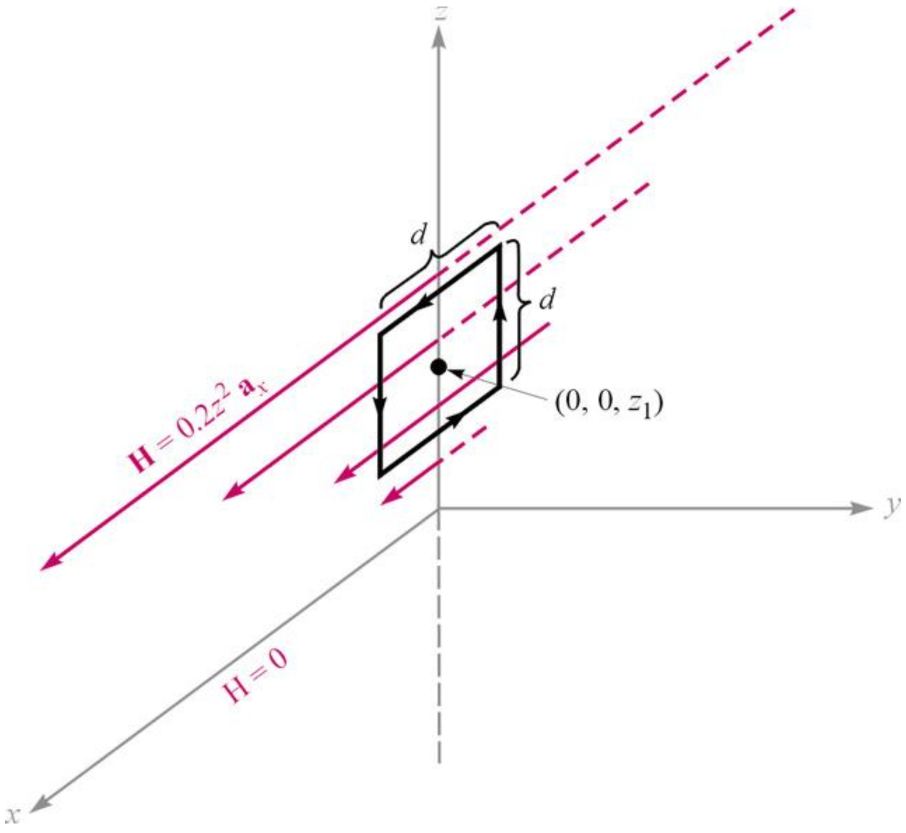
$x := 2$ $\quad$ $y := 3$ $\quad$ $z := 4$

$$DelXHx := \frac{d}{dy}H2z(x,y,z) - \frac{d}{dz}H2y(x,y,z) \qquad DelXHx = -16 \qquad \vec{ax}$$

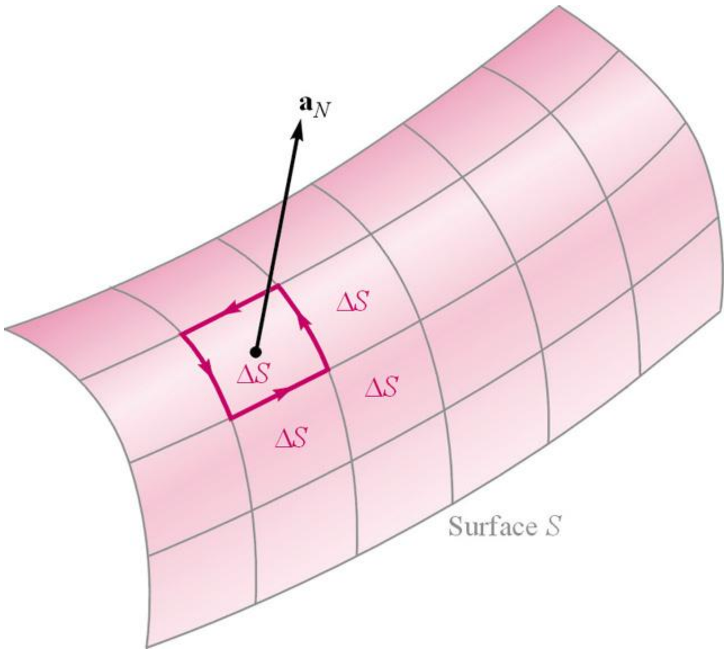
$$DelXHy := \frac{d}{dz}H2x(x,y,z) - \frac{d}{dx}H2z(x,y,z) \qquad DelXHy = 9 \qquad \vec{ay}$$

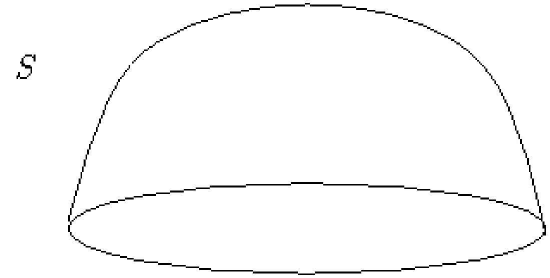
$$DelXHz := \frac{d}{dx}H2y(x,y,z) - \frac{d}{dy}H2x(x,y,z) \qquad DelXHz = 16 \qquad \vec{az}$$

Example 8.2



Stokes' Theorem



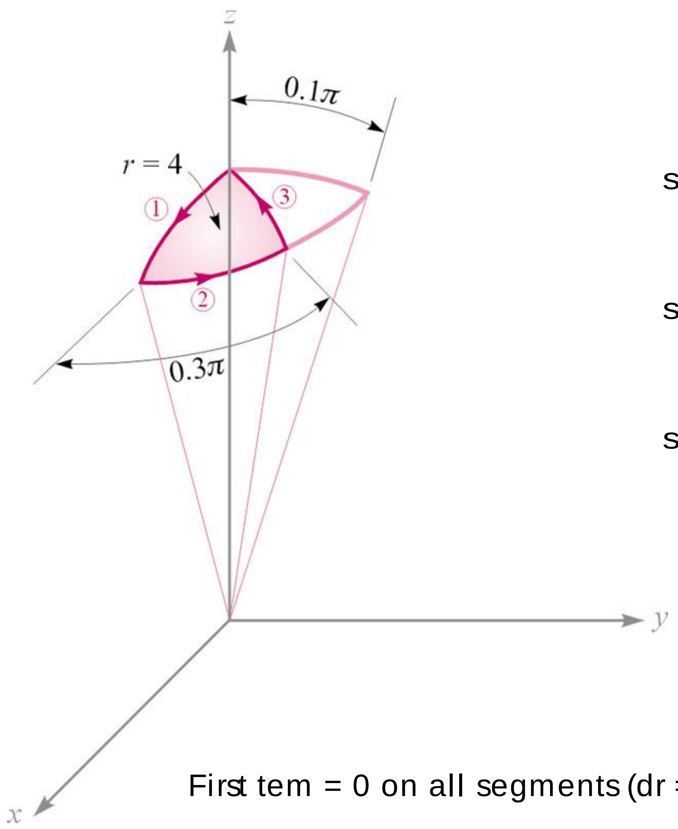


$$\oint_L \mathbf{a}\cdot d\mathbf{r} = \iint_S \nabla\times\mathbf{a}\cdot d\mathbf{S}$$

The sum of the closed line integrals about the perimeter of every Delta S is the same as the closed line integral about the perimeter of S because of cancellation on every path.

$$\oint \text{H\_dot\_ dL} = \int_S \text{(Del} \times \text{H)\_dot\_ dS}$$

Example 8.3



$$Hr(r, \theta, \phi) := 6 \cdot r \cdot \sin(\phi)$$

$$H\theta(r, \theta, \phi) := 0$$

$$H\phi(r, \theta, \phi) := 18 \cdot r \cdot \sin(\theta) \cdot \cos(\phi)$$

segment 1

$$r := 4 \quad 0 \le \theta \le 0.1 \cdot \pi \qquad \phi := 0$$

segment 2

$$r := 4 \quad \theta := 0.1 \cdot \pi \qquad 0 \le \phi \le 0.3\pi$$

segment 3

$$r := 4 \quad 0 \le \theta \le 0.1 \cdot \pi \qquad \phi := 0.3\pi$$

$$dL = dr \cdot \vec{ar} + r \cdot d\theta \cdot \vec{a\theta} + r \cdot \sin(\theta) \cdot d\phi \cdot \vec{a\phi}$$

First tem = 0 on all segments (dr = 0)

Second term = 0 on segment 2 (θ constant)

Third term = 0 on segments 1 and 3 (ϕ = 0 or constant

$$\int H\, dL = \int H\theta \cdot r\, d\theta = \int H\phi \cdot r \cdot \sin(\theta)\, d\phi = \int H\theta \cdot r\, d\theta$$

since Hθ=0 $\displaystyle\int_0^{0.3 \cdot \pi} H\phi(r, \theta, \phi) \cdot (r \cdot \sin(\theta))\, d\phi = 22.249$

Magnetic Flux and Magnetic Flux Density

$B = \mu_0 \cdot H$ $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$ $\frac{H}{m}$ permeability in free spa

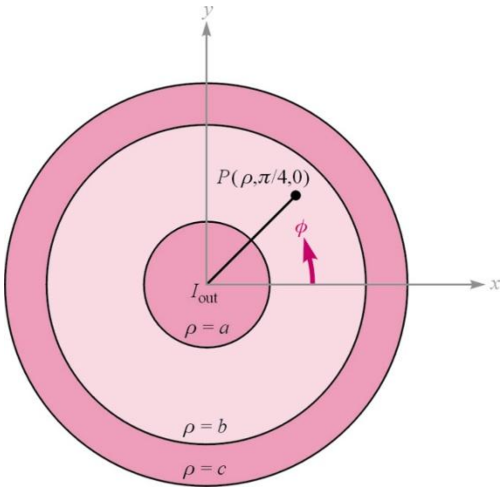
$\Phi = \int B\_dot\_dS$

$\oint B\_dot\_dS = 0$

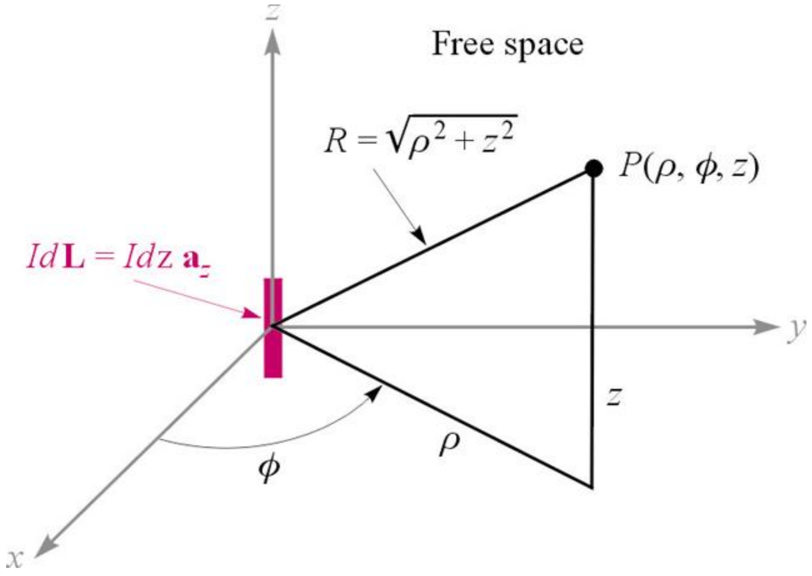
The Scalar and Vector Magnetic Potentials

$$H = -\text{Del\_}V_m \qquad\qquad J = 0$$

$$V_m = -\int_a^b \text{H\_dot\_ dL}$$



The Scalar and Vector Magnetic Potentials



Derivation of the Steady-Magnetic-Fields Laws