MAGNETOSTATIC FIELDS

Static electric fields are characterized by **E** or **D.** Static magnetic fields, are characterized by **H** or **B.** There are similarities and dissimilarities between electric and magnetic fields. As E and **D** are related according to **D =** □E for linear material space, **H** and **B** are related according to
B = □H.

A definite link between electric and magnetic fields was established by Oersted in 1820. An electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced. A magnetostatic field is produced by a constant current flow (or direct current). This current flow may be due to magnetization currents as in permanent magnets, electron-beam currents as in vacuum tubes, or conduction currents as in current-carrying wires.

The development of the motors, transformers, microphones, compasses, telephone bell ringers, television focusing controls, advertising displays, magnetically levitated high speed vehicles, memory stores, magnetic separators, and so on, involve magnetic phenomena and play an important role in our everyday life.

There are two major laws governing magnetostatic fields:

(1) Biot-Savart's law, and

(2) Ampere's circuit law.

Like Coulomb's law, Biot-Savart's law is the general law of magnetostatics. Just as Gauss's law is a special case of Coulomb's law, Ampere's law is a special case of Biot-Savart's law and is easily applied in problems involving symmetrical current distribution.

BIOT SAVART's LAW

Biot-Savart's law states that the magnetic field intensity *dH* produced at a point P, as shown in **Figure 1.1**, by the differential current element *I dl* is proportional to the product *I dl* and the sine of the angle α between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element.

That is,

$$dH \propto \frac{I\,dl\,\sin\alpha}{R^2} \qquad (1.1)$$

or

$$dH = \frac{KI\,dl\,\sin\alpha}{R^2} \qquad (1.2)$$

where, k is the constant of proportionality. In SI units, $k = 1/4\pi$. So, eq. (1.2) becomes

$$dH = \frac{I\,dl\,\sin\alpha}{4\pi R^2} \qquad (1.3)$$

From the definition of cross product equation $\mathbf{A \times B = AB \sin \theta_{AB}\, a_n}$, it is easy to notice that eq. (1.3) is better put in vector form as

$$dH = \frac{Idl \times a_R}{4\pi R^2} = \frac{Idl \times R}{4\pi R^3} \qquad (1.4)$$

where **R** in the denominator is |**R**| and **a**R = (vector **R**/|**R**|}. Thus, the direction of d**H** can be determined by the right-hand rule with the right-hand thumb pointing in the direction of the current, the right-hand fingers encircling the wire in the direction of d**H** as shown in Figure 1.2(a). Alternatively, one can use the right-handed screw rule to determine the direction of d**H**: with the screw placed along the wire and pointed in the direction of current flow, the direction of advance of the screw is the direction of d**H** as in Figure 1.2(b).

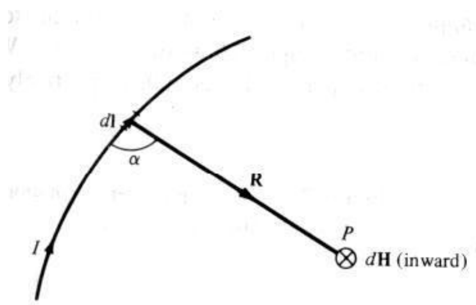

Figure 1.1: Magnetic field d**H** at P due to current element *I dl.*

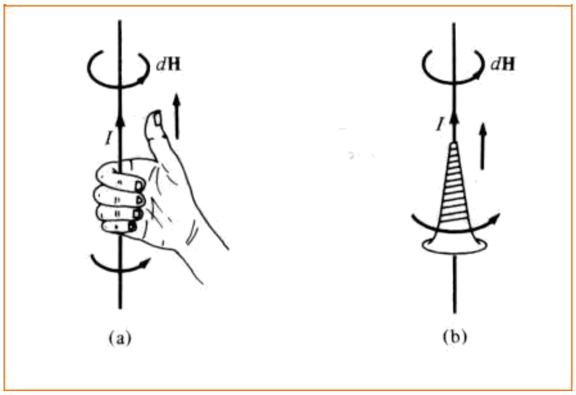

Figure 1.2: Determining the direction of d**H** using **(a)** the right-hand rule, or **(b)** the right-handed screw rule.

It is customary to represent the direction of the magnetic field intensity **H** (or current I) by a small circle with a dot or cross sign depending on whether **H** (or I) is out of, or into, the page as illustrated in Figure 1.3.

As like different charge configurations, one can have different current distributions: line current, surface current and volume current as shown in **Figure 1.4**. If we define **K** as the surface current density (in amperes/meter) and J as the volume current density (in amperes/meter square), the source elements are related as

$$I\,d\mathbf{l} \equiv \mathbf{K}\,dS \equiv \mathbf{J}\,dv \qquad (1.5)$$

Thus, in terms of the distributed current sources, Biot-Savart law as in eq. (1.4) becomes

$$H = \int_L \frac{Idl \times a_R}{4\pi R^2} \qquad \text{(Line current)} \qquad (1.6)$$

$$H = \int_S \frac{KdS \times a_R}{4\pi R^2} \qquad \text{(Surface current)} \qquad (1.7)$$

$$H = \int_V \frac{J\,dv \times a_R}{4\pi R^2} \quad \text{(Volume current)} \qquad (1.8)$$

As an example, let us apply eq. (1.6) to determine the field due to a *straight current* carrying filamentary conductor of finite length AB as in **Figure 1.5**. We assume that the conductor is along the z-axis with its upper and lower ends respectively subtending angles

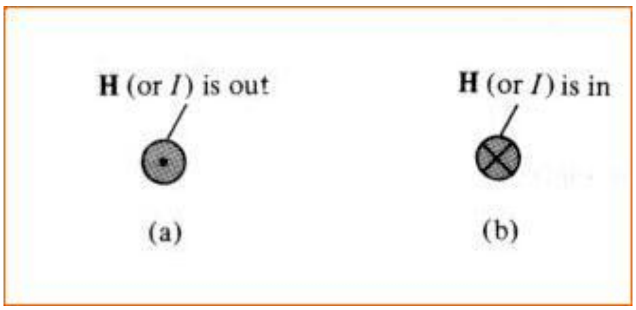

Figure 1.3: Conventional representation of H (or I) **(a)** out of the page and **(b)** into the page.

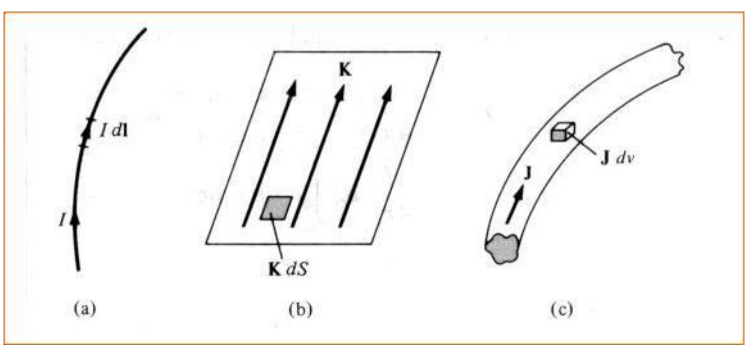

Figure 1.4: Current distributions: **(a)** line current **(b)** surface current **(c)** volume current.

□2 *and* □1 *at* P, the point at which **H** is to be determined. Particular note should be taken of this assumption, as the formula to be derived will have to be applied accordingly. If we consider the contribution d**H** at P due to an element d**l** at (0, 0, z),

$$dH \square \frac{Idl \square R}{4\square R^3} \qquad (1.9)$$

But d**l** = dz **a**z and R = □**a**□ - z**a**z , so

$$\mathbf{dl} \times \mathbf{R} = \square \, dz \, \mathbf{a}_{\square} \qquad (1.10)$$

Hence,

$$H \square \frac{I\square \, dz}{4\square \left[\square^2 \square z^2 \right]^{3/2}} a_{\square} \qquad (1.11)$$

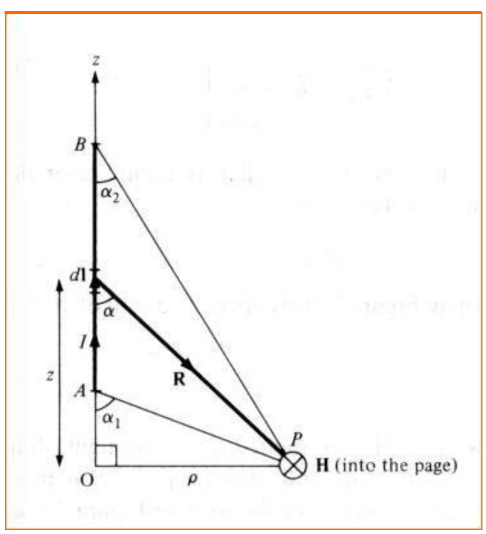

Figure 1.5: Field at point P due to a straight filamentary conductor.

Letting z = □ cot □, dz = -□ cosec^2 □ d□, equation (1.11) becomes

$$H = -\int_{\alpha_1}^{\alpha_2} \frac{I \rho^2 \operatorname{cosec}^2 \alpha \, d\alpha}{4\pi \rho^3 \operatorname{cosec}^3 \alpha} \mathbf{a}_\phi$$

$$= -\frac{I}{4\pi\rho} \mathbf{a}_\phi \int_{\alpha_1}^{\alpha_2} \sin\alpha \, d\alpha$$

Or

$$H = \frac{I}{4\pi\rho} (\cos\alpha_2 - \cos\alpha_1) \mathbf{a}_\phi \qquad (1.12)$$

The equation (1.12) is generally applicable for any straight filamentary conductor of finite length. Note from eq. (1.12) that **H** is always along the unit vector $\mathbf{a}_\phi$ (i.e., along concentric circular paths) irrespective of the length of the wire or the point of interest P. As a special case, when the conductor is *semi-infinite* (with respect to P), so that point A is now at O(0, 0, 0) while B is at (0, 0, ∞); $\alpha_1 = 90°$, $\alpha_2 = 0°$, and eq. (1.12) becomes

$$H = \frac{I}{4\pi\rho} \mathbf{a}_\phi \qquad (1.13)$$

Another special case is when the conductor is *infinite* in length. For this case, point A is at (0, 0, $-\infty$) while B is at (0, 0, ∞); $\alpha_1 = 180°$, $\alpha_2 = 0°$. So, eq. (1.12) reduces to

$$H = \frac{I}{2\pi\rho} \mathbf{a}_\phi \qquad (1.14)$$

To find unit vector $\mathbf{a}_\phi$ in equations (1.12) to (1.14) is not always easy. A simple approach is to determine $\mathbf{a}_\phi$ from

a□ □ a □a□ (1.15)

where **al** is a unit vector along the line current and **a□** is a unit vector along the perpendicular line from the line current to the field point.

Illustration: The conducting triangular loop in Figure 1.6(a) carries a current of 10 A. Find **H** at (0, 0, 5) due to side 1 of the loop.

Solution:

This example illustrates how eq. (1.12) is applied to any straight, thin, current-carrying conductor. The key point to be kept in mind in applying eq. (1.12) is figuring out □1, □2, □ and **a□**. To find **H** at (0, 0, 5) due to side 1 of the loop in Figure 1.6(a), consider Figure

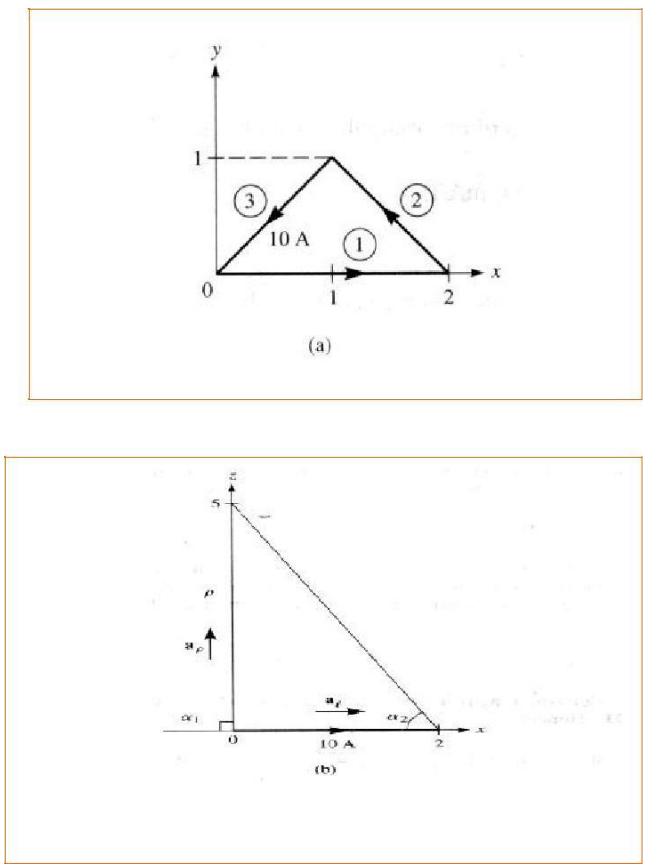

Figure 1.6: (a) conducting triangular loop **(b)** side 1 of the **loop.**

1.6(b), where side 1 is treated as a straight conductor. Notice that we join the Point of interest (0, 0, 5) to the beginning and end of the line current. Observe that α_1, α_2 and ρ are assigned in the same manner as in Figure 1.5 on which eq. (1.12) is based.

$$\cos \alpha_1 = \cos 90^\circ = 0, \qquad \cos\alpha_2 = \frac{2}{\sqrt{29}}, \qquad \rho = 5$$

To determine $\mathbf{a}_\phi$ is often the hardest part of applying eq. (1.12). According to eq. (1.15), $\mathbf{a}_l = \mathbf{a}_x$ and $\mathbf{a}_\rho = \mathbf{a}_z$, so

$$\mathbf{a}_\phi = \mathbf{a}_x \times \mathbf{a}_z = -\mathbf{a}_y$$

Hence,

$$H_1 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\mathbf{a}_\phi = \frac{10}{4\pi(5)}\left(\frac{2}{\sqrt{29}} - 0\right)(-\mathbf{a}_y)$$

$$= -59.1\ \mathbf{a}_y \text{ mA/m}$$

AMPERE'S CIRCUIT LAW

Ampere's circuit law states that the line integral of the tangential components of **H** around a closed path is the same as the net current I_{enc} enclosed by the path

In other words, the circulation of **H** equals I_{enc} ; that is,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} \qquad (1.16)$$

Ampere's law is similar to Gauss's law and it is easily applied to determine **H** when the current distribution is symmetrical. It should be noted that eq. (1.16) always holds whether the current distribution is symmetrical or not but we can only use the equation to determine **H** when symmetrical current distribution exists. Ampere's law is a special case of Biot-Savart's law; the former may be derived from the latter.

By applying Stoke's theorem to the left-hand side of eq. (1.16), we obtain

$$I_{enc} = \oint_L H \cdot dl = \int_S (\nabla \times H) \cdot dS \qquad (1.17)$$

But

$$I_{enc} = \int_S J \cdot dS \qquad (1.18)$$

Comparing the surface integrals in eqs. (7.17) and (7.18) clearly reveals that

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad (1.19)$$

This is the third Maxwell's equation to be derived; it is essentially Ampere's law in differential (or point) form whereas eq. (1.16) is the integral form. From eq. (1.19), we should observe that $\nabla \times \mathbf{H} = \mathbf{J} \neq \mathbf{0}$; that is, magnetostatic field is not conservative.

APPLICATIONS OF AMPERE'S LAW

Infinite Line Current

Consider an infinitely long filamentary current I along the z-axis as in Figure 1. 7. To determine **H** at an observation point P, we allow a closed path pass through P. This path on, which Ampere's law is to be applied, is known as an *Amperian path* (analogous to the term Gaussian surface). We choose a concentric circle as the Amperian path in view of eq. (1.14), which shows that **H** is constant provided p is constant. Since this path encloses the whole current I, according to Ampere's law

$$I = \oint H_\phi \mathbf{a}_\phi \cdot \rho \, d\phi \, \mathbf{a}_\phi = H_\phi \oint \rho \, d\phi = H_\phi \cdot 2\pi\rho$$

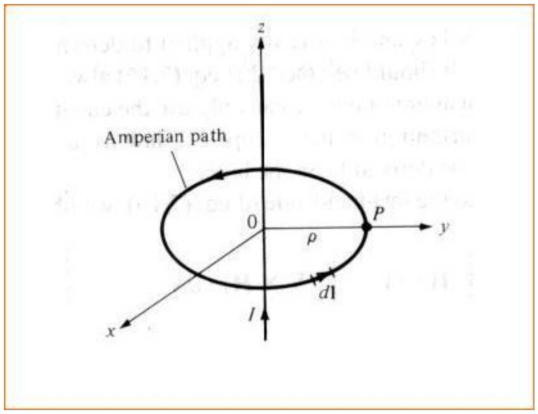

Figure 1.7: Ampere's law applied to an infinite filamentary, line current.

Or

$$\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi \quad (1.20)$$

As expected from eq. (1.14).

MAGNETIC FLUX DENSITY

The magnetic flux density **B** is similar to the electric flux density **D.** As $\mathbf{D} = \varepsilon_0 \mathbf{E}$ in free space, the magnetic flux density **B** is related to the magnetic field intensity **H** according to

$$\mathbf{B} = \mu_0 \mathbf{H} \quad (1.21)$$

where, μ_0 is a constant known as the *permeability of free space.* The constant is in henrys/meter (H/m) and has the value of

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \qquad (1.22)$$

The precise definition of the magnetic field **B,** in terms of the magnetic force, can be discussed later.

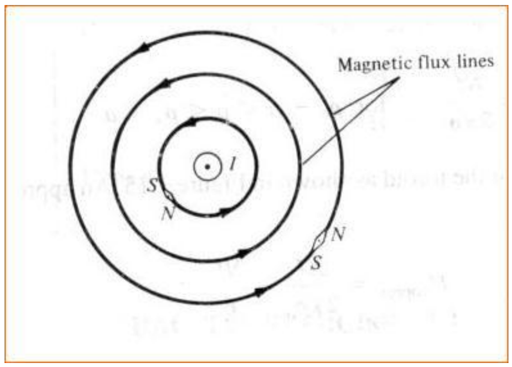

Figure 1.8: Magnetic flux lines due to a straight wire with current coming out of the page

The magnetic flux through a surface S is given by

$$\Phi = \int_S B \cdot dS \qquad (1.23)$$

Where the magnetic flux Φ is in webers (Wb) and the magnetic flux density is a webers/square meter (Wb/m^2) or teslas.

An isolated magnetic charge does not exit.

Total flux through a closed surface in a magnetic field must be zero;

that is,

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \qquad (1.24)$$

This equation is referred to as the *law of conservation of magnetic flux or Gauss''s law for magnetostatic fields just* as $\oint_S \mathbf{D} \cdot d\mathbf{S} = Q$ is Gauss's law for electrostatic fields. Although the magnetostatic field is not conservative, magnetic flux is conserved.

By applying the divergence theorem to eq. (1.24), we obtain

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B}\, dv = 0$$

Or

$$\nabla \cdot \mathbf{B} = 0 \qquad (1.25)$$

This equation is the fourth Maxwell's equation to be derived. Equation (1.24) or (1.25) shows that magnetostatic fields have no sources or sinks. Equation (1.25) suggests that magnetic field lines are always continuous.

TABLE 1.2: Maxwell's Equations for Static EM Fields

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v\, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of magnetic monopole
$\nabla \times \mathbf{E} = 0$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = 0$	Conservativeness of electrostatic field
$\nabla \times \mathbf{H} = \mathbf{J}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_s \mathbf{J} \cdot d\mathbf{S}$	Ampere's law

The **Table 1.2** gives the information related to Maxwell's Equations for Static Electromagnetic Fields.

MAGNETIC SCALAR AND VECTOR POTENTIALS

We recall that some electrostatic field problems were simplified by relating the electric Potential V to the electric field intensity **E** (**E** = $-\nabla V$). Similarly, we can define a potential associated with magnetostatic field B. In fact, the magnetic potential could be scalar V_m vector A. To define V_m and A involves two important identities:

$$\nabla \times (\nabla V) = 0 \tag{1.26}$$

$$\nabla \cdot (\nabla \times A) = 0 \tag{1.27}$$

which must always hold for any scalar field V and vector field A.

Just as $\mathbf{E} = -\nabla \mathbf{V}$, we define the *magnetic scalar potential* V_m (in amperes) as related to **H** according to

$$\mathbf{H} = -\nabla \mathbf{V_m} \text{ if } \mathbf{J} = 0 \tag{1.28}$$

The condition attached to this equation is important and will be explained. Combining eq. (1.28) and eq. (1.19) gives

$$\mathbf{J} = \nabla \times \mathbf{H} = -\nabla \times (-\nabla V_m) = 0 \tag{1.29}$$

since V_m, must satisfy the condition in eq. (1.26). Thus the magnetic scalar potential V_m is only defined in a region where J = 0 as in eq. (1.28). We should also note that V_m satisfies Laplace's equation just as V does for electrostatic fields; hence,

$$\nabla^2 V_m = 0, \ (\mathbf{J} = 0) \qquad (1.30)$$

We know that for a magnetostatic field, ∇ x **B** = **0** as stated in eq. (1.25). In order to satisfy eqs. (1.25) and (1.27) simultaneously, we can define the *vector magnetic potential* A (in Wb/m) such that

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (1.31)$$

Just as we defined

$$V = \int \frac{dQ}{4\pi\varepsilon_0 r} \qquad (1.32)$$

We can define

$$A = \int_L \frac{\mu_0 I \, dl}{4\pi R} \quad \text{for line current} \qquad (1.33)$$

$$A = \int_S \frac{\mu_0 K \, dS}{4\pi R} \quad \text{for surface current} \qquad (1.34)$$

$$A = \int_v \frac{\mu_0 J \, dv}{4\pi R} \quad \text{for volume current} \qquad (1.35)$$

Illustration 1: Given the magnetic vector potential $\mathbf{A} = -\rho^2/4 \ \mathbf{a}_z$ Wb/m, calculate the total magnetic flux crossing the surface $\phi = \pi/2$, $1 \le \rho \le 2$m, $0 \le z \le 5$m.

Solution:

$$B = \nabla \times A = -\frac{\partial A_z}{\partial \rho} a_\phi = \frac{\rho}{2} a_\phi, \qquad dS = d\rho\, dz\, a_\phi$$

$$\Psi = \iint B \cdot dS = \frac{1}{2}\int_{z=0}^{5}\int_{\rho=1}^{2} \rho\, d\rho\, dz = \frac{1}{4}\rho^2 \Big|_1^2 (5) = \frac{15}{4}$$

Ψ = 3.75 Wb

Illustration 2:

Identify the configuration in figure 1.9 that is not a correct representation of I and H.

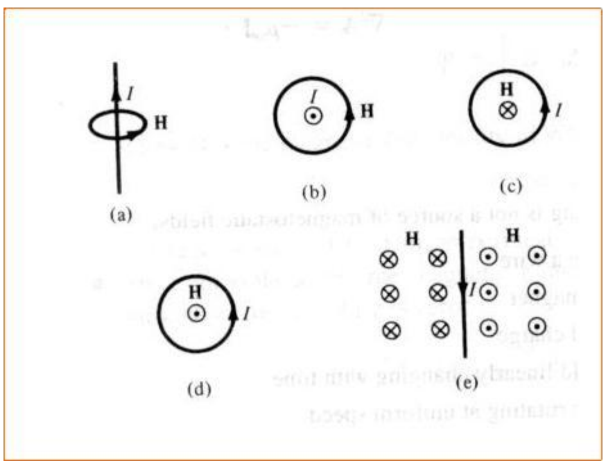

Figure 1.9: Different I and H representations (related to Illustration 2)

Solution:

Figure 1.9 (c) is not a correct representation. The direction of H field should have been outwards for the given I direction.

MAGNETIC FORCES, MATERIALS AND DEVICES

Force on a Charged Particle

According to earlier information, the electric force $\mathbf{F_e}$, on a stationary or moving electric charge *Q* in an electric field is given by Coulornb's experimental law and is related to the electric field intensity E as

$$F_e = QE \qquad (2.1)$$

This shows that if *Q* is Positive, F_e and E have the same direction.

A magnetic field can exert force only on a moving charge. From experiments, it is found that the magnetic force F_m experienced by a charge *Q* moving with a velocity **u** in a magnetic field **B** is

$$F_m = Qu \times B \qquad (2.2)$$

This clearly shows that F_m is perpendicular to both u and **B.**

From eqs. (2.1) and (2.2), a comparison between the electric force F_e and the magnetic force F_m can be made. F_e is independent of the velocity of the charge and can perform work on the charge and change its kinetic energy. Unlike F_e, F_m depends on the charge velocity and is normal to it. F_m cannot perform work because it is at right angles to the direction of motion, of the charge ($F_m.dl = 0$); it does not cause an increase in kinetic energy of the charge. The magnitude of F_m is generally small compared to F_e except at high velocities.

For a moving charge *Q* in the Presence of both electric and magnetic fields, the total force on the charge is given by

$$\mathbf{F} = \boldsymbol{F_e} + \mathbf{F_m}$$

or

$$\mathbf{F = Q\,(E + u \times B)} \qquad (2.3)$$

This is known as the *Lorentz force equation.* It relates mechanical force to electrical force. If the mass of the charged Particle moving in **E** and **B** fields is m, by Newton's second law of motion.

$$F = m\frac{du}{dt} = Q(E + u \times B) \qquad (2.4)$$

The solution to this equation is important in determining the motion of charged particles in E and B fields. We should bear in mind that in such fields, energy transfer can be only by means of the electric field. A summary on the force exerted on a charged particle is given in table 2.1.

TABLE 2.1: Force on a Charged Particle

State of Particle	E Field	B Field	Combined E and B Fields
Stationary	*QE*	-	*QE*
Moving	*QE*	*Qu x* B	*Q(E* + u x B)

The magnetic field B is defined as the force per unit current element

Alternatively, **B** may be defined from eq. (2.2) as the vector which satisfies $F_m / q = u \times \mathbf{B}$ just as we defined electric field E as the force per unit charge, F_e / q.

Force between Two Current Elements

Let us now consider the force between two elements $I_1\ dl_1$ *and* $I_2\ dl_2$. According to Biot-Savart's law, both current elements produce magnetic fields. So we may find the force $d(dF_1)$ on element $I_1\ dl_1$ due to the field $d\mathbf{B}_2$ produced by element $I_2\ dl_2$ as shown in Figure 2.1.

As per equation

$$d\mathbf{F} = I\ d\mathbf{l} \times \mathbf{B}_2$$

$$d(d\mathbf{F}_1) = I_1\ d\mathbf{l}_1 \times d\mathbf{B}_2 \tag{2.5}$$

But from Biot-Savart's law,

$$dB_2 = \frac{\mu_0 I_2\ dl_2 \times a_{R_{21}}}{4\pi R_{21}^2} \tag{2.6}$$

Hence,

$$d(dF_1) = \frac{\mu_0 I_1 dl_1 \times \left(I_2\ dl_2 \times a_{R_{21}}\right)}{4\pi R_{21}^2} \tag{2.7}$$

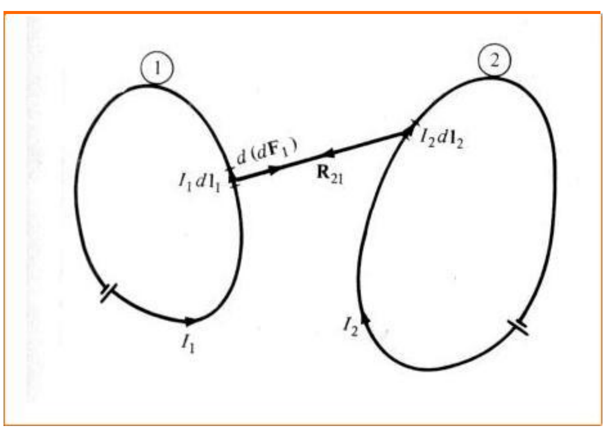

Figure 2.1: Force between two current loops.

This equation is essentially the law of force between two current elements and is analogous to Coulomb's law, which expresses the force between two stationary charges. From eq. (2.7), we obtain the total force **F_1** on current loop 1 due to current loop 2 shown Figure 2.1 as

$$F_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{dl_1 \times (dl_2 \times a_{R_{21}})}{R_{21}^2} \qquad (2.8)$$

Although this equation appears complicated, we should remember that it is based on eq. (2.5). It is eq. (8. 10) that is of fundamental importance.

The force **F_2** on loop 2 due to the magnetic field **B_1** from loop 1 is obtained from eq. (2.8) by interchanging subscripts 1 and 2. It can be shown that **$F_2 = - F_1$;** thus F_1 and F_2 obey Newton's third law that action and reaction are equal and opposite. It is worthwhile to mention that eq. (2.8) was experimentally established by Qersted and Ampete; Biot and Savart (Ampere's colleagues) actually based their law on it.

MAGNETIC TORQUE AND MOMENT

Now that we have considered the force on a current loop in a magnetic field, we can determine the torque on it. The concept of a current loop experiencing a torque in a magnetic field is of paramount importance in understanding the behavior of orbiting charged particles, d.c. motors, and generators. If the loop is placed parallel to a magnetic field, it experiences a force that tends to rotate it.

The torque T (or mechanical moment of force) on the loop is the, vector product of the force F and the moment arm r.

That is,

$$\mathbf{T} = \mathbf{r} \times \mathbf{F} \tag{2.9}$$

and its units are Newton-meters.

Let us apply this to a rectangular loop of length l and width w placed in a uniform magnetic field **B** as shown in Figure 8.5(a). From this figure, we notice that dl is parallel to **B** along sides 12 and 34 of the loop and no force is exerted on those sides. Thus

$$F = I \int_2^3 dl \times B + I \int_4^1 dl \times B$$

$$= I \int_2^l dz\, a_z \times B + I \int_l^0 dz\, a_z \times B$$

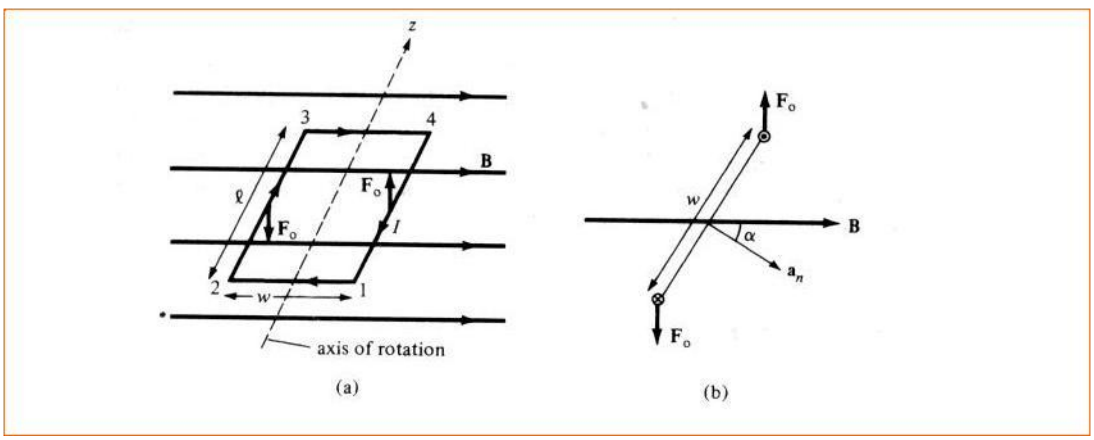

Figure 2.2: Rectangular planar loop in a uniform magnetic field.

or

$$\mathbf{F} = \mathbf{F_0} - \mathbf{F_0} = 0 \tag{2.10}$$

Where, $|\mathbf{F_0}| = \boldsymbol{I}\,\mathbf{Bl}$ because **B** is uniform. Thus, no force is exerted on the loop as a whole. However, $\mathbf{F_0}$ and $-\mathbf{F_0}$ act at different points on the loop, thereby creating a couple. If the normal to the plane of the loop makes an angle with **B**, as shown in the cross-sectional view of Figure 2.2(b), the torque on the loop is

$$|\mathbf{T}| = |\mathbf{F_0}|\, w \sin$$

or

$$T = B\, I\, l\, w \sin \qquad (2.11)$$

But lw = *S*, the area of the loop. Hence,

$$T = BIS \sin \qquad (2.12)$$

We define the quantity

$$\mathbf{m} = IS a_n \qquad (2.13)$$

as the *magnetic dipole moment* (in A/M^2) of the loop. In eq. (2.13), $\mathbf{a_n}$ is a unit normal vector to the plane of the loop and its direction is determined by the right-hand rule: fingers in the direction of current Hand thumb along $\mathbf{a_n}$.

The magnetic dipole moment is the product of current and area of the loop; its reaction is normal to the loop.

Introducing eq. (2.13) in eq. (2.12), we obtain

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \qquad (2.14)$$

3.0 STOKE'S THEOREM

Stoke's Theorem relates a line integral to the surface integral and vice-versa, that is

$$\oint_C H \cdot dL = \int_S (\nabla \times H) \cdot dS \qquad (3.1)$$

FORCE ON A MOVING CHARGE DUE TO ELECTRIC AND MAGNETIC FIELDS

If there is a charge or a moving charge, Q in an electric field, E, there exists a force on the charge. This force is given by

$$F_E = QE \qquad (3.2)$$

If a charge, Q moving with a velocity, **V** is placed in a magnetic field, **B (=μH)**, then there exists a force on the charge (Fig. 3.1). This force is given by

$$F_H = Q(V \times B) \qquad (3.3)$$

B = magnetic flux density, (wb/m^2)

V = velocity of the charge, m/s

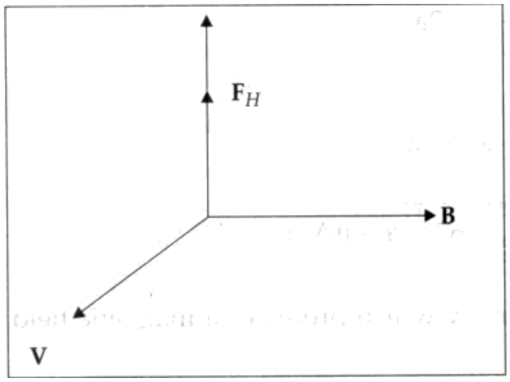

Fig. 3.1: Direction of field, velocity and force

If the charge, Q is placed in both electric and magnetic fields, then the force on the charge is

$$\mathbf{F} = Q\,(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \qquad (3.4)$$

This equation is known as **Lorentz force equation.**

Problem 1: A charge of 12 C has velocity of $\mathbf{5a_x + 2a_y} - 3a_z$ m/s. Determine **F** on the charge in the field of (a) $E = 18a_x, + 5a_y + 10a_z$ V/m

(b) $\mathbf{B} = 4a_x + 4a_y + 3a_z$ wb/m^2.

Solution:

(a) The force, **F** on the charge, Q due to E is

$$F = QE = 12\,(18a_x + 5a_y + 10a_z)$$

$$= 216a_x + 60a_y + 120a_z$$

or, $F = Q\,|E| = 12\sqrt{18^2 + 5^2 + 10^2}$

$F = 254.27$ N

(b) The force F on the charge due to B is

$F = Q[V \times B)$

Here $V = 5a_x + 2a_y - 3a_z$ m/s

$B = 4\, a_x + 4\, a_y + 3\, a_z$ wb / m^2

$F = 12\, [18a_x - 27a_y + 12a_z]$

or, $F=12\sqrt{(324 + 729 + 144)}$

$F = 415.17$ N

FORCE ON A CURRENT ELEMENT IN A MAGNETIC FIELD

The force on a current element when placed in a magnetic field, B is

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B} \qquad (3.5)$$

or,

$$F = I\, L\, B \sin\theta \text{ Newton} \qquad (3.6)$$

where θ is the angle between the direction of the current element and the direction of magnetic flux density

B = magnetic flux density, wb/m^2

I**L** = current element, Amp-m

Proof: Consider a differential charge, dQ to be moving with a velocity, V in a magnetic field, **H** = (**B**/□). Then the differential force on the charge is given by

d**F** = dQ (**V** x **B**) (3.7)

But

dQ = □□ d□

dF = □□ d□ (V x B)

= (□□ V x B) d□

But □□ V = J

d**F** = **J** d□ x **B**

Jd□ is nothing but Id**L**,

d**F** =Id**L** x **B**

or, **F** = I**L** x **B**, Newton (3.8)

Problem 2: A current element 4 cm long is along y-axis with a current of 10 mA flowing in y-direction. Determine the force on the current element due to the magnetic field if the magnetic field H = ($5a_x$/□) A/m.

Solution:

The force on a current element under the influence of magnetic field is

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

Here, $I\mathbf{L} = 10 \times 10^{-3} \times 0.04\mathbf{a}_y$

$= 4 \times 10^{-4}\ \mathbf{a}_y$

$H = (5\mathbf{a}_x/\square)$ A/m

$B = 5\mathbf{a}_x$ wb/m^2

$F = 4 \times 10^{-4}\ \mathbf{a}_y \times 5\mathbf{a}_x$

or $F = (0.4\mathbf{a}_y \times 5\mathbf{a}_x) \times 10^{-3}$

$F = -2.0\mathbf{a}_z$ mN

BOUNDARY CONDITIONS ON H AND B

1. The tangential component of magnetic field, **H** is continuous across any boundary except at the surface of a perfect conductor, that is,

   $$\mathbf{H}_{tan1} - \mathbf{H}_{tan2} = \mathbf{J}_s \qquad (3.9)$$

   At non-conducting boundaries, $J_s = 0$.

2. The normal component of magnetic flux density, **B** is continuous across any discontinuity, that is,

   $$\mathbf{B}_{n1} = \mathbf{B}_{n2} \qquad (3.10)$$

Proof: Consider Fig. 3.2 in which a differential rectangular loop across a boundary separating medium 1 and medium 2 are shown.

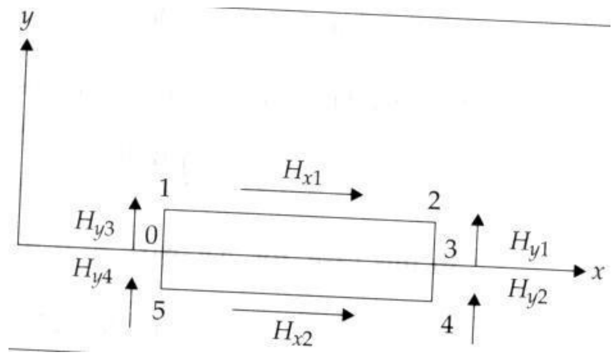

Fig. 3.2: A rectangular loop across a boundary

From Ampere's circuit law, we have

$$\oint H \cdot dL = \int_5^0 + \int_0^1 + \int_1^2 + \int_2^3 + \int_3^4 + \int_4^5$$

$$= H_{y4}\frac{\Delta y}{2} + H_{y3}\frac{\Delta y}{2} + H_{x1}\Delta x - H_{y1}\frac{\Delta y}{2} - H_{y2}\frac{\Delta y}{2} - H_{x2}\Delta x = I$$

As $\Delta y \to 0$, we get

$$\oint H \cdot dL = H_{x1}\Delta x - H_{x2}\Delta x = I$$

or,

$$H_{x1} - H_{x2} = \frac{I}{\Delta x} = J_s \qquad (3.11)$$

Here, H_{x1} and H_{x2} are tangential components in medium 1 and 2, respectively.

So, $\mathbf{H}_{tan1} - \mathbf{H}_{tan2} = \mathbf{J}_s$ (3.12)

Now consider a cylinder shown in Fig. 3.3.

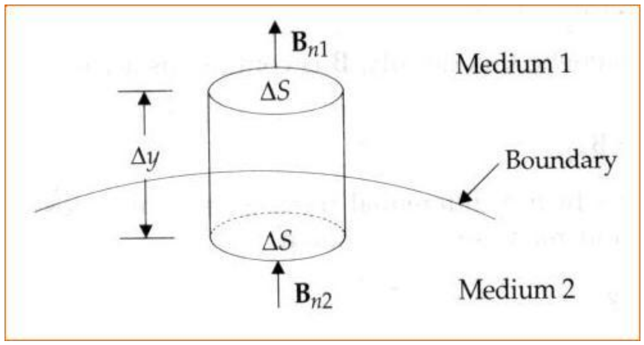

Fig. 3.3: A differential cylinder across the boundary

Gauss's law for magnetic fields is

$$\oint_s B \cdot dS = 0 \qquad (3.13)$$

In this case, for $\Delta y \to 0$

$$\oint B \cdot dS =$$

$$\oint_s B_{n1} a_y \cdot dS \quad a_y + \int_s B_{n2}\, a_y \cdot dS\,(-a_y) \qquad (3.14)$$

that is, $B_{n1}\,\Delta S - B_{n2}\,\Delta S = 0$

Therefore, $\mathbf{B}_{n1} = \mathbf{B}_{n2}$ (3.15)

Problem 3:

Two homogeneous, linear and isotropic media have an interface at $x = 0$. $x < 0$ describes medium 1 and x > 0 describes medium 2. □r1 = 2 and □r2 = 5. The magnetic field in medium 1 is $150\mathbf{a}_x - 400\mathbf{a}_y + 250\mathbf{a}_z$ A/m.

Determine:

(a) Magnetic field in medium 2

(b) Magnetic flux density in medium 1

(c) Magnetic flux density in medium 2.

Solution:

The magnetic field in medium 1 is

$$\mathbf{H}_1 = 150\mathbf{a}_x - 400\mathbf{a}_y + 250\mathbf{a}_z \text{ A/m}$$

Consider Fig. 3.4.

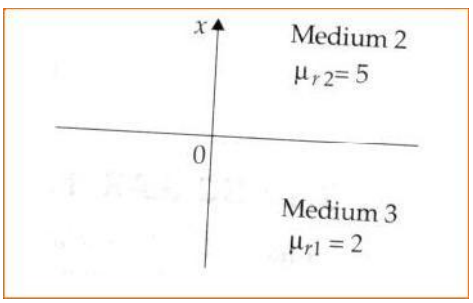

Fig. **3.4: Illustrative figure**

(a) $\mathbf{H}_1 = \mathbf{H}_{tan1} + \mathbf{H}_{n1}$

$H_{tanl} = -400\mathbf{a}_y + 250\mathbf{a}_z$ A/m H_{n1}

$= 150\mathbf{a}_x$

The boundary condition is

$H_{tan1} = H_{tan2}$

$H_{tan2} = -400\mathbf{a}_y + 50\mathbf{a}_z$ A/m

The boundary condition on **B** is $\mathbf{B}_{n1} = \mathbf{B}_{n2}$

that is, $\mu_1 H_{n1} = \mu_1 H_{n2}$

$$H_{n2} = \frac{\mu_1}{\mu_2} H_{n1}$$

$$= \frac{2}{5} \cdot 150 a_x$$

$= 60\mathbf{a}_x$

$\mathbf{H}_2 = \mathbf{H}_{tan2} + \mathbf{H}_{n2}$

(b) $\mathbf{B}_1 = \mu_1 \mathbf{H}_1$

$= \mu_0 \mu_r \mathbf{H}_1$

$= 4\pi \times 10^{-7} \times 2(150\mathbf{a}_x - 400\mathbf{a}_y + 250\mathbf{a}_z)$

$= (376.5\mathbf{a}_x - 1004\mathbf{a}_y + 627.5\mathbf{a}_z)\ \mu$wb/m^2

(c) $\mathbf{B}_2 = \mu_2 \mathbf{H}_2$

$$= 4\pi \times 10^{-7} \times 5\,(60\mathbf{a}_x - 400\mathbf{a}_y + 250\mathbf{a}_z)$$

$$= (376.98\mathbf{a}_x - 2513.2\mathbf{a}_y + 1570.75\mathbf{a}_z)\ \text{wb/m}^2$$

SCALAR MAGNETIC POTENTIAL

Like scalar electrostatic potential, it is possible to have scalar magnetic potential. It is defined in such a way that its negative gradient gives the magnetic field, that is,

$$\mathbf{H} = \nabla V_m \qquad (3.16)$$

V_m = scalar magnetic potential (Amp)

Taking curl on both sides, we get

$$\nabla \times \mathbf{H} = -\nabla \times \nabla V_m \qquad (3.17)$$

But curl of the gradient of any scalar is always zero.

So, $\nabla \times \mathbf{H} = 0$ (3.18)

But, by Ampere's circuit law $\nabla \times \mathbf{H} = \mathbf{J}$

or, $J = 0$

In other words, scalar magnetic potential exists in a region where J = 0.

$$H = -\nabla V_m \quad (J=0) \qquad (3.19)$$

The scalar potential satisfies Laplace's equation, that is, we have

$$\nabla . B = \mu_0 \nabla . H = 0 = \mu_0 \nabla . (-\nabla V_m) = 0$$

or,

$$\nabla^2 V_m = 0 \ \mathbf{(J = 0)} \qquad (3.20)$$

Characteristics of Scalar Magnetic Potential (V_m)

1. The negative gradient of V_m gives H, or $H = -\nabla V_m$
2. It exists where J = 0
3. It satisfies Laplac's equation.
4. It is directly defined as

$$V_m = -\int_A^B H \cdot dL$$

5. It has the unit of Ampere.

VECTOR MAGNETIC POTENTIAL

Vector magnetic potential exists in regions where J is present. It is defined in such a way that its curl gives the magnetic flux density, that is,

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (3.21)$$

where **A =** vector magnetic potential (wb/m).

It is also defined as

$$A = \oint \frac{\mu_0 \, IdL}{4\pi R} \quad \left(\frac{Henry \cdot Amp}{m} \right) \tag{3.22}$$

or, $$A = \int_s \frac{\mu_0 \, Kds}{4\pi R} \quad \text{(K = current sheet)} \tag{3.23}$$

or, $$A = \int_v \frac{\mu_0 \, Jdv}{4\pi R} \tag{3.24}$$

Characteristics of Vector Magnetic Potential

1. It exists even when J is present.
2. It is defined in two ways

   $B = \nabla \times A$ and

   $A = \int_v \frac{\mu_0 \, J dv}{4\pi R}$

3. $\nabla^2 \mathbf{A} = \mu_0 j$
4. $\nabla^2 \mathbf{A} = 0$ if $J = 0$
5. Vector magnetic potential, **A** has applications to obtain radiation characteristics of antennas, apertures and also to obtain radiation leakage from transmission lines, waveguides and microwave ovens.
6. A is used to find near and far-fields of antennas.

Problem 4:

The vector magnetic potential, A due to a direct current in a conductor in free space is given by $A = (X^2 + Y^2)\, \mathbf{a}_z$ wb/m^2. Determine the magnetic field produced by the current element at (1, 2, 3).

Solution:

$$\mathbf{A} = (x^2 + y^2)\,\mathbf{a}_z \ \mu\text{wb/m}^2$$

We have $\mathbf{B} = \nabla \times \mathbf{A}$

$$= 10^{-6}\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & (x^2 + y^2) \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(x^2 + y^2)\,a_x - \frac{\partial}{\partial x}(x^2 + y^2)\,a_y\right] \times 10^{-6}$$

$$= \left[(x^2 + 2y)\,a_x - (2x + y^2)\,a_y\right] \times 10^{-6}$$

$$B / at\ (1,2,3) = \left[(1 + 4)\,a_x - (2 + 4)\,a_y\right] \times 10^{-6}$$

$$= (5a_x - 6a_y) \times 10^{-6}$$

$$H = \frac{1}{\mu_0}(5a_x - 6a_y) \times 10^{-6}$$

$$= \frac{1}{4\pi \times 10^{-7}}(5a_x - 6a_y) \times 10^{-6}$$

$\mathbf{H} = (3.978\mathbf{a}_x - 4.774\mathbf{a}_y)$, A/m

FORCE AND TORQUE ON A LOOP OR COIL

Consider Fig. 3.5 in which a rectangular loop is placed under a uniform magnetic flux density, **B**.

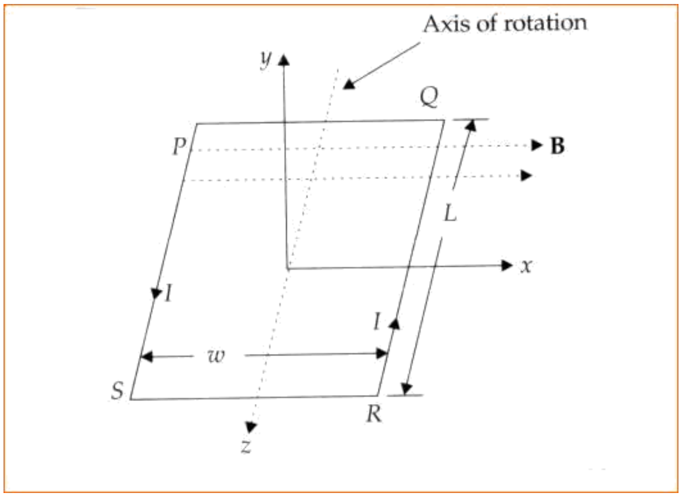

Fig. 3.5: Rectangular conductor loop in *x-z* plane

From Fig. 3.5, the force on QR due to **B** is

$$F_1 = IL \times B = -IL\mathbf{a}_z \times B\mathbf{a}_x \qquad (3.25)$$

$$F_1 = -IL\mathbf{B}\mathbf{a}_y \qquad (3.26)$$

that is, the force, F_1 on QR moves it downwards. Now the force on *PS* is

$$F_2 = I\mathbf{L} \times \mathbf{B} = -IL\mathbf{a}_z \times B\mathbf{a}_x \qquad (3.27)$$

$$F_2 = -IL\mathbf{B}\mathbf{a}_y \qquad (3.28)$$

Force, F_2 on PS moves it upwards. It may be noted that the sides PQ and SR will not experience force as they are parallel to the field, **B**.

The forces on QR *and* PS exert a torque. This torque tends to rotate the coil about its axis.

The torque, **T** is nothing but a mechanical moment of force. The torque on the loop is defined as the vector product of moment arm and force,

that is,

$$\mathbf{T} = r \times \mathbf{F}, \text{ N-m} \tag{3.29}$$

where r = moment arm

F = force

Applying this definition to the loop considered above, the expression for torque is given by

$$T = r_1 \times F_1 + r_2 \times F_2 \tag{3.30}$$

$$= \frac{w}{2} a_x \times (-ILBa_y) + \left(-\frac{w}{2} a_x\right) \times (ILBa_y) \tag{3.31}$$

$$= -BILw\mathbf{a}_z$$

or,

$$T = -BIS\mathbf{a}_z \tag{3.32}$$

where $S = wL$ = area of the loop

The torque in terms of magnetic dipole moment, **m** is

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}, \text{ N-m} \tag{3.34}$$

where $\boldsymbol{m} = I\,l\,w\,\mathbf{a}_y$

$= I\,S\,\mathbf{a}_y$

Problem 5:

A rectangular coil is placed in a field of $\mathbf{B} = (2\mathbf{a}_x + \mathbf{a}_y)$ wb/m^2. The coil is in y-z plane and has dimensions of 2 m x 2 m. It carries a current of 1 A. Find the torque about the z-axis.

Solution:

$$m = IS\,\mathbf{a}_n = 1 \times 4\mathbf{a}_x$$

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} = 4\mathbf{a}_x \times (2\mathbf{a}_x + \mathbf{a}_y)$$

$$\mathbf{T} = \mathbf{4a}_z, \text{ N-m}$$

MATERIALS IN MAGNETIC FIELDS

A material, is said to be magnetic if □$_m$ □ 0, □$_r$ = 1

A material is said to be non-magnetic if □$_m$ = 0, □$_r$ = 1.

The term 'Magnetism' is commonly discussed in terms of magnets with basic examples like north pole, compass needle, horse shoe magnets and so on.

Magnetic properties are described in terms of magnetic susceptibility and relative permeability of the materials.

Magnetic materials are classified into

1. Diamagnetic materials
2. Paramagnetic materials
3. Ferromagnetic materials

Diamagnetic Materials

A material is said to diamagnetic if its susceptibility, □$_m$ < 0 and □$_r$ □ 1.0.

Examples are copper, lead, silicon, diamond and bismuth.

Characteristics of diamagnetic materials

- Magnetic fields due to the motion of orbiting electrons and spinning electrons cancel each other.
- Permanent magnetic moment of each atom is zero.
- These materials are widely affected by magnetic field.
- Magnetic susceptibility χ_m is (-)ve.
- $\mu_r = 1$
- B = 0
- Most of the materials exhibit diamagnetism.
- They are linear magnetic materials.
- Diamagnetism is not temperature dependent.
- These materials acquire magnetisation opposite to H and hence they are called diamagnetic materials.

Paramagnetic Materials

A material for which $\chi_m > 0$ and $\mu_r \geq 1$ is said to be paramagnetic.

Examples are air, tungsten, potassium and platinum.

Characteristics of paramagnetic materials

- They have non-zero permanent magnetic moment.
- Magnetic fields due to orbiting and spinning electrons do not cancel each other.
- Paramagnetism is temperature dependent.
- χ_m lies between 10^{-5} and 10^{-3}.
- These are used in MASERS.
- $\chi_m > 0$
- $\mu_r \geq 1$

- They are linear magnetic materials.

These materials acquire magnetisation parallel to H and hence they are called paramagnetic materials.

Ferromagnetic Materials

A material for which $\chi_m >> 0$, $\mu_r >> 1$ is said to be ferromagnetic.

Examples are iron, nickel, cobalt and their alloys.

Characteristics of ferromagnetic materials

- They exhibit large permanent dipole moment.
- $\chi_m >> 0$
- $\mu_r >> 1$
- They are strongly magnetised by magnetic field.
- They retain magnetism even if the magnetic field is removed.
- They lose their ferromagnetic properties when the temperature is raised.
- If a permanent magnet made of iron is heated above its curie temperature, 770°C, it loses its magnetisation completely.
- They are non-linear magnetic materials.
- **B** = μ**H** does not hold good as μ depends on **B.**
- In these materials, magnetisation is not determined by the field present. It depends on the magnetic history of the object.

INDUCTANCE

Inductor is a coil of wire wound according to various designs with or without a core of magnetic material to concentrate the magnetic field.

Inductance, L In a conductor, device or circuit, an inductance is the inertial property caused by an induced reverse voltage that opposes the flow of current when a voltage is applied. It also opposes a sudden change in current that has been established.

Definition of Inductance, L (Henry):

The inductance, L of a conductor system is defined as the ratio of magnetic flux linkage to the current producing the flux, that is,

$$L = \frac{N\phi}{I} \ (Henry) \qquad (3.35)$$

Here N = number of turns

ϕ = flux produced

I = current in the coil

1 Henry = 1 wb/Amp

L is also defined as $(2W_H/I^2)$, or

$$L = \frac{2W_H}{I^2} \qquad (3.36)$$

where, W_H = energy in H produced by I.

In fact, a straight conductor carrying current has the property of inductance. Aircore coils are wound to provide a few pico henries to a few micro henries. These are used at IF and RF frequencies in tuning coils, interstage coupling coils and so on.

The requirements of such coils are:

- Stability of inductance under all operating conditions
- High ratio inductive reactance to effective loss resistance at the operating frequency
- Low self capacitance
- Small size and low cost
- Low temperature coefficient

STANDARD INDUCTANCE CONFIGURATIONS

Toroid

It consists of a coil wound on annular core. One side of each turn of the coil is threaded through the ring to form a Toroid (Fig. 3.6).

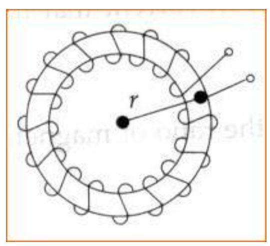

Fig. 3.6: Toroid

Inductance of Toroid, $$L = \frac{\mu_0 N^2 S}{2\pi r} \qquad (3.37)$$

Here N = number of turns

r = average radius

S = cross-sectional area

Magnetic field in a Toroid, $$H = \frac{NI}{2\pi r} \qquad (3.38)$$

I is the current in the coil.

Solenoid

It is a coil of wire which has a long axial length relative to its diameter. The coil is tubular in form. It is used to produce a known magnetic flux density along its axis.

A solenoid is also used to demonstrate electromagnetic induction. A bar of iron, which is free to move along the axis of the coil, is usually provided for this purpose. A typical solenoid is shown in Fig. 3.7.

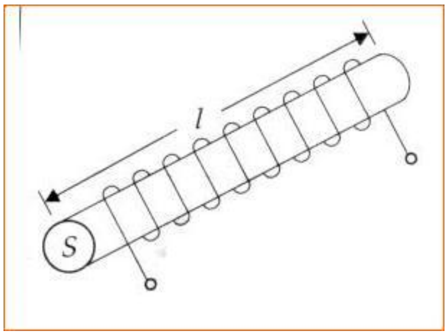

Fig. 3.7: Solenoid

The inductance, L of a solenoid is

$$L = \frac{\mu_0 N^2 S}{l} \qquad (3.39)$$

l = length of solenoid S =

cross-sectional area N =

Number of turns

The magnetic field in a solenoid is

$$H = \frac{NI}{l} \qquad (3.40)$$

I is the current

oo0oo -----------