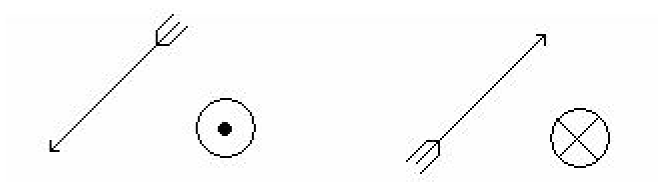
Steady state Magnetic Field

In previous chapters we have seen that an electrostatic field is produced by static or stationary charges. The relationship of the steady magnetic field to its sources is much more complicated.

The source of steady magnetic field may be a permanent magnet, a direct current or an electric field changing with time. In this chapter we shall mainly consider the magnetic field produced by a direct current. The magnetic field produced due to time varying electric field will be discussed later. Historically, the link between the electric and magnetic field was established Oersted in 1820. Ampere and others extended the investigation of magnetic effect of electricity . There are two major laws governing the magnetostatic fields are:

1. Biot-Savart Law
2. Ampere's Law

Usually, the magnetic field intensity is represented by the vector $\vec{H}$. It is customary to represent the direction of the magnetic field intensity (or current) by a small circle with a dot or cross sign depending on whether the field (or current) is out of or into the page as shown in Fig. 4.1.



$\vec{H}$ (or l) out of the page $\vec{H}$ (or l) into the page

Fig. 4.1: Representation of magnetic field (or current)

Biot- Savart Law

This law relates the magnetic field intensity *dH* produced at a point due to a differential current element $Id\vec{l}$ as shown in Fig. 4.2.

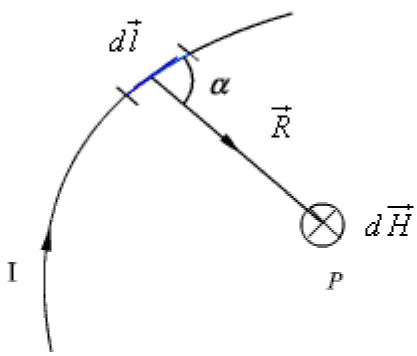


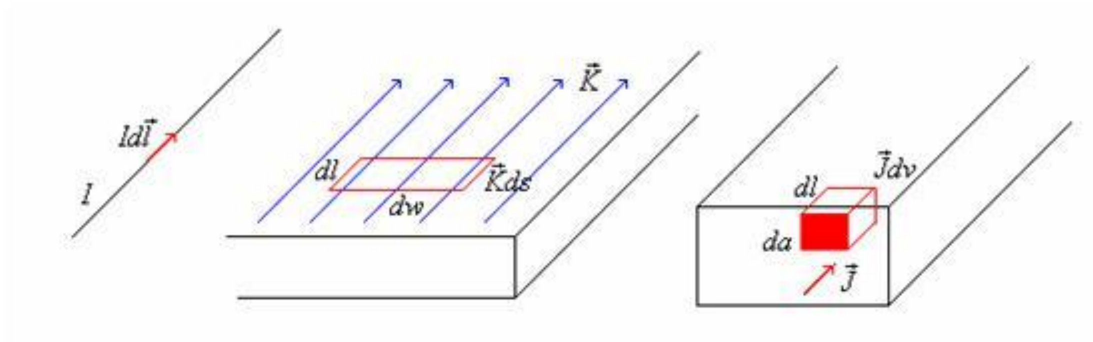
Fig. 4.2: Magnetic field intensity due to a current element

$$d\vec{H} = \frac{Id\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$ ....(4.1

$$dH = \frac{IdlSin\alpha}{4\pi R^2}$$ ....(4.1b)

where $R = |\vec{R}|$ is the distance of the current element from the point P.

Similar to different charge distributions, we can have different current distribution such as line current, surface current and volume current. These different types of current densities are shown in Fig. 4.3.



Line Curre Surface Current,Volume Current

Fig. 4.3: Different types of current distributions

By denoting the surface current density as K (in amp/m) and volume current density as J (in amp/m^2) we can write:

$$Id\vec{l} = \vec{K}ds = \vec{J}dv$$ ....(4.2)

(It may be noted that $I = Kdw = Jda$)

Employing Biot-Savart Law, we can now express the magnetic field intensity H. In terms of these current distributions.

$$\vec{H} = \int_s \frac{Kd\vec{s} \times \vec{R}}{4\pi R^3}$$ .... **for surface current** ....(4.3b)

$$\vec{H} = \int_v \frac{\vec{J}dv \times \vec{R}}{4\pi R^3}$$ .... **for volume current**......................(4.3c)

To illustrate the application of Biot - Savart's Law, we consider the following example.

Example 4.1: We consider a finite length of a conductor carrying a current $\vec{I}$ placed along z-axis as shown in the Fig 4.4. We determine the magnetic field at point P due to this current carrying conductor.

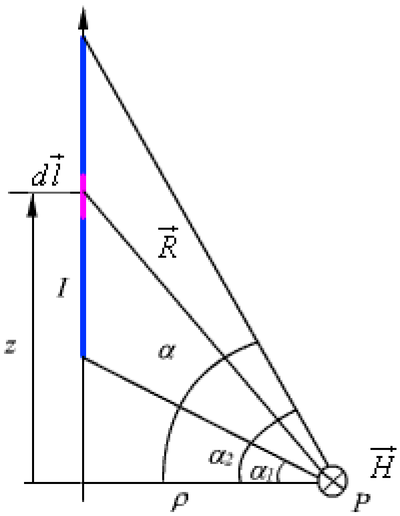


Fig. 4.4: Field at a point P due to a finite length current carrying conductor

With reference to Fig. 4.4, we find that

$$d\vec{l} = dz\,\hat{a}_z \quad and \quad \vec{R} = \rho\hat{a}_\rho - z\hat{a}_z \quad \text{...(4.4)}$$

Magnetism

Magnetism developed independently of but in parallel with the science of electrostatics. It was based on the study of naturally occurring magnetic materials (principally lodestone). The basic experimental element was the **bar magnet** comprising 2 poles, which are in reality indivisible.

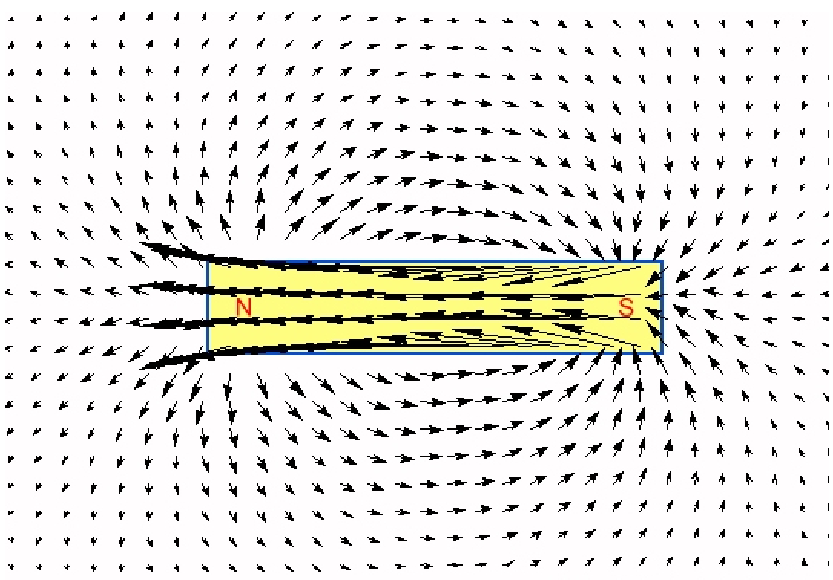


Figure Permanent magnet (from electromagnetic modeling)

Electric charges can be isolated, but magnetic poles always exist in pairs and a small bar-magnet is effectively a magnetic dipole.

Concepts:

1. Inverse square law $F = \frac{Q_1 Q_2}{4\pi \cdot \mu_0 r^2}$, where Q1 and Q2 are the magnetic pole strengths and μ_0 is the magnetic permeability of free space [4$\square$x10^{-7} H/m].

2. The concept of a **Magnetic Field** followed, having strength **H** at a point where

$$F = HdQ, \quad H = \frac{Q}{4\pi \cdot \mu_0 r^2}$$

and **H** in the **Magnetic Field Intensity.**

In a complementary way to electrostatics it was found convenient to define a new quantity **B** the **Magnetic Flux Density**, such that in vacuum:

B = $\square_0$**H**

We might note that in the presence of magnetic materials the relationship between **B** and **H** may be written

B = $\square$**H=** $\square_0 \square_r$**H**

Where: $\square_r$ Is the relative permeability of the material.

And also magnetic flux linking the surface S is defined as the total magnetic flux density passing through S, or

and from Stoke's theorem,

$$\Phi = \oint_C A \cdot dl$$

where C is the contour boarding the surface S.

Over a close surface S Gauss's Law provides (since the total pole strength is always zero)

A magnetic field not only exists around permanent magnets but can also be created by electric circuit.

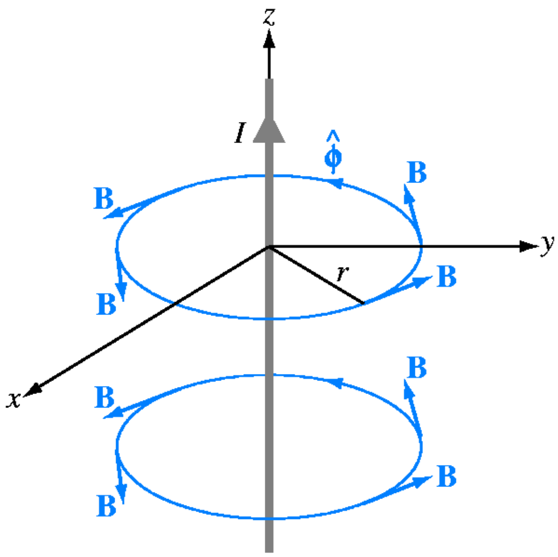


Figure The magnetic field induced by
a steady current flowing in the z-direction

Magnetic flux density B near the conductor with a current (Biot-Savart law)

where: r is the radial distance from the current and $\hat{\varphi}$ is an azimuthal unit vector, μ_0 is the magnetic permeability of free space [4□x10^{-7} H/m].

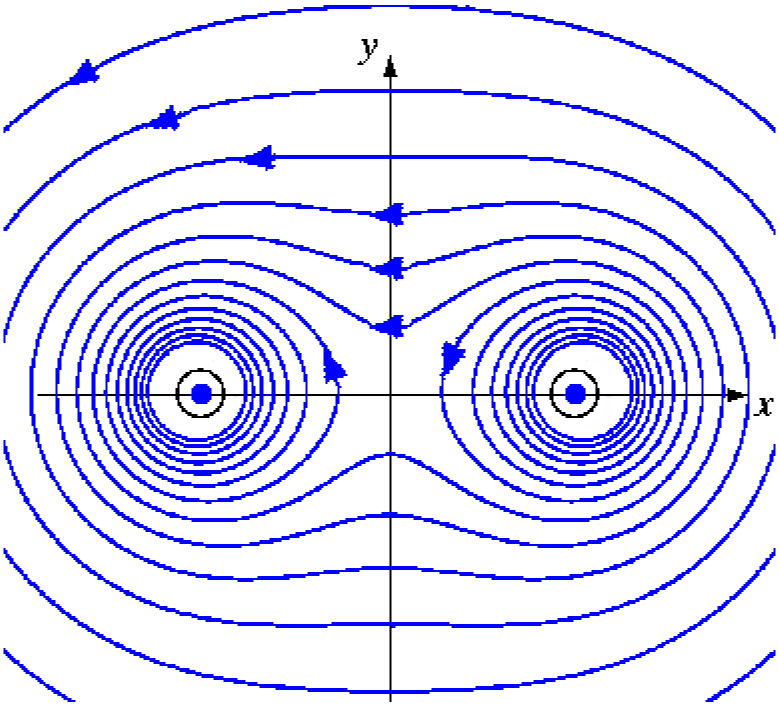


Figure Lines representing magnetic field - two parallel conductors carrying current in Z-direction

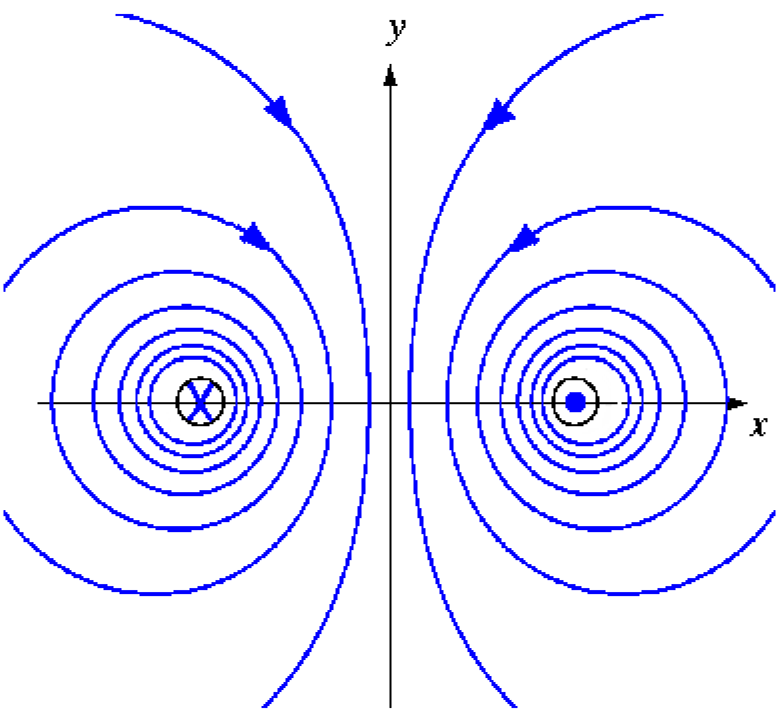


Figure Lines representing magnetic field - two parallel conductors carrying current in Z-direction and -Z-direction

See an animated model of the magnetic field from two conductors with current (results obtained from 2-dimentional electromagnetic modeling):

It was not until 1820 that a formal link was established between the sciences of **Electrostatics** and **Current Electricity and magnetism**. In that year Oersted discovered that a **magnetic compass needle was deflected in the neighborhood of an electric current – that the electric current produced a magnetic field.**

Within 3 months Ampere had developed a theory, which integrated the sciences of current electricity and magnetism. This theory is symbolized by the notion of **equivalence** of a **magnetic dipole** and a **current dipole**.

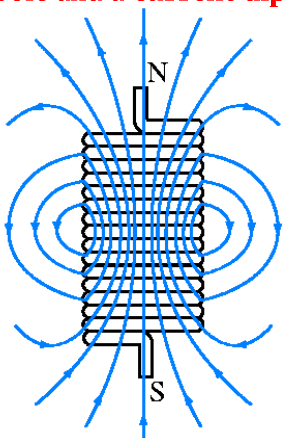


Figure Magnetic field from a solenoid

Consider now a bar magnet of pole strength Q_m and length L (magnetic dipole moment m= Q_m L) in a uniform field B. The north (+) pole experiences a force F= Q_m B to the right and the south (-) pole an equal force to the left.

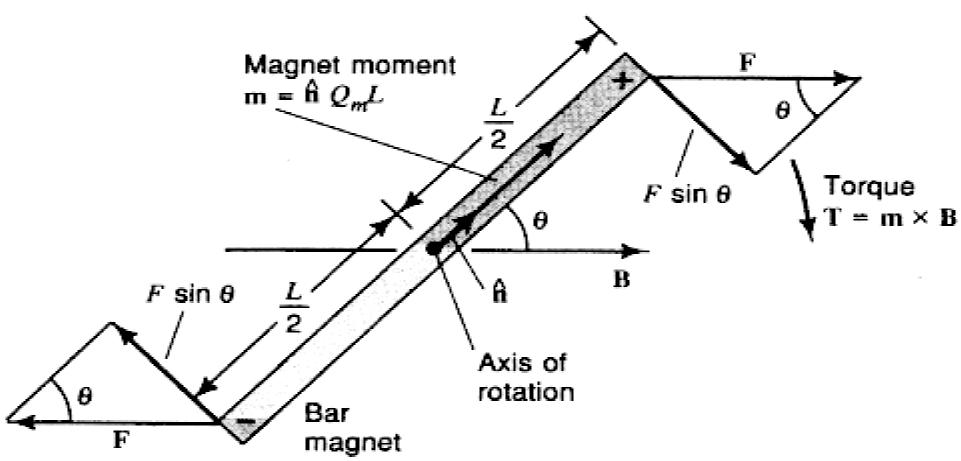


Figure Bar magnet experiences torque tending to align it with magnetic field **B**.

The torque T, or turning moment (force x distance), on the dipole is

where: F= Q_m B, [N]

L = length of dipole, [m]

□ = angle between dipole axis and B [rad/deg]

Including the value of F, we have

where: m=magnetic moment [Am^2]
In vector notation the torque is given by

$$T = m \times B = IA \times B$$

where: I = loop current, [A]
A = loop area, [m^2]
We note that the magnet and loop are equivalent if their magnetic moments are equal or

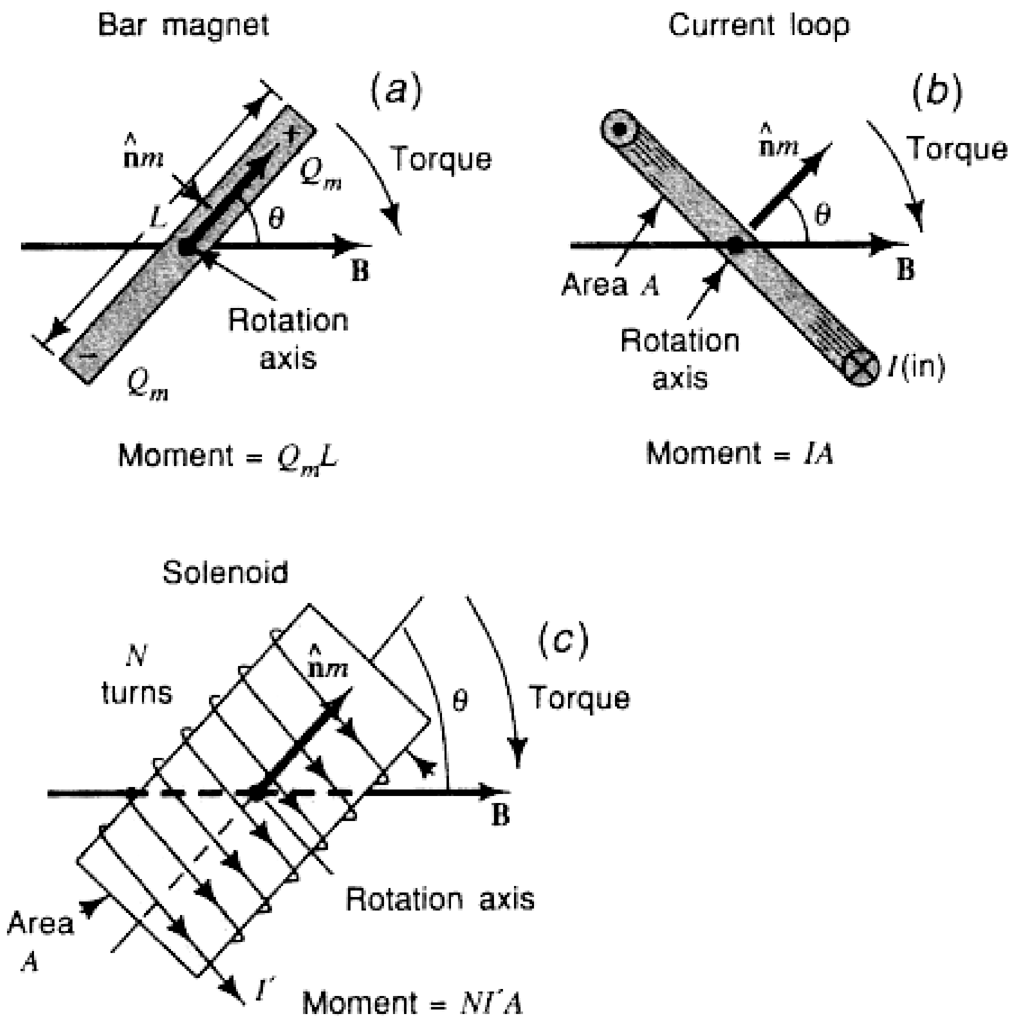


Figure Bar magnet (a), current loop (b), and solenoid (c) in a uniform field **B**. in all three cases the torque is clockwise and tends to align the magnetic moment **m** with **B.** If all moments are equal ($Q_m L$=IA=NI`A [Am^2]) the torque is the same in all three cases. The loop is shown in cross-section.
The expression for the energy stored in a magnetic field is:
(If □ is constant)
The treatment of electricity and magnetism is from an **historically based** perspective, which makes it easier to discuss the magnetic properties of matter.
There are three important results, which are all totally consistent with each other: