BJT and FET Frequency Response Characteristics:

- *Logarithms and Decibels:*

- Logarithms taken to the base 10 are referred to as *common logarithms* while logarithms taken to the base e are referred to as *natural logarithms* In summary:

$$\boxed{\text{Common logarithm: } x = \log_{10} a}$$

$$\boxed{\text{Natural logarithm: } y = \log_e a}$$

The two are related by

$$\boxed{\log_e a = 2.3 \log_{10} a}$$

- Some relationships hold true for logarithms to any base

$$\boxed{\log_{10} 1 = 0}$$

$$\boxed{\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b}$$

$$\boxed{\log_{10} \frac{1}{b} = -\log_{10} b}$$

$$\boxed{\log_{10} ab = \log_{10} a + \log_{10} b}$$

- The background surrounding the term *decibel* (dB) has its origin in the established fact that power and audio levels are related on a logarithmic basis.
- That is, an increase in power level say 4 to 16 W, does not result in an audio level increase by a factor of 16/4 = 4. It will increase by a factor of 2 as derived from the power of 4 in the following manner: $(4)^2 = 16$.
- The term *bel* was derived from the surname of Alexander Graham Bell. For standardization, the bel (B) was defined by the following equation to relate power levels P_1 and P_2:

P.L

$4 \longrightarrow 16$

A.L

4 ×

$4^2 = 2$ ✓

$$\boxed{G = \log_{10} \frac{P_2}{P_1}} \quad \text{bel}$$

- It was found, however, that the bel was too large a unit of measurement for practical purposes, so the decibel (dB) was defined such that 10 decibels=1 bel. Therefore,

$$\boxed{G_{dB} = 10 \log_{10} \frac{P_2}{P_1}} \quad \text{dB}$$

bel → decibels

- There exists a second equation for decibels that is applied frequently. It can be best described through the system with *Ri*, as an input resistance.

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_i}{V_1^2/R_i} = 10 \log_{10}\left(\frac{V_2}{V_1}\right)^2$$

power ↓ voltage

and

$$\boxed{G_{dB} = 20 \log_{10} \frac{V_2}{V_1}} \quad \text{dB}$$

$A_{v_T} = |A_{v_1}| \cdot |A_{v_2}| \ldots |A_{v_n}|$

$G_v = 20\log_{10}|A_{v_T}| = 20\log_{10}|A_{v_1}| + 20\log_{10}|A_{v_2}| + \ldots 20\log_{10}|A_{v_n}|$

- One of the advantages of the logarithmic relationship is the manner in which it can be applied to cascaded stages. In words, the equation states that the decibel gain of a cascaded system is simply the sum of the decibel gains of each stage.

$G_{dB_T} = G_{dB_1} + G_{dB_2} + \ldots\ldots G_{dB_n}$ dB.

- *General Frequency Considerations:*

f → single stage, Multistage

- The frequency of the applied signal can have a pronounced effect on the response of a single-stage or multistage network. The analysis thus far has been for the midfrequency spectrum.

f_{low} : C_C, C_E/C_S No SC X_C = High

- At low frequencies, we shall find that the coupling and bypass capacitors can no longer be replaced by the short-circuit approximation because of the increase in reactance of these elements.

- freq. Dependent Param
- stray capacitance of active device & n/w
LIMITS HIGH FREQUENCY RESPONSE

- The frequency-dependent parameters of the small-signal equivalent circuits and the stray capacitive elements associated with the active device and the network will limit the high-frequency response of the system.

- An increase in the number of stages of a cascaded system will also limit both the high- and low-frequency responses.

- For any system, there is a band of frequencies in which the magnitude of the gain is either equal or relatively close to the midband value.

- To fix the frequency boundaries of relatively high gain, $0.707A_{v_{mid}}$ was chosen to be the gain at the cutoff levels. The corresponding frequencies f_1 and f_2 are generally called the *corner, cutoff, band, break,* or *half-power frequencies.* The multiplier 0.707 was chosen because at this level the output power is half the midband power output, that is, at midfrequencies.

- The bandwidth (or passband) of each system is determined by f_1 and f_2, that is:

$$\boxed{\text{bandwidth (BW)} = f_2 - f_1}$$

- For applications of a communications nature (audio, video), a decibel plot of the voltage gain versus frequency is more useful.

- Before obtaining the logarithmic plot, however, the curve is generally normalized as shown in Fig. 9.6. In this figure, the gain at each frequency is divided by the midband value. Obviously, the midband value is then 1 as indicated. At the half-power frequencies, the resulting level is $0.707 = 1/\sqrt{2}$.

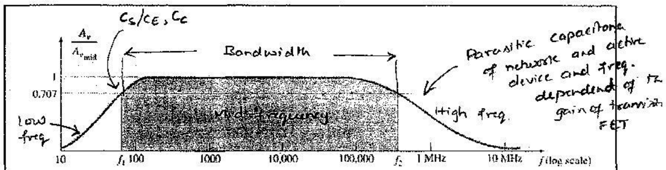


Fig. 9.6 Normalized gain versus frequency plot.

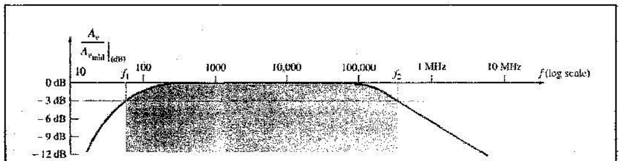
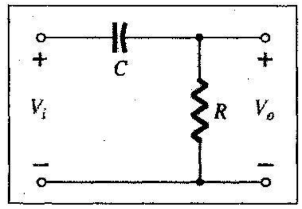


Fig. 9.7 Decibel plot of the normalized gain versus frequency plot of Fig. 9.6.

- ***Low Frequency Analysis:***

- In the low-frequency region of the single-stage amplifier, it is the R-C combinations formed by the network capacitors C_C, C_E, and C_s and the network resistive parameters that determine the cutoff frequencies.

- The analysis, therefore, will begin with the series R-C combination of the given Fig. and the development of a procedure that will result in a plot of the frequency response with a minimum of time and effort.



1) At very high frequencies,

$$X_C = \frac{1}{2\pi fC} \cong 0\ \Omega$$

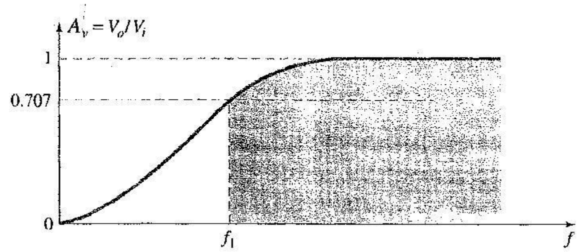
A short-circuit equivalent can be substituted for the capacitor. The result is that $V_o = V_i$ at high frequencies.

2) At $f = 0$ Hz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(0)C} = \infty\ \Omega$$

An open-circuit approximation can be applied, with the result that $V_o = 0$ V.

3) Between the two extremes, the ratio $A_v = V_o/V_i$ will vary as shown below. As the frequency increases, the capacitive reactance decreases and more of the input voltage appears across the output terminals.



- The output and input voltages are related by the voltage-divider rule in the following manner:

$$\mathbf{V_o = \frac{RV_i}{R + X_C}}$$

- The magnitude of V_o determined by

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}}$$

- For the special case where $X_C = R$,

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}} = \frac{RV_i}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}} V_i$$

$$\Rightarrow |A_v| = \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} = 0.707 = -3dB$$

- The frequency of which $X_C = R$ (***<u>the output will be 70.7% of the input</u>***) is determined as:

$$X_C = \frac{1}{2\pi f_1 C} = R \Rightarrow f_1 = \frac{1}{2\pi CR}$$

$$A_v = \frac{R}{R - jX_C} = \frac{1}{1 - j\left(\frac{X_C}{R}\right)} = \frac{1}{1 - j\left(\frac{1}{\omega CR}\right)} = \frac{1}{1 - j\left(\frac{1}{2\pi fCR}\right)}$$

$$\Rightarrow A_v = \frac{1}{1 - j\left(\frac{f_1}{f}\right)} = \underbrace{\frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}}_{\text{magnitude of } A_v} \underbrace{\angle \tan^{-1}(f_1 / f)}_{\text{phase by which } V_o \text{ leads } V_i} = -10\log\left[1 + \left(\frac{f_1}{f}\right)^2\right] dB$$

- For frequencies where $f \ll f_1$ or $(f_1/f)^2 \gg 1$, the equation above can be approximated as

$$A_v = -10\log\left[\left(\frac{f_1}{f}\right)^2\right] dB = -20\log\left(\frac{f_1}{f}\right) dB$$

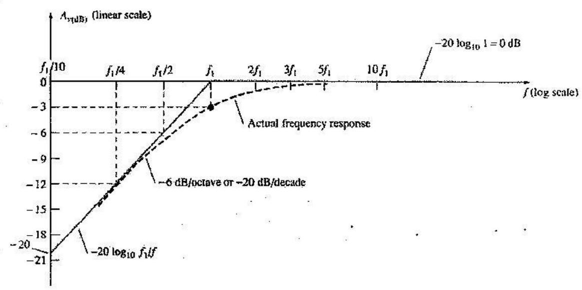
- Ignoring the previous condition for a moment, a plot on a frequency log scale will yield a result of a very useful nature for future decibel plots.

At $f = f_1 \Rightarrow -20\log(1) = 0dB$

At $f = \frac{f_1}{2} \Rightarrow -20\log(2) \cong -6dB$

At $f = \frac{f_1}{4} \Rightarrow -20\log(4) \cong -12dB$

At $f = \frac{f_1}{10} \Rightarrow -20\log(10) = -20dB$

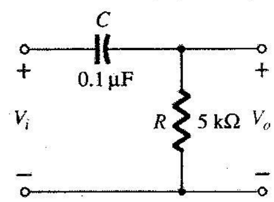


-A change in frequency by a factor of 2, equivalent to 1 octave, results in a 6-dB change in the ratio as noted by the change in gain from f1/2 to f1.

-For a 10:1 change in frequency, equivalent to 1 decade, there is a 20-dB change in the ratio as demonstrated between the frequencies of

Ex. For the given network:

(a) Determine the break frequency.
(b) Sketch the asymptotes and locate the -3-dB point.
(c) Sketch the frequency response curve.



Solution:

(a) $f_1 = \frac{1}{2\pi RC} = \frac{1}{2\pi(5\times10^3\Omega)(0.1\times10^{-6}F)} \cong 318.5Hz$

(b) See Figure below

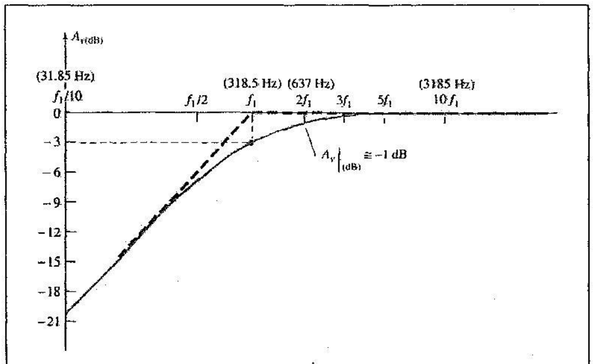
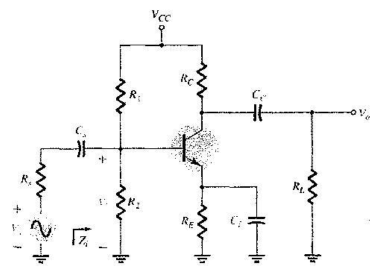


Fig. 11.14 Frequency response for the RC circuit of the Ex.

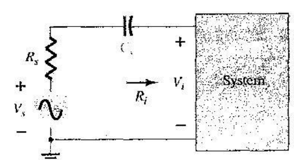
- *Low Frequency Analysis-BJT Amplifiers:*

- The analysis of this section will employ the loaded voltage-divider BJT bias configuration, but the results can be applied to any BJT configuration.
- It will simply be necessary to find the appropriate equivalent resistance for the *R-C* combination (<u>*for the capacitors C_S, C_C, and C_E, which will determine the low-frequency response*</u>).



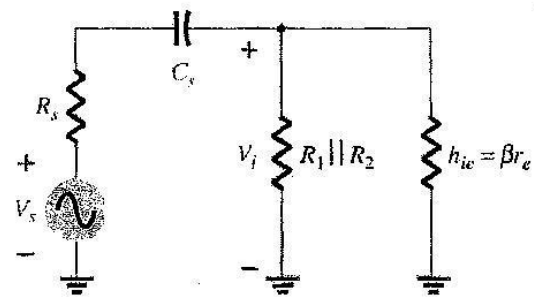
1) <u>*The effect of C_S:*</u>

- Since C_S is normally connected between the applied source and the active device, the total resistance is now $R_S + R_i$, and the cutoff frequency will be modified to be as:



$$f_{Ls} = \frac{1}{2\pi(R_s + R_i)C_s}$$

- At mid or high frequencies, the reactance of the capacitor will be sufficiently small to permit a short-circuit approximation for the element. The voltage V_i will then be related to V_s by



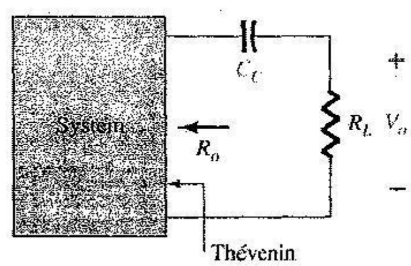
$$V_i|_{mid} = Vs\frac{Ri}{Ri + Rs}$$

- The voltage $\mathbf{V}_i$ applied to the input of the active device can be calculated using the voltage-divider rule:

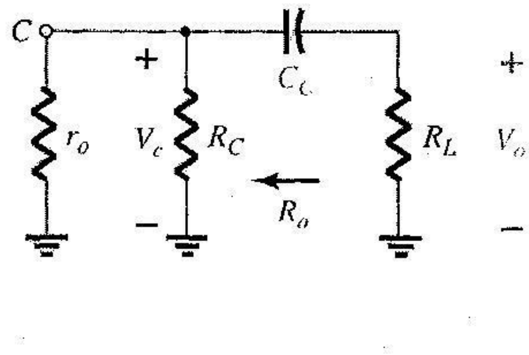
$$\mathbf{V}_i = \mathbf{V}_s \frac{R_i}{R_s + R_i - jX_{Cs}}$$

2) *The effect of C_C:*

- Since Cc the coupling capacitor is normally connected between the output of the active device and the applied load, the total resistance is now $R_o + R_L$, and the cutoff frequency will be modified to be as:

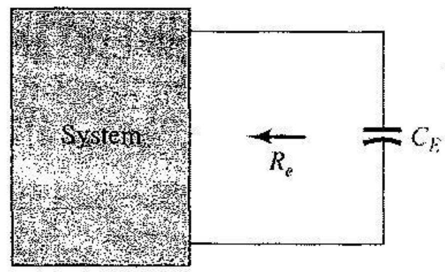


$$f_{LC} = \frac{1}{2\pi(R_o + R_L)C_C}$$



3) ***The effect of C_E:***

- To determine f_{LE}, the network "seen" by C_E must be determined as shown in the Fig. below. Once the level of R_e is established, the cutoff frequency due to C_E can be determined using the following equation:

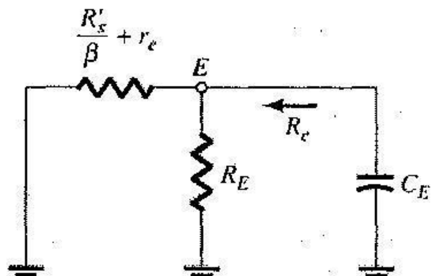


$$f_{LE} = \frac{1}{2\pi (R_e) C_E}$$

where R_e could be calculated as:

$$R_e = R_E \,//\left(\frac{R_S \,//\, R_1 \,//\, R_2}{\beta} + r_e\right)$$

where $R_S \,//\, R_1 \,//\, R_2 = R'_S$



Ex. (a) Determine the lower cutoff frequency for the voltage-divider BJT bias configuration network using the following parameters:
C_s =10μF, C_E =20μF, C_C =1μF, R_s =1kΩ, R_1 =40kΩ, R_2 =10kΩ, R_E =2kΩ, R_C =4kΩ, R_L = 2.2 kΩ, β= 100, r_o=∞, V_{CC} =20V.

(b) Sketch the frequency response using a Bode plot.

Solution:

(a) Determining r_e for dc conditions:

$$\beta R_E = (100)(2\text{ k}\Omega) = 200\text{ k}\Omega \gg 10R_2 = 100\text{ k}\Omega$$

The result is:

$$V_B \cong \frac{R_2 V_{CC}}{R_2 + R_1} = \frac{10\text{ k}\Omega(20\text{ V})}{10\text{ k}\Omega + 40\text{ k}\Omega} = \frac{200\text{ V}}{50} = 4\text{ V}$$

with

$$I_E = \frac{V_E}{R_E} = \frac{4\text{ V} - 0.7\text{ V}}{2\text{ k}\Omega} = \frac{3.3\text{ V}}{2\text{ k}\Omega} = 1.65\text{ mA}$$

so that

$$r_e = \frac{26\text{ mV}}{1.65\text{ mA}} \cong 15.76\ \Omega$$

and

$$\beta r_e = 100(15.76\ \Omega) = 1576\ \Omega = 1.576\text{ k}\Omega$$

Midband Gain $A_v = \frac{V_o}{V_i} = \frac{-R_C \| R_L}{r_e} = -\frac{(4\text{ k}\Omega)\|(2.2\text{ k}\Omega)}{15.76\ \Omega} \cong -90$

The input impedance $Z_i = R_i = R_1 \| R_2 \| \beta r_e$

$$= 40\text{ k}\Omega \| 10\text{ k}\Omega \| 1.576\text{ k}\Omega$$

$$\cong 1.32\text{ k}\Omega$$

and

$$V_i = \frac{R_i V_s}{R_i + R_s}$$

or $\frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32\text{ k}\Omega}{1.32\text{ k}\Omega + 1\text{ k}\Omega} = 0.569$

so that $A_{V_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i}\frac{V_i}{V_s} = (-90)(0.569)$

$$= \mathbf{-51.21}$$

C_s

$$R_i = R_1 \| R_2 \| \beta r_e = 40\text{ k}\Omega \| 10\text{ k}\Omega \| 1.576\text{ k}\Omega \cong 1.32\text{ k}\Omega$$

$$f_{L_S} = \frac{1}{2\pi\ (R_s + R_i)C_s} = \frac{1}{(6.28)(1\text{ k}\Omega + 1.32\text{ k}\Omega)(10\ \mu\text{F})}$$

$$f_{L_S} \cong \mathbf{6.86\ Hz}$$

C_C

$$f_{L_C} = \frac{1}{2\pi(R_C + R_L)C_C}$$

$$= \frac{1}{(6.28)(4\text{ k}\Omega + 2.2\text{ k}\Omega)(1\ \mu\text{F})}$$

$$\cong \mathbf{25.68\ Hz}$$

C_E

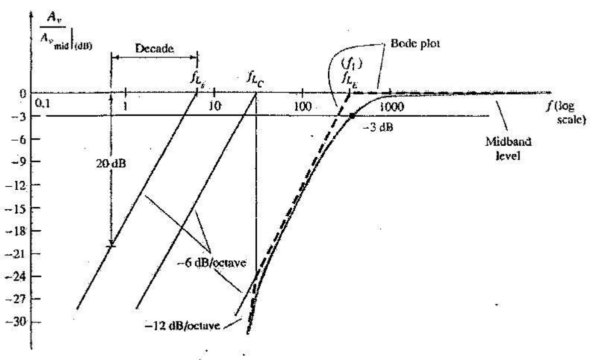
$$R_s' = R_s \| R_1 \| R_2 = 1\text{ k}\Omega \| 40\text{ k}\Omega \| 10\text{ k}\Omega \cong 0.889\text{ k}\Omega$$

$$R_e = R_E \left\| \left(\frac{R'_s}{\beta} + r_e\right) = 2\text{ k}\Omega \right\| \left(\frac{0.889\text{ k}\Omega}{100} + 15.76\ \Omega\right)$$

$$= 2\text{ k}\Omega \| (8.89\ \Omega + 15.76\ \Omega) = 2\text{ k}\Omega \| 24.65\ \Omega \cong 24.35\ \Omega$$

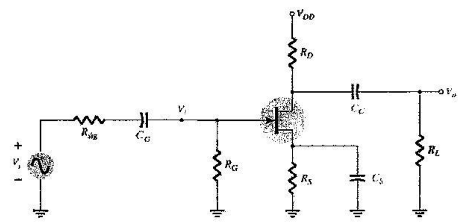
$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(24.35\ \Omega)(20\ \mu\text{F})} = \frac{10^6}{3058.36} \cong 327\text{ Hz}$$

(b)



- ***Low Frequency Analysis-FET Amplifiers:***

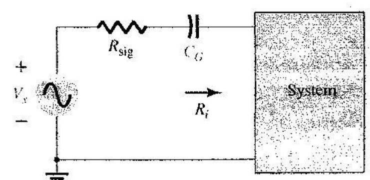
  - The analysis of the analysis of the FET amplifier in the low-frequency region will be quite similar to that of the BJT amplifier. There are again three capacitors of primary concern as appearing in the given network: *CG*, *Cc*, and *Cs*.



1) The effect of C_G:

The cutoff frequency determined by *CG* will then be

$$f_{LG} = \frac{1}{2\pi (R_G + R_{sig}) C_G}$$

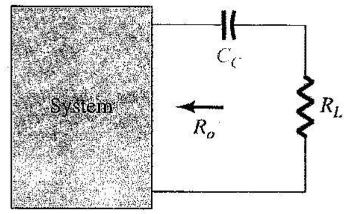


2) The effect of C_C:

The cutoff frequency determined by C_C will then be

$$f_{LC} = \frac{1}{2\pi(R_L + R_o)C_C}$$

where $R_o = R_D // r_d$



3) The effect of C_S:

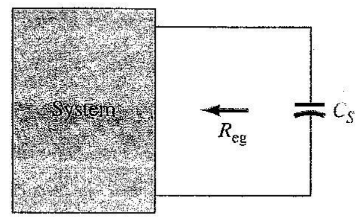
The cutoff frequency determined by C_S will then be

$$f_{LS} = \frac{1}{2\pi(R_{eg})C_S}$$

where $R_{eg} = \dfrac{R_S}{1 + \left[\dfrac{R_S(1 + g_m r_d)}{(r_d + R_G // R_L)}\right]}$

which for $r_d \cong \infty$ bcomes

$$R_{eg} = R_S // \left(\frac{1}{g_m}\right)$$



Ex. (a) Determine the lower cutoff frequency for the CS FET bias configuration network using the following parameters:
C_G =0.01μF, C_C =0.5μF, C_S =2μF, R_{sig} =10kΩ, R_G =1MΩ, R_D =4.7kΩ, R_S =1kΩ, R_L = 2.2 kΩ, I_{DSS}=8mA, V_P= -4V, r_d=∞, V_{DD} =20V.

(b) Sketch the frequency response using a Bode plot.

Solution:

(a) DC Analysis: Plotting the transfer curve of $I_D = I_{DSS}(1 - V_{GS}/V_P)^2$ and superimposing the curve defined by $V_{GS} = -I_D R_S$ will result in an intersection at $V_{GS_Q} = -2$ V and $I_{D_Q} = 2$ mA. In addition,

$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(8\text{ mA})}{4\text{ V}} = 4\text{ mS}$$

$$g_m = g_{m0}\left(1 - \frac{V_{GS_Q}}{V_P}\right) = 4\text{ mS}\left(1 - \frac{-2\text{ V}}{-4\text{ V}}\right) = 2\text{ mS}$$

C_G $\quad f_{L_G} = \dfrac{1}{2\pi\,(10\text{ k}\Omega + 1\text{ M}\Omega)(0.01\ \mu\text{F})} \cong 15.8\text{ Hz}$

C_C $\quad f_{L_C} = \dfrac{1}{2\pi\,(4.7\text{ k}\Omega + 2.2\text{ k}\Omega)(0.5\ \mu\text{F})} \cong 46.13\text{ Hz}$

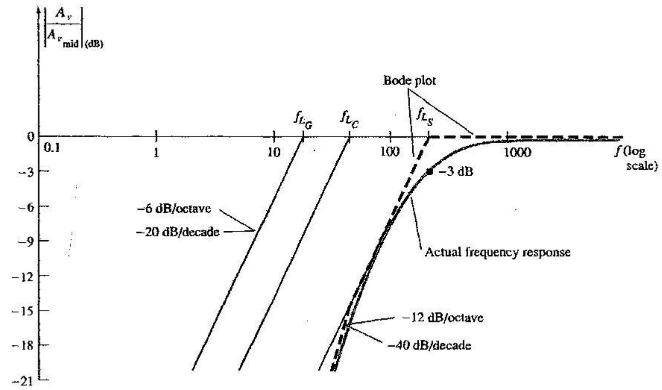
C_S $\quad R_{eq} = R_S \| \dfrac{1}{g_m} = 1\text{ k}\Omega \| \dfrac{1}{2\text{ mS}} = 1\text{ k}\Omega \| 0.5\text{ k}\Omega = 333.33\ \Omega$

$$f_{L_S} = \frac{1}{2\pi\,(333.33\ \Omega)(2\ \mu\text{F})} = 238.73\text{ Hz}$$

Since f_{L_S} is the largest of the three cutoff frequencies, it defines the low cutoff frequency for the network.

(b) The midband gain of the system is determined by

$$\begin{aligned} A_{v_{mid}} = \frac{V_o}{V_i} &= -g_m(R_D \| R_L) = -(2\text{ mS})(4.7\text{ k}\Omega \| 2.2\text{ k}\Omega) \\ &= -(2\text{ mS})(1.499\text{ k}\Omega) \\ &\cong -3 \end{aligned}$$



- ***Miller Effect capacitance:***

In the high-frequency region, the capacitive elements of importance are the interelectrode (between terminals) capacitances internal to the active device and the wiring capacitance between leads of the network. The large capacitors of the network that controlled the low-frequency response have all been replaced by their short-circuit equivalent due to their very low reactance levels.

- Miller input capacitance

$$C_{Mi} = (1 - A_v)C_f$$

where C_f is the feedback capacitance.

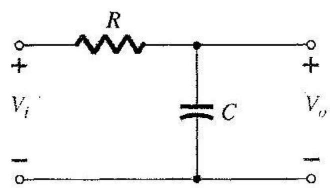
- Miller output capacitance

$$C_{Mo} = \left(1 - \frac{1}{A_v}\right)C_f \cong_{|A_v \gg 1} C_f$$

where C_f is the feedback capacitance.

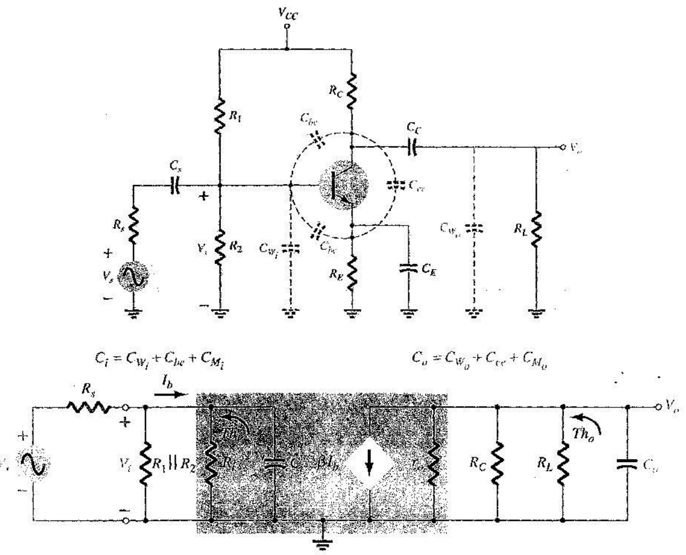
- ***High Frequency Analysis-BJT Amplifiers:***

In the high-frequency region, the *RC* network of concern has the configuration appearing in given Fig. At increasing frequencies, the reactance *Xc* will decrease in magnitude, resulting in a shorting effect across the output and a decrease in gain.



The derivation leading to the corner frequency for this *RC* configuration follows along similar lines to that encountered for the low-frequency region. The most significant difference is in the general form of A_v appearing below:

$$A_v = \frac{1}{1 + j\left(\frac{f}{f_2}\right)}$$



- In the above Fig., the various parasitic capacitances (C_{be}, C_{bc}, C_{ce}) of the transistor have been included with the wiring capacitances (C_{Wi}, C_{Wo}) introduced during construction.

- In the high-frequency equivalent model for the network, note the absence of the capacitors C_S, C_C, and C_E, which are all assumed to be in the short-circuit state at these frequencies.

- The capacitance C_i includes the input wiring capacitance C_{Wi}, the transition capacitance C_{be}, and the Miller capacitance C_{Mi}.

- The capacitance C_o includes the output wiring capacitance C_{Wo}, the parasitic capacitance C_{ce}, and the output Miller capacitance C_{Mo}.

- In general, the capacitance C_{be} is the largest of the parasitic capacitances, with C_{ce} the smallest. In fact, most specification sheets simply provide the levels of C_{be} and C_{bc} and do not include C_{ce} unless it will affect the response of a particular type of transistor in a specific area of application.

1) *For the input network, C_i:*

For the input network, the -3-dB frequency is defined by

$$f_{Hi} = \frac{1}{2\pi R_{Thi} C_i}$$

where

$$\boxed{R_{Th_1} = R_s \| R_1 \| R_2 \| R_i}$$

$$\boxed{C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc}}$$

2) *For the output network, C_o:*

$$\boxed{f_{H_o} = \frac{1}{2\pi R_{Th_2} C_o}}$$

$$\boxed{R_{Th_2} = R_C \| R_L \| r_o}$$

$$\boxed{C_o = C_{W_o} + C_{ce} + C_{M_o}}$$

Ex. For the given network, with the following parameters:
C_s =10μF, C_E =20μF, C_C =1μF, R_s =1kΩ, R_1 =40kΩ, R_2 =10kΩ, R_E =2kΩ, R_C =4kΩ, R_L = 2.2 kΩ, β= 100, $r_o=\infty$, V_{CC} =20V.
with the addition of
C_{be} =36pF, C_{bc} =4pF, C_{ce} = 1pF, C_{Wi}=6pF, C_{Wo}=8pF

(a) Determine f_{Hi} and f_{Ho}.

(b) Sketch the total frequency response for the low- and high-frequency regions.

Solution:

(a) from previous example

$$R_i = 1.32\text{ k}\Omega, \qquad A_{v_{mid}}(\text{amplifier}) = -90$$

and

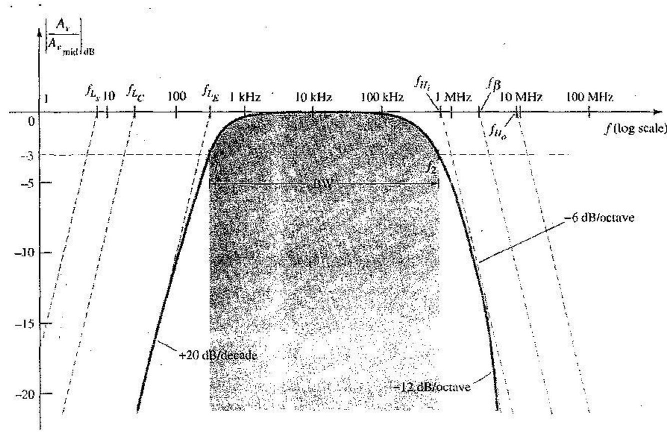
$$R_{Th_1} = R_s \| R_1 \| R_2 \| R_i = 1\text{ k}\Omega \| 40\text{ k}\Omega \| 10\text{ k}\Omega \| 1.32\text{ k}\Omega$$
$$\cong 0.531\text{ k}\Omega$$

with
$$C_i = C_{W_i} + C_{be} + (1 - A_v)C_{bc}$$
$$= 6\text{ pF} + 36\text{ pF} + [1 - (-90)]4\text{ pF}$$
$$= 406\text{ pF}$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_1} C_i} = \frac{1}{2\pi(0.531\text{ k}\Omega)(406\text{ pF})}$$
$$= \mathbf{738.24\ kHz}$$
$$R_{Th_2} = R_C \| R_L = 4\text{ k}\Omega \| 2.2\text{ k}\Omega = 1.419\text{ k}\Omega$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o} = 8\text{ pF} + 1\text{ pF} + \left(1 - \frac{1}{-90}\right) 4\text{ pF}$$
$$= 13.04\text{ pF}$$
$$f_{H_o} = \frac{1}{2\pi R_{Th_2} C_o} = \frac{1}{2\pi(1.419\text{ k}\Omega)(13.04\text{ pF})}$$
$$= \mathbf{8.6\ MHz}$$

(b)

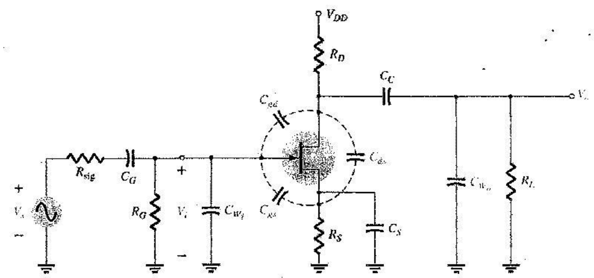


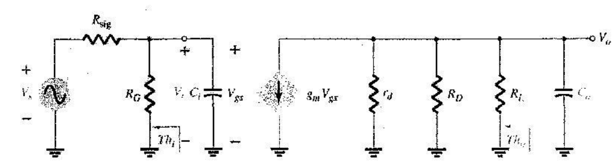
- *High Frequency Analysis-FET Amplifiers:*

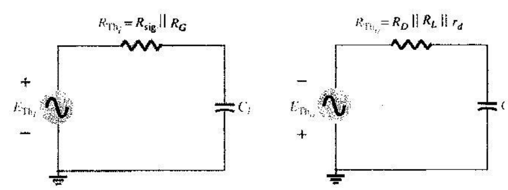
- The analysis of the high-frequency response of the FET amplifier will proceed in a very similar manner to that encountered for the BJT amplifier. There are interelectrode and wiring capacitances that will determine the high-frequency characteristics of the amplifier.

- The capacitors C_{gs} and C_{gd} typically vary from 1 to 10 pF, while the capacitance C_{ds} is usually quite a bit smaller, ranging from 0.1 to1 pF.

- At high frequencies, C_i will approach a short-circuit equivalent and V_{gs} will drop in value and reduce the overall gain. At frequencies where C_o approaches its short circuit equivalent, the parallel output voltage V_o will drop in magnitude.







$$\boxed{f_{H_i} = \frac{1}{2\pi R_{Th_1} C_i}}$$

and $$\boxed{R_{Th_1} = R_{sig} \| R_G}$$

with $$\boxed{C_i = C_{W_i} + C_{gs} + C_{M_i}}$$

and $$\boxed{C_{M_i} = (1 - A_v) C_{gd}}$$

and for the output circuit,

$$\boxed{f_{H_o} = \frac{1}{2\pi R_{Th_2} C_o}}$$

with $$\boxed{R_{Th_2} = R_D \| R_L \| r_d}$$

and $$\boxed{C_o = C_{W_o} + C_{ds} + C_{M_o}}$$

and $$\boxed{C_{M_o} = \left(1 - \frac{1}{A_V}\right) C_{gd}}$$

Ex. (a) Determine the high cutoff frequencies for the CS FET bias configuration network using the following parameters:
C_G =0.01µF, C_C =0.5µF, C_S =2µF, R_{sig} =10kΩ, R_G =1MΩ, R_D =4.7kΩ, R_S =1kΩ, R_L = 2.2 kΩ, I_{DSS}=8mA, V_P= -4V, r_d=∞, V_{DD} =20V.
C_{gd} = 2pF, C_{gs} =4pF, C_{ds} =0.5pF, C_{Wi} =5pF, C_{Wo} =6pF

Solution:

(a) From the previous example

$R_{Th_1} = R_{sig} \| R_G = 10\text{ k}\Omega \| 1\text{ M}\Omega = 9.9\text{ k}\Omega$
$A_v = -3.$

$$C_i = C_{W_i} + C_{gs} + (1 - A_v)C_{gd}$$
$$= 5\text{ pF} + 4\text{ pF} + (1 + 3)2\text{ pF}$$
$$= 9\text{ pF} + 8\text{ pF}$$
$$= 17\text{ pF}$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_1} C_i}$$

$$= \frac{1}{2\pi(9.9\text{ k}\Omega)(17\text{ pF})} = \mathbf{945.67\text{ kHz}}$$

$$R_{Th_2} = R_D \| R_L$$
$$= 4.7\text{ k}\Omega \| 2.2\text{ k}\Omega$$
$$\cong 1.5\text{ k}\Omega$$

$$C_o = C_{W_o} + C_{ds} + C_{M_o} = 6\text{ pF} + 0.5\text{ pF} + \left(1 - \frac{1}{-3}\right)2\text{ pF} = 9.17\text{ pF}$$

$$f_{H_o} = \frac{1}{2\pi(1.5\text{ k}\Omega)(9.17\text{ pF})} = 11.57\text{ MHz}$$

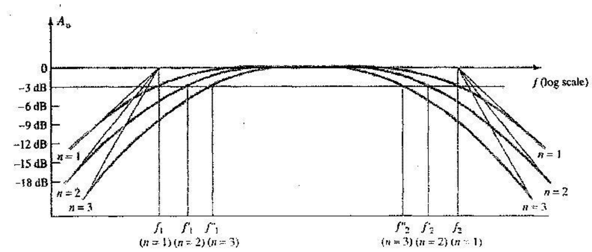
- ***Multistage Frequency Effect:***

  - **For Low Frequency region**

$$f_1' = \frac{f_1}{\sqrt{2^{\left(\frac{1}{n}\right)} - 1}}$$

  - **For High Frequency region**

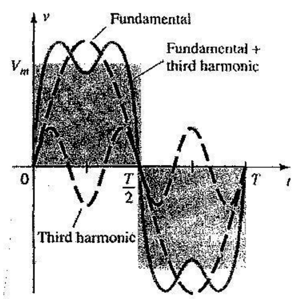
$$f_2' = \left(\sqrt{2^{\left(\frac{1}{n}\right)} - 1}\right) f_2$$

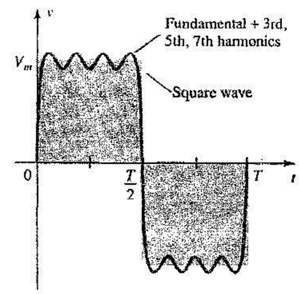


- ***Square wave Testing:***

- Experimentally, the sense for the frequency response can be determined by applying a square wave signal to the amplifier and noting the output response.
- The reason for choosing a square-wave signal for the testing process is best described by examining the Fourier series expansion of a square wave composed of a series of sinusoidal components of different magnitudes and frequencies. The summation of the terms of the series will result in the original waveform. In other words, even though a waveform may not be sinusoidal, it can be reproduced by a series of sinusoidal terms of different frequencies and magnitudes.

$$v = \frac{4}{\pi} V_m\left(\sin 2\pi f_s t + \frac{1}{3}\sin 2\pi(3f_s)t + \frac{1}{5}\sin 2\pi(5f_s)t + \frac{1}{7}\sin 2\pi(7f_s)t + \frac{1}{9}\sin 2\pi(9f_s)t + \cdots + \frac{1}{n}\sin 2\pi(nf_s)t\right)$$





- Since the ninth harmonic has a magnitude greater than 10% of the fundamental term, the fundamental term through the ninth harmonic are the major contributors to the Fourier series expansion of the square-wave function.

<u>*Ex. For a specific application (Audio amplifier with 20kHz Bandwidth), what is the maximum frequency could be amplified?*</u>

$$\frac{20\text{kHz}}{9} = 2.2\text{kHz}$$

- If the response of an amplifier to an applied square wave is an undistorted replica of the input, the frequency response (or BW) of the amplifier is obviously sufficient for the applied frequency.

- If the response is as shown in Fig. 11.60a and b, the low frequencies are not being amplified properly and the low cutoff frequency has to be investigated.

- If the waveform has the appearance of Fig. 11.60c, the high-frequency components are not receiving sufficient amplification and the high cutoff frequency (or BW) has to be reviewed.

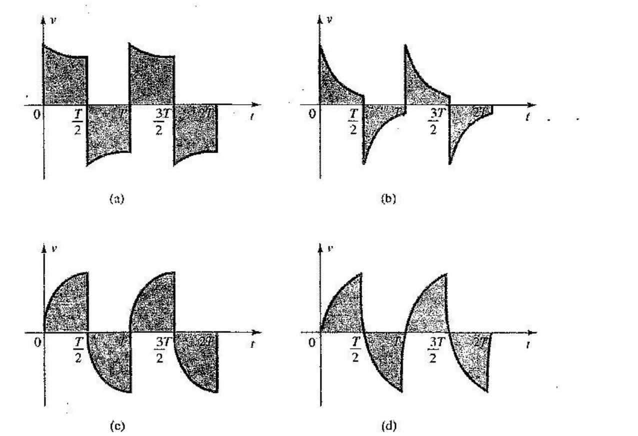
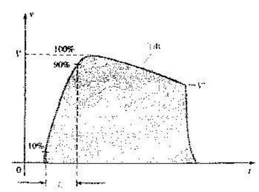


Fig. 11.60 (a) Poor low-frequency response; (b) very poor low-frequency response; (c) poor high-frequency response; (d) very poor high-frequency response.

- The actual high cutoff frequency (or BW) can be determined from the output waveform by carefully measuring the rise time defined between 10% and 90% of the peak value, as shown in the Fig. below.



- Substituting into the following equation will provide the upper cutoff frequency, and since BW = f_{Hi}- $f_{Lo} \cong f_{Hi}$, the equation also provides an indication of the BW of the amplifier.

$$BW \cong f_{Hi} = \frac{0.35}{t_r}$$

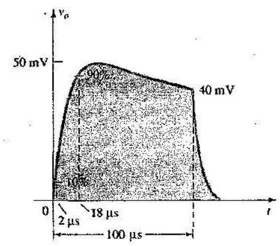
- The low cutoff frequency can be determined from the output response by carefully measuring the tilt and substituting into one of the following equations:

$$\%\text{tilt} = P\% = \frac{V - V'}{V} \times 100\%$$

$$= P = \frac{V - V'}{V} \quad \text{(decimal form)}$$

- The low cutoff frequency is then determined from

$$f_{Lo} = \frac{P}{\pi} f_s$$



Ex. The application of a 1-mV, 5-kHz square wave to an amplifier resulted in the output waveform of the given Fig.

(a) Write the Fourier series expansion for the square wave through the ninth harmonic.

(b) Determine the bandwidth of the amplifier.

Solution:

(a) $$v_i = \frac{4\text{ mV}}{\pi}\left(\sin 2\pi\ (5 \times 10^3)t + \frac{1}{3}\sin 2\pi(15 \times 10^3)t + \frac{1}{5}\sin 2\pi(25 \times 10^3)t + \frac{1}{7}\sin 2\pi(35\times10^3)t + \frac{1}{9}\sin 2\pi(45 \times 10^3)t\right)$$

(b) $t_r = 18\ \mu s - 2\ \mu s = 16\ \mu s$

$$BW = \frac{0.35}{t_r} = \frac{0.35}{16\ \mu s} = 21{,}875\text{ Hz} \cong 4.4f_s$$

(c) $$P = \frac{V - V'}{V} = \frac{50\text{ mV} - 40\text{ mV}}{50\text{ mV}} = 0.2$$

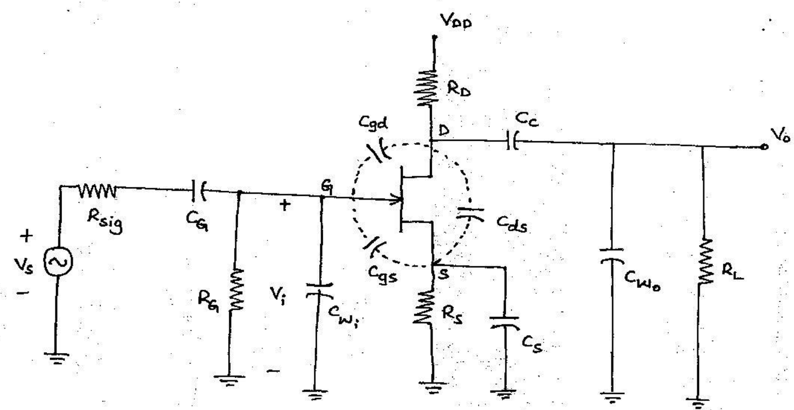
$$f_{Lo} = \frac{P}{\pi} f_s = \left(\frac{0.2}{\pi}\right)(5\text{ kHz}) = 318.31\text{ Hz}$$

High-Frequency Response – FET Amplifier

→ The analysis of the high freq. response of the FET amplifier will proceed in a similar manner to that encountered for the BJT amplifier.

→ There are electrode and wiring capacitances that will determine the high frequency characteristics of the amplifier.

→ The capacitors C_{gs} and C_{gd} typically vary from 1PF to 10 PF, while the capacitance C_{ds} is usually quite a bit smaller, ranging from 0.1 PF to 1 PF.



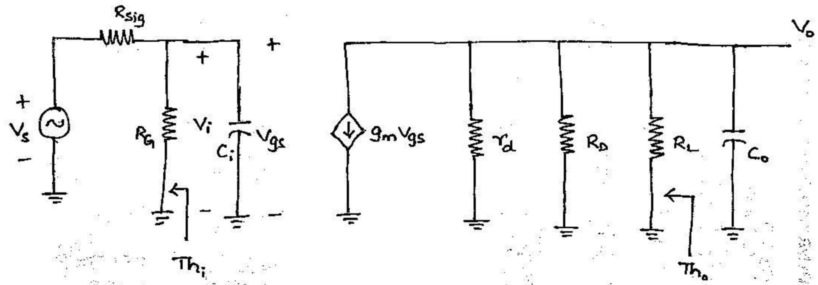
(a) Capacitive Elements that affect the High-frequency Response of a JFET amplifier

→ The network of Fig. (a) is an inverting amplifier

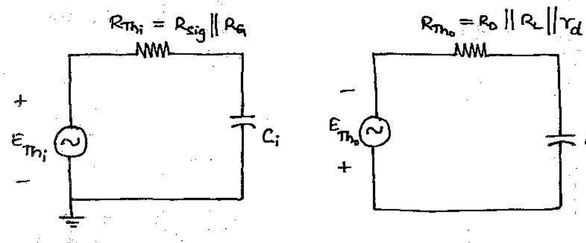
→ A Miller effect capacitance will appear in the high freq. ac equivalent network shown in Fig. (b)

→ At high frequencies, C_i will approach a short-circuit equivalent and V_{gs} will drop in value and reduces the overall gain.

→ At frequencies where C_o approaches its short circuit equivalent, the parallel output voltage V_o will drop in magnitude.



(b) High Frequency ac equivalent circuit for Fig. (a)



(c) Input circuit

(d) output circuit

Thevenin Equivalent circuits

→ The cutoff freq. defined by the input and output circuits can be obtained by first finding the Thevenin equivalent circuits shown in Fig. (c) and (d)

(i) For the input circuit, $\boxed{f_{H_i} = \dfrac{1}{2\pi R_{Th_i} C_i}}$

and $\boxed{R_{Th_i} = R_{sig} \| R_G}$

$\boxed{C_i = C_{W_i} + C_{gs} + C_{M_i}}$

and $\boxed{C_{M_i} = (1 - A_v)\, C_{gd}}$

(ii) For the output circuit,

$$\boxed{f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o}}$$

$$\boxed{R_{Th_o} = R_D \| R_L \| r_d}$$

$$\boxed{C_o = C_{W_o} + C_{ds} + C_{M_o}}$$

$$\boxed{C_{M_o} = \left(1 - \frac{1}{A_v}\right) C_{gd}}$$

Example 1 : Determine the high-cutoff frequencies for the network shown in Fig. (a) using the following parameters.

$C_G = 0.01\ \mu F$, $C_C = 0.5\ \mu F$, $C_S = 2\ \mu F$, $g_m = 2\ mS$

$R_{sig} = 10\,K\Omega$, $R_G = 1\,M\Omega$, $R_D = 4.7\,K\Omega$, $R_S = 1\,K\Omega$,

$R_L = 2.2\,K\Omega$, $I_{DSS} = 8\ mA$, $V_P = -4V$, $r_d = \infty\,\Omega$

$V_{DD} = 20V$ with the addition of

$C_{gd} = 2\,PF$, $C_{gs} = 4\ PF$, $C_{ds} = 0.5\ PF$, $C_{W_i} = 5\,PF$, $C_{W_o} = 6\,PF$

Solution :

$$R_{Th_i} = R_{sig} \| R_G = 10\,K\Omega \| 1M\Omega = 9.9\,K\Omega$$

$$A_V = -g_m (R_D \| R_L) = -2\,ms\,(4.7\,K\Omega \| 2.2\,K\Omega)$$

$$A_V = -3$$

$$C_i = C_{W_i} + C_{gs} + (1 - A_V)\, C_{gd}$$

$$= 5\,PF + 4\,PF + (1+3)\,2\,PF$$

$$C_i = 17\ PF$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} = \frac{1}{2\pi \times 9.9\,K\Omega \times 17 \times 10^{-12}\,F} = 945.67\,K

$$R_{Th_o} = R_D \parallel R_L$$

$$R_{Th_o} = 4.7k\Omega \parallel 2.2k\Omega \cong 1.5k\Omega$$

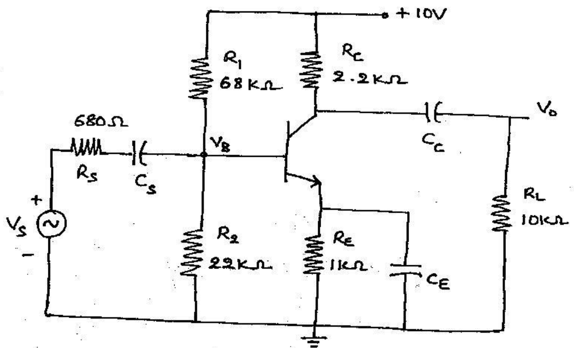
$$C_o = C_{W_o} + C_{ds} + C_{M_o} = 6\,pF + 0.5\,pF + \left(1 - \frac{1}{-3}\right) 2\,pF$$

$$C_o = 9.17\,pF$$

$$f_{H_o} = \frac{1}{2\pi (1.5k\Omega)(9.17\,pF)} = 11.57\ MHz$$

The results shows that the input capacitance with its Miller effect capacitance will determine the upper cutoff frequency.

Ⓟ Determine the high frequency response of the amplifier circuit shown below. Draw the freq. response curve.



$\beta = 100$

$C_{be} = 20$ PF

$C_{bc} = 4$ PF

$h_{ie} = 1100$

$C_{W_i} = 6$ PF

$C_{W_o} = 8$ PF

$C_C = 1$ PF

Solution :

$$V_B = \frac{V_{CC} R_2}{R_1 + R_2} = \frac{22 \times 10}{68 + 22} = 2.44V \Rightarrow \boxed{V_B = 2.44V}$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2} = \frac{22 \times 68}{22 + 68} = 16.62\,K\Omega \Rightarrow \boxed{R_B = 16.62\,K\Omega}$$

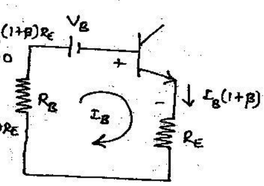
$$I_B = \frac{V_B - V_{BE}}{R_B + (1+\beta) R_E}$$

$$I_B = \frac{2.44V - 0.7V}{16.62 + (1+100)\,1K\Omega}$$

$$\boxed{I_B = 14.79\,\mu A}$$

$-V_B + V_{BE} + I_B(1+\beta)R_E + I_B R_B = 0$

$\Rightarrow I_B = \frac{V_B - V_{BE}}{R_B + (1+\beta)R_E}$



$$I_E = \beta I_B = 100 \times 14.79\,\mu A = 1.479\,mA$$

$$\boxed{I_E = 1.479\,mA}$$

$I_E = I_C + I_B$

$= \beta I_B + I_B$

$= (1+\beta) I_B$

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{1.479\,mA} = 17.58\,\Omega$$

$$\boxed{r_e = 17.58\,\Omega}$$

$$A_{V_{mid}} = \frac{-(R_L \| R_C)}{r_e} = -\frac{2.2K \| 10K}{17.58\,\Omega} = -102.58$$

$$\boxed{A_{V_{mid}} = -102.58}$$

$$R_i = R_1 \| R_2 \| \beta r_e = 68K \| 22K \| (100 \times 17.58)$$

$$\boxed{R_i = 1.59\,K\Omega}$$

$$R_{Th_i} = R_s \| R_i = 680\Omega \| 1590\Omega = 476.3\,\Omega$$

$$\Rightarrow \boxed{R_{Th_i} = 476.3\,\Omega}$$

$$C_i = C_{W_i} + C_{be} + (1 - A_{V_{mid}})\,C_{bc}$$

$$= 6\,PF + 20\,PF + [1 - (-102.58)] \times 4\,PF$$

$$\boxed{C_i = 440.32\ PF}$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} = \frac{1}{2\pi \times 476.3\Omega \times 440.32\ PF}$$

$$\boxed{f_{H_i} = 758.88\ KHz}$$

$$R_{Th_o} = R_c \| R_L = 2.2K\Omega \| 10K\Omega = 1.8K\Omega$$

$$\boxed{R_{Th_o} = 1.8\,K\Omega}$$

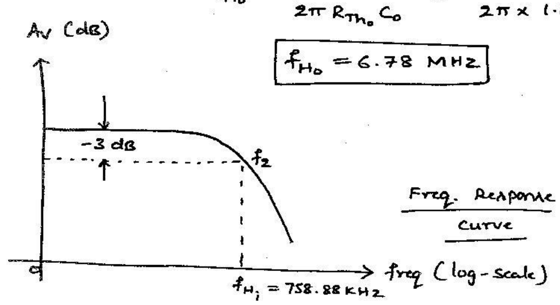
$$C_o = C_{W_o} + C_{ce} + \underbrace{C_{M_o}}_{\left(1 - \frac{1}{A_{V_{mid}}}\right) C_{bc}}$$

$$C_o = 8\ PF + 1\ PF + \left(1 - \frac{1}{-102.58}\right) \times 4\ PF$$

$$\boxed{C_o = 13.039\ PF}$$

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o} = \frac{1}{2\pi \times 1.8K \times 13.039\ PF} = 6.78\,MHz$$

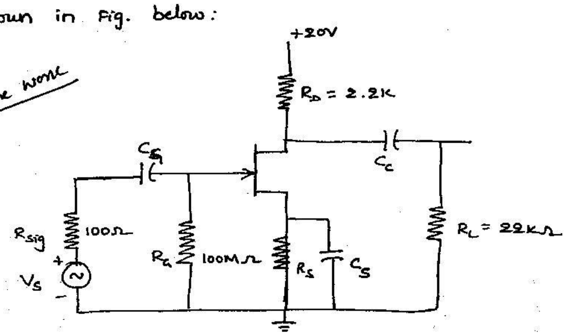
$$\boxed{f_{H_o} = 6.78\ MHz}$$



Freq. Response curve

(P) Determine the high freq. response of the amplifier circuit shown in Fig. below:

Home work



$V_{GS} = -8V$

$I_{DSS} = 80\ nA$

$g_m = 6\ ms$

$r_d = \infty$

$C_{gd} = 2\ PF$

$C_{gs} = 4\ PF$

$C_{wi} = 0$

$C_{wo} = 0$

$C_{ds} = 0$

Solution: $f_{Hi} = 53\ MHz$

$f_{Ho} = 36.74\ MHz$

(P) An amplifier consists of three identical stages in cascade, the bandwidth of overall amplifier extends from 20 Hz to 20 KHz. calculate the bandwidth of individual stage.

Solution: (i) $f_1' = \dfrac{f_1}{\sqrt{2^{1/n}-1}}$

$\underline{\underline{f_1}} = f_1'\left(\sqrt{2^{1/n}-1}\right) = 20\sqrt{2^{1/3}-1} = 10.196\ Hz$

$\rightarrow$ lower 3-dB freq. of single stage.

(ii) $f_2' = f_2\left(\sqrt{2^{1/n}-1}\right)$

$f_2 = \dfrac{f_2'}{\sqrt{2^{1/n}-1}} = \dfrac{20\times10^3}{\sqrt{2^{1/3}-1}} = 39.23\ KHz$

$\rightarrow$ upper 3dB freq. of single stage amplifier

(iii) Bandwidth $= f_2 - f_1 = 39.23\times10^3 - 10.196\ Hz$

$\boxed{BW = 39.218\ KHz}$

(P) calculate the overall lower 3-dB and upper 3-dB frequencies for a three stage amplifier having an individual $f_1 = 40\ Hz$ and $f_2 = 2\ MHz$.

Solution: overall lower 3-dB freq.: f_1

$$f_1' = \frac{f_1}{\sqrt{2^{1/n}-1}} = \frac{40}{\sqrt{2^{1/3}-1}} = 78.458 \text{ Hz}$$

overall higher 3-dB freq.: f_2'

$$f_2' = f_2\left(\sqrt{2^{1/n}-1}\right) = 2\times10^6\left(\sqrt{2^{1/3}-1}\right)$$

$$\boxed{f_2' = 1.0196 \text{ MHz}}$$

(P) A four stage amplifier has a lower 3-dB freq. for an individual stage of $f_1 = 40$ Hz and individual upper 3-dB frequency of $f_2 = 2.5$ MHz. calculate the overall lower 3-dB and upper 3-dB freq. of this full amplifier.

Solution: $f_1' = \frac{f_1}{\sqrt{2^{1/n}-1}} = \frac{40}{\sqrt{2^{1/4}-1}} = 91.95$ Hz

$$f_2' = f_2\sqrt{2^{1/n}-1} = 2.5\times10^6\times\sqrt{2^{1/4}-1} = 1.087 \text{ MHz}$$

(P) A two stage cascaded amplifier system is built with a stage voltage gain 25 and 40. Both stages have the same BW of 220 KHz. with identical lower cutoff freq. of 500 Hz. Find the overall gain band-width product.

Solution: The overall voltage gain $= A_V = A_{V_1}\times A_{V_2} = 25\times40$

$$\boxed{A_V = 1000}$$

For each stage: $f_1 = 500$ Hz, $f_2 = ?$, $BW = 220$ KHz

$$BW = f_2 - f_1 \Rightarrow 220 \text{ KHz} = f_2 - 500 \text{ Hz}$$

$$\boxed{f_2 = 219.5 \text{ KHz}}$$

overall lower 3-dB freq. $= f_1' = \frac{f_1}{\sqrt{2^{1/n}-1}} = \frac{500}{\sqrt{2^{1/2}-1}}$

$$\boxed{f_1' = 776.88 \text{ Hz}}$$

overall upper 3-dB freq. $= f_2' = f_2\left(\sqrt{2^{1/n}-1}\right)$

$$f_2' = 219.5\sqrt{2^{1/2}-1} = 141.59 \text{ KHz}$$

overall $BW = f_2' - f_1' = 141.59 \text{ KHz} - 776.88 \text{ Hz}$

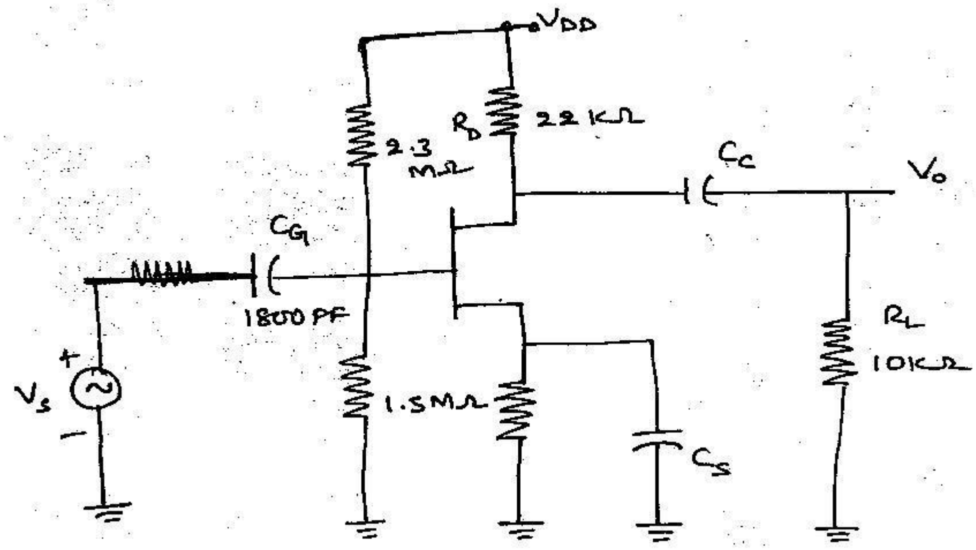
$$= 140.815 \text{ KHz}$$

overall gain bandwidth product = ...

$= 1000 \times 140.815 \text{ KHz}$

$= 140.815 \times 10^{6}$

(P) For the circuit shown in Fig. find cutoff frequencies due to C_G, C_S, and C_E and due to interelectrode capacitance, C_{gs} and C_{gd}. Given: $g_m = 0.49$ mA/V, $C_{gd} = 9.38$ PF, $C_{gs} = 1.8$ PF, $r_d = \infty\ \Omega$, $C_{ds} = C_{w_i} = C_{w_o} = 0$



Solution: $f_{L_G} = 97.378$ Hz

$A_v = -g_m (R_D \| R_L)$

$A_v = -3.369$

$f_{L_C} = 0.33$ Hz

$f_{L_S} = 1.629$ Hz

$C_{in} = C_{gd}(1 - A_v)$

$C_{in} = 40.98$ PF

$C_{out} = 12.16$ PF

$f_{out} = 1.904$ MHz

Module 3 BJT and JFET Frequency Response

Introduction

Exam Marks : 32 (with choice)
<i.e two questions, out of which student has to answer one>

→ So far,

The analysis has been limited to the particular frequency.

→ The frequency effects introduced by the larger capacitive elements of the network at low frequencies and the smaller capacitive elements of the active device at higher frequencies are now investigated.

i.e Larger capacitive elements (C/L) : Low freq. effects

Smaller capacitive elements $\underbrace{\text{(BJT / JFET)}}_{\text{active device}}$: High freq. effects.

→ The logarithmic scale will be defined and used throughout the analysis, since the analysis will extend through a wide freq. range.

→ The industry uses a decibel scale on its freq. plots. Therefore the concept of the decibel is introduced.

→ The freq. response analysis of both BJT and FETs are covered in the same chapter.

3 LOGARITHMS

→ In this chapter, there is a need to become comfortable with the logarithmic function.

→ The plotting of a variable between wide limits, comparing levels and dealing with unwieldy numbers are impractical.

→ The positive features of the logarithm function are:

- Comparing levels without dealing with unwieldy numbers
- Identifying levels of particular importance in the design
- Ease of analysis and review procedure.

→ To understand the relationship between the variables of a logarithm function, consider the following mathematical equations:

$$\boxed{\begin{array}{c} a = b^x \\ \hline x = \log_b a \end{array}} \quad \text{——— (1)}$$

Variables a, b, and x are same in each eqn.

$a = b^x$: a is determined by taking the base b to the x power

$x = \log_b a$: The x will result if the log of a is taken to the base b

Example: If $b = 10$ and $x = 2$, than

$$a = b^x = (10)^2 = 100$$

$$x = \log_b a = \log_{10} 100 = 2$$

To find the power of a number : 10,000, then

$$10{,}000 = 10^{x}$$

The level of x is determined using logarithms

i.e $x = \log_{10} 10{,}000 = 4$

$\Rightarrow x = 4$

Note: For electrical/electronic industry/scientific research, the base in the logarithm equation is chosen as either 10 or the number $e = 2.71828....$

Logarithm taken to the base 10 : referred as "Common Logarithm"

Logarithms taken to the base e : referred as "Natural Logarithms"

$$\boxed{\text{Common logarithm : } x = \log_{10} a} \quad \text{—— (2)}$$

$$\boxed{\text{Natural logarithm : } y = \log_{e} a} \quad \text{—— (3)}$$

The common and natural logarithm are related by

$$\boxed{\log_e a = 2.3 \log_{10} a} \quad \text{—— (4)}$$

In Scientific calculators use the following key :

Common logarithm : $\boxed{\text{Log}}$

Natural logarithm : $\boxed{\text{Ln}}$

Example (1)

Using the calculator, determine the logarithm of the following numbers to the base indicated:

(a) $\log_{10} 10^{6} = 6$

(b) $\log_{e} e^{3} = 3$

(c) $\log_{10} 10^{-2} = -2$

The logarithm of a number taken to a power is simply the power of the number, if the number matches the base of the logarithm

→ The logarithm of a number does not increase in the same linear fashion as the number

<u>Example:</u> $8000 = 125 \times 64$

The number 8000 is 125 times larger than 64

But, $\left.\begin{aligned} \log 8000 &= 3.903 \\ \log 64 &= 1.806 \end{aligned}\right\}$ $\log 8000 = 2.16 \times \log 64$

$\Rightarrow 3.903 = 2.16 \times 1.806$ "Non-linear relationship

$\log_{10} 10^0 = 0$

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1{,}000 = 3$

$\log_{10} 10{,}000 = 4$

$\log_{10} 100{,}000 = 5$

$\log_{10} 1{,}000{,}000 = 6$

$\log_{10} 10{,}000{,}000 = 7$

$\log_{10} 100{,}000{,}000 = 8$

This shows that logarithm of a number increases only as the exponent of a number

<u>Note:</u> The antilogarithm of a number is obtained by the func 10^x <u>or</u> e^x

§ <u>Properties of Logarithms</u> : A review

This chapter employs the common Logarithm. Therefore, we review few properties of common logarithms. The same relationship hold true for logarithms to any base.

→ $\log_{10} 1 = 0$, because $10^0 = 1$

→ $\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b$

for special case, $a = 1$ becomes

$$\log_{10} \frac{1}{b} = \underbrace{\log_{10} 1}_{=0} - \log_{10} b = -\log_{10} b$$

⇒ For any b greater than 1, the logarithm of a number less than 1 is always negative

→ $\log_{10} ab = \log_{10} a + \log_{10} b$

<u>Example</u>: Determine the logarithm of the following numbers:

(a) $\log_{10} 0.5$ (b) $\log_{10} \frac{4000}{250}$ (c) $\log_{10} (0.6 \times 30)$

Home work

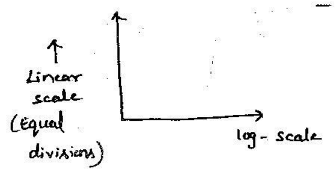
→ The use of log scales can significantly expand the range of variab of a particular variable on graph.

→ Most available graph paper:

1. Semi log (one half): one of the two scales is a lo
2. Double log (log-log): Both scales are log scales

→ <u>Semilog Scale</u>: <use semi log graph>

- Vertical scale: Linear scale with equal divisions



- observe the spacingt between the lines of a lo plot on the graph.

- $\left.\begin{array}{l}\log_{10} 2 \cong 0.3 \\ \log_{10} 1 = 0\end{array}\right\}$ ⇒ The distance from 1 t is 30% of the span

- $\left.\begin{array}{l}\log_{10} 3 = 0.4771 \\ \log_{10} 1 = 0\end{array}\right\}$ ⇒ The distance from 1 to is 48% of the span
  ⇒ Close to $\frac{1}{2}$ of distance

- 11^{ly} the $\log_{10} 4 = 0.6021$ ($\cong 60\%$)
  ⇒ The distance from 1 to 4 is 60% of the span.

- $\log_{10} 5 \cong 0.7$
  ⇒ 70% of the distance

→ Between any two digits, the same compression of the lines appears as we progress from left to right

→ The resulting numerical value and the Spacing is important to note, because the plot will typically have the tic marks, due to lack of Space

~~→ Due to lack of Space~~

→ The longer bars for this figure have the numerical values of 0.3, 3 and 30 associate with them. The next shorter bars have values of 0.5, 5, and 50 and the shortest bars 0.7, 7, and 70

5 DECIBELS

→ The concept of decibels in increasingly important in the remaining sections.

→ The term decibel has its origin in the fact that power and audio levels are related in a logarithmic basis.

→ An increase in the power level ~~by~~ from 4W to 16W does not result in an audio level increase by a factor of $16/4 = 4$ but by a factor of 2, as derived from the power of 4 in the following manner. $4^2 = 16$

→ For a change of 4W to 64W, the audio level increases by a factor of 3, since $4^3 = 64$.

In logarithmic form, the relationship can be written as

$$\log_4 64 = 3$$

→ The term bel was derived from the surname of Alexander Graham Bell. For standardization, the bel (B) was defined by the following equation to relate power levels P_1 and P_2 :

$$\boxed{G = \log_{10}\left(\frac{P_2}{P_1}\right)} \quad \text{bel}$$

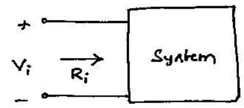
→ It was found, however, that the bel was too large a unit of measurement for practical purposes, so the decibel was defined such that 10 decibels = 1 bel. Therefore,

$$\boxed{G_{dB} = 10 \log_{10}\left(\frac{P_2}{P_1}\right)} \quad dB$$

→ There exist a second equation for decibels that is applied frequently. It can be best described through the system with R_i, the input resistance of the system.

$$G_{dB} = 10 \log_{10}\left(\frac{P_2}{P_1}\right) = 10 \log \frac{V_2^2/R_i}{V_1^2/R_i} = 10 \log_{10}\left(\frac{V_2}{V_1}\right)^2$$

$$\boxed{G_{dB} = 20 \log \left(\frac{V_2}{V_1}\right)} \quad dB$$



→ one of the advantages of the logarithmic relationship is the manner in which it can be applied to cascaded stages. In ~~other~~ words, the equation states that the decibel gain of a cascaded system is simply the sum of the decibel gains of each stage.

$$|A_{V_T}| = |A_{V_1}| \cdot |A_{V_2}| \cdot |A_{V_3}| \cdots\cdots |A_{V_n}|$$

Applying the proper logarithmic relationship results in

$$G_V = 20\log_{10}|A_{V_T}| = 20\log_{10}|A_{V_1}| + 20\log_{10}|A_{V_2}| + 20\log_{10}|A_{V_3}| + \cdots\cdots + 20\log_{10}|A_{V_n}|$$

In words, the equation is

$$G_{dB_T} = G_{dB_1} + G_{dB_2} + G_{dB_3} + \cdots\cdots + G_{dB_n}$$

→ The association between dB levels and voltage gains are as follows:

Voltage gain of 2 = dB level : +6 dB

10 = dB level : +20 dB

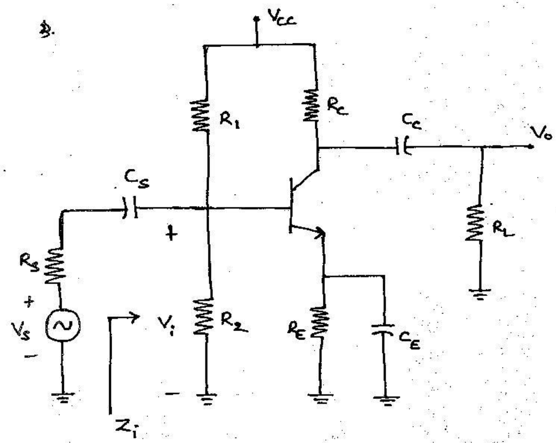
Voltage Gain $= \frac{V_o}{V_i} = A_V$	dB Level
0.5	−6 dB
0.707	−3 dB
1	0
2	6
10	20
40	32
100	40
1000	60

Comparing A_V to dB

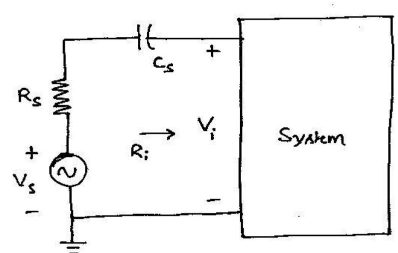
where $A_V = \frac{V_o}{V_i}$

→ The analysis of this section will employ the loaded voltage divid BJT bias configuration, but the results can be applied to any BJT configuration.

→ It will simply be necessary to find the appropriate equivalen resistance for the R-C combination. (for the capacitors C_s, C_c and C_E which will determine the low-frequency response.



(a) Loaded BJT Amplifier with capacitors that affect the low frequency response



(b) Determining the effect of C_s on the low frequency response

→ since C_s is normally connected between the applied source & the active device, the total resistance is now $R_s + R_i$

Applying Voltage divider rule :

$$V_i = \frac{R_i V_s}{R_s + R_i - jX_{C_s}}$$

The cutoff frequency defined by C_s can be determined by manipulating the above equation into a standard form

$$\frac{V_i}{V_s} = \frac{R_i}{R_s + R_i - jX_{C_s}}$$

$$\frac{V_i}{V_s} = \frac{1}{1 + \frac{R_s}{R_i} - j\frac{X_{C_s}}{R_i}}$$

$$= \frac{1}{\left(1 + \frac{R_s}{R_i}\right)\left[1 - j\frac{X_{C_s}}{R_i}\left(\frac{1}{1 + \frac{R_s}{R_i}}\right)\right]}$$

$$\frac{V_i}{V_s} = \frac{1}{\left(1 + \frac{R_s}{R_i}\right)\left(1 - j\frac{X_{C_s}}{R_i + R_s}\right)} \quad \text{—— (1)}$$

The factor,

$$\frac{X_{C_s}}{R_i + R_s} = \left(\frac{1}{2\pi f C_s}\right)\left(\frac{1}{R_i + R_s}\right) = \frac{1}{2\pi f (R_i + R_s) C_s}$$

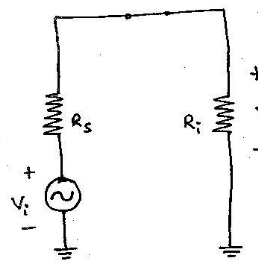
Defining $f_i = \dfrac{1}{2\pi (R_i + R_s) C_s}$

Therefore, $$\frac{X_{C_s}}{R_i + R_s} = \frac{f_i}{f} \quad \text{—— (2)}$$

Substituting (2) in (1)

we have $\dfrac{v_i}{V_s} = \dfrac{1}{\left(1+\frac{R_s}{R_i}\right)\left(1-j\frac{f_i}{f}\right)} = A_v$ —③

For the mid-band frequencies, the network appears as shown below



(c) High Freq. Equivalent

$$\boxed{A_{V_{mid}} = \frac{V_o}{V_i} = \frac{R_i}{R_s + R_i}} \text{ —④}$$

$$\frac{A_v}{A_{V_{mid}}} = \frac{1}{1-j\left(\frac{f_i}{f}\right)} \text{ —⑤}$$

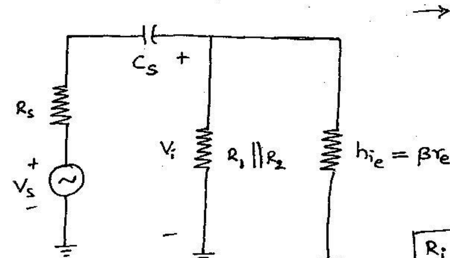
The cutoff frequency is defined by f_i

$$\boxed{f_{L_s} = \frac{1}{2\pi (R_s + R_i) C_s}} \text{ —⑥}$$

At f_L, the voltage V_o will be 70.7% of the mid-band value determined by eqn. ④, assuming the C_s is the only capacitive element controlling the low frequency response.

→ When we analyze the effects of C_s, we assume that C_E and C_C are performing their desired function

→ C_E and C_C are replaced by short circuit equivalent & the Fig. (a) reduce to Fig. (d)



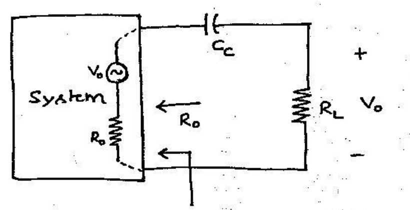
$$\boxed{R_i = R_1 \| R_2 \| \beta r_e} \text{ —⑦}$$

(d) Localized ac equivalent for C_s

2. The effect of C_c

→ Since C_c the coupling capacitor is normally connected betwe the output of the active device and the applied load.

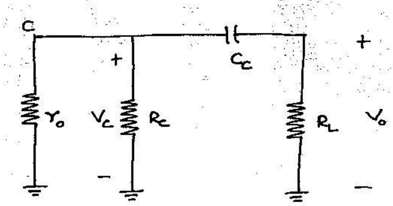
→ The RC configuration that determines the low-cut off frequency due to C_c appears as shown below



(e) Determining the effect of C_c on the low frequency response

→ The total series resistance is now $(R_o + R_L)$ and the cutoff frequency due to C_c is determined by

$$f_{L_c} = \frac{1}{2\pi (R_o + R_L) C_c} \quad \text{—} (8)$$



(f) Localized ac equivalent for C_c with $V_i = 0V$

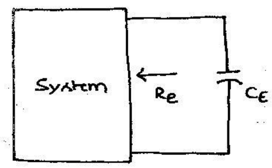
→ Ignoring the effects of C_s and C_E, the output voltage V_o will be 70.7% of its mid-band value at f_{Lc}

→ For the network shown in Fig. (a), the ac equivalent network for the output section with $V_i = 0$ appears in Fig. (f)

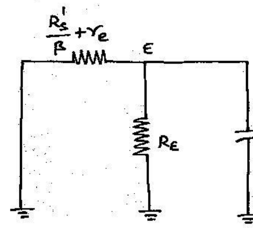
$$R_o = R_C \parallel r_o \quad \text{—— (9)}$$

3. The Effect of C_E

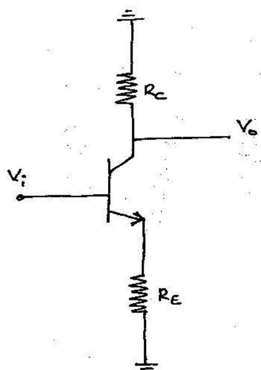
→ To determine f_{L_E}, the network seen by C_E must be determined as shown in the Fig. (g)



(g) Determining the effect of C_E on the low frequency response



(h) Localized ac equivalent of C_E



(i) Network employed to describe the effect of C_E on the amplifier gain.

→ once the level of R_e is established the cutoff frequency due to C_E can be determined using the following eqn.

$$f_{L_E} = \frac{1}{2\pi R_e C_E} \quad \text{—— (10)}$$

→ For the network of Fig. (a), the ac equivalent as seen by C_E appears in Fig. (h).

→ The value of R_e is determined by

$$R_e = R_E \parallel \left(\frac{R_s'}{\beta} + r_e\right) \quad \text{—— (11)}$$

where $R_s' = R_s \parallel R_1 \parallel R_2$

→ The effect of C_E on the gain is best described in a quantitative manner : $A_v = \frac{-R_C}{r_e + R_E}$ —— (12)

→ The max. gain is available with $R_E = 0$

→ At low freq., C_E = open circuit

⇒ All of R_E appears in the gain eqn. (12), results in a minimum gain.

→ As freq. ↑, X_{C_E} ↓

$\Rightarrow (R_E \parallel X_{C_E}) \downarrow$

R_E is effectively shorted out by C_E

The result is a maximum or mid-band gain determined by $A_V = -\frac{R_C}{r_e}$

At f_{L_E} the gain will be 3 dB below the mid-band value determined with R_E shorted out.

→ C_s, C_C and C_E will affect only the low freq. response

→ At mid-freq. level, the short circuit equivalent of the capacitance can be inserted. Although, each capacitor will affect the gain, $A_V = \frac{V_o}{V_i}$

→ The highest low freq. cutoff determined by C_s, C_C or C_E will have the greatest impact.

General Frequency Considerations

→ The frequency of the applied signal will have a pronounced effect on the response of a single stage or multistage network.

$$f \text{ (applied signal)} \begin{cases} \rightarrow \text{single-stage network} \\ \rightarrow \text{multi-stage network} \end{cases}$$

→ At low frequency, f_{low} the capacitive reactance ↑
Therefore, the coupling and bypass capacitors are no longer replaced by short-circuit.

i.e $\quad f = f_{low}, \quad X_C = \text{High}$

$\Rightarrow \quad C_C,\ C_S,$ and $C_E \neq$ Short circuit

→ The freq. dependent parameters of the small signal equivalent circuits and ~~sta~~ stray capacitive elements associated with the active device and the network will limit the high freq. response of the System.

i.e $\quad f$ dependent param of small-signal equivalent
$+$
stray capacitance of an active device & n/w
$\Downarrow$
LIMITS HIGH FREQUENCY RESPONSE

→ An ↑ in the no of stages of a cascaded System will also limit both high and low freq. responses.

→ For any system, there is a band of frequencies in which the magnitude of the gain is either equal or relatively close to the mid-band value.

→ Frequency boundaries:

$$\text{Gain at cutoff levels} = 0.707 A_{v_{mid}}$$

→ The corresponding frequencies : f_1 and f_2 are generally called the corner, cutoff, band-break or half power frequencies.

→ The ⊗lier 0.707 was chosen bec. at this level the output power is half the mid-band power output (i.e at mid-frequencies.

→ The bandwidth (or pass-band) of each system is determined by f_1 and f_2

i.e $\boxed{\text{bandwidth} = BW = f_2 - f_1}$

$$P_{o_{mid}} = \frac{|V_o^2|}{R_o} = \frac{|A_{v_{mid}} V_i|^2}{R_o}, \quad \text{since } A_{v_{mid}} = \frac{V_o}{V_i}$$

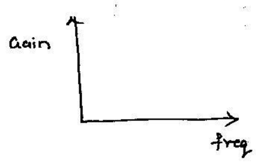
At half power frequencies

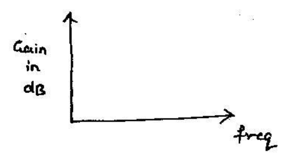
$$P_{o_{HPF}} = \frac{|0.707\, A_{v_{mid}} V_i|^2}{R_o}$$

$$P_{o_{HPF}} = 0.5 \frac{|A_{v_{mid}} V_i|^2}{R_o}$$

$$\boxed{P_{o_{HPF}} = 0.5\, P_{o_{mid}}}$$

→ For applications of a communication nature (audio, video) a decibel plot of the voltage gain versus frequency is more useful rather than gain versus frequency plot.



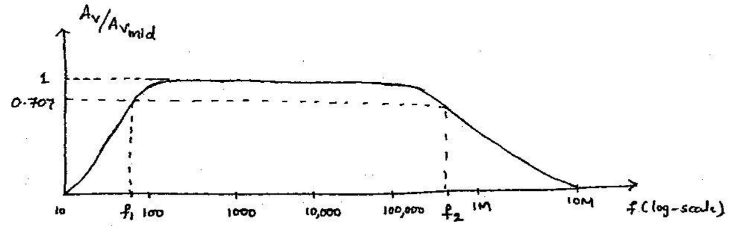


Normalization Process

→ Before obtaining the logarithmic plot, the curve is first normalized

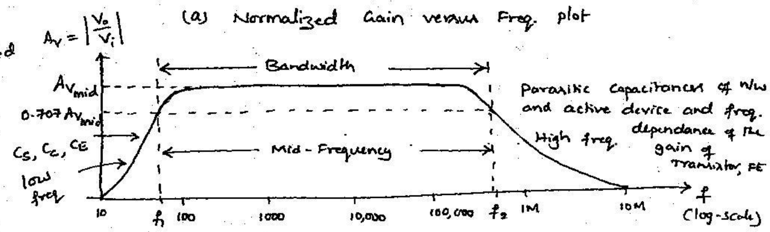
→ Normalized : A process whereby a vertical parameter is divided by a specific level or quantity sensitive to combination or or variables of a System

→ For this, the mid-band or maximum gain for the freq. range of interest is investigated.



(a) Normalized Gain versus Freq. plot

Normalized



(b) Gain versus Freq. response of an RC coupled Amplifier: unnormalized

→ The Fig. (b) is not normalized and is normalized by dividing the output voltage at each freq. by the mid-band level.

→ Note that the curve has the same shape. But the band frequencies are now defined by simply the 0.707 level and not linked to the actual mid-band level.

→ The band frequencies define a level where the gain or quantity of interest will be 70.7% of its maximum value.

→ The plot shown in Fig. (a) is not sensitive to the actual level of mid-band gain. For example, $A_{v_{mid}}$ = 50/200/300 the resulting plot would be same

LOW-FREQUENCY RESPONSE – FET AMPLIFIER

→ The analysis of the FET amplifier in the low frequency region is similar to that of BJT amplifier.

→ There are again three capacitors of primary concern:
(i) C_G (ii) C_C, and (iii) C_S

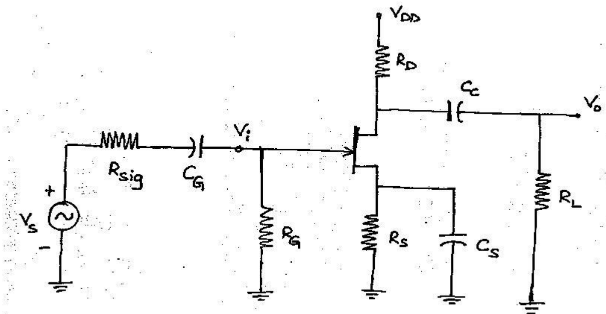


Fig. (a) Capacitive elements that affects the low frequency response of a JFET Amplifier

→ The procedures and conclusions applied to the JFET amplifier shown in Fig. (a) can be applied to other FET configurations

⊛ The Effect of C_G

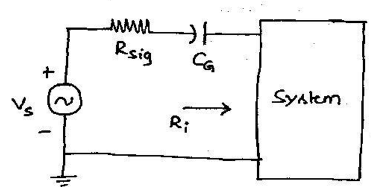


Fig (b) Determining the Effect of C_G on the low frequency response

→ Note the coupling capacitor connected between the voltage source and the active device. The ac equivalent network is shown in Fig. (b) to determine the effect of C_G on the low frequency response.

→ The cutoff freq. determined by C_G is given by

$$f_{L_G} = \frac{1}{\quad}$$

→ For the network of Fig. (b), $\boxed{R_i = R_G}$

Typically, $R_G >> R_{sig}$

⇒ The lower cutoff frequency is primarily determined by R_G and C_G

Also, R_G = Large, permits relatively low level of C_G while maintaining a low cutoff freq. level for f_{L_G}

⊛ The Effect of C_C

→ Note the C_C connected between the active device and the load of the network.

→ The cutoff freq. determined by Coupling Capacitor, C_C is given by

$$\boxed{f_{L_C} = \frac{1}{2\pi (R_o + R_L) C_C}}$$

where $R_o = R_D \| r_d$

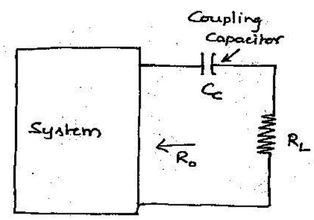


Fig. (c) Determining the Effect of C_C on the low freq. Response

⊛ The Effect of C_S

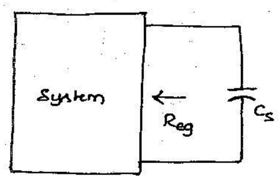


Fig. (d) Determining the Effect of C_S on the low frequency Response

The cutoff frequency determined by C_S is given by

$$\boxed{f_{L_S} = \frac{1}{2\pi (R_{eq}) C_S}}$$

where $R_{eq} = \dfrac{R_S}{1 + \left[\dfrac{R_S(1 + g_m r_d)}{r_d + R_G \| R_L}\right]}$

For $r_d \cong \infty \Omega$

$$R_{eq} = R_S \| \frac{1}{g_m}$$

Miller Effect Capacitance

→ In the high freq. region the capacitive elements that are important are the

(i) Interelectrode capacitance (Between-terminals)

→ Internal to the active device

(ii) Wiring capacitance

→ Capacitance between leads of the network.

→ The large capacitors of the network that controlled the low freq. response are replaced by the short circuit equivalent due to their very low reactance level

$$X_C = \frac{1}{2\pi f C}$$

C = large

f = low

X_C = low

⟨C_S, C_C, and C_E : SC⟩

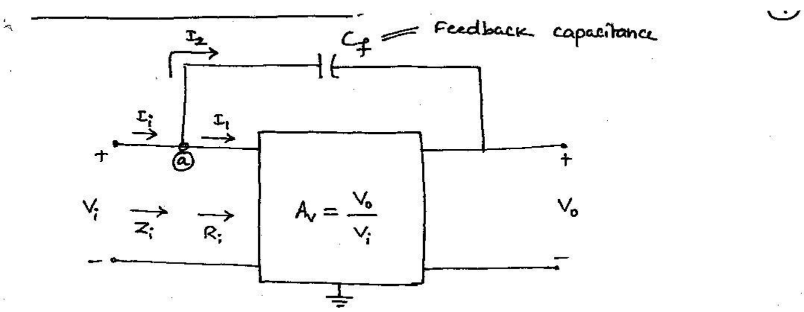
→ For inverting amplifier: (op-amp based / CE Amplifier)

→ phase shift = 180° b/w input & output

A_v = Voltage gain = Negative

The input and output capacitance of an inverting amplifier is increased by interelectrode capacitance between the input and output terminals of the device and the gain of the amplifier

i.e. $\underset{\text{Input-output capacitance}}{\underline{C_{io}\uparrow}} = f(\underset{\text{Interelectrode capacitance}}{\underline{C_{\text{interelectrode}}\uparrow}}, \underset{\text{Voltage gain}}{\underline{A_v\uparrow}})$



(a) Network Employed in the Derivation of an Equation for the Miller Input Capacitance

Applying KCL given at node a

$$I_i = I_1 + I_2 \quad \text{—— (1)}$$

using ohms law, $I_i = \frac{V_i}{Z_i}$, $I_1 = \frac{V_i}{R_i}$ —— (2)

using Nodal analysis, $I_2 = \frac{V_i - V_o}{X_{cf}}$ $V_o = A_V V_i$

$A_V = \frac{V_o}{V_i}$ = Voltage gain

$V_o = A_V V_i$

$$I_2 = \frac{V_i - A_V V_i}{X_{cf}}$$

$$I_2 = \frac{V_i(1 - A_V)}{X_{cf}} \quad \text{—— (3)}$$

Substituting (2) and (3) in (1)

$$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{V_i(1-A_V)}{X_{cf}}$$

Cancel V_i on both LHS and RHS

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{(1-A_V)}{X_{cf}}$$

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{\underbrace{\frac{X_{cf}}{(1-A_V)}}_{X_{C_M}}} \quad \text{—— (4)}$$

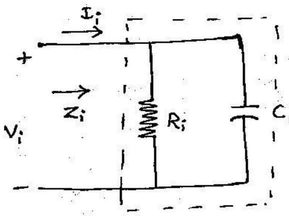
X_{C_M} = Miller Capacitive Reactance

But $\dfrac{X_{C_f}}{(1-A_v)} = \dfrac{1}{\omega \underbrace{(1-A_v) C_f}_{C_M}} = X_{C_M}$ —— ⑤

Substituting ⑤ in ④

$$\boxed{\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_M}}} \quad —— ⑥$$

→ Establishes the equivalent network of Fig. (b)



$C_M = \underbrace{(1-A_v)}_{\text{Gain}} \underbrace{C_f}_{\text{f/b Capacitor}}$

(b) Demonstrating the effect of miller Effect Capacitance

→ Eqn. ⑥ is the equivalent input impedance of the amplifier.

→ Eqn. ⑥ includes R_i, f/b capacitor magnified by the gain of the amplifier

→ In general, the miller effect input capacitance is defined by

$$C_{M_i} = (1 - A_v) C_f$$

This shows us that

"For any inverting amplifier, the input capacitance will be increased by a miller effect capacitance sensitive to the gain of the amplifier and the interelectrode capacitance between the input and output terminals of the active device"

→ At high frequencies,

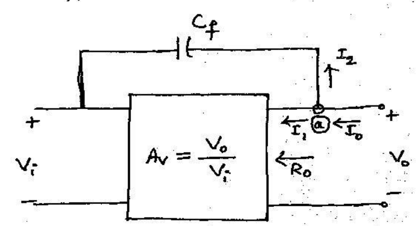
$$A_v = f(C_{M_i})$$

$A_{v_{max}} = A_{v_{mid}}$, results in a highest level of C_{M_i}

→ A positive value of A_v would result in a negative capacitance.

Miller Output Capacitance, C_{M_o}

→ The Miller effect will also increase the level of output capacitance, which must also be determined for high freq. cutoff.



(c) Network employed in the derivation of an equation for the Miller output capacitance

→ The parameters of importance to determine the output Miller effect are shown in Fig. (c)

→ Applying KCL at node (a) of Fig. (c)

$$I_o = I_1 + I_2 \quad \text{—— ①}$$

$$I_1 = \frac{V_o}{R_o} \quad \text{and} \quad I_2 \cong \frac{V_o - V_i}{X_{C_f}} \quad \text{—— ②}$$

The resistance R_o is large, therefore the first term of the equation ① may be ignored, compared to the second term.

$$\text{i.e.} \quad I_o \cong \frac{V_o - V_i}{X_{C_f}} \qquad V_i = V_o/A_v$$

$$\because A_v = \frac{V_o}{V_i}, \quad V_i = \frac{V_o}{A_v}$$

$$I_o = \frac{V_o - \frac{V_o}{A_v}}{X_{C_f}}$$

$$I_o = \frac{V_o\left(1 - \frac{1}{A_v}\right)}{\ldots}$$

$$\frac{I_o}{V_o} = \frac{1 - \frac{1}{A_V}}{X_{C_f}}$$

$$\frac{V_o}{I_o} = \frac{X_{C_f}}{1 - \frac{1}{A_V}} = \frac{1}{\omega C_f\left(1 - \frac{1}{A_V}\right)}$$

$$\boxed{\frac{V_o}{I_o} = \frac{1}{\omega C_{M_o}}}$$

Therefore, the Miller output capacitance, C_{M_o} is given by

$$\boxed{C_{M_o} = \left(1 - \frac{1}{A_V}\right) C_f}$$

If $A_V >> 1$, than the above equation becomes

$$\boxed{C_{M_o} \cong C_f}_{|A_V| >> 1}$$

Problems

1. Determine the lower cutoff freq. for the network shown in Fig. (a) using the following parameters:

$C_G = 0.01\ \mu F$, $C_C = 0.5\ \mu F$, $C_S = 2\ \mu F$, $R_{sig} = 10 K\Omega$

$R_G = 1 M\Omega$, $R_D = 4.7 K\Omega$, $R_S = 1 K\Omega$, $R_L = 2.2 K\Omega$

$V_P = -4V$, $r_d = \infty$, $V_{DD} = 20V$, $V_{GS_Q} = -2V$, $I_{D_Q} = 2\ mA$

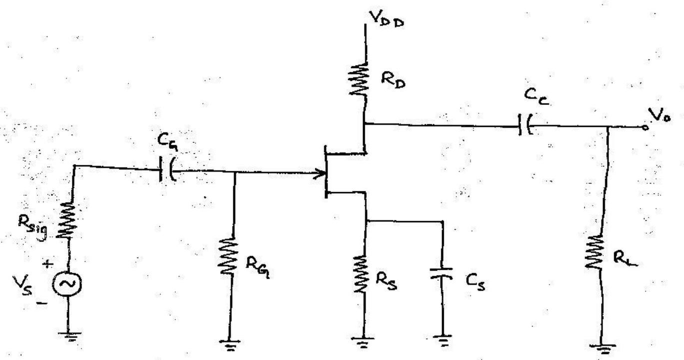


Fig. (a)

Solution: (a) $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

or $I_{DSS} = \frac{I_D}{\left(1 - \frac{V_{GS}}{V_P}\right)^2} = \frac{2\times10^{-3}}{\left(1 - \frac{-2V}{-4V}\right)^2} = \frac{2\times10^{-3}}{0.25}$

$$\boxed{I_{DSS} = 8\ mA}$$

(b) $g_{m_0} = \frac{2I_{DSS}}{|V_P|} = \frac{2\times8\times10^{-3}}{|-4V|} = 4\ ms$

$\cancel{g_{m_0} = [illegible]\ ms}$ $\boxed{g_{m_0} = 4\ ms}$

$g_m = g_{m_0}\left(1 - \frac{V_{GS_Q}}{V_P}\right) = 4\ ms\left(1 - \frac{-2V}{-4V}\right) = 2ms$

$$\boxed{g_m = 2\ ms}$$

C_G : $f_{L_G} = \dfrac{1}{2\pi (R_{sig} + R_i) C_G} = \dfrac{1}{2\pi\, (10 K\Omega + 1 M\Omega)\, (0.01 \mu F)}$

$$\boxed{f_{L_G} \cong 15.8 \; Hz}$$

C_C : $f_{L_C} = \dfrac{1}{2\pi\, (R_o + R_L)\, C_C} = \dfrac{1}{2\pi\, (4.7 K\Omega + 2.2 K\Omega)(0.5 \mu F)}$

$$\boxed{f_{L_C} \cong 46.13\, Hz}$$

C_S : $f_{L_S} = \dfrac{1}{2\pi R_{eq} C_S}$; $R_{eq} = R_s \,\|\, \dfrac{1}{g_m} = 1K\Omega \,\|\, 0.5K\Omega = 333.3$

$$f_{L_S} = \frac{1}{2\pi\, (333.33\,\Omega)\, (2 \mu F)} = 238.73\, Hz$$

$$\boxed{f_{L_S} \cong 238.73\, Hz}$$

The f_{L_S} is largest of the three cutoff frequencies, it define the low-cutoff frequency for the ~~[illegible]~~ network.

(c) The mid band gain of the system is determined by

$$A_{V_{mid}} = \frac{V_o}{V_i} = -g_m (R_D \| R_L)$$

$$= -(2ms)\, (4.7K\Omega \,\|\, 2.2K\Omega)$$

$$= -(2\, ms)\, (1.499\, K\Omega)$$

$$\boxed{A_{V_{mid}} \cong -3}$$

2. Determine the lower cutoff frequency for the BJT amplifier for the following parameters:

$C_S = 10 \mu F$, $C_E = 20 \mu F$, $C_C = 1 \mu F$

$R_S = 1K\Omega$, $R_1 = 40K\Omega$, $R_2 = 10K\Omega$, $R_E = 2K\Omega$, $R_C = 4K\Omega$

$R_L = 2.2K\Omega$, $\beta = 100$, $r_o = \infty\, \Omega$, $V_{CC} = 20V$.

(a) To determine r_e for dc conditions, first apply test condition

$$\left.\begin{aligned} \beta R_E &= 100 \times 2k\Omega = 200k\Omega \\ 10R_2 &= 10 \times 10k\Omega = 100k\Omega \end{aligned}\right\} \beta R_E >> 10R_2$$

Condition : $\beta R_E >> 10R_2$, The condition is satisfied.

The dc base voltage is determined by

$$V_B = \frac{R_2 V_{CC}}{R_1 + R_2} = \frac{(10k\Omega)(20V)}{10k\Omega + 40k\Omega} = \frac{200V}{50} = 4V$$

$$\boxed{V_B = 4V}$$

$$I_E = \frac{V_E}{R_E} = \frac{V_B - \overset{0.7V \text{ (for silicon BJT)}}{V_{BE}}}{R_E} = \frac{4V - 0.7V}{2k\Omega} = \frac{3.3V}{2k\Omega}$$

$$\boxed{I_E = 1.65 \text{ mA}}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{1.65 \text{ mA}} = 15.76\Omega$$

$$\boxed{r_e = 15.76\Omega}$$

$$\beta r_e = 100 \times 15.76\Omega = 1.576k\Omega$$

$$A_v = \frac{V_o}{V_i} = \frac{-R_C \| R_L}{r_e} = -\frac{(4k\Omega) \| (2.2k\Omega)}{15.76\Omega} = -9$$

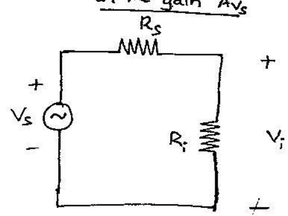
$$\boxed{A_v = -90}$$

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e$$

$$= 40k\Omega \| 10k\Omega \| 1.576k\Omega$$

$$\boxed{Z_i = 1.32k\Omega}$$

Fig. Det. the effect of R_s on the gain A_{v_s}



$$V_i = \frac{R_i V_s}{R_s + R_i}$$

$$\frac{V_i}{V_s} = \frac{R_i}{R_s + R_i} = \frac{1.32k\Omega}{1.32k\Omega + 1k\Omega}$$

$$\frac{V_i}{V_s} = 0.569$$

So that $A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s}$

$$A_{v_s} = (-90)(0.569) = -51.21$$

$$\boxed{A_{v_s} = -51.21}$$

$\underline{\underline{C_s}}$: $f_{L_s} = \frac{1}{2\pi (R_s + R_i) C_s} = \frac{1}{(6.28)(1k\Omega + 1.32k\Omega)(10\mu F)}$

$$\boxed{f_{L_s} = 6.86\ Hz}$$

$\underline{\underline{C_c}}$: $f_{L_c} = \frac{1}{2\pi (R_c + R_L) C_c} = \frac{1}{(6.28)(4k\Omega + 2.2k\Omega)(1\mu F)}$

$$\boxed{f_{L_c} = 25.68\ Hz}$$

$\underline{\underline{C_E}}$: $R_s' = R_s \| R_1 \| R_2 = 1k\Omega \| 40k\Omega \| 10k\Omega \cong 0.889$

$$R_e = R_E \| \left(\frac{R_s'}{\beta} + r_e\right) = 2k\Omega \| \left(\frac{0.889k\Omega}{100} + 15.76\Omega\right.$$

$$R_e = 2k\Omega \| (8.89\Omega + 15.76\Omega)$$

$$R_e = 2k\Omega \| 24.65\Omega$$

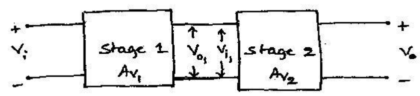
$$\boxed{R_e \cong 24.35\Omega}$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(24.35\Omega)(20\mu F)} = 327\ H$$

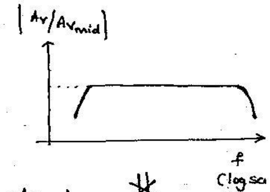
$$\boxed{f_{L_E} = 327\ Hz}$$

Multistage Frequency Effects

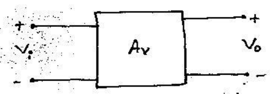
→ There will be a significant change in the overall frequency respo for the second transistor stage connected directly to the output of first stage.



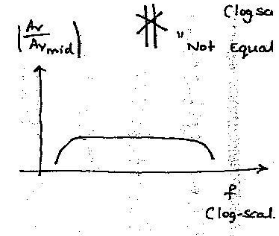
Two Stage Amplifier



$\not\parallel$ " Not Equal



Single stage Amplifier

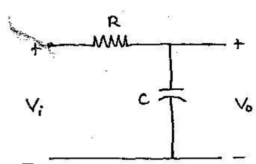


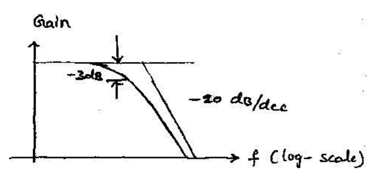
→ In the high freq. region, the output capacitance, C_o must include the following :

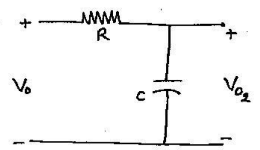
(i) The wiring capacitance, C_{W_1}
(ii) The parasitic capacitance, C_{be}
(iii) The Miller capacitance, C_{M_i}

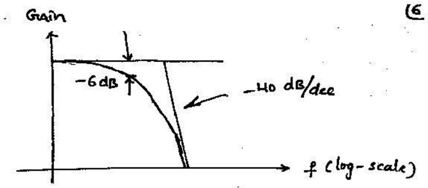
} The C_o that includes for the following stage

→ There will be additional low freq. cutoff levels due to the second stage, which will further reduce the overall gain of the system in this region.

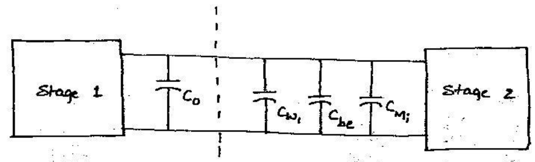








→ Output of first stage affects the input of second stage.



→ For each additional stage, the upper cutoff freq. will be determined primarily by the stage having the lowest cutoff frequency. The lower cutoff freq. is primarily determined by the stage having the highest low cutoff freq.

⇒ Therefore, the poorly designed stage can affect an otherwise well designed cascaded system.

→ The effect of increasing the number of identical stages can be clearly demonstrated using the fig. (a)

→ In each case, the upper and lower cutoff frequencies of each of the cascaded stages are identical.

→ For a single stage, the cutoff frequencies are f_1 and f_2.

→ For two identical stages in cascade, the drop-off rate in the high and low frequency regions has increased to −12 dB/octave or −40 dB/decade.

→ At f_1 and f_2, the decibel drop is now −6 dB rather than the defined band freq. gain level of −3 dB.

→ The −3 dB point has shifted to f_1' and f_2' as indicated in Fig. (a) with the resulting drop in the bandwidth.

→ A −18 dB/octave or −60 dB/decade slope will result for a three stage system of identical stages with the indicated reduction in bandwidth (f_1'' and f_2'')

i.e

f_1 : Single stage lower cutoff freq.	f_2 : Single stage upper cutoff freq.
f_1' : Two stage lower cutoff freq.	f_2' : Two stage upper cutoff freq.
f_1'' : Three stage lower cutoff freq.	f_2'' : Three stage upper cutoff freq.

f_1 and f_2 : Decibel drop = −3 dB : Slope = −20 dB/dec, −6 dB/oct

f_1' and f_2' : —"— = −6 dB : slope = −40 dB/dec, −12 dB/oct

f_1'' and f_2'' : —"— = −9 dB : slope = −60 dB/dec, −18 dB/oct

→ Assuming identical stages, we can determine an equation for each band freq. as a function of the number of stages, n

→ <u>For the low frequency region,</u>

$$A_{v_{low,(overall)}} = A_{v_{1_{low}}} \cdot A_{v_{2_{low}}} \cdot A_{v_{3_{low}}} \cdots\cdots A_{v_{n_{low}}}$$

The overall voltage gain at lower freq. is given by $A_{v_{low,(overall)}}$

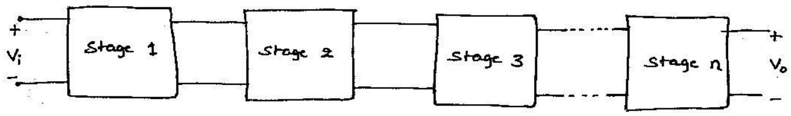


Fig. (b) : Cascade connection of n-identical stages of Amplifier

Since all the stages are identical, then

$$A_{V_{1\,low}} = A_{V_{2\,low}} = A_{V_{3\,low}} = \ldots\ldots = A_{V_{n\,low}}$$

Therefore, $A_{V_{low,(overall)}} = \cancel{A_{V_{low}}} \left(A_{V_{1\,low}}\right)^n$

The low freq. gain $A_{V_{low}}$ for one stage is given by

$$|A_{V_{low}}| = \frac{|A_{V_{mid}}|}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}$$

$$\frac{|A_{V_{low}}|}{|A_{V_{mid}}|} = \frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}$$

Therefore for n-stages in cascade connection, we have

$$\left\{\frac{|A_{V_{low}}|}{|A_{V_{mid}}|}\right\}^n = \left[\frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}\right]^n \quad \text{—— ①}$$

Now setting the magnitude of the result equal to $\frac{1}{\sqrt{2}} = -3\text{ dB}$ l.

Eqn. ① becomes for n-stages

$$\left[\frac{1}{\sqrt{1+\left(\frac{f_1}{f_1'}\right)^2}}\right]^n = \frac{1}{\sqrt{2}} = \left\{\frac{|A_{V_{low}}|}{|A_{V_{mid}}|}\right\}^n_{f = f_1'}$$

~~squaring on both sides~~

$$\sqrt{2} = \left[\sqrt{1+\left(\frac{f_1}{f_1'}\right)^2}\right]^n \quad \text{squaring on both sides}$$

$$2 = \left[1 + \left(\frac{f_1}{f_1'}\right)^2\right]^n$$

Taking n^{th} root on both sides

$$2^{1/n} = \left[1 + \left(\frac{f_1}{f_1'}\right)^2\right]^{n/n}$$

$$2^{\frac{1}{n}} = 1 + \left(\frac{f_1}{f_1'}\right)^2$$

$$\left(\frac{f_1}{f_1'}\right)^2 = 2^{1/n} - 1$$

$$\frac{f_1}{f_1'} = \sqrt{(2^{1/n} - 1)}$$

$$\boxed{f_1' = \frac{f_1}{\sqrt{(2^{1/n} - 1)}}}$$

where f_1' = lower 3-dB freq. of identical cascaded stages

f_1 = lower 3-dB freq. of single stage.

n = no of stages.

Overall Higher cutoff frequency of Multi-Stage Amplifier

→ For n-stages in cascade connection, we have

$$\left\{\frac{|A_{V_{high}}|}{|A_{V_{mid}}|}\right\}^n = \left[\frac{1}{\sqrt{1 + \left(\frac{f}{f_2}\right)^2}}\right]^n \quad \text{—— (1)}$$

→ Let f_2' be the upper cutoff freq. for n-stage amplifier in cascade. Therefore, at $f_2' = f$, we have magnitude $= \frac{1}{\sqrt{2}}$ $= -3$ dB.

$$\left\{\frac{|A_{V_{high}}|}{|A_{V_{mid}}|}\right\}^n_{f = f_2'} = \left[\frac{1}{\sqrt{1 + \left(\frac{f_2'}{f_2}\right)^2}}\right]^n = \frac{1}{\sqrt{2}}$$

$$\sqrt{2} = \left[\sqrt{1 + \left(\frac{f_2'}{f_2}\right)^2}\right]^n$$

Squaring on both sides, we get

$$2 = \left[1 + \left(\frac{f_2'}{f_2}\right)^2\right]^n$$

Taking n^{th} root on both sides, we get

$$2^{1/n} = \left[1 + \left(\frac{f_2'}{f_2}\right)^2\right]^{n/n}$$

$$2^{1/n} = 1 + \left(\frac{f_2'}{f_2}\right)^2$$

$$2^{1/n} - 1 = \left(\frac{f_2'}{f_2}\right)^2$$

$$\frac{f_2'}{f_2} = \sqrt{(2^{1/n} - 1)}$$

$$\boxed{f_2' = f_2\left(\sqrt{(2^{1/n} - 1)}\right)}$$

→ Note the presence of $\sqrt{2^{1/n} - 1}$ in both f_1' and f_2' equation. The magnitude of this factor for various valu of n are listed below

n	$\sqrt{2^{1/n} - 1}$
2	0.64
3	0.51
4	0.43
5	0.39

→ For $n=2$, consider the upper cutoff freq. $f_2' = 0.64 f_2$ or 64 % of the value obtained for a single stage

→ For $n=2$, the lower cutoff freq. $f_1' = \left(\frac{1}{0.64}\right) f_1 =$ 1.56 f_1

→ For $n=3$, $f_2 = 0.51 f_2$ or approximately one half of the value of single stage.

→ For $n=3$, $f_1' = \left(\frac{1}{0.5}\right) f_1 = 1.96\, f_1$ = Twice the Single stage value.

→ A decrease in BW is not always associated with an increase in the no of stages, If the mid-band gain can remain fixed and independent of the no of stages.

i.e <u>Single stage amplifier</u>

Gain = 100

BW = 10 KHZ = 10×10^3

GBW = 100 × 10KHz = 10^6

<u>Two stage Amplifier</u>

Gain = 10 (10 × 10 = 100)

Bandwidth = 100 KHZ

GBW = $10 \times 100 \times 10^3$ = 10^6

Same

For the two stage amplifier, the same gain can be obtained by having two stages with a gain of 10 because 10×10 = 100

High Frequency Response – BJT Amplifier

The two factors that define the $-3\,dB$ cutoff point are

1. The network capacitance (Parasitic and introduced)
2. The Frequency dependance of h_{fe} (β)

Network Parameters

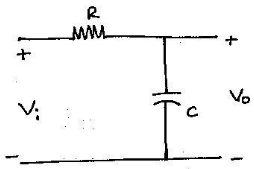


Fig. (a) RC combination that will determine a high-cutoff frequency

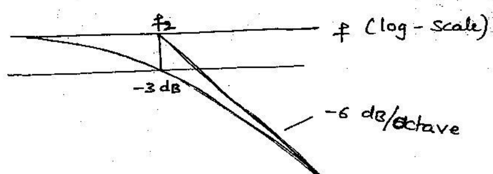
→ In the high freq. region, the RC n/w configuration appears as shown in Fig. (a)

→ At ↑ freq. X_C ↓ in magnitude resulting in a shorting effect across the output

⇒ Gain ↓

→ The derivation leading to the corner frequency for this RC configuration follows similar lines to that encountered for the low freq. region.

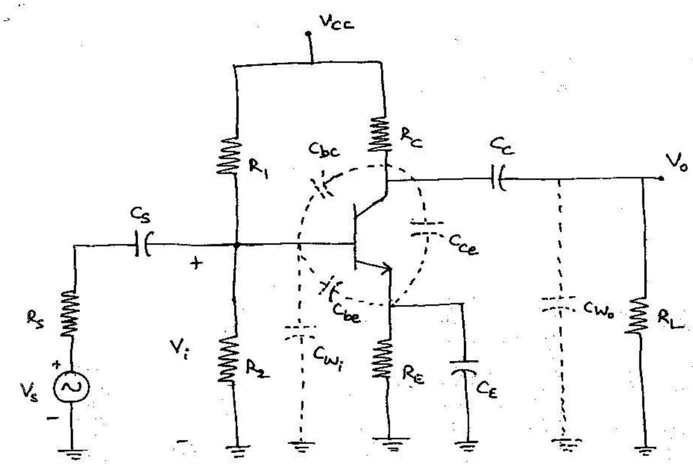
$$\boxed{A_v = \frac{1}{1 + j\left(\frac{f}{f_2}\right)}} \quad \text{—— (1)}$$



(b) Asymptotic plot as defined by Eqn. (1)

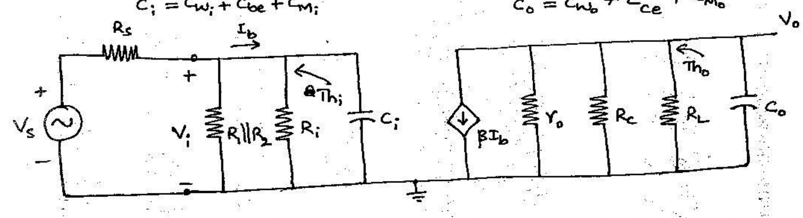
→ The magnitude plot drops off at 6 dB/octave with ↑ frequency.

→ Note that f_2 is in the denominator of the freq. ratio



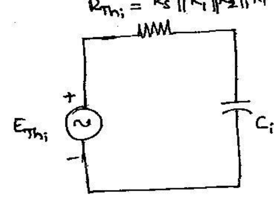
(c) The Network with the capacitors that affect the high Frequency Response

$C_i = C_{W_i} + C_{be} + C_{M_i}$ $\qquad$ $C_o = C_{W_o} + C_{ce} + C_{M_o}$

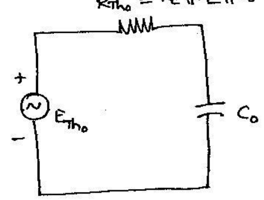


(d) High Frequency ac equivalent model for the Network of Fig. (c)

$R_{Th_i} = R_s \| R_1 \| R_2 \| R_i$ $\qquad$ $R_{Th_o} = R_c \| R_L \| r_o$



Thevenin's Equivalent of Input network of Fig. (d)



Thevenin's Equivalent of output network of Fig. (d)

→ The various parasitic capacitances (C_{be}, C_{bc}, C_{ce}) of the transistor have been included with the wiring capacitances (C_{W_i}, C_{W_o}) introduced during construction.

→ In the high frequency equivalent model for the network, note the absence of the capacitors C_S, C_C, and C_E which are all assumed to be in short-circuit state at these frequencies.

→ The capacitance C_i includes the input wiring capacitance C_{W_i}, the transition capacitance C_{be}, and the miller capacitance C_{M_i}

→ The capacitance C_o includes the output wiring capacitance C_{W_o}, the parasitic capacitance C_{ce} and the output Miller capacitance C_{M_o}

→ In general, the capacitance C_{be} is the largest of the parasitic capacitances with C_{ce} the smallest. In fact, most specification sheets simply provide the levels of C_{be} and C_{bc} and do not include C_{ce} unless it will affect the response of a particular type of transistor in a specific area of application

1. <u>For the Input Network, C_i</u>

The 3-dB freq. for the input network is defined by
$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}$$

where $R_{Th_i} = R_s \,\|\, R_1 \,\|\, R_2 \,\|\, R_i$

$$C_i = C_{W_i} + C_{be} + C_{M_i}$$

$$C_i = C_{W_i} + C_{be} + (1 - A_v)\, C_{bc}$$

2. For the output Networks, C_o

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o}$$

$$R_{Th_o} = R_C \,\|\, R_L \,\|\, r_o$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o}$$

$$C_o = C_{W_o} + C_{ce} + \left(1 - \frac{1}{A_v}\right) C_{bc}$$

$$1 \gg \frac{1}{A_v}$$

Therefore, $C_o = C_{W_o} + C_{ce} + C_{bc}$

→ At very high freq., the capacitance reactance of C_o will decrease and consequently reduce the total impedance of the output parallel branches

→ The frequencies f_{H_i} and f_{H_o} will define a -6dB/octave asymptote

→ If the parasitic capacitors were the only elements to determine the higher cutoff frequency, the lowest frequency would be the determining factor.

→ The decrease in h_{fe} or (β) with freq. must also be considered as to whether its break freq. is lower than f_{H_i} or f_{H_o}