Analog Electronics (15EC32)

Module-1 BJT AC Analysis Class : III Sem

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Syllabus: BJT AC Analysis: BJT Transistor Modeling, The re transistor model, Common emitter fixed bias, Voltage divider bias, Emitter follower configuration. Darlington connection- DC bias; The Hybrid equivalent model, Approximate Hybrid Equivalent Circuit- Fixed bias, Voltage divider, Emitter follower configuration; Complete Hybrid equivalent model, Hybrid π Model.

Objectives:

- Become familiar with the *r e* , hybrid, and hybrid p models for the BJT transistor.
- Learn to use the equivalent model to find the important ac parameters for an amplifier.
- Understand the effects of a source resistance and load resistor on the overall gain and
- characteristics of an amplifier.
- Become aware of the general ac characteristics of a variety of important BJT configurations.

Referred Text: Electronic Devices and Circuit Theory by Robert Boylestad and Louis Nashelsky

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BJT Transistor Modeling: The key to transistor small-signal analysis is the use of the equivalent circuits (models) to be introduced in this chapter.

A model is a combination of circuit elements, properly chosen, that best approximates the actual behavior of a semiconductor device under specific operating conditions.

Once the ac equivalent circuit is determined, the schematic symbol for the device can be replaced by this equivalent circuit and the basic methods of circuit analysis applied to determine the desired quantities of the network.

The *r e* model is really a reduced version of the *hybrid* p *model* used almost exclusively for high-frequency analysis. This model also includes a connection between output and input to include the feedback effect of the output voltage and the input quantities.

The ac equivalent of a transistor network is obtained by:

1. Setting all dc sources to zero and replacing them by a short-circuit equivalent
2. Replacing all capacitors by a short-circuit equivalent

3. Removing all elements bypassed by the short-circuit equivalents introduced by steps 1 and 2
4. Redrawing the network in a more convenient and logical form

THE *re* TRANSISTOR MODEL

Common-Emitter Configuration

The equivalent circuit for the common-emitter configuration will be constructed using the device characteristics and a number of approximations. Starting with the input side, we find the applied voltage *V i* is equal to the voltage *V be* with the input current being the base current *Ib* as shown in Fig. 5.8 .

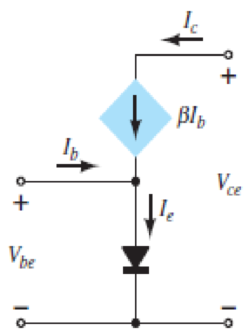


FIG. 5.12
BJT equivalent circuit.

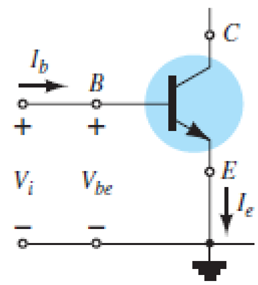


FIG. 5.8
Finding the input equivalent circuit for a BJT transistor.

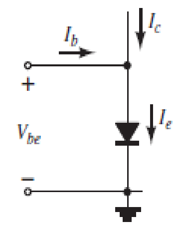


FIG. 5.10
Equivalent circuit for the input side of a BJT transistor.

The equivalent model of Fig. 5.12 can be awkward to work with due to the direct connection between input and output networks. It can be improved by first replacing the diode by its equivalent resistance as determined by the level of I_E, as shown in Fig. 5.13. Recall from Section 1.8 that the diode resistance is determined by $r_D = 26\text{ mV}/I_D$. Using the subscript *e* because the determining current is the emitter current will result in $r_e = 26\text{ mV}/I_E$.

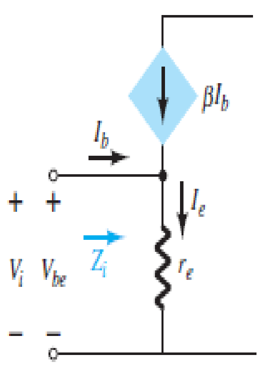


FIG. 5.13
Defining the level of Z_i.

Now, for the input side:

$$Z_i = \frac{V_i}{I_b} = \frac{V_{be}}{I_b}$$

Solving for V_{be}:

$$V_{be} = I_e r_e = (I_c + I_b)r_e = (\beta I_b + I_b)r_e$$
$$= (\beta + 1)I_b r_e$$

and

$$Z_i = \frac{V_{be}}{I_b} = \frac{(\beta + 1)I_b r_e}{I_b}$$

$$Z_i = (\beta + 1)r_e \cong \beta r_e \qquad (5.1)$$

The result is that the impedance seen "looking into" the base of the network is a resistor equal to beta times the value of r_e, as shown in Fig. 5.14. The collector output current is still linked to the input current by beta as shown in the same figure.

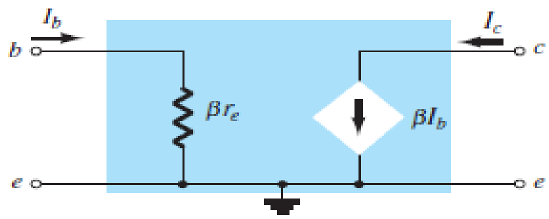


FIG. 5.14
Improved BJT equivalent circuit.

be described in detail in Section 5.19.

In any event, an output impedance can now be defined that will appear as a resistor in parallel with the output as shown in the equivalent circuit of Fig. 5.16.

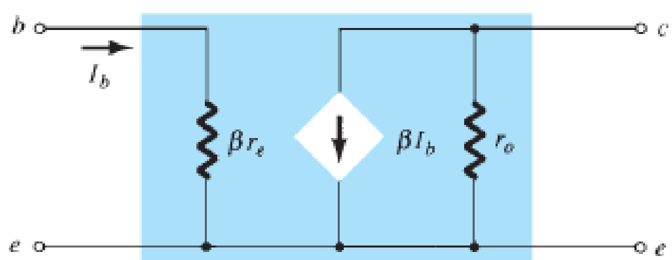


FIG. 5.16
r_e model for the common-emitter transistor configuration including effects of r_o.

WHERE $r_e = 26\ \mathrm{mV}/I_E$

$$r_o = \frac{\Delta V_{CE}}{\Delta I_C}$$

Common-Base Configuration

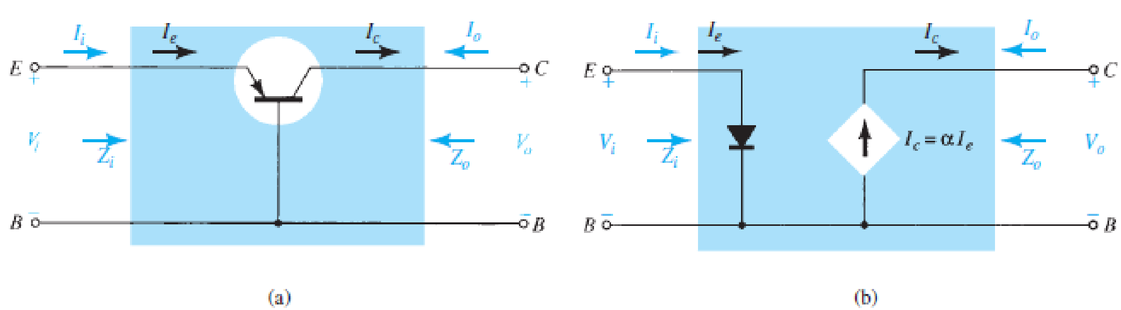


FIG. 5.17
(a) Common-base BJT transistor; (b) equivalent circuit for configuration of (a).

The output impedance r_o will typically extend into the megohm range. Because the output current is opposite to the defined I_o direction, you will find in the analysis to follow that there is no phase shift between the input and output voltages. For the common-emitter configuration there is a 180° phase shift.

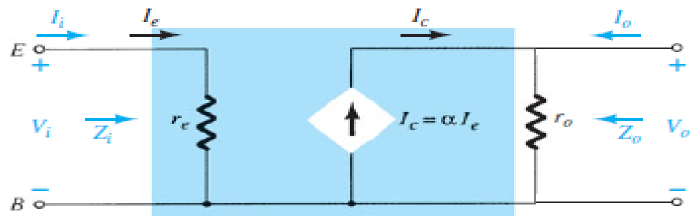


FIG. 5.18
Common base r_e equivalent circuit.

re- Common Emitter - Fixed bias configuration

• Small signal ac analysis includes determining the expressions for the following parameters in terms of Zi, Zo and AV in terms of – β, re, ro and– RB, RC.Also, finding the phase relation between input and output . The values of β, ro are found in datasheet • The value of re must be determined in dc condition as re = 26mV /IE

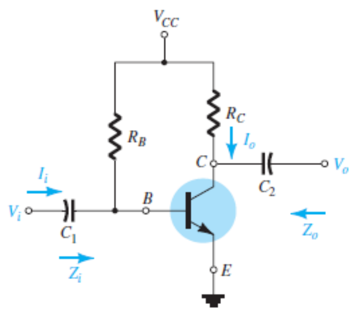


FIG. 5.20
Common-emitter fixed-bias configuration.

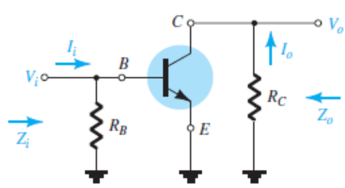


FIG. 5.21
Network of Fig. 5.20 following the removal of the effects of V_{CC}, C_1, and C_2.

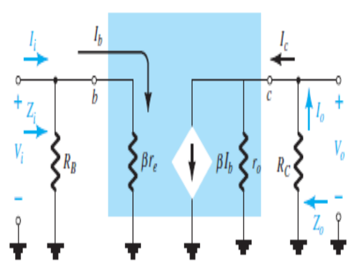


FIG. 5.22
Substituting the r_e model into the network of Fig. 5.21.

Note in Fig. 5.21 that the common ground of the dc supply and the transistor emitter terminal permits the relocation of R_B and R_C in parallel with the input and output sections of the transistor, respectively. In addition, note the placement of the important network parameters Z_i, Z_o, I_i, and I_o on the redrawn network. Substituting the r_e model for the common-emitter configuration of Fig. 5.21 results in the network of Fig. 5.22.

The small-signal ac analysis begins by removing the dc effects of V_{CC} and replacing the dc blocking capacitors C_1 and C_2 by short-circuit equivalents, resulting in the network of Fig. 5.21.

Z_i Figure 5.22 clearly shows that

$$\boxed{Z_i = R_B \| \beta r_e} \quad \text{ohms} \tag{5.5}$$

For the majority of situations R_B is greater than βr_e by more than a factor of 10 (recall from the analysis of parallel elements that the total resistance of two parallel resistors is always less than the smallest and very close to the smallest if one is much larger than the other), permitting the following approximation:

$$\boxed{Z_i \cong \beta r_e}_{R_B \geq 10\beta r_e} \quad \text{ohms} \tag{5.6}$$

Z_o Recall that the output impedance of any system is defined as the impedance Z_o determined when $V_i = 0$. For Fig. 5.22, when $V_i = 0$, $I_i = I_b = 0$, resulting in an open-circuit equivalence for the current source. The result is the configuration of Fig. 5.23. We have

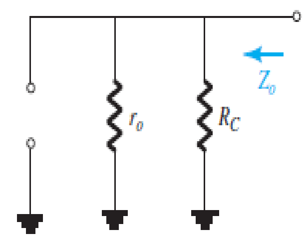


FIG. 5.23
Determining Z_o for the network of Fig. 5.22.

$$\boxed{Z_o = R_C \| r_o} \quad \text{ohms} \tag{5.7}$$

If $r_o \geq 10R_C$, the approximation $R_C \| r_o \cong R_C$ is frequently applied, and

$$\boxed{Z_o \cong R_C}_{r_o \geq 10R_C} \tag{5.8}$$

A_v The resistors r_o and R_C are in parallel, and

$$V_o = -\beta I_b (R_C \| r_o)$$

but

$$I_b = \frac{V_i}{\beta r_e}$$

so that

$$V_o = -\beta \left(\frac{V_i}{\beta r_e}\right)(R_C \| r_o)$$

and

$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{(R_C \| r_o)}{r_e}} \tag{5.9}$$

If $r_o \geq 10R_C$, so that the effect of r_o can be ignored,

$$\boxed{A_v = -\frac{R_C}{r_e}}_{r_o \geq 10R_C} \tag{5.10}$$

Note the explicit absence of β in Eqs. (5.9) and (5.10), although we recognize that β must be utilized to determine r_e.

EXAMPLE 5.1 For the network of Fig. 5.25:

a. Determine r_e.
b. Find Z_i (with $r_o = \infty\,\Omega$).
c. Calculate Z_o (with $r_o = \infty\,\Omega$).
d. Determine A_v (with $r_o = \infty\,\Omega$).
e. Repeat parts (c) and (d) including $r_o = 50\,\text{k}\Omega$ in all calculations and compare results.

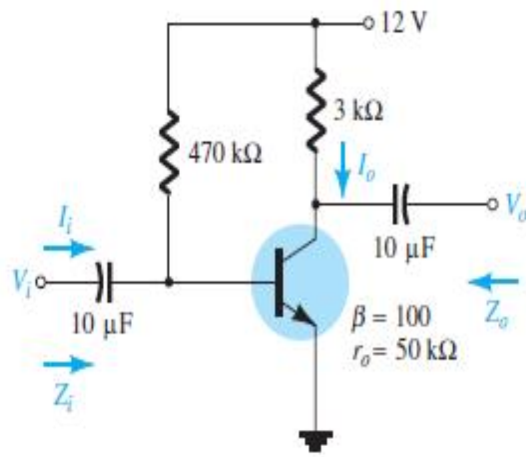


FIG. 5.25
Example 5.1.

Solution:

a. DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12\,\text{V} - 0.7\,\text{V}}{470\,\text{k}\Omega} = 24.04\,\mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04\,\mu\text{A}) = 2.428\,\text{mA}$$

$$r_e = \frac{26\,\text{mV}}{I_E} = \frac{26\,\text{mV}}{2.428\,\text{mA}} = \mathbf{10.71\,\Omega}$$

b. $\beta r_e = (100)(10.71\,\Omega) = 1.071\,\text{k}\Omega$

$$Z_i = R_B \| \beta r_e = 470\,\text{k}\Omega \| 1.071\,\text{k}\Omega = \mathbf{1.07\,k\Omega}$$

c. $Z_o = R_C = \mathbf{3\,k\Omega}$

d. $A_v = -\dfrac{R_C}{r_e} = -\dfrac{3\,\text{k}\Omega}{10.71\,\Omega} = \mathbf{-280.11}$

5.6 VOLTAGE-DIVIDER BIAS

The next configuration to be analyzed is the *voltage-divider* bias network of Fig. 5.26. Recall that the name of the configuration is a result of the voltage-divider bias at the input side to determine the dc level of V_B.

Substituting the r_e equivalent circuit results in the network of Fig. 5.27. Note the absence of R_E due to the low-impedance shorting effect of the bypass capacitor, C_E. That is, at the frequency (or frequencies) of operation, the reactance of the capacitor is so small compared to R_E that it is treated as a short circuit across R_E. When V_{CC} is set to zero, it places one end of R_1 and R_C at ground potential as shown in Fig. 5.27. In addition, note that R_1 and R_2 remain part of the input circuit, whereas R_C is part of the output circuit. The parallel combination of R_1 and R_2 is defined by

$$R' = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} \tag{5.11}$$

Z_i From Fig. 5.27

$$Z_i = R' \| \beta r_e \tag{5.12}$$

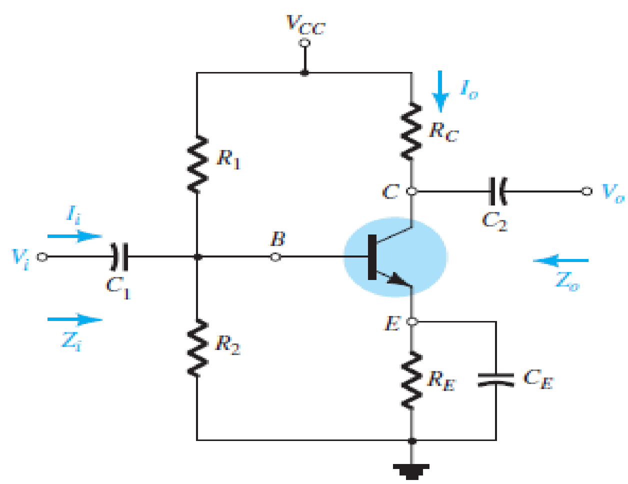


FIG. 5.26
Voltage-divider bias configuration.

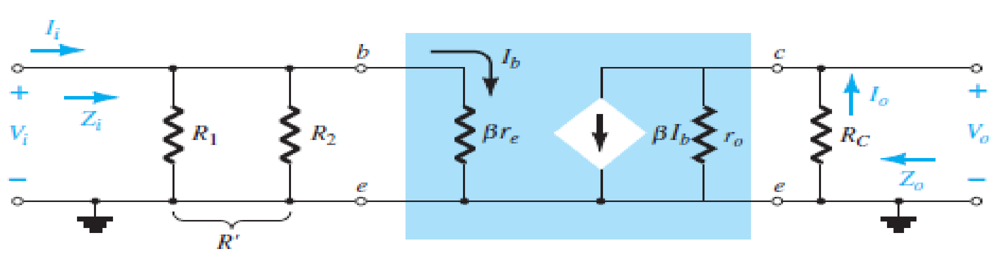


FIG. 5.27
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.26.

Z_o From Fig. 5.27 with V_i set to 0 V, resulting in $I_b = 0\ \mu A$ and $\beta I_b = 0$ mA,

$$\boxed{Z_o = R_C \| r_o} \qquad (5.13)$$

If $r_o \geq 10R_C$,

$$\boxed{Z_o \cong R_C}_{r_o \geq 10R_C} \qquad (5.14)$$

A_v Because R_C and r_o are in parallel,

$$V_o = -(\beta I_b)(R_C \| r_o)$$

and
$$I_b = \frac{V_i}{\beta r_e}$$

so that
$$V_o = -\beta\left(\frac{V_i}{\beta r_e}\right)(R_C \| r_o)$$

and
$$\boxed{A_v = \frac{V_o}{V_i} = \frac{-R_C \| r_o}{r_e}} \qquad (5.15)$$

which you will note is an exact duplicate of the equation obtained for the fixed-bias configuration.

For $r_o \geq 10R_C$,

$$\boxed{A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e}}_{r_o \geq 10R_C} \qquad (5.16)$$

Phase Relationship The negative sign of Eq. (5.15) reveals a 180° phase shift between V_o and V_i.

EXAMPLE 5.2 For the network of Fig. 5.28, determine:

a. r_e.
b. Z_i.
c. Z_o ($r_o = \infty\ \Omega$).
d. A_v ($r_o = \infty\ \Omega$).
e. The parameters of parts (b) through (d) if $r_o = 50\ k\Omega$ and compare results.

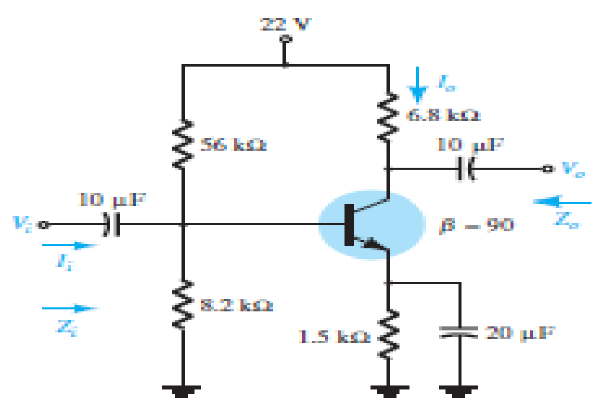


FIG. 5.28
Example 5.2.

Solution:

a. DC: Testing $\beta R_E > 10R_2$,

$$(90)(1.5\text{ k}\Omega) > 10(8.2\text{ k}\Omega)$$
$$135\text{ k}\Omega > 82\text{ k}\Omega \; (\textit{satisfied})$$

Using the approximate approach, we obtain

$$V_B = \frac{R_2}{R_1 + R_2}V_{CC} = \frac{(8.2\text{ k}\Omega)(22\text{ V})}{56\text{ k}\Omega + 8.2\text{ k}\Omega} = 2.81\text{ V}$$
$$V_E = V_B - V_{BE} = 2.81\text{ V} - 0.7\text{ V} = 2.11\text{ V}$$
$$I_E = \frac{V_E}{R_E} = \frac{2.11\text{ V}}{1.5\text{ k}\Omega} = 1.41\text{ mA}$$
$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{1.41\text{ mA}} = \mathbf{18.44\ \Omega}$$

b. $R' = R_1 \| R_2 = (56\text{ k}\Omega)\|(8.2\text{ k}\Omega) = 7.15\text{ k}\Omega$

$Z_i = R' \| \beta r_e = 7.15\text{ k}\Omega \| (90)(18.44\ \Omega) = 7.15\text{ k}\Omega \| 1.66\text{ k}\Omega$
$= \mathbf{1.35\text{ k}\Omega}$

c. $Z_o = R_C = \mathbf{6.8\text{ k}\Omega}$

d. $A_v = -\dfrac{R_C}{r_e} = -\dfrac{6.8\text{ k}\Omega}{18.44\ \Omega} = \mathbf{-368.76}$

e. $Z_i = \mathbf{1.35\text{ k}\Omega}$

$Z_o = R_C \| r_o = 6.8\text{ k}\Omega \| 50\text{ k}\Omega = \mathbf{5.98\text{ k}\Omega}$ vs. $6.8\text{ k}\Omega$

$A_v = -\dfrac{R_C \| r_o}{r_e} = -\dfrac{5.98\text{ k}\Omega}{18.44\ \Omega} = \mathbf{-324.3}$ vs. -368.76

There was a measurable difference in the results for Z_o and A_v, because the condition $r_o \geq 10R_C$ was *not* satisfied.

5.7 CE EMITTER-BIAS CONFIGURATION

The networks examined in this section include an emitter resistor that may or may not be bypassed in the ac domain. We first consider the unbypassed situation and then modify the resulting equations for the bypassed configuration.

Unbypassed

The most fundamental of unbypassed configurations appears in Fig. 5.29. The r_e equivalent model is substituted in Fig. 5.30, but note the absence of the resistance r_o. The effect of r_o is to make the analysis a great deal more complicated, and considering the fact that in

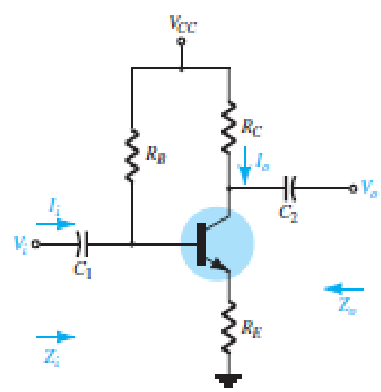


FIG. 5.29
CE emitter-bias configuration.

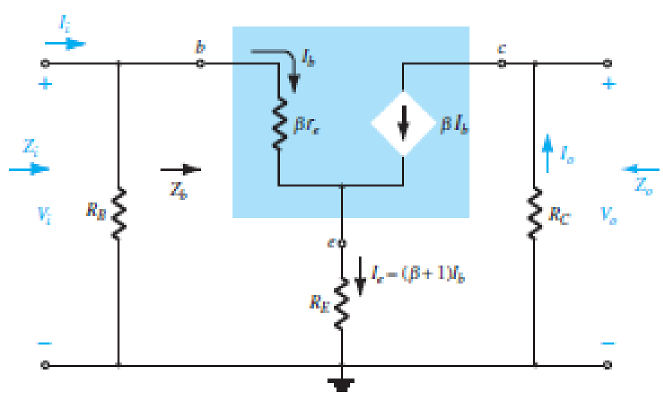


FIG. 5.30
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.29.

most situations its effect can be ignored, it will not be included in the present analysis. However, the effect of r_o will be discussed later in this section.

Applying Kirchhoff's voltage law to the input side of Fig. 5.30 results in

$$V_i = I_b\beta r_e + I_e R_E$$

or

$$V_i = I_b\beta r_e + (\beta + 1)I_b R_E$$

and the input impedance looking into the network to the right of R_B is

$$Z_b = \frac{V_i}{I_b} = \beta r_e + (\beta + 1)R_E$$

The result as displayed in Fig. 5.31 reveals that the input impedance of a transistor with an unbypassed resistor R_E is determined by

$$\boxed{Z_b = \beta r_e + (\beta + 1)R_E} \qquad (5.17)$$

Because β is normally much greater than 1, the approximate equation is

$$Z_b \cong \beta r_e + \beta R_E$$

and

$$\boxed{Z_b \cong \beta(r_e + R_E)} \qquad (5.18)$$

Because R_E is usually greater than r_e, Eq. (5.18) can be further reduced to

$$\boxed{Z_b \cong \beta R_E} \qquad (5.19)$$

Z_i Returning to Fig. 5.30, we have

$$\boxed{Z_i = R_B \| Z_b} \qquad (5.20)$$

Z_o With V_i set to zero, $I_b = 0$, and βI_b can be replaced by an open-circuit equivalent. The result is

$$\boxed{Z_o = R_C} \qquad (5.21)$$

A_v

$$I_b = \frac{V_i}{Z_b}$$

and

$$V_o = -I_o R_C = -\beta I_b R_C$$

$$= -\beta\left(\frac{V_i}{Z_b}\right)R_C$$

with

$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_b}} \qquad (5.22)$$

Substituting $Z_b \cong \beta(r_e + R_E)$ gives

$$\boxed{A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e + R_E}} \qquad (5.23)$$

and for the approximation $Z_b \cong \beta R_E$,

$$\boxed{A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{R_E}} \qquad (5.24)$$

Note the absence of β from the equation for A_v demonstrating an independence in variation of β.

Phase Relationship The negative sign in Eq. (5.22) again reveals a 180° phase shift between V_o and V_i.

5.8 EMITTER-FOLLOWER CONFIGURATION

When the output is taken from the emitter terminal of the transistor as shown in Fig. 5.36, the network is referred to as an *emitter-follower*. The output voltage is always slightly less than the input signal due to the drop from base to emitter, but the approximation $A_v \cong 1$ is usually a good one. Unlike the collector voltage, the emitter voltage is in phase with the signal V_i. That is, both V_o and V_i attain their positive and negative peak values at the same time. The fact that V_o "follows" the magnitude of V_i with an in-phase relationship accounts for the terminology emitter-follower.

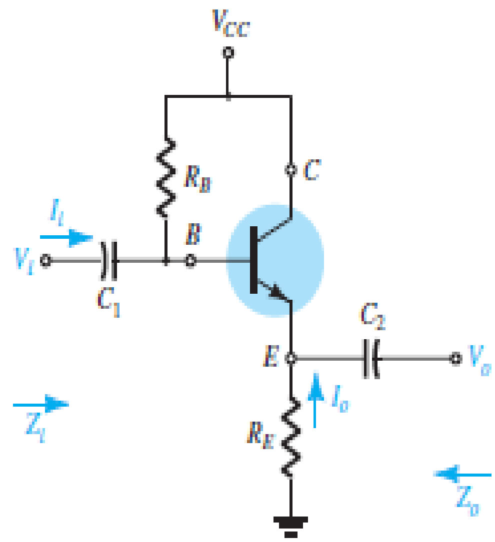


FIG. 5.36
Emitter-follower configuration.

The most common emitter-follower configuration appears in Fig. 5.36. In fact, because the collector is grounded for ac analysis, it is actually a *common-collector* configuration. Other variations of Fig. 5.36 that draw the output off the emitter with $V_o \cong V_i$ will appear later in this section.

The emitter-follower configuration is frequently used for impedance-matching purposes. It presents a high impedance at the input and a low impedance at the output, which is the direct opposite of the standard fixed-bias configuration. The resulting effect is much the same as that obtained with a transformer, where a load is matched to the source impedance for maximum power transfer through the system.

Substituting the r_e equivalent circuit into the network of Fig. 5.36 results in the network of Fig. 5.37. The effect of r_o will be examined later in the section.

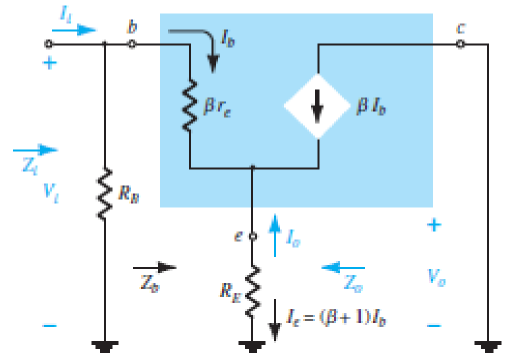


FIG. 5.37
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.36.

Z_i The input impedance is determined in the same manner as described in the preceding section:

$$Z_i = R_B \| Z_b \tag{5.31}$$

with

$$Z_b = \beta r_e + (\beta + 1)R_E \tag{5.32}$$

or

$$Z_b \cong \beta(r_e + R_E) \tag{5.33}$$

and

$$Z_b \cong \beta R_E \Big|_{R_E \gg r_e} \tag{5.34}$$

Z_o The output impedance is best described by first writing the equation for the current I_b,

$$I_b = \frac{V_i}{Z_b}$$

and then multiplying by $(\beta + 1)$ to establish I_e. That is,

$$I_e = (\beta + 1)I_b = (\beta + 1)\frac{V_i}{Z_b}$$

Substituting for Z_b gives

$$I_e = \frac{(\beta + 1)V_i}{\beta r_e + (\beta + 1)R_E}$$

or

$$I_e = \frac{V_i}{[\beta r_e/(\beta + 1)] + R_E}$$

but

$$(\beta + 1) \cong \beta$$

and

$$\frac{\beta r_e}{\beta + 1} \cong \frac{\beta r_e}{\beta} = r_e$$

so that

$$I_e \cong \frac{V_i}{r_e + R_E} \tag{5.35}$$

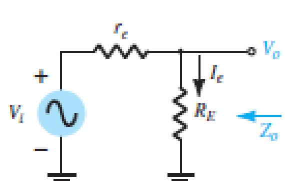


FIG. 5.38
Defining the output impedance for the emitter-follower configuration.

If we now construct the network defined by Eq. (5.35), the configuration of Fig. 5.38 results.

To determine Z_o, V_i is set to zero and

$$Z_o = R_E \| r_e \tag{5.36}$$

Because R_E is typically much greater than r_e, the following approximation is often applied:

$$Z_o \cong r_e \tag{5.37}$$

A_v Figure 5.38 can be used to determine the voltage gain through an application of the voltage-divider rule:

$$V_o = \frac{R_E V_i}{R_E + r_e}$$

and

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e} \tag{5.38}$$

Because R_E is usually much greater than r_e, $R_E + r_e \cong R_E$ and

$$A_v = \frac{V_o}{V_i} \cong 1 \tag{5.39}$$

Phase Relationship As revealed by Eq. (5.38) and earlier discussions of this section, V_o and V_i are in phase for the emitter-follower configuration.

Effect of r_o

Z_i

$$Z_b = \beta r_e + \frac{(\beta + 1)R_E}{1 + \dfrac{R_E}{r_o}} \tag{5.40}$$

If the condition $r_o \geq 10R_E$ is satisfied,

$$Z_b = \beta r_e + (\beta + 1)R_E$$

which matches earlier conclusions with

$$Z_b \cong \beta(r_e + R_E) \quad _{r_o \geq 10R_E} \tag{5.41}$$

Z_o

$$Z_o = r_o \| R_E \| \frac{\beta r_e}{(\beta + 1)} \tag{5.42}$$

Using $\beta + 1 \cong \beta$, we obtain

$$Z_o = r_o \| R_E \| r_e$$

and because $r_o \gg r_e$,

$$Z_o \cong R_E \| r_e \quad _{\text{Any } r_o} \tag{5.43}$$

A_v

$$A_v = \frac{(\beta + 1)R_E/Z_b}{1 + \dfrac{R_E}{r_o}} \tag{5.44}$$

If the condition $r_o \geq 10R_E$ is satisfied and we use the approximation $\beta + 1 \cong \beta$, we find

$$A_v \cong \frac{\beta R_E}{Z_b}$$

5.9 COMMON-BASE CONFIGURATION

The common-base configuration is characterized as having a relatively low input and a high output impedance and a current gain less than 1. The voltage gain, however, can be quite large. The standard configuration appears in Fig. 5.42, with the common-base r_e equivalent model substituted in Fig. 5.43. The transistor output impedance r_o is not included for the common-base configuration because it is typically in the megohm range and can be ignored in parallel with the resistor R_C.

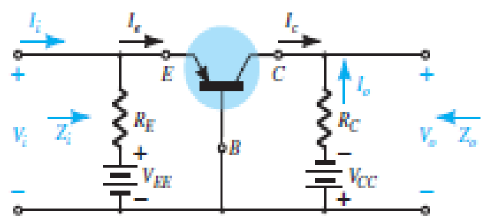


FIG. 5.42
Common-base configuration.

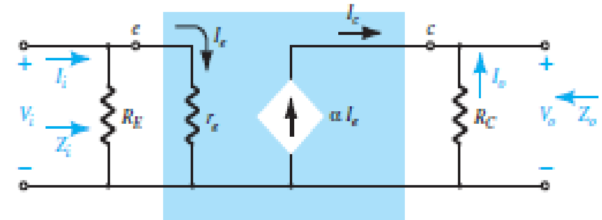


FIG. 5.43
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.44.

Z_i

$$\boxed{Z_i = R_E \| r_e} \tag{5.46}$$

Z_o

$$\boxed{Z_o = R_C} \tag{5.47}$$

A_v

$$V_o = -I_o R_C = -(-I_c)R_C = \alpha I_e R_C$$

with

$$I_e = \frac{V_i}{r_e}$$

so that

$$V_o = \alpha\left(\frac{V_i}{r_e}\right)R_C$$

and

$$\boxed{A_v = \frac{V_o}{V_i} = \frac{\alpha R_C}{r_e} \cong \frac{R_C}{r_e}} \tag{5.48}$$

A_i Assuming that $R_E \gg r_e$ yields

$$I_e = I_i$$

and

$$I_o = -\alpha I_e = -\alpha I_i$$

with

$$\boxed{A_i = \frac{I_o}{I_i} = -\alpha \cong -1} \tag{5.49}$$

Phase Relationship The fact that A_v is a positive number shows that V_o and V_i are in phase for the common-base configuration.

Effect of r_o For the common-base configuration, $r_o = 1/h_{ob}$ is typically in the megohm range and sufficiently larger than the parallel resistance R_C to permit the approximation $r_o \| R_C \cong R_C$.

5.17 DARLINGTON CONNECTION

A very popular connection of two bipolar junction transistors for operation as one "super-beta" transistor is the Darlington connection shown in Fig. 5.73. The main feature of the Darlington connection is that the composite transistor acts as a single unit with a current gain that is the product of the current gains of the individual transistors. If the connection is made using two separate transistors having current gains of β_1 and β_2, the Darlington connection provides a current gain of

$$\boxed{\beta_D = \beta_1\beta_2} \tag{5.101}$$

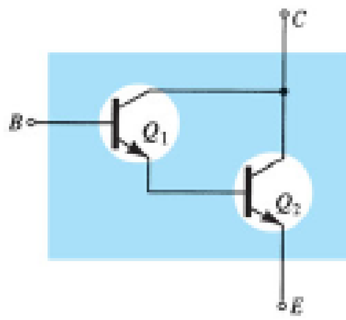


FIG. 5.73
Darlington combination.

The configuration was first introduced by Dr. Sidney Darlington in 1953. A short biography appears as Fig 5.74.

Emitter-Follower Configuration

A Darlington amplifier used in an emitter-follower configuration appears in Fig. 5.75. The primary impact of using the Darlington configuration is an input impedance much larger than

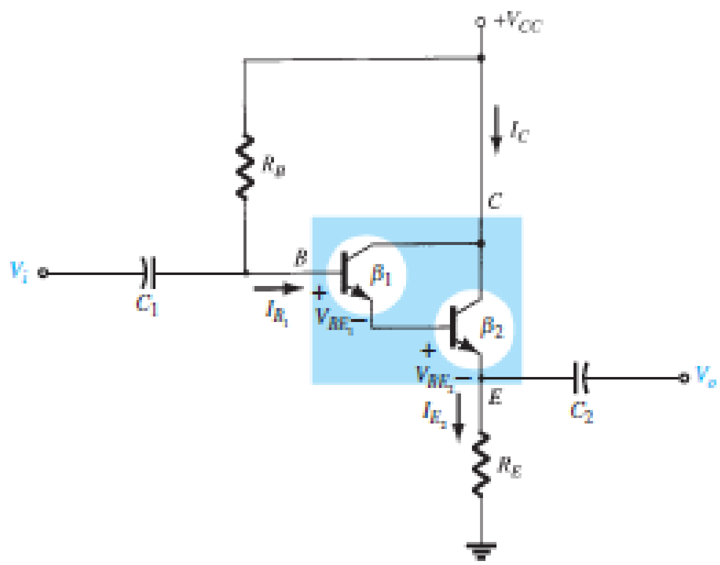


FIG. 5.75
Emitter-follower configuration with a Darlington amplifier.

that obtained with a single-transistor network. The current gain is also larger, but the voltage gain for a single-transistor or Darlington configuration remains slightly less than one.

DC Bias The case current is determined using a modified version of Eq. 4.44. There are now two base-to-emitter voltage drops to include and the beta of a single transistor is replaced by the Darlington combination of Eq. 5.101.

$$I_{B_1} = \frac{V_{CC} - V_{BE_1} - V_{BE_2}}{R_B + \beta_D R_E} \tag{5.102}$$

The emitter current of Q_1 is equal to the base current of Q_2 so that

$$I_{E_2} = \beta_2 I_{B_2} = \beta_2 I_{E_1} = \beta_2(\beta_1 I_{E_1}) = \beta_1\beta_2 I_{B_1}$$

resulting in

$$I_{C_2} \cong I_{E_2} = \beta_D I_{B_1} \tag{5.103}$$

The collector voltage of both transistors is

$$V_{C_1} = V_{C_2} = V_{CC} \tag{5.104}$$

the emitter voltage of Q_2

$$V_{E_2} = I_{E_2} R_E \tag{5.105}$$

the base voltage of Q_1

$$V_{B_1} = V_{CC} - I_{B_1} R_B = V_{E_2} + V_{BE_1} + V_{BE_2} \tag{5.106}$$

the collector-emitter voltage of Q

$$V_{CE_2} = V_{C_2} - V_{E_2} = V_{CC} - V_{E_2} \tag{5.107}$$

EXAMPLE 5.17 Calculate the dc bias voltages and currents for the Darlington configuration of Fig. 5.76.

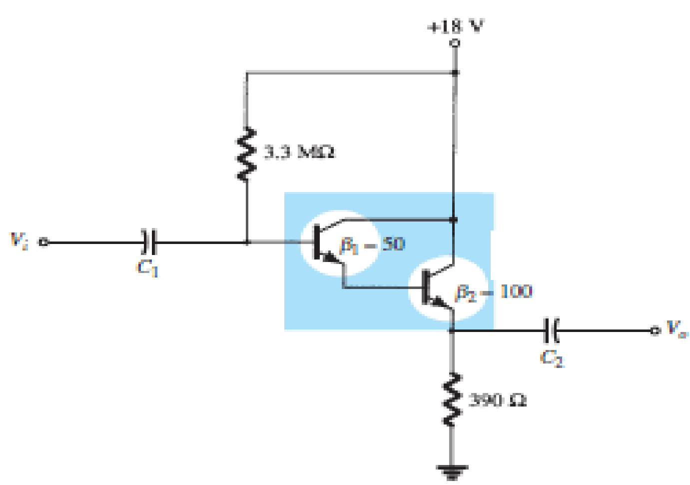


FIG. 5.76
Circuit for Example 5.17.

Solution:

$$\beta_D = \beta_1\beta_2 = (50)(100) = \mathbf{5000}$$

$$I_{B_1} = \frac{V_{CC} - V_{BE_1} - V_{BE_2}}{R_B + \beta_D R_E} = \frac{18\text{ V} - 0.7\text{ V} - 0.7\text{ V}}{3.3\text{ M}\Omega + (5000)(390\ \Omega)}$$

$$= \frac{18\text{ V} - 1.4\text{ V}}{3.3\text{ M}\Omega + 1.95\text{ M}\Omega} = \frac{16.6\text{ V}}{5.25\text{ M}\Omega} = \mathbf{3.16\ \mu A}$$

$$I_{C_2} \cong I_{E_2} = \beta_D I_{B_1} = (5000)(3.16\text{ mA}) = \mathbf{15.80\ mA}$$

$$V_{C_1} = V_{C_2} = \mathbf{18\ V}$$

$$V_{E_2} = I_{E_2}R_E = (15.80\text{ mA})(390\ \Omega) = \mathbf{6.16\ V}$$

$$V_{B_1} = V_{E_2} + V_{BE_1} + V_{BE_2} = 6.16\text{ V} + 0.7\text{ V} + 0.7\text{ V} = \mathbf{7.56\ V}$$

$$V_{CE_2} = V_{CC} - V_{E_2} = 18\text{ V} - 6.16\text{ V} = \mathbf{11.84\ V}$$

AC Input Impedance The ac input impedance can be determined using the ac equivalent network of Fig. 5.77.

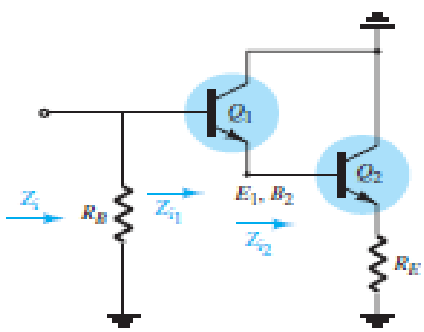


FIG. 5.77
Finding Z_i.

As defined in Fig. 5.77:

$$Z_{i_2} = \beta_2(r_{e_2} + R_E)$$

$$Z_{i_1} = \beta_1(r_{e_1} + Z_{i_2})$$

so that $$Z_{i_1} = \beta_1(r_{e_1} + \beta_2(r_{e_2} + R_E))$$

Assuming $$R_E \gg r_{e_2}$$

and $$Z_{i_1} = \beta_1(r_{e_1} + \beta_2 R_E)$$

Since $$\beta_2 R_E \gg r_{e_1}$$

$$Z_{i_1} \cong \beta_1\beta_2 R_E$$

and since $$Z_i = R_B \| Z_i$$

$$\boxed{Z_i = R_B \| \beta_1\beta_2 R_E = R_B \| \beta_D R_E} \qquad \textbf{(5.108)}$$

For the network of Fig. 5.76

$$Z_i = R_B \| \beta_D R_E$$

$$= 3.3\text{ M}\Omega \| (5000)(390\ \Omega) = 3.3\text{ M}\Omega \| 1.95\text{ M}\Omega$$

$$= \mathbf{1.38\ M\Omega}$$

5.19 THE HYBRID EQUIVALENT MODEL

The hybrid equivalent model was mentioned in the earlier sections of this chapter as one that was used in the early years before the popularity of the r_e model developed. Today there is a mix of usage depending on the level and direction of the investigation.

The r_e model has the advantage that the parameters are defined by the actual operating conditions,

whereas

the parameters of the hybrid equivalent circuit are defined in general terms for any operating conditions.

In other words, the hybrid parameters may not reflect the actual operating conditions but simply provide an indication of the level of each parameter to expect for general use. The r_e model suffers from the fact that parameters such as the output impedance and the feedback elements are not available, whereas the hybrid parameters provide the entire set on the specification sheet. In most cases, if the r_e model is employed, the investigator will simply examine the specification sheet to have some idea of what the additional elements might be. This section will show how one can go from one model to the other and how the parameters are related. Because all specification sheets provide the hybrid parameters and the model is still extensively used, it is important to be aware of both models. The hybrid

The description of the hybrid equivalent model will begin with the general two-port system of Fig. 5.93. The following set of equations (5.131) and (5.132) is only one of a number of ways in which the four variables of Fig. 5.93 can be related. It is the most frequently employed in transistor circuit analysis, however, and therefore is discussed in detail in this chapter.

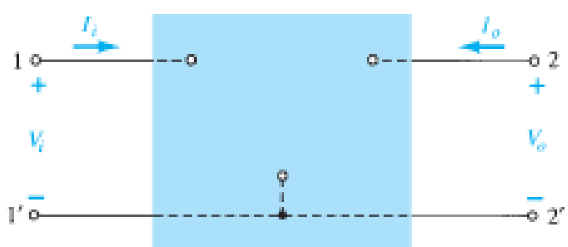


FIG. 5.93
Two-port system.

$$\boxed{V_i = h_{11}I_i + h_{12}V_o}$$

$$\boxed{I_o = h_{21}I_i + h_{22}V_o}$$

$\boldsymbol{h_{11}}$ If we arbitrarily set $V_o = 0$ (short circuit the output terminals) and solve for h_{11} in Eq. (5.133), we find

$$h_{11} = \left.\frac{V_i}{I_i}\right|_{V_o=0} \quad \text{ohms} \tag{5.135}$$

The ratio indicates that the parameter h_{11} is an impedance parameter with the units of ohms. Because it is the ratio of the *input* voltage to the *input* current with the output terminals *shorted*, it is called the *short-circuit input-impedance parameter*. The subscript 11 of h_{11} refers to the fact that the parameter is determined by a ratio of quantities measured at the input terminals.

$\boldsymbol{h_{12}}$ If I_i is set equal to zero by opening the input leads, the following results for h_{12}:

$$h_{12} = \left.\frac{V_i}{V_o}\right|_{I_i=0} \quad \text{unitless} \tag{5.136}$$

The parameter h_{12}, therefore, is the ratio of the input voltage to the output voltage with the input current equal to zero. It has no units because it is a ratio of voltage levels and is called the *open-circuit reverse transfer voltage ratio parameter*. The subscript 12 of h_{12} indicates that the parameter is a transfer quantity determined by a ratio of input (1) to output (2) measurements. The first integer of the subscript defines the measured quantity to appear in the numerator; the second integer defines the source of the quantity to appear in the denominator. The term *reverse* is included because the ratio is an input voltage over an output voltage rather than the reverse ratio typically of interest.

$\boldsymbol{h_{21}}$ If in Eq. (5.134) V_o is set equal to zero by again shorting the output terminals, the following results for h_{21}:

$$h_{21} = \left.\frac{I_o}{I_i}\right|_{V_o=0} \quad \text{unitless} \tag{5.137}$$

Note that we now have the ratio of an output quantity to an input quantity. The term *forward* will now be used rather than *reverse* as indicated for h_{12}. The parameter h_{21} is the ratio of the output current to the input current with the output terminals shorted. This parameter, like h_{12}, has no units because it is the ratio of current levels. It is formally called the *short-circuit forward transfer current ratio parameter*. The subscript 21 again indicates that it is a transfer parameter with the output quantity (2) in the numerator and the input quantity (1) in the denominator.

$\boldsymbol{h_{22}}$ The last parameter, h_{22}, can be found by again opening the input leads to set $I_1 = 0$ and solving for h_{22} in Eq. (5.134):

$$h_{22} = \left.\frac{I_o}{V_o}\right|_{I_i=0} \quad \text{siemens} \tag{5.138}$$

Because it is the ratio of the output current to the output voltage, it is the output conductance parameter, and it is measured in siemens (S). It is called the *open-circuit output admittance parameter*. The subscript 22 indicates that it is determined by a ratio of output quantities.

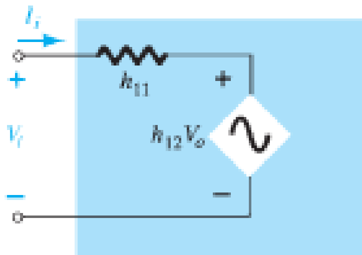


FIG. 5.94
Hybrid input equivalent circuit.

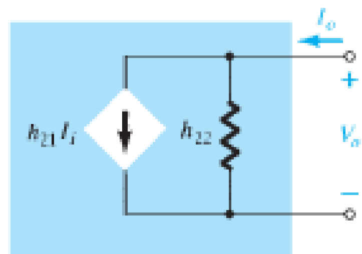


FIG. 5.95
Hybrid output equivalent circuit.

$h_{11} \rightarrow$ *i*nput resistance $\rightarrow h_i$

$h_{12} \rightarrow$ *r*everse transfer voltage ratio $\rightarrow h_r$

$h_{21} \rightarrow$ *f*orward transfer current ratio $\rightarrow h_f$

$h_{22} \rightarrow$ *o*utput conductance $\rightarrow h_o$

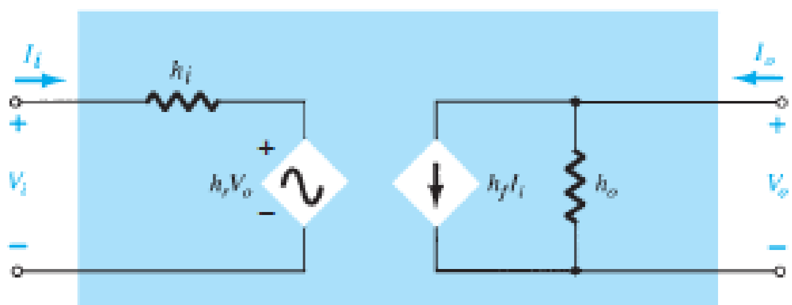
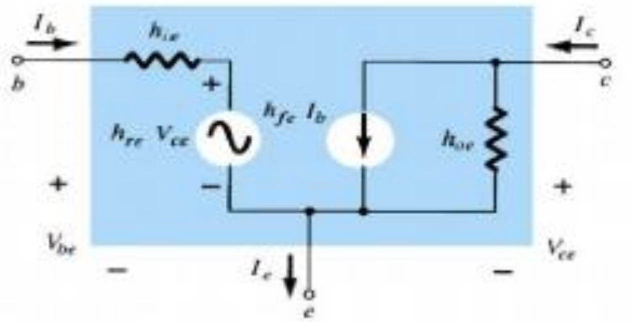


FIG. 5.96
Complete hybrid equivalent circuit.

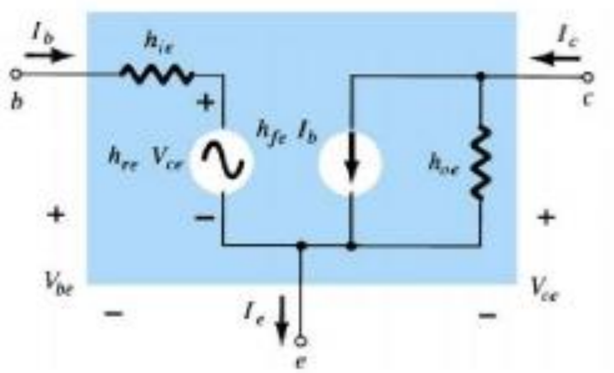
Common Emitter Configuration - hybrid equivalent circuit



(b)

- Essentially, the transistor model is a three terminal two – port system.
- The h – parameters, however, will change with each configuration.
- To distinguish which parameter has been used or which is available, a second subscript has been added to the h – parameter notation.
- For the common – base configuration, the lowercase letter b is added, and for common emitter and common collector configurations, the letters e and c are used respectively.

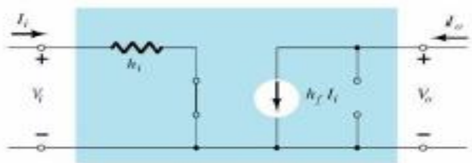
Common Base configuration - hybrid equivalent circuit

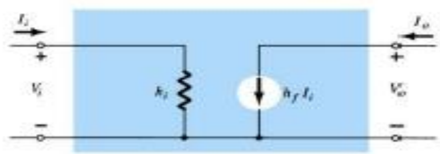


(b)

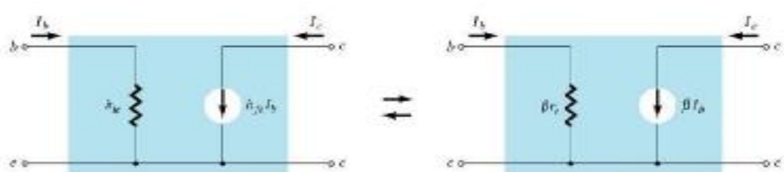
Configuration	I_i	I_o	V_i	V_o
Common emitter	I_b	I_c	V_{be}	V_{ce}
Common base	I_e	I_c	V_{eb}	V_{cb}
Common Collector	I_b	I_e	V_{be}	V_{ec}

- Normally h_r is a relatively small quantity, its removal is approximated by $h_r \cong 0$ and $h_r V_o = 0$, resulting in a short – circuit equivalent.
- The resistance determined by $1/h_o$ is often large enough to be ignored in comparison to a parallel load, permitting its replacement by an open – circuit equivalent.





h-Parameter Model v/s. r_e Model

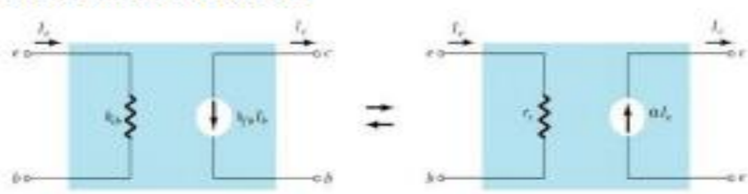


(a)

$$h_{ie} = \beta r_e$$

$$h_{fe} = \beta_{ac}$$

Common Base: r_e v/s. h-Parameter Model



(b)

Common-Base configurations - h-Parameters

$$h_{ib} = r_e$$

$$h_{fb} = -\alpha = -1$$

Fixed-Bias Configuration

For the fixed-bias configuration of Fig. 5.106, the small-signal ac equivalent network will appear as shown in Fig. 5.107 using the approximate common-emitter hybrid equivalent model. Compare the similarities in appearance with Fig. 5.22 and the r_e model analysis. The similarities suggest that the analyses will be quite similar, and the results of one can be directly related to the other.

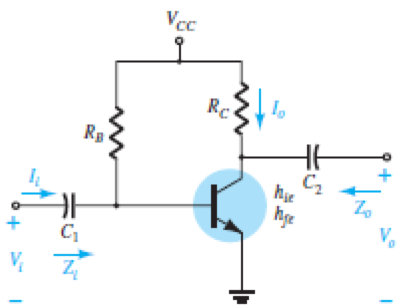


FIG. 5.106
Fixed-bias configuration.

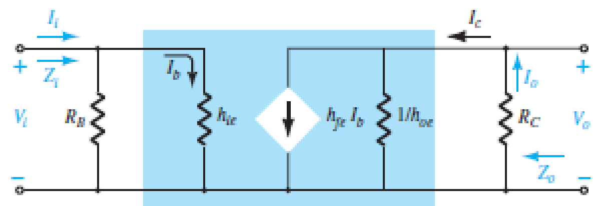


FIG. 5.107
Substituting the approximate hybrid equivalent circuit into the ac equivalent network of Fig. 5.106.

Z_i From Fig. 5.107,

$$\boxed{Z_i = R_B \| h_{ie}} \tag{5.143}$$

Z_o From Fig. 5.107,

$$\boxed{Z_o = R_C \| 1/h_{oe}} \tag{5.144}$$

A_v Using $R' = 1/h_{oe} \| R_C$, we obtain

$$\begin{aligned} V_o &= -I_o R' = -I_C R' \\ &= -h_{fe} I_b R' \end{aligned}$$

and

$$I_b = \frac{V_i}{h_{ie}}$$

with

$$V_o = -h_{fe} \frac{V_i}{h_{ie}} R'$$

so that

$$\boxed{A_v = \frac{V_o}{V_i} = -\frac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}}} \tag{5.145}$$

A_i Assuming that $R_B \gg h_{ie}$ and $1/h_{oe} \geq 10R_C$, we find $I_b \cong I_i$ and $I_o = I_c = h_{fe} I_b = h_{fe} I_i$, and so

$$\boxed{A_i = \frac{I_o}{I_i} \cong h_{fe}} \tag{5.146}$$

Voltage-Divider Configuration

For the voltage-divider bias configuration of Fig. 5.109, the resulting small-signal ac equivalent network will have the same appearance as Fig. 5.107, with R_B replaced by $R' = R_1 \| R_2$.

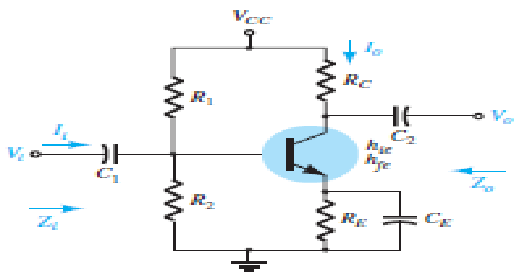


FIG. 5.109
Voltage-divider bias configuration.

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Z_i From Fig. 5.107 with $R_B = R'$,

$$Z_i = R_1 \| R_2 \| h_{ie} \tag{5.147}$$

Z_o From Fig. 5.107,

$$Z_o \cong R_C \tag{5.148}$$

A_v

$$A_v = -\frac{h_{fe}(R_C \| 1/h_{oe})}{h_{ie}} \tag{5.149}$$

A_i

$$A_i = \frac{h_{fe}(R_1 \| R_2)}{R_1 \| R_2 + h_{ie}} \tag{5.150}$$

Unbypassed Emitter-Bias Configuration

For the CE unbypassed emitter-bias configuration of Fig. 5.110, the small-signal ac model will be the same as Fig. 5.30, with βr_e replaced by h_{ie} and βI_b by $h_{fe}I_b$. The analysis will proceed in the same manner.

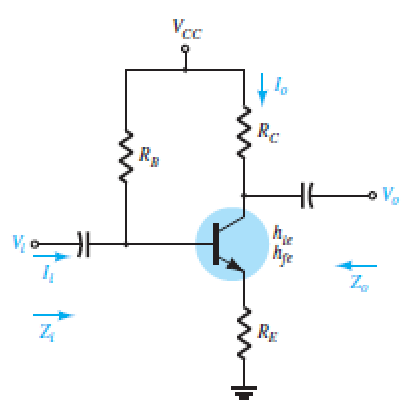

FIG. 5.110
CE unbypassed emitter-bias configuration.

Z_i

$$\boxed{Z_b \cong h_{fe} R_E} \tag{5.151}$$

and

$$\boxed{Z_i = R_B \| Z_b} \tag{5.152}$$

Z_o

$$\boxed{Z_o = R_C} \tag{5.153}$$

A_v

$$A_v = -\frac{h_{fe} R_C}{Z_b} \cong -\frac{h_{fe} R_C}{h_{fe} R_E}$$

and

$$\boxed{A_v \cong -\frac{R_C}{R_E}} \tag{5.154}$$

A_i

$$\boxed{A_i = -\frac{h_{fe} R_B}{R_B + Z_b}}$$

or

$$\boxed{A_i = -A_v \frac{Z_i}{R_C}}$$

Emitter-Follower Configuration

For the emitter-follower of Fig. 5.38, the small-signal ac model will match that of Fig. 5.111, with $\beta r_e = h_{ie}$ and $\beta = h_{fe}$. The resulting equations will therefore be quite similar.

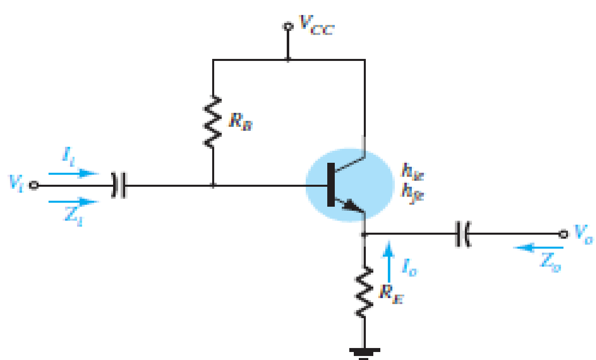

FIG. 5.111
Emitter-follower configuration.

Z_i

$$Z_b \cong h_{fe}R_E \tag{5.157}$$

$$Z_i = R_B \| Z_b \tag{5.158}$$

Z_o For Z_o, the output network defined by the resulting equations will appear as shown in Fig. 5.112. Review the development of the equations in Section 5.8 and

$$Z_o = R_E \| \frac{h_{ie}}{1 + h_{fe}}$$

or, because $1 + h_{fe} \cong h_{fe}$,

$$Z_o \cong R_E \| \frac{h_{ie}}{h_{fe}} \tag{5.159}$$

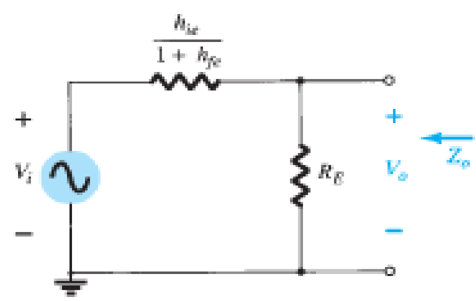

FIG. 5.112
Defining Z_o for the emitter-follower configuration.

A_v For the voltage gain, the voltage-divider rule can be applied to Fig. 5.112 as follows: APP EQ

$$V_o = \frac{R_E(V_i)}{R_E + h_{ie}/(1 + h_{fe})}$$

but, since $1 + h_{fe} \cong h_{fe}$,

$$A_v = \frac{V_o}{V_i} \cong \frac{R_E}{R_E + h_{ie}/h_{fe}} \tag{5.160}$$

A_i

$$A_i = \frac{h_{fe} R_B}{R_B + Z_b} \tag{5.161}$$

or

$$A_i = -A_v \frac{Z_i}{R_E} \tag{5.162}$$

Common-Base Configuration

The last configuration to be examined with the approximate hybrid equivalent circuit will be the common-base amplifier of Fig. 5.113. Substituting the approximate common-base hybrid equivalent model results in the network of Fig. 5.114, which is very similar to Fig. 5.44.

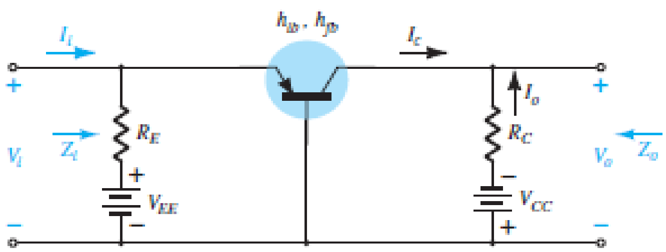

FIG. 5.113
Common-base configuration.

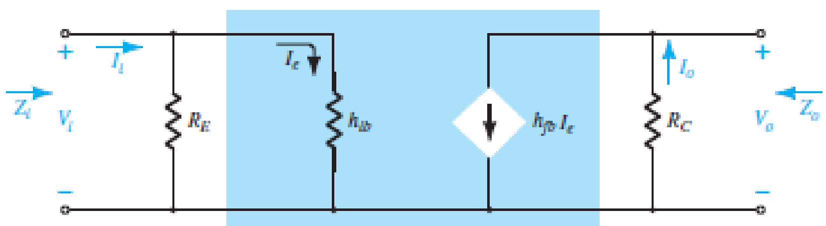

FIG. 5.114
Substituting the approximate hybrid equivalent circuit into the ac equivalent network of Fig. 5.113.

We have the following results from Fig. 5.114.

Z_i

$$Z_i = R_E \| h_{ib} \tag{5.163}$$

Z_o

$$Z_o = R_C \tag{5.164}$$

A_v

$$V_o = -I_o R_C = -(h_{fb} I_e) R_C$$

with $$I_e = \frac{V_i}{h_{ib}} \quad \text{and} \quad V_o = -h_{fb}\frac{V_i}{h_{ib}} R_C$$

so that $$A_v = \frac{V_o}{V_i} = -\frac{h_{fb} R_C}{h_{ib}}$$

A_i

$$A_i = \frac{I_o}{I_i} = h_{fb} \cong -1$$

5.21 COMPLETE HYBRID EQUIVALENT MODEL

The analysis of Section 5.20 was limited to the approximate hybrid equivalent circuit with some discussion about the output impedance. In this section, we employ the complete equivalent circuit to show the effect of h_r and define in more specific terms the effect of h_o. It is important to realize that because the hybrid equivalent model has the same appearance for the common-base, common-emitter, and common-collector configurations, the equations developed in this section can be applied to each configuration. It is only necessary to insert the parameters defined for each configuration. That is, for a common-base configuration, h_{fb}, h_{ib}, and so on, are employed, whereas for a common-emitter configuration, h_{fe}, h_{ie}, and so on, are used. Recall that Appendix A permits a conversion from one set to the other if one set is provided and the other is required.

Consider the general configuration of Fig. 5.116 with the two-port parameters of particular interest. The complete hybrid equivalent model is then substituted in Fig. 5.117 using parameters that do not specify the type of configuration. In other words, the solutions will be in terms of h_i, h_r, h_f, and h_o. Unlike the analysis of previous sections of this chapter, here the current gain A_i will be determined first because the equations developed will prove useful in the determination of the other parameters.

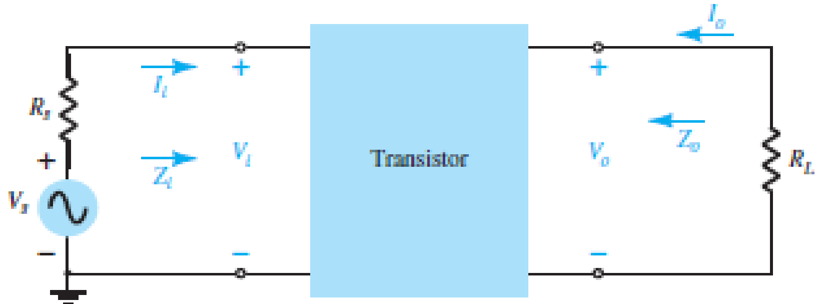

FIG. 5.116
Two-port system.

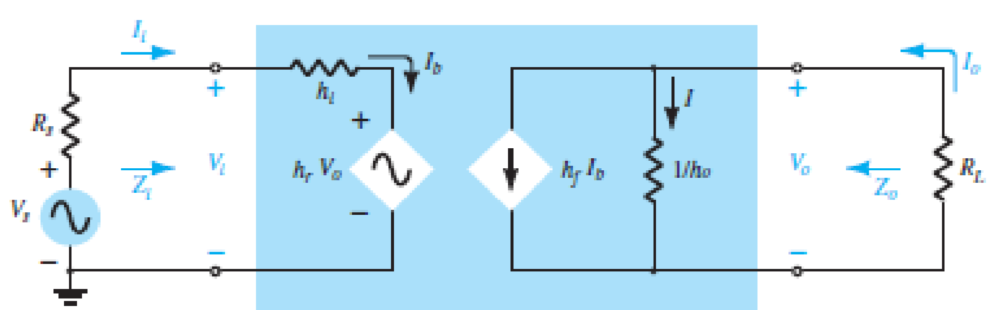

FIG. 5.117
Substituting the complete hybrid equivalent circuit into the two-port system of Fig. 5.116.

Current Gain, $A_i = I_o/I_i$

Applying Kirchhoff's current law to the output circuit yields

$$I_o = h_f I_b + I = h_f I_i + \frac{V_o}{1/h_o} = h_f I_i + h_o V_o$$

Substituting $V_o = -I_o R_L$ gives

$$I_o = h_f I_i - h_o R_L I_o$$

Rewriting the equation above, we have

$$I_o + h_o R_L I_o = h_f I_i$$

and

$$I_o(1 + h_o R_L) = h_f I_i$$

so that

$$\boxed{A_i = \frac{I_o}{I_i} = \frac{h_f}{1 + h_o R_L}} \qquad (5.167)$$

Note that the current gain reduces to the familiar result of $A_i = h_f$ if the factor $h_o R_L$ is sufficiently small compared to 1.

Voltage Gain, $A_v = V_o/V_i$

Applying Kirchhoff's voltage law to the input circuit results in

$$V_i = I_i h_i + h_r V_o$$

Substituting $I_i = (1 + h_o R_L) I_o / h_f$ from Eq. (5.167) and $I_o = -V_o/R_L$ from above results in

$$V_i = \frac{-(1 + h_o R_L) h_i}{h_f R_L} V_o + h_r V_o$$

Solving for the ratio V_o/V_i yields

$$\boxed{A_v = \frac{V_o}{V_i} = \frac{-h_f R_L}{h_i + (h_i h_o - h_f h_r) R_L}} \qquad (5.168)$$

In this case, the familiar form of $A_v = -h_f R_L / h_i$ returns if the factor $(h_i h_o - h_f h_r) R_L$ is sufficiently small compared to h_i.

Input Impedance, $Z_i = V_i/I_i$

For the input circuit,

$$V_i = h_i I_i + h_r V_o$$

Substituting

$$V_o = -I_o R_L$$

we have

$$V_i = h_i I_i - h_r R_L I_o$$

Because

$$A_i = \frac{I_o}{I_i}$$

$$I_o = A_i I_i$$

so that the equation above becomes

$$V_i = h_i I_i - h_r R_L A_i I_i$$

Solving for the ratio V_i/I_i, we obtain

$$Z_i = \frac{V_i}{I_i} = h_i - h_r R_L A_i$$

and substituting

$$A_i = \frac{h_f}{1 + h_o R_L}$$

yields

$$\boxed{Z_i = \frac{V_i}{I_i} = h_i - \frac{h_f h_r R_L}{1 + h_o R_L}} \qquad (5.169)$$

The familiar form of $Z_i = h_i$ is obtained if the second factor in the denominator $(h_o R_L)$ is sufficiently smaller than one.

Output Impedance, $Z_o = V_o/I_o$

The output impedance of an amplifier is defined to be the ratio of the output voltage to the output current with the signal V_s set to zero. For the input circuit with $V_s = 0$,

$$I_i = -\frac{h_r V_o}{R_s + h_i}$$

Substituting this relationship into the equation from the output circuit yields

$$I_o = h_f I_i + h_o V_o$$

$$= -\frac{h_f h_r V_o}{R_s + h_i} + h_o V_o$$

and

$$\boxed{Z_o = \frac{V_o}{I_o} = \frac{1}{h_o - [h_f h_r/(h_i + R_s)]}} \qquad (5.170)$$

In this case, the output impedance is reduced to the familiar form $Z_o = 1/h_o$ for the transistor when the second factor in the denominator is sufficiently smaller than the first.

5.22 HYBRID π MODEL

The last transistor model to be introduced is the hybrid π model of Fig. 5.123 which includes parameters that do not appear in the other two models primarily to provide a more accurate model for high-frequency effects.

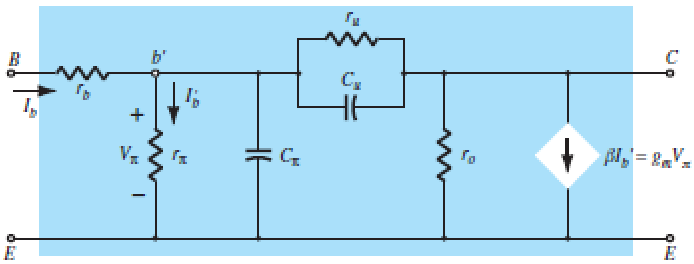

FIG. 5.123
Giacoletto (or hybrid π) high-frequency transistor small-signal ac equivalent circuit.

r_π, r_o, r_b, and r_u

The resistors r_π, r_o, r_b, and r_u are the resistances between the indicated terminals of the device when the device is in the active region. The resistance r_π (using the symbol π to agree with the hybrid π terminology) is simply βr_e as introduced for the common-emitter r_e model. That is,

$$r_\pi = \beta r_e \tag{5.171}$$

The output resistance r_o is the output resistance normally appearing across an applied load. Its value, which typically lies between 5 kΩ and 40 kΩ, is determined from the hybrid parameter h_{oe}, the Early voltage, or the output characteristics.

The resistance r_b includes the base contact, base bulk, and base spreading resistance levels. The first is due to the actual connection to the base. The second includes the resistance from the external terminal to the active region of the transistor, and the last is the actual resistance within the active base region. It is typically a few ohms to tens of ohms.

The resistance r_u (the subscript u refers to the *union* it provides between collector and base terminals) is a very large resistance and provides a feedback path from output to input circuits in the equivalent model. It is typically larger than βr_o, which places it in the megohm range.

C_π and C_u

All the capacitors that appear in Fig. 5.123 are stray parasitic capacitors between the various junctions of the device. They are all capacitive effects that really only come into play at high frequencies. For low to mid-frequencies their reactance is very large, and they can be considered open circuits. The capacitor C_π across the input terminals can range from a few pF to tens of pF. The capacitor C_u from base to collector is usually limited to a few pF but is magnified at the input and output by an effect called the Miller effect, to be introduced in Chapter 9.

$\beta I_b'$ or $g_m V_\pi$

It is important to note in Fig. 5.123 that the controlled source can be a voltage-controlled current source (VCCS) or a current-controlled current source (CCCS), depending on the parameters employed.

Note the following parameter equivalence in Fig. 5.123:

$$g_m = \frac{1}{r_e} \tag{5.172}$$

and

$$r_o = \frac{1}{h_{oe}} \tag{5.173}$$

with

$$\frac{r_\pi}{r_\pi + r_u} \cong \frac{r_\pi}{r_u} \cong h_{re} \tag{5.174}$$

Take particular note of the fact that the equivalent sources $\beta I_b'$ and $g_m V_\pi$ are both controlled current sources. One is controlled by a current at another place in the network and the other by a voltage at the input side of the network. The equivalence between the two is defined by

$$\beta I_b' = \frac{1}{r_e} \cdot r_e \beta I_b' = g_m I_b' \beta r_e = g_m (I_b' r_\pi) = g_m V_\pi$$