Load Flow Analysis

Introduction

* Power flow is the most important study in planning & expansion of power system.
* It is also the study frequently done by the utilities in planning & planning operations
* purpose of the study is to compute the steady state operating cond^ns of the system i.e. voltage magnitude & phase angles at the buses
* From these even line flow (MW & MVAr), real & reactive power supplied by generators & loading of t/fs etc can also be calculated.
* overload cond^ns can be detected
* poor voltages existing in parts of the system can be detected
* angle separation betⁿ generator buses or betⁿ any two buses gives a qualitative idea of the steady state or small disturbance stability of the power system.
* planning Engg perform LFA for diff configurations & loading cond^ns before deciding on final configuration.
* The mathematical model for the study of power flow is the set of non-linear algebraic eq^ns
* These eq^ns can be expressed as a set of real eq^ns with voltage either in the rectangular or polar form.
* n/w matrix used in deriving the above eqⁿ is generally Y_{BUS}
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* When loads and/or generators are connected at the buses, these give 5 rise to the post-constraints.

* We can specify the constraints in terms of P_i, Q_i – the net injected real & reactive power respectively – or $|V_i|$ the magnitude of voltages.

* voltage magnitude at a bus can be kept cost. through a voltage regulator as in case of a generator bus or by means of tap changing tfr in the case of a load bus

* we also need a reference bus in the system whose phase angle is zero

* at any bus there are 4 variables

  P_i, Q_i – injected real & reactive powers &

  $|V_i|$ & δ_i – voltage magnitude & the phase angle of voltage at the bus

* two of these are specified at every bus, & this gives rise to the classification of buses as follows

Real and reactive powers can now be expressed as

$$P_i \text{ (real power)} = |V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \quad i = 1, 2 \cdots n$$

$$Q_i \text{ (reactive power)} = -|V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) \quad i = 1, 2 \cdots n$$

* These two eq^{ns} represent 2n power flow equations at n buses of the power system
* Each bus is characterised by four variables : P_i, Q_i, $|V_i|$ & δ_i resulting into 4n variables
* above two eq^{n}s can be solved for 2n variables if remaining 2n variables are specified
* Practical considerations allow a power system analyst to fix a priori two variables at each bus.
* The sol^n for remaining 2n bus variables is rendered difficult as the eq^{n}s are non-linear algebraic eq^{n}s (bus voltages are involved in product form & sine & cosine terms are present)
* therefore explicit sol^n is not possible
* sol^n can only be obtained by iterative numerical techniques

Depending upon two variables specified ..., the buses are classified into three categories

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I) PQ bus /- Load bus

* at all these type of buses Powers P_i & Q_i are known

(P_{Di} & Q_{Di} are known from load forecasting & P_{Gi} & Q_{Gi} are specified)

* The unknowns are $|V_i|$ & δ_i

* A pure load bus (no generating facility at the bus) i.e. $P_{Gi} = Q_{Gi} = 0$) is a PQ bus.

) PV bus / Generation bus / Voltage controlled bus

* at this bus P_{Di} & Q_{Di} are known a prior & $|V_i|$ & P_i are specified.

* The unknown are Q_i & δ_i.

Slack bus / Swing bus / Reference bus -

* $|V_i|$ & δ_i are specified at this bus (normally δ_i is set equal to zero)

* P_i & Q_i are unknown

* normally there is only one bus of this type in a given power system.

need of slack bus

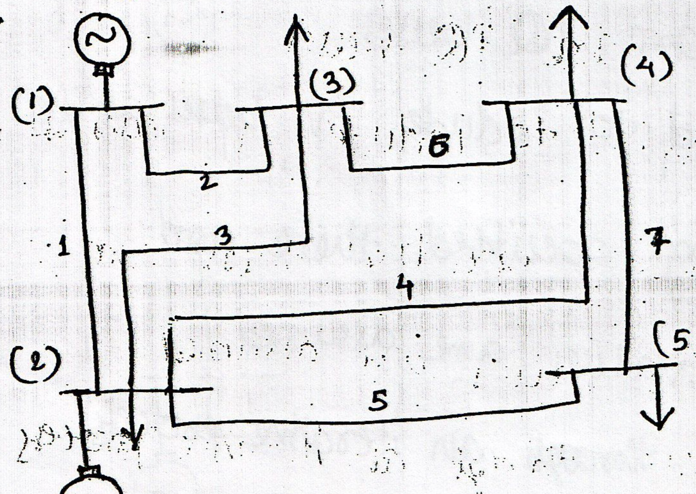
* real & reactive powers can't be fixed prior at all the buses as net complex power flow into the n/w is not known advance, the system power loss being unknown till LFA is complete.

* therefore it is necessary to have one bus at which complex power is unspecified, so that it supplies the ~~power specified~~ difference in the total system load plus losses & the sum of the complex powers specified at the remaining buses

complex power at slack bus = (Total system load + losses) − (sum of the complex powers specified at the remaining buses)

* for the same reason slack bus must be a generator bus
* The complex power allocated to this bus is determined as part of the soln.
* In order that the variations in real & reactive powers of the slack bus during the iterative process be small % of its generating capacity, the bus connected to the largest generating station is normally selected as the slack bus

e.g.



Bus No	Bus voltage	Gen. (MW)	Gen (MVAR)	Load (MW)	Load (MVAR)	Load
1	1.06+j0.0	0	0	0	0	slack
2	1.00+j0.0	40	30	20	10	
3	1.00+j0.0	0	0	45	15	load
4	1.00+j0.0	0	0	40	5	load
5	1.00+j0.0	0	0	60	10	load

G-S method

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* It is an iterative algorithm for solving a set of non-linear algebraic eq^n
* to start with, sol^n vector is assumed based on practical experience
* the eq^n is used to obtain the revised value of a particular variable by substituting in present values of the remaining variable
* The process is then repeated for all the variables thereby completing one iteration.
* Iterative process is then repeated till the sol^n vector converges within prescribed accuracy
* convergence is quite sensitive to the starting values assumed
* To apply G-S method to load flow studies, let all buses except slack are PQ buses
* method can easily adopted to include PV buses as well
* the slack bus voltage being specified, there are (n-1) bus voltages starting values are assumed
* These values are then updated through an iterative process
* voltage at i^{th} bus is obtained as follows

$$V_i = \frac{(P_i - jQ_i)}{V_i^*} \quad \text{— (1)}$$

we also have

$$J_i = \sum_{k=1}^{n} Y_{ik} \cdot V_k \quad \text{—— (2)} \quad i = 1, 2 \ldots n$$

from this eq^n

$$Y_{ii} \cdot V_i = J_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} \cdot V_k$$

&

$$V_i = \frac{1}{Y_{ii}} \left[J_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad \text{—— (3)}$$

substituting for J_i eq^n (1) in eq^n (3)

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad i = 2, 3 \ldots n \quad \text{—— (4)}$$

* during each iteration voltages at all PQ buses is updated by eq^n (4).
* V_1 is slack bus voltage & it is fixed
* Iterations are repeated till no bus voltage magnitude changes more than a prescribed value,
* The computation process is then said to converge to a sol^n

* Computation of slack bus power —

12 substituting all bus voltages computed in previous step along with V_1 in the following eqn. yields slack bus power

$$P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} \cdot V_k \qquad i = 1, 2 \ldots n \quad \text{— (4a)}$$

When PV buses are also present

At PV buses P & |V| are specified
+ Q & δ are unknown

* values of Q & δ are to be updated in every as iteration through appropriate bus eqn.

for ith PV bus

$$Q_i = -\text{Im}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\} \quad \text{— (5)}$$

$Q_2 = -\text{Im} \; V_2^* \left[Y_{21} \cdot V_1 + Y_{22} V_2 + Y_{23} V_3 + Y_{24} V_4 \right]$

* The revised value of Q_i is obtained by substituting most updated values of voltages on R.H.S

* The revised value of δ_i is obtained from

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad \text{— (6)}$$

and $\delta_i = \angle V_i$

* owing to physical limitations of P & Q generation source P_{Gi} & Q_{Gi} are constrained as follows

$$P_{Gi\,min} \leq P_{Gi} \leq P_{Gi\,max}$$

$$Q_{Gi\,min} \leq Q_{Gi} \leq Q_{Gi\,max}$$

* Voltage at PV bus can be maintained cost only if controlled Q source is available at the bus & reactive power generation required is within prescribed limits

* If $Q_i^{(r+1)} < Q_{i\,min}$, set $Q_i^{(r+1)} = Q_{i\,min}$ & treat bus i as a PQ bus & compute $V_i^{(r+1)}$ using appropriate eq^n.

* If $Q_i^{(r+1)} > Q_{i\,max}$, set $Q_i^{(r+1)} = Q_{i\,max}$ & treat bus i as PQ bus & compute $V_i^{(r+1)}$

* In order to save compute time, we can perform ~~the~~ in advance all the arithmatic operations that do not change with the iterations

Define

$$A_i = \frac{P_i - jQ_i}{Y_{ii}} \qquad i = 2, 3 \cdots n \quad — (7)$$

$$B_{ik} = \frac{Y_{ik}}{Y_{ii}} \qquad i = 2, 3 \cdots n \quad k = 1, 2 \cdots n \quad k \neq i \quad — (8)$$

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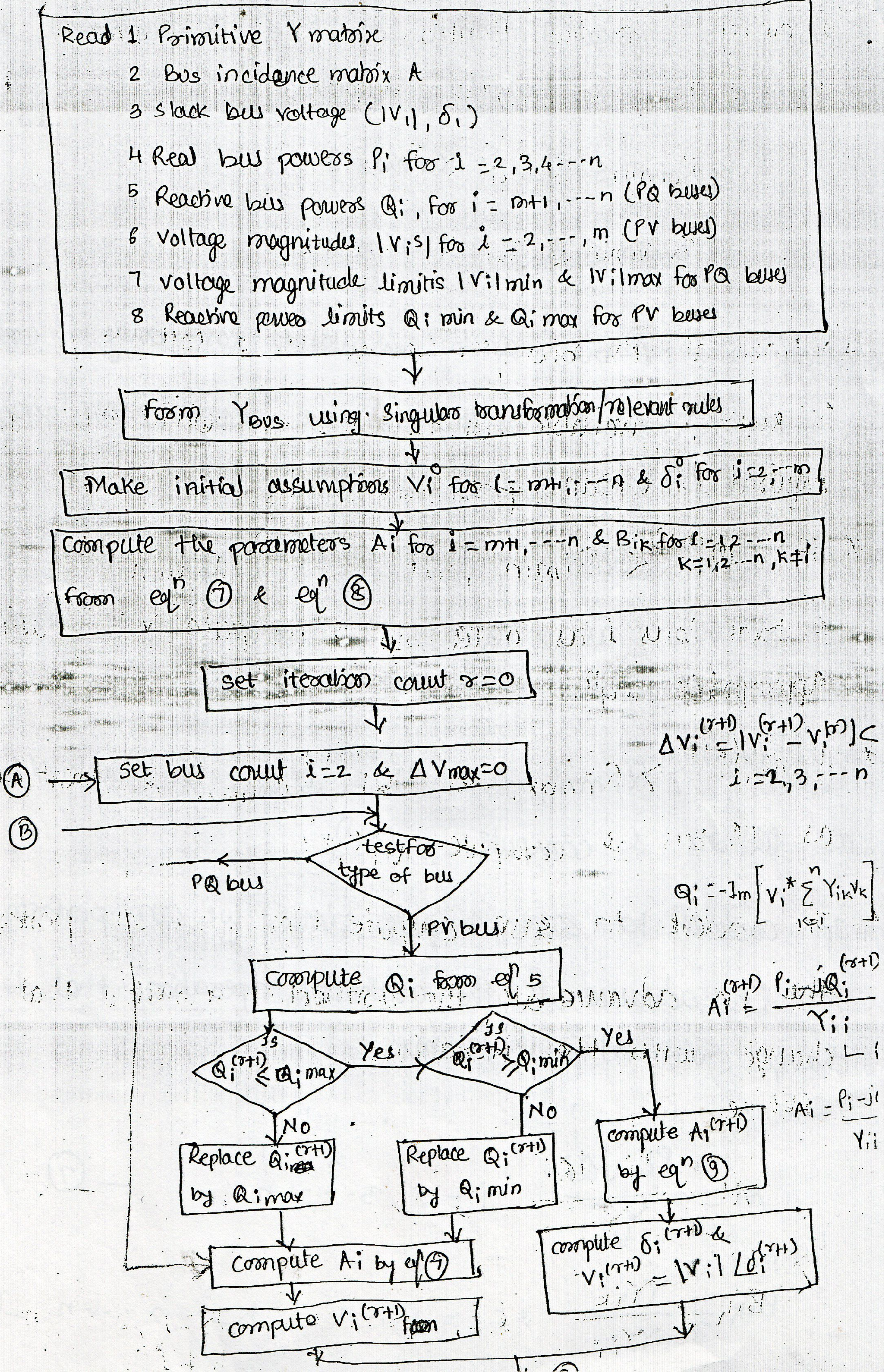
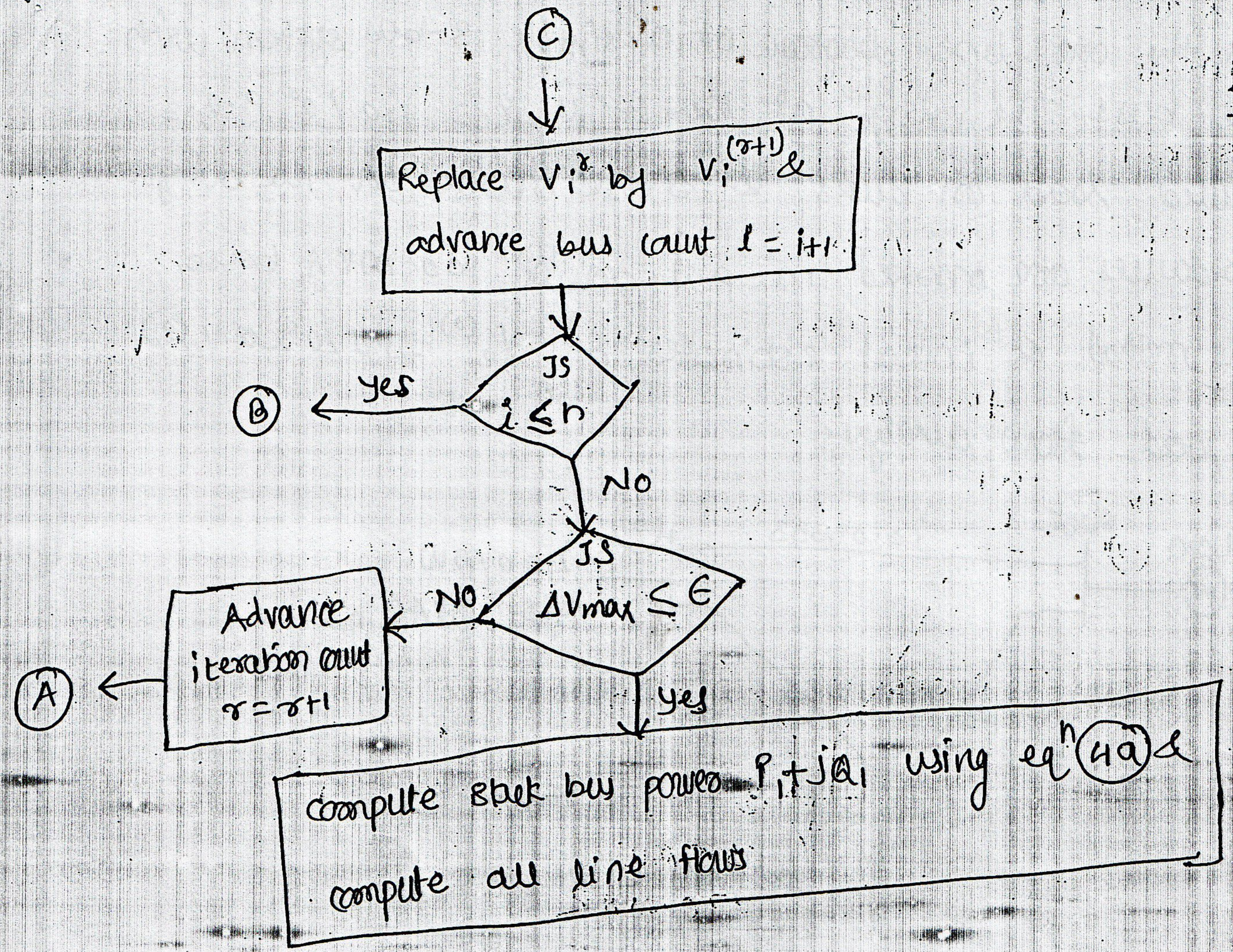

Read 1 Primitive Y matrix
2 Bus incidence matrix A
3 Slack bus voltage (|V1|, δ1)
4 Real bus powers Pi for i = 2,3,4---n
5 Reactive bus powers Qi, for i = m+1,---n (PQ buses)
6 Voltage magnitudes |Vis| for i = 2,---,m (PV buses)
7 Voltage magnitude limits |Vi|min & |Vi|max for PQ buses
8 Reactive power limits Qi min & Qi max for PV buses
form Ybus using singular transformation/relevant rules
Make initial assumptions Vi0 for i = m+1,---n & δi0 for i=2,---m
Compute the parameters Ai for i = m+1,---n & Bik for i=1,2---n, k=1,2---n, k≠i
from eqn 7 & eqn 8
set iteration count r=0
A
set bus count i=2 & ΔVmax=0
B
test for type of bus
PQ bus
PV bus
compute Qi from eqn 5
Is Qi(r+1) ≤ Qi max
Yes
Is Qi(r+1) ≥ Qi min
Yes
No
No
Replace Qi(r+1) by Qi max
Replace Qi(r+1) by Qi min
compute Ai(r+1) by eqn 9
Compute Ai by eqn 7
compute δi(r+1) & Vi(r+1) = |Vi| ∠δi(r+1)
compute Vi(r+1) from
ΔVi(r+1) = |Vi(r+1) − Vi(r)| <
i = 2,3,---n
Qi = −Im[Vi* Σ Yik Vk]
Ai(r+1) = (Pi − jQi(r+1)) / Yii
Ai = (Pi − j



Flow chart for load flow soln by the Gauss Seidel iteration method using Y_{bus}

Fig shows the one line diagram of a simple 3-bus system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.0 \angle 0°$ p.u.

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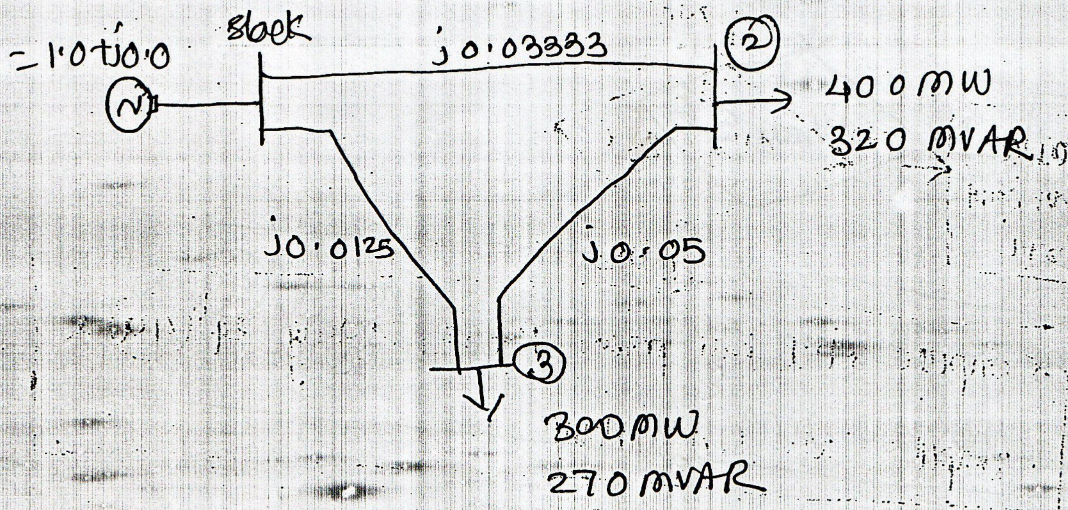
The scheduled load on buses 2 - 3 are marked on the diagram. Line impedances are marked in p.u. on a 100 MVA base. Using G-S method & initial estimates of $V_2^{(0)} = 1.0 \angle 0°$ & $V_3^{(0)}$ conduct load flow analysis



Solⁿ: line impedances are converted to line admittance as

$y_{12} = -j30.003 \qquad y_{23} = -j20 \qquad y_{13} = -j80$

$Y_{11} = y_{12} + y_{13} = -j30 - j80 = -j110$

$Y_{12} = -(-j30) = j30 \qquad Y_{13} = -(-j80)$

$Y_{22} = y_{12} + y_{23} = -j30 - j20 = -j50$

$Y_{23} = y_{23} = -(-j20)$

$Y_{33} = y_{23} + y_{13} = -j20 - j80 = -j100$

$MVA_{base} = 100$ MVA

Complex powers at PQ buses are

$$S_2 = \frac{400 + j320}{100} = (-0.4 - j3.2) \text{ P.U.}$$

$$P_{D2} = -4.0 \quad Q_{D2} = j3.2$$

llly at bus 3

$$P_{Di} = \frac{300}{100} = 3.0 \text{ PU}$$

$$Q_{Di} = \frac{270}{100} = j2.7 \text{ P.U.}$$

$$\therefore P_i = P_{Gi} - P_{Di} = 0 - 3 = -3$$

$$Q_2 = Q_{G2} - Q_{D2} = 0 - j2.7 = -j2.7$$

$$V_2^{(1)} = 0.936004 - j0.07999 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3\right]$$

$$V_3^{(1)} = 0.960201 - j0.045999$$

At bus 1

$$P_1 - jQ_1 = V_1^* \left[V_1 (y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)\right]$$

$$\approx 6.99822 - j6.995 \text{ PU}$$

Line flows & line losses –

$-2.397 + j1.92$

Line current – $I_{12} = y_{12}(V_1 - V_2) = (-j30.003)[1.0 - [0.936004 - j0.07999]] = 2.99904 - j2.997$

$$I_{21} = -I_{12} = -2.999 + j2.997$$

$$I_{13} = y_{13}(V_1 - V_3) = (-j80)(1.0 - (0.960201 - j0.045999))$$

Acceleration of convergence -

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* In GS method convergence can sometimes be speeded up by the use of acceleration factor
* at ith bus, the accelerated value of voltage at $(r+1)^{th}$ iteration is given by

$$V_i^{(r+1)} \text{(accelerated)} = V_i^{(r)} + \alpha\left(V_i^{(r+1)} - V_i^{(r)}\right)$$

* where α is a real no. called the acceleration factor.
* generally recommended value of α is = 1.6
* a wrong choice of α may slow down convergence or even cause the method to diverge.

$I_{31} = -I_{13} = -3.9992 + j\,3.99792$

$= -0.99936 + j\,0.99838$

$I_{23} = y_{23}(V_2 - V_3) = (-j20)\cdot($

$I_{32} = -y_{23} =$

The line flows are in pu & also in MW, Mvar

$S_{12} = V_1 \cdot I_{12}^* =$

$S_{21} = V_2 \cdot I_{21}^* =$

$S_{13} = V_1 \cdot I_{13}^* =$

$S_{31} = V_3 \cdot I_{31}^* =$

$S_{23} = V_2 \cdot I_{23}^* =$

$S_{32} = V_3 \cdot I_{32}^* =$

Line losses

$S_{L12} = S_{12} + S_{21} = j\,59.918$

$S_{L13} = S_{13} + S_{31} = j\,39.97$

$S_{L23} = S_{23} + S_{32} = j\,9.978$

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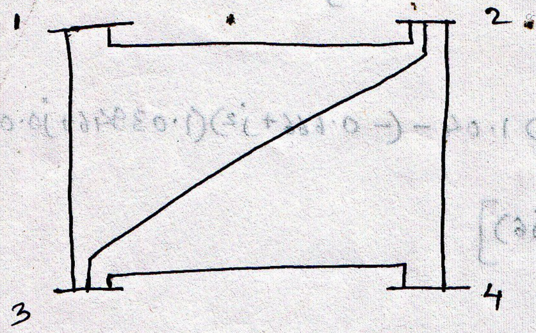
$S \quad\quad S + S \quad = \quad j\,9.978$

Perform the load flow analysis for the power system show below.



Line	R pu	G	X pu	
1-2	0.05	2	0.15	-6
1-3	0.1	1.0	0.3	-3
2-3	0.15	0.666	0.45	-2
2-4	0.1	1.0	0.3	-3
3-4	0.05	2.0	0.15	-6

$$Y_{Bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Bus	P_i pu	Q_i pu	V_i pu	Remarks
1	–	–	$1.04\angle 0^\circ$	slack
2	0.5	~~[illegible]~~	1.04	PQ
3	-1	0.5	–	PQ
4	0.3	-0.1	–	PQ

$0.2 \le Q_2 \le 1$

~~G.S~~

for bus 2

$$Q_2' = -\mathrm{Im}\left\{(V_2^0)^* Y_{21}V_1 + V_2^{0*}\left[Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4\right]\right\}$$

$$= -\mathrm{Im}\left\{1.04\,(-2+j6)\,1.04 + 1.04\left[(3.666-j11)\,1.04 + (-0.666+j2) + (-1+j3)\right]\right\}$$

$$= -\mathrm{Im}\left\{-0.0693 - j\,0.2079\right\} = 0.2079 \text{ pu}$$

$$Q_2' = 0.2079 \text{ pu}$$

$$\delta_2' = \angle\left\{\frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2'}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]\right\}$$

$$= \angle\left\{\frac{1}{3.666-j11}\left[\frac{0.5-j0.2079}{1.04-j0} - (-2+j6)(1.04+j0) - (-0.666+j2)(1+j0) - (-1+j3)(1+j0)\right]\right\}$$

$$= \left[\frac{4.2267 - j\,11.439}{3.666 - j11}\right] = \angle(1.0512 + j0.0339) = 1.05174\angle 1.847$$

$\delta_2' = 1.847$ $\quad \therefore V_2 = 1.04(\cos\delta_2' + j\sin\delta_2')$

$V_2 = 1.04\angle 1.847 = 1.03945 + j0.0335$

$$V_3' = \frac{1}{Y_{33}}\left\{\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2' - Y_{34}V_4^0\right\}$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1 - j0.5}{(1 - j0)} - (-1 + j3)\,1.04 - (-0.666 + j2)(1.03946 + j0.03351) - (-2 + j6)\right]$$

$$= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937$$

$$V_4' = \frac{1}{Y_{44}}\left\{\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2' - Y_{43}V_3'\right\}$$

$$= \frac{1}{3 - j9}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)(1.0394 + j0.0335) - (-2 + j6)(1.0317 j0.08937)\right]$$

$$V_4' = \frac{2.9671 - j8.9962}{3 - j9} = 0.9985 - j0.0031 \quad (1.0342 - j0.01502)?$$

Now suppose limits of Q_2 are revised as follows $0.25 \le Q_2 \le 1.0$ pu

calculated $Q_2 = 0.2079$ is less than Q_{2min}

$\therefore Q_2 = 0.25$ pu

now bus 2 becomes PQ bus from a PV bus

$P_2 = 0.5$, $Q_2 = 0.25$ & $V_2 \angle \delta_2 = ?$

$$\therefore V_2' = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j11}\left[\frac{0.5 - j0.25}{1 - j0} - (-2 + j6)\,1.04 - (-0.666 + j2) - (-1 + j3)\right]$$

$$= \frac{4.246 - j11.49}{3.666 - j11} = 1.0559 + j0.0341$$

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2' - Y_{34}V_4^0\right] = 1.0347 - j0.0893 \text{ pu}$$

$$V_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2' - Y_{43}V_3'\right] = 1.0775 + j0.0923 \text{ pu}$$

Newton-Raphson Method using Y_{Bus} in polar coordinates —

* N-R method is a powerful method of solving non-linear algebraic eq^ns
* It works faster & is sure to converge compared to GS method
* It is the practical method of load flow sol^n of large power n/ws
* It's only drawback is the large requirement of computer m/m
* Convergence can be considerably speeded up by performing the 1st iteration through GS method & using those values for starting N-R iterations
* The general form of N-R method is as under

consider a set of n non-linear algebraic eq^ns

$$f_i(x_1, x_2, \ldots x_n) = 0 \; ; \; i = 1, 2 \ldots n \quad — (1)$$

*Assume initial values of unknowns as $x_1^0, x_2^0, \ldots x_n^0$

let $\Delta x_1^0, \Delta x_2^0, \ldots \Delta x_n^0$ be the correction

actual sol^n = assumed value + the correction

$$= x_i^0 + \Delta x_i^0$$

$$\therefore f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \ldots \ldots x_n^0 + \Delta x_n^0) = 0 \; ; \; i = 1, 2 \ldots n \quad — (2)$$

Expanding these eq^ns in Taylor series around initial guess, we have

$$f_i(x_1^0, x_2^0 \ldots, x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right)^0 \Delta x_2^0 + \ldots + \left(\frac{\partial f_i}{\partial x_n}\right)^0 \Delta x_n^0\right] + \text{higher order terms} = 0$$

— (3)

where $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0 \cdots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the derivatives of f_i

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w.r.t. $x_1, x_2 \cdots x_n$ evaluated at $(x_1^0, x_2^0 \cdots x_n^0)$

Neglecting higher order terms we can write eqⁿ (3) in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \vdots \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & & & \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \simeq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

or in a vector form

$$F^0 + J^0 \Delta X^0 \simeq 0 \quad \text{—— (4)}$$

J^0 – known as Jacobian matrix

eqⁿ (4) can be written as

$$F^0 \simeq [-J^0] \, \Delta X^0 \quad \text{—— (5)}$$

appropriate value of correction i.e. ΔX^0 is obtained from eqⁿ (5) as

$$\Delta X^0 \simeq f^0 \cdot [J^0]^{-1}$$

updated value of x are then