1.1 Introduction

A power system mainly consists of generating stations, transmission lines and distribution systems. Generating stations and distribution systems are connected through transmission lines, which also connect one power system grid to another. A distribution system connects all loads in a particular area to the transmission lines.

A three phase power system is said to be symmetrical, when the system viewed from any phase is similar. This means that, in symmetrical systems, the self impedances of all the three phases are equal and the mutual impedances, if any, between the three phases are same. The three phase voltages (or currents) are said to be balanced if the three voltages (or currents) are equal in magnitude and have the same phase angle difference with respect to each other. In symmetrical, balanced three phase systems, the phase angle difference between the voltages (or currents) will be equal to 120° electrical.

For planning the operation, improvement and expansion of power system, it is required to make a thorough analysis of the system. This necessitates the modeling of the power system network. A complete model of a large interconnected power system representing all the three phases becomes too complicated, rendering it almost impossible for analysis. However, because of symmetry of the system and the balanced nature of the voltages, a three phase, symmetrical, balanced system can be reduced to a single phase system for the purpose of analysis. This results in considerable simplification of the three phase network.

1.2 Circuit Models of Power system components

Synchronous machines, transformers, transmission lines, static and dynamic loads are the major components of a power system. In this section, the single phase equivalent circuits of these components are discussed in brief.

a) Equivalent circuit of a synchronous machine (non-salient type)

The three-phase equivalent circuit of a synchronous generator depicting the voltage generated and the impedance of each phase is shown in fig. 1.1(a). Let us consider the generator to be balanced and perfectly symmetrical, then $E_a=E_b=E_c=E_g$ (say) and $Z_a=Z_b=Z_c=Z_s$ (say). Therefore, the three phase network can be replaced by a single phase network as shown in fig. 1.1(b).

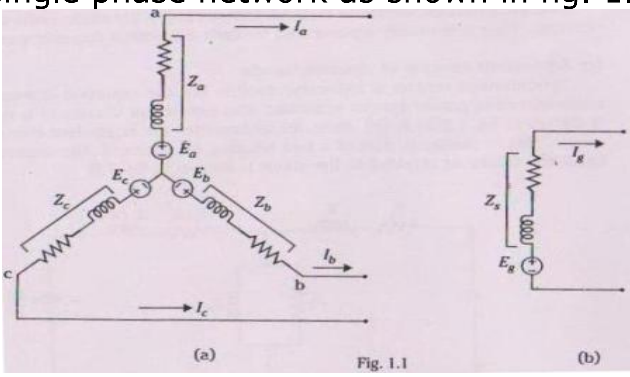
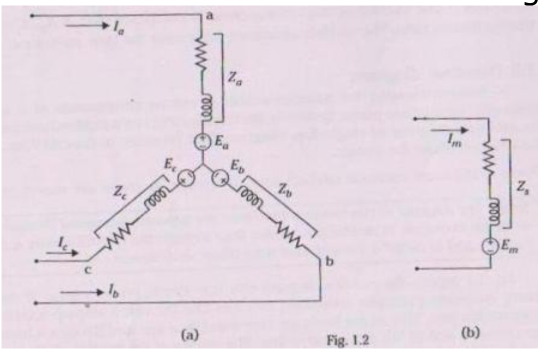


Fig. 1.1

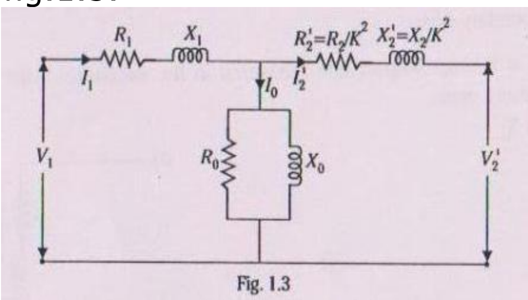
Note:

A synchronous motor receives electrical power and converts it into mechanical power. Therefore the direction of current in motor is opposite to that of generator. Hence its equivalent circuits are as shown in fig. 1.2(a) and (b)



b) Equivalent circuit of a two winding transformer

The well known equivalent circuit of a two winding transformer referred to its primary side as shown in fig.1.3.



Here, R_1 and X_1 are the resistance and reactance of the primary side, R_2 and X_2 are the resistance and reactance of the secondary side. $K=N_2/N_1=V_2/V_1$ is the voltage transform ratio. R_0 and X_0 constitute the exciting circuit of the transformer.

c) Equivalent circuit of a transmission line.

The transmission line is represented usually by its nominal p-circuit. This is shown in fig. 1.4.

Where,

Z= total series impedance of the line per phase.

Y= total shunt admittance per phase.

V_s and I_s = sending end voltage and current respectively.

V_R and I_R = receiving end voltage and current respectively.

d) Equivalent circuit of a three winding transformer.

Both the primary and secondary winding of a two winding transformer have the same kVA rating, but all three windings of a three winding transformer may have different kVA ratings.

The symbol of a three winding transformer is shown in fig. 1.5(a). The three windings are designed as primary, secondary and tertiary windings. The

impedances of these windings are connected in star to represent the single-phase equivalent circuit (with magnetizing current neglected) as shown in fig. 1.5(b). The common point is fictious and unrelated to the neutral of the system.
Let
Z_{ps}= leakage impedance measured in the primary with secondary short circuited and tertiary open.
Z_{pt}= leakage impedance measured in the primary with tertiary short circuited and secondary open.
Z_{st}= leakage impedance measured in the secondary with tertiary short circuited and primary open.

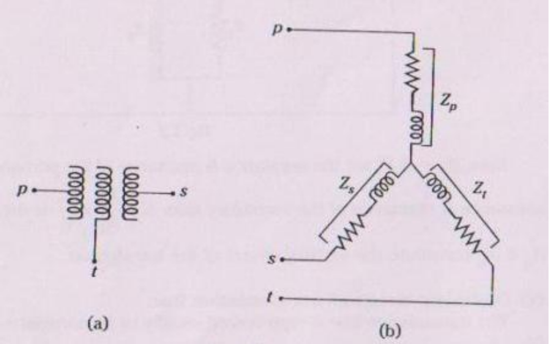


Fig. 1.5

Then, from transformer theory, it can be proved that,
$Z_{ps}=Z_p+Z_s$ 1.1
$Z_{pt}=Z_p+Z_t$ 1.2
$Z_{st}=Z_s+Z_t$ 1.3

Equations 1.1 + 1.2 – 1.3 yield,
$Z_{ps}+ Z_{pt} – Z_{st} = 2Z_p$
Or
$Z_p = ½ (Z_{ps}+ Z_{pt} – Z_{st})$ 1.4
Similarly,
$Z_s= ½ (Z_{st}+Z_{ps}–Z_{pt})$.....................1.5
And
$Z_t= ½ (Z_{st}+Z_{pt}–Z_{ps})$ 1.6

The above formulae are used to compute the impedances of the three windings.

Applications of three winding transformers
i) They are used for interconnecting three transmission lines, each working at different voltage and power levels.
ii) Find extensive utilization in high voltage laboratories.
iii) Static capacitors or synchronous condensers may be connected to tertiary windings for reactive power injection into the system.

e) Equivalent circuits of static loads

Electric furnaces, induction heaters, lamps etc are the static loads on a power system network. They are usually represented by their equivalent impedances. (Example 1.9)

f) Equivalent circuits of dynamic loads

Synchronous motors and induction motors are the common dynamic loads that are encountered in power system networks. The equivalent circuit of a synchronous motor is shown in fig. 1.2(a) and (b). Now, let us consider the equivalent circuit of an induction motor. This is similar to that of a two winding transformer. The equivalent circuit of an induction motor as referred to the stator is shown in fig. 1.6.

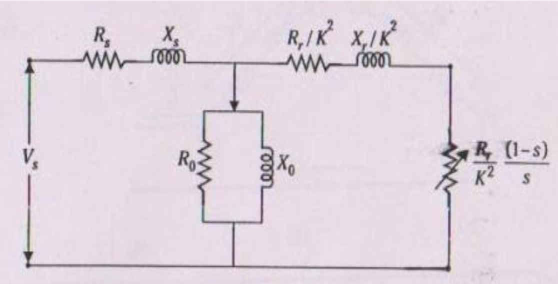


Fig. 1.6

Here,

R_s and X_s denotes stator resistance and reactance respectively. R_r and X_r are rotor resistance and reactance. The exciting or magnetizing circuit is composed of R_0 and X_0. 'K' is the voltage transformation ratio. The variable resistance represents the load on the motor.

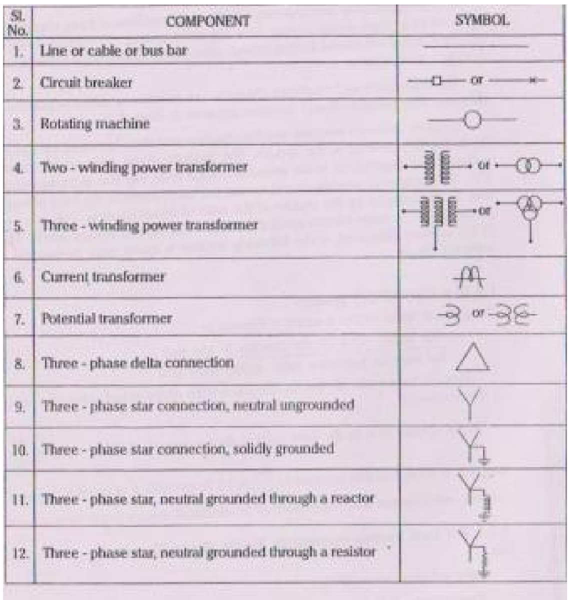
1.3 One line diagram

A diagram showing the interconnection of various components of a symmetrical, balanced, three-phase power system by standard symbols on a single-phase basis is called as one-line diagram or single-line diagram. This provides, in concise form, significant information about the system.

Some of the most common symbols used in one-line diagrams are shown in table 1.1

Note: The neutrals of synchronous machines are generally grounded through resistors or inductance coils to reduce the current flow through the neutral during a fault. The coil so used is called a ground fault neutralizer, or Petersen coil.

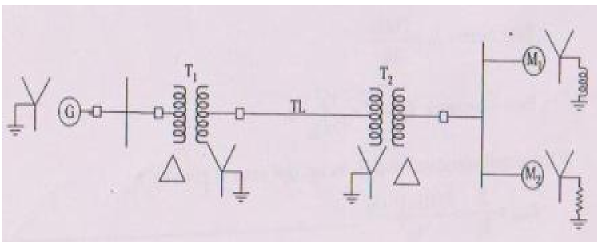
Fig. 1.7. depicts the one-line diagram of a very simple power system. It consists of a solidly grounded generator connected to a bus and through a step-up transformer to a transmission line. Two motor loads are connected to a bus and through a transformer to the opposite end of the transmission line. The ratings of the machines, their reactances and few relevant datas are also shown.



Sl. No.	COMPONENT	SYMBOL
1.	Line or cable or bus bar	
2.	Circuit breaker	or
3.	Rotating machine	
4.	Two - winding power transformer	or
5.	Three - winding power transformer	or
6.	Current transformer	
7.	Potential transformer	or
8.	Three - phase delta connection	
9.	Three - phase star connection, neutral ungrounded	
10.	Three - phase star connection, solidly grounded	
11.	Three - phase star, neutral grounded through a reactor	
12.	Three - phase star, neutral grounded through a resistor	

Table 1.1

G: 300MVA, 20kV, X”=1.2Ω
T_1: 350MVA, 230V Y/20kV Δ, X=15.2 Ω/ph
T_2: 300MVA, 230V Y/ 13.2kV Δ, X=16 Ω/ph
TL: l=64km, XTL= 0.5 Ω/km
M_1: 200MVA, 13.2kV, X”=1.6 Ω
M_2: 100MVA, 13.2kV, X”= 1.6 Ω
Fig 1.7



1.4 Impedance and reactance diagrams

The one line diagram provides a concise information about the system. The performance of the system load conditions or upon the occurrence of a short circuit cannot be directly calculated using the one-line diagram. It is necessary to obtain an equivalent circuit of the system for the purpose of analysis under the aforesaid conditions. The impedance and reactance diagram enter the screen at this juncture.

The impedance diagram is obtained by replacing each component of the power system by its single-phase equivalent circuit. The synchronous machine is represented by and emf source in series with an appropriate impedance. The transmission line replaced by its equivalent p-circuit, transformer y its equivalent circuit and loads by their equivalent impedances.

The simplified diagram got after omitting all resistances, the magnetizing circuit of the transformer and the capacitance of the transmission line in the impedance diagram is called as the reactance diagram. The simplified representation of some of the important components of power system in reactance diagram are shown in table 1.2

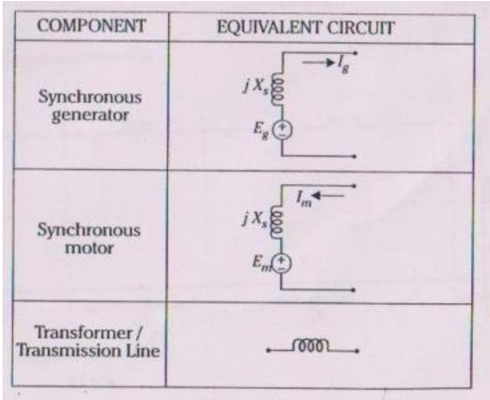


Table 1.2

Of course, omission of resistance introduces some error in the analysis, But, the results may be in almost all cases satisfactory, since the reactance of the system is much larger than its resistance.

Fig 1.8 shows the one line diagram, impedance diagram and reactance diagram of a sample power system.

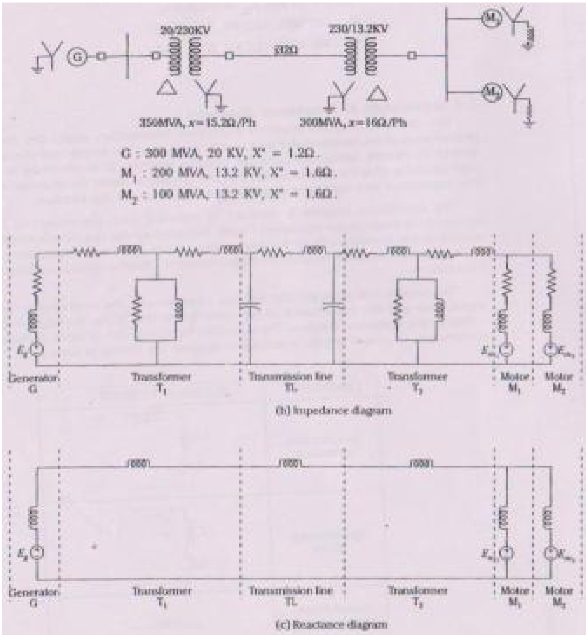


Fig. 1.8

It is re-emphasized here that only a balanced, symmetrical three phase system can be reduced to a single-phase system. In a balanced system, no current flows through the neutral and hence the current limiting impedances are not shown in the equivalent impedance and reactance diagrams.

The impedance and reactance diagram are sometimes called as positive-sequence diagram. This designation will become apparent in chap. 4.

It can be observed from the one line diagram that there are three voltage levels (13.2, 20 and 230 kV) present in the system. The analysis would proceed by transforming all voltages and impedances to ant selected voltage level, say that of transmission line (230 kV). The voltages of generators and motors are transformed in the ratio of transformation and all impedances by the square of the ratio of transformation. This is a very tedious procedure for a large interconnected network with several voltage levels. The per unit (P.U) method discussed in the following section is found quite convenient for power system analysis.

1.5 Per Unit (P.U) system

The per unit value of any quantity is defined as:

the actual value of the quantity in any unit
----
the base or reference value in the same unit

since the base value always has the same units as the actual value, the per unit value is dimensionless.

If we choose, 50A as the base current, then a current of 30 A is equal to 30/50 = 0.6 in per unit, a current of 80A is equal to 80/50= 1.6 in per unit. It is usual to express voltage, current, volt-amperes and impedance of an electrical system in per unit quantities.

1.5.1 Per Unit system applied to single phase circuits

Let,
Base voltamperes $=(VA)_B$
Base voltage $= V_B$
then
Base current $I_B= (VA)_B/V_B$
Base impedance $Z_B=V_B/I_B=V^2_B/(VA)_B$ Ω
If the actual impedance is Z Ω, its per unit value is given by
$Z_{p.u} = Z/Z_B= Z(\Omega)\times(VA)_B/ V^2_B$ 1.7

For a power system, practical choice of base value are:
Base voltamperes $= (MVA)_B$
Base voltage $= (kV)_B$
Hence,
$Z_{p.u}= Z(\Omega)\times(MVA)_B/ (kV)^2_B$.......................................1.8

1.5.2 Per unit system extended to three phase circuits

Here, rather than obtaining the per unit values using per phase base quantities, the per unit values can be obtained directly by using three phase base quantities. Let

Three phase base megavoltamperes $=(MVA)_B$

Line to line base kilovolts $= (kV)_B$

Assuming star connection (or equivalent star can always be found),

Base current $I_B=((MVA)_B / 3) / ((kV)_B / \sqrt{3}) = ((MVA)_B \times 10^6) / (((10^6 / 10^6) \times (\sqrt{3} \times \sqrt{3}) \times 10^3 (kV)_B) / \sqrt{3})$

$= (10^3 \times (MVA)_B) / (\sqrt{3} \times (kV)_B) = (1000 \times (MVA)_B) / (\sqrt{3} \times (kV)_B)$

Base impedance $Z_B= (10^3 \times (kV)_B) / (\sqrt{3} I_B) = (1000 \times (kV)_B) / (\sqrt{3} (1000 \times (MVA)_B) / (\sqrt{3} \times (kV)_B))$

$= (kV)^2_B / (MVA)_B \quad \Omega$

If the actual impedance is Z Ω, its per unit impedance is given by,

$Z_{p.u}= Z(\Omega) / Z_B = Z(\Omega) / (kV)^2_B / (MVA)_B$

or $Z_{p.u}= Z(\Omega)\times(MVA)_B / (kV)^2_B$ 1.9

Note:

Sometimes per cent values are used instead of per unit values.

Per cent value = per unit value × 100

per cent value is not convenient for use as the factor of 100 has to be carried in computations.

1.6 Change of Base Quantities

The impedance of a device or a component is usuall specified in per unit or per cent on the basis of its own rated MVA and rated kV. In a large interconnected power system, there will be various devices with different MVA and kV ratings. Hence , it will be convenient for analysis to have a common base for the entire power system. Since all impedance in any part of a system must be expressed on the common base, it is necessary to have a means of converting per unit impedances from one base to another.

Let $(kV)_{B, old}$ and $(MVA)_{B, old}$ represent old base values and $(kV)_{B, new}$ and $(MVA)_{B, new}$ represent new base values.

Then, by virtue of Eq. 1.9 , we can write

$Z_{p.u, old}$ = p.u impedance of a circuit element on old base

$= Z(\Omega)\times(MVA)_{B, old} / (kV)^2_{B, old}$ 1.10

$Z_{p.u, new}$ = p.u impedance of a circuit element on new base

$= Z(\Omega)\times(MVA)_{B, new} / (kV)^2_{B, new}$ 1.11

Dividing equation 1.11 by equation 1.10 and rearranging, we get

$Z_{p.u, new} = Z_{p.u, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$ 1.12

Eq. 1.12 can be used to convert the p.u impedance expressed on one base value (old) to another base (new).

Example 1.1: Calculate the per unit impedance of a synchronous motor rated 200kVA, 13.2kV and having a reactance of 50 Ω /ph

Solution:
Base values:
The ratings of the machine itself is considered as base values.
Therefore,
Base voltage $(kV)_B$ = 13.2kV
Base megavoltamperes $(MVA)_B$= 200 / 1000 = 0.2 MVA
Then, the reactance of the motor in p.u is given as
$X_{p.u} = X\ (\Omega)\ (MVA)_B\ /\ (kV)^2_B = 50\ \times (0.2) / (13.2\ )^2 = 0.0574$ p.u

Example 1.2 : The primary and secondary sides of a single phase 1 MVA, 4kV / 2kV transformer have a leakage reactance of 2 Ω each. Find the p.u reactance of the transformer referred to the primary and secondary side.

Solution:
Base values:
$(MVA)_B$ = 1 MVA
primary base voltage $(kV_1)_B$ = 4kV
secondary base voltage $(kV_2)_B$ = 2kV
also, it is given that $X_1 = X_2 = 2\ \Omega$.
Primary side:
the total impedances as referred to the primary side $X_{01} = X_1 + X_2'$
where,
$X_2' = X_2\ (kV_1 / kV_2)^2 = 2\ (4 / 2)^2 = 8\ \Omega$
therefore $X_{01} = 2+8 = 10\ \Omega$
$(X_{01})_{p.u} = X_{01}(\ \Omega\ ) \times (MVA)_B\ /\ (kV_1)^2_B = 10\ (\ 1\ /\ 4\ )^2 = 0.625$p.u ....a)

secondary side:
the total impedance as referred to the secondary side is $X_{02} = X_2 + X_1'$
$X_1' = X_1\ (kV_2 / kV_1)^2 = 2\ (2 / 4)^2 = 0.5\ \Omega$
therefore $X_{02} = 2+0.5 = 2.5\ \Omega$
$(X_{02})_{p.u} = X_{02}(\ \Omega\ ) \times (MVA)_B\ /\ (kV_2)^2_B = 2.5\ (\ 1\ /\ 2\ )^2 = 0.625$p.u ....b)
from eq. a and b, it can be observed that p.u reactance of the transformer referred to primary side and secondary side is the same, though their ohmic values are different.

Example 1.3: show that the per unit impedance of a transformer is the same irrespective of the side on which it is calculated.

Solution:

Base values:
Let,
$(MVA)_B$= rated MVA of the transformer.
$(kV_1)_B$= base voltage in the primary side.
$(kV_2)_B$= base voltage in the secondary side.
Also, let Z_{01} be the impedance of the transformer reffered to primary side and Z_{02} the impedance as referred to the secondary.
We have,
$(Z_{01})_{p.u} = Z_{01}(\Omega) \times (MVA)_B / (kV_1)^2_B$ ....a)
and
$(Z_{02})_{p.u} = Z_{02}(\Omega) \times (MVA)_B / (kV_2)^2_B$ ....b)
where,
$Z_{02}(\Omega)=Z_{01}(\Omega) \times ((kV_2)^2_B / (kV_1)^2_B)$ ....c)
substituting eq. c) in eq. b), we get,
$(Z_{02})_{p.u}= Z_{01}(\Omega) \times ((kV_2)^2_B / (kV_1)^2_B) \times ((MVA)_B / (kV_2)^2_B) = Z_{01}(\Omega) \times (MVA)_B / (kV_1)^2_B$
$(Z_{02})_{p.u}=(Z_{01})_{p.u}$
Thus, it is proved that the per unit impedance of a transformer is the same whether computed from primary or secondary side.

Example 1.4: three winding transformer has rating as follows:
Primary : Y connected, 6.6kV, 15MVA
secondary: Y connected, 33kV, 10MVA
tertiary : Δ connected, 2.2kV, 7.5MVA
Leakage impedance measured from primary side as Z_{ps}=j0.232 Ω , Z_{pt} = j0.29 Ω and on the secondary side Z_{st}= j8.7 Ω Find the star connected equivalent on a base of 15MVA, 6.6kV in the primary circuit. Neglect resistances.

Solution:
Base values:
primary side: $(MVA)_B$= 15MVA, $(kV_p)_B$= 6.6kV
secondary side: $(MVA)_B$= 15MVA, $(kV_s)_B$= 6.6 x 33 / 6.6 =33kV
per unit leakage impedance.
$(Z_{ps})_{p.u}= Z_{ps}(\Omega) \times (MVA)_B/(kV_p)_B^2 = j0.232 \times 15 / 6.6^2 = j0.08$p.u
$(Z_{pt})_{p.u}= Z_{pt}(\Omega) \times (MVA)_B/ (kV_p)_B^2 = j0.29 \times 15 / 6.6^2 = j0.1$p.u
$(Z_{st})_{p.u}= Z_{st}(\Omega) \times (MVA)_B/ (kV_s)_B^2 = j8.7 \times 15 / 33^2 = j0.12$p.u

Therefore,
$Z_p = ½ (Z_{ps}+Z_{pt}- Z_{st}) = ½ (j0.08+j0.1-j0.12)= j0.03$p.u
$Z_s = ½ (Z_{ps}+Z_{st}- Z_{pt})= ½ (j0.08+j0.12-j0.1)= j0.05$p.u
$Z_t = ½ (Z_{st} + Z_{pt} - Z_{ps}) = ½ (j0.12+j0.1-j0.08)=j0.07$p.u
hence, the star connected equivalent circuit is as shown in fig 1.9

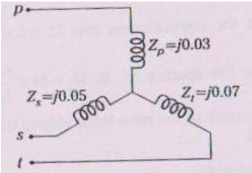


Fig. 1.9

Note:
since the resistances are neglected, the impedances are represented as pure reactances in the above equivalent circuit.

Example 1.5:
Three generators are rated as follows:
Generator 1 : 100 MVA, 33kV, reactance 10%.
Generator 2: 150MVA, 32kV, reactance 8%.
Generator 3: 110MVA, 30kV, reactance 12%.
Determine the reactance of the generator corresponding to base values of 200MVA, 35kV.

Solution:
Here, the reactances of the generators are specified on the basis of their own rated MVA and kV. We consider this as old values. Therefore,
$(X_{g1})_{p.u,\ old}$ = 10% = 0.1p.u on 100MVA, 33kV old bases
$(X_{g2})_{p.u,\ old}$ =8% = 0.08p.u on 150MVA, 32kV old bases
$(X_{g3})_{p.u,\ old}$ =12% = 0.12 p.u on 110MVA, 30kV old bases
the new base values are 200MVA, 35kV. (Given)
hence using the formula
$X_{p.u,\ new} = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
we get,
$(X_{g1})_{p.u,\ new} = 0.1 \times (200/100) \times (33^2/35^2) = 0.1777$ p.u
$(X_{g1})_{p.u,\ new} = 0.08 \times (200/150) \times (32^2/35^2) = 0.08916$ p.u
$(X_{g1})_{p.u,\ new} = 0.12 \times (200/110) \times (30^2/35^2) = 0.1603$ p.u

1.7 Advantages of per unit computations

1) Manufacturers usually specify the impedance of an apparatus in per unit or per cent value on the base of the name plate rating of the apparatus.
2) The per unit impedance of the same type of machines, may be of different ratings, lie within a narrow range. However, the ohmic values differ materially for machines of different ratings. Hence, if the per unit impedance of a generator is not known, say, then it can be chosen from a set of tabulated values.
3) The per unit impedance of transformer is the same referred to either side of it.
4) The method of connection of transformers (Y-Y, Y- Δ etc) do not effect the per unit impedance of the transformer.

5) The greatest advantage of using per unit values is that it makes the calculations relatively easier.

Note:
The per unit systems is not without its drawbacks which include:
1) Some equations that hold in the unscaled case are modified when scaled into per unit. Factors such as √3 and 3 are removed or added by the method.
2) Equivalent circuits of the components are modified, making them some what more abstract.

1.8 Per unit impedance and reactance diagrams

In these diagrams, the impedance or reactance of the components are not expressed in their ohmic values, but in per unit values expressed on a common base. This greatly simplifies the task involved in analysis.

Procedure to form per unit reactance diagram from one-line diagram.

From a one-line diagram of a power system we can directly draw the p.u reactance (or impedance) diagram by the following steps given below:

1) Choose a base kilovoltampere $(kVA)_B$ or megavoltampere $(MVA)_B$. This value remains the same for all sections of the power system and preferably is the rated kVA or MVA of one of the electrical apparatus of the system. In case of three phase power system, the $(kVA)_B$ or $(MVA)_B$ is the three phase power rating.

2) Select a base kilovolt $(kV)_B$ for one section of power system. In case of three phase power system, the $(kV)_B$ is a line value. This should preferably be the rated KV of the apparatus whose power rating has been taken as the base. The various sections of the power system works at different voltage levels and the voltage conversion is achieved by means of transformers. Hence the $(kV)_B$ of one section of the power system should be converted to a $(kV)_B$ corresponding to another section using the transformer voltage ratio. In case of three phase transformer (or a bank of three single phase transformers), line to line voltage ratio is used to transfer the $(kV)_B$ on one section to another.

$(kV)_B$ on primary winding of transformer OR $(kV)_B$ on primary side of transformer = $(kV)_B$ on secondary winding of a transformer OR $(kV)_B$ on secondary side of the transformer × (primary winding given voltage rating / secondary winding given voltage rating)

$(kV)_B$ on secondary winding of transformer OR $(kV)_B$ on secondary side of transformer = $(kV)_B$ on primary winding of a transformer OR $(kV)_B$ on primary side of the transformer × (secondary winding given voltage rating / primary winding given voltage rating)

3) In the given problem, the impedance of the components of power system are expressed either in ohms or in p.u which is calculated using the component rating as the base values. In reactance diagram, the resistance are neglected and reactances of all components are expressed on a common base. Hence, starting from one end of power system the reactances of each component should be converted to p.u reactances on the selected new base.

When the specified reactance of the component is in ohms then,
$X_{p.u} = X(\Omega) \times (MVA)_{B,new} / (kV)^2_{B,new}$

When the specified reactance of the component is in p.u on the component rating as base values, then consider the component rating as old base values and selected MVA base, calculated kV base values as new bases. Now the p.u reactance on new base can be calculated using the formula,

$$X_{p.u,\,new} = X_{p.u,\,old} \times \left((MVA)_{B,\,new} / (MVA)_{B,\,old} \right) \times \left((kV)^2_{B,\,old} / (kV)^2_{B,\,new} \right)$$

Example 1.6 : draw the reactance diagram of the system shown in fig. 1.10. the ratings of the components are

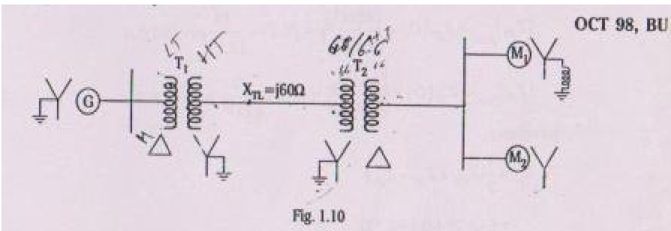


Fig. 1.10

G: 15MVA, 6.6kV, X" = 12%
T_1= 20 MVA, 6.6/66 kV, X=8%
T_2= 20 MVA, 66/6.6 kV, X=8%
M_1 & M_2: 5MVA, 6.6kV, X"=20%

solution:

Base values:
Let us consider(choose) the ratings of the generator as base values.

Therefore,

base megavoltamperes, $(MVA)_B$ = 15MVA (this is same for the entire system)
base kilovolt on the generator G, $(kV)_B$ = 6.6kV

The various sections of the power system works at different voltage levels. Hence, the base kilovolts are different in the different sections. The voltage conversion is achieved by means of transformer voltage ratio.

Base kilovolt on the secondary side of the transformer T_1 OR base kV on transmission line section =
base kilovolts on the primary side of the transformer $T_1 \times$ (its secondary winding voltage rating / its primary winding voltage rating)= $6.6 \times (66 / 6.6) = 66$ kV

Base kilovolts on the secondary winding of the transformer T_2=OR base kV on the motors M_1 & M_2 = $66 \times (6.6 / 66) = 6.6$ kV

These values are used as the new values for calculation of p.u reactances of the different components.

Reactance of generator G:

$$X_{g,\,new} = X_{g,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$$
$$= j0.12 \times (15 / 15) \times (6.6^2 / 6.6^2)$$
$$= j\,0.12 \text{ p.u}$$

This value is the same as given because, the new base values are selected on its rating.

Reactance of transformer T_1: (calculated primary side)

$$X_{T1,\,new} = X_{T1,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$$
$$= j0.08 \times (15 / 20) \times (6.6^2 / 6.6^2)$$
$$= j\,0.06 \text{ p.u}$$

Reactance of transformer T_1: (calculated secondary side)

$$X_{T1,\,new} = X_{T1,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$$
$$= j0.08 \times (15 / 20) \times (66^2 / 66^2)$$
$$= j\,0.06 \text{ p.u}$$

"It is important to know that, it is confirmed by above calculations, in calculating reactance of transformer T_1 that p.u reactance calculated on either side of the transformer is the same. Therefore we can calculate reactance of the transformer any side of it considering either primary side of the base kilovolts old and new values else secondary side base kilovolts old and new values"

Reactance of 60 Ω transmission line TL:

$$X_{TL} = X(\Omega) \times (MVA)_B / (kV)^2_B = j60 \times 15 / 66^2 = j0.207\text{p.u}$$

Reactance of transformer T_2: (calculated primary side)

$$X_{T2,\,new} = X_{T2,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$$
$$= j0.08 \times (15 / 20) \times (66^2 / 66^2)$$
$$= j\,0.06 \text{ p.u}$$

Reactance of transformer T_2: (calculated secondary side)

$X_{T2, new} = X_{T2, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.08 \times (15 / 20) \times (6.6^2 / 6.6^2)$

$= j\ 0.06$ p.u

"calculate reactance of the transformer in p.u at any one side, as we know that p.u reactance calculated is same on either side of it"

Reactance of motor M_1:

$X_{M1, new} = X_{M1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.2 \times (15 / 5) \times (6.6^2 / 6.6^2)$

$= j\ 0.6$ p.u

Reactance of motor M_2:

$X_{M2, new} = X_{M2, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.2 \times (15 / 5) \times (6.6^2 / 6.6^2)$

$= j\ 0.6$ p.u

as ratings for motor M_1 & M_2 are same,

$X_{M1, new} = X_{M2, new} = j\ 0.6$ p.u

The reactance diagram is as shown in fig 1.11

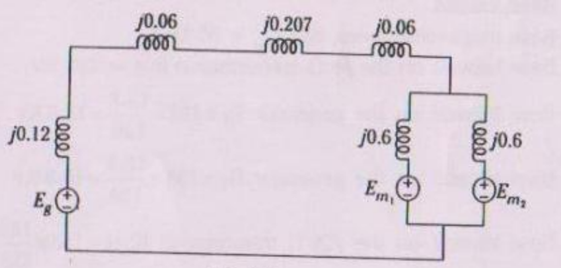


Fig. 1.11

Example 1.7:

Obtain the impedance diagram of the electrical power system shown in fig. 1.12. Mark all impedance values in per unit on a base of 50MVA, 138kV in the 40 ohm line. The machine ratings are:

G_1: 20MVA, 13.2kV, X"=15%

G_2: 20MVA, 13.2kV, X"=15%

M: 30MVA, 6.9kV, X"=20%

Three phase Y-Y transformers: 20MVA, 13.8/138kV, X=10%

Three phase Y- Δ transformers: 15MVA, 6.9/138kV, X=10%

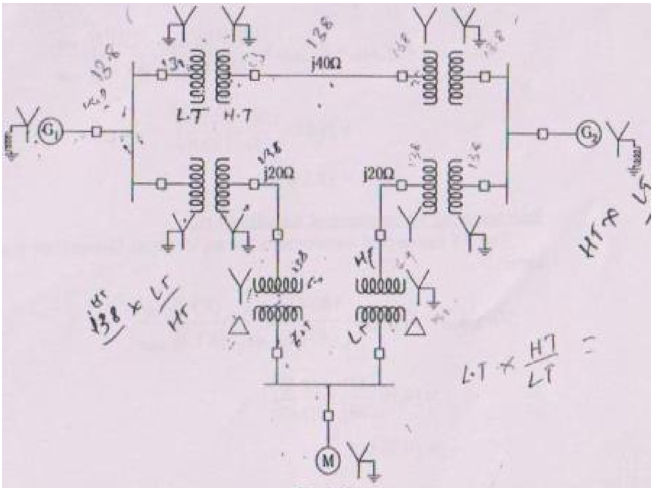


Fig. 1.12

Solution:
Given that to choose:
Base MVA= 50
Base kV on the j40 Ω transmission line = 138
We calculate,
Base kV on the generator section G1= 138 × 13.8 / 138 = 13.8
base kV on the generator section G2= 138 × 13.8 / 138 = 13.8
base kV on the j20 Ω transmission lines = 13.8 × 138 / 13.8 = 138
base kV on the motor section M = 138 × 6.9 / 138 = 6.9
Reactance of j40 ohm transmission line:
$X_{TL1} = X_{TL1}(\Omega) \times (MVA)_B / (kV)^2_B = j40 \times 50 / 138^2 = j0.105$p.u

Reactance of generators G1 & G2:
the generators G_1 & G_2 are identical. Hence their p.u reactances are the same.
$X_{G1, new} = X_{G2, new} = X_{G1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.15 \times (50 / 20) \times (13.2^2 / 13.8^2)$
$= j\,0.343$ p.u

Reactance of Y-Y connected transformers:(calculated considering primary winding old and new base kV values)
The Y-Y connected transformers are all identical. Hence their p.u reactances are the same.
$X_{TR1, new} = X_{TR1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.1 \times (50 / 20) \times (13.8^2 / 13.8^2)$
$= j\,0.25$ p.u

Reactance of j20 Ω transmission lines :
It is observed that both the sections of the j20 ohm transmission lines have the same values of reactances and same base values. Hence their p.u reactances will be the same.
$X_{TL2} = X_{TL2}(\Omega) \times (MVA)_B / (kV)^2_B = j20 \times 50 / 138^2 = j0.053$p.u

Reactance of Y- Δ connected transformers:(calculated considering primary winding old and new base kV values)

Since the Y- Δ connected transformers are all identical. Hence their p.u reactances are the same.

$X_{TR2,\ new} = X_{TR2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.1 \times (50 / 15) \times (138^2 / 138^2)$

$= j\ 0.33$ p.u

Reactance of motor M:

This motor is connected on to the secondary windings of the Y- Δ transformers (i.e Low Voltage side or Low tension side)

$X_{M1,\ new} = X_{M1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.2 \times (50 / 30) \times (6.9^2 / 6.9^2)$

$= j\ 0.33$ p.u

Thus, the impedance (reactance) diagram of the system is as in fig 1.13

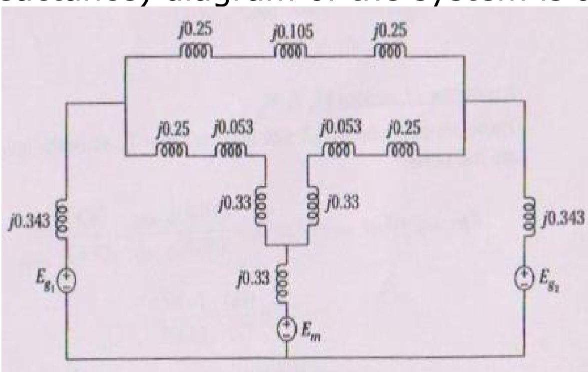


Fig. 1.13

Example 1.8:

The one-line diagram of an unloaded generator is shown in the fig 1.14 Draw the p.u impedance diagram. Choose a base of 50MVA, 13.8kV in the circuit of generator G1.

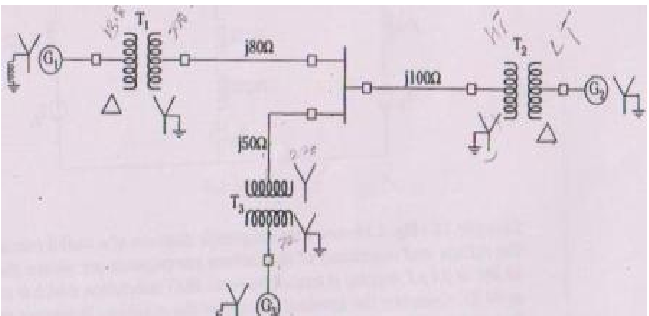


Fig. 1.14

The generators and transformers are rated as follows:

G_1: 20MVA, 13.8kV, X”=0.2 p.u

G_2: 30MVA, 18kV, X”=0.2 p.u

G_3: 30MVA, 20kV, X”=0.2 p.u

T_1: 25MVA, Y220 kV/13.8 kV Δ , X=10%

T_2: Three single phase units each rated 10MVA, 127/18kV, X=10%

T_3=35MVA, 220kV Y/22kV Y, X=10%

solution:
Base values:
given that to choose,
base MVA=50
base kV on the generator G_1=13.8
we calculate,
base kV on the j80 ohm transmission line= 13.8 × 220 / 13.8 = 220
base kV on the j50 ohm transmission line=220
base kV on the j100 ohm transmission line=220
(as all the transmission lines are connected to the same bus, so base kV on them is the same)
base kV on the generator G3= 220 × 22 / 220 = 22

The transformer T_2 is a three phase bank formed using three single phase transformers with a voltage rating of 127/18kV. In this, the HT side is star connected and LT side is delta connected.

Voltage ratio of line voltage of 3-phase transformer bank T_2= √3 × 127 / 18 = 220kV/18kV
(as primary winding is star connected $V_{line}=\sqrt{3}\, V_{ph}$, and secondary is Δ, $V_{line}=V_{ph}$)

base kV on the generator G_2= 220 × 18 / 220 = 18

Reactance of generator G_1:

$$X_{G1,\,new} = X_{G1,\,old} \times \left((MVA)_{B,\,new} / (MVA)_{B,\,old} \right) \times \left((kV)^2_{B,\,old} / (kV)^2_{B,\,new} \right)$$
$$= j0.2 \times (50 / 20) \times (13.8^2 / 13.8^2)$$
$$= j\,0.5 \text{ p.u}$$

Reactance of transformer T_1: (calculated primary side)

$$X_{T1,\,new} = X_{T1,\,old} \times \left((MVA)_{B,\,new} / (MVA)_{B,\,old} \right) \times \left((kV)^2_{B,\,old} / (kV)^2_{B,\,new} \right)$$
$$= j0.1 \times (50 / 25) \times (13.8^2 / 13.8^2)$$
$$= j\,0.2 \text{ p.u}$$

Reactance of transmission lines:

j80 ohm line,

$X_{TL1}= X_{TL1}(\Omega) \times (MVA)_B / (kV)^2_B = j80 \times 50 / 220^2 = j0.083$p.u

j100 ohm line,

$X_{TL2}= X_{TL2}(\Omega) \times (MVA)_B / (kV)^2_B = j100 \times 50 / 220^2 = j0.1033$p.u

j50 ohm line,

$X_{TL3}= X_{TL3}(\Omega) \times (MVA)_B / (kV)^2_B = j50 \times 50 / 220^2 = j0.0516$p.u

Reactance of transformer T_2: (calculated secondary side)
as this is a bank of three single phase transformers, hence,
base MVA old = 10 ×3 = 30

$X_{T2, new} = X_{T2, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.1 \times (50 / 30) \times (220^2 / 220^2)$

$= j\ 0.1667$ p.u

Reactance of generator G_2:
this is connected to the LT side of T2,

$X_{G2, new} = X_{G2, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.2 \times (50 / 20) \times (18^2 / 18^2)$

$= j\ 0.333$ p.u

Reactance of transformer T_3: (calculated secondary side of it)

$X_{T3, new} = X_{T3, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.1 \times (50 / 30) \times (22^2 / 22^2)$

$= j\ 0.143$ p.u

Reactance of generator G_3:

$X_{G3, new} = X_{G3, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.2 \times (50 / 30) \times (20^2 / 22^2)$

$= j\ 0.275$ p.u

Using the above values, the reactance diagram is as constructed in fig.1.15

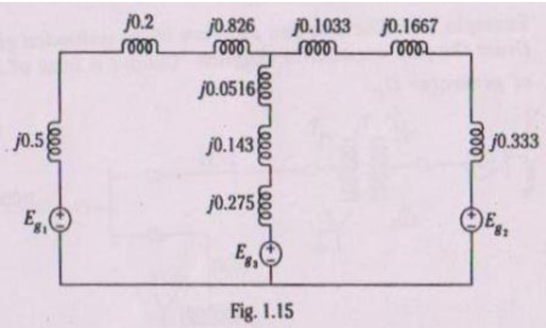


Fig. 1.15

Example 1.9:

Fig 1.16 shows the schematic diagram of a radial transmission system. The ratings and reactances of the various components are shown therin. A load of 60MW at 0.9p.f lagging is tapped from the 66kV substation which is to be maintained at 60kV. Calculated the terminal voltage of the machine. Represent the transmission line and transformer by series reactances only.

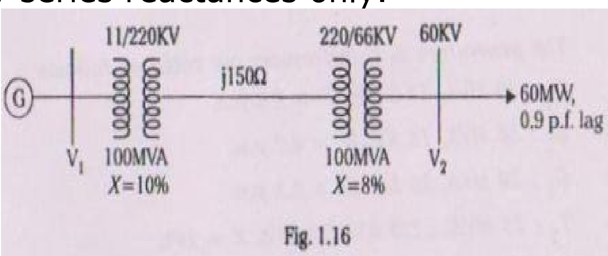


Fig. 1.16

let us choose the base MVA throughout the system be 100
choose the base kV in the transmission line j150 ohm = 220
we calculate,
base kV on the load= 220 × 66 / 220 = 66
base kV on the generator side = 220 × 11 / 220 = 11

Reactance on 11/220kV transformers:(calculated secondary side of it)

$X_{T1,\,new} = X_{T1,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$

$= j0.1 \times (100 / 100) \times (220^2 / 220^2)$

$= j\,0.1$ p.u

Reactance of j150 ohm transmission line:

$X_{TL} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = j150 \times 100 / 220^2 = j0.31$p.u

Reactance on 220/66kV transformers:(calculated primary side of it)

$X_{T2,\,new} = X_{T2,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$

$= j0.08 \times (100 / 100) \times (220^2 / 220^2)$

$= j\,0.08$ p.u

Impedance of load:
The exact value of the impedance of the load is not required in this problem. However, if inevitably to calculate, then it can be determined using the formula,

$Z = |V_L|^2 / (P - jQ)$

where,
V_L=voltage at the terminals of the laod
P=active component of the power at the load
Q=reactive component of the power at the laod.
Hence,
the reactance diagram of the system is as shown in fig1.17

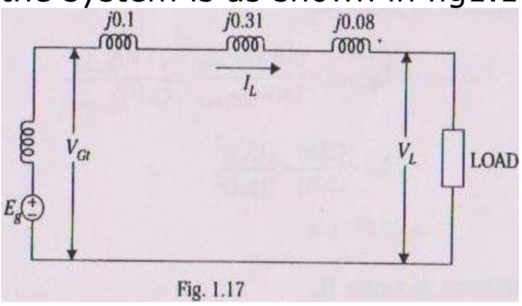


Fig. 1.17

Terminal voltage of the machine:
Let V_{gt}=terminal voltage of the machine
V_L=load voltage
I_L=load current

V_L=60kV= 60kV/66kV = 0.909p.u

$I_L = (P / (\sqrt{3} \times V_L \cos\varphi)) \angle -\cos^{-1}\varphi$

$I_L = ((60\times10^6) / (\sqrt{3} \times 60\times10^3 \times 0.9)) \angle -\cos^{-1}0.9$

$I_L = 641.5 \angle -25.48°$ A

for computation, the value of I_L should be in p.u. Hence, first we determine the base current I_B

$I_B = (1000 \times (MVA)_B) / (\sqrt{3} \times (kV)_B) = (1000 \times 100) / (\sqrt{3} \times 66) = 874.77$ A
therefore, I_L in p.u $= I_L / I_B = 641.5 \angle -25.48° / 874.77 = 0.733 \angle -25.48°$ p.u
from fig 1.17 it can be observed that,

$(V_{Gt})_{p.u} = (V_L)_{p.u} + I_L\ (X_{T1} + X_{TL} + X_{T2})$
$= 0.909 + 0.733 \angle -25.48° (j0.1 + j0.31 + j0.08)$
$= 0.909 + 0.359 \angle 64.16°$
$= 0.909 + 0.156 + j0.323$
$= 1.065 + j0.323$
$= 1.112 \angle 16.87°$ p.u

therefore,
V_{Gt} in kilovolts $= (V_{Gt})_{p.u} \times$ base kV on the generator $= 1.112 \times 11 = 13.232$kV
$|V_{Gt}| = 12.232$kV
this is the desired answer.

Example 1.10:
A 300MVA, 20kV, 3φ generator has a reactance of 20%. The generator supplies two motors M1 and M2 over a transmission line of 64km as shown in one line diagram.

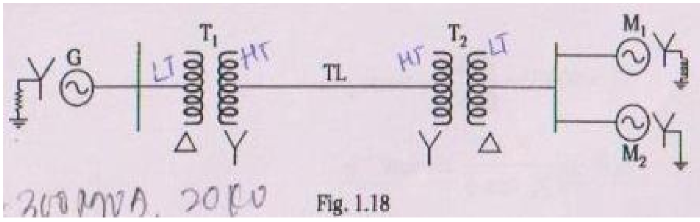


Fig. 1.18

The ratings of the components are as follows:
T_1: 350 MVA, 230KV-Y/20KV-Δ, X=10%
T_L: L=64Km, X_{TL}= j0.5Ω/Km.
T_2: Composed of three single phase transformers each rated 127/13.2 KV, 100 MVA with leakage reactance of 10%.
M_1: 200 MVA, 13.2 KV, X″= 20%
M_2: 100 MVA, 13.2 KV, X″= 20%
Select the generator ratings as the base & draw the reactance diagram with all reactance's marked in p.u. If the motors M_1 & M_2 have inputs of 120 MW & 60 MW at 13.2KV and operate at upf, find the voltage at the terminals of the generator.

Solution:
base values:
given that to choose base MVA=300
base kV on the generator =20
base kV on the transmission line = 20 × 230 / 20 = 230
base kV on the motors= 230 × 13.2 / (127√3) =13.8

Reactance of generator G:
$X_{G,\,new} = X_{G,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$

$= j0.2 \times (300 / 300) \times (20^2 / 20^2)$
$= j\,0.2$ p.u

Reactance of transformer T_1: (calculated considering secondary side of it)

$X_{T1,\ new} = X_{T1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.1 \times (300 / 350) \times (230^2 / 230^2)$
$= j\,0.085$ p.u

Reactance of transmission line TL:

$X_{TL,new} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = (j0.5 \times 64)\ 300 / 220^2 = j0.181$p.u

Reactance of transformer T_2: (calculated considering primary side)

$X_{T2,\ new} = X_{T2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.1 \times (300 / 300) \times ((127\sqrt{3})^2 / 230^2)$
$= j\,0.09$ p.u

Reactance of generator M_1:

$X_{M1,\ new} = X_{M1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.2 \times (300 / 200) \times (13.2^2 / 13.8^2)$
$= j\,0.274$ p.u

Reactance of generator M_2:

$X_{M2,\ new} = X_{M2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$
$= j0.2 \times (300 / 100) \times (13.2^2 / 13.8^2)$
$= j\,0.549$ p.u

The reactance diagram is as shown in fig 1.19

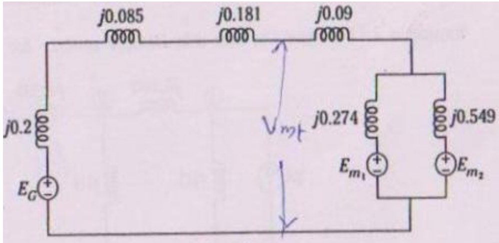


Fig. 1.19

Let,

V_{Mt}=terminal voltage at the motor ends.
V_{Gt}=terminal voltage of the generator.
The total electrical power that flows into the motors is,
P=120+60=180MW at 13.2kV, upf.
Therefore,
the current drawn by the motors,
$I_m = (180 \times 10^6) / (\sqrt{3} \times 13.2 \times 10^3 \times 1) = 7873$ A
it is required to express the current in p.u. Hence, the base current I_B=
$(1000 \times 300) / (\sqrt{3} \times 13.8) = 12551$A
therefore,
I_m in p.u=I_m/I_B=7873/12551=$0.627\angle 0°$p.u

also,

V_{mt} in p.u= 13.2∠0°/13.8=0.96∠0° p.u

from fig 1.19

it can be observed that,

$(V_{Gt})_{p.u} = (V_{Mt})_{p.u} + I_m\ (X_{T1}+X_{TL}+X_{T2})$

$=0.96\angle 0° +0.627(j0.085+j0.181+j0.09)$

$=0.96\angle 0° +j0.223$

$=0.986\angle 13.07°$

therefore,

$|V_{Gt}|=|V_{Gt}\text{ in p.u}|\times 20kV=0.986\times 20=19.72kV.$

1.9 Node Equations and Bus Admittance Matrix

The junctions formed when two or more elements (R, L, C etc) are connected are called Nodes. In a power system network, the buses can be treated as nodes and the voltages of all buses (nodes) can be solved by using the conventional nodal analysis.

Let us consider an example of equation formulation by nodal analysis method. The circuit of fig.1.20 contains four independent nodes(buses) as shown by the circled 1,2,3 and 4. These nodes are called major or principal nodes. The node O, with respect to which all voltages are measured is called as the reference node.

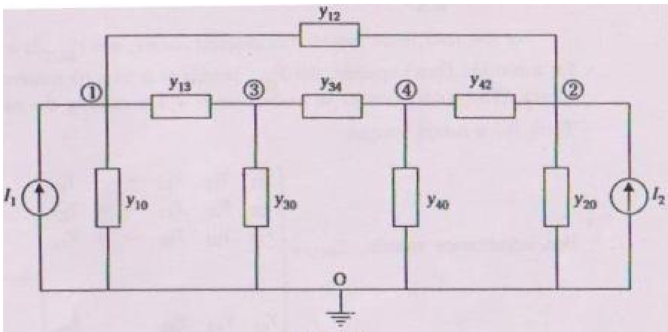


Fig. 1.20

Let V_1, V_2, V_3 & V_4 be the voltages at the respective nodes(buses). The admittances are marked as shown.

At node-1,

$I_1=(V_1-V_3)y_{13}+(V_1-V_2)y_{12}+V_1y_{10}$

$=(y_{10}+y_{12}+y_{13})V_1-y_{12}V_2-y_{13}V_3$

$=Y_{11}V_1+Y_{12}V_2+Y_{13}V_3+Y_{14}V_4$ 1.13

where,

$Y_{11}=y_{10}+y_{12}+y_{13}$

$Y_{12}=-y_{12}$

$Y_{13}=-y_{13}$

$Y_{14}=0$ 1.14

(Here y_{10} is the shunt charging admittance at node-1). Similarly, we can formulate nodal current equations at other nodes as,

$I_2=Y_{21}V_1+Y_{22}V_2+Y_{23}V_3+Y_{24}V_4$ 1.15
$I_3=Y_{31}V_1+Y_{32}V_2+Y_{33}V_3+Y_{34}V_4$ 1.16
$I_4=Y_{41}V_1+Y_{42}V_2+Y_{43}V_3+Y_{44}V_4$ 1.17
these equations can be written in a matrix form as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

..................1.18

or in a more compact form, the above equations can be written as
I=Y V...................1.19

The Y matrix above is designed as Y_{BUS}, and is called as bus admittance matrix. The node voltages are called bus voltages in power system analysis.

Thus, the performance equations 1.19 can be written as,
$I=Y_{BUS}$ V 1.20

For the four node system considered above, the Y_{BUS} is (4×4) matrix. In general, for a n-node(bus)system, the Y_{BUS} is a (n×n) matrix where n is the number of buses, for a n-bus system
Bus admittance matrix,

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & - & - & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & - & - & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & - & - & Y_{3n} \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot \\ Y_{n1} & Y_{n2} & Y_{n3} & - & - & Y_{nn} \end{bmatrix}$$

..................1.21

Each admittance Y_{ii} (i+1,2,3,..................n) is called the self admittance (or driving point admittance) of node 1 and equals the algebraic sum of all the admittances terminating on the node .(Refer eq. 1.14). Each off-diagonal term Y_{ik} (i, k=1,2,3.....n) is the mutual admittance(or the transfer admittance) between nodes i & k and equals the negative of the sum of all admittances connected directly between these nodes. Further, $Y_{ik}=Y_{ki}$ (Refer eq. 1.14)

Thus, as seen above, Y_{BUS} is a symmetric matrix (except when phase shifting transformers are involved, this case is not considered here). Furthermore, Y_{ik} =0 if the buses i & k are not connected (eg. Y_{14}=0). In a large power network, each bus (node) is connected only to a few other buses (usually to three or four buses), thus, the Y_{BUS} of a large network is very sparse i.e it has a large number of zero elements. (this may not be evident in a small system like the sample system of fig 1.20). In a large system consisting of 100 nodes, the non-zero elements may be as small as 2% of the total elements. This greatly reduces the numerical computations required for analysis. Bus admittance matrix is often used in solving load flow problems. It has gained widespread application owing to its simplicity of data preparation & handling.

1.10 Formation of Y_{BUS} by inspection

There are different techniques of assembling the Y_{BUS}, namely the method of singular transformation (MST), the K. Nagappan style and others. Here we confine ourselves to the inspection method only. Accordingly, the admittance matrix may be assembled as

i)The diagonal element of each node is the sum of the admittance connected to it.

ii)The off-diagonal element is the negated admittance between the nodes.

Example 1.11:

Find the bus admittance matrix for the circuit shown in fig.1.21

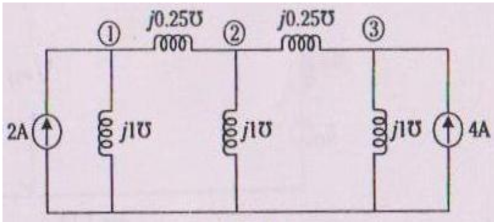


Fig. 1.21

solution:

There are three major nodes in the circuit shown. Using the inspection method, we can write,

Y_{11}=j(1+0.25)=j1.25 mho

Y_{22}=j(0.25+1+0.25)=j1.50 mho

Y_{33}=j(1+0.25)=j1.25 mho

$Y_{12}=Y_{21}$=-j0.25 mho

$Y_{23}=Y_{32}$=-j0.25 mho

$Y_{13}=Y_{31}$=0 (As they are not directly connected.

Thus, the bus admittance matrix of the system is,

$$Y_{BUS}=\begin{bmatrix} j1.25 & -j0.25 & 0 \\ -j0.25 & j1.50 & -j0.25 \\ 0 & -j0.25 & j1.25 \end{bmatrix}$$

Example 1.12:

Determine the Y_{BUS} for a four bus transmission line system the bus diagram of which is as shown in fig 1.22. The impedances and line charging admittances are as tabulated.

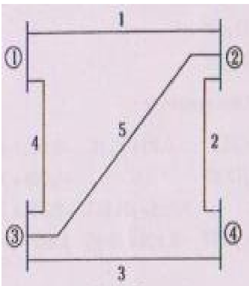


Fig. 1.22

LINE NO.	BUS-CODE p-r	IMPEDANCE Z_{pr}	LINE CHARGING ADMITTANCE FOR EACH LINE, $y_{pr}/2$
1.	1-2	j0.20	j0.04
2.	2-4	j0.25	j0.05
3.	3-4	j0.15	j0.03
4.	3-1	j0.10	j0.02
5.	2-3	j0.30	j0.06

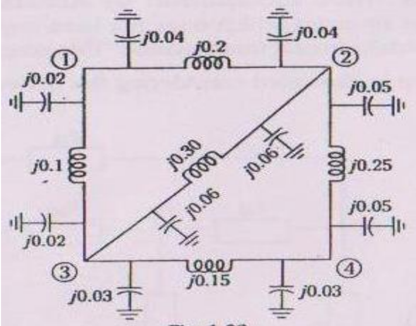


Fig. 1.23

Solution:
including the line charging admittances, the equivalent circuit is as shown in the fig.1.23
The series components are marked with their impedance values, whereas the shunt values are marked with their admittance values.
Hence,
Y_{11}= (1/j0.2)+(1/j0.1)+j0.04+j0.02= -j14.94
Y_{22}=(1/j0.2)+(1/j0.3)+(1/j0.25)+j0.04+j0.06+j0.05= -j12.18
Y_{33}=(1/j0.1)+(1/j0.3)+(1/j0.15)+j0.02+j0.03+j0.06= -j19.89
Y_{44}=(1/j0.25)+(1/j0.15)+j0.03+j0.05= -j10.58
Y_{12}=Y_{21}= -(1/j0.2)=j5
Y_{13}=Y_{31}= -(1/j0.1)= j10
Y_{14}=Y_{41}=0
Y_{23}=Y_{32}= -(1/j0.3)=j3.3
Y_{24}=Y_{42}= -(1/j0.25)=j4
Y_{34}=Y_{43}= -(1/j0.15)= j6.67
Thus,
the Y_{BUS} of the transmission system is:

$$Y_{BUS} = \begin{bmatrix} -j14.94 & j5 & j10 & 0 \\ j5 & -j12.18 & j3.3 & j4 \\ j10 & j3.3 & -j19.89 & j6.67 \\ 0 & j4 & j6.67 & -j10.58 \end{bmatrix}$$

Example 1.13:
Find the bus admittance matrix of the system shown in fig. 1.24. Given that all the lines are characterized by a series impedance of (0.1+j0.7) ohm/km and a shunt admittance of j0.35×10^{-5} mho/km. Using base values of 220kV and 100MVA, express all impedances & admittances in p.u.

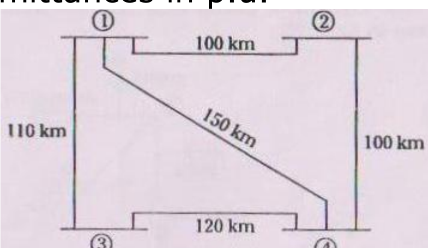


Fig. 1.24

Solution:

First, we find the total series impedance of the line. Then express it in p.u using the formula,

$Z_{p.u}= Z(\Omega)\times(MVA)_B/ (kV)^2_B$.

Here,

it is given that $(MVA)_B = 100$

and $(kV)^2_B$=220kV

then,

find the series admittance of the line.

Next,

the total shunt admittance of of the line is estimated.

Then,

it is expressed in p.u using the formula,

$Y_{p.u}= Y(mho)\times(kV)^2_B/(MVA)_B$

Carefully note that if y_{pr} is the shunt admittance(line charging admittance) of the line,

then it should be divided as $y_{pr}/2$ at both the ends of the line as shown in fig. 1.25

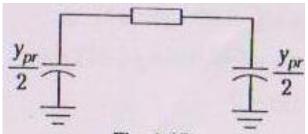


Fig. 1.25

These are tabulated as follows:

BUS-CODE	TOTAL SERIES IMPEDANCE IN Ω	TOTAL SERIES IMPEDANCE IN p.u.	TOTAL SERIES ADMITTANCE IN p.u.	TOTAL SHUNT ADMITTANCE IN ℧	TOTAL SHUNT ADMITTANCE IN p.u.
1-2	10+j70	0.02066+j0.1446	0.968-j6.77	$j350\times10^{-6}$	j0.1694
1-3	11+j77	0.023+j0.159	0.89-j6.16	$j385\times10^{-6}$	j0.1863
1-4	15+j105	0.031+j0.217	0.645-j4.52	$j525\times10^{-6}$	j0.2541
2-4	10+j70	0.02066+j0.1446	0.968-j6.77	$j350\times10^{-6}$	j0.1694
3-4	12+j84	0.0248+j0.1736	0.806-j5.65	$j420\times10^{-6}$	j0.2033

The equivalent circuit depicting the shunt admittances and series admittance is as shown in fig. 1.26

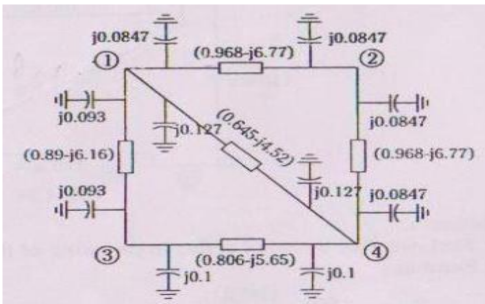


Fig. 1.26

Hence,

Y_{11}=(0.968-j6.77)+(0.89-j6.16)+(0.645-j4.52)+j0.0847+j0.127+j0.093=(2.5=j17.145)p.u

Y_{22}=(0.968-j6.77)+(0.968-j6.77)+j0.0847+j0.0847=(1.936-j13.37)p.u

Y_{33}=(0.89-j6.16)+(0.806-j5.65)+j0.093+j0.1=(1.696-j11.617)p.u
Y_{44}=(0.645-j4.52)+(0.968-j6.77)+(0.806-j5.65)+j0.0847+j0.127+j0.1=(2.419-j16.63)p.u
$Y_{12}=Y_{21}= -y_{12}=$ (-0.968+j6.77)p.u
$Y_{13}=Y_{31}= -y_{13}=$ (-0.89+j6.16)p.u
$Y_{14}=Y_{41}= -y_{14}=$ (-0.645+j4.52)p.u
$Y_{23}=Y_{32}= -y_{23}=$ 0
$Y_{24}=Y_{42}= -y_{24}=$ (-0.968+j6.77)p.u
$Y_{34}=Y_{43}= -y_{34}=$ (-0.806+j5.65)p.u
Thus, the bus admittance matrix of the system is:

$$Y_{BUS}=\begin{bmatrix} 2.5-j17.45 & -0.968+j6.77 & -0.89+j6.16 & -0.645+j4.52 \\ -0.968+j6.77 & 1.936-j13.37 & 0 & -0.968+j6.77 \\ -0.89+j6.16 & 0 & 1.696-j11.617 & -0.806+j5.65 \\ -0.645+j4.52 & -0.968+j6.77 & -0.806+j5.65 & 2.419-j16.63 \end{bmatrix}$$

Additional Examples:
Example 1.18:
A d.c series motor rated at 220kV, 100A has an armature resistance of 0.15 ohm and field resistance of 0.2 ohm. The friction and windage loss is 1650W. Calculated the efficiency of the machine. Use per unit system.
Solution:
Base values:
base kV=0.22
base current= 100A
hence,
the base power=220×100=0.022MVA
base MVA=0.022
The total resistance of the machine=R=0.15+0.2=0.35 ohm
$R_{p.u}= R(\Omega)\times(MVA)_B / (kV)^2_B$.
$= 0.35\times0.022 / 0.22^2$=0.159p.u
At rated load, the copper loss of the machine in p.u is
$I_{p.u}^2R_{p.u} =(100/100)^2\times0.159$=0.159p.u
Friction and windage loss in p.u is =1650/ (220×100) = 0.075p.u
Hence,
the total loss= 0.159+0.075=0.234p.u
Let the rated output be 1p.u
therefore,
the efficiency=1/(1+0.234)=0.81p.u

Example 1.19:
A 100 MVA, 33kV, 3φ generator has a sub-transient reactance of 155. The generator is connected to the motors through a transmission line and transformers as shown in fig 1.48. The motors have rated outputs of 30MVA, 20MVA and 50MVA at 30kV with 20% subtransient reactance each. The three-phase transformers are rated 100MVA, 32kV Δ/100kV Y with leakage reactance of 8%. The line has a

reactance of 50 ohm. Selecting the generator rating as the base in the generator circuit, draw the p.u reactance diagram.

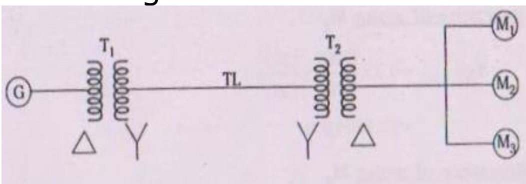


Fig. 1.48

Solution:
base values:
given that to choose,
base MVA= 100
base kV on the generator side= 33kV
we calculate,
base kV on the transmission line=33×100/32=103.125
base kV on the motor side=103.125×32/100=33
Reactance of generator G:
X_G=15%=j0.15p.u

Reactance of transformers T_1& T_2: (calculated considering primary side of T_2)
since the two transformers are one and same, their p.u reactaces are also the same.
$X_{T1, new} = X_{T2, new} = X_{T1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.08 \times (100 / 100) \times (100^2 / 103.125^2)$
$= j\,0.075$ p.u

Reactance of transmission line :
$X_{TL,p.u} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = j0.50 \times 100 / 103.125^2 = j0.47$p.u

Reactance of generator M_1:
$X_{M1, new} = X_{M1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.2 \times (100 / 30) \times (30^2 / 33^2)$
$= j\,0.5509$ p.u

Reactance of generator M_2:
$X_{M2, new} = X_{M2, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.2 \times (100 / 20) \times (30^2 / 33^2)$
$= j\,0.8264$ p.u

Reactance of generator M_3:
$X_{M3, new} = X_{M3, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$
$= j0.2 \times (100 / 50) \times (30^2 / 33^2)$
$= j\,0.3305$ p.u

Using the calculated per unit values of reactances, the p.u reactance diagram is drawn as shown in fig.1.49

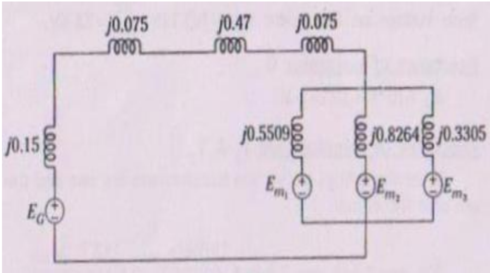


Fig. 1.49

Example 1.20:

The three parts of a single phase electric system are designated A,B,C and are connected to each other through transformers. The transformers are rated as follows:

A-B 10MVA, 13.8-138kV, leakage reactance 10%

B-C 10MVA, 69-138kV, leakage reactance 8%

if the base in circuit B is chosen as 10MVA, 138kV, find the p.u impedance of the 300 ohm resistive lead in circuit C referred to circuit C, B and A. Draw the impedance diagram of the system. Determine the voltage regulation if the voltage at the load is 66kV with the assumption that the voltage input to circuit A remains constant.

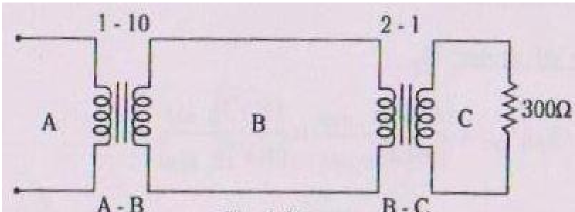


Fig. 1.50

Solution:

base values:

it is given to chose,

base MVA=10

base kV on circuit B=138

we calculate,

base kV on circuit A=138×13.8/138=13.8

base kV on circuit C=138×69/138=69

p.u reactance of the load connected in C=300×$(MVA)_B$/ $(kV)^2_B$=300×10/69^2=0.63p.u

load impedance as referred to circuit B=300×$138^2/69^2$=1200 ohm

load impedance in p.u as referred to B=1200×10/138^2=0.63p.u

similarly,

load impedance as referred to circuit C=1200×$13.8^2/138^2$=12 ohm

load impedance in p.u as referred to C=12×10/13.8^2=0.63p.u

it can be observed that the p.u impedance of the load referred to any part of the system is the same. The impedance diagram of the system is as shown in fig 1.51

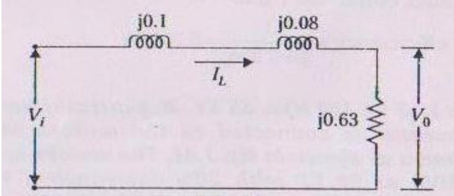


Fig. 1.51

voltage at the load, V_0=66kV/69kV=0.957 p.u
load current, I_L=0.957/0.63=1.52p.u
Voltage at input,
$V_i=I_L(j0.1+j0.08)+V0$
$=(1.52\times j0.18)+0.957=j0.2736+0.957=0.957+j0.2736=0.995\angle 15.95°$
therefore,
percentage regulation=$((|V_i|-|V_0|)/\ |V_0|)\times 100=((0.995-0.957)/0.957)\times 100=3.97\%$

Example 1.21:
Fig 1.52 shows the one-line diagram of a simple four bus system. Table below gives the line impedances identified by the buses on which these terminate. The shunt admittance at all buses are negligible.
i)Find Y_{BUS} assuming that the line shown in dotted is not connected
ii)Find Y_{BUS} if the line shown in dotted is connected.

LINE	R_1 p.u.	X_1 p.u.
1 - 2	0.05	0.15
1 - 3	0.10	0.30
2 - 3	0.15	0.45
2 - 4	0.10	0.30
3 - 4	0.05	0.15

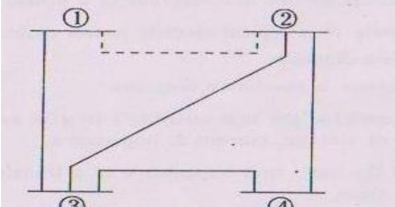


Fig. 1.52

Solution:
we need the admittance values to compute the Y_{BUS}. Hence we construct the following admittance table of the system (taking reciprocals of the impedances).

LINE	G_1 p.u.	B_1 p.u.
1-2	2.0	- 6.0
1-3	1.0	- 3.0
2-3	0.666	- 2.0
2-4	1.0	- 3.0
3-4	2.0	- 6.0

i) with the dotted line unconnected, the equivalent circuit is as shown in fig 1.53

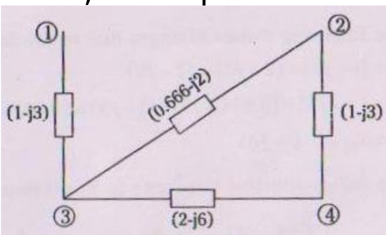


Fig. 1.53

Y_{11}=(1-j3)
Y_{22}=(0.666-j2)+(1-j3)=(1.666-j5)
Y_{33}=(1-j3)+(0.666-j2)+(2-j6)=(3.66-j11)
Y_{44}=(2-j6)+(1-j3)=(3-j9)
Y_{12}=Y_{21}=0
Y_{13}=Y_{31}= -(1-j3)= -1+j3
Y_{14}=Y_{41}=0
Y_{23}=Y_{32}= -(0.666-j2)= -0.666+j2
Y_{24}=Y_{42}= -(1-j3)= -1+j3
Y_{34}=Y_{43}= -(2-j6) = -2+j6
hence,
the Y_{BUS} of the system is,

$$Y_{BUS} = \begin{bmatrix} (1-j3) & 0 & (-1+j3) & 0 \\ 0 & (1.66-j5) & (-0.666+j2) & (-1+j3) \\ (-1+j3) & (-0.666+j2) & (3.66-j11) & (-2+j6) \\ 0 & (-1+j3) & (-2+j6) & (3-j9) \end{bmatrix}$$

ii) with the dotted line connected, the equivalent circuit becomes as in fig 1.54

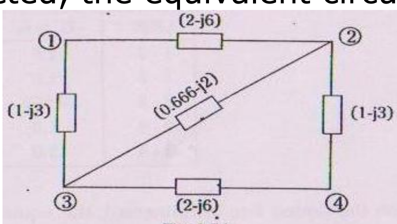


Fig. 1.54

only the following changes due to the inclusion of the line.
Y_{11}=(1-j3)+(2-j6)=(3-j9)
Y_{22}=(2-j6)+(0.666-j2)+(1-j3)=(3.666-j11)
Y_{12}=Y_{21}= -(2-j6) = -2+j6
all other values remains the same as in previous case. Hence the Y_{BUS} is,

$$Y_{BUS} = \begin{bmatrix} (3-j9) & (-2+j6) & (-1-j3) & 0 \\ (-2+j6) & (3.666-j11) & (-0.666+j2) & (-1+j3) \\ (-1+j3) & (-0.666+j2) & (3.66-j11) & (-2+j6) \\ 0 & (-1+j3) & (-2+j6) & (3-j9) \end{bmatrix}$$

-----------END-----------

2.1 Introduction

A fault in a power system is any failure which interfere with the normal operation of the system. Short circuit between lines, insulation failure of equipments or flashover of lines initiated by a lightning stroke are the main causes for these faults.

The faults occurring in a power system can be broadly classified into symmetrical faults and unsymmetrical faults.

In the case of symmetrical faults (a symmetrical short circuit involving all the three phases), the fault current is the same in all the three phases and hence the system remains balanced even after fault occurrence. Therefore, the symmetrical fault conditions can be conveniently analyzed on a single phase basis. On the other hand, the fault current is not the same in all three phases in the case of an unsymmetrical fault. Hence, such fault conditions cannot be analyzed on a single phase basis. Special tools line symmetrical components are used in such situations.

This chapter is concerned for the study of symmetrical faults short circuits. A knowledge of expected system short circuit is essential in the economic planning & design of the power system. These studies provides the engineer with information by which he can design to assure the prompt disconnection of faulted equipments with a minimum damage and a minimum of disturbance to the operation of the remaining system. We start with the discussion of the transients that occur in three phase synchronous machine due to a sudden short circuit at its terminals, when the machine is on no-load and on constant excitation.

2.2 Symmetrical short circuit of a synchronous Generator (on No-load)

A synchronous generator consists of an armature winding wound symmetrically for all the three phases on the stator and a field winding wound on the rotor. Also on the field structure are placed the damper windings which are shorted on themselves at both ends. The field winding is excited by direct current. When the rotor rotates, the armature winding(being stationary) is cut by the magnetic flux of the field winding, hence three phase alternating emfs are induced in the armature windings. In turn, alternating three phase currents are set up in these windings. These produces a rotating magnetic field which rotates at synchronous speed in the air gap.

The field structure rotates at synchronous speed along with the rotating magnetic field produced by the armature winding. During normal operating conditions, the field produced by the armature currents will be relatively stationary with respect to the field and damper windings. Hence the rotating magnetic field will not induce any voltages and currents in the field and damper windings during normal operating conditions.

But, when the alternator suddenly undergoes a symmetrical short circuit under constant excitation, the short circuited armature current changes from zero to a very high value in all the three phases. The armature current will have an a.c component as well as an offset d.c component in each of the three phases. A good way to analyse the effect of the three phase fault is to take an oscillogram of the current in one of the phases upon the occurrences of such a fault. The offset d.c component of the short circuit current will be different in each phase and hence is accounted separately on an empirical basis.

In the absence of the offset d.c component, the symmetrical short circuit current in any phase will be as shown in fig 2.1. This current will be similar in all the three phases except for a phase angle difference of 120 degree electrical.

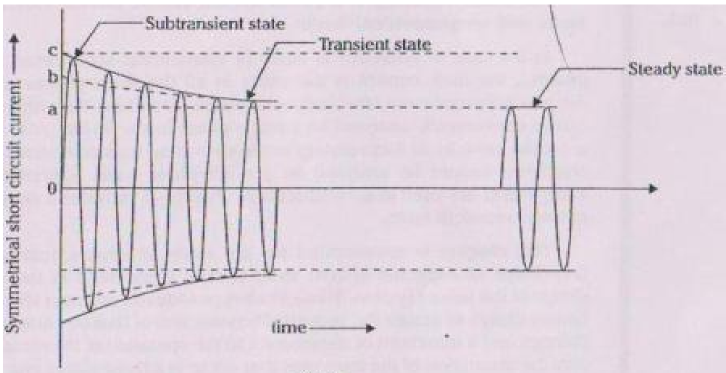


Fig. 2.1

The armature current during symmetrical short circuit can be divided into three regions namely the subtransient, transient, and steady state region. We account for the gradual decrease of current in the following paragraphs.

Under steady state short circuit conditions, the armature reaction of a synchronous generator produces a demagnetizing flux. This effect is represented as a reactance called armature reaction reactance X_a. The sum of leakage reactance X_l and the armature reactance X_a is called the synchronous reactance X_s. In case of salient pole machines the synchronous reactance is called direct axis reactance and denoted by X_d. Neglecting the armature resistance, the steady state short circuit model of an alternator on a per phase basis will be as shown in fig. 2.2

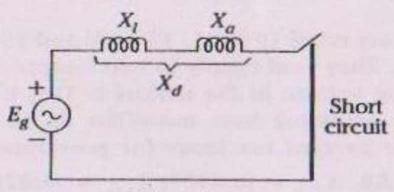


Fig. 2.2

In this case, the direct axis synchronous reactance is given as

$X_d=X_l+X_a$ 2.1

At the instant of short circuit, the offset d.c current appears in all the three phase of the stator. This transient d.c current will induce currents in the rotor field winding and damper winding by transformer action. The induced currents in these two windings will be in such a direction as to oppose the change of magnetic flux produced by the armature. This effect can be represented by two reactances in parallel with X_a as shown in fig 2.3. Here X_f is the reactance of the field winding and X_{dw} the reactance of the damper winding.

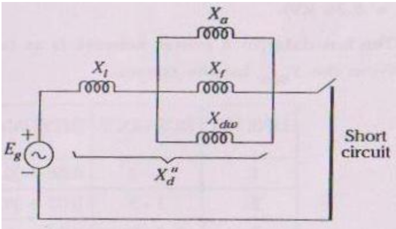


Fig. 2.3

The combined effect of all the three reactances is to reduce the total reactance of the machine and so the short circuit current is very large in this period called as the sub transient state.

The total reactance of the machine under this condition is called as the sub transient reactance X_d'' and is given by

$X_d'' =X_l+(1 / ((1/ X_a)+(1/ X_f)+(1/ X_{dw})))$ 2.2

The induced currents in both the field and damper windings decrease exponentially depending on their time constants (=L/R). The time constant of damper winding is much less than the time constant of field winding. Hence the induced currents in the damper winding dies very fast within the first few cycles effectively X_{dw} becomes open circuited and the resulting reactance is called as transient reactance X_d'. The transient state model of the alternator is shown in fig 2.4

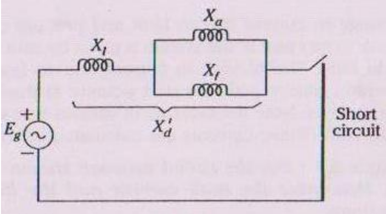


Fig. 2.4

From the figure, it can be observed that

$X_d'=X_l+(1 / ((1/ X_a)+(1/ X_f)))$ 2.3

The effect of field winding current will also die out in a short time depending on the time constant of the field winding. This effect is equivalent to open circuited X_f and thus the armature regains its normal synchronous reactance Xd. (fig 2.2)

From equations 2.1, 2.2 & 2.3, we can observe that the subtransient reactance of the machine is the smallest and steady state reactance of the machine is highest among the reactances. Therefore, $X_d''<X_d'<X_d$.

The various reactances of the synchronous machine can be estimated from the oscillogram shown in fig 2.1. The envelope of the current wave during transient period can be extrapolated backwards in time to meet the y-axis at point-b. Similarly the envelope of the current wave during steady state period can be extrapolated backwards in time to meet the y-axis at point a.

Let

|I|=RMS value of steady state current

|I'|=RMS value of transient current excluding d.c. component

|I"|=RMS value of subtransient current excluding d.c. Component

from the oscillogram of fig 2.1, we get

$|I|=oa/\sqrt{2}$; $|I'|=ob/\sqrt{2}$; $|I''|=oc/\sqrt{2}$

therefore,

$X_d''=E_g/|I''| = E_g/(oc/\sqrt{2})$ 2.4

$X_d'=E_g/|I'| = E_g/(ob/\sqrt{2})$ 2.5

$X_d=E_g/|I| = E_g/(oa/\sqrt{2})$ 2.6

2.3 Short circuit of a loaded synchronous Generator

Our analysis all along had been for a synchronous generator operating on no-load. In practice before short-circuit, the generator would mostly be on load. The determination of short circuit currents when the machine is on load involves the determination of internal voltages behind subtransient, transient and steady state reactances. These are the voltages obtained by adding vectorially the subtransient,

transient and steady state reactance voltage drops respectively due to the load current I_L to the terminal voltage V. Hence

$E_g''=V+I_L(jX_d'')$ 2.7

$E_g'=V+I_L(jX_d')$ 2.8

$E_g=V+I_L(jX_d)$ 2.9

The circuit models of the synchronous generator under the aforesaid conditions is as shown in fig 2.5a, 2.5b and 2.5c.

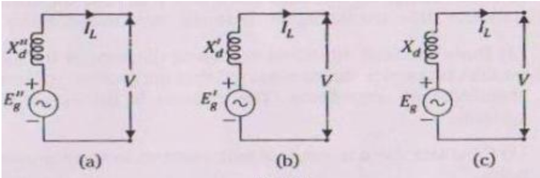


Fig. 2.5

The synchronous motors have internal emfs and reactances similar to that of a generator except that the current direction is reversed. During short circuit conditions these can be replaced by similar circuit models as shown in fig 2.5 except that the voltage behind the subtransient, transient and steady state reactance is given by

$E_m''=V-I_L(jX_d'')$ 2.10

$E_m'=V-I_L(jX_d')$ 2.11

$E_m=V-I_L(jX_d)$ 2.12

2.4 Analysis of three phase symmetrical faults

The symmetrical fault can be analysed on single phase basis using reactance diagram or by using per unit reactance diagram. The symmetrical fault analysis has to be performed separately for subtransient, transient and steady state conditions of the fault, because the reactances and internal emfs of the synchronous machines will be different in each state. Once the per unit reactance diagram of the power system is formed for a particular state (subtransient/transient/steady state) of fault condition, then the currents and voltages in the various parts of the system can be determined by any one of the following method:

i)Using Kirchoff's laws

ii)Using Thevenin's theorem.

iii)By forming the bus impedance matrix.

The first two methods are discussed in this chapter.

Symmetrical fault analysis using Kirchoff's laws.

The following procedure can be followed to directly calculate voltages and currents during symmetrical fault condition in a power system, using Kirchoff's laws,

i) Select appropriate base values and determine the prefault condition reactance diagram of the given power system. (The prefault condition reactance diagram is separately formed for subtransient, transient and steady state condition of the fault).

ii) Calculate the internal emfs of the synchronous machines and the prefault voltages at the fault point using prefault current. (load current).

Note: If the power system is unloaded (i.e if there is no prefault current), then the prefault voltage at the fault point is 1 p.u. Also the internal emfs for subtransient and transient state are the same as steady state induced emf.

iii) Draw the fault condition reactance diagram of the system. This diagram is same as prefault reactance diagram except that the fault is represented by a short circuit or by the specified fault impedance. The currents in this reactance diagram are fault condition currents.

4) Calculate the p.u value of the fault currents in various parts of the system and at the fault point.

5) The actual values of the fault currents are obtained by multiplying the p.u values by the respective base currents.

Symmetrical fault analysis using Thevenin's theorem.

The following procedure can be followed to calculate the voltages and currents during symmetrical fault using Thevenin's theorem.

1) Select appropriate base values and determine the prefault condition reactance diagram of the given power system.

2) Calculate the prefault Thevenin's voltage at the fault point using the prefault current(load current). If the system is unloaded, then the prefault voltage is 1p.u.

3) Determine the Thevenin's impedance of the system at the fault point by shorting all voltage sources.

4) Draw the Thevenin's equivalent at the fault point. Then the p.u value of fault current is given by $I_f=V_{TH}/(Z_{TH}+Z_f)$. Multiplying the p.u value by the base value gives the actual value of the fault current. Here, Z_f is the fault impedance of the system. For a solid three phase short circuit, $Z_f=0$

5) The fault current in other parts of the network are determined from the knowledge of change in current due to fault and prefault current. The fault current (i.e post fault current) in any part of the system is given by sum of prefault current

and change in current due to fault. The change in current due to fault can be estimated by connecting the Thevenin's source with reversed polarity at the fault. Replace all other sources by zero values sources. Now the currents in various part of the system are the change in currents due to fault. These currents are calculated using any conventional technique.

Example 2.1:

For the radial network shown in fig. 2.6, a three phase fault occurs at F. Determine the fault current and the line voltage at 11kV bus under fault conditions.

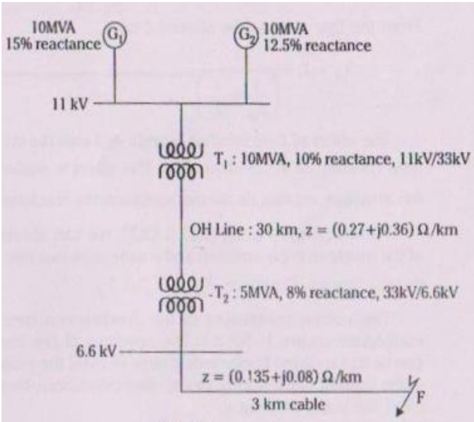


Fig. 2.6

Solution:

Base values:

Let us choose,

base MVA=100

base kV in the overhead line=33

we calculate,

base kV on the generator side=33×11/33= 11

base kV on the cable side=33×6.6/33= 6.6

Reactance of generator G_1:

$X_{G1, new} = X_{G1, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.15 \times (100 / 10) \times (11^2 / 11^2)$

$= j\ 1.5$ p.u

Reactance of generator G_2:

$X_{G2, new} = X_{G2, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.125 \times (100 / 10) \times (11^2 / 11^2)$

$= j\ 1.25$ p.u

Reactance of transformer T_1: (calculated secondary side it)HV or HT

$X_{T1,\ new} = X_{T1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.1 \times (100 / 10) \times (33^2 / 33^2)$

$= j\ 1.0$ p.u

Reactance of transformer T_2: (calculated primary side of it)HV or HT

$X_{T2,\ new} = X_{T2,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.08 \times (100 / 5) \times (33^2 / 33^2)$

$= j\ 1.6$ p.u

Impedance of O.H line:

$Z_{O.H,p.u} = Z_{O.H}(\Omega) \times (MVA)_{B,new} / (kV)^2_B$

$= (30 \times (0.27+j0.36)) \times 100 / 33^2$

$= 0.744+j0.99$ p.u

Impedance of cable:

$Z_c = Z_c(\Omega) \times (MVA)_{B,new} / (kV)^2_B$

$= (3 \times (0.135+j0.08)) \times 100 / 6.6^2$

$= 0.93+j0.55$ p.u

the prefault impedance diagram of the given system is as shown in fig. 2.7

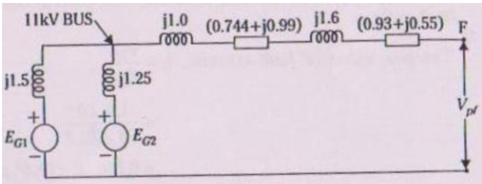


Fig. 2.7

Since the system is unloaded prior to occurrence of fault, V_{pf} is assumed as 1p.u. Thevenin's theorem is employed here to find the fault current.

$V_{TH} = V_{pf} = 1$p.u

To find Z_{TH}:

Shorting the generated voltages, we obtain the equivalent circuit of the system prior to the fault as in fig. 2.8

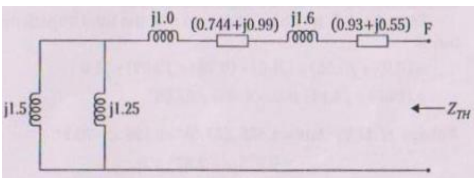


Fig. 2.8

$Z_{TH} = ((j1.5 \times j1.25) / (j1.5+j1.25)) + (j1.0+0.744+j0.99+j1.6+0.93+j0.55) = 1.674+j4.82 = 5.1\angle 70.8°$ p.u

Thus, the Thevenin's equivalent circuit of the system with respect to fault point is as shown if fig 2.9

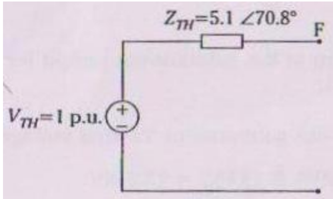


Fig. 2.9

Now short circuiting the terminals of the Thevenin's equivalent circuit as shown in fig. 2.10 is equivalent to the fault condition. The current flowing through the short circuit is the fault current.

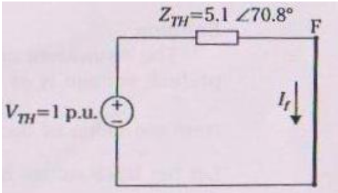


Fig. 2.10

The p.u value of fault current, $I_f=V_{TH}/Z_{TH}=(1.0\angle 0°) / (5.1\angle 70.8°) = 0.196 \angle -70.8°$ p.u

The base current,

I_b= (1000 × base power) / (√3 × base voltage)= (1000×100) / (√3 × 6.6) = 8747 A

therefore,

absolute value of fault current , I_f= 0.196 ∠-70.8° × 8747 = 1714 ∠-70.8° A

To find voltage at 11 kV bus during fault:

From fig 2.7, it can be observed that the total impedance between point F and 11kV bus is,

=(0.93+j0.55)+j1.6+(0.744+j0.99)+j1.0

=(1.674+j4.14)p.u

=4.466∠67.98°

Voltage at 11kV bus=4.466∠67.98°×0.196 ∠-70.8°=0.875∠-2.82° p.u

The absolute value of the voltage at 11kV bus=0.875∠-2.82°×11=0.9625∠-2.82°kV

Example 2.2:

A synchronous generator and motor are rated for 30,000kVA, 13.2kV and both have subtransient reactance of 20%. The line connecting them has a reactance of 10% on the base of machine ratings. The motor is drawing 20,000kW at 0.8p.f leading. The terminal voltage of the motor is 12.8kV. When a symmetrical three phase fault occurs at motor terminals, find the subtransient current in generator, motor and at the fault point.

Solution:

the equivalent circuit of the system in the subtransient period for the calculation of prefault voltage is as shown in fig 2.11.

Here the valued of the voltage sources are subtransient internal voltages.

Let we choose,

base MVA=30

base kV=13.2kV

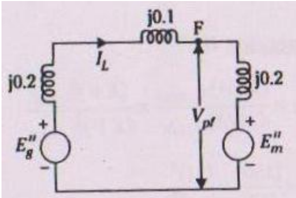


Fig. 2.11

The prefault voltage at the fault point , V_{pf}=12.8kV,

let us use this as the reference phasor per unit value of the prefault voltage,

V_{pf}=actual value/base value=12.8/13.2=0.97∠0°

base current, I_b= (1000 × base power) / (√3 × base voltage)= I_b= (1000 × 30) / (√3 × 13.2)=1312A

The load current, $I_L=(P/\sqrt{3}\ V\ \cos\varphi)\angle\cos^{-1}\varphi=20000/(\sqrt{3}\times12.8\times0.8)\angle\cos^{-1}0.8=$ 1128∠36.9° A

The load current in p.u IL=1128∠36.9°/1312= 0.8594∠36.9° p.u

Method-1:

Using Kirchoff's laws:

The subtransient voltages E_g'' and E_m'' of the fig 2.11 is calculated by Kirchoff's voltage law as shown below.

E_g'' = j0.2IL+j0.1IL+Vpf

=j0.2 (0.8594∠36.9°)+j0.1(0.8594∠36.9°)+0.97

=0.84∠14.2° p.u

E_m''=Vpf- j0.2IL

=0.97-j0.2(0.8594∠36.9°)

=1.0819∠-7.3° p.u

The equivalent circuit of the system on the occurrences of a three phase fault is as shown in fig 2.12

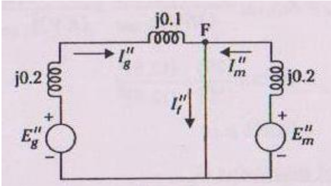


Fig. 2.12

The subtransient fault current I_f'' at the point computed by summing up the subtransient fault current in the generator. I_g'' and the subtransient fault current in the motor I_m'' i.e $I_f''=I_g''+I_m''$.

Applying KVL in the circuit of fig 2.12, we get

$j0.2I_g''+j0.1I_g''=E_g''$

or

$I_g''=E_g''/j0.3=(0.84\angle 14.2°)/ (0.3\angle 90°)=2.8\angle -75.8°$ p.u

also,

from the fig 2.12

$j0.2I_m''=E_m''$

or

$I_m''=E_m''/j0.2=(1.0819\angle -7.3°)/(0.2\angle 90°)=5.4095\angle -97.3°$ p.u

hence,

the current at the fault point $I_f''=I_g''+I_m''$

therefore,

$I_f''=2.8\angle -75.8°+5.4095\angle -97.3°=8.065\angle -90°$

Method-2:

Using Thevenin's theorem:

The prefault voltage at the fault point, $V_{pf}=0.97\angle 0°$ p.u. This is the Thevenin's voltage at the fault point.

Threfore,

$V_{TH}=0.97\angle 0°$ p.u

To compute Z_{TH}:

Short circuiting all the voltage sources in the equivalent circuit of fig.2.11, we get the circuit as shown below.

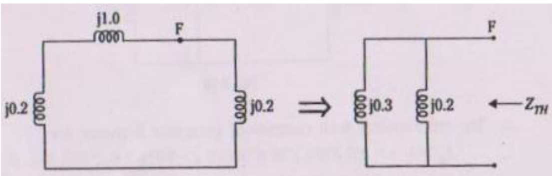


Fig. 2.13

$Z_{TH}= ((j0.3)(j0.2))/(j0.3+j0.2)=j0.12$

Hence the Thevenin's equivalent circuit of the system with respect to the fault point is as shown in fig.2.14

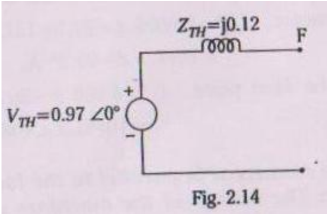


Fig. 2.14

The equivalent circuit during fault condition as shown in fig 2.15

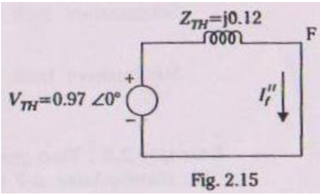


Fig. 2.15

current at the fault point, $I_f''=V_{TH}/Z_{TH}=(0.97\angle 0°)/j0.12=8.06\angle -90°$ p.u

To find subtransient fault current in motor and generator

when the fault occurs, there is a change in the current supplied by the motor and generator. This change is calculated by connecting the Thevenin's voltage with reversed polarity at the fault point as shown in fig 2.16. Here all other voltage sources are replaced by short circuit.

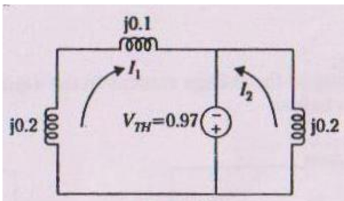


Fig. 2.16

Now,

$I_1=0.97/(j0.2+j0.1)=3.23\angle -90°$

$I_2=0.97/j0.2=4.85\angle -90°$

therefore,

the subtransient fault current of generator and motor are

$I_g''=I_L+I_1=(0.8594\angle 36.9°)+(3.23\angle -90°)=2.8\angle -75.8°$ p.u

$I_m''=I_2-I_L=(4.85\angle -90°)-(0.8594\angle 36.9°)=5.4095\angle -97.3°$ p.u

Thus, we find the currents calculated by both the methods are the same. The absolute values of the currents can be obtained by multiplying the per unit values by the base current.

Therefore,

subtransient fault current in generator, $I_g''=2.8\angle -75.8°\times 1312 = 3673.6\ \angle -75.8°$ A

subtransient fault current in motor, $I_m''=5.4095\angle -97.3°\times 1312=7097.2\angle -97.3°$ A

subtransient fault current at the fault point, $I_f''=8.065\angle 90°\times 1312= 10581.3\angle -90°$ A

Example 2.3:

Two generators are connected in parallel to the low-voltage(L.V) side of a three phase Δ-Y transformer. The ratings of the machines are

Generator G_1: 50 MVA, 13.8kV, Xd"=25%

Generator G_2: 25MVA, 13.8kV, Xd"=25%

Transformer T: 75MVA, 13.8 Δ -69 Y kV, X=10%

Before the fault occurs, the voltage on the high voltage (HV) side of the transformer is 66kV. The transformer is unloaded, and there is no circulating current between the generators. Find the subtransient current in each generator when a three phase fault occurs on the high voltage side of the transformer.

Solution:

base values:

let us choose,

base MVA= 75

base kV on HV side of transformer=69

we calculate,

base kV on the generator =69×13.8/69=13.8

Reactance of generator G_1:

$$X_{G1,\,new} = X_{G1,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$$

$$= j0.25 \times (75 / 50) \times (13.8^2 / 13.8^2)$$

$$= j\,0.375 \text{ p.u}$$

Reactance of generator G_2:

$$X_{G2,\,new} = X_{G2,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$$

$$= j0.25 \times (75 / 25) \times (13.8^2 / 13.8^2)$$

$$= j\,0.75 \text{ p.u}$$

Reactance of transformer:

X_T=j0.1

as the base values choose are of the same transformer, so its p.u reactance remains the same.

The reactance diagram of the system for the calculation of prefault values is as shown in fig 2.17

The prefault voltage on the high voltage side is 66kV. This is equal to 66/69=0.957 p.u

The equivalent subtransient reactance as visualised from the fault point is,

((j0.375×j0.75)/(j0.375+j0.75))+j0.1=j0.35 p.u

therefore,

the subtransient current in the short circuit is,

I_f''=0.957/j0.35=2.735∠-90° p.u

To find the subtransient currents in the generators:

The subtransient fault current divides between the generators inversely as the impedances of the generators.

In generator G_1:

$I_{g1}'' = 2.735\angle{-90°} \times (j0.75/j1.125) = 1.823\angle{-90°}$ p.u

In generator G_2:

$I_{g2}'' = 2.735\angle{-90°} \times (j0.375/j1.125) = 0.912\angle{-90°}$ p.u

The absolute values of the above currents can be obtained by multiplying the p.u values by the base current.

Base current,

$I_B = (1000 \times \text{base power}) / (\sqrt{3} \times \text{base voltage}) = (1000 \times 75) / (\sqrt{3} \times 13.8) = 3137.7$ A

hence the actual currents are,

$I_f'' = 2.735\angle{-90°} \times 3137.7 = 8581.6\angle{-90°}$A

$I_{g1}'' = 1.823\angle{-90°} \times 3137.7 = 5720\angle{-90°}$A

$I_{g2}'' = 0.912\angle{-90°} \times 3137.7 = 2861.6\angle{-90°}$ A

2.5 Selection of circuit breakers

When fault occur in a part of power system, heavy current flows in that part of circuit which may cause permanent damage to the equipments connected therin. Hence the faulty part should be isolated from the healthy part immediately on the occurrence of a fault. This is can be achieved by providing protective relays and circuit breakers. The protective relays sense the faulty conditions and sends signals to circuit breakers to open the circuit under faulty condition, the circuit breakers can be used as a switch.

The selection of a circuit breaker for a power system depends not only upon the current that the breaker is to carry under normal operating conditions but also upon the maximum current it may have to carry momentarily and the current is may have to interrupt at the voltage of the line in which it is placed. Hence, the choice of a circuit breaker for particular application depends on the following ratings of the circuit breaker.

1)Normal working power level specified as rated interrupting current or rated interrupting MVA.

2)The fault specified as either the rated short circuit interrupting current or rated short circuit interrupting MVA.

3)Momentary current rating.

4)Normal working voltage.

5)Speed of circuit breaker.

The speed of circuit breaker is the time between the occurrence of the fault to the extinction of the arc(when the contact opens). It is normally specified in cycle of power frequency. One cycle for 50Hz power frequency is 1/50=0.02 m sec. The standard speed of circuit breakers are 8,5,3,2 or 1 1//2 cycles.

The momentary current rating is the maximum current that may flow through a circuit breaker for a short duration. It is the current that flow during subtransient period of fault condition. In fault analysis, the subtransient fault current calculated using subtransient circuit model is the a.c component of the short circuit current. It is multiplied by a factor of 1.6 to account for the d.c offset current. This gives the maximum momentary current during fault.

The circuit breaker will open its contacts usually in the transient period and so the short circuit interrupting current rating depends on the transient currents. In fault analysis, the a.c component of the transient current obtained is multiplied by a factor 1.0 to 1.5 to get the maximum interrupting current. The factor 1.0 to 1.5 accounts for the d.c. Offset current during transient period. The circuit breaker is chosen such that its short circuit interrupting current rating is less than the calculated value. The multiplying factor to find the interrupting current depends on the speed of the circuit breaker. These are indicated in table 2.1

Speed of circuit breaker	Multiplying factor
8 cycles or more	1.0
5 cycles	1.1
5 cycles	1.2
5 cycles	1.4
1 1/2 cycles	1.5

Table 2.1

Example 2.4:

A 25MVA, 13.8kV generator with X_d''=15% is connected through a transformer to a bus that supplies four identical motors as shown in fig. 2.18. Each motor has X_d''=20% & X_d'=30% on a base of 5MVA, 6.9kV. The three phase rating of the transformer is 25MVA, 13.8-6.9 kV. With a leakage reactance of 10%. The bus voltage at the motors is 6.9kV when a three-phase fault occurs at the point P. For the fault specified determine:

a)The subtransient current in the fault.

b) The subtransient current in the breaker A.

c) The momentary current in breaker A.

d)The current to be interrupted by breaker A in 5 cycles.

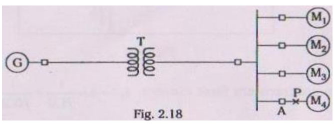


Fig. 2.18

Solution:

base values:

let we choose,

base MVA=25

base kV in the generator circuit=13.8

we calculate,

base voltage on the motor side=13.8×6.9/13.8=6.9

Reactance of generator G:

X_{dG}'' =j0.15(same as old p.u value in given because base values have been chosen on the same machine ratings)

X_{dG}' =j0.15(same as subtransient reactance as it is not specified in data).

Reactance of transformer T:

X_T=j0.1(same as old p.u value in given because base values have been chosen on the same machine ratings)

Reactances of motors:

$X_{dM,p.u,new}'' = X_{dM,p.u,old}'' \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.2 \times (25 / 5) \times (6.9^2 / 6.9^2)$

$= j\ 1.0$ p.u

$X_{dM,p.u,new}' = X_{dM,p.u,old}' \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.3 \times (25 / 5) \times (6.9^2 / 6.9^2)$

$= j\ 1.5$ p.u

The prefault voltage at the point P is 6.9kV=6.9/6.9=1p.u and the base current in the 6.9kV circuit is,

I_b= (1000 × base power) / (√3 × base voltage)= (1000 × 25) / (√3 × 6.9)=2091.8A

The reactance diagram with subtransient values of the reactance marked is shown in fig 2.19.

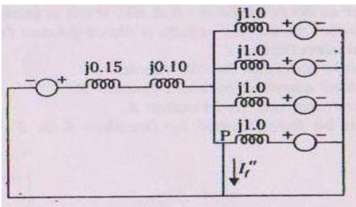


Fig. 2.19

a) therefore subtransient fault current, I_f''=4×(1/j1.0)+(1/j0.25)= -j8p.u

The absolute value of the current is I_f''= -j8 × 2091.8 = - j16734.4 A

b) therefore subtransient current in breaker A, I″= 3×(1/j1.0)+(1/j0.25)= -j7p.u

The absolute value of the current is I″= -j7 × 2091.8 = - j14642.6 A

c)To find the momentary current in the breaker A, we must account for the d.c. Offset current. This is done empirically as follows:

Momentary current through breaker A= 1.6×14642.6=23428.16 A

d)To compute the current to be interrupted by the breaker A, it is required to obtain the transient reactance model of the system. This is shown in fig 2.20

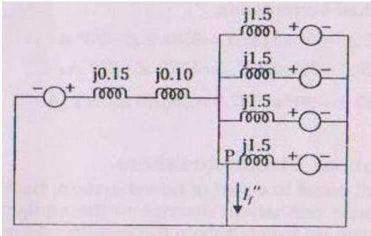


Fig. 2.20

The current to be interrupted by the breaker A now is =3×(1/j1.5)+(1/j0.25)= -j6p.u

Allowance is made for the d.c. Offset current by multiplying with a factor 1.1(see table 2.1). therefore the current to be interrupted is,

1.1×6×2091=13805.88A

Additional Examples:

Example 2.7:

A three phase, 5MVA, 6.6kV alternator with reactance of 8% is connected to a feeder of series impedance of (0.12+j0.48) ohms/phase per km. The transformer is rated at 3MVA, 6.6kV/33kV and has a series reactance of 5%. Determine the fault current supplied by the generator operating under no-load with a voltage of 6.9kV, when a three phase symmetrical fault occurs at a point 15km along the feeder.

Solution:

The single line diagram of the power system is as shown in fig 2.29. let F be the point of occurrence of the fault.

Base vaules:

Let us chose the generator rating as base values.

Therefore,

base MVA= 5

base kV on the generator=6.6

base kV on the transmission line=6.6×33/6.6=33

Reactance of generator:

X_G=8%=j0.08 p.u

Reactance of transformer T: (calculated secondary side of it)HV or HT

$X_{T, new} = X_{T, old} \times ((MVA)_{B, new} / (MVA)_{B, old}) \times ((kV)^2_{B, old} / (kV)^2_{B, new})$

$= j0.05 \times (5 / 3) \times (33^2 / 33^2)$

= j 0.833 p.u

Impedance of the feeder:

$Z_{TL,p.u} = Z_{TL}(\Omega) \times (MVA)_{B,new} / (kV)^2_B$

$= (15 \times (0.12+j0.48)) \times 5 / 33^2$

$= 0.0083+j0.033$ p.u

using these values, the prefault impedance diagram is as shown in fig 2.30

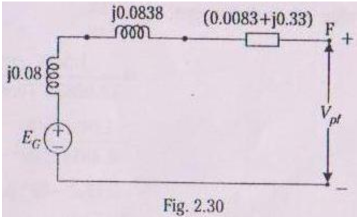


Fig. 2.30

To find E_G and V_{pf}:

Actual value of induced emf, E_G= 6.9kV

p.u value of induced emf, E_G= actual value/base value=6.9/6.9=1.0455 p.u

The prefault voltage V_{pf} at fault point F is the voltage under no-load=34.5kV therefore,

prefault voltage V_{pf}=34.5kV

The p.u value of prefault voltage, V_{pf}=actual value/base value=34.5/33=1.0455p.u

To find fault current:

The Thevenin's equivalent circuit of the system in fig 2.30 as seen from the fault point F is shown in fig 2.31. Here

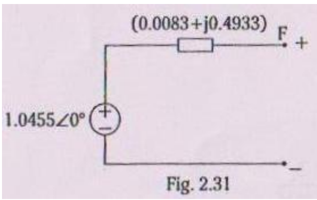


Fig. 2.31

$V_{TH}=1.045\angle 0°$

$Z_{TH}=j0.08+j0.0833+(0.0083+j0.33)=0.0083+j0.04933$

The fault in the feeder can be represented by a short circuit as shown in fig. 2.32. Now the current If through the short circuit is the fault current.

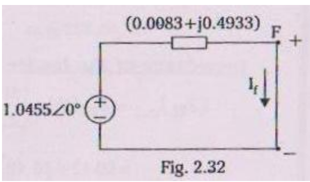


Fig. 2.32

Therefore,

p.u value of fault current, $If=V_{TH}/Z_{TH}=(1.045\angle 0°)/(0.0083+j0.04933)=2.12\angle -89°$ p.u therefore,

actual value of fault current, If= p.u value × base current

base current=(1000 × base power) / (√3 × base voltage)= (1000 × 5) / (√3 × 33)=87.47A

If=(2.12∠-89°)×87.47=185.45∠-89° A

Example 2.8:

Two synchronous motors are connected to the bus of a large system through a short transmission line as shown in fig 2.33. The ratings of the various components are:

Motors (each): 1MVA, 440V, 0.1p.u, transient reactance

line: 0.05 ohm reactance

large system: short circuit MVA at its bus at 440V is 8. when the motors are operating at 440V, calculate the short circuit current fed into a three phase fault at the motor bus.

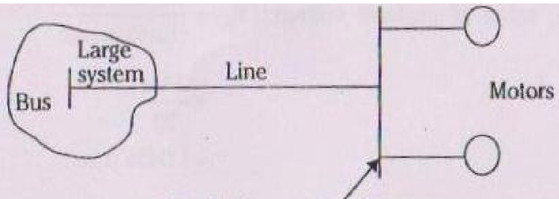


Fig. 2.33

Solution:

base values:

let us choose the motor ratings as base values

therefore,

base MVA=1

base kV on the motor side=0.44

The large system can be considered as a source of constant voltage feeding the line through an "infinite bus". The voltage rating of the bus is 440V(as given)

Reactance of Motors:

X_M=j0.1 p.u

Reactance of line:

$X_{TL,p.u}= X_{TL}(\Omega) \times (MVA)_{B,new} / (kV)^2_B$

$= j0.05 \times 1 / 0.44^2$

$=j0.258$ p.u

Hence the p.u reactance diagram is as shown in fig 2.34.

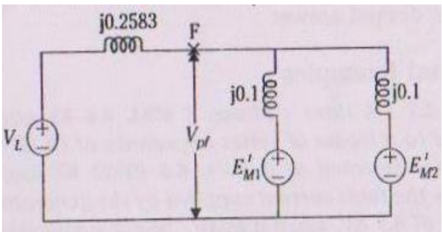


Fig. 2.34

The prefault voltage at the motor bus, V_{pf}=400V

p.u value of prefault voltage, V_{pf}=400/440=0.909p.u

p.u value of voltage at infinite bus, V_L=440/440=1p.u

The fault condition of the system is shown in fig 2.35. The total fault current I_f' is sum of the fault current I_{f1}' and I_{f2}'

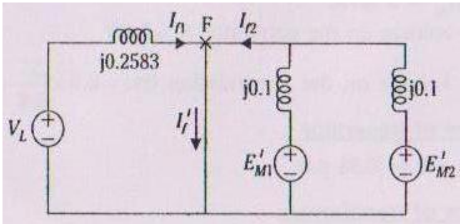


Fig. 2.35

From the fig 2.35, it can be observed that,

$I_{f1}'=V_L/j0.2583=1\angle 0°/j0.2583= -j3.87$ p.u

$I_{f2}'=V_{pf}/(j0.1/2)=0.909\angle 0°/j0.5= -j18.18$ p.u

$I_f'= I_{f1}'+I_{f2}'= -j3.87 -j\ 18.18 = -j22.05$ p.u $= 22.05\angle -90°$ p.u

Actual value of fault current = p.u value of fault current × base current

$=22.05\angle -90° \times (1000 \times 1) / (\sqrt{3} \times 0.44)$

$=28933.12 \angle -90°$ A.

Unsolved example:

2.3) A 25 MVA, 11kV, synchronous generator having a subtransient reactance of 1.5 p.u is supplying 20MW power at 0.8p.f.lagging to a synchronous motor through a transmission line. The voltage at the terminals of the generator is 10.5kV. The transmission line has a reactance of 0.5 ohm and the motor a subtransient reactance of 1.2 ohm. If a three phase fault occurs at the terminals of the motor, determine the fault current from each machine.

Ans:($I_g''=3794\angle -77°$ A. $I_m''=4301\angle -110.8°$ A).

------------END-----------

3.1 Introduction:

A symmetrical, balanced three phase system can be analysed on a single phase basis. But, an unbalanced three phase system does not permit this simplification as it involves phasors of different magnitude and phase angles in each phase. Analysis under unbalanced conditions has to be carried out on a three phase basis which is very cumbersome process. Alternatively, a more convenient method of analysing unbalanced operation is through symmetrical components.

Dr. Fortescue's theorem forms the basis of the study of symmetrical components. According to the theorem, an unbalanced system of n-related phasors can be resolved into "n" systems of balanced phasors called symmetrical components of the original phasors. The "n" phasors of each set of components are equal in length and the angles between the adjacent phasors of the set are equal. The method of symmetrical components is a general one applicable to any unbalanced polyphase system. Because of the widespread use of three phase systems, the study here is confined to three phase systems only.

3.2 Resolution of unbalanced phasors.

According to Fortescue's theorem, a set of three unbalanced phasors (voltages or currents) can be resolved into three sets of balanced phasors, each set containing three phasors. The three sets of balanced components are called positive sequence components, negative sequence components and zero sequence components. Positive sequence components consists of three balance phasors of equal magnitude, displaced from each other by 120° in phase and having the same phase sequence as the original unbalanced phasors.Negative sequence components consists of three balanced phasors of equal magnitude, displaced from each other by 120° in phase and having a phase sequence opposite to that of the original unbalanced phasors. Zero sequence components are a set of three phasors, equal to each other in all respect.

Consider three unbalanced phasors V_a, V_b and V_c as shown in fig 3.1.

Let the direction of rotation of the phasors be in the anti clockwise direction. Then, it can be observed that the phase sequence of these three unbalanced phasors is abc.

The positive sequence components V_{a1}, V_{b1} and V_{c1} shown in fig. 3.2a constituting a three phase system are equal in magnitude and are symmetrically displaced by 120°. They have the same phase sequence 'abc' as the original unbalanced phasors. The negative sequence components V_{a2}, V_{b3} and V_{c2} shown in the fig 3.2b, constituting a three phase system are equal in magnitude, symmetrically displaced by 120° and have the phase sequence acb; opposite to that of the original phasors. The zero sequence components V_{a0}, V_{b0} and V_{c0} shown in fig. 3.2c are equal in all respects. These three phasors do not constitute a three phase system. They are equivalent to three single phase phasors of equal magnitude and having zero displacement between them.

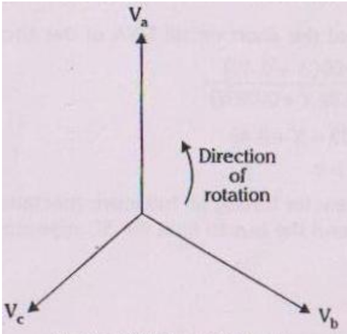
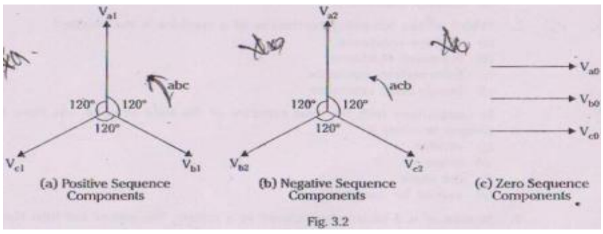


Fig. 3.1 : Unbalanced Phasor

Note:

1)Subscripts 1, 2 and 0 are used to indicate positive, negative and zero sequence components respectively.

2)The above three sets of phasors can be either voltages or currents.



(a) Positive Sequence Components (b) Negative Sequence Components (c) Zero Sequence Components

Fig. 3.2

3.3 The 'a' operator.

Because of the phase displacement of the symmetrical components of voltages and currents in a three phase system, it is convenient to have a short hand method of indicating the rotation of phasors through 120°. The letter 'a' (some books denote it as ɑ or λ also) is used to designate the operator that causes a rotation of 120° in the anticlockwise direction. This operator is a complex number of unit magnitude with an angle of 120° and is defined by the following expressions:

$a = 1\angle 120° = 1.\ e^{j120°} = \cos 120° + j\sin 120° = -0.5 + j0.866$

any phasor which is multiplied by 'a' remains unchanged in magnitude but is rotated by 120° in the anticlockwise direction.

Similarly, $a^2 = a.a = 1\angle 240° = 1.\ e^{j240°} = \cos 240° + j\sin 240° = -0.5 - j0.866$

Hence, operator 'a^2' will rotate a phasor in anticlockwise direction by 240°. This is same as rotating the phasor in clockwise direction by 120°.

It can be easily shown that,

1) $a^3 = 1$
2) $a^4 = a$
3) $1+a+a^2 = 0$
4) $a^* = a^2$, (* is conjugate)
5) $a-a^2 = j\sqrt{3}$
6) $a^2-a = -j\sqrt{3}$

These relations will be used in our discussion.

3.4 Expression for phase voltages in terms of symmetrical components.

Referring to fig. 3.2a, it can be observed that V_{b1} leads V_{a1} by 240° and the phasor V_{c1} leads V_{a1} by 120°. Since these three are also equal in magnitude we can write,

$V_{b1}=V_{a1}\angle 240°$

$V_{c1}=V_{a1}\angle 120°$

Making use of the 'a' operator, the above equations can be written as,

$V_{b1}=a^2.V_{a1}$

$V_{c1}=a.V_{a1}$ 3.1

On the same lines, referring to fig 3.2b, we get,

$V_{b2}=a.V_{a2}$

$V_{c2}=a^2.V_{a2}$ 3.2

and from fig 3.2c, it can be established that

$V_{a0}=V_{b0}=V_{c0}$ 3.3

Since three unbalanced phasors V_a, V_b and V_c can be resolved into three sets of balanced phasors, the phasor V_a, is equal to the sum of the positive sequence component V_{a1} of phase a, the negative sequence component V_{a2} of phase a and the zero sequence component V_{a0} of phase a. That is,

similarly,

$V_a=V_{a0}+V_{a1}+V_{a2}$ 3.4

$V_b=V_{b0}+V_{b1}+V_{b2}$ 3.5

$V_c=V_{c0}+V_{c1}+V_{c2}$ 3.6

Using equations 3.1, 3.2 and 3.3, the above expressions can be rewritten in terms of phase a as,

$V_a=V_{a0}+V_{a1}+V_{a2}$ 3.7

$V_b = V_{a0}+a^2.V_{a1}+a.V_{a2}$ 3.8

$V_c = V_{a0}+a.V_{a1}+a^2.V_{a2}$ 3.9

In matrix form, the above equation can be written as,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad \text{........................3.10}$$

The above equations establishes the relationship between the phase voltages of an unbalanced system and the symmetrical components.

3.5 Expression for symmetrical components in terms of phase voltages.

Let us denote,

$$[T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

then eq. 3.10 becomes,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = [T] \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

or $$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = [T]^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{.........................3.11}$$

we determine

$[T]^{-1}$= adj[T] / det[T]

In this case,

$\det[T]= 1(a^4-a^2) -1(a^2-a) +1(a-a^2)=1(a-a^2)+(a-a^2)+(a-a^2)=3(a-a^2)$

and,

$$\text{adj}[T]= \begin{bmatrix} +(a^4-a^2) & -(a^2-a) & +(a-a^2) \\ -(a^2-a) & +(a^2-1) & -(a-a^2) \\ +(a-a^2) & -(a-1) & +(a^2-1) \end{bmatrix} = \begin{bmatrix} (a-a^2) & (a-a^2) & (a-a^2) \\ (a-a^2) & a.(a-a^2) & a^2.(a-a^2) \\ (a-a^2) & a^2.(a-a^2) & a.(a-a^2) \end{bmatrix}$$

$$=(a-a^2) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

therefore,

$[T]^{-1}$= adj[T] / det[T]

$$= (1 / 3(a-a^2))\times(a-a^2) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Hence, eq. 3.11 becomes

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad ....3.12$$

in general form eq. 3.12 can be written as,

V_{a0}= (1/3) (V_a+V_b+V_c) 3.13

V_{a1}= (1/3) (V_a+a.V_b+a^2.V_c) 3.14

V_{a2}= (1/3) (V_a+a^2.V_b+a.V_c).......................3.15

The above equations gives the sequence components of voltages of phase a in terms of the phase voltages of the unbalanced system.

Equations 3.10 and 3.12 giving the transformation relationships between phase quantities and symmetrical components apply both to phase voltages and line currents of any star connected or equivalent star connected system, for line currents, the transformation is given by,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad ....3.16$$

and,

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad ....3.17$$

Note:

1) Unless otherwise mentioned, symmetrical components of voltages and currents always mean phase voltages and line currents of an equivalent star connected system.

2) Also, by the sequence components of voltages and currents, it is always meant the sequence components of voltages and currents of phase a.

Example 3.1:
Prove that a balanced set of three phase voltages will have only positive sequence components of voltages only.
Solution:
A balanced three phase system of voltages is one where in all the phase voltages are of equal magnitude and symmetrically displaced by 120°. This is shown in fig 3.3

Let Va, V_b and V_c be the balanced system of three phase voltages.
From fig. 3.3, it can be observed that,

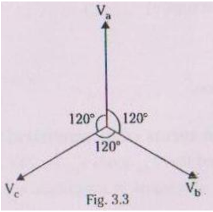


Fig. 3.3

$V_a = V_a$
$V_b = a^2.V_a$
$V_c = a.V_a$ 1

We have,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Using eq. 1 in the above matrix, we get...

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ a^2.V_a \\ a.V_a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} V_a + a^2.V_a + a.V_a \\ V_a + a^3.V_a + a^3.V_a \\ V_a + a^4.V_a + a^2.V_a \end{bmatrix}$$

putting $a^3=1$ and $a^4=a$, we get,

$$= (1/3) \begin{bmatrix} V_a + a.V_a + a^2.V_a \\ V_a + V_a + V_a \\ V_a + a.V_a + a^2.V_a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} V_a + a.V_a + a^2.V_a \\ V_a + V_a + V_a \\ V_a + a.V_a + a^2.V_a \end{bmatrix}$$

$$= (1/3) \begin{bmatrix} V_a(1+a+a^2) \\ 3V_a \\ V_a(1+a+a^2) \end{bmatrix}$$

but,

$(1+a+a^2)=0$,

$$= (1/3) \begin{bmatrix} 0 \\ 3V_a \\ 0 \end{bmatrix}$$

thus,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

comparing the terms, we obtain,

$V_{a0}=0$

$V_{a1}=V_a$

$V_{a2}=0$

This clearly indicates that a balanced set of three phase voltages will have only positive sequence voltages. The negative and zero sequence components are always absent in a balanced system. This holds good for a balanced set of currents as well.

Example 3.2:

Determine the sequence components of the three voltages, $V_a=200\angle 0°V$, $V_b=200\angle 245°V$ and $V_c=200\ \angle 105°V$

solution:

The positive sequence components of voltage is ,

V_{a1}=(1/3) (V_a + a. V_b+ a^2.V_c)

=(1/3) (200∠0°+200∠245°+120°+200 ∠105°+240°)

=(1/3) (200+ (199.24+j1743)+(193.19-j51.76))

= 0.9748-j11.44

=197.81∠-3.3° V

The negative sequence component of voltage is,

V_{a2}=(1/3) (V_a + a^2. V_b+ a.V_c)

=(1/3) (200∠0°+200∠(245°+240°)+200 ∠(105°+120°))

=(1/3) (200+(-114.72+j163.83+(-141.42-j141.42))

=-18.71+j7.47

=20.15∠158.2° V

The zero sequence component of voltage is,

V_{a0}= (1/3) (V_a + V_b+ V_c)

=(1/3) (200∠0°+200∠245°+200 ∠105°)

=(1/3)(200+(-84.52-j181.26)+(-51.76-j193.18))

=21.21+j3.97

=21.6∠16.58° V

Example 3.3: The positive, negative and zero sequence components of line currents are 20∠10° , 6∠60° and 3∠30° A respectively. Determine the line currents.

I_{a1}= 20∠10°

I_{a2}= 6∠60°

I_{a0}= 3∠30°

we have, the line current,

I_a=I_{a0}+I_{a1}+I_{a2}

=3∠30°+20∠10° +6∠60°

=27.25∠21.88° A

I_b= I_{a0}+a^2.I_{a1}+a.I_{a2}

= 3∠30°+20∠(10°+∠240°)+6∠(60°+120°)

=20.1∠-120.7° A

I_c= I_{a0}+a.I_{a1}+a^2.I_{a2}

=3∠30°+20∠(10°+120°) +6∠(60°+240°)

=13.7∠122° A

Example 3.4:

In a three phase system, $I_{a1}=100\angle 30°$ A, $I_{b2}=40\angle 90°$ A and $I_{c0}= 10\angle -30°$ A. Find the line currents.

Solution:

The sequence components of currents given in the problem are not of phase a only. Hence it is first required to express the sequence components in terms of phase a.

Consider fig 3.4. The negative sequence components of line currents are depicted in the sketch.

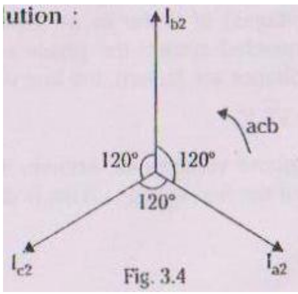


Fig. 3.4

From the fig. It can be observed that,

$I_{a2}=a^2.I_{b2}=40\angle(90°+240°)=40\angle 330°$ A

also we have

$I_{a0}=10\angle -30°$ A

$I_{a1}=100\angle 30°$ A

$I_{a2}=40\angle 330°$ A

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=10\angle -30°+100\angle 30° +40\angle 330°$

$=132.24\angle 10.89°$ A

$I_b= I_{a0}+a^2.I_{a1}+a.I_{a2}$

$= 10\angle -30°+100\angle(30°+240°)+40\angle(330°+120°)$

$=65.57\angle -82.4°$ A

$I_c= I_{a0}+a.I_{a1}+a^2.I_{a2}$

$=10\angle -30°+100\angle(30°+120°)+40\angle(330°+240°)$

$=11.32\angle 167.48°$ A

3.6 Relation between sequence components of phase and line voltages in star connected systems.

It has been emphasized previously that, unless mentioned, by sequence voltages it is always meant phase voltages (line to neutral voltages) of a star or an equivalent star connected system. It is known that in a star connected system the phase voltages are different from the line voltages, But if the phase voltages are known, the line voltages can be easily determined by using the relation $V_{LL}=\sqrt{3}V_p$.

Similarly, if the sequence components of the phase voltages are known, it should be possible to determine the sequence components of the line voltages. This is discussed in the following points.

Let Va, Vb and Vc be the phase voltages having a phase sequence abc as shown in fig 3.5.

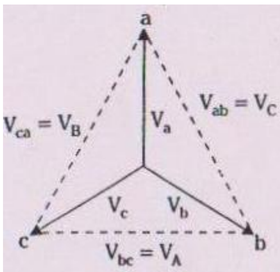


Fig. 3.5

The three line voltages of the system are Vbc, Vca and Vab. It is known from elementary vector algebra that.

$V_{bc} = V_c - V_b$

$V_{ca} = V_a - V_c$

$V_{ab} = V_b - V_a$

Let

$V_{bc} = V_A$(opposite to Vertex A)

$V_{ca} = V_B$(opposite to Vertex B)

$V_{ab} = V_C$(opposite to Vertex C)

therefore, we get

$V_A = V_{bc} = V_c - V_b$

$V_B = V_{ca} = V_a - V_c$

$V_C = V_{ab} = V_b - V_a$ 3.18

The positive sequence component of line voltage is given as

$V_{A1} = (1/3)(V_A + a.V_B + a^2.V_C)$

$= (1/3)((V_c - V_b) + a(V_a - V_c) + a^2.(V_b - V_a))$ .... in View of eq. 3.18

$= (1/3)((a(V_a + a.V_b + a^2.V_c) - a^2(V_a + a.V_b + a^2.V_c))$

$= (1/3)(a - a^2)(V_a + a.V_b + a^2.V_c)$

but, $(V_a + a.V_b + a^2.V_c) = 3.V_{a1}$

and, $(a - a^2) = j\sqrt{3}$

therefore, we get

$V_{A1} = (1/3)(j\sqrt{3})(3.V_{a1})$

Thus, $V_{A1} = j\sqrt{3}.\ V_{a1}$ 3.19

Hence, positive sequence component of line voltage is √3 times the positive sequence component of phase voltage and leads the corresponding phase voltage by 90°.

The negative sequence component of line voltage is

$V_{A2}=(1/3)(V_A+a^2.V_B+a.V_C)$

$=(1/3)((V_c-V_b)+a^2(V_a-V_c)+a(V_b-V_c))$, view of eq. 3.18

$=(1/3)\ (a^2(V_a+a^2.V_b+a.V_c)-a(V_a+a^2.V_b+a.V_c))$

$=(1/3)\ (a^2-a)\ (V_a+a^2.V_b+a.V_c)$

$=(1/3)(-j\sqrt{3})(3.V_{a2})$ [because $(V_a+a^2.V_b+a.V_c)=3.V_{a2}$ and $(a^2-a)=-j\sqrt{3}$]

Thus,

$V_{A2}=-j\sqrt{3}.V_{a2}$ 3.20

Hence, negative sequence component of line voltage is √3 times the negative sequence component of phase voltage and lags the corresponding phase voltage by 90°.

Finally, the zero sequence component of line voltage is given as,

$V_{A0}=(1/3)(V_A+V_B+V_C)$

$=(1/3)((V_c-V_b)+(V_a-V_c)+(V_a-V_b))$, in view of eq. 3.18

$=0$

Thus, $V_{A0}=0$ 3.21

Therefore, it is evident from the above equation that zero sequence component of line voltage is zero.

Note:

In similar lines as above, it can be proved that

$V_{B1}=j\sqrt{3}\ Vb1$; $V_{B2}=-j\sqrt{3}Vb2$; $V_{B0}=0$

$V_{C1}=j\sqrt{3}Vc1$; $V_{C2}=-j\sqrt{3}Vc2$; $V_{C0}=0$ 3.22

Example 3.5:

The positive and negative sequence components of phase voltages of a three phase system are Va1=230∠30° V and Va2=60∠60°V. Determine the positive and negative sequence components of line voltages and hence the line voltages.

Solution:

The positive, negative and zero sequence line voltages is given by,

$V_{A1}=j\sqrt{3}.\ V_{a1}=\sqrt{3}\ (230\angle(30°+90°))=\ 398.37\angle120°$ V

$V_{A2}=-j\sqrt{3}.V_{a2}\ =\sqrt{3}\ (60\angle(60°-90°))=\ 103.92\angle-30°$ V

It is known that zero sequence component of line voltage is zero. Thus $V_{A0}=0$.

Hence the line voltages of the system are,

$V_A=V_{A0}+V_{A1}+V_{A2}$

$=0+398.37\angle120°+103.92\angle-30°$

$=312.72\angle110°$ V

$V_B=V_{A0}+a^2.V_{A1}+a.V_{A2}$

$=0+398.37\angle(120°+240°)+103.92\angle(-30°+120°)$

$=411.7\angle14.62°$ V

$V_C=V_{A0}+a.V_{A1}+a^2.V_{A2}$

$=0+398.37\angle(120°+120°)+103.92\angle(-30°+240°)$

$=491.13\angle-126°$ V

3.7 Relation between sequence components of phase and line currents in delta connected systems.

In a star connected system, the line currents are the same as that of the phase currents. But, this is not the case in a delta connected system. Here, the phase currents are different from the line currents. Like wise, the sequence components of line currents are different from the sequence components of phase currents.

Consider a delta connected three phase system where in the line currents Ia, Ib and Ic are entering the delta connected system as shown in fig 3.8

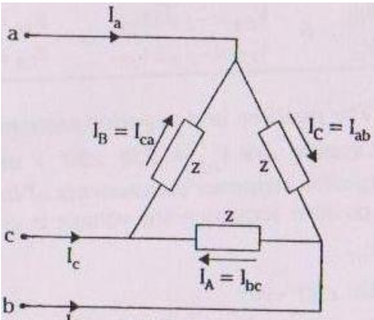


Fig. 3.8

The phase currents (currents in delta winding) are I_{ab}, I_{bc}, and I_{ca}. Let us designate $I_{ab}=I_C$, $I_{bc}=I_A$ and $I_{ca}=I_B$ (opposite to respective vertices).

Now, applying KCL to the system shown in fig 3.8, we get

$I_a=I_C-I_B$

$I_b=I_A-I_C$

$I_c=I_B-I_A$..........3.23

Then, the sequence component of line current are

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)((I_C-I_B)+a(I_A-I_C)+a^2.(I_B-I_A))$

$=(1/3)((a(I_A+a.I_B+a^2.I_C)-a^2(I_A+a.I_B+a^2.I_C))$

$=(1/3)(a-a^2)(I_A+a.I_B+a^2.I_C)$

but, $(I_A+a.I_B+a^2.I_C)=3.I_{A1}$

and, $(a-a^2)=j\sqrt{3}$

therefore, we get

$I_{a1}=(1/3)(j\sqrt{3})(3.I_{A1})$

$I_{a1}= j\sqrt{3} .I_{A1}$ 3.24

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)((I_C-I_B)+a^2(I_A-I_C)+a(I_B-I_C))$, view of eq. 3.18

$=(1/3) (a^2(I_A+a^2.I_B+a.I_C)-a(I_A+a^2.I_B+a.I_C))$

$=(1/3) (a^2-a) (I_A+a^2.I_B+a.I_C)$

$=(1/3)(-j\sqrt{3})(3.I_{A2})$ [because $(I_A+a^2.I_B+a.I_C)=3.I_{A2}$ and $(a^2-a)=-j\sqrt{3}$]

Thus,

$I_{a2}=-j\sqrt{3}.I_{A2}$ 3.25

From equations 3.24 and 3.25, it can be inferred that the line currents in delta system is √3 times the currents. The positive sequence line current leads the respective phase current by 90° whereas the negative sequence line current lags the negative sequence phase currents by 90°.

Finally, the zero sequence components of the line current is

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)((I_C-I_B)+(I_A-I_C)+(I_A-I_B))$, in view of eq. 3.18

$=0$

Thus, $I_{a0}=0$ 3.26

The above result indicates that zero sequence currents are absent in the lines. In general, it can be shown that zero sequence component of line current is absent in any three wire system. This will be made clear in section 3.8. also the zero sequence line current $I_{a0}=0$, does not mean that the zero sequence phase current I_{A0} is also zero.

Note:

1)In similar lines as above, it can be proved that when the line currents are entering the data windings

$I_{b1}=j\sqrt{3}\ I_{B1}$; $I_{b2}=-j\sqrt{3}\ I_{B2}$; $I_{b0}=0$

$I_{c1}=j\sqrt{3}\ I_{C1}$; $I_{c2}=-j\sqrt{3}\ I_{C2}$; $I_{c0}=0$..................3.27

2)when the line currents I_a, I_b and I_c are leaving the delta connected windings as shown in fig.3.9, then it can be proved that

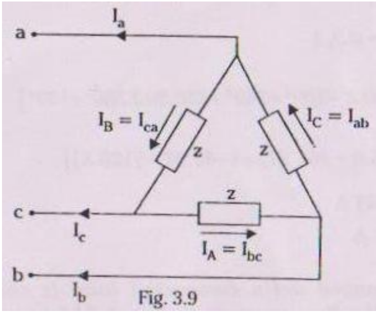


Fig. 3.9

$I_{a1}=-j\sqrt{3}\ I_{A1}$; $I_{a2}=j\sqrt{3}I_{A2}$; $I_{a0}=0$

$I_{b1}=-j\sqrt{3}\ I_{B1}$; $I_{b2}=j\sqrt{3}I_{B2}$; $I_{b0}=0$

$I_{c1}=-j\sqrt{3}\ I_{C1}$; $I_{c2}=j\sqrt{3}I_{C2}$; $I_{c0}=0$ 3.28

Example 3.7:

In a three phase, three wire system, the line currents are $Ia=100\angle0°$A and $Ib=100\angle-100°$ A. Determine the sequence components of line currents.

Solution:

In three wire system always,

$I_a+I_b+I_c = 0$

$I_c= -(I_a+I_b)$

$= -(100\angle0°+100\angle-100°)$

$=128.56\angle-130°$ A

Therefore, the sequence components of line currents are

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)(100\angle0°+100\angle-100°+128.56\angle-130°)$

$=0$ A (as expected)

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(100\angle0°+100\angle(-100°+120°)+128.56\angle(-130°+240°)$

$=108.5\angle10°$ A

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(100\angle0°+100\angle(-100°+240°)+128.56\angle(-130°+120°)$

$=20.5\angle-110°$ A

Example 3.8:

A balanced delta connected load is connected to a three phase symmetrical supply. The line currents are each 10A in magnitude. If fuse in one of the lines blows out, determine the sequence components of line current.

Solution:

Let us assume that the fuse blows in line c. Then with the current in line a as reference, the diagram of the circuit is as shown in fig. 3.10

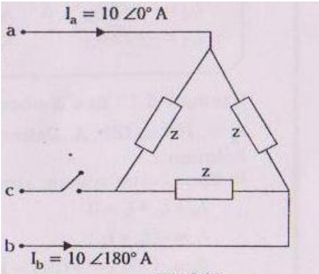


Fig. 3.10

Since this is a three wire system, we have

$I_a+I_b+I_c=0$

but,

$I_c=0$, as fuse blows out.

Therefore,

$I_b= -I_a$

if $I_a=10\angle 0°$ A, then

$I_b= -10\angle 0° = 10\angle 180°$ A

Hence, the positive sequence components of line current is

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(10\angle 0°+10\angle(180°+120°)+0)$

$=5.78\angle -30°$ A.

The negative sequence component of line current is

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(10\angle 0°+10\angle(180°+240°)+0)$

$=5.78\angle 30°$ A.

The zero sequence component of line current is absent in any three wire system. Thus $I_{a0}=0$ A

Example 3.9:

A delta connected balanced resistive load is connected across an unbalanced three phase supply as shown in fig 3.11. Find the symmetrical components of line current and delta current.

Solution:

$I_a+I_b+I_c=0$

or,

$I_c=-(I_a+I_b)$

$=-(10\angle 30°+15\angle -60°)$

$=18\angle 154°$ A.

Hence, symmetrical components of line currents are

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(10\angle 30°+15\angle(-60°+120°)+18\angle(154°+240°))$

$=13.94\angle 41.86°$ A

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(10\angle 30°+15\angle(-60°+240°)+18\angle(154°+120°))$

$=4.65\angle 248°$ A

$I_{a0}=(1/3)(I_a+I_b+I_c)=0$ A

Here, the line currents are entering the delta connected load. Therefore, the sequence components of delta currents are,

$I_{A1}=(I_{a1} / j\sqrt{3}) = (13.94\angle(41.86°-90°) /\sqrt{3}) = 8.05\angle -48.14°$ A

$I_{A2}=(I_{a2} / -j\sqrt{3}) = (4.65\angle(248°+90°) /\sqrt{3}) = 2.68\angle 338°$ A

3.8 Effect of neutral in the system

consider a star connected system as shown in fig.3.13

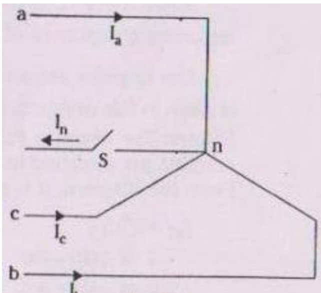


Fig. 3.13

Let the unbalanced line currents I_a, I_b and I_c. There are two possible cases here. One with the switch 's' closed i.e with the presence of the neutral wire. This forms a four wire system. The other with the switch 's' open forms a three wire system.

Let us consider both the cases independently.

Case i):

Four wire system.

Now, the current can flow through the neutral wire. Applying KCL at node 'n', we get the current through the neutral as

$I_n=I_a+I_b+I_c$ 3.20

but, we have

$I_a=I_{a0}+I_{a1}+I_{a2}$

$I_b=I_{a0}+a^2.I_{a1}+a.I_{a2}$

and,

$I_c=I_{a0}+a.I_{a1}+a^2.I_{a2}$

Using these results in eq. 3.29 yields,

$I_n=3.I_{a0}+I_{a1}(1+a+a^2)+I_{a2}(1+a+a^2)$

$=3I_{a0}+0+0$, as $(1+a+a^2)=0$

or $I_n=3.I_{a0}$ 3.30

From eq. 3.30 it can be deduced that positive and negative sequence currents do not flow in the neutral wire. On the other hand, the neutral current is equal to thrice the zero sequence currents in a four wire system.

Case ii):

Three wire system

In this case, the neutral wire is not made available so that

$I_n= 0$ 3.31

Hence eq. 3.30 yields

$I_{a0} = 0$ 3.32

That is, zero sequence currents are absent in three wire system.

Note:

A delta connected system is also a three wire system. Hence, the zero sequence component of line current $I_{a0}=0$. This has been proved in section 3.7

Example 3.11:

In a three phase, three wire system, if $I_{a1}=100\angle30°$ A, $I_{b2}=40\angle90°$ A, find the line currents of the system.

Solution:

Since there is a three wire system, the zero sequence component of line current $I_{a0}=0$.

The negative sequence component of phase 'b' is given in this problem. Hence, we determine Ia2 as follows. The negative sequence components of line currents are depicted in fig. 3.14.

From the diagram, it is clear that

$I_{a2}=a^2.\ I_{b2}$

$=40\angle(90°+240°)$

$=40\angle330°$ A

Thus, the sequence components are

$I_{a0}=0$

$I_{a1}=100\angle 30°$ A

$I_{a2}=40\angle 330°$ A

Hence the line currents of the system are

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=0+100\angle 30°+40\angle 330°$

$=124.89\angle 13.9°$ A

$I_b=I_{a0}+a^2.I_{a1}+a.I_{a2}$

$=0+100\angle(30°+240°)+40\angle(330°+120°)$

$=60\angle -90°$ A

$I_c=I_{a0}+a.I_{a1}+a^2.I_{a2}$

$=0+100\angle(30°+120°)+40\angle(330°+240°)$

$=124.89\angle 166.1°$ A

Example 3.12:

In a three phase system supplying power to a Y-load, the line currents when the neutral of the supply is not connected to the neutral of the load are Ia=20∠0° A and Ib=20∠-100° A. When the neutrals are connected, the current through the neutral wire is found to be 12∠-30° A. Determine the line currents under this situation.

Solution:

case i) when neutral of load is isolated from neutral of supply

In this case, $I_{a0}=0$

$I_a=I_{a1}+I_{a2}=20\angle 0°$ 1

$I_b=a^2.I_{a1}+a.I_{a2}=20\angle -100°$ 2

$I_c=a.I_{a1}+a^2.I_{a2}=-(I_a+I_b)= -(20\angle 0°+20\angle -100°) = 25.7\angle 130°$ 3

case ii) when the neutrals are connected

Here, it is given that I_n, the neutral current is 12∠-30°.

therefore,

$3.I_{a0} =12\angle -30°$

$I_{a0}=4\angle -30°$

Let I_a', I_b' and I_c' be the new values of line currents in this case, we get

$I_a'=I_{a0}+(I_{a1}+I_{a2})$

$= 4\angle -30°+20\angle 0°$, from result 1

$=23.53\angle -4.87°$ A

$I_b'=I_{a0}+(a^2.I_{a1}+a.I_{a2})$

$=4\angle -30°+20\angle -100°$, from result 2

$=21.69\angle{-90°}$ A

$I_c'=I_{a0}+(a.I_{a1}+a^2.I_{a2})$

$=4\angle{-30°}+25.7\angle{130°}$, from result 3=$22\angle{126.48°}$ A

3.9 Phase shift of symmetrical components in Y-Δ transformer bank.

Positive and negative sequence voltages and currents undergo a phase angle change in passing through a Y-Δ transformer (or a bank of three single phase transformers). This phenomenon is called as phase shift.

3.9.1 Voltage relations

Consider a transformer connection as shown in fig 3.15

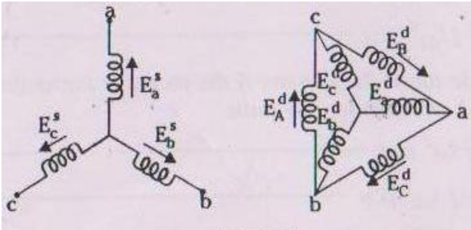


Fig. 3.15

Let E_a^s, E_b^s and E_c^s be the phase voltages on the Y-side of the transformer and E_a^d, E_b^d and E_c^d be the voltages across the windings on the Δ side of the transformer. Here, superscripts "s" and "d" stand for star side and delta side respectively.

We know from the principle of transformer, that the voltages across the various windings wound on any core will all be in phase, since these voltages are all produced due to rate of change of a common magnetic flux in the core. In this case, therefore, the phase voltage on the Y-side of the transformer should be in phase with the voltage across phase voltage on the Y-side of the transformer should be in phase with the voltage across the corresponding phase winding on the Δ side. If 'n' is the turns ratio, then we can write

$E_a^s=n.E_A^d$

$E_b^s=n.E_B^d$

$E_c^s=n.E_C^d$

Hence, the sequence components are also related as,

$E_{a1}^s=n.E_{A1}^d$ 3.33

$E_{a2}^s=n.E_{A2}^d$ 3.34

$E_{a0}^s=n.\ E_{A0}^d=0$ 3.35

The line voltages on the delta side of the transformer are equal to the voltages across the phase windings (on the delta side) of the transformer. But the phase voltages of the equivalent star on the delta side are different from the line voltages. Let E_a^d, E_b^d and E_c^d be the phase voltages of the equivalent star on the delta side. These are related to the line voltages (on the delta side) as follows.

$E_{A1}^d = j\sqrt{3}\ E_{a1}^d$ ....3.36 (refer equations 3.19 & 3.20)

and $E_{A2}^d = -j\sqrt{3}\ E_{a2}^d$ 3.37

Using these equations in 3.33 and 3.34, we get

$E_{a1}^s = n.E_{A1}^d = +j\sqrt{3}\ n.\ E_{a1}^d$ 3.38

and $E_{a2}^s = n.\ E_{A2}^d = -j\sqrt{3}.n.E_{a2}^d$ 3.39

Hence, we conclude that the positive sequence components of phase voltages on the star side of the transformer lead the corresponding positive components of the phase voltages (of the equivalent star) on the delta side by 90°. The same is true for line voltages on both sides of the transformer. The negative sequence components of the phase voltages on the star side of the transformer lags behind the corresponding negative sequence components of the equivalent phase voltages on the delta side by 90°. The relations 3.38 and 3.39 are vectorially represented in fig 3.16a and 3.16b.

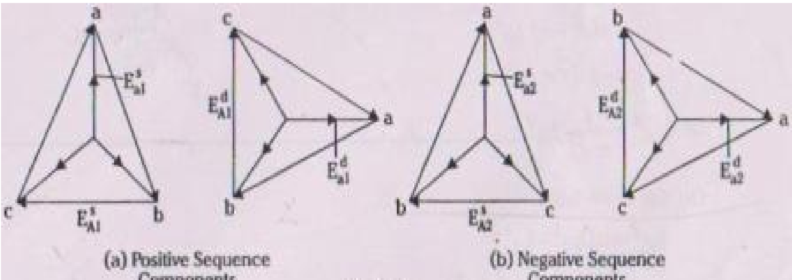


Fig. 3.16

3.9.2 Current relations

consider a star delta transformer connection as shown in fig 3.17

Let I_a^s, I_b^s and I_c^s be the line currents in the star side, I_a^d, I_b^d and I_c^d the line currents in delta side I_A^d, I_B^d and I_C^d the currents in the delta windings.

If 'n' is the turns ratio, then from the theory of transformers, we can write that, $I_A^d = n.I_a^s$ 3.40

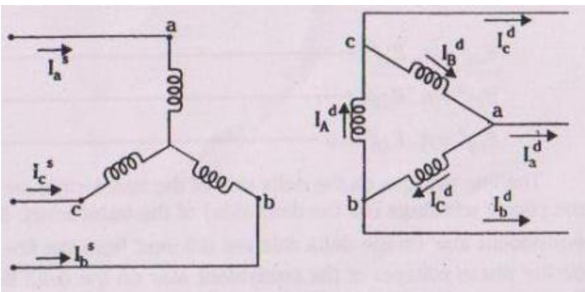


Fig. 3.17

$I_B^d = n.I_b^s$ 3.41

$I_C^d = n.I_c^s$ 3.42

Applying KCL to the nodes on the delta side, we can establish that

$I_a^d = I_B^d - I_C^d = n(I_b^s - I_c^s)$

$I_b^d = I_C^d - I_A^d = n(I_c^s - I_a^s)$, from equations 3.40 , 3.41 and 3.42

$I_c^d = I_A^d - I_B^d = n(I_a^s - I_b^s)$

Considering only positive sequence currents, the above relation becomes

$I_{a1}^d = n(I_{b1}^s - I_{c1}^s) = n(a^2.I_{a1} - a.I_{a1}^s) = n(a^2 - a)I_{a1}^s = -j\sqrt{3}\ n.I_{a1}^s$

or $I_{a1}^s = (j/n.\sqrt{3})I_{a1}^d$ 3.43

on the same lines

$I_{a2}^d = n(I_{b2}^s - I_{c2}^s) = n(a.I_{a2}^s - a2.I_{a2}^s) = n(a - a2)I_{a2}^s = j\sqrt{3}.n.I_{a2}^s$

or $I_{a2}^s = (-j/n\sqrt{3})I_{a2}^d$ 3.44

From 3.43 and 3.44, it can be inferred that the positive sequence component of the line current on the star side of the transformer leads the corresponding positive sequence component of line currents on the delta side by 90° and the negative sequence components of the line current on the star side of the transformer lags behind the corresponding negative sequence component of the line current on the delta side by 90°.

Note:

1)The turns ratio 'n' of a transformer is defined as,

n=number of primary turns / number of secondary turns

=primary voltage / secondary voltage

=secondary current / primary current

2)If each voltage is expressed in per unit with its own voltage as the base voltage, then equation 3.38 and 3.39 can be written as,

$E_{a1}^s = jE_{a1}^d$ p.u 3.45

$E_{a2}^s = -jE_{a2}^d$ p.u 3.46

3)Similarly, the per unit for a Y-Δ transformer

$I_{a1}^s = jI_{a1}^d$ p.u 3.47

$I_{a2}^s = -jI_{a2}^d$ p.u 3.48

4)If delta side forms the primary and the star side forms the secondary, that is in the case of Δ-Y transformer, we have

$I_{a1}^d = jI_{a1}^s$ p.u 3.49

$I_{a2}^d = -jI_{a2}^s$ p.u 3.50

$E_{a1}^d = jE_{a1}^s$ p.u 3.51

$E_{a2}^d = -jE_{a2}^s$ p.u 3.52

3.10 Complex power in terms of symmetrical components

If the symmetrical components of currents and voltages are known, then the power in a three phase circuit can be computed directly from the components.

The total complex power flowing into a three phase circuit is given as

$S=P+jQ = V_a.I_a*+V_b.I_b*+V_c.I_c*$

where,

S=Total complex power,(* indicates conjugate)

P=Active power

Q=Reactive power

In matrix from, the above equation can be expressed as,

$$S = P + jQ = [V_a \quad V_b \quad V_c]\begin{bmatrix} I_a* \\ I_b* \\ I_c* \end{bmatrix}$$

..........................3.53

But,

$$[V_a \quad V_b \quad V_c] = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T = \left\{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}\right\}^T$$

$$= \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}^T \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$ 3.54

since $([A].[B])^T = [A]^T.[B]^T$

and,

$$\begin{bmatrix} I_a* \\ I_b* \\ I_c* \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \left\{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}\right\}^*$$

since $([A].[B])* = [A]*.[B]*$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^* \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

Now $a*=a^2$

$(a^2)*=a$, Using these, we get

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

Substituting equation 3.52 and 3.53 in 3.51 we get,

$$S=(P+jQ)=\begin{bmatrix}V_{a0}\\V_{a1}\\V_{a2}\end{bmatrix}^T\begin{bmatrix}1&1&1\\1&a^2&a\\1&a&a^2\end{bmatrix}.\begin{bmatrix}1&1&1\\1&a&a^2\\1&a^2&a\end{bmatrix}\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}^*$$

$$=\begin{bmatrix}V_{a0}&V_{a1}&V_{a2}\end{bmatrix}\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}\begin{bmatrix}I_{a0}\\I_{a1}\\I_{a2}\end{bmatrix}^*$$

$S=P+jQ = 3\{V_{a0}.I_{a0}^*+V_{a1}.I_{a1}^*+V_{a2}.I_{a2}^*\}$ VA 3.56

Thus, if the symmetrical components of currents and voltages are known, then the power consumed by a three phase circuit can be computed from these components.

Note:

1)In terms of active and reactive powers, the above equation can be written as

$P=3\{|V_{a0}|\ |I_{a0}|\ \cos\theta_0 +|V_{a1}|\ |I_{a1}|\ \cos\theta_1+|V_{a2}|\ |I_{a2}|\ \cos\theta_2\}$ W 3.57

and

$Q=3\{|V_{a0}|\ |I_{a0}|\ \sin\theta_0 +|V_{a1}|\ |I_{a1}|\ \sin\theta_1+|V_{a2}|\ |I_{a2}|\ \sin\theta_2\}$ VAR 3.58

2)If V_B is the base voltage and I_B the base current of the system, then the complex power in pu is given as,

$S_{p.u}=3(V_{a0}\ I_{a0}^*+V_{a1}\ I_{a1}^*+V_{a2}\ I_{a2}^*)\ (1\ /\ V_B\ I_B)$ 3.59

or $S_{p.u}=S\ /S_B$ 3.60

Where S_B=base power of the system $=3V_BI_B$

3)If the symmetrical components of voltages and currents are given in pu directly, then the total 3 phase power is given as

$S_{p.u}=V_{a0}\ I_{a0}^*+V_{a1}\ I_{a1}^*+V_{a2}\ I_{a2}^*$ 3.61

Example 3.13:

The sequence components of the phase voltages are $V_{a1}=200\angle30°$, $V_{a2}=60\angle60°$ and $V_{a0}=20\angle-30°$ V. The line currents are $I_{a1}=20\angle10°$, $I_{a2}=5\angle20°$ and $I_{a0}=3\angle-10°$ A. Determine the three phase power in kVA and p.u. If the base power is 1kVA.

Solution:

The three phase complex power is given as

$S_{p.u}=3(V_{a0}\ I_{a0}^*+V_{a1}\ I_{a1}^*+V_{a2}\ I_{a2}^*)$

$=3\{(200\angle30°)(20\angle-10°)+(60\angle60°)(5\angle-20°)+(20\angle-30°)(3\angle10°)\}$

$=(12.13+j4.62)$ kVA

We have $S_{p.u}=S\ /S_B$ = (12.13+j4.62) kVA / 1kVA = (12.13+j4.62) p.u

Example 3.14:

In a three phase system, the sequence quantities are $V_{a1}=(0.9+j0.2)$p.u; $V_{a2}=(0.1+j0.1)$p.u; $V_{a0}=(0.1+j0.05)$p.u and $I_{a1}=(0.9-j0.1)$ p.u; $I_{a2}=(0.2-j0.1)$p.u; $I_{a0}=(0.05-j0.02)$p.u. Find the three phase complex power in p.u and in MAV on a base of 100MVA. Also compute the active and reactive powers.

Solution:

Here, the sequence components are given in p.u Hence, the total three phase power is given as,

$$S_{p.u}=(V_{a0}\,I_{a0}^*+V_{a1}\,I_{a1}^*+V_{a2}\,I_{a2}^*)$$
$$=\{(0.1+j0.05)(0.05-j0.02)^*+(0.9+j0.2)(0.9-j0.1)^*+(0.1+j0.1)(0.2-j0.1)^*\}$$
$$=\{(0.1+j0.05)(0.05+j0.02)^*+(0.9+j0.2)(0.9+j0.1)^*+(0.1+j0.1)(0.2+j0.1)^*\}$$
$$=(0.817+j0.3126)\text{p.u}$$

Next, $S=S_{p.u}\times S_B$

$$=(0.817+j0.3126)\times 100\text{ MVA}=(81.7+j31.26)\text{MVA}$$

We have, $S=P+jQ=(81.7+j31.26)$MVA

Therefore,

The active power is P=81.7MW

The reactive power is Q=31.26MVAR

Example 3.15:

In a three phase four wire system, the sequence voltages and currents are: $V_{a1}=0.9\angle 10°$p.u; $V_{a2}=0.25\angle 110°$p.u; $V_{a0}=0.12\angle 300°$p.u and $I_{a1}=0.75\angle 25°$ p.u; $I_{a2}=0.15\angle 170°$p.u; $I_{a0}=0.1\angle 330°$p.u. Find the complex power in p.u. If the neutral gets disconnected, find the new power.

Solution:

The total three phase power in pu is given as,

$$S_{p.u}=(V_{a0}\,I_{a0}^*+V_{a1}\,I_{a1}^*+V_{a2}\,I_{a2}^*)$$
$$=(0.12\angle 300°)(0.1\angle 330°)^*+(0.9\angle 10°)(0.75\angle 25°)^*+(0.25\angle 110°)(0.15\angle 170°)^*$$
$$=(0.12\angle 300°)(0.1\angle -330°)+(0.9\angle 10°)(0.75\angle -25°)+(0.25\angle 110°)(0.15\angle -170°)$$
$$=(0.68-j0.212)\text{p.u}$$

When the neutral gets opened, then $I_{a0}=0$. Hence the new power is,

$$S_{p.u}'=(V_{a1}\,I_{a1}^*+V_{a2}\,I_{a2}^*)$$
$$=(0.9\angle 10°)(0.75\angle 25°)^*+(0.25\angle 110°)(0.15\angle 170°)^*$$
$$=(0.9\angle 10°)(0.75\angle -25°)+(0.25\angle 110°)(0.15\angle -170°)$$
$$=(0.67-j0.206)\text{p.u.}$$

--------END--------

4.1 Introduction:

In previous chapter, we have discussed the sequence components for voltages and currents. Let us now consider the impedances offered to positive, negative and zero sequence currents in symmetrical circuits. In symmetrical circuits, currents of a given sequence produce voltage drops of the same sequence only. Before attempting a proof for this, let us define sequence impedances.

The impedances offered by the circuit for the flow of positive sequence currents is called as positive sequence impedance. Similarly, the impedance offered by the circuit for the flow of negative sequence currents is negative sequence impedance and zero sequence currents is zero sequence impedance.

4.2 Sequence impedance of a symmetrical circuit:

In this section, we show that in a symmetrical circuit, currents of a given sequence produce a voltage drop of the same sequence only.

Consider a three phase symmetrical circuit as shown in fig 4.1. Let I_a, I_b and I_c be the currents in each line. These currents return via the neutral(ground) impedance Z_n as shown.

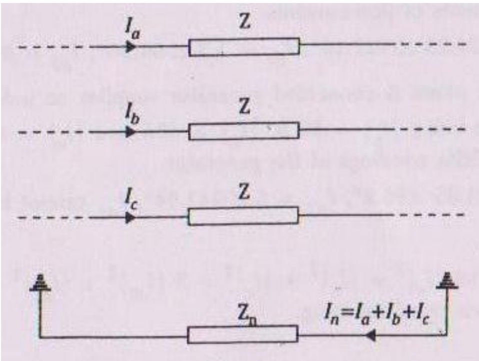


Fig. 4.1

The voltage drops are computed as follows:

$V_a=I_a.Z+I_n.Z_n$

$=I_a.Z+(I_a+I_b+I_c)Z_n$

$=I_a.(Z+Z_n)+I_b.Z_n+I_cZ_n$ 4.1

Similarly,

$V_b=I_a.Z_n+I_b(Z+Z_n)+I_c.Z_n$ 4.2

and $V_c=I_a.Z_n+I_b.Z_n+I_c(Z_n+Z_n)$ 4.3

In matrix form, the above equations can be expressed as:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad ....4.4$$

Expressing the voltages and currents by their sequence components, we get.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

or,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} Z+3Z_n & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad ....4.5$$

Namely, positive sequence, negative sequence and zero sequence in this manner the three sequences may be solved individually. Once the problem is solved in terms of symmetrical components. It can be transformed back to the actual circuit conditions.

This gives relationships

$V_{a0}=(Z+3Z_n)I_{a0}$

$V_{a1}=Z.I_{a1}$

$V_{a2}=Z.I_{a2}$ 4.6

The above equations indicate that in symmetrical circuits, currents of given sequence produce voltage drops of the same sequence only, i.e the sequence impedances are uncoupled in case of symmetrical circuits. Accordingly the problem can be effectively broken down into three separate systems.

Also, as per definition of sequence impedances, we have the three sequence impedances as,

$Z_0=V_{a0}/I_{a0}=Z+3Z_n$=Zero sequence impedance 4.7

$Z_1=V_{a1}/I_{a1}=Z$=positive sequence impedance 4.8

$Z_2=V_{a2}/I_{a2}=Z$=negative sequence impedance 4.9

From the above results, we can conclude that for a symmetrical static circuit (like that of transformers of fully transposed transmission lines).

i) The positive sequence impedance is the same as the negative sequence impedance.

ii) Zero sequence impedance is much larger than the positive (or negative) sequence impedance. In the absence of the neutral, $Z_n = \infty$

therefore,

$Z_0 = \infty$ and hence $I_{a0} = V_{a0}/Z_0 = V_{a0}/\infty = 0$, as expected.

Example 4.4:

Across a star connected symmetrical impedance load of 10Ω in each phase and a neutral impedance of 3.33Ω, an unbalanced three phase supply with $V_a = 220\angle 0°$, $V_b = 200\angle 110°$ and $V_c = 180\angle -110°$ is applied. Determine the line currents using symmetrical components.

Solution:

since the circuit is symmetrical, we have

$Z_1 = Z_2 = 10\ \Omega$

and, $Z_0 = Z + 3Z_n = 10 + 3(3.33) = 20\ \Omega$

carefully observe the phase voltages shown in fig 4.2

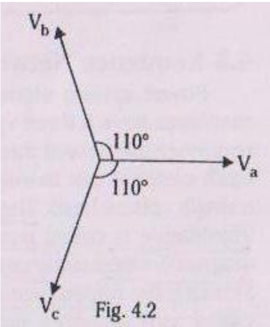


Fig. 4.2

It can be seen that phase sequence is acb. Hence in the equations for the determination of sequence components of phase voltages, the subscripts b and c should be interchanged.

Hence,

$V_{a1} = (1/3)\ (V_a + a.\ V_c + a^2.V_b)$

$= (1/3)\ (220\angle 0° + 180\angle(-110° + 120°) + 200\angle(110° + 240°))$

$= 198.07\angle -0.33°$ volts.

$V_{a2} = (1/3)\ (V_a + a^2.\ V_c + a.V_b)$

$= (1/3)\ (220\angle 0° + 180\angle(-110° + 240°) + 200\angle(110° + 120°))$

$= 9.56\angle -147.7°$ volts.

$V_{a0} = (1/3)\ (V_a + V_c + V_b)$

$= 220\angle 0° + 180\angle -110° + 200\angle 110°$

$= 30.64\angle 11.77°$ volts.

Now,

$I_{a1} = V_{a1}/Z_1$

$=(198.07\angle -0.33°)/10$

$=19.807\angle -0.33°$ A

$I_{a2}=V_{a2}/Z_2$

$=(9.56\angle -147.7°)/10$

$=0.956\angle -147.7°$ A

$I_{a0}=V_{a0}/Z_0$

$=(30.64\angle 11.77°)/20$

$=1.532\angle 11.77°$ A

The line currents are,

$I_a=I_{a0}+I_{a1}+I_{a2}$

$=1.532\angle 11.77°+ 19.807\angle -0.33 +0.956\angle -147.7°$

$=20.49\angle -0.87°$ A

$I_b= I_{a0}+a.I_{a1}+a^2 .I_{a2}$ (for phase sequence acb)

$=1.532\angle 11.77°+ 19.807\angle(-0.33°+120°) +0.956\angle(-147.7°+240°)$

$=20.27\angle 114.4°$ A

$I_c= I_{a0}+a^2 .I_{a1}+a.I_{a2}$

$=1.532\angle 11.77°+ 19.807\angle(-0.33°+240°) +0.956\angle(-147.7°+120°)$

$=18.85\angle -113.86°$ A

Example 4.2:

Prove that a three phase symmetrically wound alternator generators only positive sequence components of voltages.

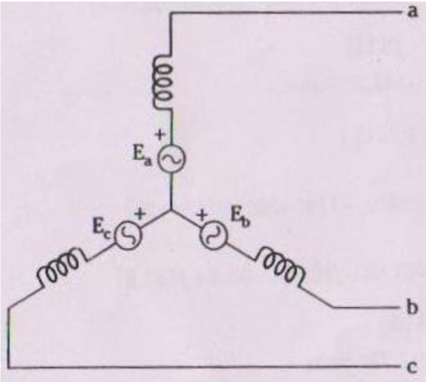


Fig. 4.3

Solution:

Fig. 4.3 depicts an unbalanced synchronous generator. E_a, E_b, E_c are the induced emfs of the three phases. Since the windings are symmetrical, the induced emfs are perfectly balanced.

Let,

$|E_a|=|E_b|=|E_c|=V_p$

Then, if follows that (assuming a abc phase sequence)

$E_a=V_p\angle 0°$

$E_b=V_p\angle -120°$

$E_c=V_p\angle 120°$

Hence the sequence components of voltages are,

$E_{a0}=(1/3)(E_a+E_b+E_c)$

$=(1/3)(V_p\angle 0°+V_p\angle -120°+V_p\angle 120°)$

$=0$

$E_{a1}=(1/3)(E_a+a.E_b+a^2E_c)$

$=(1/3)(V_p\angle 0°+V_p\angle(-120°+120°)+V_p\angle(120°+240°))$

$=V_p$

$=E_a$

$E_{a2}=(1/3)(E_a+a^2.E_b+a.E_c)$

$=(1/3)(V_p\angle 0°+V_p\angle(-120°+240°)+V_p\angle(120°+120°))$

$=0$

From the results obtained above, it can be inferred that a three phase symmetrically wound alternator generators only positive sequence components of voltages.

4.3 Sequence Networks of power system elements:

Power system elements namely transmission lines, transformers and synchronous machines have a three phase symmetry because of which when currents of a particular sequence are passed through these elements, voltage drops of the same sequence appear. Each element can therefore be represented by three decoupled sequence networks on a single phase basis. The impedance or reactance diagram formed using positive sequence impedance is called positive sequence network. Similarly the impedance or reactance diagram formed using negative sequence impedance is called negative sequence network. Similarly the impedance or reactance diagram formed using zero sequence impedance is called zero sequence network.

The sequence networks are very useful in the analysis of unsymmetrical faults in the power system. In unsymmetrical fault analysis of a power system, the positive, negative and zero sequence networks of the system are determined and then they are interconnected suitably to represent the various fault conditions. The sequence currents and voltages during the fault are then calculated from which actual fault currents and voltages can be found. Before proceeding to these fault analysis, we must know the equivalent circuit presented by the power elements to the flow of positive, negative and zero sequence currents respectively.

4.4 Sequence impedances and networks of synchronous generator:

Consider the three phase equivalent circuit of a synchronous generator shown in fig.4.4. The neutral of the generator is grounded through a reactor(impedance Z_n). When the generator is delivering a balanced load or under symmetrical fault conditions, the neutral current is zero. But when the generator is delivering an unbalanced load or during unsymmetrical faults neutral current in flows to neutral from ground via Z_n.

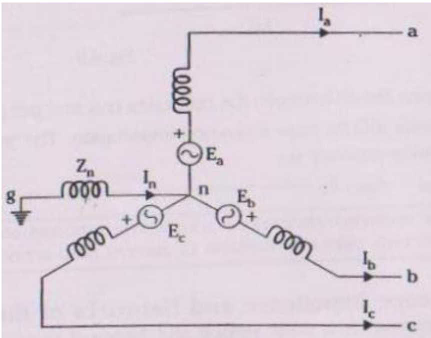


Fig. 4.4

Here, E_a, E_b and E_c are the induced emfs of the three phases. I_a, I_b and I_c are the currents flowing in the lines when a fault(not shown in the figure) takes place at machine terminals. Since a synchronous machine is designed with symmetrical windings, it induces emfs of positive sequence only (This fact is proved in example 4.2).Fig.4.5a shows the three phase positive sequence network model of the synchronous generator Z_n does not appear in the model as I_n=0 for positive sequence currents. E_{a1}, E_{b1} and E_{c1} are the positive sequence generated voltages and Z_1 is the positive sequence impedance. Because of the balanced and symmetrical nature of the system, the three phase system can be replaced by a single phase network as shown in fig 4.5b.

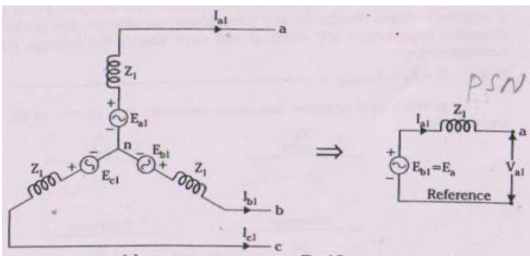


Fig. 4.5

Using the notation E for generated voltage and V for the terminal voltage, the equation that holds good for positive sequence network is,

$V_{a1}=E_{a1}-I_{a1}Z_1=E_a-I_{a1}Z_1$ 4.10

The per phase positive sequence impedance Z_1 in the above case is the subtransient, transient or steady state reactance of the machine depending on whether subtransient, transient or steady state conditions are being studied.

The negative sequence network models of a synchronous generator on a three phase and single-phase basis are shown in fig 4.6a and 4.6b respectively. Negative sequence voltages are not present in the equivalent circuits, as they are not generated in the synchronous machine. However, due to a fault or an unbalanced load, negative sequence currents can flow in the machine. Since negative sequence currents do not flow in the neutral, Z_n does not appear in the model.

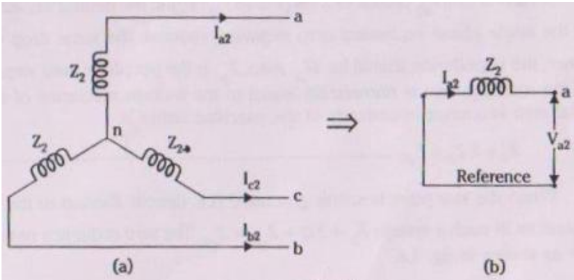


Fig. 4.6

The equation that holds good for the negative sequence network is,

$V_{a2}=-I_{a2}Z_2$ 4.11

It is known that the phase sequence of negative sequence currents is opposite to that of the phase sequence of positive sequence currents. With the flow of negative sequence currents in the stator of rotating field is created which rotates in the opposite direction to that of the positive sequence field and, therefore, at double synchronous speed with respect to the rotor. Currents at double the stator frequency are therefore induced in rotor field and damper winding. In sweeping over the rotor surface, the negative sequence mmf is alternately presented with reluctances of direct and quadrature axis. The negative sequence impedance presented by the machine with this consideration is often defined as,

$Z_2=j((X_g''+X_d'')/2)$; $|Z_2|<|Z_1|$ 4.12

It is the zero sequence currents that flow in the neutral during an unbalance or faulty condition ($I_n=3.I_{a0}$ refer section 3.8). The impedance offered by the synchronous generator to zero sequence currents depend on grounding of the neutral (star point). For a synchronous generator whose star point is grounded through an impedance Z_n, the zero sequence network models are as shown in fig.4.7.

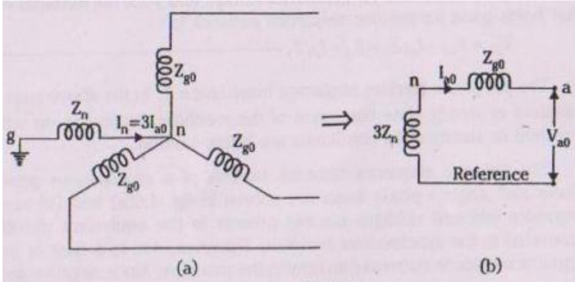


Fig. 4.7

A current of $3.I_{a0}$ produces a drop of $(3\ I_{a0}.Z_n)$ in the neutral impedance Z_n. To show in the single phase equivalent zero sequence network the same drop where current I_{a0} flows, the impedance should be $3Z_n$. Also, Z_{g0} is the per phase zero sequence impedance of the machine and is numerically equal to the leakage reactance of the machine. The total zero sequence impedance of the machine hence is,

$Z_0=3Z_n+Z_{g0}$ 4.13

When the star point is solidly grounded i.e directly shorted to the ground), $Z_n=0$. Therefore in such a system $Z_0=3(0)+Z_{g0}=Z_{g0}$. The zero sequence networks in this case are as shown in fig 4.8.

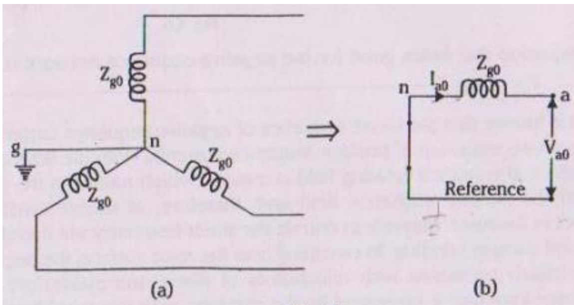


Fig. 4.8

When the star point is grounded, $Z_n=\infty$, Therefore eq. 4.13 becomes $Z_0=\infty+Z_{g0}=\infty$. The zero sequence networks for a ungrounded generator is as shown below.

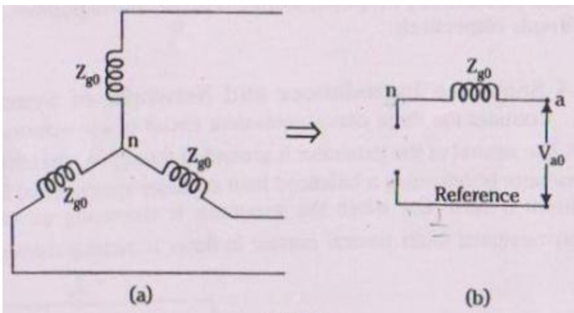


Fig. 4.9

The open circuit between the reference bus and per phase zero sequence impedance Z_{g0} represents infinite zero sequence impedance. The general equation applicable for a zero sequence network is,

$V_{a0} = -I_{a0}.Z_0$ 4.14

Note:

The sequence networks of synchronous motors are same as that of synchronous generators only with the direction of current flow reversed.

4.5 Sequence Impedance and networks of three phase transformer.

A transformer is a static device and hence it does not have anything like a phase sequence by itself, unlike that of a three phase synchronous generator or a synchronous motor. For this reason, the impedance of a transformer is independent of phase sequence of applied voltages. Hence for any three phase transformer, the positive and negative sequence impedances are identical and each equals the leakage reactance of the transformer. i.e

$Z_1 = Z_2 = X_{leakage}$ 4.15

The positive and negative sequence networks are shown in fig. 4.10a and 4.10b respectively.

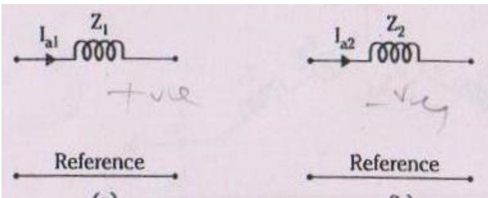


Fig. 4.10

The zero sequence impedance will be equal to positive (or negative) sequence impedances if there is a path for zero sequence current and will be infinity if there is no path for zero sequence currents. The following general observations can be made for zero sequence currents in transformers.

1) If the neutral point (star point) in the star connected winding is not grounded, then there is no path for zero sequence currents in the legs of star connection. i.e, the zero sequence currents flows in the star connected winding and in the lines connected to the winding only when the neutral point is grounded.

2) No zero sequence currents can flow in the lines connected to a delta connection as no return path is available for these currents. Zero sequences currents can however, flow in the legs of a delta-such currents are cause by the presence zero sequence voltages in the delta connection.

Based on the above observations the zero sequence network of three phase transformer can be obtained for any configuration. There is, however, a mechanical way of obtaining the zero sequence networks for any configuration of a three phase transformer (this as well caters the conditions specified above).

The general circuit for any configuration is given in fig4.11.

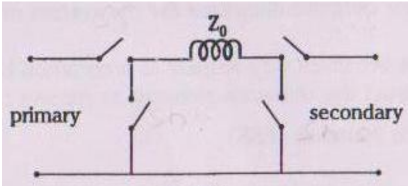


Fig. 4.11

Z_0 is the per phase zero sequence impedance of the winding of the transformer. There are two series and two shunt switches. One series and one shunt switch for both the sides separately. The series switch of a particular side is closed if it is a star connection with its neutral grounded and the shunt switch is closed if that side is delta connected, otherwise they are left open.

Using the above general rule, the zero sequence network for some of the configurations of three phase transformer are presented in table 4.1.

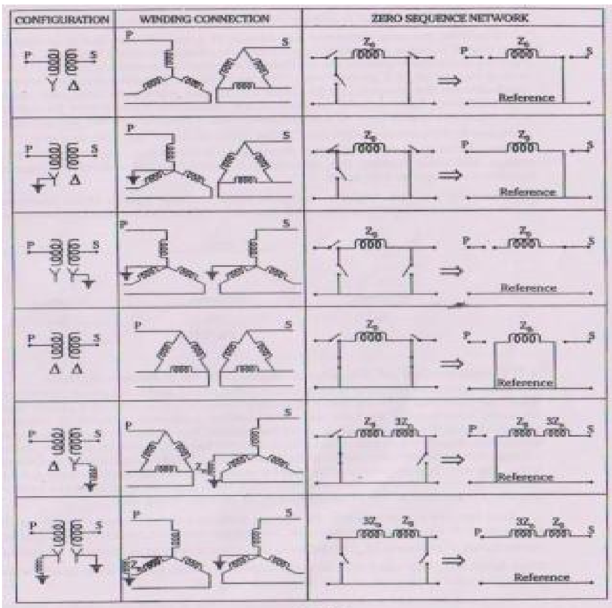


TABLE 4.1

4.6 Sequence impedances and networks of transmission lines.

The series impedance of a transmission line, since it is a static apparatus, is the same for both positive and negative sequence currents. Hence the positive and negative sequence impedances of a transmission line are identical and can be obtained as follows:

$Z_1=Z_2=X_L$ ohms/phase/unit length 4.16

Where,

$X_L=\omega.L$, L=inductance/phase/unit length. This can be computed(for any symmetrically or unsymmetrically spaced transposed single circuit or double circuit transmission line) using the formula.

$L=2(10^{-7})\log_e(GMD/GMR)$ H/m/phase 4.17

Here GMD is the Geometric Mean Distance of the spacings of the conductors between the phase and GMR is the Geometric Mean Radius of each of the three phases.

However, the zero sequence impedance of a transmission line is of different nature from that of positive and negative sequence impedances. Since the three zero sequence currents of the three phases are in phase, it necessitates a return path either in the ground or in a neutral or in a ground wire. This involves the understanding of current flow and distribution in the ground (earth). Since the ground impedance is heavily dependent on soil conditions, it is essential to make some simplifying assumptions for calculating the zero sequence impedance of a transmission line. Carson's formulas are generally used to compute Z_0 of transmission line. These formulas relate the zero sequence impedance of transmission lines to the physical dimensions of conductors, earth's resistivity, operation voltage and other factors.

Once the positive, negative and zero sequence impedances of the transmission line is known, the corresponding sequence networks can be easily drawn as shown in fig 4.12.

4.7 Construction of sequence networks of a power system.

In the previous sections the sequence networks of important power system elements have been discussed. Using these, complete sequence networks of a power system can be easily constructed. To start with, the positive sequence network is constructed by examination of the one-line diagram of the system. The transition from positive sequence network to negative sequence network is straight forward. Since the positive and negative sequence impedances are identical for static elements (like transformers and transmission lines), the only change necessary in positive sequence network to obtain negative sequence network is in respect of synchronous machines. Each machine is represented by its negative sequence impedance, the negative sequence voltage being zero.

Zero sequence subnetworks for various parts of a system can be easily combined to form complete zero sequence network. No voltage source is present in the zero sequence network. Any impedance (Z_n) included in generator, motor or transformer neutral becomes three times ($3Z_n$) its value in a zero sequence network. Special care needs to be taken of transformers in respect of zero sequence network.

The procedure for drawing sequence networks is illustrated through the following examples.

Example 4.3:
A 250 MVA, 11kV, 3 phase generator is connected to a large system through a transformer and a line as shown in fig below.

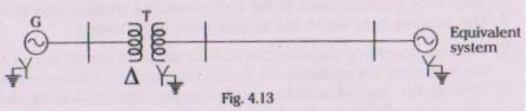


Fig. 4.13

Generator: $X_1 = X_2 = 0.15$ p.u, $X_0 = 0.1$ p.u
Transformer: $X_1 = X_2 = X_0 = 0.12$ p.u
Line: $X_1 = X_2 = 0.25$ p.u, $X_0 = 0.75$ p.u
Equivalent system: $X_1 = X_2 = X_0 = 0.15$ p.u
Draw the sequence network diagrams for the system and indicate all per unit values.
Solution:
all reactances are give with respect to a common base in this problem. Hence we can directly construct the sequence network as follows:

Positive sequence network (PSN):

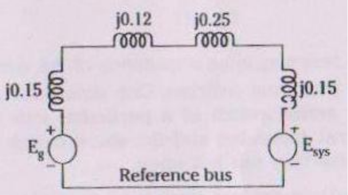


Fig. 4.14

Negative sequence network (NSN):

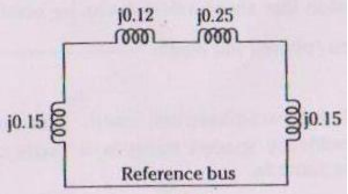


Fig. 4.15

Zero sequence network (ZSN):

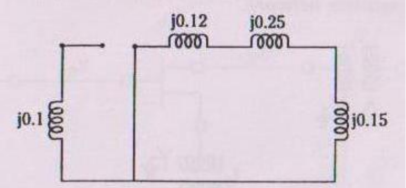


Fig. 4.16

Example 4.4:

Draw the positive, negative and zero sequence networks for the power system shown in fig 4.17.

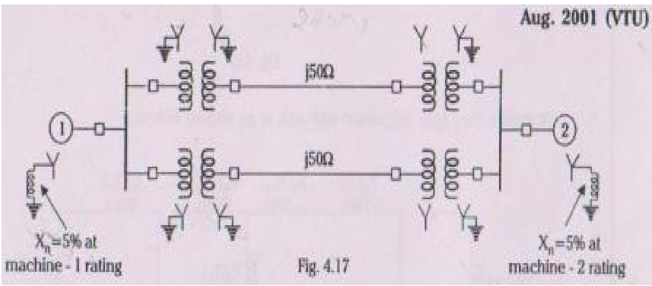


Fig. 4.17

Choose a base of 50MVA, 220kV in the 50Ω transmission lines and mark all reactances in p.u. The ratings of the generators and transformers are:
Generator 1: 25MVA, 11kV, X"=20%.
Generator 2: 25MVA, 11kV, X"=20%.
Three phase transformer (each): 20MVA, 11 Y/220 Y kV, X=15%.
The negative sequence reactance of each synchronous machine is equal to the subtransient reactance. The zero sequence reactance of each machine is 8%. Assume that the zero sequence reactance of lines are 250% of their positive sequence reactances.

Solution:
base values:
We choose a given,
base MVA=50
base kV on 50Ω transmission lines=220
base kV on generator 1=220(11/220)=11
base kV on generator 2=220(11/220)=11

sequence reactances of generators:
Since the ratings of the machines are the same, their reactances are also the same.
Positive sequence reactance=X''_{G1}=subtransient reactance on new base.

$X''_{G1,\ new} = X''_{G1,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.2 \times (50 / 25) \times (11^2 / 11^2)$

$= j\,0.4$ p.u

Negative sequence reactance=X''_{G1}=subtransient reactance on new base.

=j0.4 p.u (as per given data)

Zero sequence reactance=XG0=8% on new base

$= j0.08 \times (50 / 25) \times (11^2 / 11^2)$

=j0.16 p.u

p.u value of generator neutral reactance=XGn=5% on new base.

$= j0.05 \times (50 / 25) \times (11^2 / 11^2)$

=j0.1 p.u.

Sequence reactances of transformers:

Since the ratings of all transformers are identical, their sequence reactances should be one and the same. Also, all the three sequence reactances of transformers are the same.

Hence positive sequence reactance=Negative sequence reactance
=Zero sequence reactance

$X_{p.u,\ new} = X_{p.u,\ old} \times ((MVA)_{B,\ new} / (MVA)_{B,\ old}) \times ((kV)^2_{B,\ old} / (kV)^2_{B,\ new})$

$= j0.15 \times (50 / 20) \times (220^2 / 220^2)$

$= j\ 0.375$ p.u

Sequence reactance of transmission lines:

Since the transmission line is a static apparatus, its positive and negative sequence reactances are one and the same.

Hence, $X_{TL1}=X_{TL2}= X_{TL}(\ \Omega\ ) \times (MVA)_B / (kV)^2_B = j50 \times 50 / 220^2 = j0.052$p.u

However, it is given that the zero sequence reactances of transmission lines are equal to 250% of their positive sequence reactances.

Therefore,

$X_{TL\ .Zero}=2.5(j0.052)=j0.13$p.u

Using these computed values, the sequence networks are drawn as below.

Positive sequence network (PSN):

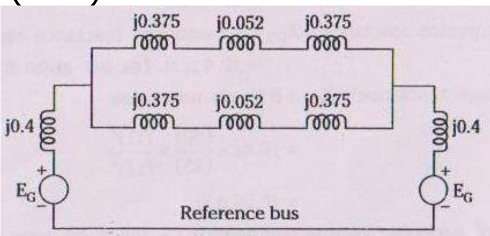


Fig. 4.18

Negative sequence network (NSN):

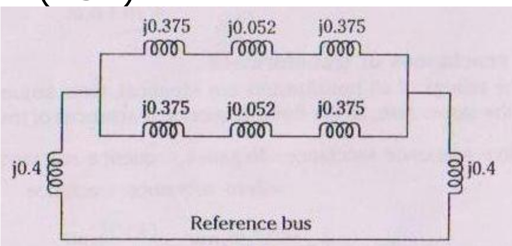


Fig. 4.19

Zero sequence network (ZSN):

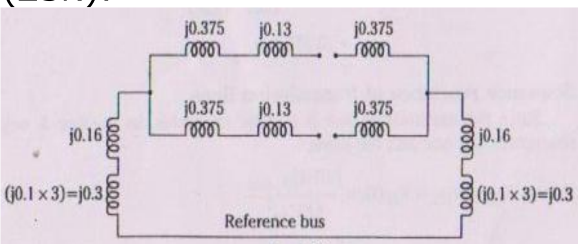


Fig. 4.20

Note:
While drawing zero sequence networks, carefully follow the steps.

Example 4.5:
For the power system whose one line diagram is shown in fig 4.21. Sketch the zero sequence network.

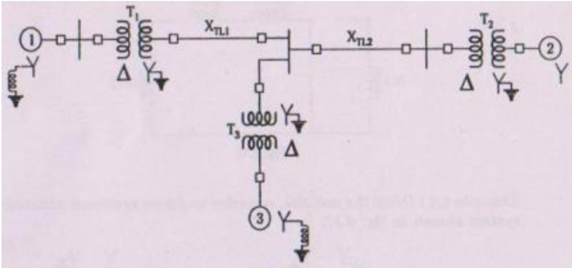


Fig. 4.21

Solution:
The zero sequence network is as shown below.

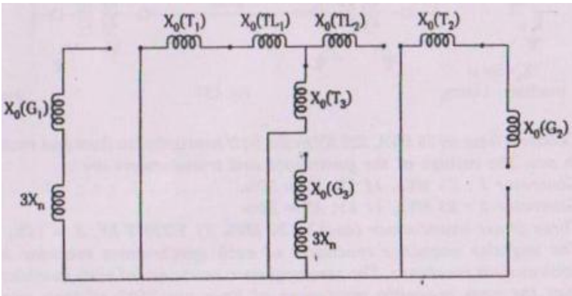


Fig. 4.22

Example 4.8:
Draw the sequence networks of the simple power system shown in fig. 4.24.

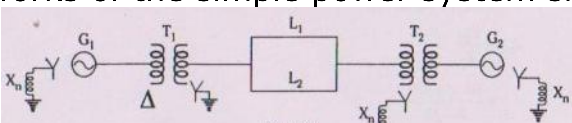


Fig. 4.24

Positive sequence network (PSN):

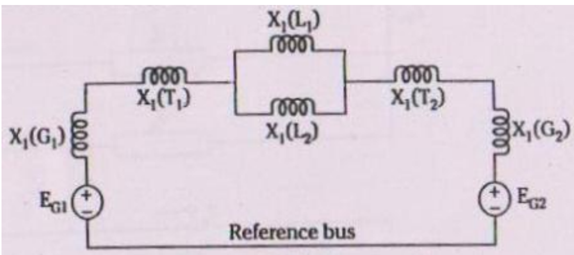


Fig. 4.25

Negative sequence network (NSN):

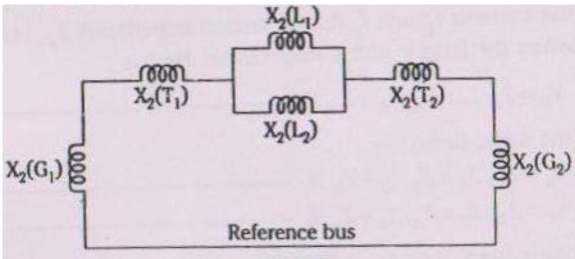


Fig. 4.26

Zero sequence network (ZSN):

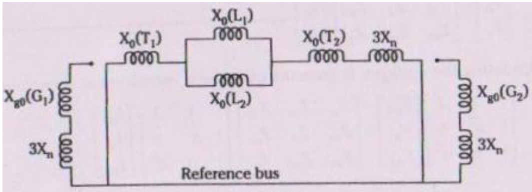


Fig. 4.27

Example 4.9:
The one line diagram of a power system is shown in fig 4.28.
The ratings of the devices are as follows:
G_1 & G_2: 104MVA, 11.8kV, $X_1=X_2=0.2$p.u; $X_0=0.1$p.u.
T_1 & T_2: 125MVA, 11Y-220Y kV, $X_1=X_2=X_0=0.1$p.u.
T_3 & T_4:120MVA, 230Y-6.9Y kV, $X_1=X_2=X_0=0.12$p.u.
M_1: 175MVA, 6.6kV, $X_1=X_2=0.3$p.u; $X_0=0.15$p.u.
M_2: 50MVA, 6.9kV, $X_1=X_2=0.3$p.u; $X_0=0.1$p.u.
Transmission line reactances: $X_1=X_2=30\Omega$; $X_0=60\Omega$.

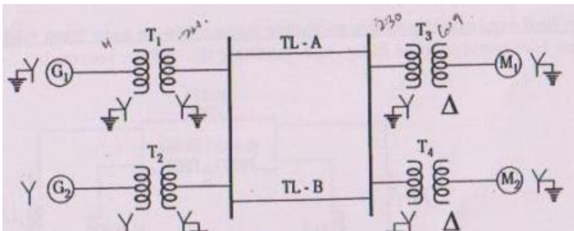


Fig. 4.28

Draw the sequence impedance diagram in p.u on a base of 200MVA, 220kV in the transmission lines. Also, find the equivalent positive sequence impedance as seen from the mid point of line-B.
Solution:
Base values:
We choose from give,
base MVA=200 MVA
base kV on the transmission lines=220

base kV on the generators G_1 & G_2=220(11/220)=11
base kV on the motors M_1 & M_2=220(6.9/230)=6.6
Reactances of G_1 & G_2:
$X_1 = X_2 = X_{p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.2 \times (200 / 104) \times (11.8^2 / 11^2)$
$= j\,0.44$ p.u
$X_0 = j0.1 \times (200 / 104) \times (11.8^2 / 11^2)$
$= j0.22$p.u

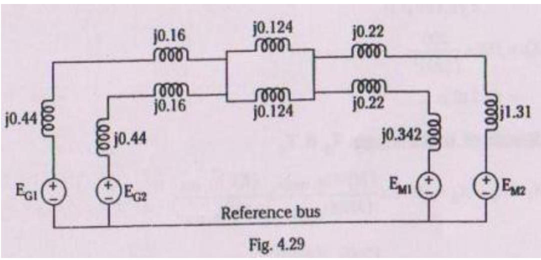
Reactances of transformers T_1 & T_2:(calculated on primary side of them),
$X_1 = X_2 = X_0 = X_{p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.1 \times (200 / 125) \times (11^2 / 11^2)$
$= j\,0.16$ p.u

Reactance of transmission lines:
$X_{1TL} = X_{2TL} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B = j30 \times 200 / 220^2 = j0.124$p.u
$X_{0TL} = j60 \times 200 / 220^2 = j0.248$p.u

Reactances of transformers T_3 & T_4:(calculated on primary side of them),
$X_1 = X_2 = X_0 = X_{p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.12 \times (200 / 120) \times (230^2 / 220^2)$
$= j\,0.22$ p.u

Reactances of M_1:
$X_1 = X_2 = X_{p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.3 \times (200 / 175) \times (6.6^2 / 6.6^2)$
$= j\,0.342$ p.u
$X_0 = j0.15 \times (200 / 175) \times (6.6^2 / 6.6^2)$
$= j0.171$p.u

Reactances of M_2:
$X_1 = X_2 = X_{p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.3 \times (200 / 50) \times (6.9^2 / 6.6^2)$
$= j\,1.31$ p.u
$X_0 = j0.1 \times (200 / 50) \times (6.9^2 / 6.6^2)$
$= j0.4372$ p.u
Using these values, the sequence networks are drawn as below.
Positive sequence network (PSN):



Negative sequence network (NSN):

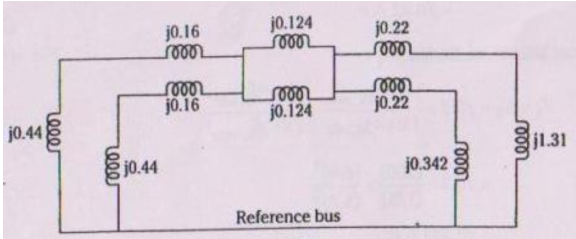


Fig. 4.30

Zero sequence network (ZSN):

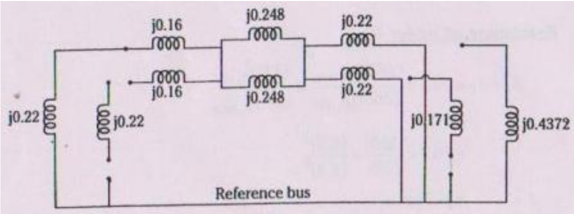


Fig. 4.31

To find equivalent positive sequence impedance as seen from mid point of line-B: The PSN is redrawn as in fig 4.32(shorting the voltage sources)

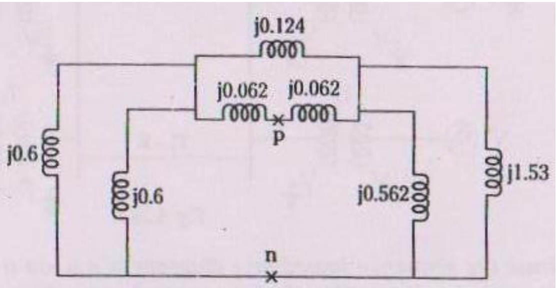


Fig. 4.32

The mid point of line-B is marked as 'P' and point 'n' is a point in the reference. Using elementary circuit theory, the network of fig 4.32 can be simplified as shown in fig 4.33

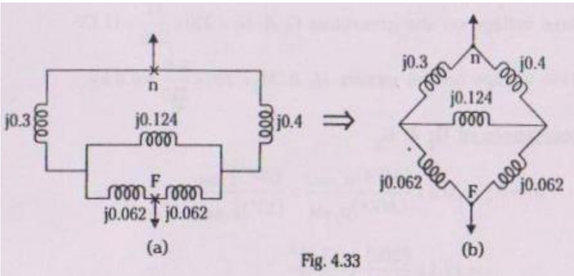


Fig. 4.33

Consider the delta connected network of fig 4.33b, it cab be converted to its equivalent star connection as shown below.

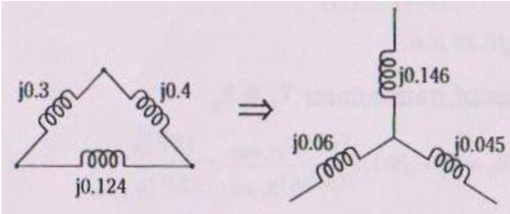


Fig. 4.34

Putting this back into fig 4.33b and simplifying, we get

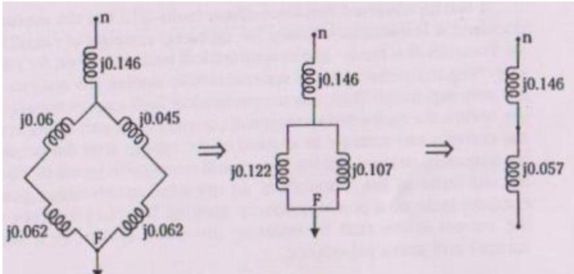


Fig. 4.35

Hence the equivalent positive sequence impedance as seen from the point P is, (j0.146+j0.057)=j0.203 p.u.

------------END-----------

5.1 Introduction:

The concept of faults has been already introduced in chapter 2 which was dedicated to the treatment of symmetrical faults. In this chapter, we shall deal with unsymmetrical faults. The unsymmetrical faults are basically categorized into two types, namely,

1)Shunt type of faults and

2)Series type of faults.

Shunt type of fault involves short circuit between conductors or between the conductors and ground. They are characterized by an increase in current and fall in voltage and frequency in the faulted phase. Shunt type of faults are in turn classified as:

1)Single line to ground (LG) fault

2)Line to line (LL) fault

3)Double line to ground (LLG) fault.

When one or two lines in a thee phase system get opened while other lines or line remain intact, such faults are called as series type of faults. They are characterized by increase in voltage and frequency and fall in current in the faulted phase series type of faults cab be grouped as:

1)One conductor open fault

2)Two conductor open fault

We will individually consider each of these faults in this chapter. Before that, let us look into the typical relative frequencies of different kinds of faults in a power system in order of decreasing severity.

Symmetrical faults (3L) -5%

Double line to ground (LLG) faults -10%

Double line (LL) faults -15%

Single line to ground (LG) fault- 70%

It can be observed that three phase faults (3L) has the maximum severity, though its occurrence is infrequent. Hence the rapturing capacity of circuit breakers are calculated on the basis of a three phase symmetrical fault. However, for relay setting, single phase switching and performing the system stability studies, the analysis of unsymmetrical faults are very important. Since any unsymmetrical fault causes unbalanced currents to flow in the system, the method of symmetrical components is very useful in an analysis to determine the currents and voltages in all parts of the system after the occurrence of the fault. Also the sequence networks of the system will come quite handy in this process. First, we shall discuss fault at the terminals of an unloaded synchronous generator. Then, we shall consider faults on a power system by applying Thevenin's theorem, which allows us to find the current in the fault by replacing the entire system by a single generator (voltage source) and series impedance.

5.2 Fault calculations of synchronous generator.

Consider a balanced three phase synchronous generator (alternator), which is subjected to some unsymmetrical fault F at the terminal, as shown in fig 5.1.

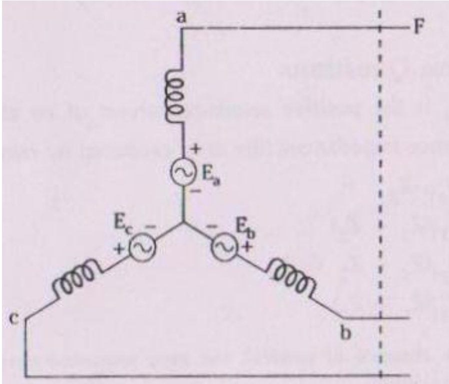


Fig. 5.1

The fault may be unsymmetrical one. But to the left of the fault point F, the system (alternator) is completely symmetrical. Hence, in such a system, currents of a given sequence produce voltage drops of the same sequence only. The sequence impedances are uncoupled. Since the generates balanced voltages only (positive sequence voltages only), the following equations are applicable to a synchronous generator, even during an unsymmetrical fault.

$V_{a1}=E_a-I_{a1}. Z_1$ 5.1

$V_{a2}=-I_{a2}. Z_2$ 5.2

$V_{a0}=-I_{a0}.Z_0$ 5.3

These equations cab be called the system equations. For any fault at the terminals of synchronous generator, the quantities that are to be determined are the three sequence currents (I_{a1}, I_{a2}, I_{a0}) and the three sequence terminal voltages (V_{a1}, V_{a2}, V_{a0}). Out of the six unknowns, only three quantities are linearly independent. Hence to determine these three linearly independent quantities, three terminal conditions are to be specified for any type of fault at the terminals of the generator.

Before proceeding to the analysis of faults at the terminals of an unloaded generator, it is good enough to remember that the single phase representation of the positive sequence network of a synchronous generator consists of positive sequence generated emf E_{a1} in series with positive sequence impedance Z_1(fig 4.5b). The negative sequence network consists of negative sequence impedance Z_2 with no negative sequence generated voltage (fig 4.6b). The zero sequence network consists of zero sequence impedance Z_0 with no zero sequence generated voltage (fig 4.7b).

5.2.1 Single line to ground (LG) fault on an unloaded generator:

The circuit diagram for an LG fault on an unloaded star connected generator with its neutral grounded through a reactance is shown in fig 5.2. Here it is assumed that phase a is shorted to ground directly. The condition at the fault are represented by the following terminal conditions.

Terminal conditions:

$V_a=0$ 5.4

$I_b=0$ 5.5

$I_c=0$ 5.6

These three terminal conditions in terms of line currents and phase voltage are to be transformed to conditions in terms of symmetrical components.

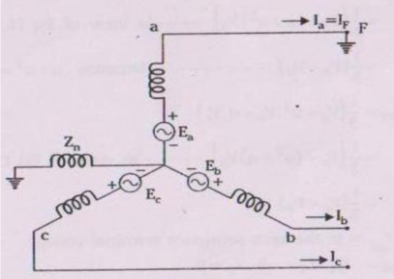


Fig. 5.2

Symmetrical components relations:

Since two conditions are available regarding the line currents, it is convenient to transform them to conditions in terms of symmetrical components.

$I_{a0}=(1/3)(I_a+I_b+I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

so $I_{a1}=I_{a2}=I_{a0}=(1/3).I_a$ 5.7

The terminal conditions $V_a=0$ gives,

$V_{a0}+V_{a1}+V_{a2}=0$ 5.8

As per eq. 5.7, all sequence currents are equal and as per eq. 5.8, the sum of sequence voltage equals zero. Therefore, these equations suggest a series connection of sequence networks through a short circuit as shown in fig 5.3.

Interconnection of sequence networks:

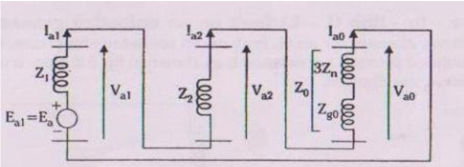


Fig. 5.3

Sequence quantities:

The following relations can be directly obtained from fig 5.3

$I_{a1}=I_{a2}=I_{a0}=E_a / (Z_1+Z_2+Z_0)$ 5.9

$V_{a1}=E_{a1} - I_{a1}.Z_1= E_a - (E_a/(Z_1+Z_2+Z_0)).Z_1$

$=E_a((Z_2+Z_0) / (Z_1+Z_2+Z_0))$ 5.10

$V_{a2}=-I_{a2}.Z_2= -(E_a.Z_2 /(Z_1+Z_2+Z_0))$ 5.11

$V_{a0}= -I_{a0}.Z_0 = -(E_a.Z_0 /(Z_1+Z_2+Z_0))$ 5.12

Fault current;

The fault current If in this case is equal to the current in phase a i.e I_a. Hence the fault current is given as,

$I_f=I_a=3.I_{a0}$ ....in view of eq. 5.7

$=3(E_a /(Z_1+Z_2+Z_0))$ 5.13

In case the neutral of the generator is not grounded, then

$Z_0=Z_g0+3Z_n=Z_{g0}+\infty=\infty$. Therefore, the fault current in such a condition is,

$If=3(E_a /(Z_1+Z_2+\infty)=0$ 5.14

Thus, it can be inferred that fault current in the system is zero if the neutral is not grounded in the case of an LG fault.

5.2.2 Line to line (L-L) fault on an unloaded generator:

The circuit diagram for an LL fault on an unloaded star connected generator with its neutral grounded through a reactance is as shown in fig 5.4. Here it is assumed that phase b and phase c are shorted.

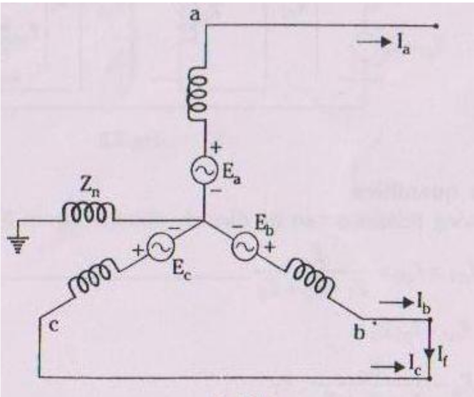


Fig. 5.4

Terminal conditions:

The condition at the fault are expressed by the following terminal conditions:

$I_a=0$ 5.15

$I_b+I_c=0$ i.e. $I_c= -I_b$ 5.16

$V_b=V_c$ 5.17

Symmetrical components relations:

Since there are two conditions regarding current, analysing them first, we get

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)(0+I_b-I_b)$

$=0$ ....in view of eq. 5.16

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(0+a.I_b-a^2.I_b)$

$=(1/3)(a-a^2).I_b$

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(0+a^2.I_b-a.I_b)$

$=(1/3)(a^2-a).I_b$

So, we have,

$I_{a0}=0$ 5.18

$I_{a2}= -I_{a1}$ 5.19

Regarding sequence terminal voltages,

$V_{a1}=(1/3)(V_a+a.V_b+a^2.V_c)$

$=(1/3)(V_a+(a+a^2)V_b)$ ....in view of eq. 5.17

$=(1/3)(V_a-V_b)$ ....because $a+a^2= -1$

$V_{a2}=(1/3)(V_a+a^2.V_b+a.V_c)$

$=(1/3)(V_a+(a^2+a)V_a)$ ....in view of eq.5.17

$=(1/3)(V_a-V_b)$

Since $I_{a0}=0$; the zero sequence terminal voltage

$V_{a0}= -I_{a0}.Z_0= -0.Z_0 =0$

so, we have

$V_{a0}=0$ 5.20

$V_{a1}=V_{a2}$ 5.21

Equations 5.19 and 5.21 suggest parallel connection of positive and negative sequence networks. Since $I_{a0}=V_{a0}=0$, the zero sequence networks is connected separately and shorted on itself as shown in the following diagrams.

Interconnection of sequence networks:

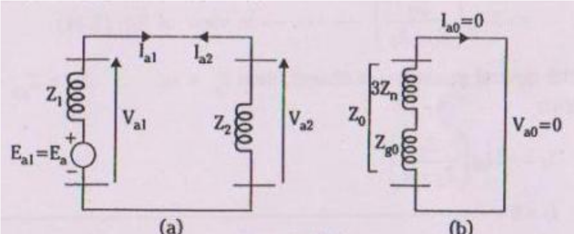


Fig. 5.5

Sequence quantities:

The following relations can be directly obtained from fig 5.5

$I_{a1}= -I_{a2} =E_a / (Z_1+Z_2)$ 5.22

$I_{a0}=V_{a0}=0$ 5.23

$V_{a1}=V_{a2}=E_a - I_{a1}.Z_1 = E_a (Z_2/Z_1+Z_2))$ 5.24

Fault current:

The fault current in this case is,

$I_f=I_b$ (or I_c)

$=I_{a0}+a^2.I_{a1}+a.I_{a2}$

$=0+(a^2-a)I_{a1}$

$= -j\sqrt{3}I_{a1}$

or $|I_f|=\sqrt{3}\ I_{a1} = \sqrt{3}(E_a / (Z_1+Z_2))$ 5.25

In case the neutral is not grounded, then $Z_0=Z_{g0}+3Z_n=Z_{go}+\infty=\infty$. But since the expression for fault current is independent of the value of Z_0, the presence or absence of a grounded neutral at the generator does not affect the fault current.

5.2.3 Double line to ground (LLG) fault on an unloaded generator:

The circuit diagram for an LLG fault on an unloaded star connected alternator having grounded neutral is shown in fig 5.6. We assume that the fault takes place in phases b and c.

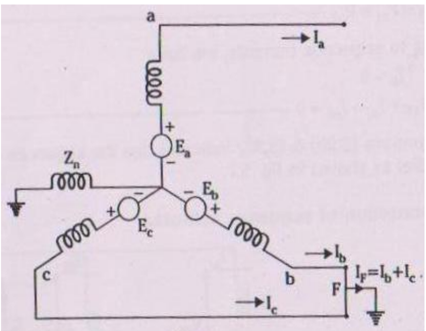


Fig. 5.6

Terminal conditions:

The conditions at the fault are expressed by the following equations:

$V_b=0$ 5.26

$V_c=0$ 5.27

$I_a=0$ 5.28

Symmetrical components relations:

Since there are the condition regarding the voltages analysing (transforming to symmetrical components) then first we get.

$V_{a0}=(1/3)(V_a+V_b+V_c)$

$=(1/3)(V_a+0+0)$

$=(1/3).V_a$

$V_{a1}=(1/3)(V_a+a.V_b+a^2.V_c)$

$=(1/3)(V_a+0+0)$

$=(1/3).V_a$

$V_{a2}=(1/3)(V_a+a^2.V_b+a.V_c)$

$=(1/3)(V_a+0+0)$

$=(1/3).V_a$

so, $V_{a0}=V_{a1}=V_{a2}$ 5.29

coming to sequence currents, we have,

$I_a=0$

i.e $I_{a0}+I_{a1}+I_{a2} = 0$ 5.30

Equations 5.29 and 5.30 indicates that the sequence networks should be connected in parallel as shown in fig. 5.7.

Interconnection of sequence networks:

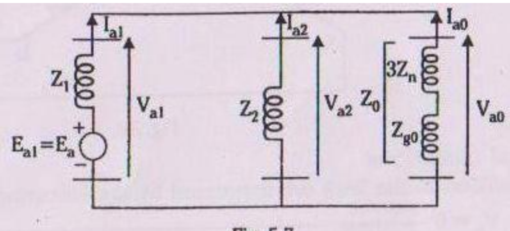


Fig. 5.7

Sequence quantities:

The following relations can be directly obtained from the fig 5.7

$V_{a1}=V_{a2}=V_{a0}=E_a=I_{a1}.Z_1$ 5.31

$I_{a1}=E_a/ [Z_1+\{Z_2Z_0/(Z_2+Z_0)\}]$ 5.32

I_{a2}= $-I_{a1}$.$[Z_0/(Z_2+Z_0)]$ 5.33

I_{a0}= $-I_{a1}$.$[Z_2/(Z_2+Z_0)]$ 5.34

Equations 5.33 and 5.34 are direct consequences of current division formula.

Fault Current:

The fault current I_f in this case is given by,

$I_f=I_b+I_c$

$=(I_{a0}+a^2.I_{a1}+a.I_{a2}) + (I_{a0}+a.I_{a1}+a^2.I_{a2})$

$=2I_{a0}+(a+a^2)I_{a1}+(a+a^2)I_{a2}$

$=2I_{a0} - I_{a1} - I_{a2}$ because $(a+a^2)= -1$

$=2I_{a0} -(I_{a1}+I_{a2})$

It can be observed from fig 5.7 that $(I_{a1}+I_{a2})= -I_{a0}$.

Substituting this in the expression for fault current, we get

$I_f=2I_{a0} -(-I_{a0})$

$=3I_{a0}$ 5.35

$=-3.I_{a1}.[Z_2/(Z_2+Z_0)]$ --------------in view of eq. 5.34

If the neutral grounding is absent, then $Z_n=\infty$.

Therefore, $Z_0=Z_{g0}+3Z_n=Z_{g0}+\infty=\infty$.

Hence,

$I_f = -3I_{a1}.[Z_2/(Z_2+\infty)]$

Therefore, $I_f=0$ 5.36

Thus, it can be inferred that fault current in the system is Zero, if the neutral is not grounded in the case of LLG fault.

5.3 Fault through impedances:

All the faults discussed in the preceding section consisted of direct short circuits between line and between one or two line to ground. In these cases, the impedance between the fault points is considered as zero. There may be situation in which the fault path includes an impedance between the faulted points. In these situation the analysis is carried similar to that of the previous section, except that the fault impedance is concluded at appropriate points in the circuits obtained by connecting sequence networks. Hence the theory is not elaborated in much detail.

5.3.1 Single line to ground (LG) fault on an unloaded generator through a fault impedance:

The circuit diagram for an LG fault on an unloaded generator through a fault impedance Z_f is shown in fig 5.8.

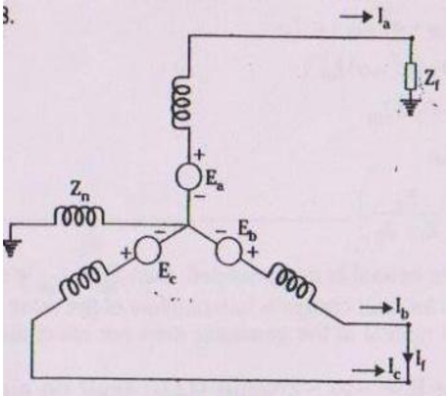


Fig. 5.8

Terminal conditions:

$V_a = I_a . Z_f$ 5.37

$I_b = 0$ 5.38

$I_c = 0$ 5.39

Symmetrical components relations:

The following relations can be obtained from the terminal conditions

$I_{a0} = (1/3)(I_a + I_b + I_c)$

$= (1/3)(I_a + 0 + 0)$

$= (1/3).I_a$

$I_{a1} = (1/3)(I_a + a.I_b + a^2.I_c)$

$= (1/3)(I_a + 0 + 0)$

$= (1/3).I_a$

$I_{a2} = (1/3)(I_a + a^2.I_b + a.I_c)$

$= (1/3)(I_a + 0 + 0)$

$= (1/3).I_a$

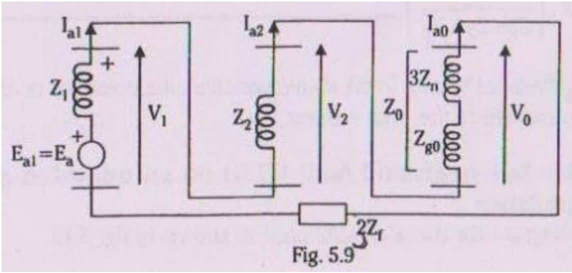
So $I_{a1} = I_{a2} = I_{a0} = (1/3).I_a$ 5.40

The terminal condition $V_a = I_a.Z_f$ gives

$V_{a0} + V_{a1} + V_{a2} = I_a.Z_f = 3.I_{a0}.Z_f$ 5.41

As per equations 5.40 and 5.41 all sequence currents are equal and the sum of sequence voltages equals $2.I_{a0}.Z_f$. Therefore, these equations suggest a series connection of sequence networks through an impedance $3.Z_f$ as shown in fig 5.9.

Interconnection of sequence networks:



Sequence quantities:

The following equations can be directly obtained from the fig 5.9

$I_{a0}=I_{a1}=I_{a2}=E_a / (Z_1+Z_2+Z_0+3Z_f)$ 5.42

$V_{a1}=E_a-I_{a1}.Z_1=E_a.(Z_2+Z_0+3Z_f) /(Z_1+Z_2+Z_0+3Z_f)$ 5.43

$V_{a2}= -I_{a2}.Z_2 = -E_aZ_2 / (Z_1+Z_2+Z_0+3Z_f)$ 5.44

$V_{a0}= -I_{a0}Z_0= -E_aZ_0 / (Z_1+Z_2+Z_0+3Z_f)$ 5.45

Fault current:

The fault current in this case is given as,

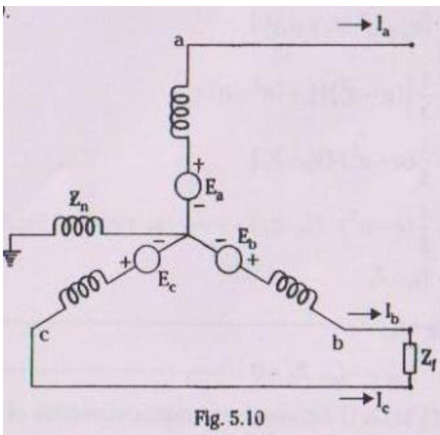
$I_f=I_a=3.I_{a0}=3 [E_a / (Z_1+Z_2+Z_0+3Z_f)]$ 5.46

From the above expression, it can be observed that the fault current is reduced by the fault impedance. Even in this case, if the neutral is left ungrounded, $Z_n=\infty$

i.e $Z_0=\infty$ and hence $I_f=0$.

5.3.2 Line to line (LL) fault on an unloaded generator through a fault impedance:

The circuit diagram for an LL fault on an unloaded generator through a fault impedance Zf is shown in fig 5.10.



Terminal conditions:

$I_a=0$ 5.47

$I_b+I_c=0$; $I_c=-I_b$ 5.48

$V_b=V_c+I_b.Z_f$ 5.49

Symmetrical components relations:

The following relations can be obtained from the terminal conditions

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)(I_a+I_b-I_b)$

$=0$

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(0+a.I_b-a^2.I_c)$

$=(1/3)(a-a^2)I_b$

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(0+a^2 .I_b-a.I_c)$

$=(1/3)(a^2-a)I_b$

so, $I_{a0}=0$ 5.50

$I_{a1}= -I_{a2}$ 5.51

Next,

$V_{a1}=(1/3)(V_a+a.V_b+a^2.V_c)$

$V_{a2}=(1/3)(V_a+a^2.V_b+a.V_c)$

Therefore,

$V_{a1}-V_{a2}= (1/3) [(a-a^2)V_b+(a^2-a)V_c]$

$=(1/3) [(a-a^2)(V_b-V_c)]$

$=(1/3) (a-a^2)(I_b.Z_f)$ ------------in view of eq. 5.49

$= I_{a1}.Z_f$

Thus, $V_{a1}=V_{a2}+I_{a1}.Z_f$ 5.52

Since, $I_{a0}=0$, $V_{a0}= -I_{a0}.Z_0=0$ 5.53

Equations 5.51 and 5.52 suggest parallel connection of positive and negative sequence networks through a series impedance Z_f as shown in fig 5.11. Since $I_{a0}=V_{a0}=0$, the zero sequence network is connected separately through a short as shown in fig 5.12.

Interconnection sequence networks:

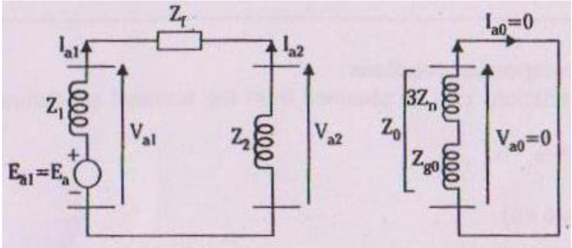


Fig. 5.11 Fig. 5.12

Sequence quantities:

The following equations can be directly obtained from the diagrams shown above.

$I_{a1}= -I_{a2} =E_a / (Z_1+Z_2+Z_f)$ 5.54

$I_{a0}=V_{a0}=0$ 5.55

$V_{a1}=E_a - I_{a1}Z_1$

$=E_a.(Z_2+Z_f) /(Z_1+Z_2+Z_f)$ 5.56

$V_{a2}= -I_{a2}.Z_2$

$= - E_a.Z_2 /(Z_1+Z_2+Z_f)$ 5.57

Fault current:

In this case the fault current is equal to the current in phase-b (or phase c). Hence

$I_f=I_b=I_{a0}+a^2.I_{a1}+a.I_{a2}$

$=0+a^2.I_{a1} -a.I_{a1}$

$=(a^2 - a)I_{a1}$

$= - j\sqrt{3}I_{a1}$

or $|I_f|=\sqrt{3}.I_{a1}$

$=\sqrt{3}.E_a / (Z_1+Z_2+Z_f)$ 5.58

Since Z_0 does not appear in the above equation, the presence or absence of a grounded neutral does not affect the fault current.

5.3.3 Double line to ground fault (LLG) on an unloaded generator through a fault impedance:

The circuit diagram for the aforesaid case is shown in fig 5.13

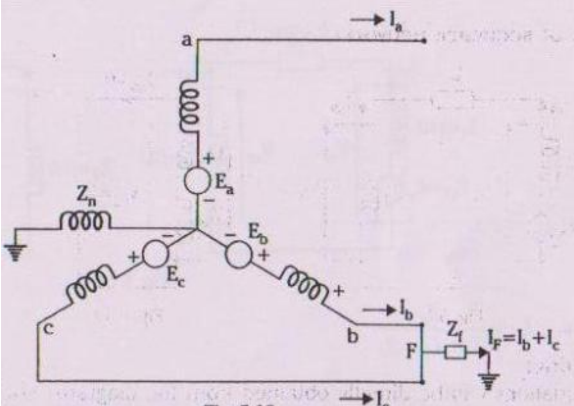


Fig. 5.13

Terminal conditions:

$I_a=0$ 5.59

$V_b=(I_b+I_c)Z_f$ 5.60

$V_c=(I_b+I_c)Z_f$ 5.61

Symmetrical component relations:

Consider,

$V_{a1}=(1/3)(V_a+a.V_b+a^2.V_c)$

$=(1/3)[(V_a+(a+a^2)V_b]$

$=(1/3)(V_a-V_b)$

$V_{a2}=(1/3)(V_a+a^2.V_b+a.V_c)$

$=(1/3)[(V_a+(a^2+a)V_b]$

$=(1/3)(V_a-V_b)$

$V_{a0}=(1/3)(V_a+V_b+V_c)$

$=(1/3)(V_a+2.V_b)$

Thus,

$V_{a1}=V_{a2}$ 5.62

$V_{a0}-V_{a2}=(1/3).3V_b$

$=V_b$

$=(I_b+I_c)Z_f$ ....from eq. 5.60

$=3.I_{a0}.Z_f$ --------This will be proved to the expression for fault current.

Thus, $V_{a0}=V_{a2}+3.I_{a0}.Z_f$ 5.63

The condition $I_a=0$ gives $I_{a0}+I_{a1}+I_{a2}=0$ 5.64

Equations 5.62 , 5.63 and 5.64 suggest the connection of sequence network as shown in fig 5.14.

Interconnection of sequence networks:

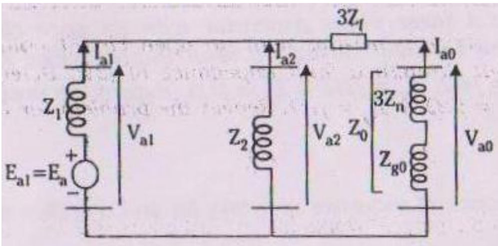


Fig. 5.14

$I_{a1} = E_a / [Z_1+\{Z_2.(3Z_f+Z_0)/(Z_2+3Z_f+Z_0)\}]$ 5.65

$I_{a2} = -I_{a1}.\ (Z_0+3Z_f)/(Z_0+Z_2+3Z_f)$ 5.66

$I_{a0} = -I_{a1}.Z_2/(Z_0+Z_2+3Z_f)$;using current division eq.)...........................5.67

Equations 5.66 and 5.67 are obtained from the current division fromula.

Fault current:

In this case, the fault current is given as,

$I_f=I_b+I_c$

$=(I_{a0}+a^2.I_{a1}+a.I_{a2}) + (I_{a0}+a.I_{a1}+a^2.I_{a2})$

$=2I_{a0}+(a+a^2)I_{a1}+(a+a^2)I_{a2}$

$=2I_{a0} - I_{a1} - I_{a2}$ because $(a+a^2)= -1$

$=2I_{a0} -(I_{a1}+I_{a2})$

It can be observed from fig 5.64 that $(I_{a1}+I_{a2})= -I_{a0}$.

Substituting this in the expression for fault current, we get

$I_f=2I_{a0} -(-I_{a0})$

$=3I_{a0}$ 5.68

$=-3.I_{a1}.[Z_2/(Z_0+Z_2+3Z_f)]$ --------------in view of eq. 5.69

In absence of the neutral grounding, $Z_n=\infty$. That is, $Z_0=\infty$ and Hence, $I_f=0$.

Note:

1)Instead of remembering the various results for a particular fault, it is often easier to visualize the connection of sequence networks to represent the fault and then proceed suitably.

2)The fault current in the case of faults involving ground(LG,LLG) is given as,

$I_f=3.|I_{a0}|$

3)In case of LL fault, the fault current is given as $If=\sqrt{3}.|I_{a1}|$

Example 5.1:

A three phase generator with an open circuit voltage of 400V is subjected to an LG fault through a fault impedance of j2Ω. Determine the fault current if Z_1=j4Ω, Z_2=j2Ω and Z_0=j1Ω. Repeat the problem for LL and LLG fault.

Solution:

i)LG fault:

The interconnection of sequence networks for an LG fault is shown in fig. 5.15

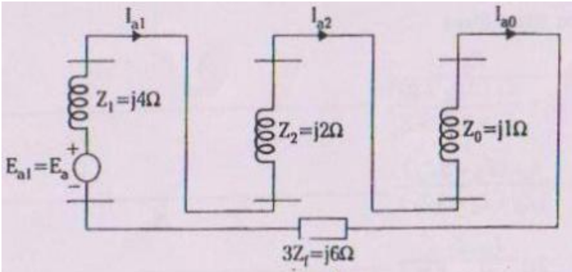


Fig. 5.15

In this case,

$I_{a1}=i_{a2}=I_{a0}=E_a / (Z_1+Z_2+Z_0+3Z_f)$

$=(400\angle 0° /\sqrt{3}) / j(4+2+1+6)$

$=-j17.765$ A

Fault current=$I_f=3.|I_{a0}|$

$=3(17.765)$

$=53.295$ A

ii)LL fault:

The interconnection of sequence networks to represent an LL fault is shown if fig 5.16.

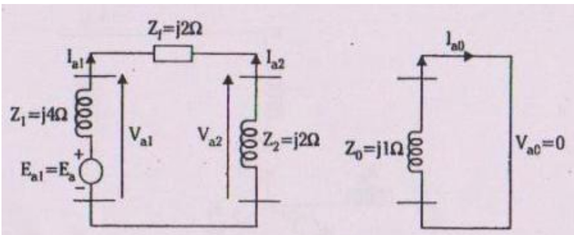


Fig. 5.16

Here, $I_{a1}=E_a / (Z_1+Z_2+Z_f)$

$=(400\angle 0° /\sqrt{3}) / j(4+2+2)$

$= -j28.87$ A

Therefore, fault current, $I_f = \sqrt{3}.|I_{a1}|$

$= \sqrt{3}(28.87)$

$= 50A.$

Iii) LLG fault:

The sequence networks are interconnected as shown in fig 5.17

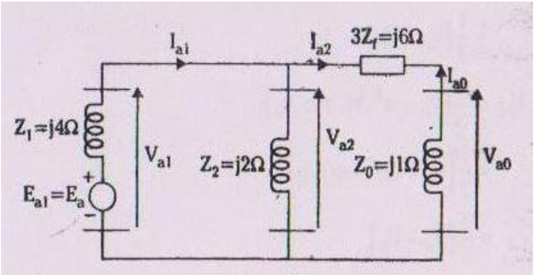


Fig. 5.17

Here, $I_{a1} = E_a / [Z_1+\{Z_2.(Z_0+3Z_f)/(Z_2+Z_0+3Z_f)\}]$

$= (400\angle 0° /\sqrt{3}) / [4+\{2(1+6)/(2+1+6)\}]$

$= -j41.57$ A

Therefore, using current division equation,

$I_{a0} = -I_{a1}.Z_2/(Z_2+Z_0+3Z_f)$

$= -j41.57 (2/ ((2+1+6))$

$= j9.24$ A

Hence, the fault current is,

$I_f = 3.|I_{a0}|$

$= 3(9.24)$

$= 27.72$ A

-------------END-------------

5.4 Unsymmetrical faults on power system:

The unsymmetrical faults on the power system are analyzed using Thevenin's theorem. The Thevenin's equivalent of positive, negative and zero sequence networks are obtained with respect to the fault point.

The prefault voltage at the fault point is the Thevenin's voltage of positive sequence network. The negative and zero sequence components of prefault voltage at the fault point is absent.

Let,

Z_1= Thevenin's impedance of positive sequence network.

Z_2= Thevenin's impedance of negative sequence network.

Z_0= Thevenin's impedance of zero sequence network.

V_{TH}=prefault voltage at the fault point.

=Thevenin's impedance of positive sequence network.

Thevenin's equivalent of positive, negative and zero sequence networks of the power system with respect to the fault point will be as shown in fig 5.29, 5.30 and 5.31 respectively.

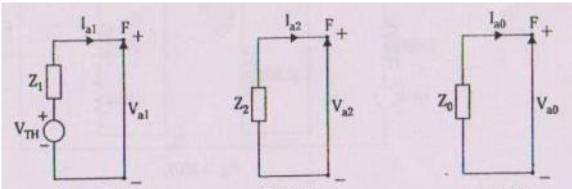


Fig. 5.29 Fig. 5.30 Fig. 5.31

Using Using Kirchoff's law to the circuits shown below, we get

$V_{a1}=V_{TH}-I_{a1}.Z_1$ 5.70

$V_{a2}= -I_{a2}.Z_2$ 5.71

$V_{a0}= -I_{a0}.Z_0$ 5.72

These equations are similar to that of a synchronous generator. They are useful in the analysis of unsymmetrical faults on the power system. We shall now consider the various types of unsymmetrical faults on a general power system.

5.4.1 Single line to ground (LG) fault:

fig 5.32 shows an LG fault at F in a power system through a fault impedance Z_f. The phases are so labeled that the fault occurs on phase a.

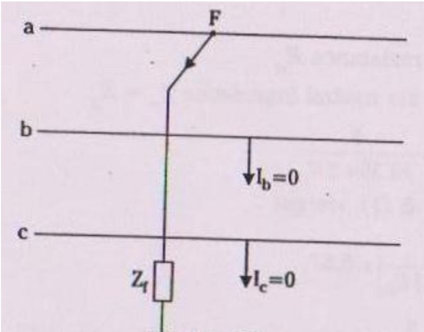


Fig. 5.32

Terminal conditions:

$V_a=I_a.Z_f$ 5.73

$I_b=0$ 5.74

$I_c=0$ 5.75

Symmetrical component relations:

The following relations can be obtained from the terminal conditions.

$I_{a0}=(1/3)(I_a+I_b+I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

so $I_{a1}=I_{a2}=I_{a0}=(1/3).I_a$ 5.76

The terminal condition $V_a=I_a.Z_f$ gives

$V_{a0}+V_{a1}+V_{a2}=I_a.Z_f=3I_{a0}.Z_f$ 5.77

Equations 5.76 and 5.77 suggest a series connection of sequence networks through a impedance $3.Z_f$ as shown in fig 5.33

Interconnection of sequence networks:

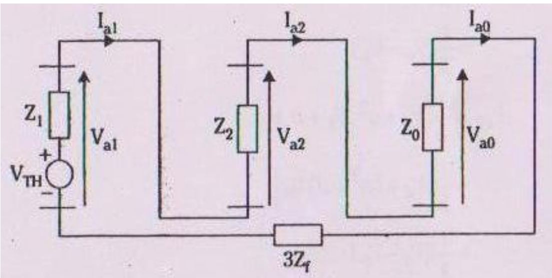


Fig. 5.33

Fault current:

The fault current in this case in given as,

$I_f=I_a=3I_{a0}= 3V_{TH} / (Z_1+Z_2+Z_0+3Z_f)$ 5.78

Note:

In the absence of fault impedance, replace Z_f by zero in the above calculations.

5.4.2 Line to line (LL) fault:

Fig 5.34 shows a LL fault at F in a power system on phase b and c through a fault impedance Z_f

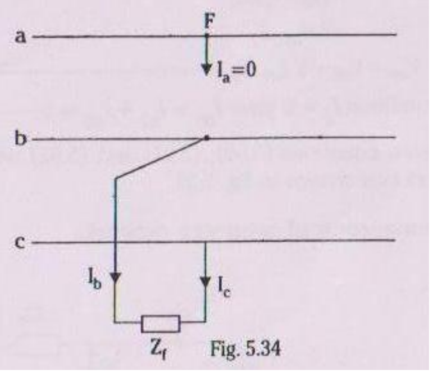


Fig. 5.34

Terminal conditions:

$I_a=0$ 5.79

$I_b+I_c=0$; $I_c=-I_b$ 5.80

$V_b=V_c+I_b.Z_f$ 5.81

Symmetrical components relations:

The following relations can be obtained from the terminal conditions

$I_{a0}=(1/3)(I_a+I_b+I_c)$

$=(1/3)(I_a+I_b-I_b)$

$=0$

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(0+a.I_b-a^2.I_c)$

$=(1/3)(a-a^2)I_b$

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(0+a^2 .I_b-a.I_c)$

$=(1/3)(a^2-a)I_b$

so, $I_{a0}=0$ 5.82

$I_{a1}= -I_{a2}$ 5.83

Next,

$V_{a1}=(1/3)(V_a+a.V_b+a^2.V_c)$

$V_{a2}=(1/3)(V_a+a^2.V_b+a.V_c)$

Therefore,

$V_{a1}-V_{a2}= (1/3) [(a-a^2)V_b+(a^2-a)V_c]$

$=(1/3) [(a-a^2)(V_b-V_c)]$

$=(1/3) (a-a^2)(I_b.Z_f)$

$= I_{a1}.Z_f$

Thus, $V_{a1}=V_{a2}+I_{a1}.Z_f$ 5.84

Since, $I_{a0}=0$, $V_{a0}= -I_{a0}.Z_0=0$ 5.85

Equations 5.83 and 5.85 suggest parallel connection of positive and negative sequence networks through a series impedance Z_f as shown in fig 5.35. Since $I_{a0}=V_{a0}=0$, the zero sequence network is connected separately and a shorted as shown in fig 5.36.

Interconnection sequence networks:

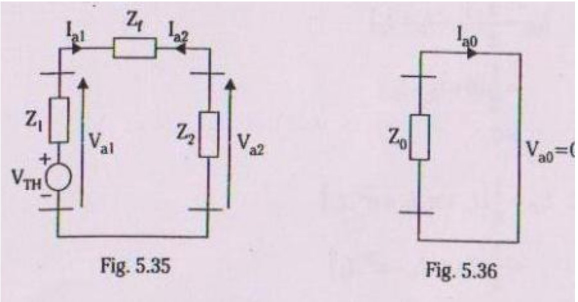


Fig. 5.35 Fig. 5.36

Fault current:

$I_f=I_b=I_{a0}+a^2.I_{a1}+a.I_{a2}$

$=0+a^2.I_{a1} -a.I_{a1}$

$=(a^2 - a)I_{a1}$

$= - j\sqrt{3}I_{a1}$

or $|I_f|=\sqrt{3}.I_{a1}$

$=\sqrt{3}.V_{TH} / (Z_1+Z_2+Z_f)$ 5.86

Note:

In the absence of fault impedance, replace Z_f by zero in the above calculations.

5.4.3 Double line to ground fault (LLG):

Fig 5.37 shows an LLG fault at F in a power system. The fault may in general have an impedance Z_f as shown.

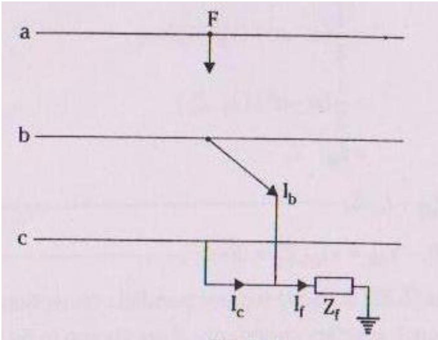


Fig. 5.37

Terminal conditions:

$I_a=0$ 5.87

$V_b=(I_b+I_c)Z_f$ 5.88

$V_c=(I_b+I_c)Z_f$ 5.89

Symmetrical component relations:

Consider,

$V_{a1}=(1/3)(V_a+a.V_b+a^2.V_c)$

$=(1/3)[(V_a+(a+a^2)V_b]$

$=(1/3)(V_a-V_b)$

$V_{a2}=(1/3)(V_a+a^2.V_b+a.V_c)$

$=(1/3)[(V_a+(a^2+a)V_b]$

$=(1/3)(V_a-V_b)$

$V_{a0}=(1/3)(V_a+V_b+V_c)$

$=(1/3)(V_a+2.V_b)$

Thus,

$V_{a1}=V_{a2}$ 5.90

$V_{a0}-V_{a2}=(1/3).3V_b$

$=V_b$

$=(I_b+I_c)Z_f$

$=3.I_{a0}.Z_f$ --------This will be proved to the expression for fault current.

Thus, $V_{a0}=V_{a2}+3.I_{a0}.Z_f$ 5.91

The condition $I_a=0$ gives $I_{a0}+I_{a1}+I_{a2}=0$ 5.92

From equations 5.90 , 5.91 and 5.92 we can draw connection of sequence networks as shown in fig 5.38

Interconnection of sequence networks:

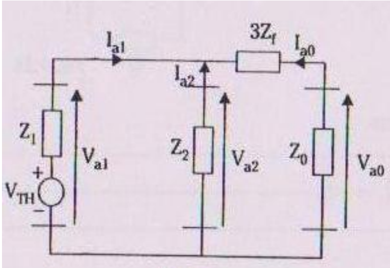


Fig. 5.38

Fault current:

In this case, the fault current is given as,

$I_f = I_b + I_c$

$= (I_{a0} + a^2.I_{a1} + a.I_{a2}) + (I_{a0} + a.I_{a1} + a^2.I_{a2})$

$= 2I_{a0} + (a+a^2)I_{a1} + (a+a^2)I_{a2}$

$= 2I_{a0} - I_{a1} - I_{a2}$ because $(a+a^2) = -1$

$= 2I_{a0} - (I_{a1} + I_{a2})$

It can be observed that $(I_{a1}+I_{a2}) = -I_{a0}$.

Substituting this in the expression for fault current, we get

$I_f = 2I_{a0} - (-I_{a0})$

$= 3I_{a0}$

$= -3.I_{a1}.[Z_2/(Z_0+Z_2+3Z_f)]$ 5.93

Note:

In the absence of fault impedance, Z_f is replaced by zero in the calculations.

The steps briefed below are used in solving the following problems:

1)The positive, negative and zero sequence networks for the given system are drawn.

2)The Thevenin's equivalent circuit of each of the networks with respect to the fault point is calculated.

3)These networks are interconnected suitably to simulate the particular type of fault condition.

Example 3.8:

A synchronous motor is receiving 10MW of power at 0.8pf lag at 6kV. An LG fault takes place at the middle point of the transmission line as shown in fig 5.39. Find the fault current. The ratings of the generator motor and transformer are as under.

Generator: 20MVA, 11kV, X_1=0.2p.u; X_2=0.1p.u; X_0=0.1p.u.
Transformer T_1: 18MVA, 11.5Y-34.5Y kV, X=0.1p.u.
Transmission line: $X_1=X_2=5\Omega$; $X_0=10\Omega$.
Transformer T_2: 15MVA, 6.9Y-34.5Y kV, X=0.1p.u.
Motor : 15MVA, 6.9kV, X_1=0.2p.u; $X_2=X_0$=0.1p.u.

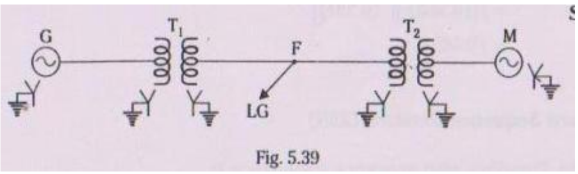


Fig. 5.39

Solution:
base values:
Let we choose,
base MVA=20
base kV on the generator=11
we calculate,
base kV on the transmission line=11(34.5/11.5)=33
base kV on the motor= 33(6.9/34.5)=6.6

Sequence reactances of generator:

$X_1 = X_{1,p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.2 \times (20 / 20) \times (11^2 / 11^2)$
$= j\,0.2$ p.u

$X_2 = X_{2,p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.1 \times (20 / 20) \times (11^2 / 11^2)$
$= j\,0.1$ p.u

$X_0 = X_{0,p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.1 \times (20 / 20) \times (11^2 / 11^2)$
$= j\,0.1$ p.u

Sequence reactances of transformer T_1: (calculated primary side of it)

$X_1 = X_2 = X_0 = X_{p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$
$= j0.1 \times (20 / 18) \times (11.5^2 / 11^2)$
$= j\,0.12$ p.u

Sequence reactances of transmission line:

$X_{1TL} = X_{2TL} = X_{TL}(\Omega) \times (MVA)_B / (kV)^2_B$
$= 5 \times 20 / 33^2$
$= 0.092$p.u

$X_{0TL} = 10 \times 20 / 30^2$
$=0.184$p.u

Sequence reactances of transformer T_2: (calculated secondary side of it)

$X_1 = X_2 = X_0 = X_{p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$

$= j0.1 \times (20 / 15) \times (6.9^2 / 6.6^2)$

$= j\,0.146$ p.u

Sequence reactances of motor:

$X_1 = X_{1,p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$

$= j0.2 \times (20 / 15) \times (6.9^2 / 6.9^2)$

$= j\,0.29$ p.u

$X_2 = X_0 = X_{2,p.u,\,old} \times ((MVA)_{B,\,new} / (MVA)_{B,\,old}) \times ((kV)^2_{B,\,old} / (kV)^2_{B,\,new})$

$= j0.1 \times (20 / 15) \times (6.9^2 / 6.9^2)$

$= j\,0.145$ p.u

Positive sequence Network (PSN):

Using the calculated values of positive sequence impedances, the PSN is drawn as shown in fig 5.40.

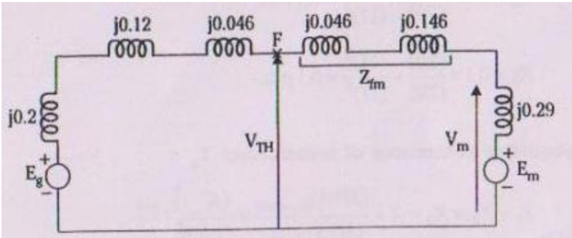


Fig. 5.40

To find the voltage at the fault point (V_{TH}):

The current drawn by the motor $I_m = ((10\times10^6) / (\sqrt{3}\times6\times10^3\times0.8)) \angle -\cos^{-1}0.8$

$= 1202.8\angle -36.87°$ A

The base current in the motor $(I_m)_B = (1000\times20) / (\sqrt{3}\times6.6)$

$= 1749.55$ A

Therefore,

I_m in p.u $= I_m/(I_m)_B = (1202.8/ 1749.55) \angle -36.87°$

$= 0.687\angle -36.87°$ p.u

Vm in p.u $= 6/6.6 = 0.909\angle 0°$ p.u

$V_{TH} = V_m + I_m .\ Z_{fm}$

$= 0.909 + ((0.687\angle -36.87°)(0.182\angle 90°))$, where $Z_{fm} = j0.046 + j0.146 = 0.182\angle 90°$

$= 0.909 + 0.132\angle 53.13°$

$= 0.909 + 0.0792 + j0.106$

$= 0.9882 + j0.106$

$= 0.994\angle 6.1°$

To find the Thevenin's impedance Z_{1TH}

The Thevenin's impedance as seen from point F is,

$Z_{1TH} = j[(0.2+0.12+0.046)||(0.046+0.146+0.29)]$

$= j(0.366||0.482)$

$= j0.208$ p.u

Hence the equivalent PSN of the system is as shown below:

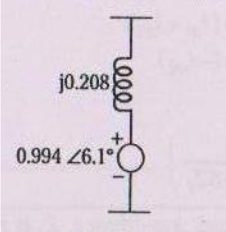


Fig. 5.41

Negative sequence Network (NSN):

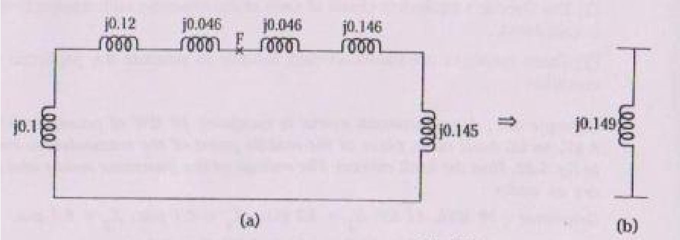


Fig. 5.42

The Thevenin's equivalent impedance with respect to the fault point is:

Z_{2TH}=j[(0.1+0.12+0.092)||(0.046+0.146+0.145)]

=j(0.266||0.337)

=j0.149 p.u

Zero sequence network (ZSN):

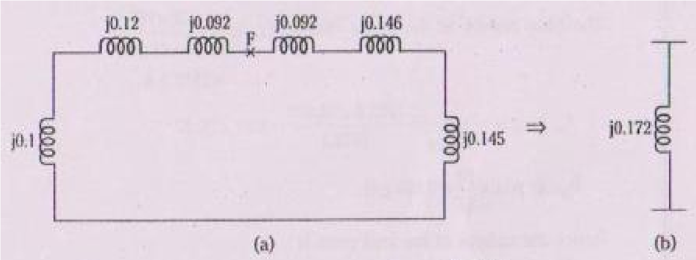


Fig. 5.43

The Thevenin's zero sequence impedance is,

Z_{0TH}=j[(0.1+0.12+0.092)||(0.092+0.146+0.145)]

=j(0.312||0.383)

=j0.172 p.u

Interconnection of sequence networks:

The sequence networks are connected as shown in fig 5.44 to represent LG fault.

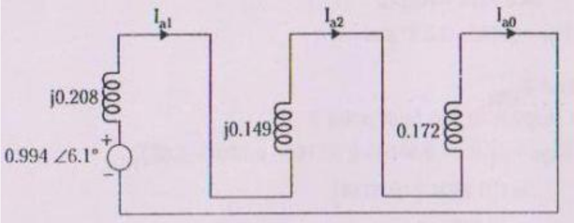


Fig. 5.44

Here,

$I_{a1}=I_{a2}=I_{a0}=$ (0.994∠6.1°) / j(0.208+0.149+0.172)
=1.88∠-83.9° p.u

Hence fault current:

$|I_f|=3.|I_{a0}|$

=3(1.88)

=5.64 p.u

Fault current in amperes is:

$=|I_f|_{p.u} \times (I_{TL})_B$

=5.64×((1000×20) / (√3×33))

=1973.49 A

5.5 Series type of faults:

We have so far discussed the various shunt type of faults that occur in a power system. But unsymmetrical faults in the form of open conductors (series type) also do take place in power system. It is required to determine the sequence components of line currents and the voltages across the broken ends of the conductors.

Fig 5.56 shows a system wherein an open conductor fault takes place.

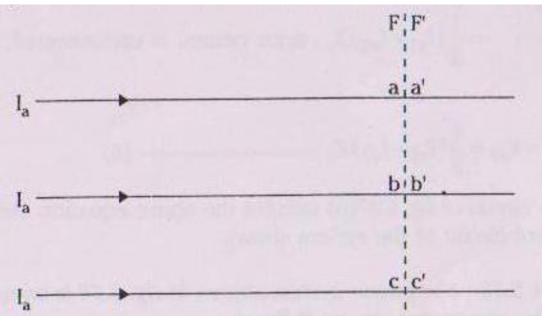


Fig. 5.56

The ends of the system on the sides of the fault are identified as F, F', while the conductor ends are denoted by aa', bb' and cc'. The voltage across the conductors are denoted by V_{aa}', V_{bb}' and V_{cc}'. The symmetrical components of these voltages are $(V_{aa}')_1$, $(V_{aa}')_2$,$(V_{aa}')_0$. The sequence networks as seen from the two ends FF' of the system are schematically shown in fig 5.57.

These are suitably interconnected depending on the type of fault (one or two conductors open).

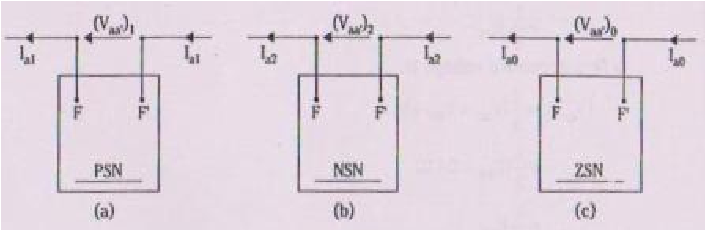


Fig. 5.57

i) One conductor open fault:

Let us assume that the conductor 'a' of a system gets opened as shown in fig 5.58.

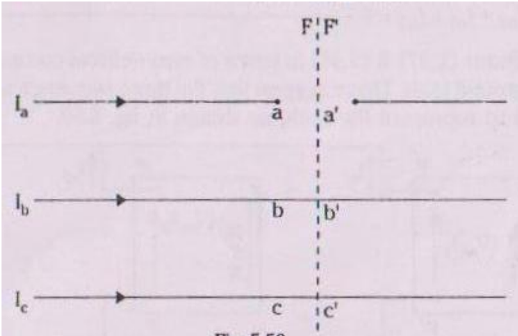


Fig. 5.58

Terminal conditions:

As seen from the two ends F and F'. The terminal conditions that are applicable are:

$I_a=0$ 5.94

$V_{bb}'=0$ 5.95

$V_{cc}'=0$ 5.96

Symmetrical components relations:

$(V_{aa'})_1=(1/3)(V_{aa'}+a.V_{bb'}+a^2.V_{cc'})$

$=(1/3)(V_{aa'}+0+0)$

$=(1/3)(V_{aa'})$

$(V_{aa'})_2=(1/3)(V_{aa'}+a^2.V_{bb'}+a.V_{cc'})$

$=(1/3)(V_{aa'}+0+0)$

$=(1/3)(V_{aa'})$

$(V_{aa'})_0=(1/3)(V_{aa'}+V_{bb'}+V_{cc'})$

$=(1/3)(V_{aa'}+0+0)$

$=(1/3)(V_{aa'})$

Thus,

$(V_{aa'})_1=(V_{aa'})_2=(V_{aa'})_0=(1/3)V_{aa'}$ 5.97

The condition $I_a=0$ gives the result

$I_{a0}+I_{a1}+I_{a0}=0$ 5.98

Condition 5.97 and 5.98 in terms of symmetrical components are similar to a double line to ground fault. These suggest that the three sequence networks should be connected in parallel to represent the fault, as shown in fig 5.59.

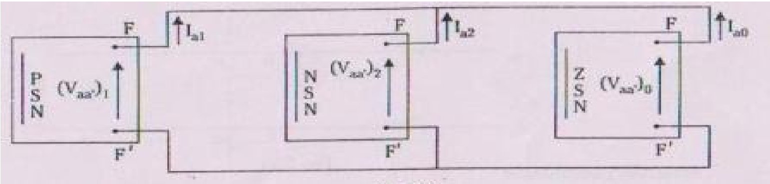


Fig. 5.59

ii)Two conductors open fault:

Let us assume that the two conductors b and c get open at the points F, F' as shown in fig 5.60.

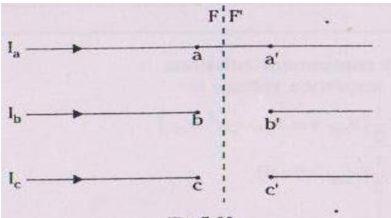


Fig. 5.60

Terminal conditions:

As seen from the points F and F', the terminal conditions that are applicable to this fault are:

$I_b=0$ 5.99

$I_c=0$ 5.100

$V_{aa'}=0$ 5.101

Symmetrical components relations:

consider,

$I_{a0}=(1/3)(I_a+I_b+I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)=(1/3)(I_a+0+0)=(1/3).I_a$

so $I_{a1}=I_{a2}=I_{a0}=(1/3).I_a$ 5.102

The terminal conditions $V_{aa'}=0$ gives the result,

$(V_{aa'})_0+(V_{aa'})_1+(V_{aa'})_2=0$ 5.103

These conditions are similar to those of line to ground fault and suggest that the three sequence networks be connected in series and shorted as shown in fig5.61.

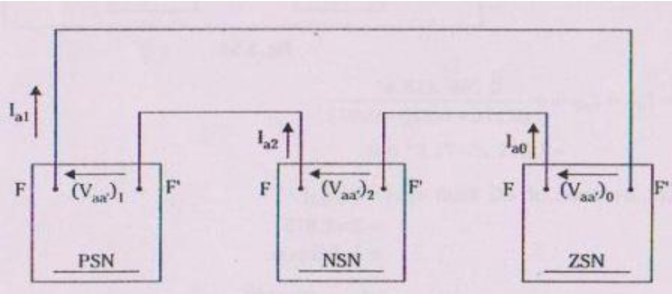


Fig. 5.61

-----------END------------

7.1 Introduction:

Stability of a large interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance. Conversely, instability denotes a condition of loss of synchronization in the system. This will result in wild fluctuation of currents and voltages within the power system network which is obviously undesirable. Hence, stability considerations form an important aspect in the study of power systems.

7.2 Some definitions:

Stability:

Stability, when used with reference to a power system, is that attribute of the system, or part of the system, which enables it to develop restoring forces between the elements thereof equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements.

Steady state stability:

This is the stability of the system under consideration subjected to a gradual or relatively slow change in load.

Transient state stability:

This refers to the stability of the system subjected to a sudden large disturbance. The large disturbance may be brought about by a sudden large change in load, faults in system or loss of generation in the system.

Dynamic stability:

This denotes the artificial stability given to a system by the action of automatic control device like fast acting voltage regulators and governors.

Steady state stability limit (SSSL):

This refers to the maximum flow of power possible through a particular point in the system without loss of stability when the power is increased gradually.

Transient state stability stability (TSL):

This refers to the maximum flow of power possible through a particular point in the system without the loss of stability when a sudden disturbance occurs.

Infinite bus:

A system having a constant voltage and a constant frequency regardless of the load on it is called an infinite bus-bar system or an infinite bus. Physically, it is impossible to have an infinite bus-bar system. This is just considered for the purpose of analysis.

7.3 Steady state stability:

The study of steady state stability is basically concerned with the determination of the maximum power flow possible through the power system, without loss of synchronism (stability). The formation of power angle equation plays a vital role in the study of steady state stability.

7.3.1 Power angle equation of synchronous machine:

The synchronous machine is one of the most important element of a power system. A synchronous generator converts mechanical power into electrical form and feeds it into the power system network. On the other hand, a synchronous motor draws electrical power from the network and converts it into mechanical form. There are basically two types of synchronous machines, the round rotor or Non-salient type and the salient pole type.

a)Power-angle equation of a non salient pole synchronous machine:

In the case of a non salient pole synchronous machine, the rotor consists of a cylindrical structure having a number of slots at intervals along the outer periphery of the cylinder for accommodating the field coils. Here, the air gap is uniform all along the rotor periphery and hence the flux linkage is also uniform. Therefore, the machine offers the same reactance for the flow of armature current at all places. This reactance is called as the synchronous reactance (X_s) of the machine.

The single phase equivalent reactance diagram of a non salient pole synchronous generator connected through a transmission line to an infinite bus is shown in fig 6.1.

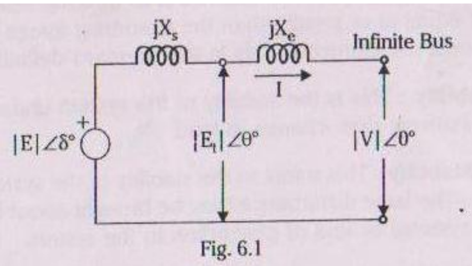


Fig. 6.1

Let,

$E\angle\delta$=Generated voltage in the machine

δ=Load angle or Torque angle or power angle

X_s=Synchronous reactance of the machine

$E_t=\angle\theta$=Voltage at the terminals of the machine

X_e=reactance of the transmission line

$V\angle 0°$=Voltage at infinite bus(taken as reference)

I=load current.

Let us suppose that the machine is operating for a large power factor load, that is, the load current lags the infinite bus voltage (reference) by an angle φ. The corresponding phasor diagram of the system is shown in fig. 6.2.

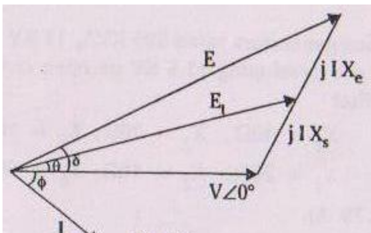


Fig. 6.2

Referring to the phasor diagram, we can relate the phasors as,

$E=V+j.\ I(X_s+X_e)$

or

$I=(E-V) / j(X_s+X_e)$ 6.1

The net power delivered by the machine is given as

$P=Re[V.I^*]$

Substituting Eq. 6.1 in the above equation, we get

$P=Re\ [V.\ \{(E-V) / j(X_s+X_e)\}^*]$

$=Re[|V|\angle 0°.\{(|E|\angle\ \delta-|V|\angle 0°) / (X_s+X_e)\angle 90°\}^*]$

$=Re[|V|\angle 0°.\{(|E|\angle\ -\delta-|V|) / (X_s+X_e)\angle -90°\}]$

$=[\{(|V|.|E|)/ (X_s+X_e)\}.\ \cos(90°-\ \delta)] - [\{|V|2 / (X_s+X_e)\}.\ \cos 90°]$

$=[(|V|.|E|)/ (X_s+X_e)].\ \sin\delta$

Thus,

$P=[(|V|.|E|)/ (X_s+X_e)].\ \sin\delta$ 6.2

Eq. 6.2 shows that, the power transferred depends upon the generated voltage E, bus voltage V, system reactance and the torque angle δ. This equation is called as the power angle equation of a non salient pole synchronous machine. A graphical plot showing the variation of electrical power P against the load angle δ for fixed values of E,V and reactance is called as the power angle curve. This is shown in fig 6.3

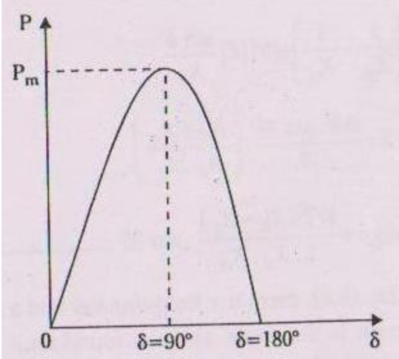


Fig. 6.3

The maximum power transfer occurs at δ=90°. The corresponding power is

$P_m = (|V|.|E|)/(X_s+X_e)$ 6.3

For values of δ>90°, the power output of the machine reduces successively and finally the machine may stall. Hence, Pm at which maximum power transfer occurs is called as the steady state stability limit (SSSL) of the machine. The machine operation is stable in the region 0°<δ<90° i.e the slope of the curve (dP/dδ)>0. This term (dP/dδ) is called as synchronizing power coefficient or machine stiffness. The condition (dP/dδ)>0 is called as the stability criterion. The SSSL is reached when (dP/dδ)=0 and if (dP/dδ)<0 the system is unstable.

Note:

1)If the synchronous machine is concerned directly to an infinite bus, i.r $X_e=0$, then eq. 6.2 becomes,

$P=[(|V|.|E|)/X_s].\sin\delta$ 6.4

2)Pm, the maximum power is also called as pull out power of the machine.

b) Power angle equation of a salient pole synchronous machine.

A salient pole machine has a number of projecting (salient) poles. Hence, the air gap is non uniform along the rotor periphery. It is least along the axis of the main poles (called the direct axis) and maximum along the axis of the inter polar region (called the quadrature axis). Hence flux linkages is non uniform. Correspondingly, the machine offers a direct axis reactance X_d and quadrature axis reactance X_q for the flow of armature current. A circuit model of the machine cannot be easily drawn. However, the phasor diagram of the machine neglecting its armature resistance is shown in fig. 6.6.

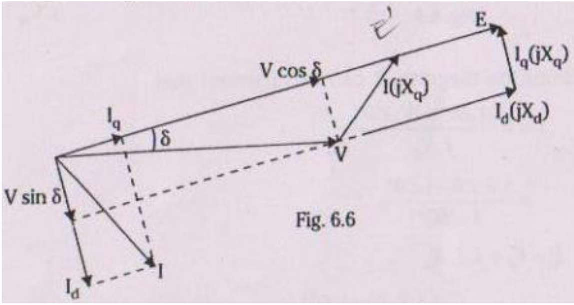


Fig. 6.6

Here,

E∠ δ=Generated voltage in the machine

V∠0°=Voltage at infinite bus(taken as reference)

δ=Load angle or Torque angle or power angle

X_d=Synchronous reactance of the machine

X_q=quadrature axis synchronous reactance

X_e=reactance of the transmission line

I=current delivered at a lagging power factor of cos φ.

Referring to the diagram, we can write the expression for power developed as,

$P=|V| \cos \delta . |I_q| + |V| \sin\delta . |I_d|$ 6.5

also, from fig 6.6 we observe that,

$|I_q.X_q| = |V.\sin\delta|$

or,

$|I_q| = V.\sin\delta / X_q$ 6.6

again,

$|I_d.X_d| = |E-V\cos\delta|$

or,

$|I_d| = |E|-|V|.\cos\delta / X_d$ 6.7

Using equations 6.6 and 6.7 in eq. 6.5 we get,

$P=[|V|.\cos\delta.\{(|V|.\sin\delta)/X_q\}] + [(|V|.\sin\delta).(|E|-|V|\cos\delta)/X_d]$

$=[|V|^2.(\sin2\delta/2.X_q)] + [(|V|.|E|.\sin\delta)/X_d] - [(|V|^2.\sin2\delta)/2.X_d]$

$=|V|^2.(\mathrm{Sin}2\delta/2).((1/X_q)-(1-X_d))+|V|.|E|.(\mathrm{Sin}\delta/X_d)$

$=(|V|.|E|/X_d).\mathrm{Sin}\delta + |V|^2.(\sin2\delta/2).((X_d-X_q)/(X_d.X_q))$

Thus,

$P=[(|V|.|E|)/X_d].\sin\delta + |V|^2((X_d-X_q)/2.X_d.X_q).\sin2\delta$ 6.8

As evident from eq.6.8, there is a fundamental and a second harmonic component of power. The first term is the same as far a round rotor machine with $X_s=X_d$. This constitutes the major part of power transfer. The second term is quite small (10-20%)compared to the first term and is known as reluctance power.

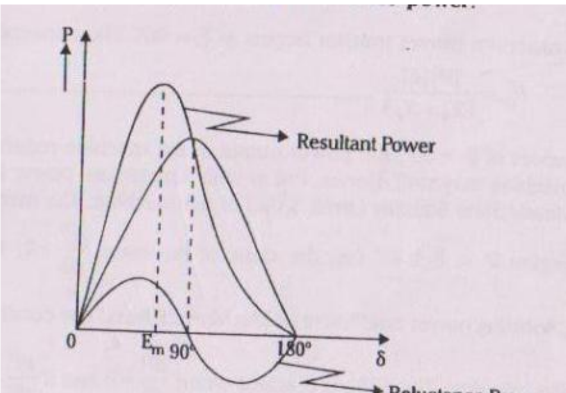


Fig. 6.7

The power angle curve of the machine is shown in fig 6.7. it is noticed that the maximum power output (SSSL) occurs at δ<90°(about 70°). This value of δ at which the power flow is maximum can be computed by equating the synchronizing power coefficient i.e dP/dδ to zero.

7.3.5 Methods of improving SSSL:

For a two machine system, we have

SSSL= $|E_g|.|E_m| / |X|$

As indicated by the equation, the SSSL can be increased by

i)increasing either of the voltages $|E_g|$ or $|E_m|$. This can be achieved by increasing the excitation to the generator or motor or both.

ii)Reducing the reactance between the transmission and receiving points. If the transmission lines are of sufficiently high reactance, the stability limit cab be raised by using two parallel lines which incidentally also increases the reliability of the system. Series capacitors are sometimes employed in lines to get better voltage regulation and to raise the SSSL by decreasing the line reactance. The use of bundled conductors is another method of reducing the line reactance and hence improving the SSSL.

7.4 Transient stability:

The transient stability refers to the maximum power flow possible through a point without losing stability with sudden and large changes in the power system. Following a sudden disturbance on a power system, the rotor speeds and rotor angular differences undergo fast changes whose magnitudes are dependent on the severity of disturbance. For a large disturbance, changes in angular differences δ may be so large that the machines may fall out of step. Thus, the transient stability of a system predominantly depends upon the dynamics of the synchronous machine.

7.4.1 Dynamics of a synchronous machine:

The kinetic energy of a rotor is,

$K.E = (1/2).I.\omega^2$

Where,

I=moment of inertia in $kg.m^2$

ω=angular speed in rad/sec

The angular momentum is,

$M=I.\omega$

therefore,

$K.E=(1/2).M.\omega$ joules

Rather than having the moment of inertia of a synchronous machine for dynamic studies, it is more convenient to use per unitized quantity called inertia constant H. This is defined as the ration stored energy of a machine at synchronous speed to the rated apparent power of the machine, i.e.

H=(stored energy in megajoules)/ (machine rating in mega volt-amperes) ...6.19

Let G=rating of the machine in MVA, then

GH=stored energy in mega joules

Hence, GH=K.E

$=(1/2).I.\omega^2$

$=(1/2)M.\omega$ joules (MJ)

Where,

$\omega=2\Pi f$ elec-rad/sec

$=360.f$ elec.deg/sec

or

$GH=(1/2).M.(360.f)$

therefore,

$M=GH/180.f$ MJ-sec./elect.deg 6.20

M is also called as the inertial constant eq. 6.20 relates the two inertia constants of the machine. For stability studies it is necessary to determine M which depends upon the size and speed of the machine, but instead H has a characteristic value of range of values for each class of machines. Typical values of H are indicated below:

cylindrical rotor alternator: 4-10

salient pole alternators: 2-3

Salient pole synchronous motors: .5-2

Modern power systems have many interconnected generators stations each with several generators and many loads. The machines located at any one point in a system usually act in unison. It is, therefore, common practice in stability studies to consider all machines at one point as a single equivalent machine, having a rating equal to the sum of the ratings of several machines considered to act together. The inertia constant M of the equivalent machines is the sum of the inertia constants of the individual machines, i.e.

$M_{eq} = M_1 + M_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + M_n$

or $H_{eq}.\ G_{base} = H_1G_1` + H_2G_2 + \ldots\ldots + H_nG_n$

or $H_{eq} = H_1G_1/G_{base} + H_2G_2/G_{base} + \ldots\ldots\ldots\ldots + H_nG_n/G_{base}$ 6.21

7.4.2 Swing equation:

The load angle or the torque angle δ depends upon the loading of the machine. Larger the loading, larger is the value of the torque angle. If some load is added or removed from the shaft of the synchronous machine, the rotor well decelerate or accelerate respectively with respect to the synchronously rotating stator field and a relative motion begins. It is said that the rotor is swinging with respect to the stator field. The equation describing the relative motion of the rotor (load angle δ) with respect to the stator field as a function of time is called as swing equation.

Consider the generator shown in fig 6.26. It receives mechanical power Ps at torque Ts and rotor speed ω via shaft from the prime mover. It delivers electrical power Pe to the power system network via the bus bars. The generator develops electromechanical torque Te in opposition to Ts.

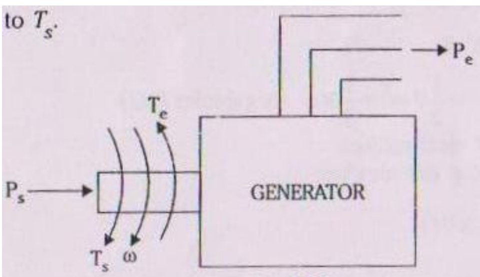


Fig. 6.26

Assuming that winding and friction losses to be negligible, the accelerating torque on the rotor is given by'

$T_a = T_s - T_e$ 6.22

Multiplying by ω on both sides, we get

$\omega.T_a = \omega.T_s - \omega.T_e$

but

$\omega.T_a=P_a$=accelerating power

$\omega.T_s=P_s$=mechanical power input

$\omega.T_e=P_e$=electrical power output assuming that power loss is negligible.

Therefore, we get

$P_a=P_s-P_e$ 6.23

Under steady state conditions, $P_a=P_e$, so that $P_a=0$.

When Ps.Pe balance is disturbed, the machine undergoes dynamics governed by

$P_a=T_a.\omega=I.\alpha.\omega=M.(d^2\theta/dt^2)$ 6.24

where $=d^2\theta/dt^2$ is the angular acceleration of the rotor.

Since the angular position of the motor is continually varying with time, it is more convenient to measure the angular position and velocity with respect to a synchronously rotating axis fig 6.27.

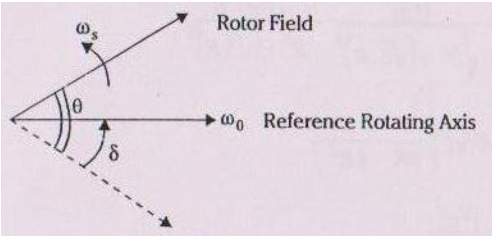


Fig. 6.27

From the fig 6.25, it can be inferred that

$\delta=\theta- \omega 0t$ 6.25

where, ω_0=angular velocity of the reference rotating axis.

δ=rotor angular displacement with respect with respect to the stator field.

Taking time derivatives of eq. 6.25

$(d\delta/dt)= (d\theta/dt)-\omega_0$

and $(d^2\delta/dt^2)=d^2\theta/dt^2$ 6.26

combining equation 6.23 and 6.24 and 6.26, we get

$M.(d^2\theta/dt^2)=P_a=P_s-P_e$ 6.27

This equation is called as the swing equation of the synchronous machine. Whe the machine is connected to the infinite bus bars, then $P_e=[(|E| .|V|)/ X].\sin\delta=P_m.\sin\delta$.

Or $M.(d^2\delta/dt^2)= P_s -P_m.\sin\delta$. 6.28

7.4.3 Swing curve:

The solution of swing equation gives the relation between rotor angle δ as a function of time t. The plot of δ versus t is called as swing curve. The exact solution of the swing equation is however a very tedious task. Normally, step by step method or any other numerical solution techniques like Euler's method, Runge-Kutta's method are used for solving the swing equation. The swing curve is used to determine the stability of the system. In case δ increases indefinitely, it indicates instability. Whereas if if reaches a maximum and starts decreasing, it shows that the system will not lose stability since the oscillations will be damped out with time. A sample swing curve is shown in fig 6.28.

For the stability of the system, $d\delta/dt=0$ 6.29

The system will be unstable if $d\delta/dt>0$ for a sufficiently long time (normally more than 1sec)

7.4.4 Equal area criterion (EAC):

This provides a qualitative assessment of transfer stability of a synchronous system. It helps in deciding whether a system is stable or not under transient conditions, without solving the swing equation.

Consider the swing equation of a single machine connected to an infinite bus,

$M.(d^2\theta/dt^2)=P_a$

Multiplying both sides of the equation by (2/M).(dδ/dt), we get

$2.(d\delta/dt).(d^2\delta/dt^2)=(2/M).P_a.(d\delta/dt)$ because $d(x^2)/dt=2.x.\ dx/dt$

or

$(d(d\delta/dt)^2/dt)=(2/M).P_a.(d\delta/dt)$

Integrating with respect to t we obtain,

$$(d\delta/dt)^2=(2/M)\int_{\delta o}^{\delta} Pa.\left(\frac{d\delta}{dt}\right).dt$$

$$=(2/M)\int_{\delta o}^{\delta} Pa.d\delta$$

$$\text{or } (d\delta/dt)=\sqrt{\frac{2}{M}\int_{\delta o}^{\delta} Pa.d\delta}$$

for the system to be stable, $(d\delta/dt)=0$

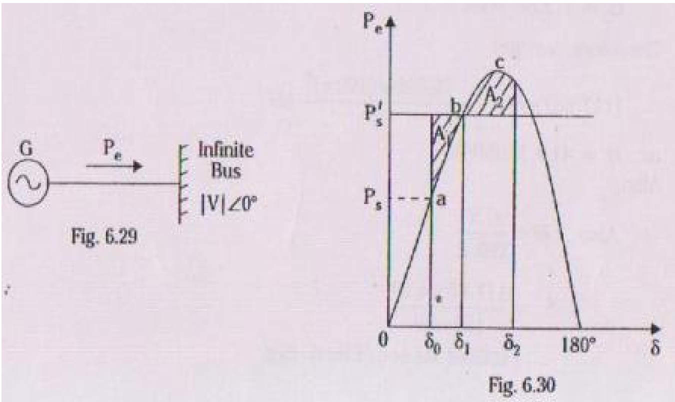
$$\text{i.e } \sqrt{\frac{2}{M}\int_{\delta o}^{\delta} Pa.d\delta}=0$$

$$\text{or } \int_{\delta o}^{\delta} Pa.d\delta=0 \qquad \text{.............................6.30}$$

The physical meaning of integration is the estimation of the area under the curve. The above integral indicates zero area. Thus, the system is stable if the area under P_a-δ curve reduces to zero for some value of δ. This is possible only when P_a has both positive (accelerating) and negative (decelerating) powers. For a stable system, the positive area under P_a-δ curve must be equal to the negative area and hence, this is called Equal Area Criterion for stability.

7.4.5 Applications of Equal area criterion:

Fig 6.29 shows the one line diagram of a synchronous generator connected to an infinite bus.



Let us consider the case of sudden change (increase) in the mechanical input. Fig 6.30 shows the plot of P_e-δ, the power angle curve with the system operating at point a corresponding to input P_s. Let the mechanical input be suddenly increased to P_s' as shown. The accelerating power $P_a(=P_s'-P_e)$ causes the rotor to accelerate. Hence the rotor angle δ increases, the electrical power transfer increases, reducing P_a, till a point b at which P_a=0. The rotor angle δ, however, continues to increase because of the inertia of the rotor and Pa becomes negative causing the rotor to decelerate. At some point c where A_1=area A_2 or $\int_{\delta o}^{\delta} Pa.d\delta$ =0, the rotor velocity dδ/dt becomes zero (this corresponds to the synchronous speed) and then starts to become negative owing to continuing negative P_a. The rotor angle thus reaches the maximum value δ_2 and then starts to decrease.

From fig 6.30 areas A_1 and A_2 are given by,

$$A_1 = \int_{\delta o}^{\delta 1} (Ps' - Pe).d\delta$$

$$A_2 = \int_{\delta 1}^{\delta 2} (Pe - Ps').d\delta$$

For the system to be stable, it should be possible to find angle δ_2 such that $A_1 = A_2$ as P_s' is increased, a limiting condition is finally reached when area A_1 equals the entire area A_2 above the line P_s' as shown in the fig 6.31. Under this condition δ_2 acquires the maximum value δ_m such that,

$\delta_2 = \delta_m = 180° - \delta_1$ 6.31

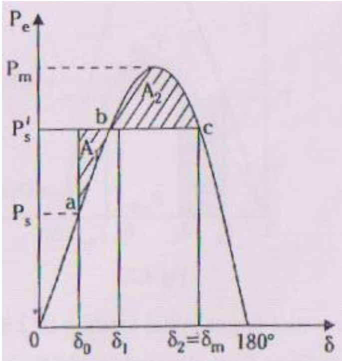


Fig. 6.31

Now, the system is said to be critically stable. Any further increase in P_s' means the area available for A_2 is less than A_1, so that the system becomes unstable. It has thus been shown by the use of EAC that there is an upper limit to sudden increase in mechanical input (P_s'-P_s) for the system in question to remain stable. The power P_s' is transient stability Limit (TSL) of the system. As clearly visible from the fig 6.31, TSL (P_s') is less than SSSL (P_m) of the system.

b)Sudden loss of one of the parallel lines:

Let us consider a single machine connected to an infinite bus through two parallel lines as in fig 6.32. Circuit model of the system is given in fig 6.33.

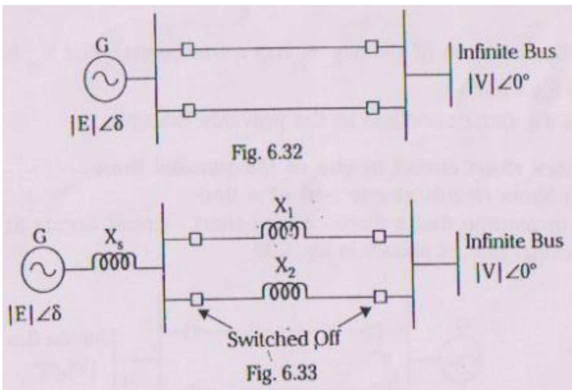


Fig. 6.32

Fig. 6.33

The transient stability of the system when one of the lines is suddenly switched OFF from the system, while operating under steady load conditions is now being considered.

Case i)Before switching OFF, the power angle equation is,

P_{e1}= [(|E| .|V|)/ {X_s+(X_1||X_2)}].sinδ=P_{m1}.sinδ curve1

case ii)On switching OFF line -2, the power angle equation is

P_{e2}= [(|E| .|V|)/ (X_s+X_1)].sinδ=P_{m2}.sinδ curve2

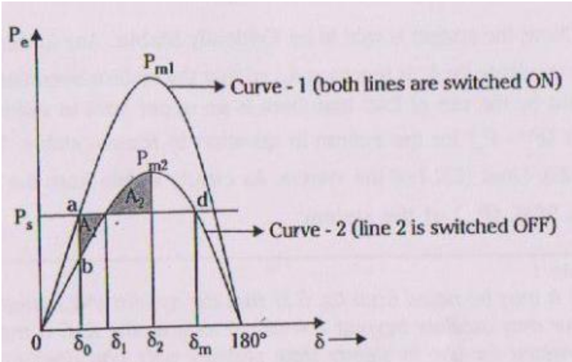


Fig. 6.34

Fig 6.34 shows the two curves wherein P_{m2}<P_{m1} as (X_s+X_1)>(X_s+(X_1||X_2)). As soon as line-2 was switched OFF, the original operating point a on curve-1 is shifted to a point b on curve-2. Accelerating energy corresponding to area A_1 is put into rotor followed by decelerating energy. If an area A_2 equal to A_1 is found above the P_s line, the system will be stable, and finally operates at C corresponding to a new rotor angle $\delta_1>\delta_0$.

For the limiting case of stability, δ_2 has a maximum value δ_m is given by

$\delta_2=\delta_m=180°-\delta_1$

Which is the same condition as the previous example.

c)Sudden short circuit in one of the parallel lines:

case i) short circuit at one end of a line:

Let us assume that a three phase short circuit occurs at the end of line-2 of a double circuit line as shown in fig 6.35.

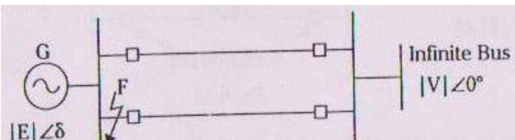
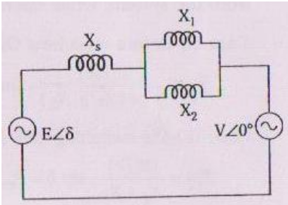
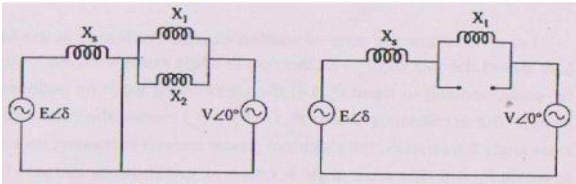


Fig. 6.35

The equivalent reactance diagrams before, during and after fault clearance are shown in fig 6.36a, 6.36b and 6.36c respectively.



(a)



(b) Fig. 6.36 (c)

Before the occurrence of a fault, the power angle curve is given by,

$P_{e1} = [(|E| .|V|)/ \{X_s+(X_1||X_2)\}].\sin\delta = P_{m1}.\sin\delta$

Upon occurrence of a three phase fault at the end of line-2, there is no power flow as seen from the fig i.e $P_{e2}=0$.

The circuit breakers at the two ends of the faulted line open at the t_1 (corresponding to angle δ_1), called the clearing time, dis-connecting the faulted line. The power flow is now restored via the healthy line-1. With power angle curve is given as,

$P_{e3} = [(|E| .|V|)/ (X_s+X_1)].\sin\delta = P_{m3}.\sin\delta$

obviously, $P_{m3}<P_{m1}$. The rotor now starts to decelerate as shown in fig 6.37. The system, will be stable if a decelerating area A_2 can be found equal to the accelerating area A_1 before δ reaches the maximum allowable value δ_m.

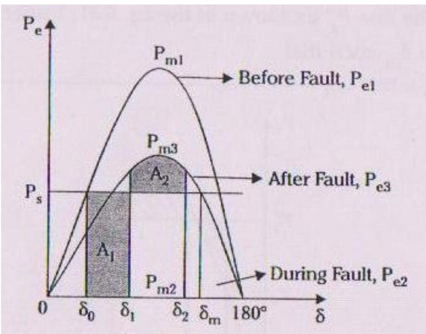


Fig. 6.37

It easily follows that larger initial loading (P_s) increases A_1 for a given clearing angle δ_1 and therefore, quicker fault clearing would be needed to maintain stable operation.

Case ii)Short circuit away from line ends:

When a three phase fault occurs away from line ends (say in the middle of a line). There is some impedance between the paralleling buses and the fault. Therefore, some power is transmitted while the fault is still on the system. The one line diagram of the system is shown in fig 6.38.

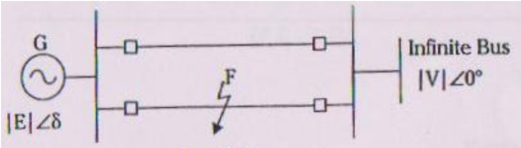


Fig. 6.38

The equivalent circuit before occurrence of fault is shown in fig 6.39.

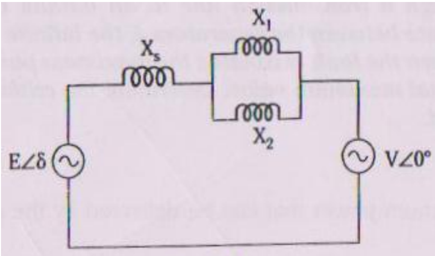


Fig. 6.39

The power angle curve is given by,

$P_{e1}= [(|E| .|V|)/ \{X_s+(X_1||X_2)\}].\sin\delta=P_{m1}.\sin\delta$

circuit model of the system during fault is shown in fig 6.40.

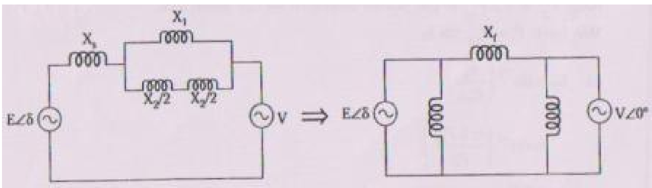


Fig. 6.40

Here, X_f= transfer reactance of the system. The power angle curve during fault is therefore given by

$P_{e2}= [(|E| .|V|)/ X_f].\sin\delta=P_{m2}.\sin\delta$

After the clearing of the fault by opening of the circuit breakers, the equivalent circuit is as shown in fig 6.41.

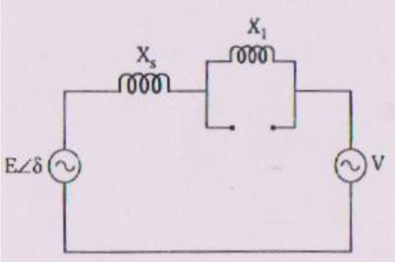


Fig. 6.41

The Power angle curve is given by,

$P_{e3}= [(|E| .|V|)/ (X_s+X_1)].\sin\delta=P_{m3}.\sin\delta$

The power angle curves corresponds to P_{e1}, P_{e2} and P_{e3} are shown in fig 6.42.

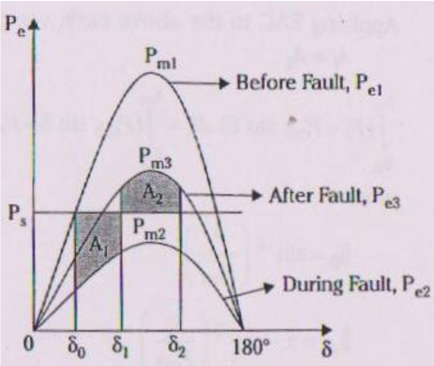


Fig. 6.42

The system is stable only if it is possible to find an area A_2 equal A_1.

7.4.6 Critical clearing angle and critical clearing time:

In the previous case if, P_s is increased, then δ_1 increase, area A_1 increases and to find $A_2=A_1$, δ_2 is increased till it has a value δ_m, the maximum allowable limit for stability. Then the system is said to be critically stable. The angle δ_1 is then called as the critical clearing angle (δ_{cc}). The time corresponding to this is called the critical clearing time (t_{cc}). The critical clearing angle can be determined from the EAC. However, the critical clearing time cannot be obtained from EAC. It is possible to estimate critical clearing time using the swing curve. This time is very much essential in designing the protective circuit breakers for the system.

The case of critical stability of a system is shown in fig 6.43.

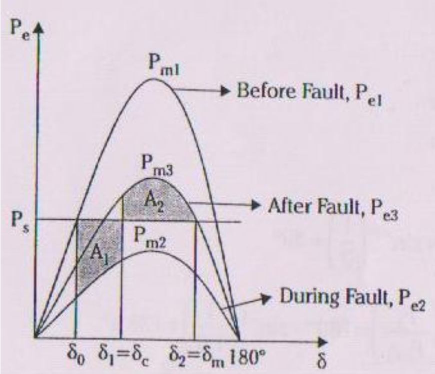


Fig. 6.43

Applying EAC to the above case, we get

$A_1=A_2$

$$\int_{\delta 0}^{\delta cc}(Ps-Pm2.sin\delta).d\delta = \int_{\delta cc}^{\delta m}(Pm3.sin\delta-Ps).d\delta$$

where,

$\delta_0=\sin^{-1}(P_s/P_{m1})$

$\delta_m=\Pi-\sin^{-1}(P_s/P_{m3})$

Integrating, we get

$$(P_s.\delta+P_{m2}.\cos\delta)\Big|_{\delta 0}^{\delta cc} = (-P_{m3}.\cos\delta-P_s\delta)\Big|_{\delta cc}^{\delta m}$$

or

$P_s(\delta_{cc}-\delta_0)+P_{m2}(\cos\delta_{cc}-\cos\delta_0)+P_s(\delta_m-\delta_{cc})+P_{m3}(\cos\delta_m-\cos\delta_{cc})=0$

or

$\cos\delta_{cc}=[P_s.(\delta_m-\delta_0)-P_{m2}.\cos\delta_0+P_{m3}.\cos\delta_m] / (P_{m3}-P_{m2})$ 6.32

The angles in the above equation are in radians. If the angles are in degrees, the equation modifies as below. (Π over 180°)

$$\text{Cos}\delta_{cc}=\frac{[\frac{\Pi}{(180°)}Ps.(\delta m-\delta 0)-Pm2.cos\delta 0+Pm3.cos\delta m]}{(Pm3-Pm2)} \quad ....6.33$$

7.4.7 Methods of improving transient stability:

From the swing equation we have $(d^2\delta/dt^2)=(P_a/M)$ i.e the acceleration of the rotor $(d^2\delta/dt^2)$ is inversely proportional to the angular momentum of the machine when accelerating power is a constant. This means that higher the value of M, slower will be the change in the rotor angle of the machine and thus allows a longer time for the circuit breakers to isolate the fault before the machine passes through the critical clearing angle. However, to achieve higher value of M, a heavier rotor is required which inturn increases the the cost of the machine. Therefore, this method cannot be employed in practice because of economic reasons.

The methods often employed in practice to improve system stability are:
1)Increase of system voltages.
2)Reduction of transfer reactance.
3)Use of high speed circuit breakers and auto-reclosing breakers.

It is observed from eq. 6.12 that $P_m=|E_g| .|E_m|)/ X$. Thus, by increasing the system voltages or by reducing the system transfer reactance, the maximum power transfer (stability) can be increased. The system voltages can be increased by the use of high speed excitation systems (AVRs). The reactance of a transmission line can be decreased.
i)by reducing the conductor spacing.
ii)by increasing conductor diameter.
iii)by the use of bundled conductors.
iv)by increasing the number of parallel lines or
v)by using series capacitors in the transmission lines.

The quicker a breaker operates, the faster the fault is removed from the system and better is the tendency of the system to store to normal operations.

Recent trends:

A brief account of some of the recent methods of maintaining stability is given below.

a)HVDC links:

A d.c transmission line does not have any stability problem in itself because, d.c operation is an asynchronous operation of the machines. Hence, the use of HVDC links to connect separate a.c systems improves the stability of the systems.

b)Braking Resistors:

For improving the stability of a system when a large load is suddenly lost, a resistive load called a braking resistor is connected at or near the generator terminals. This load compensates for at least some of the reduction of load on the generators and so reduces the acceleration ($d^2\delta/dt^2$) of the machines.

c)Fast valving:

In this method, the stability of a unit is improved by decreasing the mechanical input power to the turbine. When a fault occurs in the system, a control scheme detects the difference between a mechanical input and reduced electrical output of the generator, initiates the closing of a turbine valve to reduce the power input.

d)Full load rejection technique:

Sometimes, to maintain stability it becomes inevitable to take a faulty unit out of service. However, the loss of a major unit for a long time can be seriously hazardous for the remaining system. To avoid this, a full load rejection scheme could be utilized after the unit is separated from the system. To do this, the unit has to be equipped with a large bypass system. After the system has recovered from the shock caused by the fault, the unit could by synchronized and reloaded. The main disadvantage of this method is the extra cost of a large bypass system.

----------END-----------

8.1 Introduction:

The operation of three phase induction motor supplied with balanced voltages has been dealt in the previous semesters. In this chapter, we analyse the performance of a three phase induction motor under unbalanced conditions. The method of symmetrical components will be suitably exploited in study.

8.2 Performance of a three phase induction motor under unbalanced supply voltages:

Let us assume that unbalanced voltages are applied across the stator of a 3-phase symmetrically wound induction motor. It is known that the unbalanced voltages can be resolved into positive, negative and zero sequence components of voltages. The effect of applying unbalanced voltages can be obtained by superposing the effects due to positive, negative and zero sequence components of voltages.

The positive sequence voltages constitute a balanced three phase system having the same phase sequence as that of the original unbalanced phasors. They induce positive sequence rotor currents which circulate a s times the supply frequency (f_r=s.f) and produce a positive torque. The negative sequence voltages also constitute a balanced three phase system, but having opposite phase sequence as that of the original unbalanced phasors. They induce negative sequence rotor currents at (2-s) times the supply frequency in the reverse direction(f_r'=(2-s)f) and hence produce a negative torque. The zero sequence voltages do not constitute a three phase system. Hence they do not produce any net torque.

Now let us find expressions for positive and negative torques, the net torque and net power in an induction motor under unbalanced supply voltages. The analysis applies to a star connected (or equivalent star connected) induction motor. A delta connected stator winding of an induction motor, can however be transformed to an equivalent star connected stator winding for the purpose of analysis.

When positive sequence voltages are applied to a symmetrical star (or equivalent star) connected stator of a three phase induction motor, as already stated, the induced currents in the rotor circulate at slip s times the supply frequency. The approximate equivalent circuit of an induction motor on a single phase basis omitting the exciting circuit and assuming the voltage transformation ration as unity is shown in fig 7.1

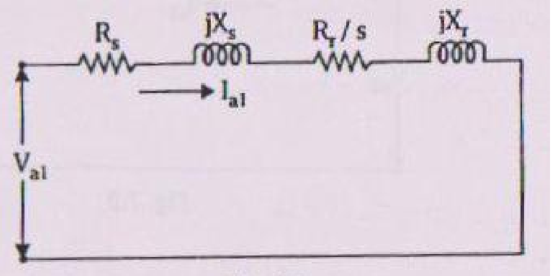


Fig. 7.1

Here,
V_{a1}=positive-sequence component of phase voltage
R_s=stator resistance per phase
X_s=stator reactance per phase
R_r=rotor resistance per phase as referred to stator.
X_r=rotor reactance per phase as referred to stator.

Note:
In this case, the rotor impedance is same as the rotor impedance referred to stator as we have assumed voltage transformation ratio as unity.

The equivalent positive sequence current is given as
$I_{a1}=V_{a1}/[(R_s+(R_r/s))^2+(X_s+X_r)^2]^{1/2}$ 7.1
Therefore,
Rotor copper loss/ph $=I_{a1}^2\times R_r$
In the case of an induction motor, we know that
torque in synchronous =rotor input
=rotor copper loss / s
=power / (1-s)
Using the above relations, we get the positive torque produced per phase due to positive sequence currents as
$T_1=I_{a1}^2\times R_r/s$ syn. Watts 7.2
and positive shaft power is,
$P_1=T_1(1-s)$
$=I_{a1}^2\times R_r/s\ (1-s)$ watts 7.3
Where, I_{a1} is defined through equation 7.1
When negative sequence voltages are applied to the stator, the negative sequence currents produced in the rotor circulate at (2-s) times the supply frequency but in

opposite direction. The approximate negative sequence equivalent circuit is shown in fig 7.2.

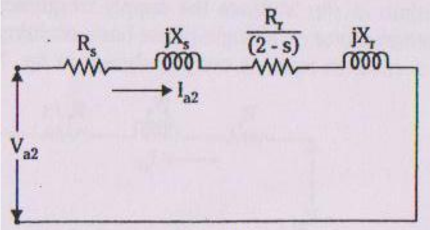


Fig. 7.2

On similar lines as derived above, we get the negative sequence current as:

$I_{a2}=V_{a2} /[(R_s+(R_r/(2\text{-}s)))^2+(X_s+X_r)^2]^{1/2}$ 7.1

The negative torque produced per phase is

$T_2=- I_{a2}^2 .(R_r/(2\text{-}s))$ syn. Watts 7.5

and negative shaft power is

$P_2=T_2(1\text{-}s)= -I_{a2}^2 .(R_r/(2\text{-}s)).(1\text{-}s)$ watts 7.6

The net three phase torque produced is thrice the sum of positive and negative i.e

$T=3(T_1+T_2)$

$=3.[\{I_{a1}^2 .(R_r/s)\} -\{I_{a2}^2 .(R_r/(2\text{-}s)\}]$ syn. Watts 7.7

similarly, the net three phase power produced is given by

$P=3(P_1+P_2)$

$=3.[\{I_{a1}^2 .(R_r/s)(1\text{-}s)\} -\{I_{a2}^2 .(R_r/(2\text{-}s).(1\text{-}s)\}]$

$=3.[\{I_{a1}^2 .(R_r/s)\} -\{I_{a2}^2 .(R_r/(2\text{-}s)\}].(1\text{-}s)$ 7.8

$=T(1\text{-}s)$ 7.9

Thus it can be observed that due to the negative torque produced by the negative sequence components of the unbalanced voltages, the net torque and hence the net power output is reduced. In addition to this, the negative sequence currents circulating in the rotor at (2-s) times the supply frequency, produce a large amount of coreless than their counter part of positive sequence currents which circulates at s times the supply frequency. This may some times result in overheating of rotor core. Hence, the performance of a three phase induction motor deteriorates when supplied with unbalanced voltages.

8.3 Single phasing of an induction motor:

Single phasing means the opening of one wire (or leg) of a three phase circuit whereupon the remaining legs at once becomes single phase. When a three phase circuit is functioning normally, there are three distinct currents flowing in the circuit. As is known, one of the three phases act as a return path for the other two. Obviously, an open circuit in one wire kills two phases and there will be only one current or phase working, even though two wires are left intact. The usual cause of single phasing is due to the blowing out of a fuse in the switch gear because of over loading of the machine.

Consider a star (or equivalent star) connected stator of a three phase induction motor. Let us assume that an open circuit occurs in phase a. This is diagrammatically shown in fig 7.7.

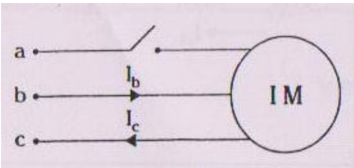


Fig. 7.7

The terminal conditions are,

$I_a=0$ (open)

$I_b=-I_c=I$ (say)

In terms of symmetrical components, we can write

$I_{a1}=(1/3)(I_a+a.I_b+a^2.I_c)$

$=(1/3)(0+a.I-a^2.I)$

$=((a-a^2)/3).I$

$=(j/\sqrt{3}).I$

similarly,

$I_{a2}=(1/3)(I_a+a^2.I_b+a.I_c)$

$=(1/3)(0+a^2.I-a.I)$

$=((a^2-a)/3).I$

$=(-j/\sqrt{3}).I$

It can be observed that,

$I_{a1}=-I_{a2}=(j/\sqrt{3}).I$ 7.10

The only voltages that is known to us in this case is the line to line voltage between the lines b and c, that are not open.

Hence,

$V_{bc}=V_A$

$=V_c-V_b$

$=(V_{a0}+a.V_{a1}+a^2.V_{a2}) - (V_{a0}+a^2.V_{a1}+a.V_{a2})$

$=(a-a^2)(Va1-Va2)$

or $V_{a1}-V_{a2}=V_A/(j/\sqrt{3})$

$V_{a1}-V_{a2}=V_A/\sqrt{3}$ 7.11

=Magnitude of the phase voltage.

An equivalent circuit connecting the two sequence networks and satisfying equations 7.10 and 7.11 is as shown in fig 7.8.

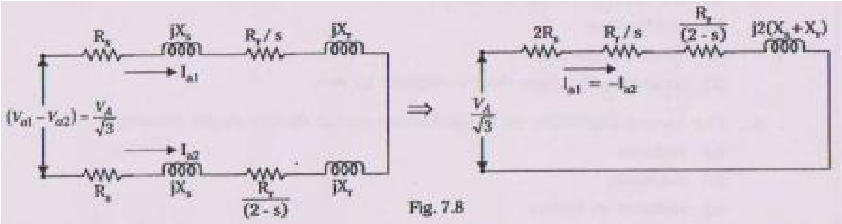


Fig. 7.8

From the circuit, it can be observed that,

$I_{a1}= -I_{a2}= [V_A/\sqrt{3}] / [\{2R_s+(R_r/s)+(R_r/(2-s)\}+\{2(X_s+X_r\}^2]^{1/2}$ 7.12

The net three phase torque is given by,

$T=3.I_{a1}^2[(R_r/s)-(R_r/(2-s)]$

$=6.\ I_{a1}^2.Rr.[(1-s)/\{s.(2-s)\}]$ syn. Watts 7.13

The net three phase shaft power output is (1-s) times the torque in syn. Watts. Therefore,

$P=6.I_{a1}^2.Rr.[(1-s)^2/\{s.(2-s)\}]$ watts............................7.14

Equations 7.13 and 7.17 indicates that, for s=1(at standstill position), the net torque and the net power output are zero. This indicates that a stationary motor will not start with one line open. In fact, due to heavy standstill current, it is likely to burn out quickly unless immediately disconnected.

For 0<s<1, i.e during the running condition of the motor, the net torque and net power output is positive. This implies that a running motor will continue running with one-line open. However, if the motor is very heavily loaded, then it will stop under single phasing. Since it can neither restart not blow out the remaining fuses, the burn out of the machine is very prompt.

-------END--------