## Design of IIR filters from analog filters

There are several techniques to convert analog filter specfn into digital filter. If the conversion technique is to be effective, it should posses the following desirable properties

(1) The $j\Omega$ axis in s-plane should map into unit circle in z-plane. Hence there will be direct relationship between two frequency variables in two domains.

(2) The left half of s-plane should map into inside of the unit circle in z-plane. Hence stable analog filter will be converted into stable digital filter.

There are 4 most widely used methods for digitizing the analog filter into digital filter. Those are

(1) Approximation of derivative
(2) the impulse invariant transformation
(3) the bilinear transformation
(4) the matched z-transform technique

(I) Design of IIR filter using impulse invariance technique:

z-trf of infinite impulse response is given by

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$H(z)\Big|_{z=e^{sT}} = \sum_{n=0}^{\infty} h(n) e^{-sTn}$$

Real part of analog pole determines the radius of z-plane. & imaginary part of analog pole determines ~~radius~~ angle of ~~z~~-plane. digital pole.

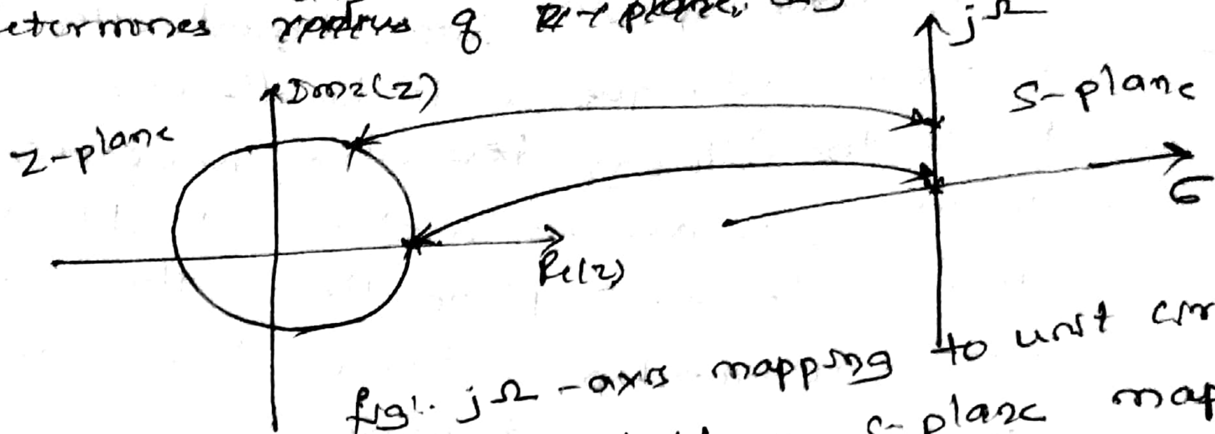
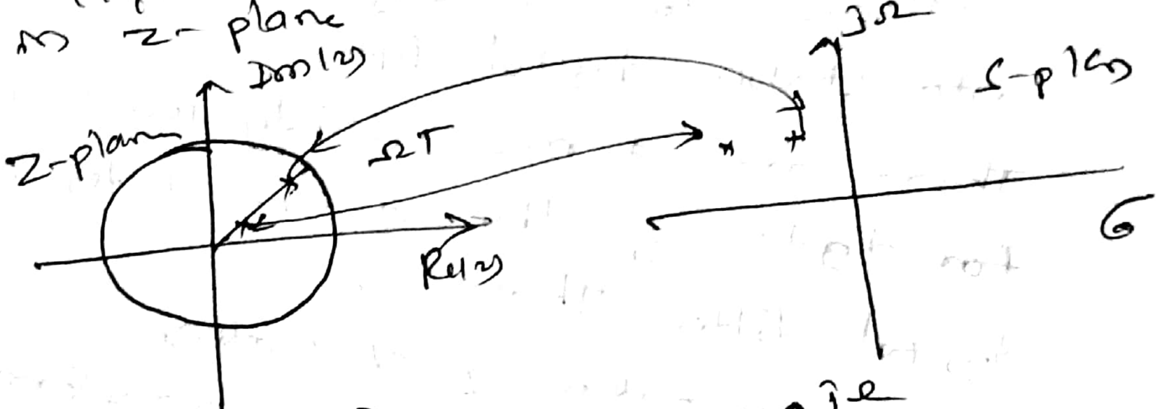
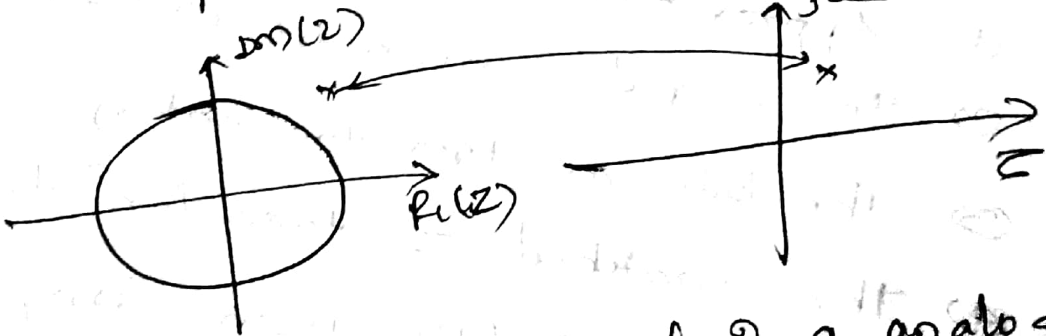



fig: jΩ-axis mapping to unit circle.

All poles in right half of s-plane map to digital poles outside the unit circle & -ve real parts maps inside the unit circle (left half) in z-plane




Let $H_a(s)$ is system fun of analog filter be Expressed as $H_a(s) = \sum_{k=1}^{N} \frac{C_k}{s - p_k}$

$p_k$ are poles of analog filter. $C_k$ are coefficients in the partial fraction Exp...

If the sampling rates are T. Then z-transform H(z) is given by

$$H(z) = \sum_{k=1}^{N} \frac{Tc_k}{1-e^{p_kT}z^{-1}}$$

But due to aliasing the impulse invariant method is appropriate for the design of lowpass & bandpass filters only. The method is unsuccessful for implementing digital filters such as high pass filters.

<u>Steps to Design Digital filter using Impulse invariant method.</u>

(1) For the given specifications find $H_a(s)$, i.e transfer fun of an analog filter

(2) Select the sampling rate of digital filter T seconds per sample

(3) Express analog filter transfer functions as the sum of single pole filters.

$$H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k}$$

(4) Compute z-transform of digital filter using the formula $H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_kT}z^{-1}}$

For high sampling rates

$$H(z) = \sum_{k=1}^{N} \frac{Tc_k}{1-e^{p_kT}z^{-1}}$$

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Ex For the analog transfer fun $H(s) = \frac{2}{(s+1)(s+2)}$ determine $H(z)$ using impulse invariant method. Assume $T = 1$ sec.

Sol:

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Using partial fraction we can write

$$H(s) = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$= \frac{A(s+2) + B(s+1)}{(s+1)(s+2)}$$

$A = 1$, $B = -2$

$$H(s) = \frac{2}{s-(-1)} - \frac{2}{s-(-2)}$$

Using impulse invariance technique we have

$$H(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k} \text{ then } H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}$$

$(s - p_k)$ is transformed to $1 - e^{p_k T} z^{-1}$

$\therefore p_1 = -1$, $p_2 = -2$.

$$H(z) = \frac{2}{1 - e^{-T} z^{-1}} - \frac{2}{1 - e^{-2T} z^{-1}}$$

For $T = 1$ sec,

$$H(z) = \frac{2}{1 - e^{-1} z^{-1}} - \frac{2}{1 - e^{-2} z^{-1}}$$

$$= \frac{2}{1 - 0.3678 z^{-1}} - \frac{2}{1 - 0.1353 z^{-1}}$$

$$= \frac{0.465 z^{-1}}{1 - 0.503 z^{-1} + 0.04976 z^{-2}}$$ //

Ex An analog filter has a transfer function $H(s) = \frac{10}{s^2+7s+10}$. Design a digital filter equivalent to this using impulse invariant method for $T = 0.2$ sec

$$H(s) = \frac{10}{s^2+7s+10} = \frac{10}{s^2+5s+2s+10}$$

$$= \frac{10}{(s+5)(s+2)} = \frac{A}{s+5} + \frac{B}{s+2}$$

$$= \frac{A(s+2)+B(s+5)}{(s+5)(s+2)}$$

$A = -3.33$, $B = 3.33$

$$H(s) = \frac{-3.33}{(s+5)} + \frac{3.33}{(s+2)}.$$

$$= \frac{-3.33}{(s-(-5))} + \frac{3.33}{(s-(-2))}$$

For high sampling rates

$$H(z) = \sum_{k=1}^{N} \frac{T c_k}{1-e^{p_k T} z^{-1}}.$$

$$= T\left[\frac{-3.33}{1-e^{-5T} z^{-1}} + \frac{3.33}{1-e^{-2T} z^{-1}}\right]$$

$$= 0.2\left[\frac{-3.33}{1-e^{-5\times0.2} z^{-1}} + \frac{3.33}{1-e^{-2\times0.2} z^{-1}}\right]$$

$$= 0.2\left[\frac{-0.666}{1-0.3678 z^{-1}} + \frac{0.666}{1-0.67 z^{-1}}\right]$$

$$= \frac{0.2012 z^{-1}}{1-1 \ldots}$$

Design a third order Butterworth digital filter using impulse invariant technique. Assume sampling period T = 1 sec

Sol$^n$ - N = 3. Transfer fun of a normalized butterworth filter is given by

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{A}{(s+1)} + \frac{B}{(s+0.5+j0.866)} + \frac{C}{(s+0.5-j0.866)}$$

$$A = (s+1) \cdot \frac{1}{(s+1)(s^2+s+1)}\Bigg|_{s=-1} = \frac{1}{\quad}$$

$$\frac{1}{(s+1)(s^2+s+1)} = \frac{A(s+0.5+j0.866)(s+0.5-j0.866) + B(s+1)(s+0.5-j0.866) + C(s+1)(s+0.5+j0.866)}{(s+1)(s^2+s+1)}$$

$$1 = A(s+0.5+j0.866)(s+0.5-j0.866)$$

$$A = \frac{1}{(s+0.5+j0.866)(s+0.5-j0.866)}\Bigg|_{s=-1}$$

$$= \frac{1}{(-1+0.5+j0.866)(-1+0.5-j0.866)}$$

$$= \frac{1}{(-0.5+j0.866)(-0.5-j0.866)}\Bigg|_{s=-1}$$

$$= \frac{1}{(-0.5)^2-(j0.866)^2} = \frac{1}{0.25+0.7}$$

$$\frac{1}{(\;)(\;)} = \frac{B(s+1)(s+0.5-j0.866)}{(\;)(\;)}$$

$$B = \frac{1}{(s+1)(s+0.5-j0.866)}\Bigg|_{s=-0.5-j0.866}$$

$$= \frac{1}{(-0.5-j0.866+1)(-0.5-j0.866+0.5-j0.866)} = \frac{1}{(0.5-j0.866)(-1.732j)}$$

$$= \frac{1}{-0.866j-1.499}$$

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Design of IIR Filter by using Bilinear transformation.

The bilinear transformation is a mapping technique that transform's the $j\Omega$ axis into the unit circle in z-plane. All points in LHP of s-plane are mapped inside the unit circle in z-plane & all points in RHP of s-plane are mapped outside the unit circle in z-plane.

Consider the analog filter with system function

$$H(s) = \frac{b}{s+a} \quad \text{--- (1)}$$

which can be written as

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

$$sY(s) + aY(s) = bX(s) \quad \text{--- (2)}$$

This can be characterized by differential Eq$^n$

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

The z- transform of above differential Eq$^n$ is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{bT}{2}(1+z^{-1})}{1+\frac{aT}{2} - \left(1-\frac{aT}{2}\right)z^{-1}}$$

Dividing Nr & Dr by $\frac{T}{2}(1+z^{-1})$ we get

$$H(z) = \frac{b}{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+a} \quad \text{--- (3)}$$

By comparing Eq (1) & (3), the mapping from s plane to z- plane can be obtained as

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \quad \text{--- (4)}$$

$$\Omega = \frac{2}{T}\frac{\sin\omega}{(1+\cos\omega)} = \frac{2}{T}\frac{2\sin\frac{\omega}{2}\cdot\cos\frac{\omega}{2}}{2\cos^2\frac{\omega}{2}},$$

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2} \quad \text{--- (5)}$$

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$$\omega = 2\tan^{-1}\frac{\Omega T}{2} \quad \text{—} (6)$$

**Warping Effect**

Let $\Omega$ & $\omega$ represents the frequency variable in analog filter & desired digital filter respectively.

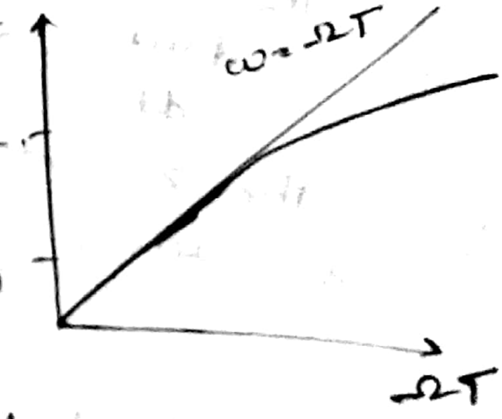
$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

For small value of $\omega$

$$\Omega = \frac{2}{T}\cdot\frac{\omega}{2} = \frac{\omega}{T}$$ $\because$ for small value of $\theta$ $\tan\theta = \theta$.

$$\omega = \Omega T$$

For low frequencies the relationship between $\Omega$ & $\omega$ are linear. Hence digital filter have the same amplitude response of analog filter. But for high frequencies the relation between $\omega$ & $\Omega$ becomes non linear which is shown in figure.




The distortion introduced by ... in frequency scale of digital filter to that of analog filter, is known as warping effect.

The warping effect can be eliminated by prewarping the analog filter. This can be done by finding prewarping analog frequency using the formula.

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

$\therefore$

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2}.$$

Steps to design filter using Bilinear transform technique

1) From the given specification, find prewarping analog frequencies using the formula

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2},$$

2) Using the analog frequencies find H(s) of analog filter.

3) Select the sampling rate of digital filter, i.e. T seconds per sample

4) Substitute $s = \frac{2}{T} \frac{(1-z^{-1})}{(1+z^{-1})}$ into the transfer function found in step 2.

Ex Apply bilinear transformation $H(s) = \frac{2}{(s+1)(s+2)}$ with T = 1 sec find H(z)

Sol:

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Substitute $s = \frac{2}{T} \frac{(1-z^{-1})}{(1+z^{-1})}$ in H(s) to get H(z).

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{2}{(s+1)(s+2)}\Big|_{s=\frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}}$$

$$= \frac{2}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right\}\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right\}}$$

$$= \frac{2(1+z^{-1})^2}{(3-z^{-1})\cdot 4} = \frac{(1+z^{-1})^2}{(6-2z^{-1})}$$

$$= \frac{0.166(1+z^{-1})^2}{(1-0.33z^{-1})}$$

Ex Determine $H(z)$ that result when the bilinear transformation is applied to $H_a(s)$.

$$H_a(s) = \frac{s^2+4.525}{s^2+0.692s+0.504}$$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$ Assume T = 1 sec

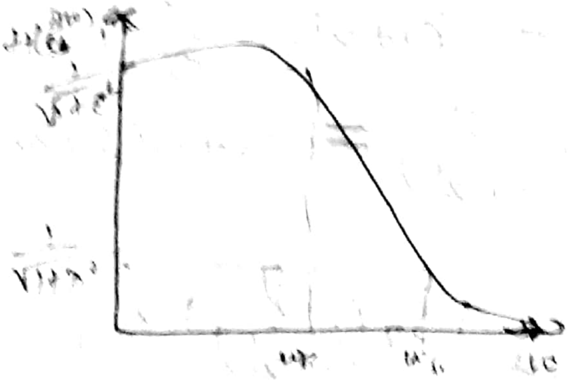
$$H(z) = \frac{\left(\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2+4.525}{\left[\left(\frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}\right)^2+0.692\left(\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)+0.504\right]}$$

$$= \frac{1.4479+0.1783z^{-1}+1.4479z^{-2}}{1-1.1875z^{-1}+0.529z^{-2}}$$

Ex Design a Butterworth filter using the bilinear transform for following specification.

$$0.8 \le |H(e^{j\omega})| \le 1, \quad 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2, \quad 0.6\pi \le \omega \le \pi.$$




Digital LPF.

$\omega_p = 0.2\pi$, $\omega_s = 0.6\pi$

Assume T = 1 sec

Prewarping frequencies are

$$\Omega_s = 2\tan\frac{\omega_s}{2}$$

$$= 2\tan\frac{0.6\pi}{2} \approx 2.752$$

$$\Omega_p = 2\tan\frac{\omega_p}{2}$$

$$= 2\tan\frac{0.2\pi}{2} = 0.6498$$

$$k = \frac{\Omega_p}{\Omega_s} = \frac{0.6498}{2.752} = 0.236$$

$$\frac{1}{\sqrt{1+\epsilon^2}} = 0.8, \quad \epsilon = 0.75, \quad \frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\lambda = 4.899$$

$$N = \frac{\log \lambda/\varepsilon}{\log 1/k} = \frac{\log 4.899/0.75}{\log 1/0.236} = 1.3 \leq 2.$$

$H(s)$ for $N=2$, Butterworth filter is

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}$$

$$\Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}} = \frac{\Omega_p}{\left(\sqrt{10^{0.1\alpha_p}-1}\right)^{1/N}}$$

$$\Omega_c = \frac{0.6498}{(0.75)^{1/2}} = 0.75 \qquad \Omega_c \neq 1$$

$$H_a(s) = \frac{1}{s^2+\sqrt{2}s+1}\Bigg|_{s \to s/0.75}$$

$$= \frac{(0.75)^2}{s^2+1.065+0.5625}$$

For Bilinear transformation

$$H(z) = H(s)\Bigg|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} \qquad (T = 1\,sec)$$

$$= \frac{(0.75)^2}{\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2+\sqrt{2}\times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.5625}$$

$$= \frac{0.5625\,(1+z^{-1})^2}{4(1-z^{-1})^2+2.12\,(1-z^{-2})+0.5625\,(1+z^{-1})^2}$$

$$= \frac{0.5625\,(1+z^{-1})^2}{6.6825-6.875z^{-1}+2.44z^{-2}}$$

$$= \frac{0.084\,(1+z^{-1})^2}{1-1.0288z^{-1}+0.3651z^{-2}}$$

Eg Find Pole & Zero locations of an analog Chebyshev digital filter type-II filter for the following specifications. Use Bilinear transformation

$$\frac{-1}{\alpha_p} \le |H(e^{j\omega})|_{dB} \le 0, \quad 0 \le |\omega| \le 0.2\pi$$

$$|H(e^{j\omega})|_{dB} \le \frac{-20}{\alpha_s}, \quad |\omega| \ge 0.3\pi.$$

The Prewarped analog frequencies are given by

$$\frac{\Omega_p T}{2} = \tan\frac{\omega_p T}{2} = \tan\frac{0.2\pi}{2} = 0.32492$$

$$N = \frac{\cosh^{-1}\sqrt{10^{0.1\alpha_s}-1 / 10^{0.1\alpha_p}-1}}{\cosh^{-1}\Omega_s/\Omega_p}$$

$$\frac{1}{\sqrt{1+\epsilon^2}}$$

$$= \frac{\cosh^{-1}\sqrt{10^{0.1\times 20}-1 / 10^{0.1\times 1}-1}}{\cosh^{-1}\frac{0.50953}{0.32492}} = \frac{3.66}{1.021} = 3.59 \simeq 4$$

$N = 4$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}, \quad k = 1,2,3,4$$

$\phi_1 = 112.5°$, $\phi_2 = 157.5°$, $\phi_3 = 202.5°$, $\phi_4 = 247.5°$

$$S_k = \frac{j\Omega_s}{\sin\phi_k}, \quad k = 1,2,3,4$$

$$S_1 = \frac{j \times 0.50953}{\sin 112.5°} = j\,0.55151$$

$$S_2 = \frac{j\,0.50953}{\sin 157.5°} = j\,1.3314$$

$$S_3 = \frac{j\,0.50953}{\sin 202.5°} = -j\,1.3314$$

$$S_4 = \frac{j\,0.50953}{\sin 247.5°} = -j\,0.55151$$

$\mu = \lambda + \sqrt{1+\lambda^2}$

$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = \sqrt{99}$

$\mu = 9.9498 + 10 = 19.95$

$a = \Omega_p \left[\frac{\mu^{1/p} - \mu^{-1/p}}{2}\right]$

$a = 0.2664$

$b = \Omega_p \left[\frac{\mu^{1/p} + \mu^{-1/p}}{2}\right] = 0.4202$

$\sigma_1 = a\cos\phi_1 = -0.102$ $\quad \Omega_1 = b\sin\phi_1 = 0.3882$

$\sigma_2 = a\cos\phi_2 = -0.2461$ $\quad \Omega_2 = b\sin\phi_2 = 0.1608$

$\sigma_3 = a\cos\phi_3 = -0.2481$ $\quad \Omega_3 = b\sin\phi_3 = -0.1608$

$\sigma_4 = a\cos\phi_4 = -0.102$ $\quad \Omega_4 = b\sin\phi_4 = -0.3882$

$x_k = \frac{\Omega_s \sigma_k}{\sigma_k^2 + \Omega_k^2}$ $\quad y_k = \frac{\Omega_s \Omega_k}{\sigma_k^2 + \Omega_k^2}$, $k = 1, 2, 3, 4$

$x_1 = -0.3226$ $\quad y_1 = -1.22775$

$x_2 = -1.45$ $\quad y_2 = -0.948$

$x_3 = -1.45$ $\quad y_3 = 0.948$

$x_4 = -0.3226$ $\quad y_4 = 1.2275$

Zeros are located at $s_k$, ie $\pm 0.5515$, $\pm j1.33$

Poles are located at $\pm j1.2275$, $\& -1.45 \pm j0$

Realize the system with difference Eqn

$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$

in cascade form

$$H(z) = \frac{Y(z)}{X(z)}$$

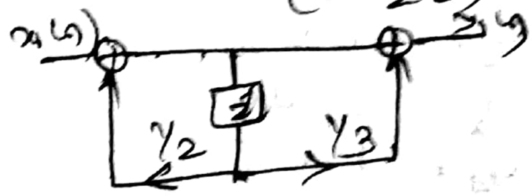
$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{3}x(n-1)$$

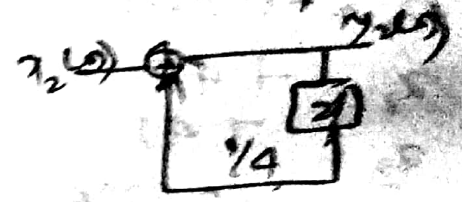
$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z) + \frac{1}{3}z^{-1}X(z)$$

$$Y(z)\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right) = X(z)\left(1 + \frac{1}{3}z^{-1}\right)$$

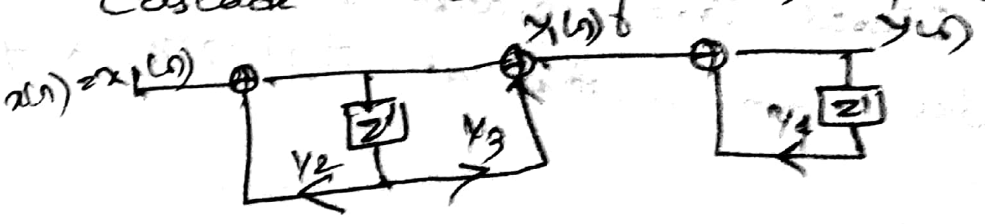
$$\frac{Y(z)}{X(z)} = \frac{(1 + \frac{1}{3}z^{-1})}{(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2})} = \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})} = H_1(z)\cdot H_2(z)$$

$$H_1(z) = \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{1}{2}z^{-1})}, \quad H_2(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}$$




Cascade Realzn of $H_1(z)$ & $H_2(z)$ is



Realize the system given by difference Eqn

$y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7x(n) - 0.252\, x(n-2)$ in parallel form

Sol:

$y(n) + 0.1\, y(n-1) - 0.72\, y(n-2) = 0.7x(n) - 0.252\, x(n-2)$

$Y(z) + 0.1 z^{-1} Y(z) - 0.72 z^{-2} Y(z) = 0.7X(z) - 0.252 z^{-2} X(z)$

$Y(z)(1 + 0.1 z^{-1} - 0.72 z^{-2}) = X(z)(0.7 - 0.252 z^{-2})$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

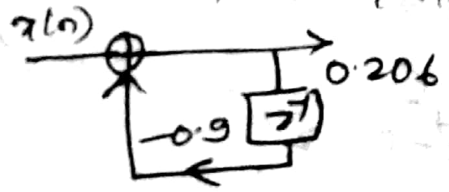
Above Eqn must be converted in the form $H(z) = c + \frac{C_1}{1 - P_1 z^{-1}} + \frac{C_2}{1 - P_2 z^{-1}} + \cdots$

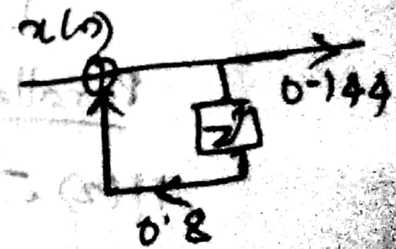
Divide Nr by Dr

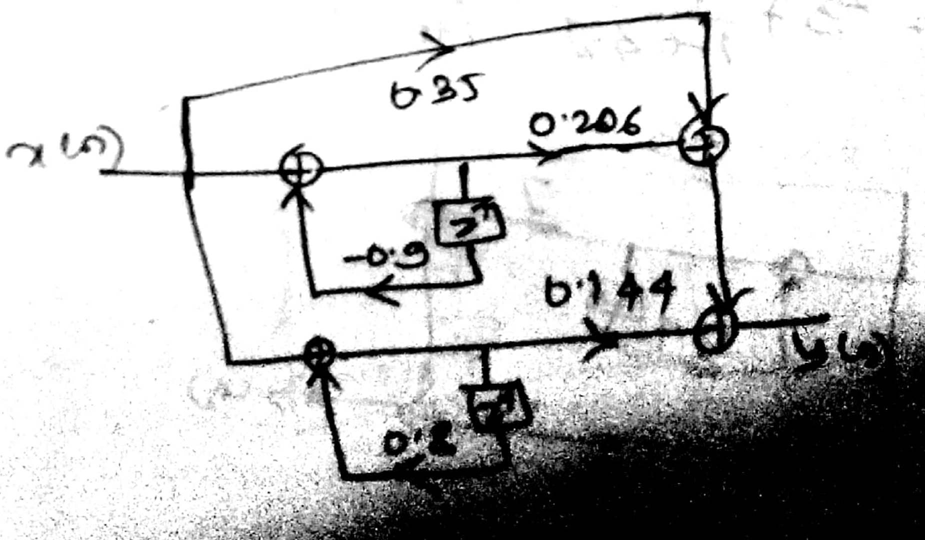
$$H(z) = 0.35 + \frac{0.35 - 0.035 z^{-1}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

$$= 0.35 + \frac{0.206}{1 + 0.9 z^{-1}} + \frac{0.144}{1 - 0.8 z^{-1}}$$

$H(z) = c + H_1(z) + H_2(z)$

$H_1(z) =$ 


$H_2(z)$ 

Obtain the DF-I & DF-II, Cascade & Parallel form realiz^n for the system

$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

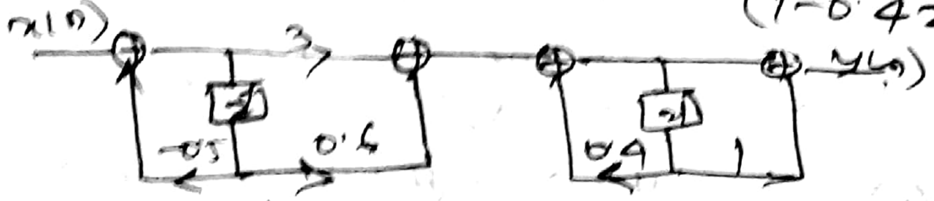
$Y(z) + 0.1z^{-1}Y(z) - 0.2z^{-2}Y(z) = 3X(z) + 3.6z^{-1}X(z) + 0.6z^{-2}X(z)$

$Y(z)(1 + 0.1z^{-1} - 0.2z^{-2}) = X(z)(3 + 3.6z^{-1} + 0.6z^{-2})$

$$\frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= \frac{(3 + 0.6z^{-1})(1 + z^{-1})}{(1 + 0.5z^{-1})(1 - 0.4z^{-1})}$$

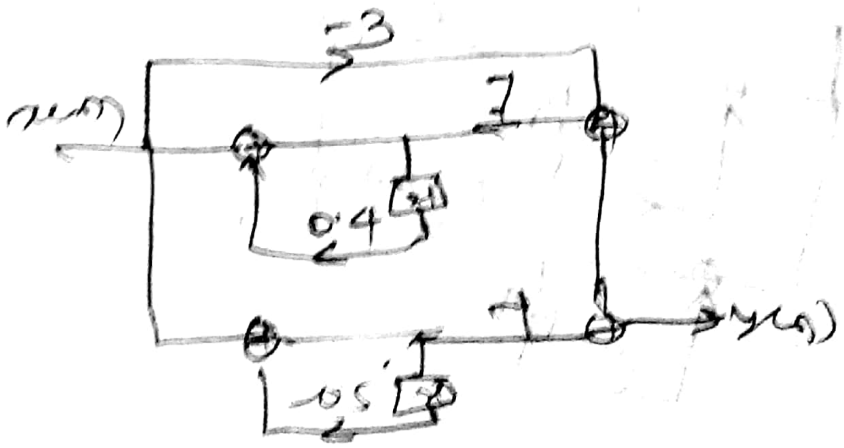
$$H_1(z) = \frac{(3 + 0.6z^{-1})}{(1 + 0.5z^{-1})}, \quad H_2(z) = \frac{(1 + z^{-1})}{(1 - 0.4z^{-1})}$$



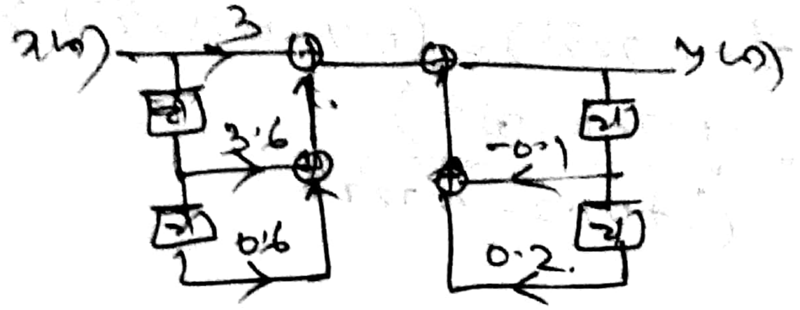

Parallel form

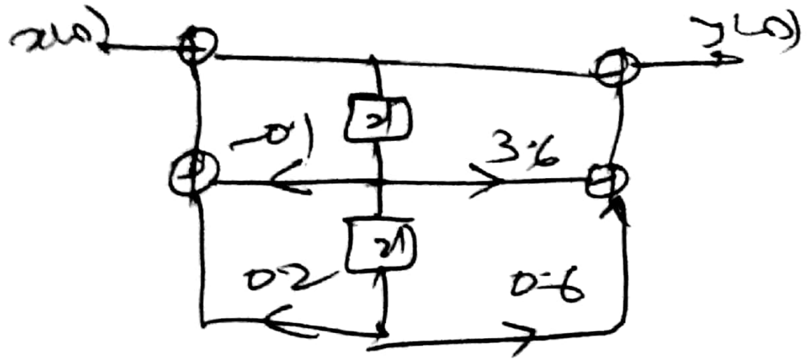
$$H(z) = -3 + \frac{3.3z^{-1} + 6}{1 + 0.1z^{-1} - 0.2z^{-2}} = -3 + \frac{A}{(1 - 0.4z^{-1})} + \frac{B}{(1 + 0.5z^{-1})}$$

$$= -3 + \frac{7}{1 - 0.4z^{-1}} - \frac{1}{1 + 0.5z^{-1}}$$



$y(n) = -0.1\, y(n-1) + 0.2\, y(n-2) + 3x(n) + 3.6x(n-1)$
$+ 0.6x(n-2)$



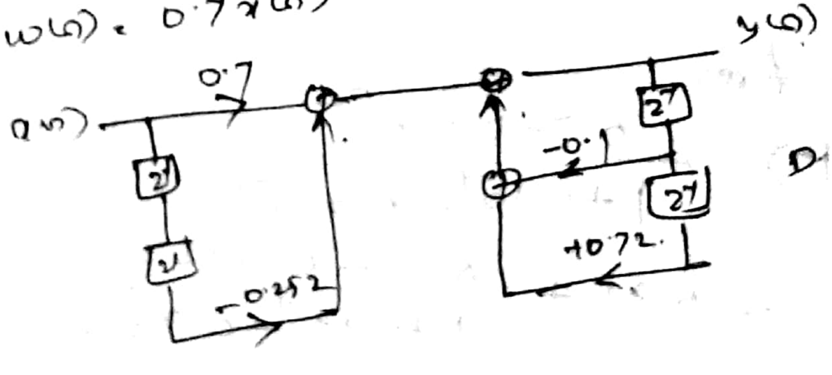




Realize the s/m by difference eqn
$y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7x(n) - 0.252x(n-2)$
in parallel form

$y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + w(n)$

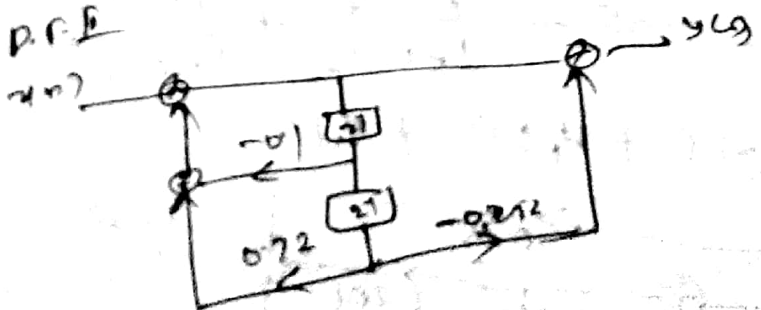
$y(n) = -0.1\, y(n-1) - 0.252x(n-2)$

$w(n) = 0.7x(n) - 0.252x(n-2)$



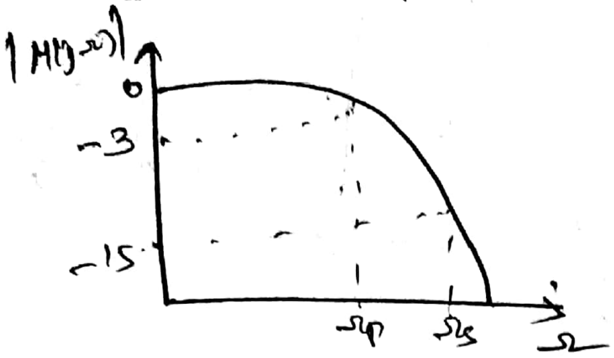

D.F.I

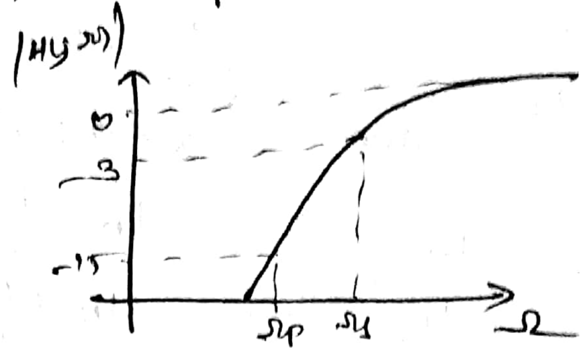
D.F.II



Ex for the given specs$^n$ $\alpha_p = 3dB$, $\alpha_s = 15dB$, $\Omega_p = 1000$ rad/sec

$\Omega_s = 500$ rad/sec Design High pass filter.




$\Omega_c = \Omega_p = 500$ rad/sec $\quad \alpha_p = 3dB$

$\Omega_s = 1000$ rad/sec $\quad \alpha_s = 15dB$

$$N = \cancel{\frac{\log \lambda/}{\log \epsilon}} \quad \frac{\log \lambda/\epsilon}{\log 1/k}$$

$$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = \lambda = \sqrt{10^{0.1 \times 15} - 1} = 5.533$$

$$\epsilon = \sqrt{10^{0.1\alpha_p} - 1} = \epsilon = \sqrt{10^{0.1 \times 3} - 1} = 1$$

$$k = \Omega_p / \Omega_s = 0.5 = \frac{500}{1000} = 0.5$$

$$N = \frac{\log 5.333}{\log 0.5} = 2.466 \cong 3.$$

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$\boxed{\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{500}{(10^{0.1 \times 3} - 1)^{1/2}}}$$

Assume $\Omega_c = 1000$ rad/sec

$s \to 1000/s$

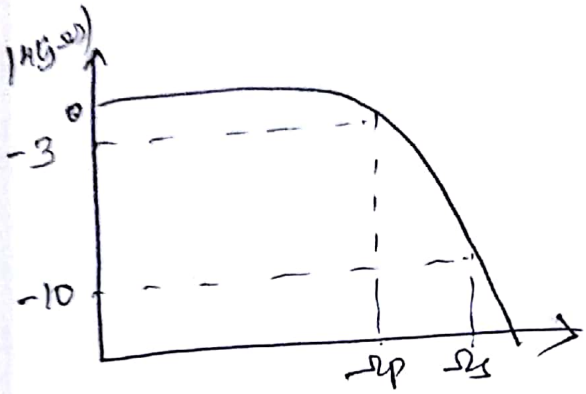
$$H_a(s) = H(s) \Big|_{s \to 1000/s}$$

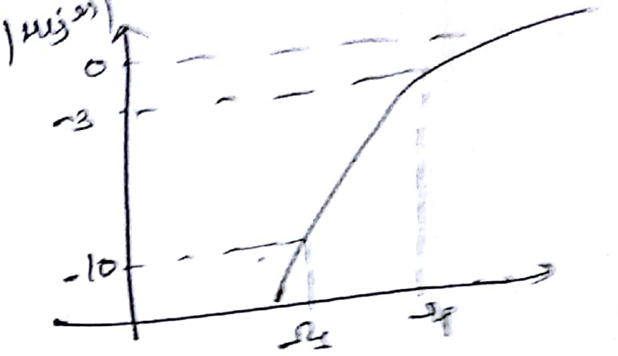
$$H_a(s) = \frac{1}{(s+1)(s^2+s+1)}\Bigg|_{s \to 1000/s}$$

$$\frac{s^3}{(s+1000)(s^2+1000s+1000^2)}$$

Q. Using Bilinear transform, design a high pass filter 1dB passband with cutoff frequency of 1000 Hz at 3dB & down 10dB at 350Hz. The sampling frequency is 5000Hz.

Soln. In HP filter $\omega_c = \omega_p = 2\pi f_p = 2\pi \times 1000 = 2000\pi$ rad/sec




$\omega_c = \omega_p = 2 \times \pi \times 1000 = 2000\pi$ rad/sec

$\alpha_p = 3$ dB, $\alpha_s = 10$dB , $\omega_s = 2\times\pi\times 350 = 700\pi$ rad/sec

$$T = \frac{1}{f} = \frac{1}{5000} = 2\times 10^{-4} \text{ sec}$$

The characteristics are monotonic in both passband & stopband. ∴ filter is Butterworth filter

Prewarping the digital frequencies we have

$$\Omega p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2\times 10^{-4}} \tan\left(\frac{2000\pi \times 2\times 10^{-4}}{2}\right) = 7265 \text{ rad/sec}$$

$$\Omega s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times 10^{-4}} \tan\left(\frac{700\pi \times 2\times 10^{-4}}{2}\right) = 2235 \text{ rad/sec}$$

First design LPF & then transform into HPF.

$$N = \frac{\log\sqrt{10^{0.1\alpha_s}-1 / 10^{0.1\alpha_p}-1}}{\log \Omega s/\Omega p} = \frac{\log\sqrt{10^{0.1\times 10}-1 / 10^{0.1\times 3}-1}}{\log \frac{7265}{2235}} = \frac{\log 3}{\log 3.25}$$

$= 0.932 \approx 1$

$H(s) = \frac{1}{1+s}$ for $\Omega_c = 1$ rad/sec

$\Omega_c = \Omega p = 7265$ rad/sec

$$s \to \frac{\Omega c}{s} = \frac{7265}{s}$$

Tat fn of HPF $H(s) = \frac{1}{s+1}\Big|_{s = \frac{7265}{s}} = \frac{s}{s+7265}$

Dn ~~getting~~ $H(z) = H(s)\Big|_{s = \frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}} = \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265}$ = [illegible]

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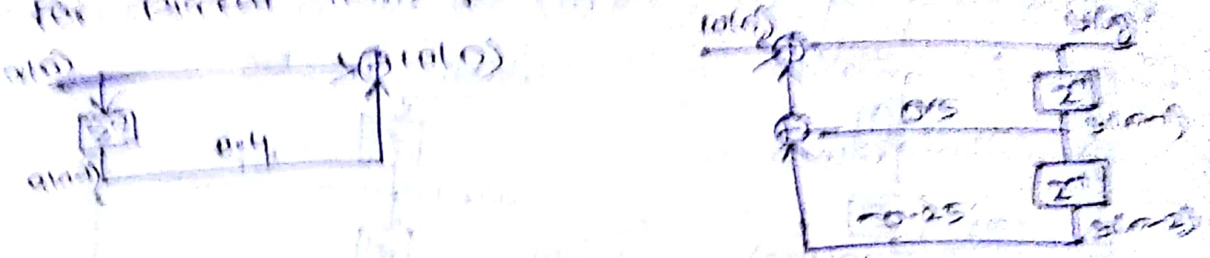
Obtain Direct form I realization for the system described by difference eqn $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

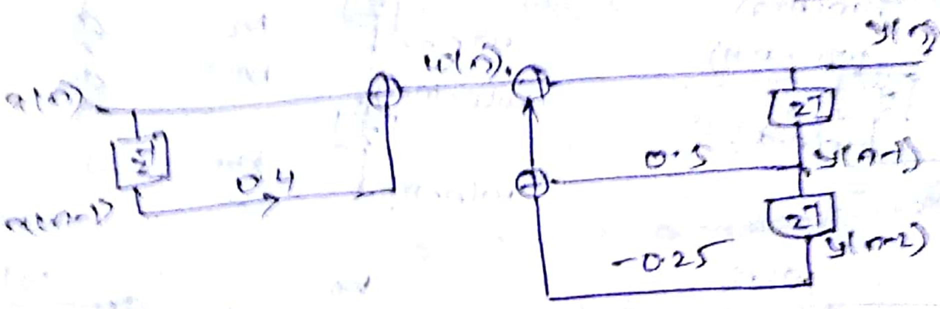
Soln

$$w(n) = x(n) + 0.4x(n-1) \longleftarrow (1)$$

$$y(n) = 0.5y(n-1) - 0.25y(n-2) + w(n) \longleftarrow (2)$$

For direct form I realization consider Eq (1) & (2)




<u>Direct - form - II realization</u>

Consider the difference Eqn of the form

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \longrightarrow (1)$$

Eqn can be exprd ...

System fcn of above difference

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} \longleftarrow (2)$$

Multiply & divide by Eq (2) by $W(z)$

$$\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)} \qquad \text{Here } \frac{W(z)}{X(z)} = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+a_1z^{-1}+a_2z^{-2}+\dots\dots a_N z^{-N}}$$

$$W(z) + W(z)a_1z^{-1} + W(z)a_2z^{-2} + \dots\dots W(z)a_Nz^{-N} = X(z)$$

$$(W(z)(1+a_1z^{-1}+a_2z^{-2}+\dots\dots a_Nz^{-N}) = X(z))$$

$$X(z) - W(z) - W(z)a_1z^{-1} + W(z)a_2z^{-2} + \dots\dots W(z)a_Nz^{-N}$$

$$X(z) - a_1z^{-1}W(z) - a_2z^{-2}W(z) - \dots\dots a_Nz^{-N}W(z) = W(z) \longrightarrow (3)$$

then $\frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k} = b_0z^0 + b_1z^{-1} + b_2z^{-2} + \dots\dots b_mz^{-m}$

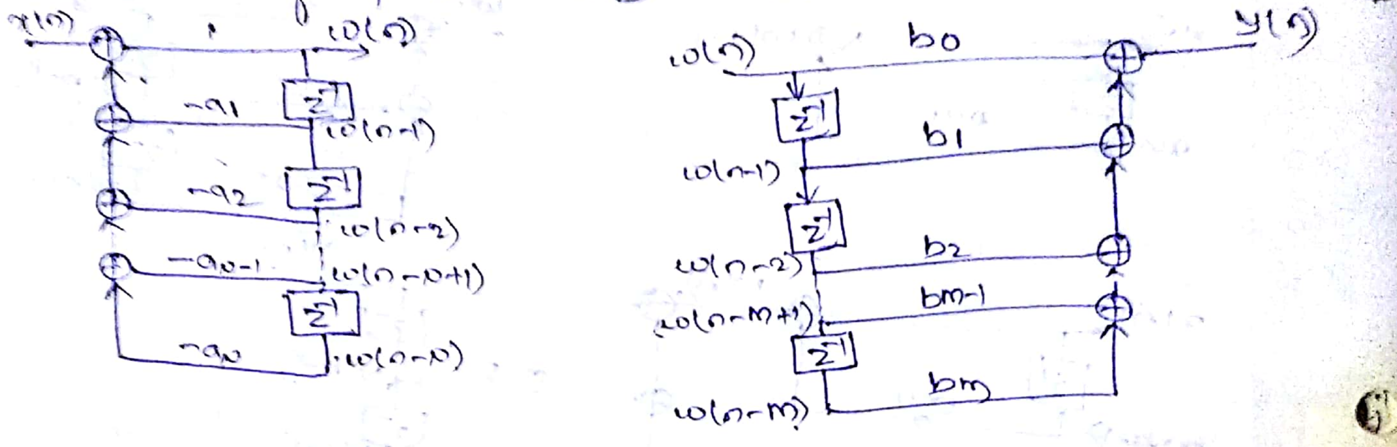
$$Y(z) = b_0W(z) + b_1z^{-1}W(z) + b_2z^{-2}W(z) + \dots\dots b_mz^{-m}W(z) \longrightarrow (4)$$

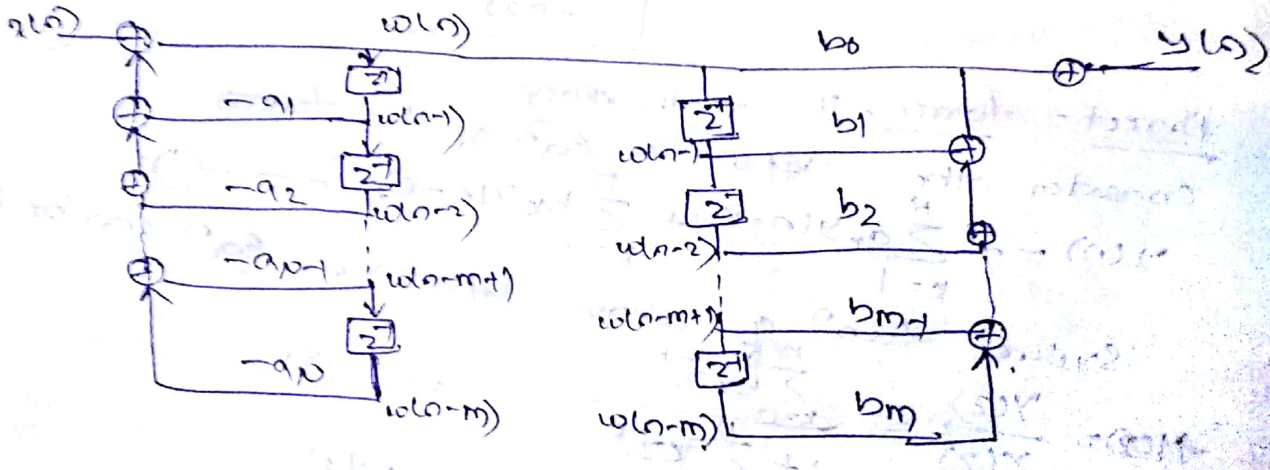
Now Eq (3) & (4) can be Expressed in difference ...

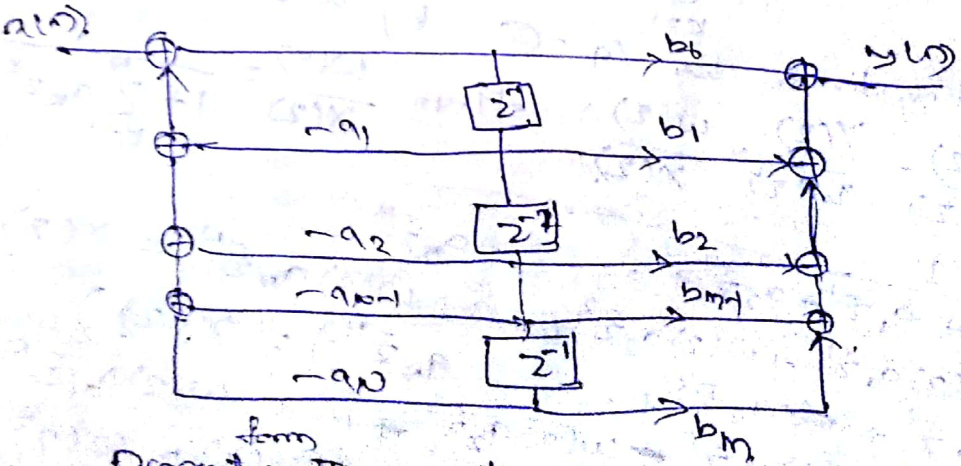
$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) - \dots - a_N w(n-N) \quad \text{——} (5)$$

$$\rightarrow y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \dots + b_M w(n-M) \quad \text{——} (6)$$

From eq (5) & (6) we observed that some delay terms $w(n-1)$, $w(n-2)$ are used to express $w(n)$ & $y(n)$.
Realization of eq (5) & (6) are given in figs (1) & (2) resp.




Direct form-II realization

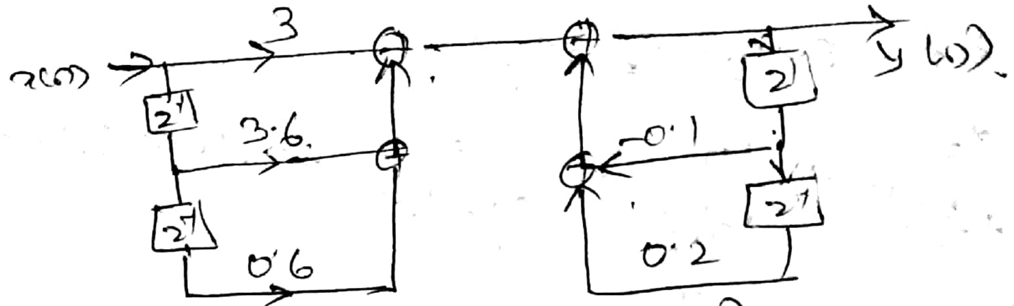
By observing the direct form-II realization it reduces the no of m/m locations. Hence it is said to be canonic.

Eg: Obtain direct form-I, direct form-II, cascade & Parallel form realization for following system

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

Sol:

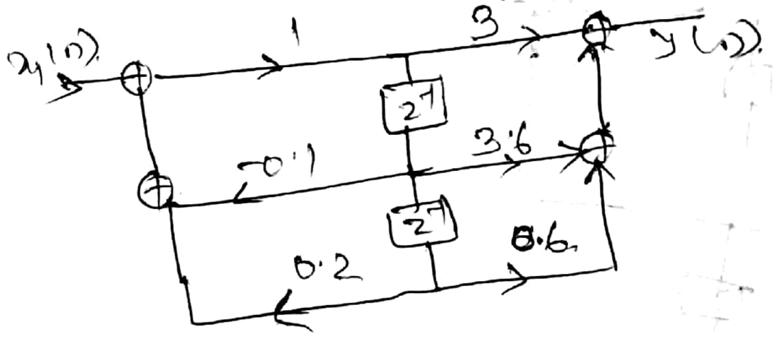
$$y(n) = -0.1y(n-1) + 0.2y(n-2) + w(n)$$




Direct form-I realization

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

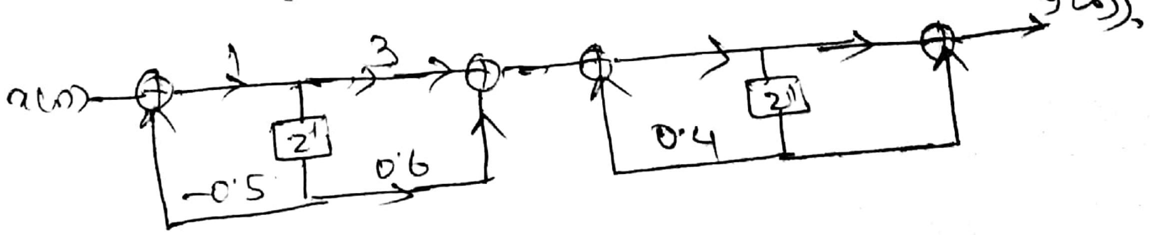
$-0.2z^{-2}+0.1z^{-1}+1$




Cascade form

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(3+0.6z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

$$H_1(z) = \frac{(3+0.6z^{-1})}{(1+0.5z^{-1})} \quad \& \quad H_2(z) = \frac{(1+z^{-1})}{(1-0.4z^{-1})}$$




Cascade form

Parallel form.

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$\begin{array}{r} -3 \\ -0.2z^{-2}+0.1z^{-1}+1 \,\big)\, 0.6z^{-2}+3.6z^{-1}+3 \\ 0.6z^{-2}-0.3z^{-1}-3 \\ \hline 3.9z^{-1}+6 \end{array}$$

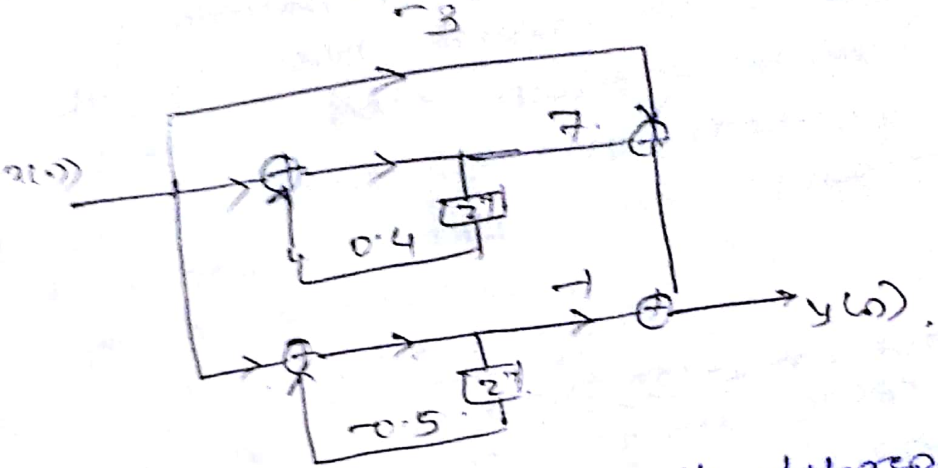
$$H(z) = -3 + \frac{3.9z^{-1}+6}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= -3 + \frac{A}{1-0.4z^{-1}} + \frac{B}{1+0.5z^{-1}}$$

$A = 7,\ B = -1$

$$= -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}$$

$$= C + H_1(z) + H_2(z)$$




Realize the system with difference Eqn $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ in cascade f

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$H_1(z) = \frac{(1+\frac{1}{3}z^{-1})}{(1-\frac{1}{2}z^{-1})}, \quad H_2(z) = \frac{1}{(1-\frac{1}{4}z^{-1})}$$