Signal Processing

It is an operation that changes characteristics of a signal. It deals with modification of signal to that result is in more desirable form. The main objective is to extract information.

Signals processing is concerned with representing signals in mathematical terms and extracting the information by carrying out algorithmic operations on the signal.

Basically there are two types of signal processing systems.
(1) Analog signal processing
(2) Digital signal processing.

Analog Signal Processing

In science & Engineering field most of the signals are analog in nature. These signals are signals are functions of a continuous variable such as time or space. To process such signals devices like amplifiers, filter, frequency analyzers are required. The analog signal

The system that processes the signal by to extract information present in the signal processing analog system is known as analog signal processing system.

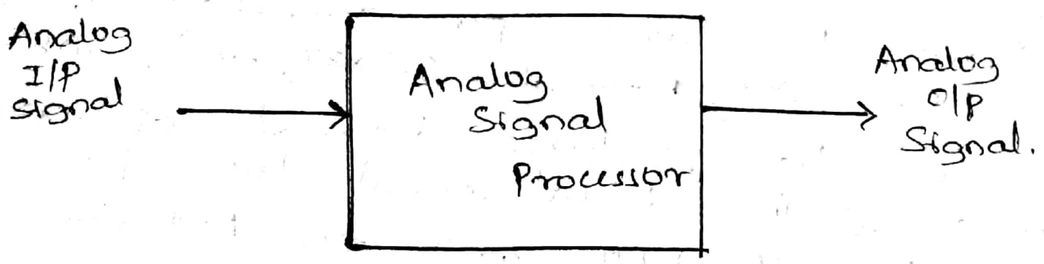

fig. Analog Signal Processing system.

Analog signals are continuous in both time and amplitude. In Real world current, voltage, pressure, temperature, light intensity are analog signals

(2) <u>Digital Signal Processing</u>

It deals with mathematical operations on digital signals

Defn:- "Working with digital signals to modify them to transmit them, Extract information, to receive or to process in any other ways signals or manipulate them is known as digital signal processing."

<u>Block diagram Representation of DSP.</u>

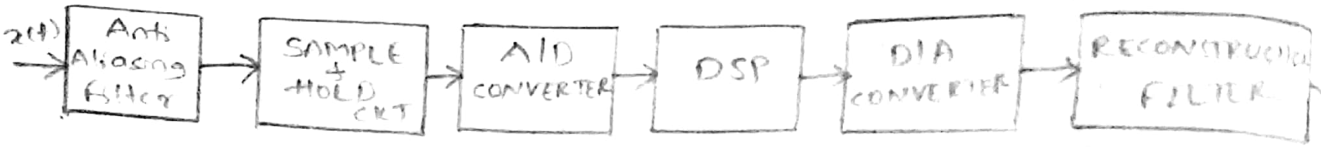

fig: Digital Signal Processing system.

x(t) = Analog i/p Signal y(t) = Analog processed o/p

Here i/p x(t) is from transducer (may be...). This i/p signal is applied to antialiasing filter. This is low pass filter used to remove high frequency component. & also to band limits the signal. This is also helpful to select sampling frequency (fs).

Sample and Hold circuit keeps i/p signal constant during conversion of analog signal to digital form.

Depends on analog i/p (0 to +10V for unipolar or -5V to +5V if bipolar), ADC gives the N-bit binary number. This converted signal can be processed using digital techniques.

DSP may be large programmable digital computer or µP (Intel 80XX, Motorola 68XXX, etc). which is programmed to perform desired operations on the i/p signal. The digital signal from processor is applied to i/p of DAC. The o/p of DAC continuous but not smooth. To eliminate these high frequency components the o/p DAC is connected to Reconstruction filter. Hence this filter produces the smooth continuous Analog signal.

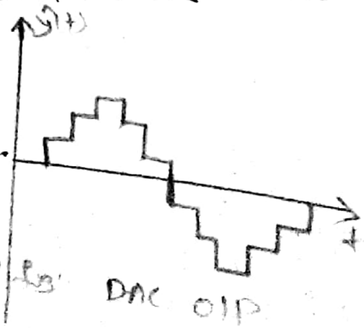

fig: DAC o/p

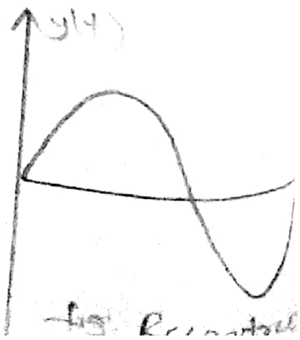

fig: Reconstructed

Advantages of DSP over Analog processing.

(1) Computers can be made accurate to required degree, by choosing their word length according to desired accuracy.

(2) Sensitivity of digital computer to electrical noise is quite low.

(3) Speed of computations can be enhanced by using advances in technology — Greater CPU and memory speed and parallel computing.

(4) Digital storage is less expensive and flexible.

(5) Change in processing functions can be easily made through changes in programming.

(6) Digital information can be enhanced to have security and privacy through special coding techniques.

(7) Very low frequency (VLF) signals can be easily processed.

(8) Digital Realization is cheaper.

(9) Non Linear and time varying operations can be accomplished via programming.

Limitations

(1) System complexity increase because of A/D, D/A and their associated filters.

(2) In processing RF signals (Radio frequency) DSP cannot meet the speed requirements. Signals having extremely wide bandwidths requires fast sampling rate A/D converters ($f_s \geq 2f_m$). But there is a practical limitation in the speed of operation of A/D converters & DS processors.

(3) Software development and testing costs are very high and DSP chip contains more 4 lakhs transistors dissipates more power (1 watt)

(PTO)

Areas of Applications of DSP

1) communication:- Detection, filtering, encoding & decoding, like telephone dialing appl^n, Modems, Data compression, video conferencing, Cellular phone, FAX etc.

(2) Image processing:- compression, enhancement, analysis and recognition.

(3) Speech processing:- Noise filtering, coding, auto..., speech recognition, speaker verification and identification, In speech synthesis technique (conversion of text into speech).

(4) Instrumentation:- Digital filters, Robot control, pro...

(5) Medical:- In medical diagnostic instruments like, computerised Tomography (CT), X-ray scanning, spectrum analysis of ECG & EEG signals to detect disorders in heart & brain. In patient monitoring.

(6) Military:- Radar signal processing, Navigation, secure information etc.

(7) Consumer electronics:- Digital audio/TV, Education toys, FM stereo applications, sound recording app.

(8) Detection of Underground nuclear Explosion, and Earthquake monitoring.

(9) Anti Aliasing means removing signal components that have a higher frequency than is able to be properly resolved by recording (sampling device). Anti Aliasing is technique that minimizes distortion representing high resolution signal at a lower res. Used in digital photography, computer graphics, digital audio etc //

Frequency Domain Sampling

Today many application's demand the processing of signals in frequency domain. For example frequency content, periodicity, energy and power spectrum etc. can be better analyzed in frequency domain. Hence by using fourier transform (FT) and discrete fourier transform (DFT) signals are transformed from time domain to frequency domain. When required analysis and processing is performed in frequency domain then signals are transformed back to time domain by Inverse discrete fourier transform (IDFT).

Discrete ~~time~~ Fourier Series

Let $x_p(n)$ is periodic signal with fundamental period N. over the interval ($-\pi$ to π) or $(0, 2\pi)$ which is unique over the components separated by $\frac{2\pi}{N}$ radians & consists of N frequency components. $x_p(N)$ is weighted sum of complex exponentials. for all N.

$$x_p(n) = x_p(n+lN)$$

These periodic complex exponentials are of the form.

$$e^{j2\pi kn/N} = e^{j2\pi k(n+lN)/N} \quad \text{where } l \text{ is an integer.}$$

According to periodicity property of DTFT there are finite no of harmonic's with frequency $\frac{2\pi}{N}k$, $(k=0,1,2\ldots N-1)$

$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_p(k)\, e^{j2\pi nk/N}, \quad n=0,\pm 1,\ldots \qquad (1)$$

∴ Periodic sequence where $\{X_p(k), k=0,\pm 1\ldots\}$ are called discrete fouriers Series coefficients.

To obtain fourier coefficient we multiply both sides of Eq (1) by $e^{-j(2\pi/N)mn}$ & summing the product from $n=0$ to $n=N-1$ then

$$\sum_{n=0}^{N-1} x_p(n)\, e^{-j(2\pi/N)mn} = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{n=0}^{N-1} X_p(k)\, e^{j2\pi nk/N} \cdot e^{-j(2\pi/N)mn}$$

$$= \cancel{\frac{1}{N}\sum_{k=0}^{N-1} X_p(k)}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} X_p(k)\, e^{j2\pi/N(k-m)n} \qquad (2)$$

(P.T.O)

Interchanging the order of summation in RHS of

$$\sum_{n=0}^{N-1} x_p(n)\, e^{-j(2\pi/N)mn} = \frac{1}{N}\sum_{k=0}^{N-1} X_p(k) \sum_{n=0}^{N-1} e^{j(2\pi/N)(k-m)n} \quad \text{--- (3)}$$

But $$\sum_{n=0}^{N-1} e^{j(2\pi/N)(k-m)n} = N \quad \text{if } (k-m)=0, \pm N \pm 2N \ldots$$
$$= 0 \quad \text{other wise}$$

then we get

$$\sum_{n=0}^{N-1} x_p(n)\, e^{-j(2\pi/N)mn} = X_p(m) \quad \text{--- (4)}$$

Fourier series coefficients $X_p(k)$ can be obtained by changing m to k in Eq – (4).

$$X_p(k) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j2\pi kn/N} \quad \text{--- (5)}$$

Eq – (5) is called eqn for Discrete fourier series

$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_p(k)\, e^{j2\pi nk/N} \quad \text{--- (6)}$$

Eq – (6) is called as eqn for Inverse discrete fourier series

Hence Both $x_p(n)$ & $X_p(k)$ both are periodic with period of N samples. & can be represented as

$$DFS\left[x_p(n)\right] = X_p(k)$$

Discrete fourier Transform

DFT fourier transform of the signal is.

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n} \quad \text{--- (1)}$$

Here Ω is frequency & continuous func. fun^n but $X(\Omega)$ is continuous (0 – 2π). $x(n)$ is discrete signal in digital processor. This can't be evaluated this problem. $X(\Omega)$ is sam... Hence To avoid $X(\Omega)$ is periodic with period 2π uniformly. N samples are taken from 0 – 2π with spa... between successive samples is $2\pi/N$. we get

Put $\Omega = (2\pi/N)k$ in Eq – (1)

$$X(2\pi k/N) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi}{N}kn} \qquad k = 0, 1, 2 \ldots N-1$$

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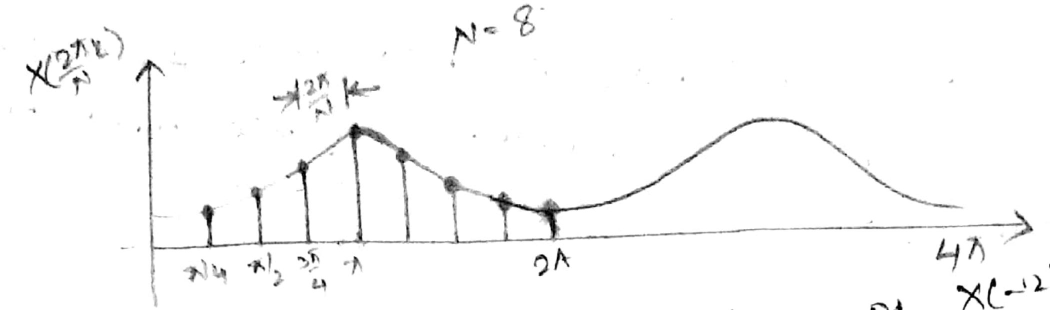


$X(\Omega)$.

fig:- Frequency domain sampling of $X(\Omega)$. k is index for samples. Here $X(\Omega)$ repeats after 2π. $N=8$ samples are taken over the period $0-2\pi$ & $X(\Omega)$ is calculated only at discrete values of Ω.

Finding minimum value of N.

Here n varies from $-\infty$ to ∞ in below Eqn.

$$X(\tfrac{2\pi}{N}k) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi}{N}k\cdot n}$$

Divide the Σ into individual summation's containing only N samples of $x(n)$.

$$X(\tfrac{2\pi}{N}k) = \cdots + \sum_{n=-N}^{-1} x(n)\, e^{-j\frac{2\pi}{N}kn} + \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} + \sum_{n=N}^{2N-1} x(n)\, e^{-j\frac{2\pi}{N}kn} + \cdots$$

Considering only one summation the above Eqn can rewritten as

$$X(\tfrac{2\pi}{N}k) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

Now change n to $n-lN$ only for inner summation then.

$$X(\tfrac{2\pi}{N}k) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi}{N}k(n-lN)}$$

$$= \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi}{N}kn} \cdot e^{j2\pi kl}$$

Here $e^{j2\pi kl} = 1$ $\because$ k & l both are integers & $2\pi kl$ is integer multiple of 2π.

$$\therefore X(\tfrac{2\pi}{N}k) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi}{N}kn}$$

Now change the order of summation then

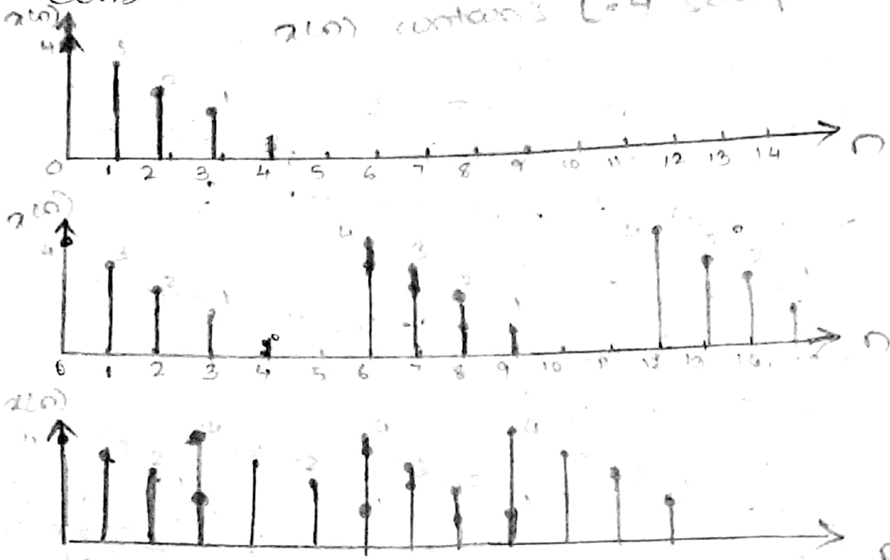
$$X(\tfrac{2\pi}{N}k) = \sum_{n=0}^{N-1} \sum_{l=-\infty}^{\infty} x(n-lN)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} x_p(n)\, e^{-j2\pi kn/N} \qquad \text{Here } k = 0, 1, 2 \cdots N-1$$

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$
$$= \cdots + x(n+2N) + x(n+N) + x(n) + x(n-N) + x(n-2N) \cdots$$

$x_p(n)$ is periodic repetition of $x(n)$ with N samples. $x_p(n)$ is periodic signal, as shown below.

Consider some non periodic signal
$x(n)$ contains $L=4$ samples



$N=6$ $L=4$
$N>L$ No
$N=3$ $N<L$
Two components

When $N>L$ ($N=6$, $L=4$) signal repeats & there is no overlap.
When $N<L$ ($N=3$ $L=4$) two samples overlap at each ... aliasing occurs. To avoid aliasing number of samples in frequency domain spectrum must be greater than number of samples in time domain sequence. $N \geq L$.

Consider Eqn
$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x_p(n) e^{-j2\pi kn/N}$$
k & n both are ... $k=0,1,2 \ldots N-1$

If no of samples in $x(n)$ are less than N then there is no aliasing. Then above Eqn

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{L-1} x(n) e^{-j2\pi kn/N} \qquad k=0,1,\ldots N-1.$$

To avoid aliasing $N>L$ Then above Eqn becomes

$$\boxed{X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k=0,1,2 \ldots N-1}$$

Above Eqn is Eqn for Discrete fourier transform
$X(k)$ is shorthand for $X\left(\frac{2\pi}{N}k\right)$, k indicates index of frequency. Here values of $X\left(\frac{2\pi}{N}k\right)$ are addressed by value of k only. $X(k)$ is also called N-point DFT.

WKT $x_p(n)$ is a periodic Extension of $x(n)$ with period N. & can be Expressed in fourier series Expansion as

$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_p(k)\, e^{j2\pi kn/N}$$

fourier series coefficients $X_p(k)$ of periodic sequence

$x_p(n)$ is periodic sequence with period N.

$$\left.\begin{aligned} \therefore\ x_n &= x_p(n) \\ X(k) &= X_p(k) \end{aligned}\right\}\quad 0 \le k < N-1$$

$= 0$ otherwise

$\therefore$ Then $x_p(n)$ eqn becomes

$$\boxed{x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}}$$

This eqn is called inverse discrete fourier transform (IDFT). Both are represented by notations as shown below.

$$X(k) = DFT[x(n)]$$
$$x(n) = IDFT[X(k)]$$

Q:- Compute DFT of the sequence whose values for one period is given by $x(n) = \{1, 1, -2, -2\}$.

DFT eqn is $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$ $\quad k = 0, 1, 2 \cdots N-1$

Assume $N = L = 4$ $\quad k = 0, 1, 2 \cdots 3$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi kn/4} = \sum_{n=0}^{3} x(n)\, e^{-j\pi kn/2}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$\underset{k=0}{X(0)} = \sum_{n=0}^{3} x(n)\, \cancel{e^{-j2\pi n/4}} = x(0) + x(1) + x(2) + x(3) = \{1+1-2-2\} = -2$$

$$\underset{k=1}{X(1)} = \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2} = x(0) + x(1)\, e^{-j\pi/2} + x(2)\, e^{-j2\pi/2} + x(3)\, e^{-j3\pi/2}$$

$$\boxed{\begin{aligned} e^{-j\alpha} &= \cos\alpha - j\sin\alpha \\ e^{j\alpha} &= \cos\alpha + j\sin\alpha \end{aligned}}$$

$$= 1 + 1\cdot e^{-j\pi/2} - 2\cdot e^{-j\pi} - 2e^{-j3\pi/2}$$

$$= 1 + (\cos\pi/2 - j\sin\pi/2) - 2(\cos\pi - j\sin\pi) - 2(\cos 3\pi/2 - j\sin 3\pi/2)$$

$$= 1 + (0 - j) - 2(-1 - 0) - 2(0 - j(-1))$$

$$= 1 - j + 2 - 2j$$

$$X(1) = 3 - 3j$$

$\sin 180 = 0$
$\cos 360 = 1$
$\sin 360 = 0$

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j\pi n}$$

$k=2$

$$= x(0) + x(1) e^{-j\pi} + x(2) e^{-j2\pi} + x(3) e^{-j3\pi}$$

$$= 1 + 1(\cos\pi - j\sin\pi) + (-2 \cdot \cos 2\pi - j\sin 2\pi) + (-2 \cdot \cos 3\pi - j\sin 3\pi)$$

$$= 1 + (-1 - j \cdot 0) + (-2 \cdot 1 - j \cdot 0) + (-2 \cdot -1 - j \cdot 0)$$

$$= 1 - 1 - \cancel{2} - \cancel{0} + \cancel{2} - \cancel{0}$$

$$= 0$$

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j3\pi n/2}$$

$k=3$

$$= x(0) + x(1) e^{-j3\pi/2} + x(2) e^{-j6\pi/2} + x(3) e^{-j9\pi/2}$$

$$= 1 + 1 \cdot \cos 3\pi/2 - j\sin 3\pi/2 - 2 \cdot \cos 6\pi/2 - j\sin 6\pi/2 - 2\cos 9\pi/2$$

$$= 1 + (0 - (-j)) - 2(-1 - j\,0) - 2(0 - j \cdot 1)$$

$$= 1 + j + 2 - 2j \quad = 1 + j + 2 + 2j$$

$$= \cancel{2 + 2 + 2j} \qquad 3 + 3j.$$

$$X(k) = \{-2, 3-3j, 0, 3+3j\} \quad //$$

Twiddle factor (W_N)

It is a vector on the unit circle & it is a complex quantity of N equally spaced samples. It is & periodic with period equal to N.

$$W_N = e^{-j\frac{2\pi}{N}}$$

* The sequence W_N^n for $0 \le n \le N-1$ lies on the complex plane circle of unit radius in the complex plane equally spaced beginning at zero phases are equally spaced beginning at zero.

By using Twiddle factor DFT pair can be shown as

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn} = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} \qquad 0 \le k$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(K) W_N^{-nk} \; ; \qquad 0 \le n \le N-1$$

Magnitude of twiddle factor is given by

$$|W_N| = \left| e^{-j2\pi/N} \right| = \left| \cos\frac{2\pi}{N} - j\sin\frac{2\pi}{N} \right|$$

& phase angle $\angle e^{-j2\pi/N} = -2\pi$

Consider W_N^{kn} where $kn = r$. Values of W_N^r for $N = 8$ & $r = 0, 1, \dots 16$ are tabulated below.

W_N^r is a periodic fun of r with period N.

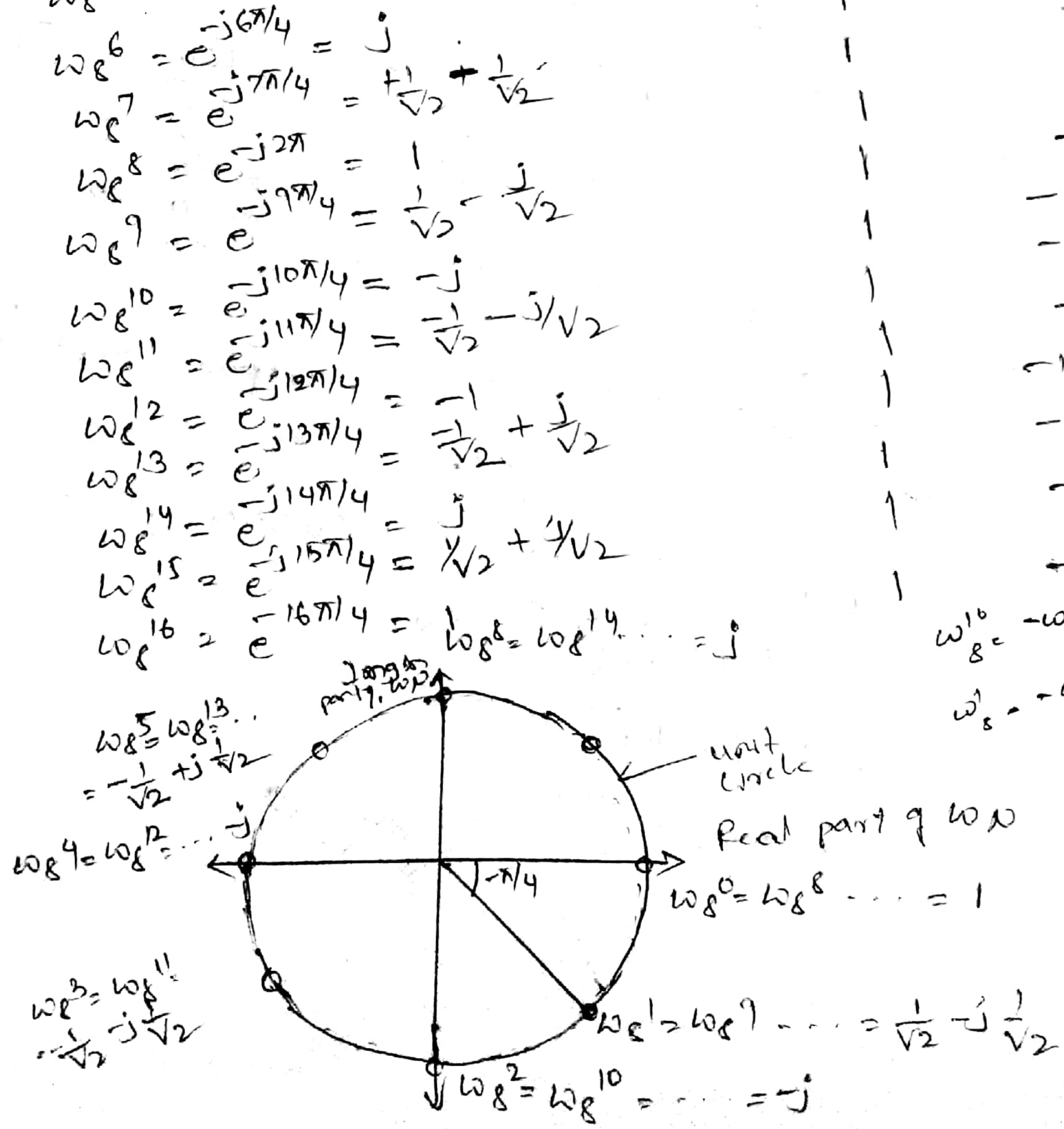
$kn = r$ $\quad W_N^r = (e^{-j2\pi/N})^{kn} = (e^{-j2\pi/8})^r = (e^{-j\pi/4})^r$

$W^r = -W^{r+\frac{N}{2}}$ symmetry proper

$kn = r$	W_8^r	Magnitude	Phase angle
0	$W_8^0 = 1$	1	0
1	$W_8^1 = e^{-j\pi/4} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $= \sqrt{(1/\sqrt{2})^2 + (1/\sqrt{2})^2}$ $= \sqrt{1/2 + 1/2} = 1$		$\phi = \tan^{-1}(\frac{-b}{a})$ $= \tan^{-1}(1)$ $= -45$
2	$W_8^2 = e^{-j\pi/2} = 0 - j1 = -j$	1	$-j = -\pi/2$
3	$W_8^3 = e^{-j3\pi/4} = \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$	1	$-3\pi/4$
4	$W_8^4 = e^{-j\pi} = -1$	1	$-\pi$
5	$W_8^5 = e^{-j5\pi/4} = -1/\sqrt{2} + \frac{1}{\sqrt{2}}$	1	$-5\pi/4$
6	$W_8^6 = e^{-j6\pi/4} = j$	1	$-3\pi/2$
7	$W_8^7 = e^{-j7\pi/4} = +\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$	1	$-7\pi/4$
8	$W_8^8 = e^{-j2\pi} = 1$	1	-2π
9	$W_8^9 = e^{-j9\pi/4} = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$	1	$-9\pi/4$
10	$W_8^{10} = e^{-j10\pi/4} = -j$	1	$-5\pi/2$
11	$W_8^{11} = e^{-j11\pi/4} = \frac{-1}{\sqrt{2}} - j/\sqrt{2}$	1	$-11\pi/4$
12	$W_8^{12} = e^{-j12\pi/4} = -1$	1	-3π
13	$W_8^{13} = e^{-j13\pi/4} = \frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$	1	$-13\pi/4$
14	$W_8^{14} = e^{-j14\pi/4} = j$	1	$-7\pi/2$
15	$W_8^{15} = e^{-j15\pi/4} = 1/\sqrt{2} + j/\sqrt{2}$	1	$-15\pi/4$
16	$W_8^{16} = e^{-16\pi/4} = W_8^8 = W_8^{14} \dots = j$		-4π

$W_8^{10} = -W_8^{10+4} = -W_8^{14}$

$W_8^1 = -W^{1+4}$



Let us represent sequence $x(n)$ as vector x_N of N samples

$$x_N = \begin{array}{c} n=0 \\ n=1 \\ \vdots \\ n=N-1 \end{array} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}_{N\times 1}$$

& $X(K)$ can be represented as a vector X_N of N samples

$$X_N = \begin{array}{c} k=0 \\ k=1 \\ \vdots \\ k=N-1 \end{array} \begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix}_{N\times 1}$$

Value of W_N^{kn} can be represented as matrix $[W_N]$ of size $N\times N$ as

$$W_N = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ \vdots \\ k=N-1 \end{array} \begin{array}{ccccc} n=0 & n=1 & n=2 & \cdots & n=N-1 \\ \left. W_N^{kn} \right|_{k=0,n=0} = W_N^0 & \left. W_N^{kn} \right|_{k=0,n=1} = W_N^0 & \left. W_N^{kn} \right|_{k=0,n=2} = W_N^0 & \cdots & \left. W_N^{kn} \right|_{k=0,n=N-1} \\ \left. W_N^{kn} \right|_{k=1,n=0} = W_N^0 & \left. W_N^{kn} \right|_{k=1,n=1} = W_N^1 & \left. W_N^{kn} \right|_{k=1,n=2} = W_N^2 & \cdots & \left. W_N^{kn} \right|_{k=1,n=N-1} \\ \left. W_N^{kn} \right|_{k=2,n=0} = W_N^0 & \left. W_N^{kn} \right|_{k=2,n=1} = W_N^2 & \left. W_N^{kn} \right|_{k=2,n=2} = W_N^4 & \cdots & \left. W_N^{kn} \right|_{k=2,n=N-1} = W_N^{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ \left. W_N^{kn} \right|_{k=N-1,n=0} = W_N^0 & \left. W_N^{kn} \right|_{k=N-1,n=1} = W_N^{N-1} & \left. W_N^{kn} \right|_{k=N-1,n=2} = W_N^{2N-1} & \cdots & \left. W_N^{kn} \right|_{k=N-1,n=N-1} \end{array}$$

$$W_N = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \cdots & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ W_N^0 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & & W_N^{(N-1)(N-1)} \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & \end{bmatrix}_{N\times N}$$

$\therefore$ N point DFT can be represented in matrix form as

$$X_N = [W_N]\,x(n)$$

||ly IDFT Eqn can be Expressed as

$$x_N = \frac{1}{N}[W_N^*]\,X_N$$

where $[W_N] = W_N^{kn} = W_N^{r}$

$$[W_N^*] = W_N^{-kn}$$

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Zero Padding

Consider a signal with length L i.e $x(n) = \{x(0), x(1), \ldots x(L-1)\}$. For L samples minimum no's of equally spaced frequency points can be calculated from $0 - 2\pi$. If we want to find N-point DFT $(N > L)$ of the sequence $x(n)$, we have to add $(N-L)$ zeros to the sequence $x(n)$ to improve the frequency resolution. This is known as Zero padding.

$x(n) = \{x(0), x(1), \ldots x(L-1), 0, 0, \ldots 0\}$

To find N frequency points between 0 to 2π we are adding $(N-L)$ zeros to the sequence. Then we can get better display of the frequency spectrum. & with zero padding the DFT can be used in linear filtering.

Compute 8 point DFT of the sequence $x(n)$ given below $x(n) = (1, 1, 1, 1, 0, 0, 0, 0)$. also calculate magnitude & phase of $X(k)$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad k = 0, 1, \ldots N-1$$

recomputing

For 8 point DFT eqn can be $\qquad k = 0, 1, \ldots 7$

$$X(k) = \sum_{n=0}^{7} x(n)\, W_8^{kn}$$

In matrix form

$$X(k) = [W_N]\, x_N$$

$$X_8 = [W_8]\, x_8$$

$$[W_8] = W_8^{kn} = e^{-j2\pi/N\, kn} = e^{-j\pi/4\, kn}$$

$W_8^0 = e^{-j\pi/4 \times 0} = 1 = W_8^8 = W_8^{16} = W_8^{24} = W_8^{32} = W_8^{40} = 1$

$W_8^1 = e^{-j\pi/4 \times 1} = \cos(\pi/4) - j\sin(\pi/4) = \frac{1}{\sqrt2} - j\frac{1}{\sqrt2} = W_8^9 = W_8^{17} = W_8^{25} = W_8^{33} = W_8^{41} = W_8^{49}$

$W_8^2 = e^{-j\pi/2} = \cos(\pi/2) - j\sin\pi/2 = -j = W_8^{10} = W_8^{18} = W_8^{26} = W_8^{34} = W_8^{42}$

$W_8^3 = e^{-j3\pi/4} = \cos(3\pi/4) - j\sin 3\pi/4) = \frac{-1}{\sqrt2} - j\frac{1}{\sqrt2}$

$W_8^4 = e^{-j\pi} = \cos(\pi - j\sin\pi) = -1$

$W_8^5 = e^{-j5\pi/4} = -\frac{1}{\sqrt2} + j\frac{1}{\sqrt2}$

$W_8^6 = e^{-j6\pi/4} = +j$

$W_8^7 = e^{-j7\pi/4} = \frac{1}{\sqrt2} + j\frac{1}{\sqrt2}$

$$X_8 = [W_8]^{nk}\, x_8$$

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5)\\ X(6)\\ X(7)\end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0\\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7\\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14}\\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21}\\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28}\\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35}\\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42}\\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix}1\\1\\1\\1\\0\\0\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & +j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2}\\ 1 & -j & -1 & j & 1 & -j & -1 & j\\ 1 & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & +j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2}\\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1\\ 1 & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & j & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2}\\ 1 & +j & -1 & -j & 1 & +j & -1 & -j\\ 1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & +j & -\frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & -\frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1+1+0+0+0+0\\ 1+\cancel{\frac{1}{\sqrt2}}-j\frac{1}{\sqrt2}-j-\cancel{\frac{1}{\sqrt2}}-j\frac{1}{\sqrt2}+0+0+0+0\\ 1-j-1+j+0+0+0+0\\ 1-\frac{1}{\sqrt2}-j\frac{1}{\sqrt2}+j+\frac{1}{\sqrt2}-j\frac{1}{\sqrt2}+0+0+0+0\\ 1-1+1-1+0+0+0+0\\ 1-\frac{1}{\sqrt2}+j\frac{1}{\sqrt2}-j+\frac{1}{\sqrt2}+j\frac{1}{\sqrt2}+0+0+0+0\\ 1+j-1-j+0+0+0+0\\ 1+\frac{1}{\sqrt2}+j\frac{1}{\sqrt2}+j-\frac{1}{\sqrt2}+j\frac{1}{\sqrt2}+0+0+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 4\\ 1-2j\frac{1}{\sqrt2}-j\\ 0\\ 1+j-2j\frac{1}{\sqrt2}\\ 0\\ 1-j+2j\frac{1}{\sqrt2}\\ 0\\ 1+j+j\frac{1}{\sqrt2} \end{bmatrix} = \begin{bmatrix} 4\\ 1-j(1+\sqrt2)\\ 0\\ 1+j(1-\sqrt2)\\ 0\\ 1-j(1-\sqrt2)\\ 0\\ \ldots \end{bmatrix} = \begin{bmatrix} 4\\ 1-j2.414\\ 0\\ 1-j0.414\\ 0\\ 1+j0.414\\ 0\\ \ldots \end{bmatrix}$$

Real & Imaginary parts are

$$\begin{bmatrix} X_R(0) \\ X_R(1) \\ X_R(2) \\ X_R(3) \\ X_R(4) \\ X_R(5) \\ X_R(6) \\ X_R(7) \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad \& \quad \begin{bmatrix} X_I(0) \\ X_I(1) \\ X_I(2) \\ X_I(3) \\ X_I(4) \\ X_I(5) \\ X_I(6) \\ X_I(7) \end{bmatrix} = \begin{bmatrix} 0 \\ -(1+\sqrt{2}) \\ 0 \\ (1-\sqrt{2}) \\ 0 \\ -(1-\sqrt{2}) \\ 0 \\ (1+\sqrt{2}) \end{bmatrix}$$

8 point DFT magnitude

$$|X(k)| = \sqrt{[X_R(k)]^2 + [X_I(k)]^2}$$

$$= \sqrt{|1|^2 + (1+\sqrt{2})^2} = 2.613$$

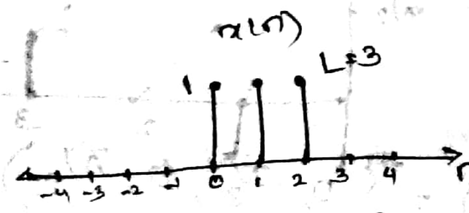
phase $\angle X(1) = \tan^{-1} \frac{X_I(1)}{X_R(1)} = \tan^{-1} \frac{(1+\sqrt{2})}{1} = 1.178.$

Ex. Find the DFT of a sequence

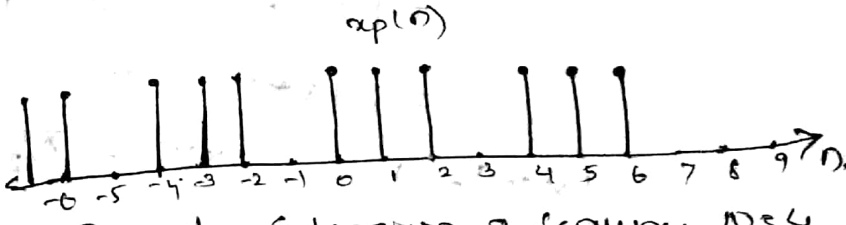
$x(n) = 1$ for $0 \le n \le 2$

$= 0$ otherwise

for (i) $N=4$ (ii) $N=8$ plot $|X(k)|$ & $\angle X(k)$.



Given sequence $L=3$



Periodic Extension of sequence $N=4$.

In above periodic Extension ($N=4$) one zero is added.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}. \quad k=0,1,2 \ldots N-1$$

from fig 3.5 b

$x(n) = \{1, 1, 1, 0\}.$

$N=4$ $$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/4} = \sum_{n=0}^{N-1} x(n) e^{-j\pi kn/2} \quad k=0,1,2 \ldots N-1$$

$k=0$ $$X(0) = \sum_{n=0}^{N-1} x(n) = x(0)+x(1)+x(2)+x(3) = 1+1+1+0 = 3$$

$|X(0)| = 3$ $\quad \angle X(0) = 0$

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$k=1 \cdot X(1) = \sum_{n=0}^{3} x(n) e^{-j\pi n/2}$

$= x(0)\cancel{e^{-j\pi/2}} + x(1) e^{-j\pi/2} + x(2) e^{-j\pi} + x(3) e^{-j3\pi/2}$

$= 1 + 1\cdot(\cos\pi/2 - j\sin\pi/2) + (\cos\pi - j\sin\pi) + 0$

$= 1 + (0 - j) + (-1 - 0) + 0$

$= \cancel{1} - j - \cancel{1} = -j$

$|X(1)| = 1 \qquad \angle X(1) = -90^\circ$

$k=2 \quad X(2) = \sum_{n=0}^{3} x(n) e^{-j\pi n}$

$= x(0) + x(1) e^{-j\pi} + x(2) e^{-j2\pi} + x(3) e^{-j3\pi}$

$= 1 + (\cos\pi - j\sin\pi) + 1(\cos 2\pi - j\sin 2\pi) + 0$

$= 1 + (-1 - 0) + 1(1 - 0) + 0$

$= 1 - 1 + 1 = 1$

$|X(2)| = 1 \qquad \angle X(2) = 0$

$k=3 \quad X(3) = \sum_{n=0}^{3} x(n) e^{-j3\pi n/2}$

$= x(0) + x(1) e^{-j3\pi/2} + x(2) e^{-j6\pi/2} + 0$

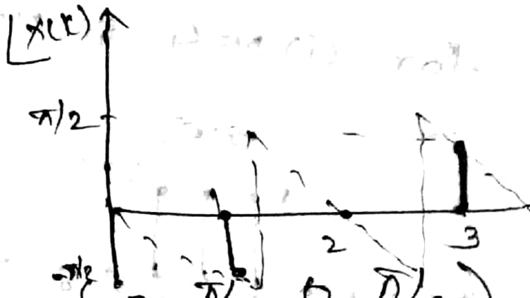
$= 1 + (\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}) + 1\cos(3\pi - j\sin 3\pi)$

$= 1 + (0 + j) + (-1 - 0)$

$= 1 + j - 1 = j$

$\angle X(3) = \pi/2$

$|X(3)| = 1$



$|X(k)| = \{3, 1, 1, 1\} \qquad \angle X(k) = \{0, -\pi/2, 0, \pi/2\}$

For $N=8$ periodic extension can be obtained by adding 5 zeros ($N-L$ zeros)

$x(0) = x(1) = x(2) = 1 \quad x(n) = 0 \text{ for } 3 \le n \le 7$

$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/8} = \sum_{n=0}^{N-1} x(n) e^{-j\pi nk/4}$

$k=0 \quad X(0) = \sum_{n=0}^{7} x(n) e^{-j\pi n/4} = x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6)$

$= 1 + 1 + 1 + 0 + 0 + 0 + 0 + 0 = 3$

$|X(0)| = 3 \qquad \angle X(0) = 0$

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$k=1$ $\quad X(1) = \sum_{n=0}^{7} x(n)\, e^{-j\pi n/4}$

$= x(0) + x(1) e^{-j\pi/4} + x(2) e^{-j\pi/2} + 0 + \cdots$

$= 1 + (\cos\pi/4 - j\sin\pi/4) + 1(\cos\pi/2 - j\sin\pi/2)$

$= 1 + (0.707 - j0.707) + (0 - j)$

$X(1) = 1.707 - j1.707$

$|X(1)| = 2.414$ $\quad \angle X(1) = \tan^{-1}\frac{-1.707}{1.707} = -\pi/4$

$= \sqrt{1.707^2 + 1.707^2}$

$k=2$ $\quad X(2) = -j$

$|X(2)| = 1$ $\quad \angle X(2) = \tan^{-1}\left(\frac{-1}{0}\right) = -\pi/2$

$k=3$ $\quad X(3) = \sum_{n=0}^{7} x(n)\, e^{-j3\pi n/4}$

$= 0.293 + j0.293$

$|X(3)| = \sqrt{0.293^2 + 0.293^2} = 0.414$ $\quad \angle X(3) = \tan^{-1}\frac{0.293}{0.293} = \pi/4$

$k=4$ $\quad X(4) = \sum_{n=0}^{7} x(n)\, e^{-\pi n} = 1 - 1 + 1 = 1$

$|X(4)| = 1$ $\quad \angle X(4) = 0$

$k=5$ $\quad X(5) = \sum_{n=0}^{7} x(n)\, e^{-j5\pi/4} = 1 - 0.707 + j0.707 - j = 0.293 - j0.293$

$|X(5)| = 0.414$ $\quad \angle X(5) = -\pi/4$

$k=6$ $\quad X(6) = \sum_{n=0}^{7} x(n)\, e^{-j3\pi n/2} = 1 + j - 1 = j$

$|X(6)| = 1$ $\quad \angle X(6) = +\pi/2$

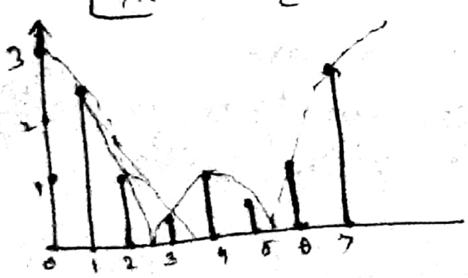
$k=7$ $\quad X(7) = \sum_{n=0}^{7} x(n)\, e^{-j7\pi n/4} = 1.707 + j1.707.$

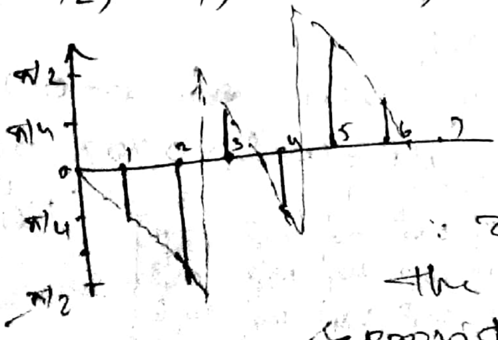
$|X(7)| = 2.414.$ $\quad \angle X(7) = \pi/4.$

$|X(k)| = \{\cancel{3, 1, \ldots}, 1, 0.414, 1, 0.414, 1, 2.414\}$

$= \{3, 2.414, 1, 0.414, 1, 0.414, 1, 2.414\}$

$\angle X(k) = \{0, -\pi/4, -\pi/2, \pi/4, 0, -\pi/4, \pi/2, \pi/4\}$





By increasing N: increase the resolution & possible to extrapolate frequency spectrum.

∴ zero padding gives the high density spectrum & provides better displayed version for plotting.

Ex Find DFT of the following signals (i) $x(n) = \delta(n)$
(ii) $x(n) = a^n$

(i) $x(n) = \delta(n)$ $\qquad$ $\delta(n) = 1$ for $n = 0$
$\qquad\qquad\qquad\qquad\qquad\qquad = 0$ for $n \neq 0$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} \delta(n)\, e^{-j2\pi kn/N} = x(0) \cdot e^{0} = 1 \times 1$$

$$= 1$$

(ii) $x(n) = a^n$

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N}$$

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$= \sum_{n=0}^{N-1} a^n \cdot e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} (a \cdot e^{-j2\pi k/N})^n$$

$$X(K) = \frac{1 - a^N e^{-j2\pi k}}{1 - a e^{-j2\pi k/N}}$$

Here $e^{-j2\pi k} = \cos(2\pi k) - j\sin 2\pi k$
$= 1 - j0 = 1$

$$X(K) = \frac{1 - a^N \cdot 1}{1 - a e^{-j2\pi k/N}} \quad \text{—— (1)}$$

Eqn (1) is called DFT of Exponential sequence a^n.

for Example $x(n) = (0.5)^n$ & $n = 0, 1, 2, 3$.
$a = 0.5$
Here $N = 4$.

$$X(K) = \frac{1-a^N}{(1 - a e^{-j2\pi k/N})} = \frac{1-(0.5)^4}{(1 - 0.5 \cdot e^{-j2\pi k/4})}$$

$$= \frac{0.9375}{(1 - 0.5 \cdot e^{-j\pi k/2})}$$

Ex Find the 4-point DFT of the Sequence

$$x(n) = \cos \frac{n\pi}{4}$$

Soln Four samples of $x(n)$ for $n = 0, 1, 2, 3$ are

$x(0) = \cos(0) = 1$, $x(1) = \cos\frac{\pi}{4} = 0.707$, $x(2) = \cos\frac{2\pi}{4} = 0$, $x(3) = \cos\frac{3\pi}{4} = -0.707$

$X(K) = [W_N]\, x_N$ $\qquad$ $W_4^{kn} = e^{-j2\pi/4} = e^{-j\pi/2} = 1$

$$X(K) = \begin{array}{c|c} & \begin{array}{cccc} n=0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} k=0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \end{array}$$

$$W_{4N}^{r} = W_N^{kn} = W_4^{r} = e^{-j\frac{2\pi}{N}kn} = e^{-j\frac{2\pi}{N}r}$$

$$W_4^0 = e^{-j\frac{2\pi}{4}0} = 1$$

$$W_4^1 = e^{-j\frac{2\pi}{4}1} = e^{-j\pi/2} = \cos(\pi/2) - j\sin\pi/2 = 0 - j = -j$$

$$W_4^2 = e^{-j\frac{2\pi\times 2}{4}} = e^{-j\pi} = \cos(\pi) - j\sin\pi = -1 - 0 = -1$$

$$W_4^3 = e^{-j\frac{6\pi}{4}} = e^{-j3\pi/2} = \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} = j$$

$$W_4^4 = W_4^0 = 1$$

$$W_4^5 = W_4^1 = -j$$

$$W_4^6 = W_4^2 = -1$$

$$W_4^7 = W_4^3 = j$$

$$W_4^8 = W_4^4 = W_4^0 = 1$$

$$W_4^9 = W_4^5 = W_4^1 = -j$$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \times \begin{bmatrix} 1 \\ 0.707 \\ 0 \\ -0.707 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 0.707 + 0 - 0.707 \\ 1 - j0.707 + 0 - j0.707 \\ 1 - 0.707 + 0 + 0.707 \\ 1 + j0.707 + 0 + j0.707 \end{bmatrix}$$

$$X(k) = \begin{bmatrix} 1 \\ 1 - j1.414 \\ 1 \\ 1 + j1.414 \end{bmatrix}$$

$\therefore$ Required DFT $\triangleq \{1, 1-j1.414, 1, 1+j1.414\}$ //

Ex Find $x(n)$ of $X(k) = \{1-j2, -1, 1+j2\}$ using formula for obtaining DFT.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[W_N^{kn}\right]^*$$

$$x(n) = \frac{1}{N}\left[W_N^{kn}\right]^* X_N$$

$$W_N^{-kn} = W_3^{-kn} = W_3^{-r} =$$

$$W_3 = \begin{array}{c|ccc} & 0 & 1 & 2 \\ 0 & W_3^0 & W_3^0 & W_3^0 \\ 1 & W_3^0 & W_3^1 & W_3^2 \\ 2 & W_3^0 & W_3^2 & W_3^4 \end{array}$$

$$W_3^{-0} = \left[\quad\right]$$

$$W_3^{-0} = e^{j\frac{2\pi r}{N}} = e^{j\frac{2\pi 0}{3}} = 0$$

$$W_3^{-1} = e^{j\frac{2\pi}{3}} = \cos(2\pi/3) - j\sin$$

$$W_3^{-2} = e^{j\frac{4\pi}{3}} = \cos\left(\frac{4\pi}{3}\right) - j\sin$$

$$W_3^{-0} = W_3^{-3} = 0$$

$$W_3^{-1} = W_3^{-4} =$$

Ex Consider two periodic sequences $x(n)$ & $y(n)$. $x(n)$ has period N and $y(n)$ has a period M. The sequence $w(n)$ is defined as $w(n) = x(n) + y(n)$. (i) show that $w(n)$ is periodic with MN point seq. (ii) Also show that $W(k)$ represents M·N point DFT of

Soln (i) To prove $w(n) = w(n+MN)$. $w(n)$ is periodic with period

$w(n+MN) = x(n+MN) + y(n+MN)$

& $x(n) = x(n+N) = x(n+2N) = x(n+3N) + \ldots = x(n+MN)$ for all integer multiple of N

$y(n) = y(n+M) = y(n+2M) = y(n+3M) = \ldots = y(n+MN)$ for all integer multiple of M.

$w(n+MN) = x(n) + y(n) = w(n)$. It is periodic with MN.

(ii) DFT of $w(n)$ with length MN will be

DFT of $w(n)$

$$W(k) = \sum_{n=0}^{MN-1} w(n)\, W_M^{kn}, \quad k = 0, 1, \ldots MN-1$$

$\therefore W(k)$ is MN point DFT of $w(n)$.

Relation of DFT with Fourier transform

Q17. Fourier transform $X(e^{j\omega})$ of a finite duration sequence $x(n)$ having length N:

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n} \quad \text{— (1)}$$

Here $X(e^{j\omega})$ is a continuous funⁿ of ω.

DFT of $x(n)$ is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} \quad \text{— (2)} \quad k=0,1,2 \ldots N-1$$

By comparing Eq (1) & (2) DFT of $x(n)$ is sampled version of fourier transform of sequence & is given by

$$X(k) = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}} \quad k=0,1,2 \ldots N-1.$$

//

Relation of DFT with Z-transform

Relation of DFT with N finite duration with z-transform

→ consider $x(n)$ with N

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} \quad \text{— (1)}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad \text{— (2)}$$

$$X(z) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}\right] z^{-n}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1}\left(e^{j\frac{2\pi k}{N}} \cdot z^{-1}\right)^n$$

$X(z)$ is sampled at N equally spaced points $z_k = e^{j2\pi k/N}$ at $k=0,1,\ldots N$. on the unit circle.

Then z transform at these points

$$X(z)\Big|_{z=e^{j2\pi k/N}} = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j2\pi kn/N}$$

If $x(n)$ has N number of samples then above Eqⁿ becomes

$$X(z) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

above Eqⁿ is DFT Eqⁿ. //

Then RHS of are related of

∴ DFT & z transform

$$X(k) = X(z)\Big|_{z=e^{j2\pi k/N}}$$

// If z-transform is evaluated on unit circle at evenly spaced points then its DFT. // on unit circle it becomes fourier transform

... form is evaluated. on unit circle

Ex Compute IDFT of the sequence

$X(K) = (2, 1+j, 0, 1-j)$ $\quad N=4$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(K) e^{+j\frac{2\pi kn}{N}}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(K) \cdot W_N^{-kn} \qquad 0 \le n \le 3$$

1, 2, 3, 11, 14, 16, 17, 15, 21, 25, 29, 32, 34, 37, 39, 40, 43, 44

$$x(n) = \frac{1}{4}\sum_{k=0}^{3} X(K) W_4^{-kn}$$

$n=0$

$$x(0) = \frac{1}{4}\left[X(0) W_4^0 + X(1) W_4^0 + X(2) + X(3)\right]$$

$$= \frac{1}{4}\left[2 + 1+j + 0 + 1-j\right] = 1$$

$n=1$

$$x(1) = \frac{1}{4}\sum_{k=0}^{3} X(K) e^{j\frac{2\pi k}{N}} = \frac{1}{4}\sum_{k=0}^{3} X(K) \cdot e^{j\frac{\pi k}{2}}$$

$$= \frac{1}{4}\left[X(0) + X(1) \cdot e^{j\pi/2} + X(2) e^{j\pi} + X(3) e^{j3\pi/2}\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(\cos\pi/2 + j\sin\pi/2) + 0 + (1-j)(\cos 3\pi/2 + j\sin 3\pi/2)\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(0+j) + (1-j)(0-j)\right]$$

$$= \frac{1}{4}\left[2 + (\cancel{j} + j^2) + (\cancel{j} + j^2)\right]$$

$$x(1) = \frac{1}{4}\left[2 + (-1) + (-1)\right] = \frac{1}{4}\left[2 + (-2)\right] = 0$$

$$x(2) = \frac{1}{4}\sum_{k=0}^{3} X(K) e^{j\pi k}$$

$$= \frac{1}{4}\left[X(0) + X(1) e^{j\pi} + X(2) e^{j2\pi} + X(3) e^{j3\pi}\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(\cos\pi + j\sin\pi) + 0 + (1-j)(\cos 3\pi + j\sin 3\pi)\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(-1 + j\times 0) + 0 + (1-j)(-1+0)\right]$$

$$= \frac{1}{4}\left[2 - 1 - \cancel{j} - 1 + \cancel{j}\right] = 0$$

$$x(3) = \frac{1}{4}\sum_{k=0}^{3} X(K) e^{j\frac{3\pi k}{2}}$$

$$= \frac{1}{4}\left[X(0) + X(1) e^{j3\pi/2} + X(2) e^{j3\pi} + X(3) e^{j9\pi/2}\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(\cos 3\pi/2 + j\sin 3\pi/2) + 0 + (1-j)(\cos\tfrac{9\pi}{2} + j\sin 9\pi/2)\right]$$

$$= \frac{1}{4}\left[2 + (1+j)(0-j) + (1-j)(0+j)\right]$$

$$= \frac{1}{4}\left[2 - \cancel{j} - j^2 + \cancel{j} - j^2\right] = \frac{1}{4}\left[2+1+1\right] = 1$$

$$x(n) = \{1, 0, 0, 1\}$$

Properties of DFT

(1) **Periodicity**

If $X(k)$ is a N-point DFT of finite duration sequence $x(n)$ then

$$x(n+N) = x(n) \quad \text{for all } n$$
$$X(k+N) = X(k) \quad \text{for all } k.$$

(2) **Linearity**

$$DFT\{a x_1(n) + b x_2(n)\} = a X_1(k) + b X_2(k) \quad \therefore \; k=0,1,\dots N-1$$

with $X_1(k)$ & $X_2(k)$ are the DFT's of sequences $x_1(n)$ & $x_2(n)$ respectively, with length N.

Proof:

$$\underset{n=0,1,\dots N-1}{x(n)} \longrightarrow \boxed{DFT} \longrightarrow \underset{k=0,1,\dots N-1}{X(k)}$$

Here i/p is $x(n)$ & o/p is $X(k)$ both are having same length N. Hence N is known as transform length for the DFT operation.

WKT
$$DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

Let $x(n) = a x_1(n) + b x_2(n)$ then

$$DFT\{a x_1(n) + b x_2(n)\} = \sum_{n=0}^{N-1} [a x_1(n) + b x_2(n)] W_N^{kn}$$
$$= a\sum_{n=0}^{N-1} x_1(n) W_N^{kn} + b\sum_{n=0}^{N-1} x_2(n) W_N^{kn}$$
$$= a X_1(k) + b X_2(k) \quad 0 \le k \le N-1$$

$\therefore$ According to linearity property

$$a(x_1(n) + b x_2(n)) \overset{DFT}{\longleftrightarrow} a X_1(k) + b X_2(k).$$

Ex: Find the 4 point DFT of the sequence. Use linearity property

$$x(n) = \cos(\tfrac{\pi}{4} n) + \sin(\tfrac{\pi}{4} n)$$

Ex: Given $N=4$

$$W_N^{kn} = e^{-j\frac{2\pi kn}{N}} = e^{-j\frac{2\pi}{N} r}$$
$$W_4^{r} = e^{-j\frac{\pi}{2} r}$$

1, 16, 17, 21, 25, 36, 37.

$$W_4^0 = 1$$
$$W_4^1 = \cos(\pi/2) - j\sin\pi/2 = -j$$
$$W_4^2 = e^{-j\pi} = \cos\pi - j\sin\pi = (-1) = -1$$
$$W_4^3 = e^{-j\frac{3\pi}{2}} = \cos\tfrac{3\pi}{2} - j\sin\tfrac{3\pi}{2} = 0 + j = j$$

Given $x_1(n) = \cos\frac{\pi}{4}n$ $\qquad x_2(n) = \sin\frac{\pi}{4}n$

$x_1(0) = 1$ $\qquad x_2(0) = 0$

$x_1(1) = 0.707$ $\qquad x_2(1) = 0.707$

$x_1(2) = 0$ $\qquad x_2(2) = 1$

$x_1(3) = -0.707$ $\qquad x_2(3) = 0.707$.

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) W_N^{kn}$$

$$X_1(0) = \sum_{n=0}^{3} x_1(n) W_4^{kn}$$

$$= [x_1(0) + x_1(1) + x_2(1) + x_3(1)]$$

$$= [1 + 0.707 + 0 - 0.707 = 1$$

$$X_1(1) = \sum_{n=0}^{3} x_1(n) W_4^{kn}$$

$$= [x_1(0) + x_1(1) W_4^1 + x_1(2) W_4^2 + x_1(3) W_4^3]$$

$$= [1 + 0.707(-j) + 0 + 0.707 j]$$

$$= [1 - j1.414]$$

$$X_1(2) = [x_1(0) + x_1(1) W_4^2 + x_1(2) W_4^2 + x_1(3) W_4^6]$$

$$= 1 \cancel{\ldots}$$

$$X_1(3) = 1 + \frac{1}{\sqrt{2}} W_4^3 - \frac{1}{\sqrt{2}} W_4^9$$

$$= 1 + j1.414$$

$$X_2(k) = \sum_{n=0}^{3} x_2(n) W_4^{kn}$$

$$X_2(0) = x_2(0) + x_2(1) + x_2(2) + x_2(3)$$

$$= 1 + 0.707 + 0.707 = 2.414$$

$$X_2(1) = 0.707 W_4^1 + W_4^2 + 0.707 W_4^3 = -1.$$

$$X_2(2) = 0.707 W_4^2 + W_4^0 + 0.707 W_4^6 = -0.414$$

$$X_2(3) = 0.707 W_4^3 + W_4^6 + 0.707 W_4^9 = -1.$$

Finally applying linearity property

$$X(k) = \text{DFT}\{x_1(n) + x_2(n)\}$$

$$= X_1(k) + X_2(k)$$

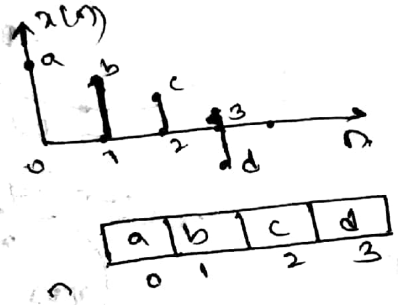
$$= \{X_1(0) + X_2(0),\ X_1(1) + X_2(1);\ X_1(2) + X_2(2),\ X_1(3) + X_2 \ldots$$

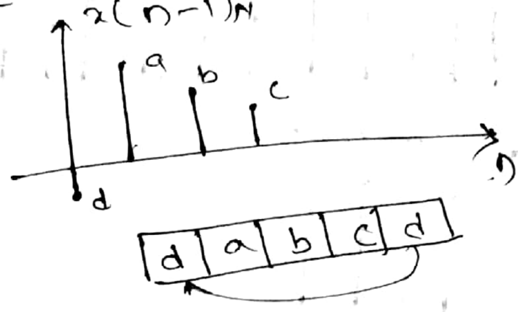
$$= \{3.414, -j1.414, 0.586, j1.414\}$$

Circular shift and Circular Symmetry (shifted)

Consider sequence $x(n)$ for all n. Its translated version of $x(n)$ is $x(n-n_0)$ where n_0 represents the no of indices that sequence $x(n)$ is translated to right. If $x(n)$ is a periodic for finite length sequence, then fundamental form of sequence with N, then fundamental form of sequence is a circular translation useful in mathematical sense of finite length translation. When periodic signal on a circular sequence repeats periodically (modulo N) from axis is

$$x_p(n) = x[(n \text{ modulo } N)]$$





Properties

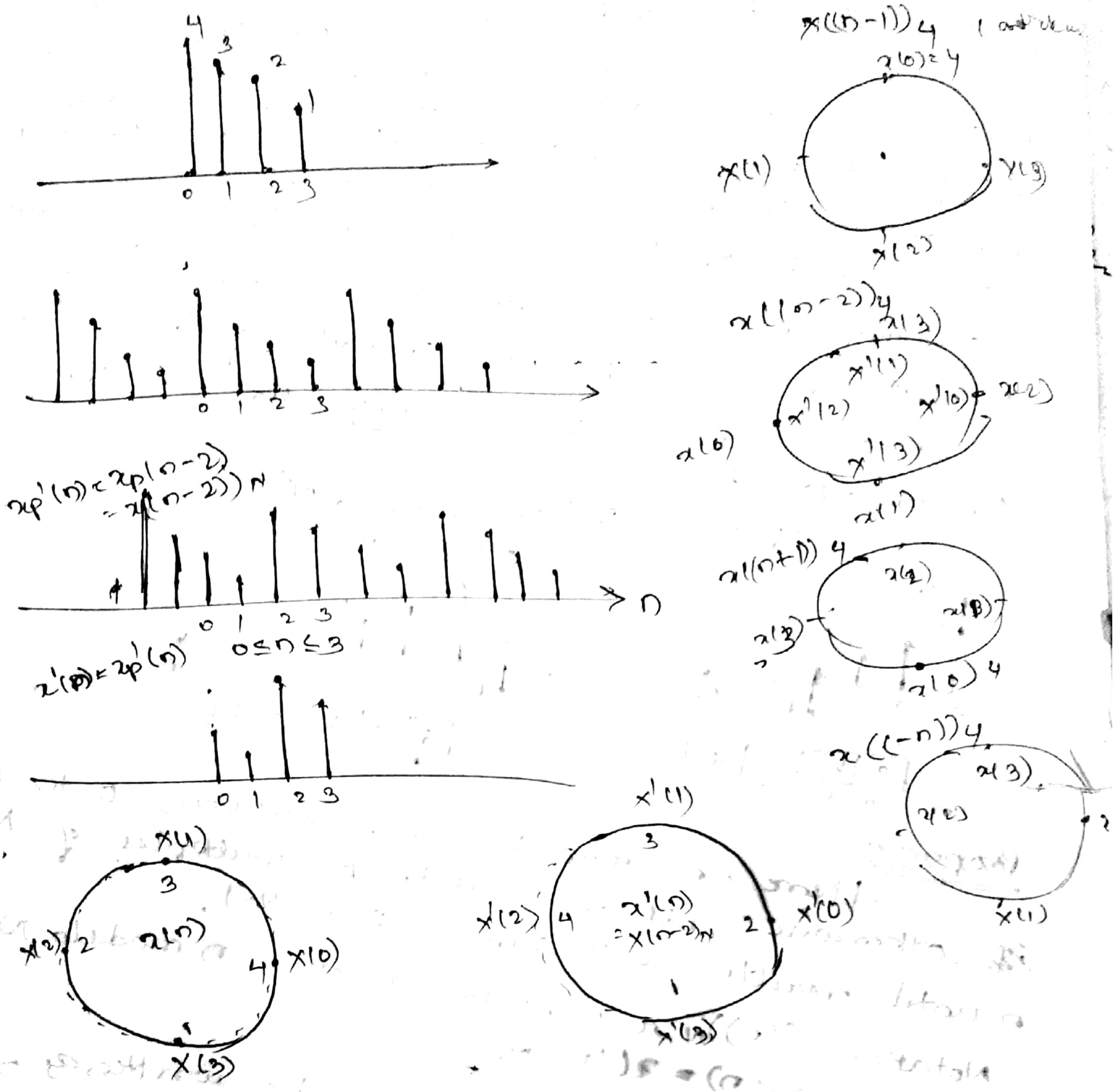
Here n must be n between 0 to $N-1$. If otherwise add or substract multiples of N from n until result is in b/w 0 to $N-1$. To denote n modulo N, notation $((n))_N$ is used.

$$x_p(n) = x((n))_N.$$

$x_p(n)$ can be written as $x_p(n-$ shifted version of $x_p(n)$ towards right. Here k represent no of shifts of $x_p(n)$. Then $x_p'(n)$ is shifted version

$$x_p'(n) = x_p(n-k) = \sum_{l=-\infty}^{\infty} x(n-lN-k).$$

Consider sequence $x(n)$ & its periodic extension x_p. Then $x_p'(n)$ is obtained by shifting $x_p(n)$ by k units right.



Property

$x((n))_N$ ⇒ N point sequence plotted across circle in +ve direction (anticlockwise)

$x((n-k))_N$ ⇒ x(n) shifted in anticlockwise direction (+ve by k samples which shows delay

$x((n+k))_N$ ⇒ x(n) shifted clockwise (-ve dirⁿ) by k sampl (advancing operation)

$x((-n))_N$ ⇒ Circular folding x(n) plotted across circle in clockwise dirⁿ. (-ve dirⁿ).

Circular folding

It generates $x((-n))_N$ from $x(n)$.

Consider $x(n) = (1, 2, 3, 4)$, $0 \le n \le 3$
$\uparrow n=0$

$x(-n) = (4, 3, 2, 1)$
$\uparrow n=0$

$x((-n))_N = x((-n))_4 = x(4-n)$, $0 \le n < 3$

$= x(4), x(3), x(2), x(1)$

$x((n))_N = (x(0), x(3), x(2), x(1))$

Here $x(n+N) = x(n)$, $x(4) = x(0)$

$= (1, 4, 3, 2)$
$\uparrow n=0$

1.2. . 15.35..

Time
Circular shift of sequence

$x(n) \xrightarrow{} \boxed{DFT} \xrightarrow{} X(k)$

$n = 0, 1, \ldots N-1$ $\quad$ $k = 0, 1, \ldots N-1$

Def^n

If $DFT[x(n)] = X(k)$,

then $DFT[x((n-m))_N] = e^{-j2\pi km/N} X(k)$

$$DFT[x((n-m))_N] = \sum_{n=0}^{N-1} x((n-m))_N \, e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{m-1} x((n-m))_N \, e^{-j2\pi kn/N} + \sum_{n=m}^{N-1} x((n-m))_N \, e^{-j2\pi kn/N} \quad \text{---} (1)$$

But $x((n-m))_N = x(N-m+n) = x(n-m+N)$

$$\therefore \sum_{n=0}^{m-1} x((n-m))_N \cdot e^{-j2\pi kn/N} = \sum_{n=0}^{m-1} x(N-m+n) \cdot e^{-j2\pi kn/N}$$

$n = m-1$
$m-1$
$N-m$
$N-m+m-1$

Now put $l = (N-m+n)$ or $l = N-m$.

$n = l - N + m$

upper limit $n = m-1$ $\therefore m-1 = l-N+m$

$m-1 = l-N+1$

$m-m = l-N+1$

$0 = l - N + 1$

$l = (N-m+n)$

$n = l - N + m$

$0 = l = N-m$

$m-1 = l-N+m$

$$= \sum_{l=N-m}^{N-1} x(l) \cdot e^{-j2\pi k(-N+m+l)/N}$$

$$\sum_{n=0}^{m-1} x((n-m))_N \cdot e^{-j2\pi kn/N} = \sum_{N-m}^{N-1} x(l) \cdot e^{-j2\pi k(l+m)/N} \quad \therefore e^{j2\pi k} = 1 \quad \text{for } k = 0, 1, 2 \ldots \quad \text{---} (2)$$

IIly

$$\sum_{n=m}^{N-1} x((n-m))_N \, e^{-j2\pi kn/N} = \sum_{l=0}^{N-1-m} x(l) \cdot e^{-j2\pi k(l+m)/N} \quad \text{---} (3)$$

Put $n - m = l$ $\quad$ $n = l + m$ $\quad$ $n = N-1 = l + m$

If $n = m \therefore l = 0$ $\quad$ if $n = m$ $\quad$ $l = N-1-m$

$n = l + m$ $\quad$ $m = l + m$

$l = 0$

Substituting Eq-(2) & (3) in Eq (1) we get.

$$DFT[x((n-m))_N] = \sum_{l=N-m}^{N-1} x(l).e^{-j2\pi k(m+l)/N} + \sum_{l=0}^{N-1-m} x(l).e^{-j2\pi k(m+l)/N}$$

$$= e^{-j2\pi km/N} \sum_{l=0}^{N-1} x(l).e^{-j2\pi kl/N}$$

$$= e^{-j2\pi km/N} \sum_{n=0}^{N-1} x(n).e^{-j2\pi kn/N}$$

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$$\boxed{DFT[x((n-m))_N] = e^{-j2\pi km/N} X(k)}$$ //

Or $x((n-m))_N \xleftrightarrow{DFT} W_N^{km} X(k)$ //.

Ex. Find the 4 point DFT of the sequence $x(n) = (1,-1,1,-1)$. Also using time shift property, find the DFT of the sequence, $y(n) = x((n-2))_4$.

Soln Here $N=4$

WKT $W_N^{kn} = W_4^0 = e^{-j\frac{2\pi}{N}n} = e^{-j\frac{2\pi}{N}\times 0} = 1$

$W_4^1 = e^{-j\frac{2\pi}{4}} = e^{-j\pi/2} = \cos(\pi/2) - j\sin\pi/2 = 0-j = -j$

$W_4^2 = e^{-j\pi} = \cos\pi - j\sin\pi = -1$

$W_4^3 = e^{-j\frac{6\pi}{4}} = e^{-j3\pi/2} = \cos 3\pi/2 - j\sin 3\pi/2 = 0+j = j.$

$X(k) = DFT\{x(n)\} \quad 0 \le k \le 3$

$= \sum_{n=0}^{3} x(n) W_4^{kn}$

$k=0$ $X(0) = x(0)W_4^0 + x(1) + x(2) + x(3) = 1-1+1-1 = 0$

$X(1) = x(0)W_4^0 + x(1)W_4^1 + x(2)W_4^2 + x(3)W_4^3$

$= 1\times 1 + (-1)\times(-j) + (1)(-1) + (-1)\times j$

$= 1 + j - 1 - j = 0$

$X(2) = 4$

$X(3) = 0$

$y(n) = x((n-2))_N = x((n-2))_4$

Using the Circular time shift property

DFT of $y(n)$ is $Y(k)$

$Y(k) = e^{-j2\pi km/N} X(k) = W_4^{2k} X(k).$

$k=0,1,2,3$ $Y(0) = W_4^0 X(0) = 0$ $Y(1) = W_4^2 X(1) = (-1)0 = 0$ $Y(2) = W_4^4 X(2) = 1\times 4$

$Y(3) = W_4^6 X(3) = (-1)\times 0 = 0$

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If DFT $[x(n)] = X(k)$ then

$$DFT[x(n) \cdot e^{j2\pi l n/N}] = X((k-l))_N$$

Soln

$$DFT[x(n) \cdot e^{j2\pi l n/N}] = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N} \cdot e^{j2\pi l n/N}$$

$$= \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi n(k-l)/N}$$

$$= \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi n(N+k-l)/N}$$

26.10.

$$= X(N+k-l)$$

$$DFT[x(n) \cdot e^{j2\pi l n/N}] = X((k-l))_N$$

Time reversal of the sequence $x((-n))_N$:

The time reversal of an N-point sequence $x(n)$ is attained by rounding the sequence $x(n)$ around the circle in clockwise direction.

ie $x((-n))_N = x(N-n)$, $0 \le n \le N-1$

If DFT $[x(n)] = X(k)$

then DFT $[x((-n))_N] =$ DFT $[x(N-n)] = X((-k))_N = X(N-k)$

Pf

$$DFT[x(N-n)] = \sum_{n=0}^{N-1} x(N-n) \cdot e^{-j2\pi kn/N}$$

change n to m then put $m = N-n$

then above eqn becomes

$n = N-1$

$m = N-n$

$N-n = N-1$

$n = N-m$

$N-1 = N-m$

$$DFT[x(N-m)] = \sum_{m=0}^{N-1} x(m) \cdot e^{-j2\pi k(N-m)/N}$$

$$= \sum_{m=0}^{N-1} x(m) \cdot e^{-j2\pi kN/N} \cdot e^{j2\pi km/N}$$

$e^{j2\pi k} = 1$ for $k = 0, 1, ...$

$$= \sum_{m=0}^{N-1} x(m) \cdot e^{j2\pi km/N}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{-j2\pi m(N-k)/N}$$

$$DFT[x(N-m)] = X(N-k)$$

Complex conjugate property

If $DFT[x(n)] = X(K)$

then $DFT[x^*(n)] = X^*(N-K) = X^*((-K))_N$.

Proof: $DFT[x^*(n)] = \sum_{n=0}^{N-1} x^*(n) \cdot e^{-j2\pi kn/N}$

$$\left[\sum_{n=0}^{N-1} x(n) \cdot e^{j2\pi kn/N}\right]^*$$

$$= \left[\sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi n(N-K)/N}\right]^*$$

$$= X^*(N-K)$$

Ily. $DFT[x^*(N-n)] = X^*(K)$

Proof: $IDFT[X^*(K)] = \frac{1}{N}\sum_{K=0}^{N-1} X^*(K)\, e^{j2\pi kn/N}$

$$= \frac{1}{N}\left[\sum_{K=0}^{N-1} X(K) \cdot e^{-j2\pi kn/N}\right]^*$$

$$= \frac{1}{N}\left[\sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi K(N-n)/N}\right]^*$$

$$= x^*(N-n)$$

$\therefore DFT[x^*(N-n)] = X^*(K)$ //

Circular convolution

If $x_1(n) \xleftrightarrow[N]{DFT} X_1(K)$. & $x_2(n) \xleftrightarrow[N]{DFT} X_2(K)$

then $x_1(n) \circledN x_2(n) \xleftrightarrow[N]{DFT} X_1(K) \cdot X_2(K)$.

Property shows that circular convolution of two sequences in time domain is equal to the multiplication of two DFT's.

Proof wkT $X_1(K) = \sum_{n=0}^{N-1} x_1(n) \cdot e^{-j2\pi kn/N}$, $K=0,1,\dots N-1$.

$X_2(K) = \sum_{l=0}^{N-1} x_2(l) \cdot e^{-j2\pi kl/N}$, $K=0,1,\dots N-1$

Here two indices l & n are different for $X_1(K)$ & $X_2(K)$

Then $X_3(K) = X_1(K) \cdot X_2(K)$. —(1)

Take IDFT Eqn for $X_3(k)$ then

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k).e^{j2\pi km/N} \quad \text{--- (2)}$$

Substituting Eq (1) in Eq (2) we get

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_1(k).X_2(k).e^{j2\pi km/N}$$

Then put the ~~....~~ values of $X_1(k)$ & $X_2(k)$

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{N-1} x_1(n).e^{-j2\pi kn/N} \cdot \sum_{l=0}^{N-1} x_2(l).e^{-j2\pi kl/N} \cdot e^{j2\pi km/N}\right]$$

All above summations are different ∵ of different indices

then by arranging above Eqn we get

$$= \frac{1}{N}\sum_{n=0}^{N-1} x_1(n)\sum_{l=0}^{N-1} x_2 l\left[\sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N}\right] \text{--- (3)}$$

Then consider Eqn $= N$ --- (4)

$$\sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N} \cdot \sum_{k=0}^{N-1} a^k \quad \text{Put } a = e^{j2\pi(m-n-l)/N}$$

for $a = 1$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N \\ \frac{1-a^N}{1-a} \quad \text{for } a \neq 1 \end{cases}$$

consider $(m-n-l) = N, 2N, 3N \ldots$ multiples of N. $= 1$.

the $a = 1$

$$a = e^{j2\pi N/N} = e^{j2\pi 2N/N} = e^{j2\pi 3N/N} = \ldots \quad \text{for } a = 1$$

$$\sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N} = \sum_{k=0}^{N-1} a^k = N.$$

when $a \neq 1$

If $a \neq 1$. ∴ $\sum_{k=0}^{N-1} a^k = \frac{1-a^N}{1-a}$

$$= \frac{1 - e^{j2\pi k(m-n-l)}}{1 - e^{j2\pi k(m-n-l)/N}}$$

If $e^{j2\pi k(m-n-l)} = 1$ always ∴

$$\sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N} = \frac{1-1}{1 - e^{j2\pi k(m-n-l)/N}} = 0$$

$$\therefore \sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N} = \begin{cases} N & \text{when } (m-n-l) \text{ is multiple of } N \\ 0 & \text{otherwise} \end{cases} \quad \text{--- (4)}$$

(P.T.O)

Substituting Eq - 4 in Eq - 3 we get

$$x_3(m) = \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) \cdot \sum_{l=0}^{N-1} x_2(l) \cdot N$$ when $(m-n-l)$ is multiple of N

$$= \sum_{n=0}^{N-1} x_1(n) \cdot \sum_{l=0}^{N-1} x_2(l)$$

Then if $(m-n-l)$ is multiple of N then

$(m-n-l) = -pN$ p is some integer, may be +ve or -ve.

$\therefore (m-n+pN) = l$.

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n) \cdot x_2(m-n+pN)$$

$x_2(m-n+pN)$ represent periodic sequence with period N. & delayed by n. i.e. $x_2(m)$ shifted circularly by n samples.

$\therefore x_2(m-n+pN) = x_2(m-n, \text{ modulo } N) = x_2((m-n))_N$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n) \cdot x_2((m-n))_N, \quad m = 0, 1, \ldots N-1.$$

$\therefore$ ~~$x_1(k)$~~.

$x_3(m) = x_1(n) \, (N) \, x_2(n)$ — 5

Eq - 5 shows the circular convolution ~~the~~ of circular shift of $x_2(n)$.

~~$x_2(m)$~~ ~~shift~~

Circular correlation

For complex valued sequences $x(n)$ & $y(n)$

If DFT $[x(n)] = X(k)$ & DFT $[y(n)] = Y(k)$

then DFT $[\tilde{r}_{xy}(l)] = $ DFT $\left[\sum_{n=0}^{N-1} x(n) \cdot y^*((n-l))_N\right] = X(k) \cdot Y^*(k)$

where $\tilde{r}_{xy}(l)$ is the circular correlation sequence

multiplication of two sequences

If DFT $[x_1(n)] = X_1(k)$, & DFT $[x_2(n)] = X_2(k)$

then DFT $[x_1(n) \cdot x_2(n)] = \frac{1}{N}[X_1(k) \, (N) \, X_2(k)]$

Parseval's theorem

If DFT $[x(n)] = X(k)$, & DFT $[y(n)] = Y(k)$

then $$\sum_{n=0}^{N-1} x(n) y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot Y^*(k)$$

Multiplication of two sequences

If $DFT[x_1(n)] = X_1(k)$

& $DFT[x_2(n)] = X_2(k)$

then $DFT[x_1(n) \cdot x_2(n)] = \frac{1}{N}[X_1(k) \, (N) \, X_2(k)]$

Parsaval's theorem

If $DFT[x(n)] = X(k)$

& $DFT[y(n)] = Y(k)$

then $DFT[x(n)] = X(k)$

& $DFT[y(n)] = Y(k)$

then $\sum_{n=0}^{N-1} x(n) \cdot y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot Y^*(k)$

- Comparison between Linear convolution with Circular conv

Circular Convolution	Linear convolution
*) If $x_1(n)$ is sequence of L no of samples & $x_2(n)$ with M no of samples after convol^n $y(n)$ will contain $N = Max(L, M)$ samples	If $x_1(n)$ is sequence of L no samples & $x_2(n)$ with M no of samples after convolution $y(n)$ will contain $N = L+M-1$ samples
*) Zero padding is necessary to find the response of a filter	Zero Padding is not necessary to find the response of linear filter
*) ~~Linear~~ ~~can~~ Circular convolution cannot be used to find the response of a filter	Linear convolution can be used to find the response of linear filter

Basically two methods are used to find the circular convolution of two sequences Those are

(1) Concentric Circle method

(2) Matrix Multiplication method

Concentric Circle method

Following steps are used to find out circular convolution of two given sequency $x_1(n)$ & $x_2(n)$ i.e $x_3(n) = x_1(n) Ⓝ x_2(n)$.

(1) Draw N equally spaced samples of $x_1(n)$ on an outer circle with in anticlockwise direction

(2) ~~st~~ with the same reference point draw N equally spaced samples of $x_2(n)$ in inner circle in clock wise direction

(3) O/P is obtained by multiplying samples on two circles & adding the product.

(4) Rotate inner circle with one sample at a time in anti clock wise dir^n & again take product & find the O/P

(5) Repeat the above steps until inner circle first sample lines up with first sample of Exterior circle once again..

Matrix Multiplication method

Consider two sequences $x_1(n)$ & $x_2(n)$. ⓓ circular convolution form as two sequences representing in matrix form

$$\begin{bmatrix} x_2(0) & x_2(N-1) & x_2(N-2) & \cdots & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(N-1) & \cdots & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & \cdots & x_2(4) & x_2(3) \\ \vdots & \vdots & \vdots & & & \\ x_2(N-2) & x_2(N-3) & x_2(N-4) & \cdots & x_2(0) & x_2(N-1) \\ x_2(N-1) & x_2(N-2) & x_2(N-3) & \cdots & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ \vdots \\ x_1(N-2) \\ x_1(N-1) \end{bmatrix} = \begin{bmatrix} x_3(\\ x_3(\\ x_3(\\ \vdots \\ x_3(\\ x_3 \end{bmatrix}$$

Circular shift samples of $x_2(n)$ are represented in $N \times N$ matrix form.

Ex Given the 8 point DFT of the sequence

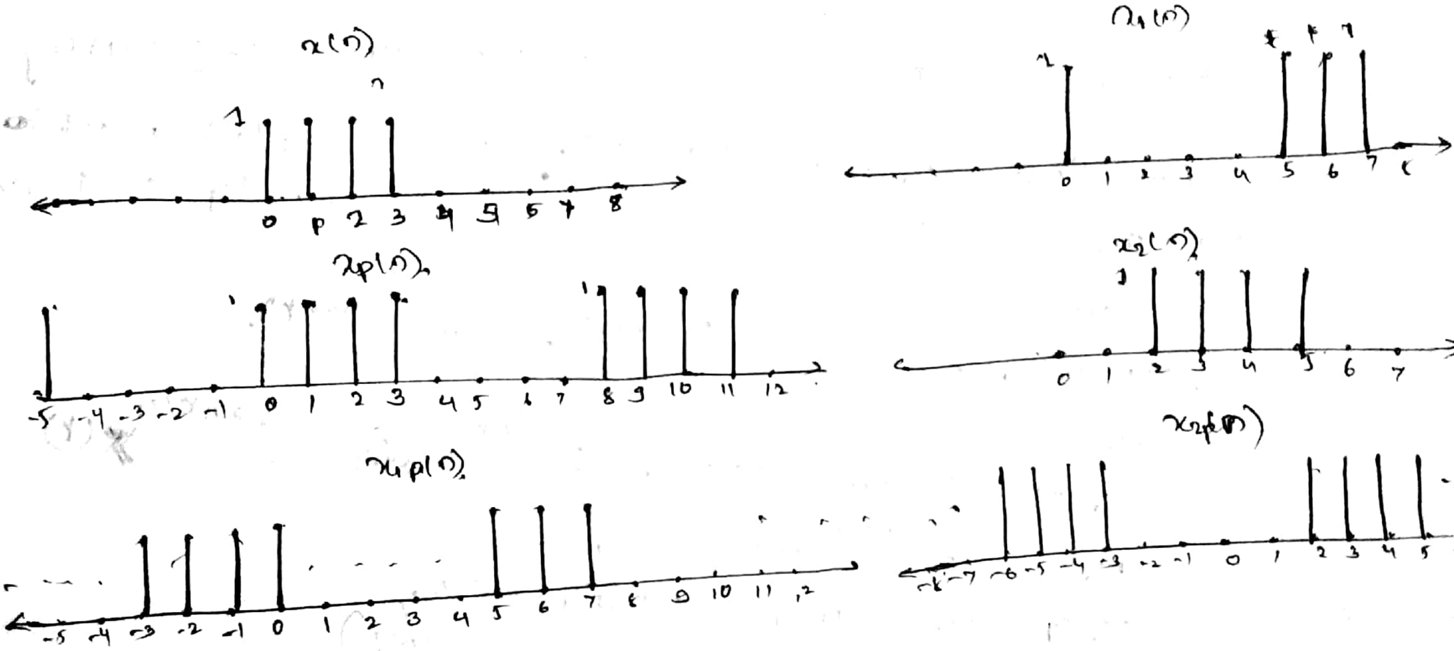
$$x(n) = 1 \quad 0 \le n \le 3$$
$$= 0 \quad 4 \le n \le 7$$

03

Compute DFT of

$$x_1(n) = \begin{cases} 1 & n=0 \\ 0 & 1 \le n \le 4 \\ 1 & 5 \le n \le 7 \end{cases} \quad ; \quad x_2(n) = \begin{cases} 0 & 0 \le n \le 1 \\ 1 & 2 \le n \le 5 \\ 0 & 6 \le n \le 7 \end{cases}$$

Sol?



Find the DFT of $x(n)$ Now.

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi nk/N}, \quad k = 0, 1, \ldots (N-1)$$

For N=8

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi nk/8} \quad k = 0, 1, \ldots 7$$

for k=0

$$X(0) = \sum_{n=0}^{7} x(n) = x(0)+x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)$$
$$= 1+1+1+1+0+0+0+0$$
$$X(0) = 4$$

k=1

$$X(1) = \sum_{n=0}^{7} x(n) \cdot e^{-j2\pi n/8} = \sum_{n=0}^{7} x(n) \cdot e^{-j\pi n/4}$$
$$= x(0) \cdot 1 + x(1) \cdot e^{-j\pi/4} + x(2) \cdot e^{-j\pi/2} + x(3) \cdot e^{-j3\pi/4} + 0+0+0+0$$
$$= 1 + \cos(\pi/4) - j\sin\pi/4 + \cos\pi/2 - j\sin\pi/2 + \cos(3\pi/4) - j\sin 3\pi/4$$
$$= 1 + 0.707 - j0.707 + 0 - j + (-0.707 - j0.707)$$
$$= 1 - j2.414$$

k=2

$$X(2) = \sum_{n=0}^{7} x(n) \cdot e^{-j\pi n/2} = 0$$

$$X(4) = \sum_{n=0}^{7} x(n) \cdot e^{-j\pi n} = 0$$

04

$$X(5) = \sum_{n=0}^{7} x(n) \cdot e^{-j5\pi n/4} = 1+j0.414$$

$$X(6) = \sum_{n=0}^{7} x(n) \cdot e^{-j3\pi n/4} = 0$$

$$X(7) = \sum_{n=0}^{7} x(n) \cdot e^{-j7\pi n/4} = 1+j2.414$$

$$X(k) = \{4, 1-j2.414, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414\}$$

But Here $x_1(n) = x((n+3))_N$ means $x_1(n)$ is obtained by shifting the sequence $x(n)$ circularly 3 times in clockwise direction.

$$x((n-m))_N = e^{-j2\pi km/N} X(k) = W_N^{km} \cdot X(k).$$

Here $m = -3$

$$X_1(k) = W_N^{km} X(k) = e^{-j2\pi k \times -3/8} X(k) = e^{j3\pi k/4} X(k)$$

$$X_1(k) = e^{j3\pi k/4} X(k). = 4$$

$$X_1(7) = (1-j2.414) \cdot \cos\frac{3\pi}{4} + j\sin 3\pi/4$$

$$= (1-j2.414(-0.707+j0.707))$$

$$= \cancel{-0.707 - j1.707}.$$

$$= \cancel{(2.61 \angle 2.61)}$$

$$2.613 \angle -67.498 \cdot 0.999 \angle 135$$

$$= 2.61 \angle 67.502$$

$$X_1(1) = 0.998 + j2.411$$

$$X_1(2) = X(2) \cdot e^{j3\pi/2} = 0$$

$$X_1(3) = X(3) \cdot e^{j9\pi/4} = (1-j0.414)(0.707+j0.707)$$

$$= 1+j0.414$$

$$X_1(4) = X(4) \cdot e^{j3\pi} = 0$$

$$X_1(5) = X(5) \cdot e^{j15\pi/4} = (1+j0.414)(0.707-j0.707)$$

$$= 1-j0.414$$

$$X_1(6) = 0$$

$$X_1(7) = 1-j2.414$$

$\{4, 0.998+j2.411, 0, 1+j0.414, 0, 1-j0.414, 0, 1-j2.413\}$

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$x_2(n)$ can be obtained by shifting the sequence $x(n)$ circularly two time anticlockwise direction.

$m = 2$

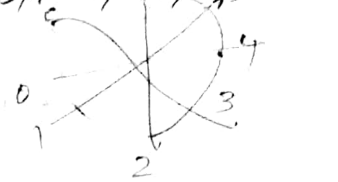
$DFT[x((n-m))_N] = X(k) \cdot e^{-j2\pi km/N} = X(k) \cdot W_N^{km}$

$Y_2(k) = e^{-j2\pi km/N} X(k)$

$= e^{-j2\pi k \times 2/8} X(k)$

$= e^{-j4\pi k/8} X(k) = e^{-j\pi k/2}$

$= e$



$X_2(k) = X(k)\, e^{-j\pi k/2}$

$X_2(0) = X(0) \cdot 1 = 4$

$X_2(1) = -2.414 - j$

$X_2(2) = 0$

$X_2(3) = X(3) \cdot e^{-j3\pi/2}$

$= 0.414 + j$

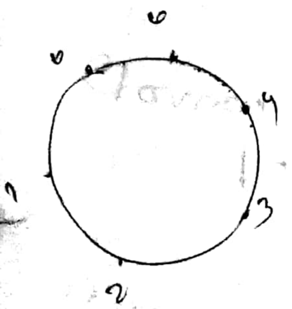
$X_2(4) = 0$

$X_2(5) = X(5) \cdot e^{-j5\pi/2} = 0.414 - j$

$X_2(6) = 0$

$X_2(7) = X(7) \cdot e^{-j7\pi/2} = -2.414 + j$

$X_2(k) = \{4, -2.414 - j, 0, 0.414 + j, 0, 0.414 - j, 0, -2.414 + j\}$



Linear Convolution from Circular Convolution

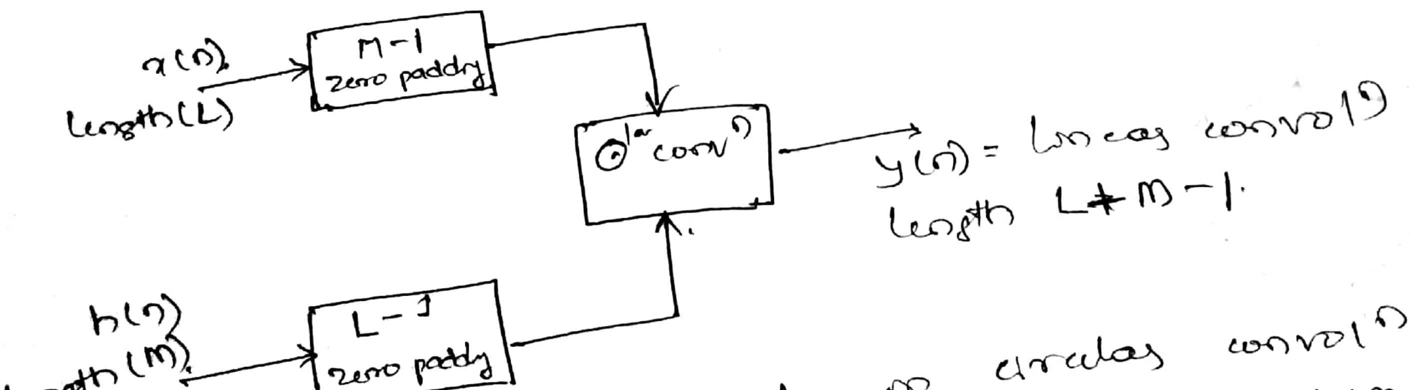
06 Consider two finite duration sequences $x(n)$ & $h(n)$, $x(n)$ represents signal to be filtered & $h(n)$ is impulse response of the system. Duration of $x(n)$ is L samples & $h(n)$ is M samples. Linear convol^n of $x(n)$ & $h(n)$ is given by

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k) \quad \text{——} (1).$$

$y(n)$ is finite duration sequence of $L+M-1$ samples. Circular convol^n of $x(n)$ & $h(n)$ gives N samples where $N = \text{Max}(L, M)$

If $M < L$ then zero padding is req^d to find circular convolution. i.e addition of $(L-M)$ zero's to sequence $h(n)$.

Block diagram.



To obtain no of samples in circular convol^n both $x(n)$ & $h(n)$ must be of $L+M-1$ length equal to $L+M-1$ then ~~To get y(k)~~ then this can be done by zero padding. $x(n)$ & $h(n)$ are multiplied to get y(n) $(L+M-1)$ point DFT of $x(n)$ & $h(n)$ by taking inverse. $y(n)$ can be obtained

By increasing length of sequences $x(n)$ & $h(n)$ to $(L+M-1)$ & applying circular convolution gives the same result as of linear convol^n.

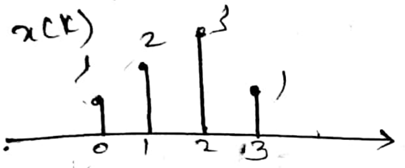
Q. Determine the O/P response $y(n)$ if $h(n) = \{1, 1, 1\}$, $x(n) = \{1, 2, 3, 1\}$ by using
(1) Linear convol^n (2) Circular convol^n
(3) Circular convol^n with zero padding —

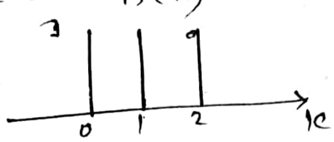
07

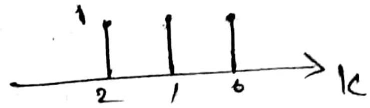
Sol^n
$x(n) = \{1, 2, 3, 1\}$ $\quad$ $L = 4$
$h(n) = \{1, 1, 1\}$. $\quad$ $M = 3$

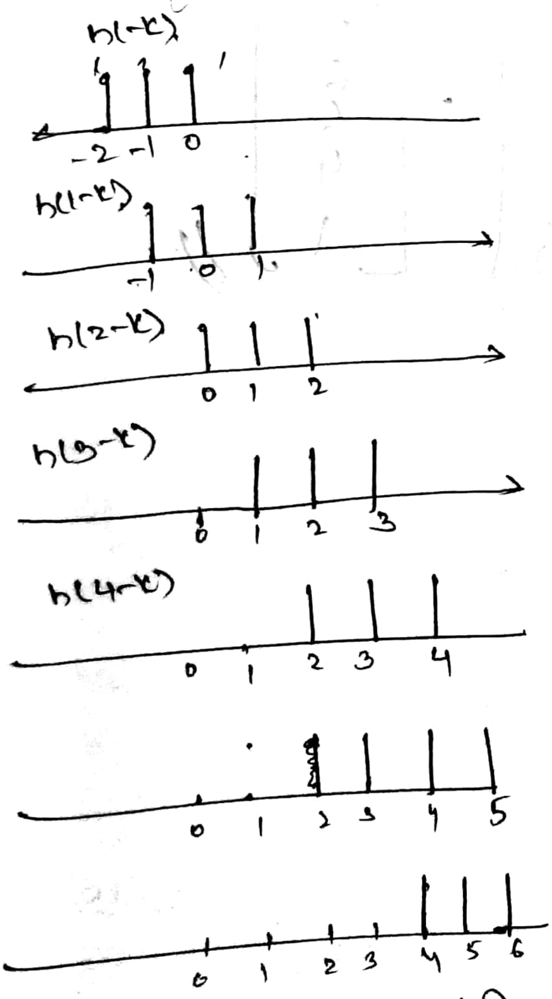
No of samples in Linear convol^n $L + M - 1 = 4 + 3 - 1 = 6$.

(1) Linear convol^n = $y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$.









$y(0) = \sum_{k=-\infty}^{\infty} x(k) h(-k) = 1$

$y(1) = \sum_{k=-\infty}^{\infty} x(k) h(1-k) = 1 + 2 = 3$

$y(2) = \sum_{k=-\infty}^{\infty} x(k) h(2-k) = 1 + 2 + 3 = 6$

$y(3) = \sum_{k=-\infty}^{\infty} x(k) h(3-k) = 2 + 3 + 1 = 6$

$y(4) = \sum_{k=-\infty}^{\infty} x(k) h(4-k) = 3 + 1 + 0 = 4$

$y(5) = \sum_{k=-\infty}^{\infty} x(k) h(5-k) = 1 + 0 = 1$

$y(6) = \sum_{k=-\infty}^{\infty} x(k) h(6-k) = 0$.

$y(n) = \{1, 3, 6, 6, 4, 1\}$

(ii) Circular convol^n $x(n) = \{1, 2, 3, 1\}$. $h(n) = \{1, 1, 1, 0\}$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+3+1 \\ 1+2+0+1 \\ 1+2+3+0 \\ 2+1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 6 \\ 6 \end{bmatrix}$$

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Filtering Long duration sequences

If long duration sequence $x(n)$ is to be processed then it must be divided into blocks ∵ long duration sequence is not practical to store. Then successive blocks are processed separately one at a time & results are combined to give desired o/p sequence which is same as sequence obtained by linear convolⁿ.

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Commonly two methods are used for filtering of the sectioned data. Those are
(1) Overlap save method
(2) Overlap Add method.

(1) Overlap save method

Consider i/p sequence is divided into blocks of data of size $N = L+M-1$. To make data sequence length $N = L+M-1$ each block contains last $(M-1)$ data points of previous block. But for 1st block $(M-1)$ points are set to zero. & also consider impulse response of length M.

$$\therefore x_1(n) = \{\underbrace{0, 0, 0, \dots 0}_{(M-1)\ \text{zeros}}, x(0), x(1) \dots x(L-1)\}$$

$$x_2(n) = \{\underbrace{x(L-M+1), \dots x(L-1)}_{\substack{(M-1)\ \text{data points} \\ \text{from } x_1(n)}}, \underbrace{x(L), \dots x(2L-1)}_{L\ \text{new data points}}\}$$

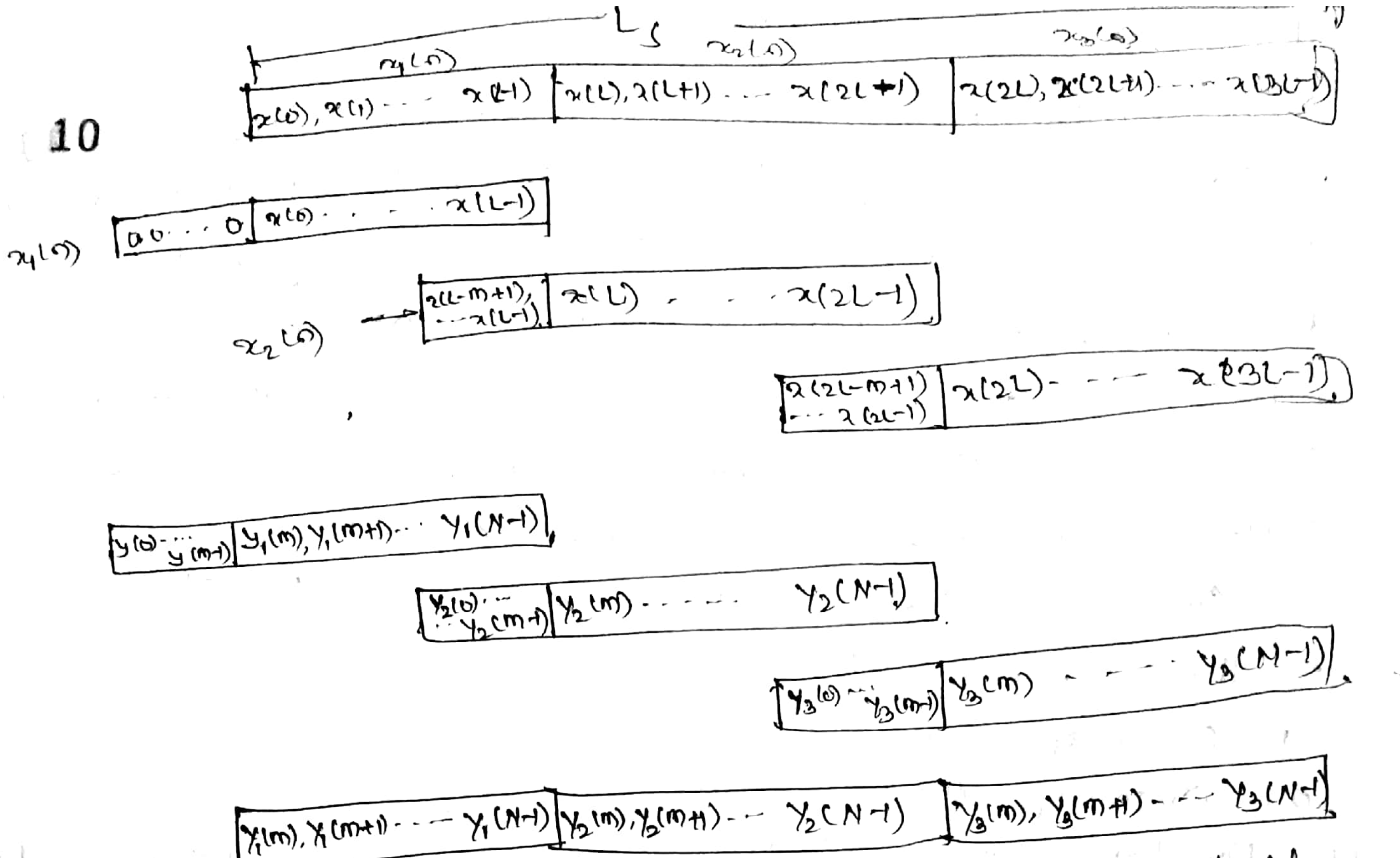
$$x_3(n) = \{x(2L-M+1), \dots x(2L-1), x(L) \dots x(3L-1)\}$$ & so on.

$h(n)$ & length is increased by similar way appending $(L-1)$ zeros with $h(n)$.

∴ Circular convolⁿ of $x_i(n)$ with $h(n)$.

$$y_i(n) = x_i(n) \circledast h(n).$$

Here first $(M-1)$ points of $y_i(n)$ i.e. $(x_i(n) \circledast h(n))$ discarded due to aliasing but remaining points are identical to linear convolⁿ. Then remaining points from successive sections are abutted to construct the final filtered o/p.



Because of aliasing Effect in Circular shift first (M-1) samples are overlap with O/P data block. Hence 1st (M-1) samples are discarded but ~~correct O/P samples~~ but last L. samples are correct O/P samples such blocks are fitted one after the other to get final O/P.

Ex: Let $L_s = 15$. $h(n)$ length is 3. & length of Each block is 5. Then I/P sequence is divided into no of block are

$$x_1(n) = \{\underbrace{0, 0}_{\substack{(M-1)\text{ zeros} \\ 2 \text{ zeros}}}, x(0), x(1), x(2)\}$$

$$x_2(n) = \{\underbrace{x(1), x(2)}_{\text{last 2 data points of previous block}}, x(3), x(4), x(5)\}$$

$$x_3(n) = \{x(4), x(5), x(6), x(7), x(8)\}$$

$$x_4(n) = \{x(7), x(8), x(9), x(10), x(11)\}$$

$$x_5(n) = \{x(10), x(11), x(12), x(13), x(14)\}$$

$$x_6(n) = \{x(13), x(14), 0, 0, 0\}$$

Then 5 point Circular convolution of $x_i(n)$ & $h(n)$ are performed by adding 2 zeros to $h(n)$ & $(M-1)$ points are discarded from $y_i'(n)$

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$y_1(n) = x_1(n) \circledN h(n) = \{\underbrace{y_1(0), y_1(1)}_{\text{discard}}, y_1(2), y_1(3)\ y_1(4)\}$

$y_2(n) = x_2(n) \circledN h(n) = \{\underbrace{y_2(0). y_2(1)}_{\text{discard}}. y_2(2)\ y_2(3)\ y_2(4)\}$

$y_3(n) = x_3(n) \circledN h(n) = \{y_3(0). y_3(1). y_3(2)\ y_3(3)\ y_3(4)\}$

$y_4(n) = y_4(n) \circledN h(n) = \{y_4(0). y_4(1). y_4(2)\ y_4(3)\ y_4(4)\}$

$y_5(n) = y_5(n) \circledN h(n) = \{y_5(0). y_5(1). y_5(2). y_5(3)\ y_5(4)\}$

$y_6(n) = y_6(n) \circledN h(n) = \{y_6(0). y_6(1). y_6(2)\ y_6(3)\ y_6(4)\}$

Then o/p blocks are abutted together g

$y(n) = \{y_1(2). y_1(3). y_1(4). y_2(2). y_2(3). y_2(4), y_3(2)\ y_3(3). y_3(4)$
$y_4(2). y_4(3). y_4(4). y_5(2). y_5(3), y_5(4). y_6(2)\ y_6(3).\}$

<u>Overlap – Add method</u>

Consider sequence of length L_s & $h(n)$ of length M. L_s is divided into no of data blocks of Length L & (M−1) zeros are appended to it to make data size (L+M−1). Then

$x_1(n) = \{x(0), x(1) \cdots x(L-1). \underbrace{0\ 0 \cdots}_{(M-1)\text{ zeros appended}}\}$

$x_2(n) = \{x(L), x(L+1) \cdots x(2L-1), 0\ 0 \cdots\}$

$x_3(n) = \{x(2L), x(2L+1) \cdots x(3L-1) \ldots \underbrace{0\ 0 \cdots}_{(M-1)\text{ zeros appended}}\}$

Illy (L−1) zeros are added to h(n) to compute N point circular convolⁿ. Last (M−1) points of each block are overlapped & added to first (M−1) points of next block. Hence it is called as overlap Add method

O/P blocks are

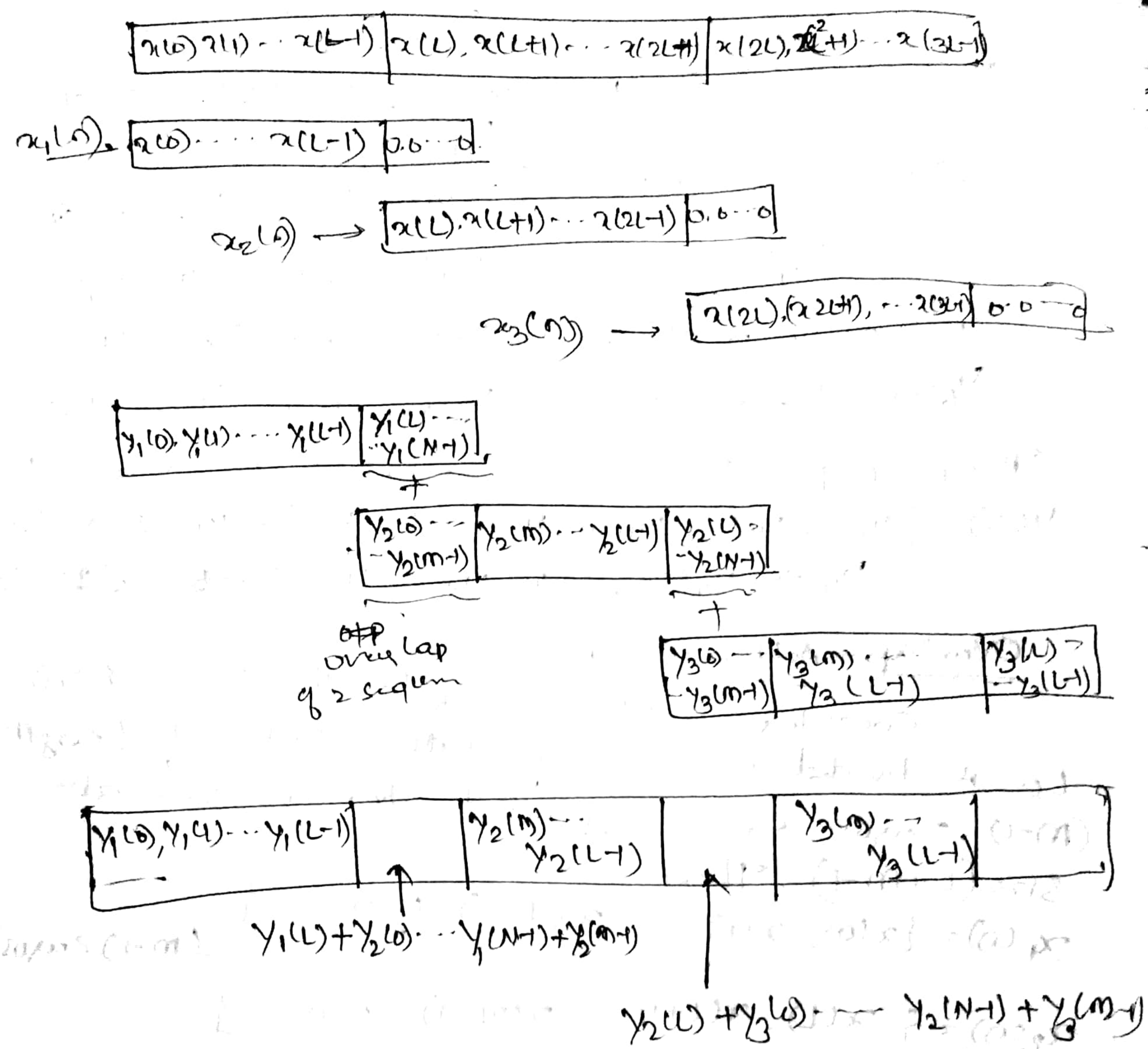
$y_1(n) = \{y_1(0), y_1(1) \cdots y_1(L-1), y_1(L), \cdots y_1(N-1)\}$

$y_2(n) = \{y_2(0). y_2(1), \cdots y_2(L-1), y_2(L) \cdots y_2(N-1)\}$

$y_3(n) = \{y_3(0). y_3(1) \cdots y_3(L-1)\ y_3(L) \cdots y_3(N-1)\}$

∴ O/P sequence □

$y(n) = \{y_1(0), y_1(1) \cdots y_1(L-1), y_1(L)+y_2(0) \cdots y_1(N-1)+y_2(M-1),$
$y_2(M) \cdots y_2(L)+y_3(0), y_2(L+1)+y_3(1) \cdots y_3(N-1)$



Here $(M-1)$ overlapping samples of o/p blocks are not discarded & no aliasing effect due to circular shift. Thus $(M-1)$ ~~blocks~~ samples of current block are added to 1st $(M-1)$ samples of next block

Jun/July-08

A long sequence is filtered through a filter of impulse response $h(n)$ to give the o/p $y(n)$ for the i/p $x(n)$. Given $x(n)$ & $h(n)$ as follows compute $y[n]$ using overlap & add method. **13**

$x(n) = [1,1,1,1,1,3,1,1,4,2,1,1,3,1,1,1]$ $h(n) = [1,-1]$

Use only five point circular convol^n in your approach.

5 point DFT the $N = L+M-1$

$5 = L+2-1$

$L = L = 4$

$x_1(n) = [1,1,1,1,0]$ Ⓝ $[1,-1,0,0,0]$

$x_2[n] = [1,3,1,1,0]$ Ⓝ $[1,-1,0,0,0]$

$x_3[n] = [4,2,1,1,0]$ Ⓝ $[1,-1,0,0,0]$

$x_4[n] = [3,1,1,1,0]$ Ⓝ $[1,-1,0,0,0]$

5 13 ... 26 32 44 48 54 58 61

1, 14, 18, 19, 21, 26, 27, 31, 39, 40, 42

$$y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$y_2(n) = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ -1 \\ 0 \\ -1 \end{bmatrix}$$

$N = L+M-1$

$= 16+2-1$

$N = 17$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$[1, 0, 0, 0, -1]$
$\quad 1 \quad 2 \quad -2 \quad 0 \quad -1$
$\quad\quad 4 \quad -2 \quad -1 \quad 0 \quad -1$
$\quad\quad\quad 3 \quad -2 \quad 0 \quad 0 \quad -1$

$= [1, 0, 0, 0, 0, 2, -2, 0, 3, -2, -1, 0, 2, -2, 0, 0, -1]$

14 Using linear convolution find $y(n) = x(n) * h(n)$ for the sequence $x(n) = (1, 2, -1, 2, 3, -2, -3, -1)$

Determine the response of an LTI (y(n)) with $h(n) = \{1, -1, 2\}$ for an i/p $x(n) = \{1, 0, 1, -2, 1, 2, 3, -1, 0, 2\}$ Employ over lap add method with block length $L = 4$.

$N = L + M - 1$

$L = 4$. $M = 3$. $N = 6$.

$h_1(n) = \{1, -1, 2, 0, 0, 0\}$

$x_1(n) = \{1, 0, 1, -2, 0, 0\}$

$x_2(n) = \{1, 2, 3, -1, 0, 0\}$

$x_3(n) = \{0, 2, 0, 0, 0, 0\}$

o/p $N = L_1 + M - 1$
$= 10 + 3 - 1$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 0 & 2 & -1 \\ -1 & 1 & 0 & 0 & 0 & 2 \\ 2 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \\ 4 \\ -4 \end{bmatrix}$$

$y_2(n) = \{1, 1, 3, 0, 7, -2\}$

$y_3(n) = \{0, 2, -2, 4, 0, 0\}$

$y(n) = \{$ ~~1, -1, 3, 0~~ $1, -1, 3, -3, 4+1, -4+1, 3, 0, 7+0, -2+(-2), 4\}$

$= \{1, -1, 3, -3, 5, -3, 3, 0, 7, 0, -2, 4\}$ //

```
1 -1 3 -3 4 -4
            1 1 3 0 7 -2
                        0 2 -2 4 0 0
------------------------------------------
1 -1 3 -3 5 -3 3 0 7 0 -2 4} 0 0}
```

08/07/06

Ex

Using the overlap - save method compute $y(n)$ of a FIR filter with impulse response $h(n) = \{3, 2, 1\}$ & i/p $x(n) = \{2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1\}$

Use only 8 point circular convolⁿ in your approach

15

$x(n) = \{2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1\}$

Ex

$L_1 = 12$ $h(n) = \{3, 2, 1\}$, $M = 3$

length of block $N = L + M - 1$

$8 = L + M - 1 = L + 3 - 1$

$L = 6$

$x_1(n) = \{0, 0, 2, 1, -1, -2, -3, 5\}$

$x_2(n) = \{-3, 5, 6, -1, 2, 0, 2, 1\}$ $h(n) = \{3, 2, 1, 0, 0, 0, 0, 0\}$

$x_3(n) = \{2, 1, 0, 0, 0, 0, 0, 0\}$

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ -1 \\ -2 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -3+10 \\ 5 \\ 6 \\ 4-3 \\ 2+2-3 \\ -1-2-6 \\ -1-4-9 \\ -2-6+15 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 6 \\ 1 \\ 1 \\ -7 \\ -14 \\ 7 \end{bmatrix}$$

$y_2(n) = \{-5, 10, 25, 14, 10, 3, 8, 7\}$

$y_3(n) = \{6, 7, 4, 1, 0, 0, 0, 0\}$

$N = L_1 + M - 1$

$= 12 + 2 - 1$

$= 13$

$y(n) = \{6, 7, 1, -7, -14, 7$

$-14, 7$

$75, 61, 1, -7,$

$6\ 7\ 4\ 1\ 0\ 0\ 0\ 0$

67

$\frac{24 \\ -6}{18}$

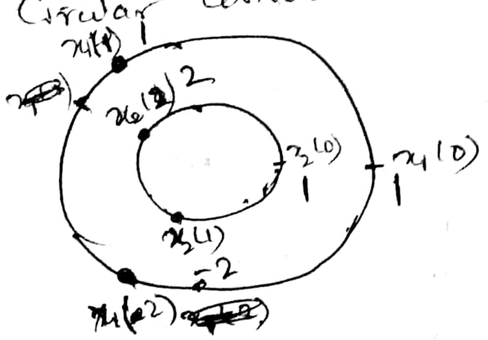
$N = L_s + M - 1$

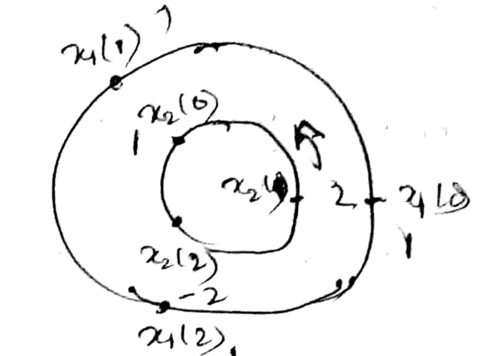
Scanned by CamScanner

16 For $x_1(n)$ & $x_2(n)$ given below compute $x_1(n) \circledast_N x_2(n)$. Take $N=3$

$x_1(n) = (1,1,1)$, $x_2(n) = (1,-2,2)$.

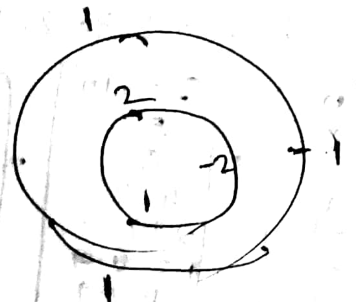
Circular convolution method





$y(0) = 1\times1 + 1\times2 + 1\times-2$
$= 1+2-2 = 1$

$y(1) = 1\times2 + 1\times1 + 1\times-2$
$= 2+1-2 = 1$



$y(2) = 1\times-2 + 1\times2 + 1\times1$
$= -2+2+1 = 1$

$y(k) = \{1, 1, 1\}$ //

Matrix multiplⁿ method

$$\begin{bmatrix} 1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\times1 + 2\times1 + -2\times1 \\ -2\times1 + 1\times1 + 2\times1 \\ 2\times1 + -2\times1 + 1\times1 \end{bmatrix} = \begin{bmatrix} 1+2-2 \\ -2+1+2 \\ 2-2+1 \end{bmatrix}$$

$= [1, 1, 1]$

Stockhams method

$X_1(K) = \sum_{n=0}^{2} x_1(n) \omega_3^{kn} = 1$

$W_N^{r} = e^{-j2\pi r/N} = e^{-j2\pi\times0/3} = 1$

$W_3^{k} = e^{-j2\pi/3} = \cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3}$
$= 0.814 - j0.580$
$100\angle{-0.32}$

~~$X_1(K) = 1 + 1\times W_3^{k} + 1\times W_3^{2k}$~~ $0 \le k \le 2$

~~$X_2(K) = \sum_{n=0}^{2} x_2(n)\, W_3^{kn} = 1 - 2W_3^{k} + 2W_3^{2k}$~~ $0 \le k \le 2$

$W_3^{2} = e^{-j2\pi\times2/3} = 0$
$= \cos 4\pi/3 - j\sin 4\pi/3$
$= \cos 240 - j\sin 240$
$= 0.325 - j0.94$
$= 0.999 \angle 1.23$

$X_1(0) = \sum_{n=0}^{2} x_1(n) W_3^{0} = x_1(0)W_3^0 + x_1(1) + x_1(2) = 1+1+1 = 3$

$X_1(1) = \sum_{n=0}^{2} x_1(n) W_3^{n} = x_1(0)W_3^0 + x_1(1)W_3^1 + x_1(2)W_3^2$

$= 1\times1 + 1\times(0.814 - j0.580) + 1\times(0.325 - j0.45)$

$= 1 + (0.814 - j0.580) + 0.325 - j0.45 = 1.139 - j1.03 = 1.53\angle 0.73$

$X_1(2) = \sum_{n=0}^{2} x_1(n) W_3^{2n} = x_1(0)W_3^0 + x_1(1)W_3^2 + x_1(2)W_3^4 = 1\times1 + 1\times(0.325 - j0.94) + 1\times(0.814 - j0.550)$

$Y(k) = X_1(k) \cdot X_2(k)$

$= (1 + \omega_3^k + \omega_3^{2k}) \times (1 - 2\omega_3^k + 2\omega_3^{2k})$

$= 1 - 2\omega_3^k + 2\omega_3^{2k} + \omega_3^k - 2\omega_3^{2k} + 2\omega_3^{3k} + \omega_3^{2k} - 2\omega_3^{3k} + 2\omega_3^{4k}$ 17

$Y(k) = 1 - \cancel{2\omega_3^k} + \cancel{2\omega_3^{2k}} + \omega_3^k - \cancel{2\omega_3^{2k}} + \cancel{2\omega_3^{3k}} + \omega_3^{2k} - \cancel{2\omega_3^{4k}} + \cancel{2\omega_3^{4k}}$

$= 1 + \omega_3^k + \omega_3^{2k} +$

$y(n) =$

$y(n) = 1 + \omega_3^1 + \omega_3^2 = 1 + 1\delta(n-1)$

$= 1 + 1 - 0.62 +$

Find 4 point circular convolution of seq

$x_1(n) = \{1 \cdot 2 \cdot 3 \cdot 1\}$ $x_2(n) = \{4, 3 \cdot 2 \cdot 2\}$ $0 \le k \le 3$

$X_1(k) = \sum_{n=0}^{3} x_1(n) \omega_4^{kn} = 1 + 2\omega_4^k + 3\omega_4^{2k} + \omega_4^{3k}$

$X_2(k) = \sum_{n=0}^{3} x_2(n) \omega_4^{kn} = 4 + 3\omega_4^k + 2\omega_4^{2k} + 2\omega_4^{3k}$

$Y(k) = X_1(k) \cdot X_2(k)$

$= (1 + 2\omega_4^k + 2\omega_4^{2k} + \omega_4^{3k})(4 + 3\omega_4^k + 2\omega_4^{2k} + 2\omega_4^{3k})$

$= 4 + 3\omega_4^k + 2\omega_4^{2k} + 2\omega_4^{3k} + 8\omega_4^k + 6\omega_4^{2k} + 4\omega_4^{3k} + 4\omega_4^{4k}$

$= 4 + 3\omega_4^k + 2\omega_4^{2k}$

$+$

$\omega_4^{4k} = \omega_4^{0k} = 1$, $\omega_4^{5k} = \omega_4^k$, $\omega_4^{6k} = \omega_4^{2k}$ $0 \le k \le 3$

$=$

$Y(k) = 17 + 19\omega_4^k + 22\omega_4^{2k} + 19\omega_4^{3k}$

$y(n) = 17 + 19\delta(n-1) + 22\delta(n-2) + 19\delta(n-3)$

$= (17, 19, 22, 19)$

$$\begin{bmatrix} 4 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+4+6+3 \\ 3+8+6+2 \\ 2+6+12+2 \\ 2+4+9+4 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

18 Using linear convol^n find $y(n) = x(n) * h(n)$ for seq $x(n) = (1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1)$ & $h(n) = (1, 2)$. Compare the result by solving the pbm using (a) Overlap-save (b) Overlap Add method

$y(n) = x(n) * h(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$

Sol^n

Overlap - Save

$M = 2$, $L = 3$

Assu^n $N = L + M - 1$

$N = 3 + 2 - 1 = 4$.

$x_1(n) = (\underbrace{0}_{M-1\ \text{zero}}, \underbrace{1, 2, -1}_{3\ \text{data}})$, $x_2(n) = (-1, 2, 3, -2)$

$x_3(n) = (-2, -3, -1, 1)$, $x_4(n) = (1, 1, 2, -1)$

$x_5(n) = (-1, 0, 0, 0)$ $h(n) = (1, 2, 0, 0)$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \\ 3 \end{bmatrix}$$

$y_2(n) = (-5, 0, 7, 4)$ $y_3(n) = (0, -7, -7, -1)$

$y_4(n) = (-1, 3, 4, 3)$. $y_5(n) = (-1, -2, 0, 0)$

$\cancel{-2}$ 1, 4, 3
$\cancel{-5}$, 0, 7, 4
$\cancel{0}$ -7, -7, -1
$\cancel{-1}$, 3, 4, 3
$\cancel{-1}$, -2, 0, 0

$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$

$N = L_s + M - 1 = 13 + 2 - 1 = 14$

$N = N_1 = L + M - 1$
4 + 1

0 1 4 3 0
$\cancel{0}$ 7 4 -7 -7
$\cancel{-7}$ -1 3 4 3
$\cancel{3}$, -2 | 0 0 0

$x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$

$x_1(n) = (1, 2, -1, 0)$ $\quad x_2(n) = (2, 3, -2, 0)$

$x_3(n) = (-3, -1, 1, 0)$. $\quad x_4(n) = (1, 2, -1, 0)$.

$h(n) = \{1, 2, 0, 0\}$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \\ -2 \end{bmatrix}$$

$y_2(n) = \{2, 7, 4, -4\}$

$y_3(n) = (-3, -7, -1, 2)$

$y_4(n) = (1, 4, 3, -2)$

$$\begin{array}{cccccccccccccc} 1 & 4 & 3 & -2 & & & & & & & & & & \\ & & & +2 & +7 & +4 & -4 & & & & & & & \\ & & & & & & -3 & -7 & -1 & 2 & & & & \\ & & & & & & & & & 1 & 4 & 3 & -2 \\ \hline \end{array}$$

$\{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\} = y(n)$

DEC 07/08 10M

(32) 54.48 29/12/13

20 Using overlap-save method, compute $y(n)$ of a FIR filter with impulse response $h(n) = \{3, 2, 1\}$ 5 7 19

2(b)

$x(n) = \{2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1\}$. Use only 8-point circular convol^n in your approach.

$N = L+M-1$

$8 = L+3-1 \Rightarrow L = 6$ — 1M

$x_1(n) = \{0, 0, 2, 1, -1, -2, -3, 5\}$

$x_2(n) = \{-3, 5, 6, -1, 2, 0, 2, 1\}$ $h(n) = \{3, 2, 1, 0, 0, 0, 0, 0\}$ — 2M

$x_3(n) = \{2, 1, 0, 0, 0, 0, 0, 0\}$

$y_1(n) = x_1(n) \circledast h_1(n) = \{7, 5, 6, 7, 1, -7, -14, 7\}$ — 2M

$y_2(n) = x_2(n) \circledast h(n) = \{-3, 10, 25, 14, 10, 3, 8, 7\}$ — 2M

$y_3(n) = x_3(n) \circledast h(n) = \{6, 7, 4, 1, 0, 0, 0, 0\}$ — 2M

7, 5, 6, 7, 1, -7, -14, 7

-5, 10, 25, 14, 10, 3, 8, 7

6, 7, 4, 1, 0, 0, 0, 0

$y(n) = \{6, 7, 1, -7, -14, 7, 25, 14, 10, 3, 8, 7, 4, 1\}$

DEC 2011 5M

c) Given the sequences $x(n) = \cos\frac{\pi n}{2}$ & $h(n) = 2^n$. Compute 4-point circular convolution.

$x(n) = \cos(\frac{\pi n}{2}) = \{1, 0, -1, 0\}$

$h(n) = 2^n = \{1, 2, 4, 8\}$

$y(n) = x(n) \circledast_N h(n)$

$$\begin{bmatrix} 1 & 8 & 4 & 2 \\ 2 & 1 & 8 & 4 \\ 4 & 2 & 1 & 8 \\ 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-4 \\ 2-8 \\ 4-1 \\ 8-2 \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ 3 \\ 6 \end{bmatrix}$$

$y(n) = \{-3, -6, 3, 6\}$

↑

10 8M

a) Explain the concept of Overlap-Add method, with necessary dgm — Expl^n – 4M

Eq^n – 2M

Flow dgm – 2M

~~consider~~ a

$x(n) = \{1, 0, 1, -2, 1, 2, 3, -1, 0, 2\}$ $h(n) = \{1, -1, 2\}$

Solve by using overlap Add method for 6 point circ conv

$N = L+M-1$

$6 = L+3-1$ $L = 4$. $h(n) = \{1, -1, 2, 0, 0, 0\}$ — (2m)

$x_1(n) = \{1, 0, 1, -2, 0, 0\}$
$x_2(n) = \{1, 2, 3, -1, 0, 0\}$
$x_3(n) = \{0, 2, 0, 0, 0, 0\}$ (3m)

$y_1(n) = \{1, -1, 3, -3, 4, 4)$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 0 & 2 & -1 \\ -1 & 1 & 0 & 0 & 0 & 2 \\ 2 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \\ 3 \\ -3 \\ 4 \\ -4 \end{bmatrix}$$

$y_1(n) = \{1, -1, 3, -3, 4, -4\}$
$y_2(n) = \{1, 1, 3, 0, 7, -2\}$
$y_3(n) = \{0, 2, -2, 4, 0, 0\}$ (3m)

$y(n) =$ 1, −1, 3, −3, 4, −4
1, 1, 3, 0, 7, −2
0, 2, −2, 4, 0, 0 (2m)

$\{1, -1, 3, -3, 5, -3, 3, 0, 7, 0, -2, 4\}$

A sequence $x(n) = (1, 2, -1, 3, 3, -3, -3, -1)$ is filtered through a filter having impulse response $h(n) = (3, 2)$, find the filter o/p using overlap add
~~$x_1(n) = (1, 2, -1, 3, 0)$, $x_2(n) = (3, -3, -3, -1, 0)$~~

A long sequence $x(n)$ is filtered thru a filter with impulse res $h(n)$ to yield the o/p $y(n)$ if $x(n) = (1, 1, 1, 1, 1, 3, 1, 1, 4, 2, 1, 1, 3, 1)$ $h(n) = (1, -1)$, compute $y(n)$ using overlap save technique. Use 5 pt circ

$N = L+M-1 = 14+2-1 = 15$

$y_1(n) = [-1, 1, 0, 0, 0]$
$y_2(n) = [0, 0, 2, -2, 0]$
$y_3(n) = [0, 3, -2, -1, 0]$
$y_4(n) = [1, 2, -3, -1, 0]$

$y(n) = [1, 0, 0, 0, 0, 2, -2, 0, 3, -2, -1, 0, 2, -2, -1]$ 0)

Ex: Compute DFT of a seq $(-1)^n$ for $N=4$

$x(n)=(-1)^n$, $x(0)=1$, $x(1)=-1$, $x(2)=1$, $x(3)=-1$

32

$X(k)=\sum_{n=0}^{N-1} x(n)\cdot e^{-j2\pi kn/N}$ $\quad k=0,1,2,\ldots N-1$

$X(0)=\sum_{n=0}^{3} x(n) = x(0)+x(1)+x(2)+x(3) = 1-1+1-1=0$

$X(1)=\sum_{n=0}^{3} x(n)\cdot e^{-j2\pi n/4} = 1+j-1-j=0$

$X(2)=\sum_{n=0}^{3} x(n)\cdot e^{-j4\pi n/4} = 1+1+1+1=4$

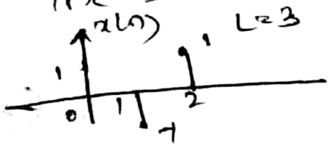
$X(3)=\sum_{n=0}^{3} x(n)\, e^{-j6\pi n/4} = 1-j-1+j=0$ $\quad X(k)=\{0,0,4,0\}$

Ex: Use the DFT to compute the linear convolution of the signal shown below

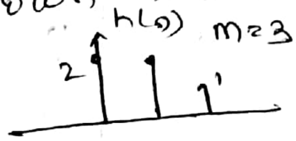
$L=M=3$

$N=L+M-1$
$=3+2=5$

$x(n)$, $L=3$



$h(n)$, $M=3$



$x(n)=\{1,-1,1,0,0\}$, $h(n)=\{2,2,1,0,0\}$

$X(k)=\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$

$=\sum_{0}^{4} x(n)\cdot e^{-j2\pi kn/5}$

$X(0)=1$

$X(1)=\sum_{n=0}^{4} x(n)\cdot e^{-j2\pi n/5} = -0.118+j0.364$

$X(2)=\sum_{n=0}^{4} x(n)\cdot e^{-j4\pi n/5} = 2.118+j1.5387$

$X(3)=\sum_{n=0}^{4} x(n)\cdot e^{-j6\pi n/5} = 2.118-j1.5387$

$X(4)=\sum_{n=0}^{4} x(n)\cdot e^{-j8\pi n/5} = -0.118-j0.364$

$H(k)=\sum_{n=0}^{4} h(n)\cdot e^{-j2\pi nk/5}$

$H(0)=5$ $\quad H(2)=0.691-j0.223$

$H(1)=1.809-j2.489$ $\quad H(4)=1.809+j2.489$

$H(3)=0.691+j0.223$

$Y(k)=X(k)\cdot H(k)=$ IDFT $Y(k)$ $\quad y(n)=\frac{1}{5}\sum_{k=0}^{N-1} Y(k)\, e^{j2\pi kn/5}$ $\quad n=0,1,2,3,4$

$y(n)=2$ ...

Q3 For $x_1(n)$ & $x_2(n)$ & N compute $x_1(n) \circledast x_2(n)$

(a) $x_1(n) = \delta(n) + \delta(n-1) + \delta(n-2)$, $N=3$

$x_2(n) = 2\delta(n) - \delta(n-1) + 2\delta(n-2)$

$x_1(n) = \{1, 1, 1\}$ $x_2(n) = \{2, -1, 2\}$ $N=3$

$$\begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \quad y(n) = \{3.3.3\}$$

(b) $x_1(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$. $N=5$

$x_2(n) = \delta(n) - \delta(n-2) + \delta(n-4)$

$x_1(n) = \{1, 1, -1, -1\}$. $x_2(n) = \{1, 0, -1, 0, 1\}$

$= \{1, 1, -1, -1, 0\}$

$$\begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -3 \\ -2 \\ 2 \end{bmatrix}$$

Ex Compute IDFT of the sequence $X(K) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\}$

$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi Kn/N}$ $n = 0.1 \cdots N-1 = \frac{1}{8}\sum_{K=0}^{7} X(K) \cdot e^{j\pi Kn/4}$, $n=0 \to 7$

$x(0) = \frac{1}{8}\sum_{K=0}^{7} X(K) \cdot e^{0} = \frac{1}{8}[5+0+1-\cancel{j}+0+1+0+1+\cancel{j}+0] = 9/8$

$x(1) = \frac{1}{8}\sum_{K=0}^{7} X(K) \cdot e^{j\pi K/4} = \frac{1}{8}[X(0) + X(1) \cdot e^{j\pi/4} + X(2)e^{j\pi/2} + X(3)e^{j3\pi/4}$
$+ X(4)e^{j\pi} + X(5)e^{j5\pi/4} + X(6)e^{j3\pi/2} + X(7)e^{j7\pi/4}]$

$= \frac{1}{8}[5 + 0 + (1-j) \times (\cos\pi/2\ j\sin\pi/2) + 0 + 1 \times (\cos\pi\ j\sin\pi) + 0 + (1+j)(\cos 3\pi/2\ j\sin$

$(0+1)$

$= \frac{1}{8}[5 + (1-j)(0-j) + (-1) + (1+j)(0+1)$

$= \frac{1}{8}[5 \cancel{-j} - 1 \cancel{-1} + 1 \cancel{+j}] - 0.5$

$4/8 = \frac{1}{2} = 0.5$

$X(2) = \frac{1}{8}\sum_{K=0}^{7} X(K) \cdot e^{j2\pi K/4}$

$= \frac{1}{8}[X(0) + X(1)$