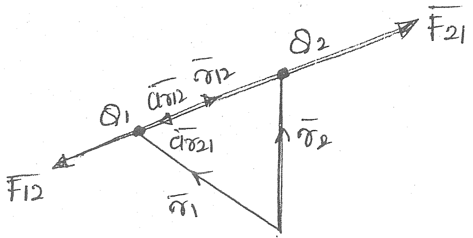
Fourth Semester BE. Degree Examination, June/July 2013

Field Theory

(1) a) State and explain Coulomb's Law of force between two point charges also indicate the units of quantities in the force equation

→ It states that " The electrostatic force of attraction or repulsion between the two point charges is directly propotional to the product of magnitude of charges and is inversly propotional to the square of distance between them."



Consider the two point charges Q_1 and Q_2 as shown in fig separated by distance 'r'. The charge Q_1 exerts a force on Q_2 while Q_2 also exerts force on Q_1 the force acting along line joining Q_1 and Q_2 the force exerted betn them is repulsive if the polarities are of same type and attractive if charge polories are different

$$F \propto Q_1 Q_2$$

$$F \propto \frac{1}{|r_{12}|^2}$$

$$F \propto \frac{Q_1 Q_2}{|r_{12}|^2}$$

$$F = K \frac{Q_1 Q_2}{|r_{12}|^2}$$

where K is constant of propotionality and value is $\frac{1}{4\pi\epsilon}$

$$\therefore \quad F = \frac{Q_1 Q_2}{4\pi\epsilon\, |r_{12}|^2}$$

Inverted form $F = \dfrac{Q_1 Q_2}{4\pi\epsilon\, |r_{12}|^2}\, \bar{a}_{r12}$

From above fig we have

$$\bar{r}_1 + \bar{r}_{12} = \bar{r}_2$$

$$\therefore \bar{r}_{12} = \bar{r}_2 - \bar{r}_1$$

and, $|\bar{r}_{12}| = \sqrt{r_2^2 + r_1^2}$

Now $\quad a_{r12} = \dfrac{\bar{r}_{12}}{|\bar{r}_{12}|} = \dfrac{\bar{r}_2 - \bar{r}_1}{\sqrt{r_2^2+r_1^2}}$

Then $\quad \vec{F} = \dfrac{Q_1 Q_2}{4\pi\varepsilon\left(\sqrt{r_2^2+r_1^2}\right)^2} \cdot \dfrac{\bar{r}_2 - \bar{r}_1}{\sqrt{r_2^2+r_1^2}}$

$$\therefore \boxed{\vec{F_{21}} = \frac{Q_1 Q_2}{4\pi\varepsilon\,(r_2^2+r_1^2)^{3/2}}\left(\bar{r}_2 - \bar{r}_1\right)} \quad \text{—— (1)}$$

$|||^{ly}$ Force on Q_1 due to Q_2 is given by

$$\boxed{\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon\,(r_1^2+r_2^2)^{3/2}}\left(\bar{r}_1 - \bar{r}_2\right)} \quad \text{—— (2)}$$

From (1) & (2) $\quad \vec{F_{21}} = -\vec{F_{12}}$

<u>Units :</u>

$Q_1\, Q_2$ = coulombs

$|r_1|$ = meters

ε_0 = Absolute Permittivity = 8.85×10^{-12} F/m

ε_r = Relative Permittivity value depends on type of medium

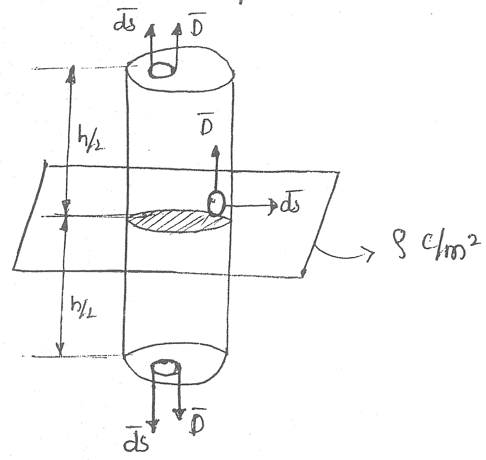
ε_r = 1 (for air medium).

b) State and apply Gauss law to obtain the expression for electric feild intencity due to an infinite sheet charge with a surface charge dencity ρ C/m² and area A. m²

Gauss Law: " The total normal electric flux passing through any closed surface is equal to the charge enclosed by that surface"

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed}$$

Electric Feild Intencity due to infinite sheetcharge:



Consider an uniform sheet charge with surface charge dencity of ρ C/m² and area as $A \cdot m^2$

$\therefore$ From Gauss law $\oint_S \bar{D} \cdot \bar{ds} = Q_{enclosed}$

For different surfaces $\oint_{TS} \bar{D}\, \bar{ds} + \int_{BS} \bar{D} \cdot \bar{ds} + \int_{CS} \bar{D} \cdot \bar{ds} = Q_{enclosed}$. with cylindrical co-s

$$\oint_{ts} D_z \bar{a}_z \, r\, dr\, d\phi\, \bar{a}_z + \int_{BS} D_z (-\bar{a}_z)\, r\, dr\, d\phi\, (-\bar{a}_z) + 0 = Q_{enclosed}$$

$$\oint_S D_z\, r\, dr\, d\phi + \int_S D_z\, r\, dr\, d\phi = Q_{enclosed}$$

$$D_z \int_{r=0}^{A} r\, dr \cdot \int_0^{2\pi} d\phi + D_z \int_{r=0}^{A} r\, dr \int_0^{2\pi} d\phi = Q_{enclosed}$$

$$D_z \left[\frac{r^2}{2}\right]_0^A \left[\phi\right]_0^{2\pi} + D_z \left[\frac{r^2}{2}\right]_0^A \left[\phi\right]_0^{2\pi} = Q_{enclosed}$$

$$D_z \left(\frac{A^2}{\cancel{2}}\right) \cancel{2}\pi + D_z \left(\frac{A^2}{\cancel{2}}\right) \cancel{2}\pi = Q_{enclosed}$$

$$D_z\, 2\cancel{\pi}\, \cancel{A^2} = \rho\, \cancel{\pi}\, \cancel{A^2}$$

$$D_z = \frac{\rho}{2}$$

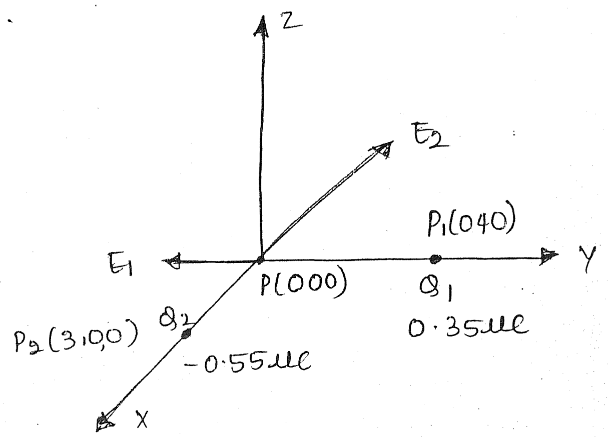
$$\bar{D} = D_z \bar{a_z}$$

$$\therefore \quad \bar{D} = \frac{\rho}{2} \bar{a_z}$$

WKT $\quad \bar{D} = \varepsilon \bar{E}$

$$\therefore \quad \boxed{\bar{E} = \frac{\rho}{2\varepsilon} \bar{a_z}}$$

c) Find: i) Electric Feild intensity and ii) Electric Flux denesity at the origin due to $Q_1 = 0.35\mu c$ at $(0,4,0)m$ and $Q_2 = -0.55\mu c$ at $(3,0,0)m$.



$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2} \bar{a_r}$$ From electric Feild intencity

$$\bar{E_1} = \frac{Q_1}{4\pi\varepsilon |\bar{P_1P}|^2} \bar{a_{P_1P}}$$

$\bar{P_1P} = 0 a_x - 4\bar{a_y} + 0\bar{a_z} = -4\bar{a_y}$

$|\bar{P_1P}| = 4$

$$\therefore \quad \bar{a_{P_1P}} = \frac{\bar{P_1P}}{|\bar{P_1P}|} = \frac{-4\bar{a_y}}{4} = -\bar{a_y}$$

$$\bar{E_1} = \frac{0.35 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 4^2} (-\bar{a_y})$$

$$\bar{E_1} = -196.6 \bar{a_y} \quad N/c$$

$$\bar{E_2} = \frac{Q_2}{4\pi\varepsilon |\bar{P_2P}|^2} \bar{a_{P_2P}}$$

$\bar{P_2P} = -3\bar{a_x} + 0\bar{a_y} + 0\bar{a_z} = -3\bar{a_x}$

$|\bar{P_2P}| = 3$

$$\bar{a_{P_2P}} = \frac{\bar{P_2P}}{|\bar{P_2P}|} = \frac{-3\bar{a_x}}{3} = -\bar{a_x}.$$

$$\bar{E_2} = \frac{-0.55 \times 10^{-6}}{4\pi \times 8.85 \times 10^{-12} \times 3^2} (-\bar{a_x})$$

$$\bar{E_2} = 549.42 \bar{a_x} \quad N/c$$

We have the relation $\bar{D} = \epsilon\bar{E}$ or $\bar{D} = \frac{Q}{4\pi r^2}\bar{a_r}$

Magnetic Flux density $\bar{D} = \epsilon\bar{E}$

$\therefore \bar{D_1} = \epsilon\bar{E_1}$

$= 8.85\times10^{-12} \times (-196.6\,\bar{a_y})$

$\bar{D_1} = -1.744\times10^{-9}\,\bar{a_y}$

$D_2 = \epsilon E_2$

$= 8.85\times10^{-12} \times 549.42\,\bar{a_x}$

$= 4.86\times10^{-9}\,\bar{a_x}$

$\therefore \bar{D} = \bar{D_1} + \bar{D_2}$

$\bar{D} = (4.86\,\bar{a_x} - 1.744\,\bar{a_y})\times10^{-9}$

$|\bar{D}| = \sqrt{4.86^2 + (-1.744)^2 \times (10^{-9})^2}$

$|\bar{D}| = 5.1620\times10^{-9}\ C/m^2$

2 a) Explain with mathematical expressions i) Potential difference ii) Absolute potential. iii) Potential gradiant.

<u>i) Potential Difference</u>

In electric feild, work done in moving a charge from B to A is given by

$$W = -Q\int_B^A \bar{E}\cdot\bar{dl}$$

If charge is selected as unit test charge then from above eq^n we get the work done in moving a unit charge from B to A in the feild $\bar{E}$ this work done in moving a unit charge from B to A in the feild $\bar{E}$ is called Potential Difference between the point B and A is denoted as V

$$\therefore \text{Potential Difference } V = -\int_B^A \bar{E}\cdot\bar{dl}$$

Thus " work done per unit charge in moving unit charge from B to A in the feild $\bar{E}$ is called potential difference between the point B & A

$$\boxed{V_{AB} = -\int_B^A \bar{E}\cdot\bar{dl}}$$

ii) Absolute potential

Absolute potential can be expressed of various points in the feild such absolute potential are measured wrt specified refference position, such a refference position is assumed to be at zero

Consider a potential difference due to moment of unit charge B to A in a feild of point charge is given by

$$V_{AB} = \frac{Q}{4\pi \epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

Now if the charge is moved from point ∞ to A i,e $r_B = \infty$

$$\frac{1}{r_B} = \frac{1}{\infty} = 0$$

$$\therefore V_{AB} = \frac{Q}{4\pi \epsilon_0 r_A} \, V \quad \text{---- potential at point A}$$

which is also called absolute potential of point A

$|||^{ly}$ absolute potential of point B is given as

$$V_B = \frac{Q}{4\pi \epsilon_0 r_B} \, V$$

iii) Potential Gradiant

Consider an electric feild due to a positive charge placed at the origin of sphere then

$$V = -\int \bar{E}\cdot d\bar{l} = \frac{Q}{4\pi \epsilon_0 r}$$

The potential decreases as distance of point from the charge increases $\therefore V_{AB} = \Delta V = -\bar{E}\cdot d\bar{l}$

The rate of change of potential wrt the distance is called the potential gradiant

$$\boxed{\frac{dv}{dL} = \lim_{\Delta L \to 0} \frac{\Delta V}{\Delta L} = \text{Potential Gradiant}}$$

(b) Derive and expression for equation of continuety of current

The continuety equation of current is based on the principle of conservation of charge which states that "charge can niether be cleated nor be distryed"

Consider a closed surface 's' with c/n denuity $\bar{J}$ then the total c/n crossing the surface is given as

$$I = \oint_S \bar{J} \cdot \bar{ds} \quad \text{--- (1)}$$

It has been mentioned eirlier that the c/n means the flow of +ve charges hence I is constituted due to outward flow of +ve charges from closed surface 's' according to the principle of conservation of Q

Q_i = charge within closed surface

$-\frac{dQ_i}{dt}$ = rate of decrease of charge inside the closed surface

The (-ve) charge indicates decrease in charge

Due to principle of conservation of charge this rate of decrease in same as rate of outword flow of charge which is a c/n

$$I = \oint_S \bar{J} \cdot \bar{ds} = -\frac{dQ_i}{dt} \quad (2) \text{ Intigral form of c/n continuety}$$

The negative sign in eqn indicates outward flow of c/n from closed surface so the eqn (2) indicating outword flow of c/n

$$\oint_S \bar{J} \cdot \bar{ds} = -I = +\frac{dQ_i}{dt}$$

The point form of continuety equation can be obtained from integral form from using divergence theorem

$$\oint_S \bar{J} \cdot \bar{ds} = \int_{vol} (\nabla \cdot \bar{J})\, dv$$

$$-\frac{dQ_i}{dt} = \int_V (\nabla \cdot \bar{J})\, dv$$

But $\quad Q_i = \int_V \rho_v \, dv$

where ρ_v = volume charge dencity

$$\therefore \int_v (\nabla \cdot \bar{J})\, dv = \frac{d}{dt}\left[\int_v \rho_v dv\right] = -\int_v \frac{\partial \rho_v}{\partial t} dv.$$

for constant surface the derivitive becomes partial derivitive

$$\therefore \int_v (\nabla \cdot \bar{J})\, dv = \int_v -\frac{\partial \rho_v}{\partial t}\, dv$$

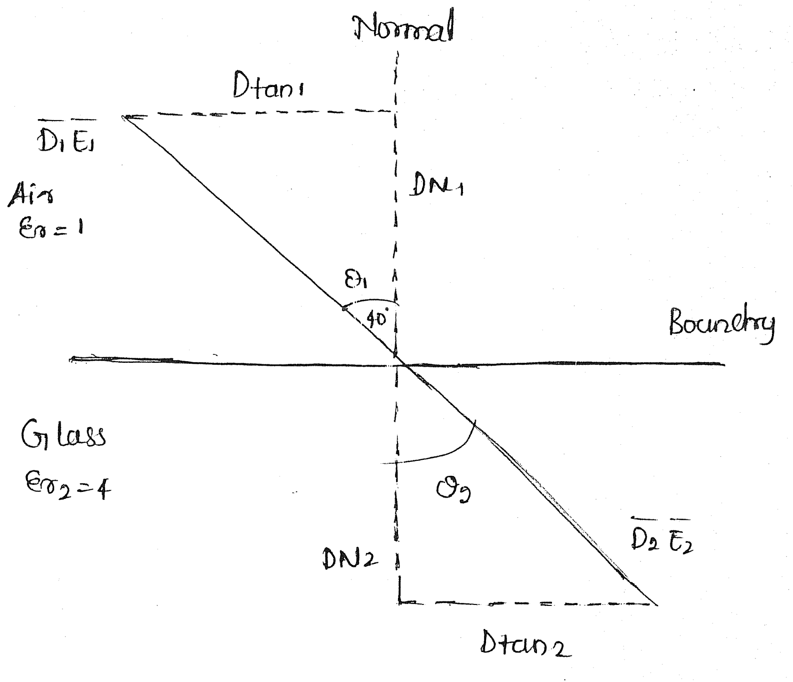
If the relation for any volume is ~~true~~ true for incrimental volume

$$(\nabla \cdot \bar{J})\, dv = \frac{\partial \rho_v}{\partial t}\, dv.$$

$$\boxed{\nabla \cdot \bar{J} = \frac{\partial \rho_v}{\partial t}}$$

This is point form of continuety equation of current

c) At the boundary between glass ($\varepsilon_r = 4$) and air the lines of electric feild make an angle of 40° with normal to the boundry If the electric flux dencity in air is $0.25\,\mu C/m^2$, determine the oreantation of magnetude of i) electric flux dencity and ii) electric feild intencity in glass



The arrangement as shown in above fig For boundry betn the two dielectrics

$$D_{N1} = D_{N2}$$

and $\dfrac{E_{N1}}{E_{N2}} = \dfrac{\varepsilon_2}{\varepsilon_1}$

Now $D_1 = 0.25\ \mu C/m^2$

$$\cos\theta_1 = \frac{D_{N1}}{D_1}$$

$$\cos 40^\circ = \frac{D_{N1}}{0.25\mu}$$

$\therefore\ D_{N1} = 0.1915\ \mu C/m^2$

As $D_{N2} = D_{N1}$

$\therefore\ D_{N2} = 0.1915\ \mu C/m^2$

$$D_1 = \sqrt{(D_{N1})^2 + (D_{tan1})^2}$$

$$0.25 = \sqrt{(0.1915)^2 + (D_{tan1})^2}$$

$\therefore\ D_{tan1} = 0.1607\ \mu C/m^2$

But $\dfrac{D_{tan1}}{D_{tan2}} = \dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{\varepsilon_{r1}}{\varepsilon_{r2}}$

$$D_{tan2} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} \times D_{tan1} = \frac{4}{1} \times 0.1607 = 0.6428\ \mu C/m^2$$

$$D_2 = \sqrt{(D_{N2})^2 + (D_{tan2})^2}$$

$$D_2 = 0.6707\ \mu C/m^2$$

and $\cos\theta_2 = \dfrac{D_{N2}}{D_2} = \dfrac{0.1915}{0.6707} = 0.2855$

$\therefore\ \theta_2 = 73.41^\circ$

3 (a) Derive Poisson's and Laplace equations starting from gauss Law

→ When charge and potentials are given at same boundry regions then potential and electric feild intenuity at any point may be obtained by using poissons and laplace equations

Consider Gauss Law in point form

$$\nabla \cdot D = \rho_v \quad \text{——} (1)$$

where ρ_v = volume charge dencity and
D = Electric flux dencity

WKT Homogenious Isotopic equation $|\bar{D}| = \epsilon \bar{E}$

then eqn (1) becomes

$$\nabla \cdot \epsilon \bar{E} = \rho_v$$

$$\nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon}$$

we have $E = -\nabla V$

$$\therefore \nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon}$$

$$-\nabla^2 V = \frac{\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}} \quad \text{——} (2)$$

Equation (2) is called Poisson's equation.

For dieletric medium $\rho_v = 0$ then eqn (2) can be rewritten as

$$\boxed{\nabla^2 V = 0} \quad \text{——} (3)$$

equation (3) is called laplace equation

where ∇^2 = Laplace operator

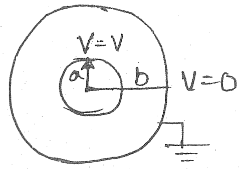
$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \longrightarrow \text{Rectangular coordinate}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \longrightarrow \text{Cylendrical coordinate}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial\phi^2} \longrightarrow \text{Spherical coordinate}$$

(b) Using laplace equation derive an expression for capacitance of concentric spherical capacitor the inner spherical conductor is of radius 'a' and potential 'V' while outer conductor of radius 'b' and potential zero.

→



The concentric conductor are shown above fig at $r=b$ and $V=0$ hence outer potential is shown at zero potential

The electric feild intecncity E will be only in only radial direc^n hence V is changing only in radial distance r and not the function of θ & ϕ

equation for $\nabla^2 V$ is spherical coordinate.

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial\phi^2} = 0$$

As their is no potential variation along θ and ϕ axis $\therefore$

$$\frac{\partial V}{\partial\theta} = \frac{\partial V}{\partial\phi} = 0.$$

From laplace equation $\nabla^2 V = 0$

i,e $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0$$

Intigrating on bothsides wrt r

$$\int \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$\therefore \quad r^2 \frac{\partial V}{\partial r} = C_1$$

$$\frac{\partial V}{\partial r} = \frac{C_1}{r^2}$$

Intigrating again wrt r on b.s

$$V = \frac{-C_1}{r^2} + C_2 \quad \text{——— (1)}$$

Substituting the boundary conditions

$r = a,\ V = V$

$r = b;\ V = 0$

$$V = \frac{-C_1}{a} + C_2$$

$$0 = \frac{-C_1}{b} + C_2$$

$$- \quad + \quad -$$

$$V = -C_1\left(\frac{1}{a} \cancel{+} \frac{1}{b}\right)$$

$$-V = -C_1\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$\therefore \quad C_1 = \frac{V(ab)}{(a-b)}$$

$$C_2 = \frac{C_1}{b} = \frac{V(ab)}{b(a-b)}$$

Substituting in potential equation (1)

$$V = \frac{-V(ab)}{(a-b)r} + \frac{V(ab)}{b(a-b)}$$

we have $\bar{E} = -\nabla V$

$$\therefore \quad \bar{E} = -\left[\frac{\partial V}{\partial r}\bar{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}a_\phi\right]$$

as feild is varying only along r-axis $\therefore \theta = \phi = 0$

$$\therefore \bar{E} = -\left(\frac{\partial}{\partial r}\left[\frac{-V(ab)}{r(a-b)} + \frac{Va}{(a-b)}\right]\bar{a_r}\right)$$

$$\therefore \bar{E} = -\left[\frac{V(ab)}{(a-b)r^2} + 0\right]\bar{a_r}$$

$$\bar{E} = \frac{-V(ab)}{r^2(a-b)}\bar{a_r}$$

$$|E| = \frac{V(ab)}{r^2(a-b)}\cdot\bar{a_r}$$

WKT $Q = \frac{\rho_s}{\cancel{A}} A$

also $Q = |D| A$

$$Q = \epsilon |E| A$$

$$\therefore Q = \epsilon \frac{V(ab)}{r^2(a-b)} 4\pi r^2$$

$$Q = \frac{4\pi\epsilon V(ab)}{(a-b)}$$

capacitance $\boxed{C = \frac{Q}{V_0} = \frac{4\pi\epsilon V(ab)}{V_0(a-b)}}$

(c) Determine wheather or not following potential feilds satisfies Laplace equation (i) $V = 2x^2 - 3y^2 + z^2$ (ii) $r^2 + z^2$

→ $V = 2x^2 - 3y^2 + z^2$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \longrightarrow \text{Rectangular coordinate}$$

$$\nabla^2 V = \frac{\partial^2}{\partial x^2}(2x^2 - 3y^2 + z^2) + \frac{\partial^2}{\partial y^2}(2x^2 - 3y^2 + z^2) + \frac{\partial}{\partial z^2}(2x^2 + 3y^2 + z^2)$$

$$= \frac{\partial}{\partial x}(4x) + \frac{\partial}{\partial y}(-6y) + \frac{\partial}{\partial z}(2z)$$

$$= 4 - 6 + 2$$

$$\nabla^2 V = 0$$

From Laplace equation $\nabla^2 V = 0$

It satisfies Laplace equation.

$V = r^2 + z^2$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2} \longrightarrow \text{Cylindrical coordinates}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}(r \times 2r) + \frac{1}{r}(0) + 2$$

$$= \frac{1}{r} \times 4r + 0 + 2$$

$$\nabla^2 V = 6$$

As laplace equation $\nabla^2 V = 0$

So it does not satisfies laplace equation

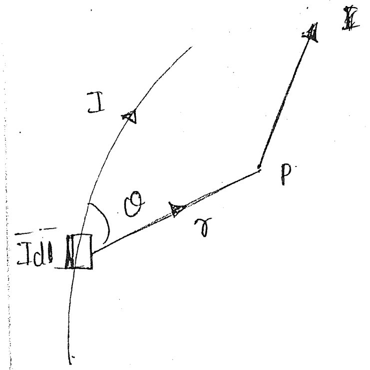
4 (a) Write Explanatory note on Biot Savart Law

→ This law states that " The magnetic feild intencity at point produced by differential c/n element is

a) directly propotional to the product of c/n I & differential c/n element dl

b) is directly propotional to the sin of angle between the element and the line joining point 'p' to element and

c) is Inversly propotional to the squore of distance R betn the Point P and c/n element (i)



$$d\bar{H} \propto I|d\bar{l}|$$

$$d\bar{H} \propto \sin\theta$$

$$d\bar{H} \propto \frac{1}{r^2}$$

$$d\bar{H} \propto \frac{I\, d\bar{l}\, \sin\theta}{r^2}$$

$$d\bar{H} = \frac{k I d\bar{l} \sin\theta}{r^2}$$

where k is constant of propotionality $k = \frac{1}{4\pi}$

$$\therefore d\bar{H} = \frac{I\,|d\bar{l}|\sin\theta}{4\pi r^2}\,\bar{a_r} \quad \Big| \because \bar{a_r} = \frac{\bar{r}}{|\bar{r}|}$$

(b) Discuss the concept of scalar and vector magnetic potential and arrive the expression for poissons equation in magnetostatics

→ Scalar Magnetic Potential

If V_m is the scalar magnetic potential then it must satisfies the equation $\nabla \times \nabla V_m = 0$ —— (1)

But scalar magnetic potential is related to magnetic feild intencity $\bar{H}$ as $\bar{H} = -\nabla V_m$

using equation (1)

$$\nabla \times (-\bar{H}) = 0 \qquad \text{i,e} \quad \nabla \times \bar{H} = 0$$

But $\quad \nabla \times \bar{H} = J \qquad$ i,e $\quad \bar{J} = 0$

The scalar magnetic potential V_m can be defined as for source free region where J i,e current dencity is zero

$$\bar{H} = -\nabla V_m \qquad \text{only for } \bar{J} = 0$$

$\parallel^{ly}$ to ther relation between $\bar{E}$ and electric scalar potential magnetic scalar potential can be expressed in terms of $\bar{H}$ as

$$V_{m\,a,b} = -\int_b^a \bar{H} \cdot d\bar{l} \quad \text{--- specified path}$$

Vector Magnetic Potential.

The magnetic potential denoted as $\bar{A}$ and measured in Wb/m it has to satifies the eqn that divergence of curl of a vector is always zero

$$\nabla \cdot (\nabla \times \bar{A}) = 0 \quad \text{--- } \bar{A} \text{ - Vector magnetic potentaal}$$

$$\nabla \cdot \bar{B} = 0 \quad \text{--- From Laplace in scalar magnetic Potential}$$

$$\bar{B} = \nabla \times A$$

Thus curl of vector magnetic potential is flux dencity

$$\nabla \times \bar{H} = \bar{J}$$

$$\nabla \times \frac{B}{\mu_0} = \bar{J} \quad \text{---} \quad [\because \bar{B} = \mu_0 \bar{H}]$$

$$\nabla \times \bar{B} = \mu_0 \bar{J}$$

$$\nabla \times \nabla \times \bar{A} = \mu_0 \bar{J}$$

Using vector identity to express left hand side we can write.

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 A = \mu_0 \bar{J}$$

$$\therefore \bar{J} = \frac{1}{\mu_0}\left[\nabla \times \nabla \times \bar{A}\right] = \frac{1}{\mu_0}\left[\nabla(\nabla \cdot \bar{A}) - \nabla^2 A\right] \quad \text{---} \quad (2)$$

Thus if vector magnetic potential is known ~~as~~ then C/n density $\bar{J}$ can be obtained for defining $\bar{A}$ the C/n density need to zero.

Poisson's Equation in Magnetostatics

In vector algebra the vector can fully defined if its curl and divergence are defined. For a vector magnetic potential $\bar{A}$. curl is defined as ~~$\nabla \times H$~~ $\nabla \times \bar{A} = \bar{B}$ which is known but to completely define $\bar{A}$ its divergence must be known assume $\nabla \cdot A$ the divergence A is zero this is consistant with some other conditions to be

Using eq^n (2)

$$J = \frac{1}{\mu_0}\left[-\nabla^2 \bar{A}\right]$$

$$\boxed{\nabla^2 A = -\mu_0 J}$$

This is called Poissons equation in Magnetostatics.

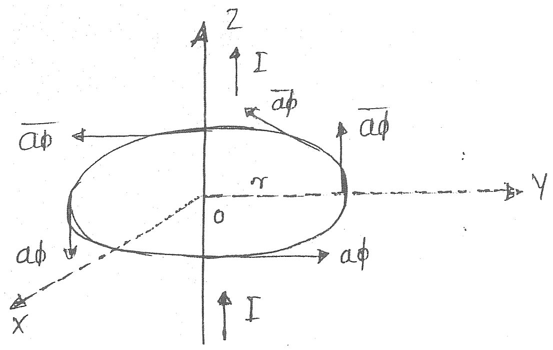
(c) State and proove Amperes circuit law and apply it to straight solid conductor to calculate the magnetic feild intencity.

→ "The line intigral of magnetic feild intencity $\bar{H}$ around a closed path is exactly equal to the direct C/n enclosed by that path" $\oint \bar{H} \cdot d\bar{l} = I_{enclosed}$

Proof

Consider a long straight conductor carrying c/n I placed along Z-axis as shown in below fig consider a circular closed path of radius 'r' which encloses the straight conductor carrying c/n I the point P is a ⊥ler distance from the conductor consider $\bar{dl}$ at point P which is in $\bar{a\phi}$ direction tangential to circular path at p.

$$d\bar{l} = r d\phi \, a\phi$$



From Biot savart's law

$$H = \frac{I}{2\pi r} a\phi$$

$$\therefore \oint_l \bar{H} \cdot \bar{dl} = \int_l \frac{I}{2\pi r} \bar{a\phi} \left(dr \, \bar{a_r} + r d\phi \, \bar{a\phi} + dz \, \bar{a_z} \right)$$

$$= \int_l \frac{I}{2\pi r} r d\phi$$

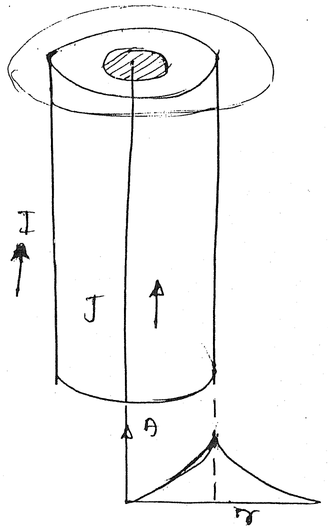
$$= \frac{I}{2\pi} \int_0^{2\pi} d\phi$$

$$= \frac{I}{2\pi} \times 2\pi$$

Thus $\boxed{\oint_l \bar{H} \cdot \bar{dl} = I \text{ enclosed}}$

Magnetic feild Intencity due to straight & Solid conductor

Consider a straight solid conductor of length h and it carrying the current of I with current dencity J. Let a be the crossectional area of solid conductor. Then magnetic feild incity can be obtained inside & outside the conductor as shown in below fig



$H = ?$ at $r < 0$ (H is inside the condr)

$$\oint_1 \bar{H} \cdot \bar{dl} = I_{enc}$$

$$\oint_1 H\phi\, \bar{a\phi}\; r\, d\phi\, a\phi = I_{enc}$$

$$\oint_1 H\phi\; r\, d\phi = I_{enc}$$

$$H\phi\, r \int_0^{2\pi} d\phi = I_{enc}$$

$$H\phi\, r\, 2\pi = J \times \pi r^2$$

$$H\phi = \frac{Jr}{2\pi}$$

$$\bar{H} = H\phi\, a\phi$$

$$= \frac{J \cdot r}{2\pi}\, a\phi$$

$$\boxed{\bar{H} = \frac{I}{2\pi a^2}\, a\phi}$$

$H = ?$ at $r > 0$ (outside the condr)

$$\oint_1 \bar{H} \cdot \bar{dl} = I_{enc}$$

$$\oint_1 H\phi\; a\phi\; r\, d\phi\, a\phi = I_{enc}$$

$$\oint_1 H\phi\; r\, d\phi = I_{enc}$$

$$H\phi\, r\, 2\pi = I_{enc}$$

$$H\phi = \frac{I}{2\pi r}$$

$$\bar{H} = H\phi\, a\phi$$

$$\boxed{\bar{H} = \frac{I}{2\pi r} \cdot a\phi}$$

(5) a) Find the ~~force or~~ expression for force on a differential C/n carrying elements.

→ Force exerted on a differential C/n element of charge dQ moving in a steady magnetic feild is given by

$$d\vec{F} = dQ\, v \times \bar{B} \;\; N \quad \text{——} (1)$$

The C/n dencity $\bar{J}$ can be expressed in terms of velocity of voleme charge dencity as,

$$\bar{J} = \rho_v \bar{V} \quad \text{——} (2)$$

But differential element charge can be expressed in terms of volume charge denuity as

$$dQ = \rho_v \, dv$$

Substituting the value of dQ in eq^n (1)

$$\therefore \; d\vec{F} = \rho_v \, dv \, \bar{V} \times B$$

Expresssing dF interms of $\bar{J}$ using eq^n (2) we can write.

$$d\vec{F} = \bar{J} \times \bar{B} \;\; dv \quad \text{——} (3)$$

we have the relation $\bar{J}\, dv = \bar{K}\, ds = I\, d\bar{l}$

Then force ordents on surface C/n dencity is given by

$$d\vec{F} = K \times \bar{B} \; ds \quad \text{——} (5)$$

Simillerly Force exerted on differential C/n element is given by

$$d\vec{F} = (I\, d\bar{l} \times B)$$

Intigrating eq^n (4) over volume.

$$\vec{F} = \int_V \bar{J} \times \bar{B} \; dv$$

Intigrating eq^n (5) over closed or open surface.

$$F = \int_S \bar{K} \times \bar{B} \; d\bar{s}$$

$|||^{ly}$ intigrating eqn ⑥ over closed path

$$\vec{F} = \oint I\, d\bar{l} \times \bar{B}$$

If the conductor is straigh the $\bar{B}$ is uniform along it then intigrating eqn ⑥ we get simple expression for force as

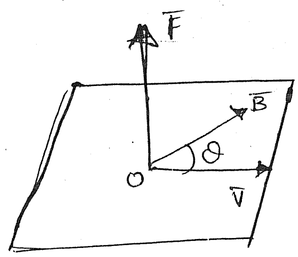
$$\boxed{\vec{F} = I\bar{L}\times\bar{B}}$$

magnitude of the Force is given by

$$\boxed{F = BIL\sin\theta}$$

(b) Define Lorent'z force equation and metion the applications of its solution

→ Lorentz Force : It is the product of charge and sum of Electric feild & cross product of velocity of charge with flux dencity (N)



we know that $F_e = Q\bar{E}$ —— ①

$$\vec{F_m} = Q|V||B|\sin\theta$$

$$\vec{F_m} = Q(V\times\bar{B}) \text{ —— ①}$$

From ① & ② $\vec{F} = \vec{F_e} + \vec{F_m}$

$$= Q\bar{E} + Q(V\times B)$$

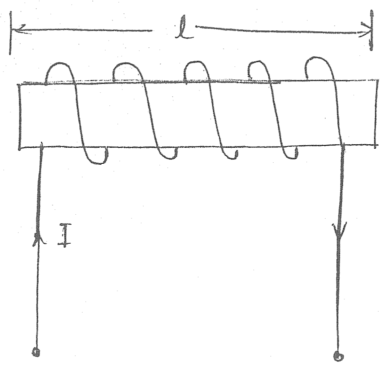
$$= Q(\bar{E} + (\bar{V}\times\bar{B}))$$

$$\boxed{\vec{F} = Q(E + (\bar{V}\times\bar{B}))}$$ Lorentz force equation

* it is used to find the total net force acting on a charge when Electric feild, flux dencity is given where charge is moving with specified velocity of Q C of charge.

(c) Calculate the inductance of solinoid of 200 turns wound tightly on cylindrical tube of length 60cm and diameter 6cm with air as media derive the expression used

→ <u>Inductance of solinoid</u>



Consider a solinoid of N number turns as shown in above fig let current flowing through solinoid I amps let 'l' be the length of solinoid and 'A' be the cross sectional area

Feild intencity inside the solinoid

$$H = \frac{NI}{l} \quad A/m \quad \text{——} (1)$$

The total flux linkage is given by

Total flux linkage $= N\phi = N(B)(A)$

$$\phi = N(\mu H)A$$

Total flux linkge $\phi = \mu NHA$

we have $H = \frac{NI}{l}$ From (1)

$\therefore$ Total flux linkage $\phi = \mu N\left(\frac{NI}{l}\right)A$

$$\therefore \phi = \frac{\mu N^2 IA}{l}$$

Indutance of solinoid $L = \frac{\text{Total flux linkage}}{\text{Total C/n}}$

$\therefore\ L = \frac{\mu N^2 I A}{l I}$

$\therefore\ \boxed{L = \frac{\mu N^2 A}{l}}$ or

$\boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$

Given : $N = 200$ turns

$\mu = \mu_0 = 4\pi \times 10^{-7}$ wb/A.m

$d = 6cm = 6 \times 10^{-2} m$ hence $r = \frac{d}{2} = 3 \times 10^{-2} m$

$l = 60 cm = 60 \times 10^{-2} m$

The inductance of solinoid is

$L = \frac{\mu N^2 A}{l}$

$= \frac{4\pi \times 10^{-7} \times (200)^2 \times \pi (3 \times 10^{-2})^2}{60 \times 10^{-2}}$

$L = 2.3687 \times 10^{-4} H$

$\boxed{L = 0.2368\ mH}$

6(a) With neccissity relation ships, Explain Faraday's law's electromagnetic induction for both static and time varying feilds.

Farday's I^{st} law

Whanever a changing flux linking with the coil an emf gets induced in the coil is known as Farday's I^{st} law

Fardy's II^{nd} Law

Magnitude of Induced emf is equal to the rate of change of Flux linkage

Farady's III^{rd} Law (Lenz's Law)

"The emf induced in the coil is opposite to own couse for it" the (−) minus sign represent the induced emf is in opposite direction of applied voltage.

$$e \propto -\frac{d\phi}{dt}$$

$$e = -N\frac{d\phi}{dt}$$ where N= no. of conductors

$$\oint_l \bar{E}\cdot\bar{dl} = -\frac{d}{dt}\int_s \bar{B}\cdot\bar{ds}$$

$$\boxed{\oint_l \bar{E}\cdot\bar{dl} = -\frac{\partial}{\partial t}\int_s \bar{B}\cdot\bar{ds}}$$ — ① Integral form of Fardays law

From Stokes theorem we have

$$\oint_l \bar{H}\cdot\bar{dl} = \oint_s (\nabla\times\bar{H})\cdot\bar{ds}$$

$$\oint_l \bar{E}\cdot\bar{dl} = \oint_s (\nabla\times\bar{E})\bar{ds}$$

$$\oint_s \cdot(\nabla\times\bar{E})\,ds = \frac{\partial}{\partial t}\int_s \bar{B}\cdot\bar{ds}$$ $\because$ From ①

$$\boxed{\nabla\times\bar{E} = -\frac{\partial B}{\partial t}}$$

is called Fadday's law in point form or Maxwell's I^{st} equation.

(b) Starting From Forday's Law of electromagnetic Induction derive $\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$

→ From Forday's law of induction " Indireeed emf is equal to the rate of change of magnetic flux linking with the closed circuit"

$$e = -N \frac{d\phi}{dt} \text{ volts} \quad \text{---} (1)$$

where N = number of turns

e = induced emf

Let us assume single turn circuit for N=1

$$\therefore \quad e = -\frac{d\phi}{dt} \text{ volts} \quad \text{---} (2)$$

The minus sign in both eq^ns (1) & (2) indicates the direc^n of induced emf in the coil is ~~very~~ opposite to very own couse for it i,e applied voltage which is Lenz's law

Let us consider the faraday's law of induced emf is scalar quantity measured in volts thus induced emf is given as

$$e = \oint \bar{E} \cdot \bar{dl} \quad \text{---} (3)$$

The above equation is voltage about closed path

The magnetic flux passing through the surface area is

$$\phi = \int_S \bar{B} \cdot \bar{ds}$$

where B = magnetic flux density

eq^n (2) can be rewritten as

$$e = -\frac{d}{dt} \int_S \bar{B} \cdot \bar{ds} \quad \text{---} (4)$$

From eq^n (3) & (4) we get

$$e = \oint \bar{E} \cdot \bar{dl} = -\frac{d}{dt} \cdot \int_S \bar{B} \cdot \bar{ds} \quad \text{---} (5)$$

Their are two conditions for induced emf the closed ckt in which the induced emf is stationary varying with time From (5) $\bar{B}$ may be changing coordinates as well as time

$$\oint \bar{E} \cdot \bar{dl} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot \bar{ds}$$

This is ~~th~~ similler to the transformer action

$$\int_S (\nabla \times \bar{E}) \cdot \bar{ds} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot \bar{ds}$$

Assuming both the intigrals ~~so~~ are take ~~closed~~ Identical surface

$$(\nabla \times \bar{E})\, \bar{ds} = -\frac{\partial \bar{B}}{\partial t} \cdot \bar{ds}$$

Hence, Finally

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

(c) Find the frequency at which conduction current dencity and displacement current dencity are equal in medium with $\sigma = 2 \times 10^{-4}$ S/m and $\varepsilon_r = 81$

→ we have the Condution c/n dencity equation as

$$\left|\frac{J_C}{J_D}\right| = \frac{\sigma}{\omega\varepsilon} = \frac{2 \times 10^{-4}}{2\pi f \times 81 \times \varepsilon_0}$$

$$\therefore \boxed{f = 44.38 \text{ kHz}}$$

7 (a) What is uniform plane wave? Explain its propegation in free space with neccessity equation

→ An electromagnetic wave travelling in plane with uniform velocity is called as uniform plane wave.

$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t}$ · Maxwell's third equation

$\nabla \times \bar{H} = \bar{J}c + Jc$

$= \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}$ —— (2)

Taking Curl on both sides

$\nabla \times (\nabla \times \bar{E}) = -\mu \, \nabla \times \frac{\partial \bar{H}}{\partial t}$

$\nabla \times (\nabla \times \bar{H}) = -\mu \frac{\partial}{\partial t} (\nabla \times \bar{H})$

From vector Identity $\nabla(\nabla \cdot \bar{E}) - \nabla^2 E = \nabla \times \nabla \times \bar{E}$

$\nabla(\nabla \bar{E}) - \nabla^2 \bar{E} - \mu \frac{\partial}{\partial t} \left(\sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}\right)$

$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\mu \sigma \frac{\partial \bar{E}}{\partial t} - \mu \varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}$

$\nabla(\nabla \cdot \bar{E}) = \nabla^2 \bar{E} - \mu \sigma \frac{\partial \bar{E}}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2}$

$\nabla^2 \bar{E} - \mu \sigma \frac{\partial \bar{E}}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = \nabla \left(\frac{\rho_v}{\varepsilon}\right)$

For free space and charge free region

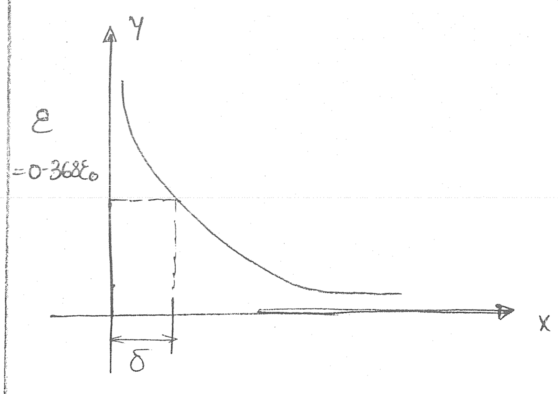
$$\boxed{\nabla^2 \bar{E} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = 0}$$

Wave equation in free space

(b) Define Skin depth and depth of pennitration.

→ Skin effect can be defined as the

Depth of Pennetaration



$$\mathcal{E} = \mathcal{E}_0 e^{-\alpha x}$$

$$\alpha \ \mu \ \gamma = \delta = \frac{1}{\alpha}$$

$$\mathcal{E} = \mathcal{E}_0 e^{-\alpha \frac{1}{\alpha}}$$

$$= \mathcal{E}_0 e^{-1}$$

$$\therefore \mathcal{E} = 0.3686 \ \mathcal{E}_0$$

α for good conductor

$$\alpha = \sqrt{\frac{\mu \omega \varepsilon}{2}}$$

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\frac{\mu \omega \varepsilon}{2}}}$$

$$\delta = \sqrt{\frac{2}{\omega \mu \varepsilon}}$$

$$\delta = \sqrt{\frac{2}{2\pi f \mu \varepsilon}}$$

$$\therefore \boxed{\delta = \frac{1}{\sqrt{\pi f \mu r}}}$$

(c) For copper conductivity is 58 MS/m find skin depth at a frequency of 10 MHz

Given $\sigma = 58 \ M\mho/m$

$f = 10 \ MHz$

$\mu_0 = 4\pi \times 10^{-7}$ $\qquad \mu_r = 1$

Depth of Pennitration or skin dept can be given as

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$= \frac{1}{\sqrt{3.142 \times 10 \times 10^{6} \times 4\pi \times 10^{-7} \times 58 \times 10^{6}}}$$

$$= 2.0895 \times 10^{-5}$$

$$\delta = 20.89\ \mu m$$

8 (a) With neccessity equation explain standing wave equation ratio

→ Consider a uniform plane wave travelling in Loss less medium with no reflected wave present in medium. It is observed that relative amplitude of feild same at every point It is obivious that instantaneous feild under goes phase change of $\beta (z_2 - z_1)$ when the probe is moved ~~to~~ from $z = z_1$ to $z = z_2$ along a wave thus we can conclude an unattinuated travelling wave shows equal voltage amplitude charecteristics Now consider uniform plane wave travelling in loss less medium it gets relflected by back the perfect condr The result in generating standing wave equation when voltage probe is located at interferance of $z=0$ & at every intigral no. of half wavelength from the interference in medium 1 $(z<0)$. the o/p of the probe will be zero when positions of probe is interchanged the amplitued of feild varies as $\sin |\beta z|$ as shown in fig

(b) Find wheater the wet morshy soil choreeterised by $\sigma = 10^{-2}$ S/m $\epsilon_r = 15$ and $\mu = 1$ may be consiederd as conductor a dieleetric or niether for the freqeuncies i) 60 Hz, ii) 1 MHz iii) 100 MHz (iv) 10 GHz

$\frac{\sigma}{\omega\epsilon} >> 1$ For good conductor

$\frac{\sigma}{\omega\epsilon} << 1$ For good lossy dielectric conductor

$\frac{\sigma}{\omega\epsilon} = 0$ For Perfect dieletric

(i) 60 Hz = f

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-2}}{2\pi \times 60 \times 15 \times \epsilon_0} = 200\cdot 8 \times 10^{3} >> 1$$

It is good conductor at 60 Hz

(ii) f = 1 MHz

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-2}}{2\pi \times 1M \times 15 \times \epsilon_0} = 11\cdot 98 >> 1$$

Hence it is good conductor

(iii) f = 100 MHz

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-2}}{2\pi \times 100M \times 15 \times \epsilon_0} = 0\cdot 1198 << 1$$

It is lossy dielectric conductor

(iv) f = 10 GHz

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-2}}{2\pi \times 10G \times 15 \times \epsilon_0} = 0\cdot 0011 \simeq 0$$

For this frequeney soil is perfect.