Load flow Analysis

Introduction

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* Power flow is the most important study in planning & expansion of power system.
* It is also the study frequently done by the utilities in planning & planning operations
* purpose of the study is to compute the steady state operating cond$^n$s of the system i.e. voltage magnitudes & phase angles at the buses
* From these even line flow (MW & MVAr), real & reactive power supplied by generator & loading of ffrs etc can also be calculated.
* overload cond$^n$s can be detected
* poor voltages existing in parts of the system can be detected
* angle seperation bet$^n$ generator buses or bet$^n$ any two buses gives a qualitative idea of the steady state or small disturbance stability of the power system.
* planning Engg perform LFA for diff configurations & loading cond$^n$s before deciding on final configuration.
* The mathematical model for the study of power flow is the set of non-linear algebraic eq$^n$s
* These eq$^n$s can be expressed as a set of real eq$^n$s with voltage either in the rectangular or polar form.
* n/w matrix used in deriving the above eq$^n$ is generally $Y_{bus}$
* $Y_{BUS}$ is preferred b'cos it's sparse structure, and efficient ordering technques enable fast sol$^n$s using only nonzero entries.

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* $Y_{BUS}$ is preferred b'cos it's sparse structure and efficient programming techniques enable fast solⁿs using only nonzero entries.

* When loads and/or generators are connected at the buses, these give rise to the post-constraints. 5

* We can specify the constraints in terms of $P_i$, $Q_i$ - the net injected real & reactive power respectively - or $|V_i|$ the magnitude of voltages.

* voltage magnitude at a bus can be kept cost. through a voltage regulator as in case of a generator bus or by means of tap changing tfr in the case of a load bus

* we also need a reference bus in the system whose phase angle is zero

* at any bus there are 4 variables

  $P_i$, $Q_i$ - injected real & reactive powers &

  $|V_i|$ & $\delta_i$ - voltage magnitude & the phase angle of voltage at the bus

* two of these are specified at every bus, & this gives rise to the classification of buses as follows

Real and reactive powers can now be expressed as

$$P_i \text{ (real power)} = |V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$i = 1, 2 \cdots n$$

$$Q_i \text{ (reactive power)} = -|V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$i = 1, 2 \cdots n$$

* These two eq$^{ns}$ represent $2n$ power flow equations at $n$ buses of the power system
* Each bus is characterised by four variables : $P_i$, $Q_i$ $|V_i|$ & $\delta_i$ resulting into $4n$ variables
* above two eq$^n$s can be solved for $2n$ variables if remaining $2n$ variables are specified
* Practical considerations allow a power system analyst to fix a priori two variables at each bus.
* The sol$^n$ for remaining $2n$ bus variables is rendered difficult as the eq$^n$s are non-linear algebraic eq$^n$s (bus voltages are involved in product form & sine & cosine terms are present)
* therefore explicit sol$^n$ is not possible
* sol$^n$ can only be obtained by iterative numerical techniques

Depending upon two variables specified priori, the buses are classified into three categories

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I) PQ bus /- Load bus

* at all these type of buses Powers $P_i$ & $Q_i$ are known

($P_{Di}$ & $Q_{Di}$ are known from load forecasting & $P_{Gi}$ & $Q_{Gi}$ are specified)

* The unknowns are $|V_i|$ & $\delta_i$

* A pure load bus (no generating facility at the bus) i.e. $P_{Gi} = Q_{Gi} = 0$) is a PQ bus.

) PV bus / Generation bus / Voltage controlled bus

* at this bus $P_{Di}$ & $Q_{Di}$ are known a prior & $|V_i|$ & $P_i$ are specified.

* The unknown are $Q_i$ & $\delta_i$.

Slack bus / Swing bus / Reference bus -

* $|V_i|$ & $\delta_i$ are specified at this bus (normally $\delta_i$ is set equal to zero)

* $P_i$ & $Q_i$ are unknown

* normally there is only one bus of this type in a given power system.

Need of slack bus

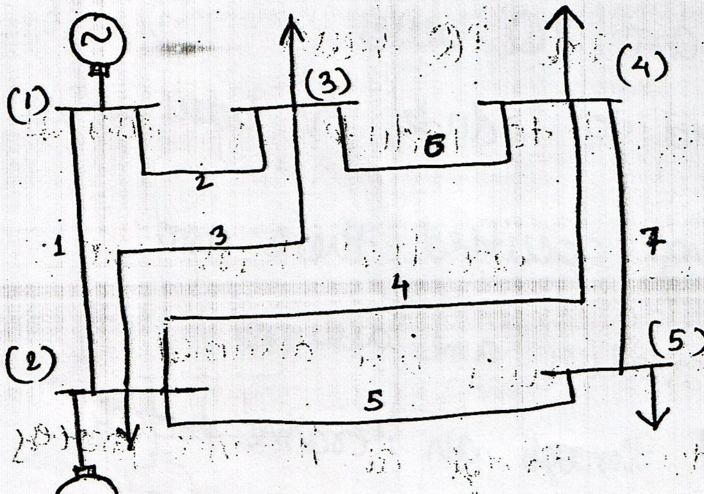
* real & reactive powers can't be fixed priori at all the buses as net complex power flow into the n/w is not known advance, the system power loss being unknown till LFA is complete.

* therefore it is necessary to have one bus at which complex power is unspecified, so that it supplies the ~~[illegible]~~ difference in the total system load plus losses & the sum of the complex powers specified at the remaining buses

complex power at slack bus = (Total system load + losses) − (sum of the complex powers specified at the remaining buses)

* for the same reason slack bus must be a generator bus
* The complex power allocated to this bus is determined as part of the soln.
* In order that the variations in real & reactive powers of the slack bus during the iterative process be small % of its generating capacity, the bus connected to the largest generating station is normally selected as the slack bus

e.g.




| Bus No | Bus voltage | Gen. (MW) | Gen (MVAR) | Load (MW) | Load (MVAR) | Load |
|---|---|---|---|---|---|---|
| 1 | 1.06+j0.0 | 0 | 0 | 0 | 0 | slack |
| 2 | 1.00+j0.0 | 40 | 30 | 20 | 10 | |
| 3 | 1.00+j0.0 | 0 | 0 | 45 | 15 | load |
| 4 | 1.00+j0.0 | 0 | 0 | 40 | 5 | load |
| 5 | 1.00+j0.0 | 0 | 0 | 60 | 10 | load |

# G-S Method

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* It is an iterative algorithm for solving a set of non-linear algebraic eq$^n$
* to start with, sol$^n$ vector is assumed based on practical experience
* the eq$^n$ is used to obtain the revised value of a particular variable by substituting in present values of the remaining variable
* The process is then repeated for all the variables thereby completing one iteration.
* Iterative process is then repeated till the sol$^n$ vector converges within prescribed accuracy
* convergence is quite sensitive to the starting values assumed
* To apply G-S method to load flow studies, let all buses except slack are PQ buses
* method can easily adopted to include PV buses as well
* the slack bus voltage being specified, there are (n-1) bus voltages starting values are assumed
* These values are then updated through an iterative process
* voltage at $i^{th}$ bus is obtained as follows

$$I_i = \frac{(P_i - jQ_i)}{V_i^*} \rightarrow (1)$$

we also have

$$J_i = \sum_{k=1}^{n} Y_{ik} \cdot V_k \quad \text{—— (2)} \quad i = 1, 2 \ldots n.$$

from this eq$^n$

$$Y_{ii} \cdot V_i = J_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} \cdot V_k$$

&

$$V_i = \frac{1}{Y_{ii}} \left[ J_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad \text{—— (3)}$$

substituting for $J_i$ eq$^n$ (1) in eq$^n$ (3)

$$V_i = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad i = 2, 3 \ldots n \quad \text{—— (4)}$$

* during each iteration voltages at all PQ buses is updated by eq$^n$ (4).
* $V_1$ is slack bus voltage & it is fixed
* Iterations are repeated till no bus voltage magnitude changes more than a prescribed value.
* The computation process is then said to converge to a sol$^n$

* Computation of slack bus power —

12 Substituting all bus voltages computed in previous step along with $V_1$ in the following eqn. yields slack bus power

$$P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} \cdot V_k \qquad i = 1, 2 --- n \quad — (4a)$$

When PV buses are also present

At PV buses P & |V| are specified

+ Q & δ are unknown

* Values of Q & δ are to be updated in every as iteration through appropriate bus eqn.

for ith PV bus

$$Q_i = -\text{Im}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\} \quad — (5)$$

$Q_2 = -\text{Im} \; V_2^* [Y_{21} \cdot V_1 + Y_{22} V_2 + Y_{23} V_3 + Y_{24} V_4]$

* The revised value of $Q_i$ is obtained by substituting most updated values of voltages on R.H.S

* The revised value of $\delta_i$ is obtained from

$$V_i = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad — (6)$$

and $\delta_i = \angle V_i$

* owing to physical limitations of P & Q generation source $P_{Gi}$ & $Q_{Gi}$ are constrained as follows

$$P_{Gi\,min} \le P_{Gi} \le P_{Gi\,max}$$

$$Q_{Gi\,min} \le Q_{Gi} \le Q_{Gi\,max}$$

* Voltage at PV bus can be maintained cost only if controlled Q source is available at the bus & reactive power generation required is within prescribed limits

* If $Q_i^{(r+1)} < Q_{i\,min}$, set $Q_i^{(r+1)} = Q_{i\,min}$ & treat bus i as a PQ bus & compute $V_i^{(r+1)}$ using appropriate eq$^n$.

* If $Q_i^{(r+1)} > Q_{i\,max}$, set $Q_i^{(r+1)} = Q_{i\,max}$ & treat bus i as PQ bus & compute $V_i^{(r+1)}$

* In order to save compute time, we can perform ~~the~~ in advance all the arithmatic operations that do not change with the iterations

Define

$$A_i = \frac{P_i - jQ_i}{Y_{ii}} \qquad i = 2, 3 \cdots n \qquad — (7)$$

$$B_{ik} = \frac{Y_{ik}}{Y_{ii}} \qquad i = 2, 3 \cdots n \quad k = 1, 2 \cdots n \quad k \ne i \qquad — (8)$$

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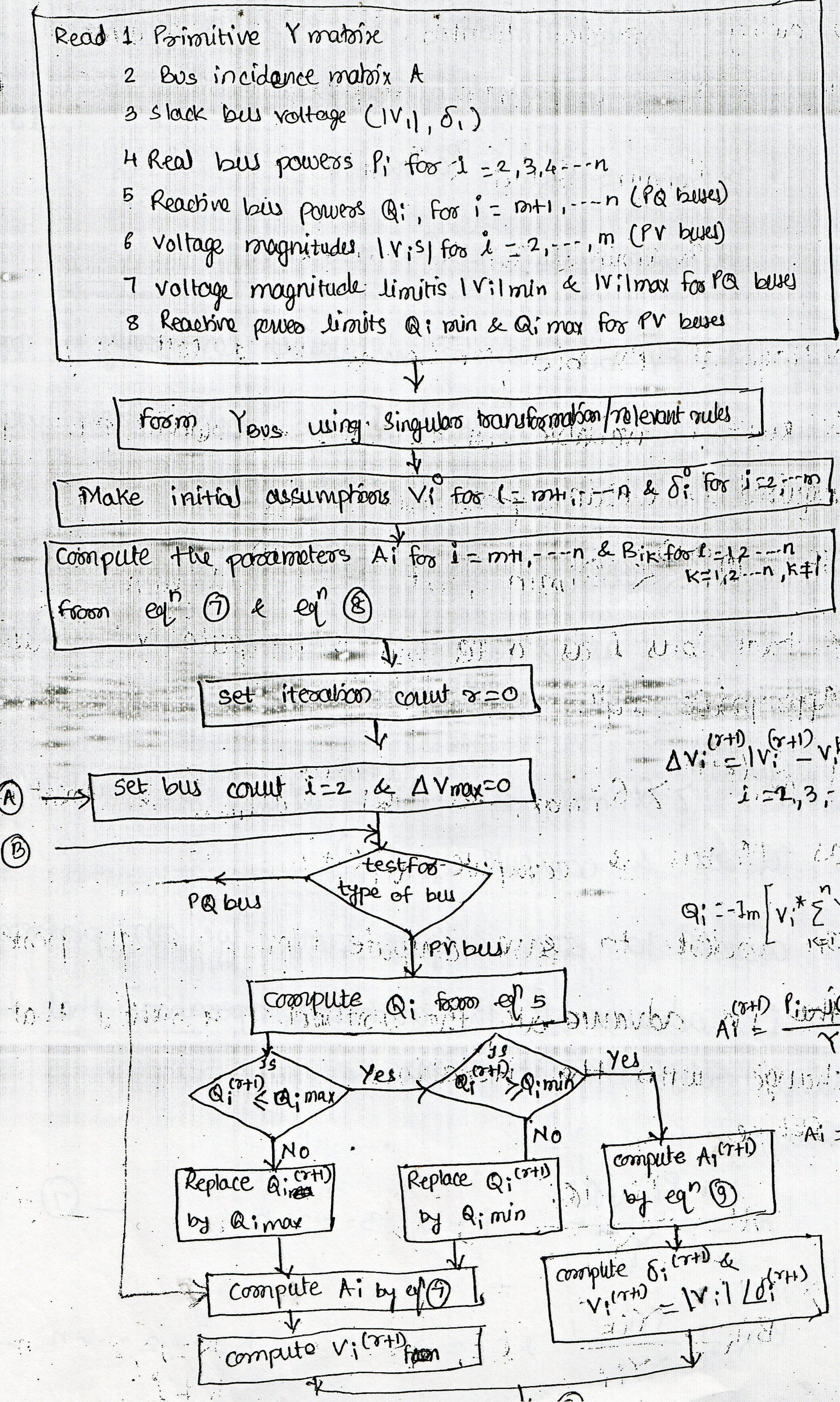


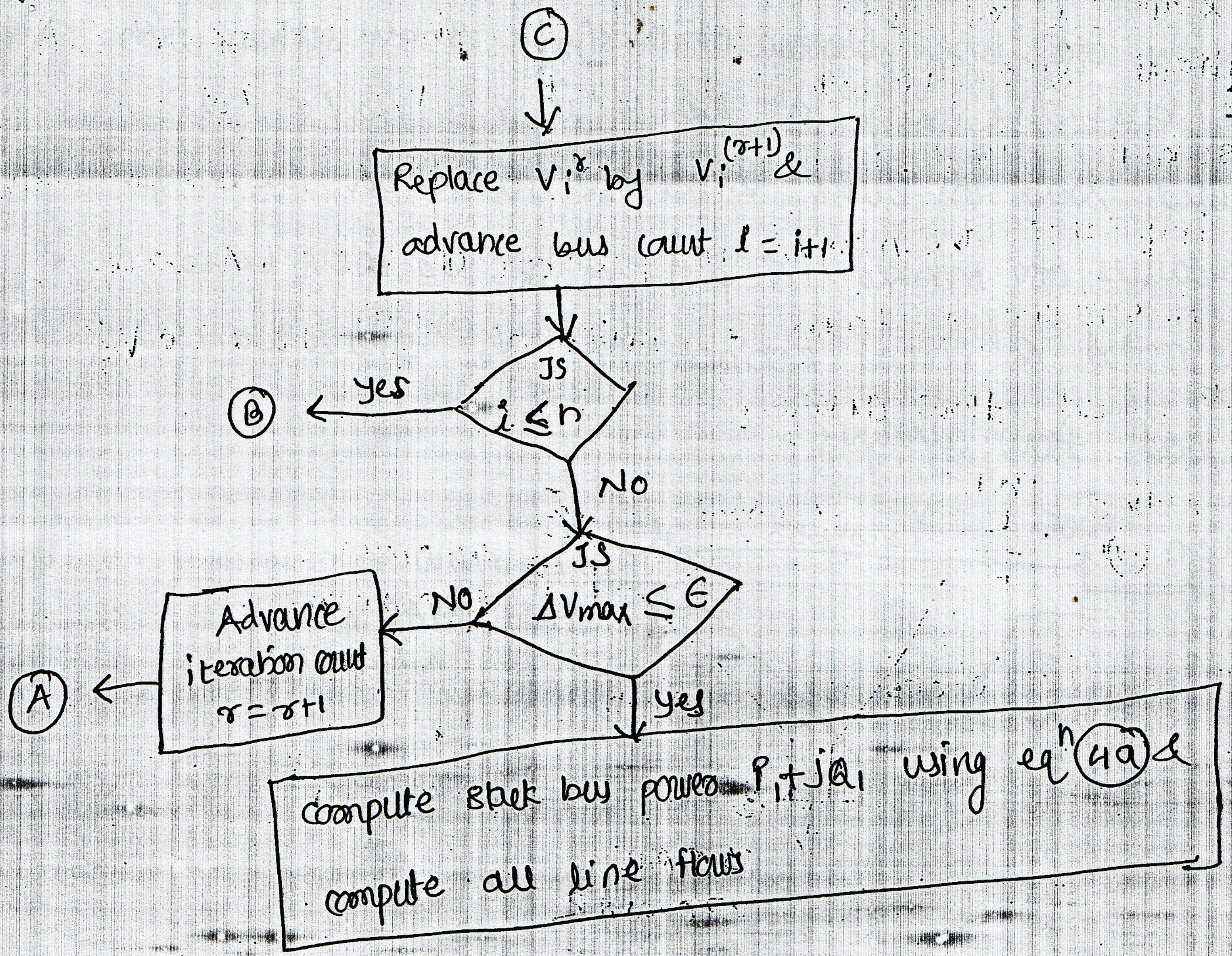

$\Delta V_i^{(r+1)} = |V_i^{(r+1)} - V_i^{(r)}| <$

$i = 2, 3, --- n$

$Q_i = -Im\left[V_i^* \sum_{k=1}^{n} Y_{ik} V_k\right]$

$A_i^{(r+1)} = \frac{P_i - jQ_i^{(r+1)}}{Y_{ii}}$

$A_i = \frac{P_i - j}{Y_{ii}}$




Flow chart for load flow sol^n by the Gauss seidel iteration method using $Y_{BUS}$

Fig shows the one line diagram of a simple 3-bus system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.0 \angle 0^\circ$ p.u.

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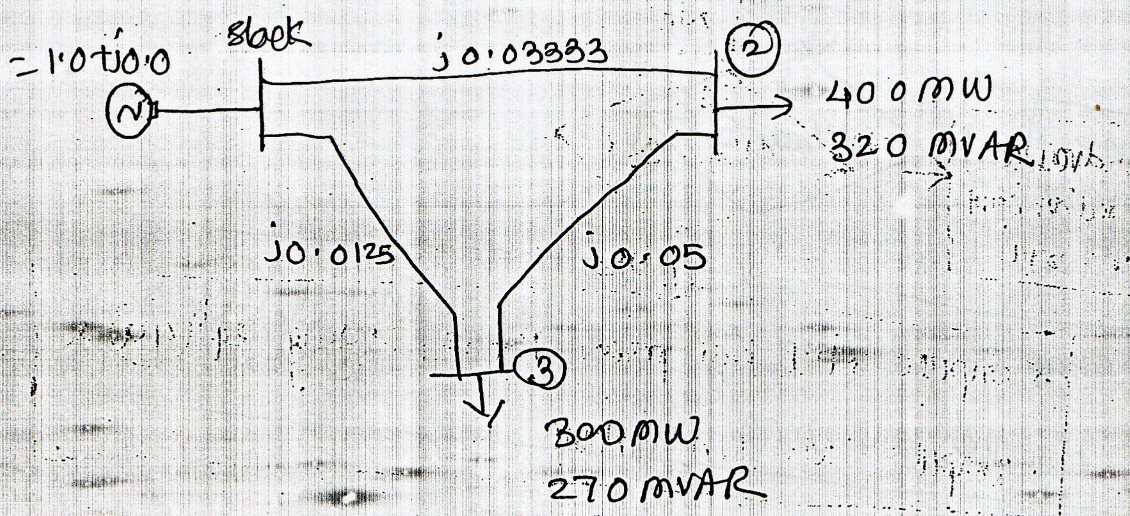
The scheduled load on buses 2 - 3 are marked on the diagram. Line impedances are marked in p.u. on a 100 MVA base. Using G-S method & initial estimates of $V_2^{(0)} = 1.0 \angle 0^\circ$ & $V_3^{(0)}$ conduct load flow analysis.




Solⁿ: line impedances are converted to line admittance as

$y_{12} = -j30.003$ $\quad y_{23} = -j20$ , $\quad y_{13} = -j80$

$Y_{11} = y_{12} + y_{13} = -j30 - j80 = -j110$

$Y_{12} = -(-j30) = j30$ $\quad Y_{13} = -(-j80)$

$Y_{22} = y_{12} + y_{23} = -j30 - j20 = -j50$

$Y_{23} = y_{23} = -(-j20)$

$Y_{33} = y_{23} + y_{13} = -j20 - j80 = -j100$

MVA base = 100 MVA

Complex powers at PQ buses are

$$S_2 = \frac{400 + j320}{100} = (-0.4 - j3.2) \text{ P.U.}$$

$P_{D2} = -4.0 \quad Q_{D2} = j3.2$

|||y at bus 3

$$P_{Di} = \frac{300}{100} = 3.0 \text{ PU}$$

$$Q_{Di} = \frac{270}{100} = j2.7 \text{ P.U.}$$

$\therefore P_i = P_{Gi} - P_{Di} = 0 - 3 = -3$

$Q_2 = Q_{G2} - Q_{D2} = 0 - j2.7 = -j2.7$

$$V_2^{(1)} = 0.936004 - j0.07999 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3\right]$$

$$V_3^{(1)} = 0.960201 - j0.045999$$

At bus 1

$$P_1 - jQ_1 = V_1^*\left[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)\right]$$

$$\approx 6.99822 - j6.995 \text{ PU}$$

Line flows & line losses –

Line current – $I_{12} = y_{12}(V_1 - V_2) = (-j30.003)[1.0 - [0.936004 - j0.07999]] = 2.39904 - j2.997$

$-2.397 + j1.92$

$I_{21} = -I_{12} = -2.999 + j2.997$

$I_{13} = y_{13}(V_1 - V_3) = (-j80)(1.0 - (0.960201 - j0.045999))$

## Acceleration of convergence -

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* In GS method convergence can sometimes be speeded up by the use of acceleration factor

* at ith bus, the accelerated value of voltage at $(r+1)^{th}$ iteration is given by

$$V_i^{(r+1)} \text{(accelerated)} = V_i^{(r)} + \alpha(V_i^{(r+1)} - V_i^{(r)})$$

* where $\alpha$ is a real no. called the acceleration factor.

* generally recommended value of $\alpha$ is $\simeq 1.6$

* a wrong choice of $\alpha$ may slow down convergence or even cause the method to diverge.

---

$I_{31} = -I_{13} = -3.9992 + j\,3.99792$

$= -0.99936 + j\,0.99838$

$I_{23} = y_{23}(V_2 - V_3) = (-j20)\cdot($

$I_{32} = -y_{23} =$

The line flows are in pu & also in MW, Mvar

$S_{12} = V_1 \cdot I_{12}^* =$

$S_{21} = V_2 \cdot I_{21}^* =$

$S_{13} = V_1 \cdot I_{13}^* =$

$S_{31} = V_3 \cdot I_{31}^* =$

$S_{23} = V_2 \cdot I_{23}^* =$

$S_{32} = V_3 \cdot I_{32}^* =$

Line losses

$S_{L12} = S_{12} + S_{21} = j\,59.918$

$S_{L13} = S_{13} + S_{31} = j\,39.97$

$S_{L23} = S_{23} + S_{32} = j\,9.978$

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$S_{13} = V_1 \cdot I_{13}^* =$

$S_{31} = V_3 \cdot I_{31}^* =$

$S_{23} = V_2 \cdot I_{23}^* =$

$S_{32} = V_3 \cdot I_{32}^* =$

Line losses

$S_{L12} = S_{12} + S_{21} = j\,59.918$

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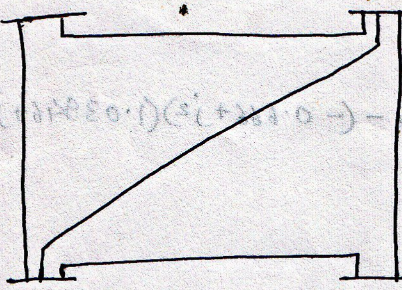
$S \quad S \quad + S \quad = j\,9.978$

Perform the load flow analysis for the power system show below.




| Line | R pu | G | X pu | B |
|---|---|---|---|---|
| 1-2 | 0.05 | 2 | 0.15 | -6 |
| 1-3 | 0.1 | 1.0 | 0.3 | -3 |
| 2-3 | 0.15 | 0.666 | 0.45 | -2 |
| 2-4 | 0.1 | 1.0 | 0.3 | -3 |
| 3-4 | 0.05 | 2.0 | 0.15 | -6 |

$$Y_{Bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

| Bus | $P_i$ pu | $Q_i$ pu | $V_i$ pu | Remarks |
|---|---|---|---|---|
| 1 | — | — | $1.04\angle 0°$ | slack |
| 2 | 0.5 | ~~[illegible]~~ | 1.04 | PQ |
| 3 | -1 | 0.5 | — | PQ |
| 4 | 0.3 | -0.1 | — | PQ |

$0.2 \le Q_2 \le 1$

~~Giv~~

for bus 2

$$Q_2' = -Im\{(V_2^0)Y_{21}V_1 + V_2^{0*}[Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4]\}$$

$$= -Im\left[\{1.04(-2+j6)1.04 + 1.04[(3.666-j11)1.04 + (-0.666+j2) + (-1+j3)]\}\right]$$

$$= -Im\{-0.0693 - j\,0.2079\} = 0.2079 \text{ pu}$$

$$Q_2' = 0.2079 \text{ pu}$$

$$\delta_2' = \angle\left\{\frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2'}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]\right\}$$

$$= \angle\left\{\frac{1}{3.666-j11}\left[\frac{0.5-j0.2079}{1.04-j0} - (-2+j6)(1.04+j0) - (-0.666+j2)(1+j0) - (-1+j3)(1+j0)\right]\right\}$$

$$= \left[\frac{4.2267 - j\,11.439}{3.666 - j11}\right] = \angle(1.0512 + j0.0339) = 1.05174\angle 1.847$$

$\delta_2' = 1.847$ $\quad\therefore V_2 = 1.04(\cos\delta_2' + j\sin\delta_2')$

$V_2 = 1.04\angle 1.847 = 1.03945 + j0.0335$

$$V_3' = \frac{1}{Y_{33}}\left\{\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2' - Y_{34}V_4^0\right\}$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1 - j0.5}{(1 - j0)} - (-1+j3)\,1.04 - (-0.666+j2)(1.03946+j0.03351) - (-2+j6)\right]$$

$$= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937$$

$$V_4' = \frac{1}{Y_{44}}\left\{\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2' - Y_{43}V_3'\right\}$$

$$= \frac{1}{3 - j9}\left[\frac{0.3 + j0.1}{1 - j0} - (-1+j3)(1.0394+j0.0335) - (-2+j6)(1.0317\,j0.08937)\right]$$

$$V_4' = \frac{2.9671 - j8.9962}{3 - j9} = 0.9985 - j0.0031 \quad (1.0342 - j0.01502)?$$

Now suppose limits of $Q_2$ are revised as follow $0.25 \le Q_2 \le 1.0$ pu

calculated $Q_2 = 0.2079$ is less than $Q_{2\,min}$

$\therefore Q_2 = 0.25$ pu

now bus 2 becomes PQ bus from a PV bus

$P_2 = 0.5, \quad Q_2 = 0.25 \quad \& \quad V_2 \angle\delta_2 = ?$

$$\therefore V_2' = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j11}\left[\frac{0.5 - j0.25}{1 - j0} - (-2+j6)\,1.04 - (-0.666+j2) - (-1+j3)\right]$$

$$= \frac{4.246 - j11.49}{3.666 - j11} = 1.0559 + j0.0341$$

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2' - Y_{34}V_4^0\right] = 1.0347 - j0.0893 \text{ pu}$$

$$V_4' = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2' - Y_{43}V_3'\right] = 1.0775 + j0.0923 \text{ pu}$$

## Newton-Raphson Method using $Y_{BUS}$ in polar coordinates —



* N-R method is a powerful method of solving non-linear algebraic eq^ns
* It works faster & is sure to converge compared to GS method
* It is the practical method of load flow sol^n of large power n/ws
* It's only drawback is the large requirement of computer m/m
* Convergence can be considerably speeded up by performing the 1st iteration through GS method & using those values for starting N-R iterations
* The general form of N-R method is as under

consider a set of n non-linear algebraic eq^ns

$$f_i(x_1, x_2, \dots, x_n) = 0 \quad ; \quad i = 1, 2 \dots n \quad \text{—(1)}$$

* Assume initial values of unknowns as $x_1^0, x_2^0, \dots, x_n^0$

let $\Delta x_1^0, \Delta x_2^0, \dots \Delta x_n^0$ be the correction

actual sol^n = assumed value + the correction

$$= x_i^0 + \Delta x_i^0 \quad \text{—(2)}$$

$$\therefore f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots \dots x_n^0 + \Delta x_n^0) = 0 \; ; \; i = 1, 2 \dots n$$

Expanding these eq^ns in Taylor series around initial guess, we have

$$f_i(x_1^0, x_2^0 \dots, x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right)^0 \Delta x_2^0 + \dots + \left(\frac{\partial f_i}{\partial x_n}\right)^0 \Delta x_n^0\right] + \text{higher order terms} = 0 \quad \text{—(3)}$$

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where $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0 - - - \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the derivatives of $f_i$

w.r.t. $x_1, x_2 - - - x_n$ evaluated at $(x_1^0, x_2^0, - - - x_n^0)$

Neglecting higher order terms we can write eqⁿ ③ in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \vdots \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & & & \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \simeq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

or in a vector form

$$F^0 + J^0 \Delta X^0 \simeq 0 \quad \text{— ④}$$

$J^0$ – Known as Jacobian matrix

eqⁿ ④ can be written as

$$F^0 \simeq [-J^0] \Delta X^0 \quad \text{— ⑤}$$

appropriate value of correction i.e. $\Delta X^0$ is obtained from eqⁿ ⑤ as

$$\Delta X^0 \simeq f^0 . [J^0]^{-1}$$

updated value of X are then

**e-Notes of Lecture Sessions of e-Classes conducted by**

**Dr. M.S. Raviprakasha**
**Professor of E&E Engg.,**
**Malnad College of Engineering, Hassan**

## Subject: COMPUTER TECHNIQUES IN POWER SYSTEMS

**Code: EE72** **No. of Hrs.: 33**

### SUBJECT EXPERTS:

- **Dr. MS Raviprakasha**, Professor of E&EE, MCE, Hassan
- **Dr. K Umarao,** Prof. & HOD of E&EE, RNSIT, Bangalore

### LECTURE SCHEDULE:

- **Dr. M.S. Raviprakasha:** Chapters # 1, 3 (Introduction, Linear graph theory, $Z_{BUS}$ Building, Load flow analysis (15 Hrs.): (Plus 2 concluding Sessions)
- **Dr. K Umarao**: Chapter # 2,4,5 (Load frequency control, economic operation of power systems, transient stability studies) (15 Hrs.) : (Plus 1 concluding Session)
- 

### PROGRAMME SCHEDULE:

- **17 Hrs**. of Classes by Dr. MS Raviprakasha, MCE, Hassan
- **16 Hrs.** of Classes by Dr. K Umarao, RNSIT, Bangalore
- **3 Classes per week**: Wednesday, Thursday and Friday (All classes at 11am -12 noon)
- **3 concluding sessions** to discuss on solutions to the recent question papers of VTU, VIIEEE class on the selected subject of CTPS (EE72).

## CONTENTS (CHAPTER-WISE)

## Chapter 3

## LOAD FLOW STUDIES

### PART A: REVIEW OF SOLUTION OF EQUATIONS

- Solution of linear, nonlinear and differential equations
- Importance of iterative methods
- Flow diagrams for the iterative methods
- Comparison of methods
- Examples

### PART B: BASIC LOAD FLOW ANALYSIS

- Classification of buses
- Importance of slack bus
- Static load flow equations
- Gauss Siedel Method: for systems with PQ, PV and Constrained PV buses
- Limitations of Gauss Siedel method
- Importance of acceleration factor
- Newton Raphson method in polar coordinates
- Concept of Jacobian matrix
- Comparison of load flow methods
- Examples

### PART C: ADVANCED LOAD FLOW ANALYSIS

- Decoupled load flow
- Fast decoupled load flow analysis
- Representation of TCUL transformers
- Examples

## Chapter 4

## ECONOMIC OPERATION OF POWER SYSTEMS

- Optimal distribution of loads within a plant
- Langrangian multipliers
- Transmission loss as a function of plant generation

- Determination of loss coefficients
- Automatic economic load dispatch
- Examples

## Chapter 5

## TRANSIENT STABILITY STUDIES

- Basic Stability terms
- Swing Equation & Swing Curve
- Numerical solution of differential equations
- Modified Euler's method
- Runge-Kutta IV order method
- Milne's predictor corrector method
- Representation of synchronous machine, loads, etc. for TS studies
- Network performance equations
- Solution techniques and flow charts
- Examples

## EXPECTED PATTERN OF QUESTION PAPER

**One** question each on chapter 2 and 4, and **Two** questions each on chapters 1, 3 and 5.
**Note: Five questions out of 8 are to be answered in full.**

## TEXTS/ REFERENCES:


1. **Stagg and El Abiad, Computer methods in power system analysis**, MH.
2. **MA Pai,** Computer techniques in power Systems, TMH..
3. **K Umarao,** Computer Techniques and Models in Power Systems, IK International Publishing House Pvt. Ltd., New Delhi.
4. **RN Dhar,** Computer techniques in power system operation and control.


## PREREQUISITE SUBJECTS:

1. Power System Analysis
2. Synchronous Machines
3. Numerical analysis

## ELECTRICAL POWER SYSTEMS-

## THE STATE-OF-THE-ART:

### *To Begin With ……*

| No. | TOPICS | SUB-TOPICS |
|---|---|---|
| 1. | **Representation of Power systems** | - p.u. React./ Imp. Diagram |
| 2. | **Electric Power System** | - Generation - Machines<br>- Transmission - Distribution - Utilization<br>– Tariffs |
| 3. | **Fault studies** | - Sym. Faults<br>- Sym. Components<br>- Seq. Imps. / Networks<br>- Unsymmetrical Faults |
| 4. | **System Stability** | - SSS, TS, DS<br>- Angle Stability<br>- Solution of Equations<br>- EAC, Clarke's Diagram |
| 5. | **Linear Graph Theory (Linear Equations)** | - Incidence Matrices<br>- Singular/ NS Transformations<br>- Network Matrices<br>- $Z_{BUS}$ Building |
| 6. | **Power Flow Studies (NL Equations)** | - Buses, $Y_{BUS}$ Advs., Loads flow equations<br>- Iterative Methods<br>- GS, NR, FDLF & DCLF |

## *Present Scenario ……*

| No. | TOPICS | SUB-TOPICS |
|---|---|---|
| 7. | **Reactive Power Management** | - Importance of VArs<br>- Compensation Devices, Sizing, Placement, Design, Optimality,<br>- VAr Dispatch<br>- VAr Co-ordination |
| 8. | **Gen. Expansion Planning** | - Optimality<br>- Load Prediction: Short, Medium and Long Term Forecasting |
| 9. | **Operation and Control** | -EMS: EMC, SLDC,RLDC<br>-ALFC, Voltage Control<br>-Tie-line Power Control |
| 10. | **System Reliability** | - Requirements<br>- Methods |
| 11. | **Economic Operation** | - Unit Commitment<br>- Parallel Operation<br>- Optimal Load Dispatch<br>- Constraints |
| 12. | **Instrumentation** | - CTs, PTs |
| 13. | **State Estimation** | - SCADA<br>- Bad Data Elimination<br>- Security/ Cont. Studies |

….. and so on.

## *Future Trends ……*

| No. | TOPICS | SUB-TOPICS |
| --- | --- | --- |
| 14. | **Voltage Stability** | - Importance<br>- Angle/ Voltage stability |
| 15. | **Power System Simulators** | - Requirements<br>- Control Blocks<br>- Data-Base Definition |
| 16. | **Energy Auditing** | - Deregulation |
| 17. | **Demand Side Management** | - Time of Use Pricing |
| 18. | **Renewable Energy** | - The Paradigm |
| 19. | **Sparsity Oriented Programming** | - Sparsity: $Y_{BUS}$<br>- Ordering Schemes<br>- LU- Factorization: Fills<br>- Pivoting<br>- UD Table Storage |
| 20. | **Recent Computer Applications** | - AI<br>- Expert Systems<br>- ANN,<br>- Genetic Algorithms<br>- Fuzzy Logic, Etc. |

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# CHAPTER- 1-A

# INCIDENCE AND NETWORK MATRICES

**[CONTENTS:** Definitions of important terms, Incidence matrices: Element node incidence matrix and Bus incidence matrix, Primitive networks and matrices, Performance of primitive networks, Frames of reference, Singular transformation analysis, Formation of bus admittance matrix, examples]

**INTRODUCTION**

The solution of a given linear network problem requires the formation of a set of equations describing the response of the network. The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components. In the bus frame of reference the variables are the node voltages and node currents.

The independent variables in any reference frame can be either currents or voltages. Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix. The formulation of the appropriate relationships between the independent and dependent variables is an integral part of a digital computer program for the solution of power system problems. The formulation of the network equations in different frames of reference requires the knowledge of graph theory. Elementary graph theory concepts are presented here, followed by development of network equations in the bus frame of reference.

**ELEMENTARY LINEAR GRAPH THEORY: IMPORTANT TERMS**

The geometrical interconnection of the various branches of a network is called the *topology* of the network. The connection of the network topology, shown by replacing all its elements by lines is called a *graph.* A *linear graph* consists of a set of objects called *nodes* and another set called *elements* such that each element is identified with an ordered pair of nodes. An *element* is defined as any line segment of the graph irrespective of the characteristics of the components involved. A graph in which a direction is assigned to each element is called an *oriented graph* or a *directed graph*. It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions. Elements are indicated by numbers and the nodes by encircled numbers. The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

**Connected Graph :** This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1, with:

| | |
|---|---|
| e = number of elements = 6 | l = number of links = e-b = 3 |
| n = number of nodes = 4 | Tree = T(1,2,3) and |
| b = number of branches = n-1 = 3 | Co-tree = T(4,5,6) |

**Sub-graph :** sG is a sub-graph of G if the following conditions are satisfied:

- sG is itself a graph
- Every node of sG is also a node of G
- Every branch of sG is a branch of G

For eg., sG(1,2,3), sG(1,4,6), sG(2), sG(4,5,6), sG(3,4),.. are all valid sub-graphs of the oriented graph of Fig.1c.

**Loop :** A sub-graph L of a graph G is a loop if

- L is a connected sub-graph of G
- Precisely two and not more/less than two branches are incident on each node in L

In Fig 1c, the set{1,2,4} forms a loop, while the set{1,2,3,4,5} is not a valid, although the set(1,3,4,5) is a valid loop. The KVL (Kirchhoff's Voltage Law) for the loop is stated as follows: *In any lumped network, the algebraic sum of the branch voltages around any of the loops is zero.*

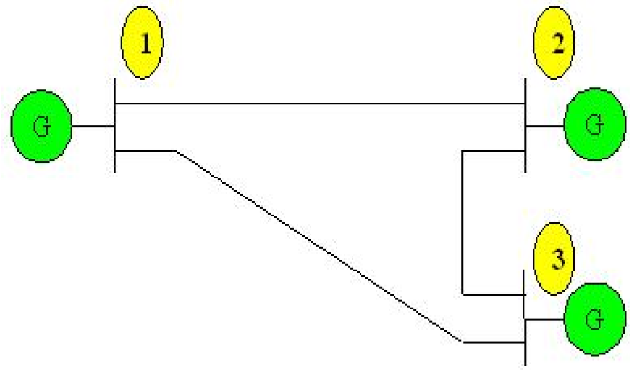



**Fig 1a. Single line diagram of a power system**

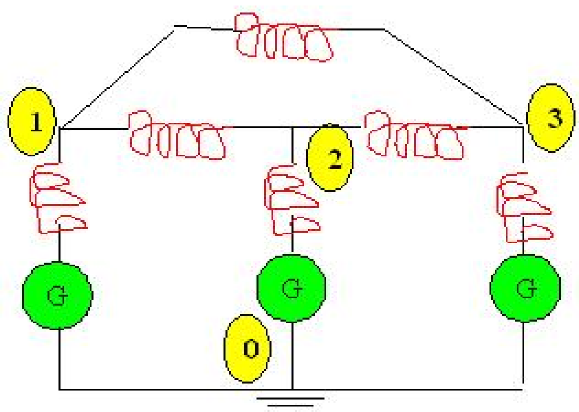



**Fig 1b. Reactance diagram**

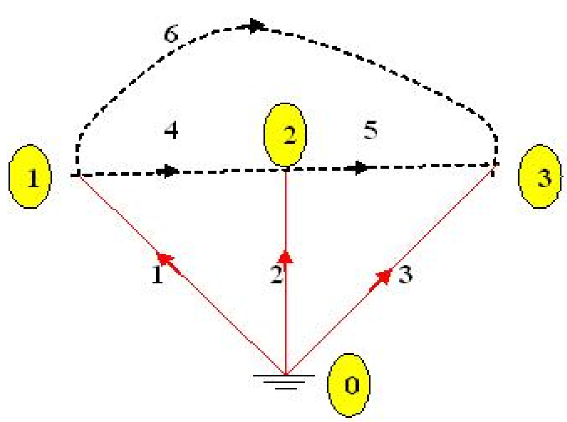



**Fig 1c. Oriented Graph**

**Cutset :** It is a set of branches of a connected graph G which satisfies the following conditions :

- The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
- The removal of all but one of the branches of the set, leaves the remaining graph connected.

Referring to Fig 1c, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected sub-graphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: *In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.*

**Tree:** It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with *n* nodes,

**The number of branches: b** = n-1 **(1)**

For the graph of Fig 1c, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

**Co-Tree :** The set of branches of the original graph G, not included in the tree is called the *co-tree*. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called *links*. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With e as the total number of elements,

**The number of links:** $\boldsymbol{l}$ = e – b = e – n + 1 **(2)**

For the graph of Fig 1c, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

| **Tree** | T(1,2,3) | T(1,4,6) | T(2,4,5) | T(2,5,6) |
|---|---|---|---|---|
| **Co-Tree** | T(4,5,6) | T(2,3,5) | T(1,3,6) | T(1,3,4) |

**Basic loops:** When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

**Basic cut-sets:** Cut-sets which contain only one branch and remaining links are called *basic cutsets* or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

**Examples on Basics of LG Theory:**

**Example-1:** *Obtain the oriented graph for the system shown in Fig. E1. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.*

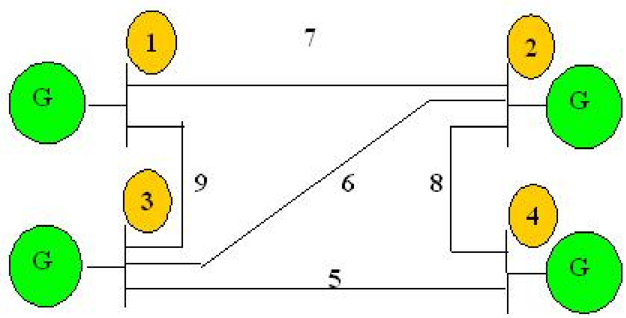



**Fig. E1a. Single line diagram of Example System**

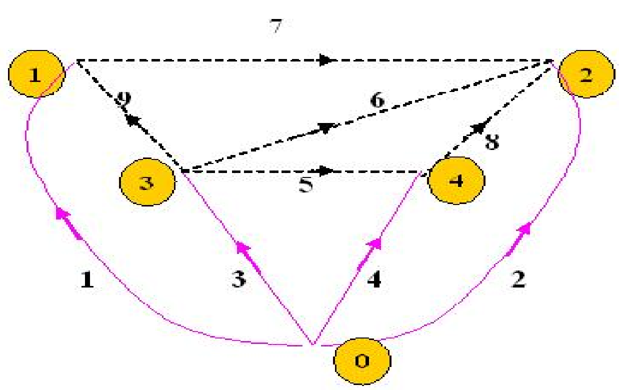



**Fig. E1b. Oriented Graph of Fig. E1a.**

For the system given, the oriented graph is as shown in figure E1b. some of the valid Tree graphs could be T(1,2,3,4), T(3,4,8,9), T(1,2,5,6), T(4,5,6,7), etc. The basic cut-sets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.E1a and tree, T(1,2,3,4) are as shown in Figure E1c and Fig.E1d respectively.

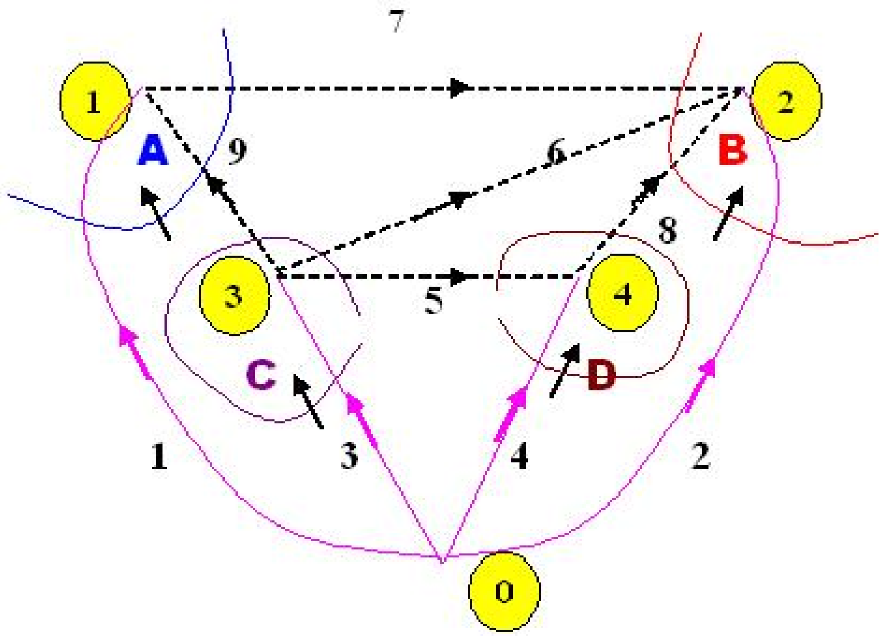



**Fig. E1c. Basic Cutsets of Fig. E1a.**

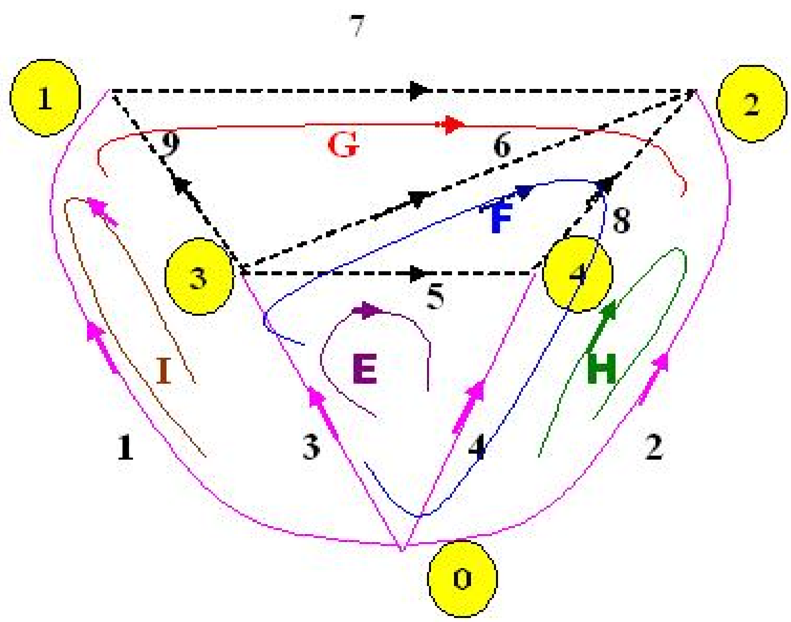



**Fig. E1d. Basic Loops of Fig. E1a.**

## INCIDENCE MATRICES

**Element–node incidence matrix:** $\hat{A}$

The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, $\hat{A}$. An element $a_{ij}$ of $\hat{A}$ is defined as under:

$a_{ij}$ = 1 if the branch-i is incident to and oriented away from the node-j.

= -1 if the branch-i is incident to and oriented towards the node-j.

= 0 if the branch-i is not at all incident on the node-j.

Thus the dimension of $\hat{A}$ is $e \times n$, where e is the number of elements and *n* is the number of nodes in the network. For example, consider again the sample system with its oriented graph as in fig. 1c. the corresponding element-node incidence matrix, is obtained as under:

$\hat{A}$ =

| Nodes / Elements | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **1** | 1 | -1 | | |
| **2** | 1 | | -1 | |
| **3** | 1 | | | -1 |
| **4** | | 1 | -1 | |
| **5** | | | 1 | -1 |
| **6** | | 1 | | -1 |

It is to be noted that the first column and first row are not part of the actual matrix and they only indicate the element number node number respectively as shown. Further, the sum of every row is found to be equal to zero always. Hence, the rank of the matrix is less than *n*. Thus in general, the matrix $\hat{A}$ satisfies the identity:

$$\sum_{j=1}^{n} a_{ij} = 0 \quad \forall \; i = 1,2,\ldots..e. \qquad (3)$$

**Bus incidence matrix: A**

By selecting any one of the nodes of the connected graph as the reference node, the corresponding column is deleted from $\hat{A}$ to obtain the bus incidence matrix, A. The dimensions of A are e$\times$*(n-1)* and the rank is *n-1*. In the above example, selecting node-0 as reference node, the matrix A is obtained by deleting the column corresponding to node-0, as under:

**A =**

| Buses / Elements | 1 | 2 | 3 |
|---|---|---|---|
| **1** | -1 | | |
| **2** | | -1 | |
| **3** | | | -1 |
| **4** | 1 | -1 | |
| **5** | | 1 | -1 |
| **6** | 1 | | -1 |

=

| | |
|---|---|
| $A_b$ | **Branches** |
| $A_l$ | **Links** |

It may be observed that for a selected tree, say, T(1,2,3), the bus incidence matrix can be so arranged that the branch elements occupy the top portion of the A-matrix followed by the link elements. Then, the matrix-A can be partitioned into two sub matrices $A_b$ and $A_l$ as shown, where,

(i) $A_b$ is of dimension (bxb) corresponding to the branches and

(ii) $A_l$ is of dimension (lxb) corresponding to links.

A is a rectangular matrix, hence it is singular. $A_b$ is a non-singular square matrix of dimension-b. Since A gives the incidence of various elements on the nodes with their direction of incidence, the KCL for the nodes can be written as

$$A^T \bar{i} = 0 \qquad \textbf{(4)}$$

where $A^T$ is the transpose of matrix A and $\bar{i}$ is the vector of branch currents. Similarly for the branch voltages we can write,

$$\bar{v} = A \ \bar{E}_{bus} \qquad \textbf{(5)}$$

**<u>Examples on Bus Incidence Matrix:</u>**

**Example-2:** *For the sample network-oriented graph shown in Fig. E2, by selecting a tree, T(1,2,3,4), obtain the incidence matrices A and* $\hat{A}$. *Also show the partitioned form of the matrix-A.*

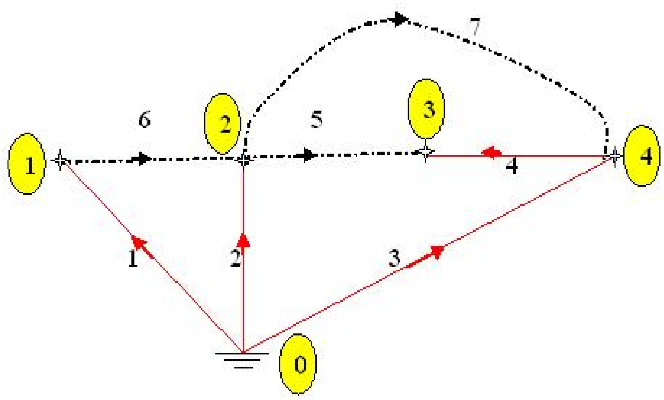



**Fig. E2. Sample Network-Oriented Graph**

nodes

$\hat{A}$ = Elements

| $e \backslash n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1 | 1 | −1 | 0 | 0 | 0 |
| 2 | 1 | 0 | −1 | 0 | 0 |
| 3 | 1 | 0 | 0 | 0 | −1 |
| 4 | 0 | 0 | 0 | −1 | 1 |
| 5 | 0 | 0 | 1 | −1 | 0 |
| 6 | 0 | 1 | −1 | 0 | 0 |
| 7 | 0 | 0 | 1 | 0 | −1 |

buses

**A** = Elements

| $e \backslash b$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | −1 | 0 | 0 | 0 |
| 2 | 0 | −1 | 0 | 0 |
| 3 | 0 | 0 | 0 | −1 |
| 4 | 0 | 0 | −1 | 1 |
| 5 | 0 | 1 | −1 | 0 |
| 6 | 1 | −1 | 0 | 0 |
| 7 | 0 | 1 | 0 | −1 |

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two sub-matrices as under:

$$A_b = \text{branches}\overset{\text{buses}}{\begin{bmatrix} b \backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \end{bmatrix}}$$

$$A_l = \text{links}\overset{\text{buses}}{\begin{bmatrix} l \backslash b & 1 & 2 & 3 & 4 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

**Example-3:** *For the sample-system shown in Fig. E3, obtain an oriented graph. By selecting a tree, T(1,2,3,4), obtain the incidence matrices A and* $\hat{A}$. *Also show the partitioned form of the matrix-A.*

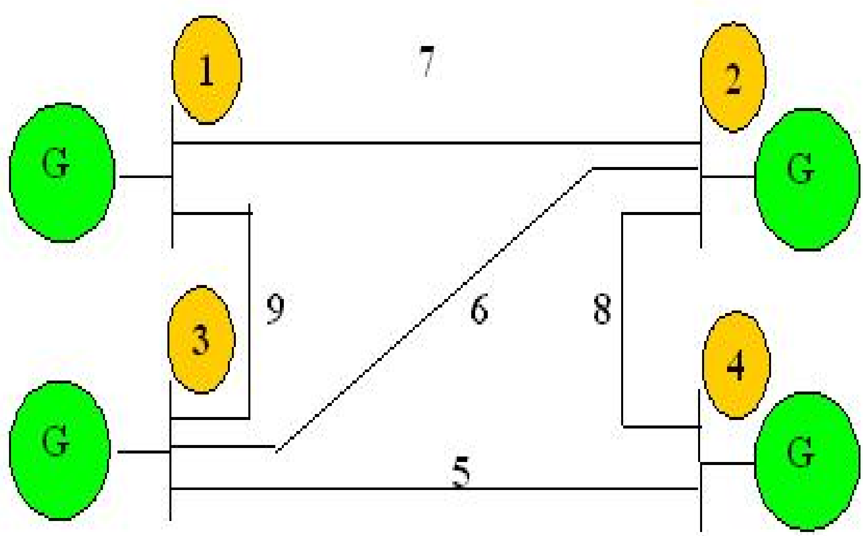



**Fig. E3a. Sample Example network**

Consider the oriented graph of the given system as shown in figure E3b, below.

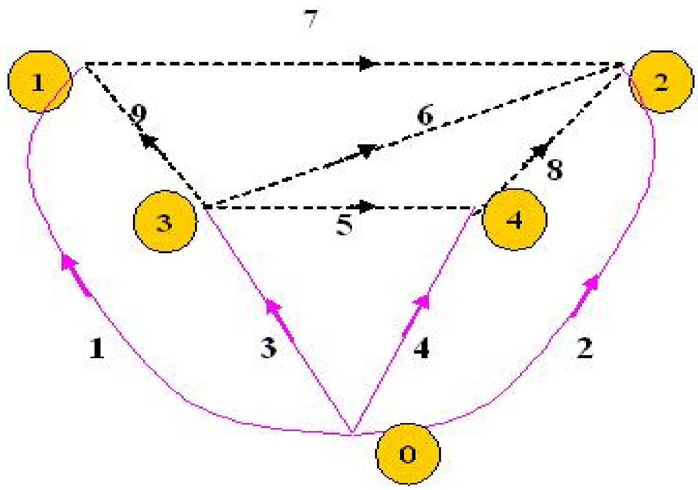



**Fig. E3b. Oriented Graph of system of Fig-E3a.**

Corresponding to the oriented graph above and a Tree, T(1,2,3,4), the incidence matrices Â and A can be obtained as follows:

$\hat{A}$ =

| e\n | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1 | 1 | -1 | | | |
| 2 | 1 | | -1 | | |
| 3 | 1 | | | -1 | |
| 4 | 1 | | | | -1 |
| 5 | | | | 1 | -1 |
| 6 | | | -1 | 1 | |
| 7 | | 1 | -1 | | |
| 8 | | | -1 | | 1 |
| 9 | | -1 | | 1 | |

**A =**

| e\b | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -1 | | | |
| 2 | | -1 | | |
| 3 | | | -1 | |
| 4 | | | | -1 |
| 5 | | | 1 | -1 |
| 6 | | -1 | 1 | |
| 7 | 1 | -1 | | |
| 8 | | -1 | | 1 |
| 9 | -1 | | 1 | |

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two sub-matrices as under:

$\mathbf{A_b}$ =

| e\b | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -1 | | | |
| 2 | | -1 | | |
| 3 | | | -1 | |
| 4 | | | | -1 |

$\mathbf{A_l}$ =

| e\b | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 5 | | | 1 | -1 |
| 6 | | -1 | 1 | |
| 7 | 1 | -1 | | |
| 8 | | -1 | | 1 |
| 9 | -1 | | 1 | |

## PRIMITIVE NETWORKS

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

**General representation of a network element:** In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

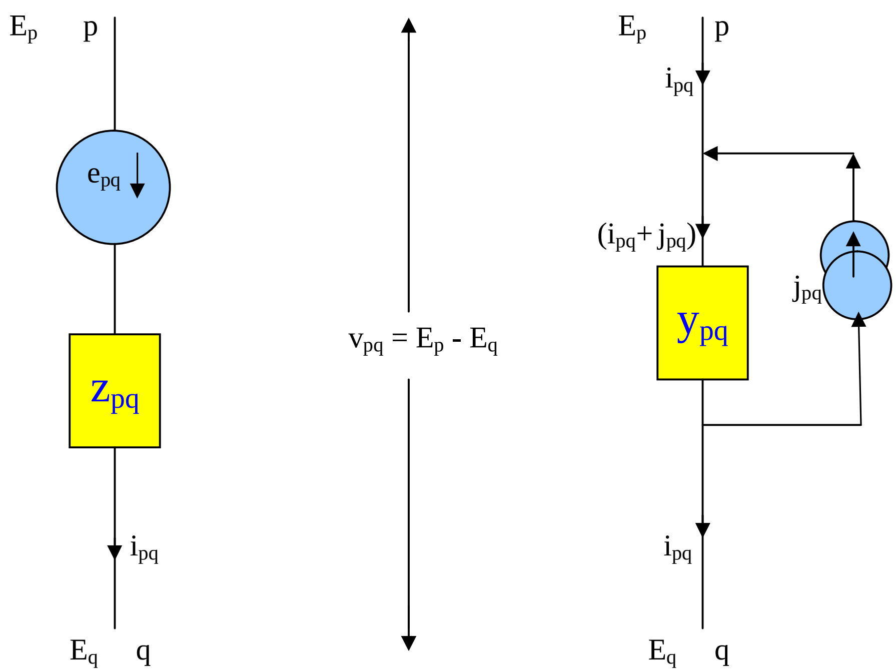



**Fig.2 Representation of a primitive network element**

**(a) Impedance form (b) Admittance form**

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

$v_{pq}$ = voltage across the element p-q,

$e_{pq}$ = source voltage in series with the element p-q,

$i_{pq}$= current through the element p-q,

$j_{pq}$= source current in shunt with the element p-q,

$z_{pq}$= self impedance of the element p-q and

$y_{pq}$= self admittance of the element p-q.

**Performance equation:** Each element p-q has two variables, $v_{pq}$ and $i_{pq}$. The performance of the given element p-q can be expressed by the performance equations as under:

$v_{pq} + e_{pq} = z_{pq}i_{pq}$ (in its impedance form)

$i_{pq} + j_{pq} = y_{pq}v_{pq}$ (in its admittance form) **(6)**

Thus the parallel source current jpq in admittance form can be related to the series source voltage, epq in impedance form as per the identity:

$j_{pq} = - y_{pq} e_{pq}$ **(7)**

A set of non-connected elements of a given system is defined as a *primitive Network and a*n element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$v + e = [z]\, i$

$i + j = [y]\, v$ **(8)**

**Primitive network matrices:**

A diagonal element in the matrices, [z] or [y] is the self impedance $z_{pq\text{-}pq}$ or self admittance, $y_{pq\text{-}pq}$. An off-diagonal element is the mutual impedance, $z_{pq\text{-}rs}$ or mutual admittance, $y_{pq\text{-}rs}$, the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix, [y] can be obtained also by inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

**Examples on Primitive Networks:**

**Example-4:** *Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to:* $Z_1=Z_2=0.2$, $Z_3=0.25$, $Z_4=Z_5=0.1$ *and* $Z_6=0.4$ *units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.*

**A =**

| | | |
|---|---|---|
| -1 | 0 | 0 |
| 0 | -1 | 0 |
| 0 | 0 | -1 |
| 1 | -1 | 0 |
| 0 | 1 | -1 |
| 1 | 0 | -1 |

**Solution:**

The element node incidence matrix, $\hat{A}$ can be obtained from the given A matrix, by pre-augmenting to it an extra column corresponding to the reference node, as under.

$\hat{A}$ **=**

| | | | |
|---|---|---|---|
| 1 | -1 | 0 | 0 |
| 1 | 0 | -1 | 0 |
| 1 | 0 | 0 | -1 |
| 0 | 1 | -1 | 0 |
| 0 | 0 | 1 | -1 |
| 0 | 1 | 0 | -1 |

Based on the conventional definitions of the elements of $\hat{A}$, the oriented graph can be formed as under:

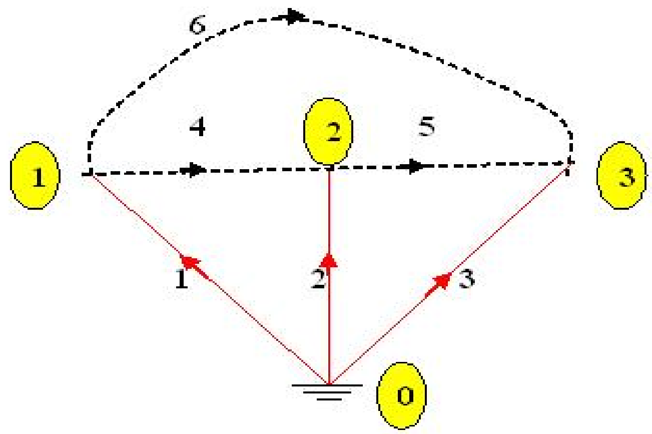



**Fig. E4 Oriented Graph**

Thus the primitive network matrices are square, symmetric and diagonal matrices of order e=no. of elements = 6. They are obtained as follows.

**[z] =**

| | | | | | |
|---|---|---|---|---|---|
| **0.2** | 0 | 0 | 0 | 0 | 0 |
| 0 | **0.2** | 0 | 0 | 0 | 0 |
| 0 | 0 | **0.25** | 0 | 0 | 0 |
| 0 | 0 | 0 | **0.1** | 0 | 0 |
| 0 | 0 | 0 | 0 | **0.1** | 0 |
| 0 | 0 | 0 | 0 | 0 | **0.4** |

And

**[y] =**

| | | | | | |
|---|---|---|---|---|---|
| **5.0** | 0 | 0 | 0 | 0 | 0 |
| 0 | **5.0** | 0 | 0 | 0 | 0 |
| 0 | 0 | **4.0** | 0 | 0 | 0 |
| 0 | 0 | 0 | **10** | 0 | 0 |
| 0 | 0 | 0 | 0 | **10** | 0 |
| 0 | 0 | 0 | 0 | 0 | **2.5** |

**Example-5:** *Consider three passive elements whose data is given in Table E5 below. Form the primitive network impedance matrix.*

**Table E5**

| **Element number** | **Self impedance** ($z_{pq-pq}$) | | **Mutual impedance,** ($z_{pq-rs}$) | |
|---|---|---|---|---|
| | **Bus-code, (p-q)** | **Impedance in p.u.** | **Bus-code, (r-s)** | **Impedance in p.u.** |
| **1** | 1-2 | j 0.452 | | |
| **2** | 2-3 | j 0.387 | 1-2 | j 0.165 |
| **3** | 1-3 | j 0.619 | 1-2 | j 0.234 |

**Solution:**

[z] =

| | **1-2** | **2-3** | **1-3** |
|---|---|---|---|
| **1-2** | **j 0.452** | **j 0.165** | **j 0.234** |
| **2-3** | **j 0.165** | **j 0.387** | **0** |
| **1-3** | **j 0.234** | **0** | **j 0.619** |

**Note:**

- The size of [z] is e× e, where e= number of elements,
- The diagonal elements are the self impedances of the elements
- The off-diagonal elements are mutual impedances between the corresponding elements.
- Matrices [z] and [y] are inter-invertible.

## FORMATION OF $Y_{BUS}$ AND $Z_{BUS}$

The bus admittance matrix, $Y_{BUS}$ plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. $Z_{BUS}$ Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

**Frames of Reference:**

*Bus Frame of Reference*: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$E_{BUS} = Z_{BUS} I_{BUS}$$
$$I_{BUS} = Y_{BUS} E_{BUS} \quad \textbf{(9)}$$

*Branch Frame of Reference*: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{BR} = Z_{BR} I_{BR}$$
$$I_{BR} = Y_{BR} E_{BR} \quad \textbf{(10)}$$

*Loop Frame of Reference*: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{LOOP} = Z_{LOOP} I_{LOOP}$$
$$I_{LOOP} = Y_{LOOP} E_{LOOP} \quad \textbf{(11)}$$

Of the various network matrices refered above, the bus admittance matrix ($Y_{BUS}$) and the bus impedance matrix ($Z_{BUS}$) are determined for a given power system by the rule of inspection as explained next.

**Rule of Inspection**

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3 (V_1-V_3) + Y_6 (V_1 – V_2)$

At node 2: $I_2 = Y_2V_2 + Y_5 (V_2-V_3) + Y_6 (V_2 – V_1)$

At node 3: $0 = Y_3 (V_3-V_1) + Y_4V_3 + Y_5 (V_3 – V_2)$ **(12)**

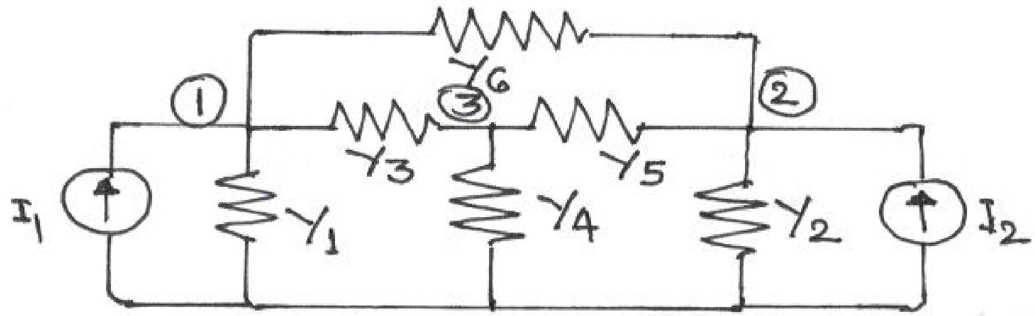



**Fig. 3 Example System for finding $Y_{BUS}$**

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} = \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad \textbf{(13)}$$

In other words, the relation of equation (9) can be represented in the form

$$I_{BUS} = Y_{BUS} E_{BUS} \qquad \textbf{(14)}$$

Where, $Y_{BUS}$ is the bus admittance matrix, $I_{BUS}$ & $E_{BUS}$ are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, $Y_{BUS}$ of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

*Diagonal elements*: A diagonal element ($Y_{ii}$) of the bus admittance matrix, $Y_{BUS}$, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

*Off Diagonal elements*: An off-diagonal element ($Y_{ij}$) of the bus admittance matrix, $Y_{BUS}$, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$Y_{ii} = \Sigma\, y_{ij} \quad (j = 1,2,\ldots\ldots n)$$

$$Y_{ij} = -\, y_{ij} \quad (j = 1,2,\ldots\ldots n) \qquad \textbf{(15)}$$

For i = 1,2,….n,  n = no. of buses of the given system, $y_{ij}$ is the admittance of element connected between buses i and j and $y_{ii}$ is the admittance of element connected between bus i and ground (reference bus).

**Bus impedance matrix**

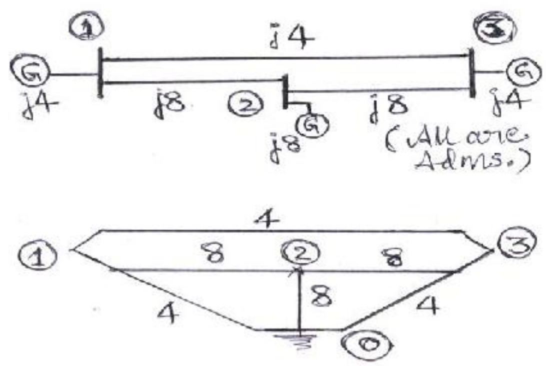
In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible.

**Note:** It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

## <u>Examples on Rule of Inspection:</u>

**Example 6:** *Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection*

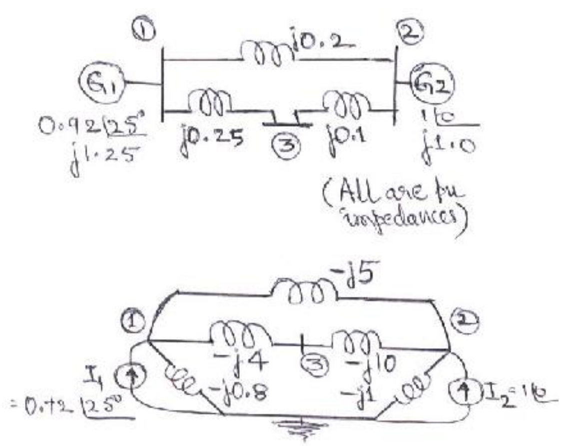
$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$

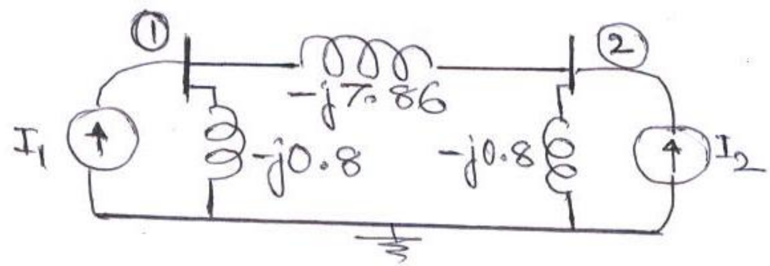


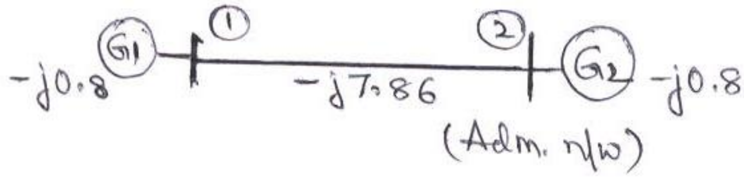

**Example 7**: *Obtain $Y_{BUS}$ for the impedance network shown aside by the rule of inspection. Also, determine $Y_{BUS}$ for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1}$$






$$\mathbf{Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C}$$

$$\mathbf{Y_{BUS} = j\begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}}$$

## SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

### Bus admittance matrix, $Y_{BUS}$ and Bus impedance matrix, $Z_{BUS}$

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1 nodes are present, out of which one of them is designated as the reference node). For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$$I_{BUS} = Y_{BUS} E_{BUS} \quad \textbf{(17)}$$

Where $E_{BUS}$ = vector of bus voltages measured with respect to reference bus

$I_{BUS}$ = Vector of currents injected into the bus

$Y_{BUS}$ = bus admittance matrix

The performance equation of the primitive network in admittance form is given by

$$i + j = [y] v$$

Pre-multiplying by $A^t$ (transpose of A), we obtain

$$A^t i + A^t j = A^t [y] v \quad \textbf{(18)}$$

However, as per equation (4),

$$A^t i = 0,$$

since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff's law is zero. Similarly, $A^t j$ gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

$$A^t j = I_{BUS} \quad \textbf{(19)}$$

Thus from (18) we have, $I_{BUS} = A^t [y] v$ **(20)**

However, from (5), we have

$$v = A E_{BUS}$$

And hence substituting in (20) we get,

$$I_{BUS} = A^t [y] A E_{BUS} \quad \textbf{(21)}$$

Comparing (21) with (17) we obtain,

$$Y_{BUS} = A^t [y] A \quad \textbf{(22)}$$

The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

$$Z_{BUS} = Y_{BUS}^{-1} \quad \textbf{(23)}$$

Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

## Examples on Singular Transformation:

**Example 8:** *For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.*

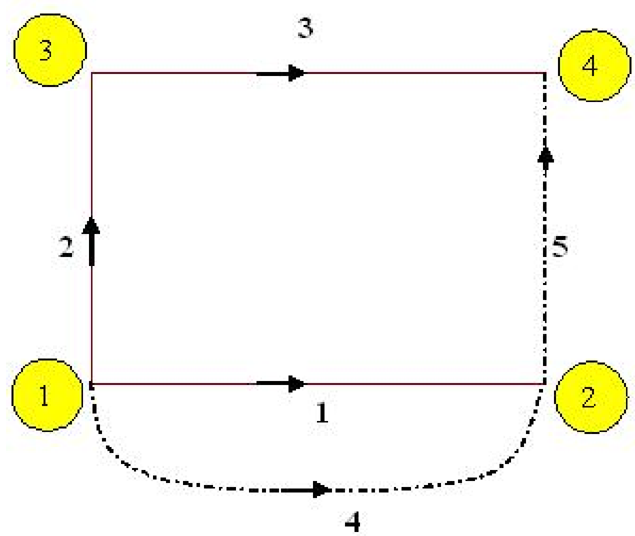



**Fig E8 System for Example-8**

**Table E8: Data for Example-8**

| Elements | Self impedance | Mutual impedance |
|---|---|---|
| **1** | j 0.6 | - |
| **2** | j 0.5 | j 0.1(with element 1) |
| **3** | j 0.5 | - |
| **4** | j 0.4 | j 0.2 (with element 1) |
| **5** | j 0.2 | - |

**Solution:**

The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by,

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix $[y] = [z]^{-1}$ and given by,

$$[y] = \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t\,[y]\,A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

## SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances or admittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by $(n-1)$ nodal equations, where $n$ is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

# Chapter-1-B

# FORMATION OF BUS IMPEDANCE MATRIX

**[CONTENTS:** *Node elimination by matrix algebra, generalized algorithms for $Z_{BUS}$ building, addition of BRANCH, addition of LINK, special cases of analysis, removal of elements, changing the impedance value of an element, examples*]

**NODE ELIMINATION BY MATRIX ALGEBRA**

Nodes can be eliminated by the matrix manipulation of the standard node equations. However, *only those nodes at which current does not enter or leave the network can be considered for such elimination*. Such nodes can be eliminated either in one group or by taking the eligible nodes one after the other for elimination, as discussed next.

**CASE-A: Simultaneous Elimination of Nodes:**

Consider the performance equation of the given network in bus frame of reference in admittance form for a n-bus system, given by:

$$\mathbf{I_{BUS} = Y_{BUS}\, E_{BUS}} \qquad \mathbf{(1)}$$

Where $I_{BUS}$ and $E_{BUS}$ are n-vectors of injected bus current and bus voltages and $Y_{BUS}$ is the square, symmetric, coefficient bus admittance matrix of order n.

Now, of the n buses present in the system, let p buses be considered for node-elimination so that the reduced system after elimination of p nodes would be retained with m (= n-p) nodes only. Hence the corresponding performance equation would be similar to (1) except that the coefficient matrix would be of order m now, i.e.,

$$\mathbf{I_{BUS} = Y_{BUS}^{new}\, E_{BUS}} \qquad \mathbf{(2)}$$

Where $Y_{BUS}^{new}$ is the bus admittance matrix of the reduced network and the vectors $I_{BUS}$ and $E_{BUS}$ are of order m. It is assumed in (1) that $I_{BUS}$ and $E_{BUS}$ are obtained with their elements arranged such that the elements associated with p nodes to be eliminated are in the lower portion of the vectors. Then the elements of $Y_{BUS}$ also get located accordingly so that (1) after matrix partitioning yields,

$$
\begin{array}{c} \mathbf{I_{BUS\text{-}m}} \\ \mathbf{I_{BUS\text{-}p}} \end{array} = \begin{array}{c} \\ m \\ p \end{array} \begin{array}{cc} m & p \\ \mathbf{Y_A} & \mathbf{Y_B} \\ \mathbf{Y_C} & \mathbf{Y_D} \end{array} \quad \begin{array}{c} \mathbf{E_{BUS\text{-}m}} \\ \mathbf{E_{BUS\text{-}p}} \end{array} \tag{3}
$$

Where the self and mutual values of $Y_A$ and $Y_D$ are those identified only with the nodes to be retained and removed respectively and $Y_C=Y_B^t$ is composed of only the corresponding mutual admittance values, that are common to the nodes m and p.

Now, for the p nodes to be eliminated, it is necessary that, each element of the vector $I_{BUS\text{-}p}$ should be zero. Thus we have from (3):

$$
\begin{aligned} I_{BUS\text{-}m} &= Y_A\, E_{BUS\text{-}m} + Y_B\, E_{BUS\text{-}p} \\ I_{BUS\text{-}p} &= Y_C\, E_{BUS\text{-}m} + Y_D\, E_{BUS\text{-}p} = 0 \end{aligned} \tag{4}
$$

Solving, $$\mathbf{E_{BUS\text{-}p} = - Y_D^{-1} Y_C\, E_{BUS\text{-}m}} \tag{5}$$

Thus, by simplification, we obtain an expression similar to (2) as,

$$\mathbf{I_{BUS\text{-}m}} = \{Y_A - Y_B Y_D^{-1} Y_C\}\, \mathbf{E_{BUS\text{-}m}} \tag{6}$$

Thus by comparing (2) and (6), we get an expression for the new bus admittance matrix in terms of the sub-matrices of the original bus admittance matrix as:

$$\mathbf{Y_{BUS}^{new} = \{Y_A - Y_B Y_D^{-1} Y_C\}} \tag{7}$$

This expression enables us to construct the given network with only the necessary nodes retained and all the unwanted nodes/buses eliminated. However, it can be observed from (7) that the expression involves finding the inverse of the sub-matrix $Y_D$ (of order p). This would be computationally very tedious if p, the nodes to be eliminated is very large, especially for real practical systems. In such cases, it is more advantageous to eliminate the unwanted nodes from the given network by considering one node only at a time for elimination, as discussed next.

**CASE-B: Separate Elimination of Nodes:**

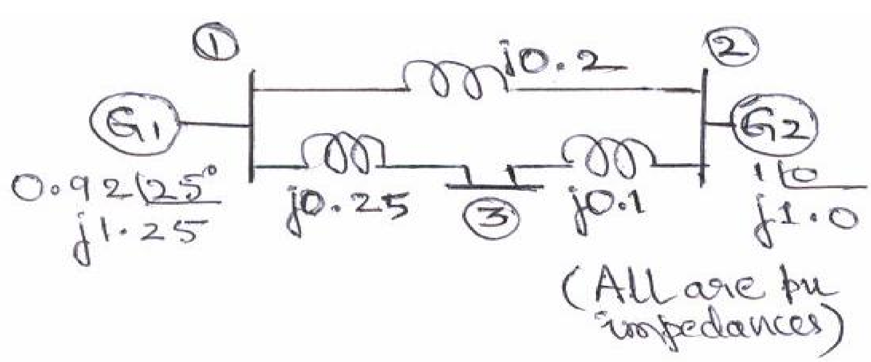
Here again, the system buses are to be renumbered, if necessary, such that the node to be removed always happens to be the last numbered one. The sub-matrix $Y_D$ then would be a single element matrix and hence it inverse would be just equal to its own reciprocal value. Thus the generalized algorithmic equation for finding the elements of the new bus admittance matrix can be obtained from (6) as,

$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\, Y_{nj} / Y_{nn}} \qquad \forall\ i,j = 1,2,\ldots\ldots n. \qquad \textbf{(8)}$$

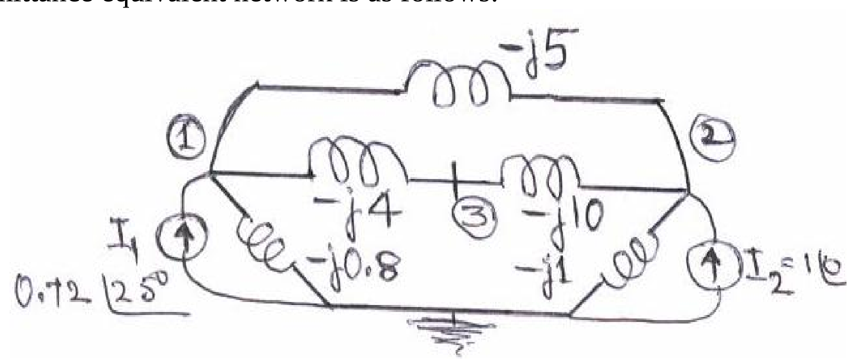
Each element of the original matrix must therefore be modified as per (7). Further, this procedure of eliminating the last numbered node from the given system of n nodes is to be iteratively repeated p times, so as to eliminate all the unnecessary p nodes from the original system.

**<u>Examples on Node elimination:</u>**

**Example-1:** *Obtain $Y_{BUS}$ for the impedance network shown below by the rule of inspection. Also, determine $Y_{BUS}$ for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*




The admittance equivalent network is as follows:



The bus admittance matrix is obtained by RoI as:

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

The reduced matrix after elimination of node 3 from the given system is determined as per the equation:

$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$Y_{BUS}^{new} =$

| *n/n* | *1* | *2* |
|---|---|---|
| *1* | -j8.66 | j7.86 |
| *2* | j7.86 | -j8.66 |

Alternatively,

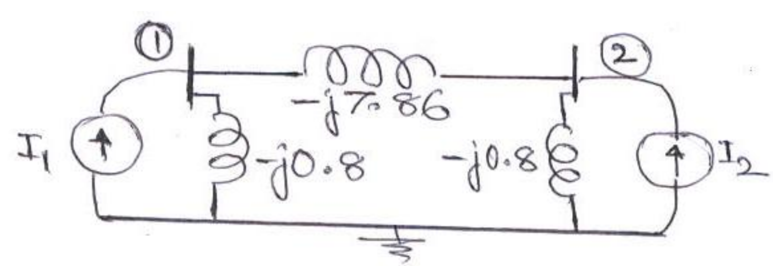
$$Y_{ij}^{new} = Y_{ij}^{old} - Y_{i3}\, Y_{3j} / Y_{33} \qquad \forall\ i,j = 1,2.$$

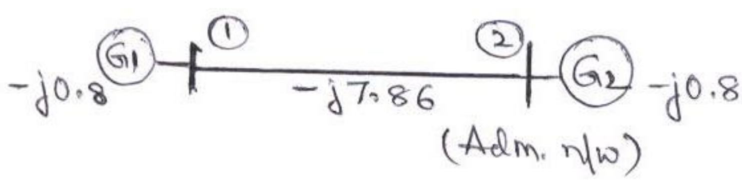
$$Y_{11} = Y_{11} - Y_{13}Y_{31}/\, Y_{33} = -j8.66$$

$$Y_{22} = Y_{22} - Y_{23}Y_{32}/\, Y_{33} = -j8.66$$

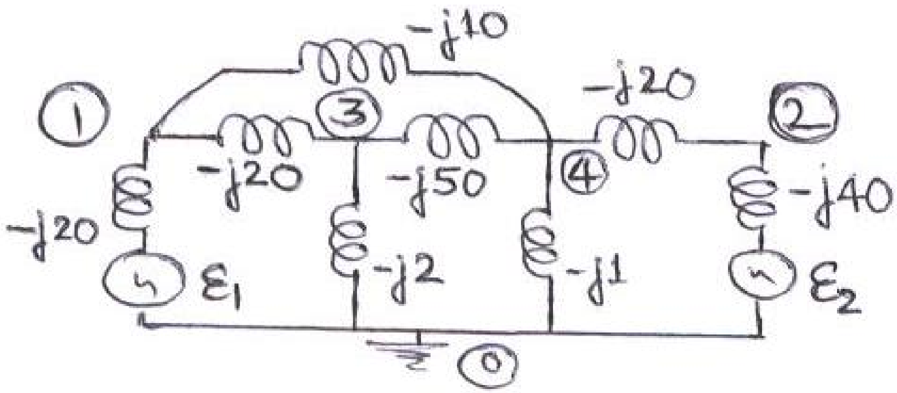
$$Y_{12} = Y_{21} = Y_{12} - Y_{13}Y_{32}/Y_{33} = j7.86$$

Thus the reduced network can be obtained again by the rule of inspection as shown be low.



**Example-2:** *Obtain $Y_{BUS}$ for the admittance network shown below by the rule of inspection. Also, determine $Y_{BUS}$ for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*




$Y_{BUS}=$

| n/n | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | **-j50** | **0** | **j20** | **j10** |
| 2 | **0** | **-j60** | **0** | **j72** |
| 3 | **j20** | **0** | **-j72** | **j50** |
| 4 | **j10** | **j72** | **j50** | **-j81** |

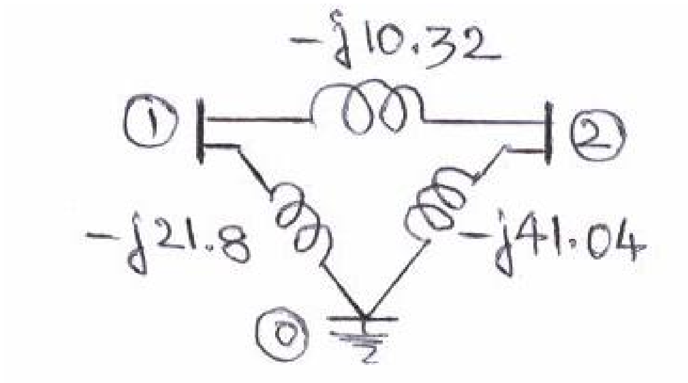
$$= \begin{vmatrix} \mathbf{Y_A} & \mathbf{Y_B} \\ \mathbf{Y_C} & \mathbf{Y_D} \end{vmatrix}$$

$$\mathbf{Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C}$$

$Y_{BUS}^{new}=$

| n/n | 1 | 2 |
|---|---|---|
| 1 | **-j32.12** | **j10.32** |
| 2 | **j10.32** | **-j51.36** |

*Thus the reduced system of two nodes can be drawn by the rule of inspection as under:*




## $Z_{BUS}$ building

## FORMATION OF BUS IMPEDANCE MATRIX

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\overline{E}_{bus} = [Z_{bus}]\bar{I}_{bus} \tag{9}$$

When expanded so as to refer to a *n* bus system, (9) will be of the form

$$\begin{aligned}
E_1 &= Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots + .Z_{1k}I_k \ldots + Z_{1n}I_n \\
&\vdots \\
&\vdots \\
E_k &= Z_{k1}I_1 + Z_{k2}I_2 + \ldots\ldots + Z_{kk}I_k + \ldots + Z_{kn}I_n \\
&\vdots \\
&\vdots \\
E_n &= Z_{n1}I_1 + Z_{n2}I_2 + \ldots\ldots + Z_{nk}I_k + \ldots + Z_{nn}I_n
\end{aligned} \tag{10}$$

Now assume that the bus impedance matrix $Z_{bus}$ is known for a partial network of *m* buses and a known reference bus. Thus, $Z_{bus}$ of the partial network is of dimension $m \times m$. If now a new element is added between buses *p* and *q* we have the following two possibilities:

(i) *p* is an existing bus in the partial network and *q* is a new bus; in this case *p*-*q* is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both *p* and *q* are buses existing in the partial network; in this case *p*-*q* is a **link** added to the p-network as shown in Fig 1b.

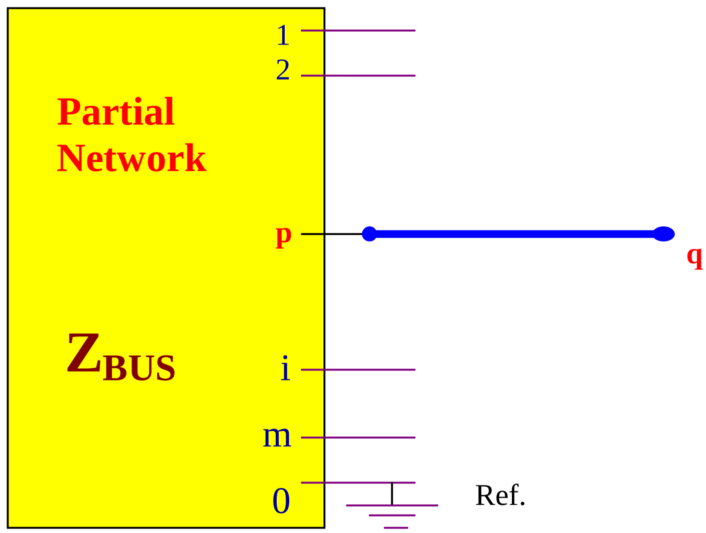



**Fig 1a. Addition of branch p-q**

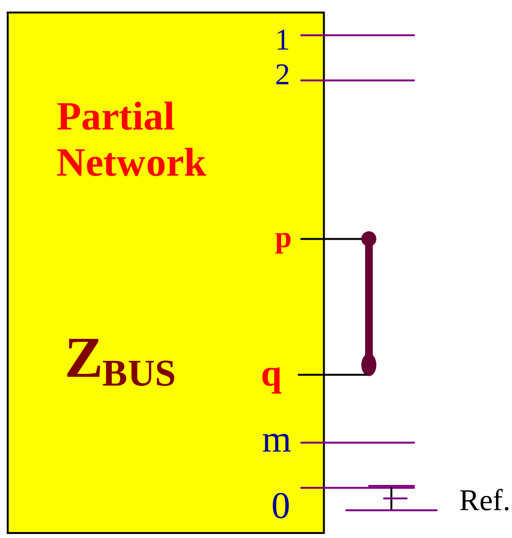



**Fig 1b. Addition of link p-q**

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix.

If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

## ADDITION OF A BRANCH

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \qquad (11)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

**Vector $y_{pq-rs}$ is not equal to zero** and $Z_{ij} = Z_{ji}$ $\forall$ i,j=1,2,…m,q (12)

**To find $Z_{qi}$:**

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

$E_k = Z_{ki} I_i = Z_{ki}$ $\forall$ k = 1, 2,…i…...p,….m, q (13)

Hence, $E_q = Z_{qi}$ ; $E_p = Z_{pi}$ ………

Also, $E_q=E_p - v_{pq}$ ; so that $Z_{qi} = Z_{pi} - v_{pq}$ $\forall$ i =1, 2,…i…...p,….m, $\neq$q (14)

**To find $v_{pq}$:**

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \tag{15}$$

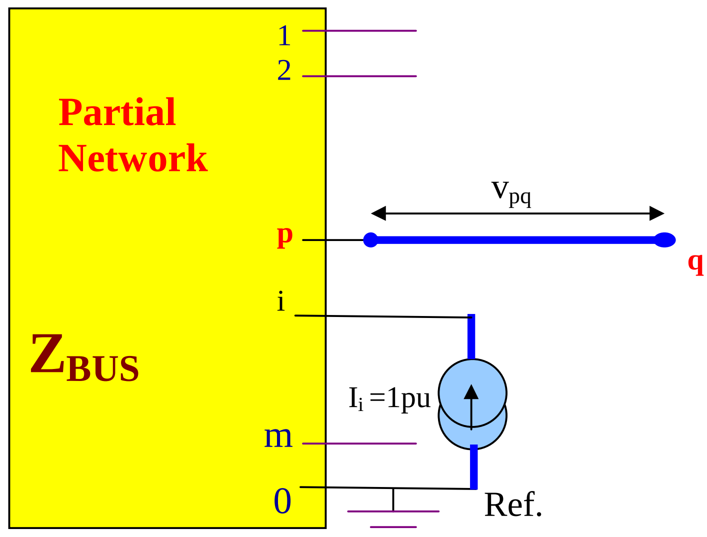



**Fig.2 Calculation for $Z_{qi}$**

where $i_{pq}$ is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

$v_{pq}$ is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \tag{16}$$

Solving, $v_{pq} = -\frac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \qquad (17)$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \quad i = 1,2......m; i \neq q \qquad (18)$$

**To find zqq:**

The element $Z_{qq}$ can be computed by injecting a current of 1pu at bus-q, $I_q$ = 1.0 pu. As before, we have the relations as under:

$E_k = Z_{kq} I_q = Z_{kq}$ $\quad\forall$ k = 1, 2,…i…....p,….m, q (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$ ; Also, $E_q = E_p - v_{pq}$ ; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs}\bar{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \qquad (21)$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rq} - \bar{Z}_{sq}\right)}{y_{pq,pq}} \qquad (22)$$

**Special Cases**

*The following special cases of analysis concerning $Z_{BUS}$ building can be considered with respect to the addition of branch to a p-network.*

**Case (a)**: If there is no mutual coupling then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then $E_p$=0. thus,

$Z_{pi} = 0 \qquad i = 1,2......m : i \neq q$

And $\qquad Z_{pq} = 0.$

Hence, from (18) (22) $\qquad Z_{qi} = 0 \qquad i = 1,2.......m; i \neq q$

And $\qquad Z_{qq} = z_{pq,pq}$ \ (23)

**Case (b)**: If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1,2....m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \tag{24}$$

## ADDITION OF A LINK

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \tag{25}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

**Vector $\mathbf{y_{pq\text{-}rs}}$ is not equal to zero** and **$\mathbf{Z_{ij} = Z_{ji}}$** $\forall$ i,j=1,2,…m,l. (26)

**<u>To find $Z_{li}$:</u>**

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, $e_l$ in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki} I_i = Z_{ki} \qquad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots.m, l \tag{27}$$

Hence, $e_l = E_l = Z_{li}$ ; $E_p = Z_{pi}$ ; $E_p = Z_{pi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$ ;

So that $Z_{li} = Z_{pi} - Z_{qi} - v_{pq}$ $\forall$ i=1,2,…i.…p,...q,….m, ≠l (28)

**To find $v_{pq}$:**

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \quad (29)$$

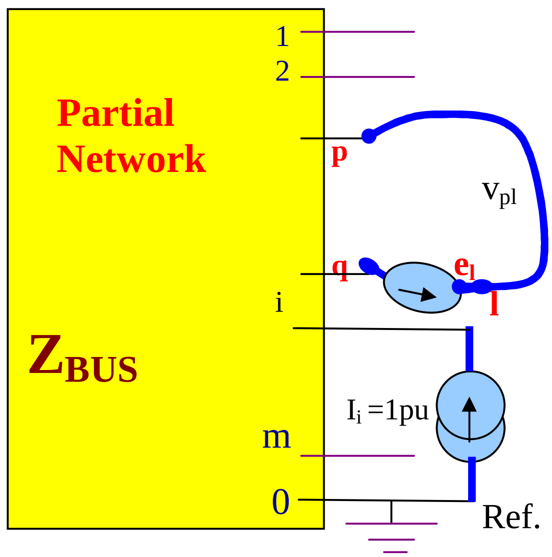



**Fig.3 Calculation for $Z_{li}$**

where $i_{pl}$ is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

$v_{pl}$ is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$ .

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl}v_{pl} + \bar{y}_{pl,rs}\bar{v}_{rs} = 0 \qquad (30)$$

Solving, $v_{pl} = -\frac{\bar{y}_{pl,rs}\bar{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \qquad (31)$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $y_{pl,pl} = y_{pq,pq}$ (32)

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2......m; i \neq l \qquad (33)$$

**<u>To find $Z_{ll}$:</u>**

The element $Z_{ll}$ can be computed by injecting a current of 1pu at bus-l, $I_l$ = 1.0 pu. As before, we have the relations as under:

$E_k = Z_{kl} I_l = Z_{kl}$ $\quad \forall$ k = 1, 2,…i…...p,…q,….m, l (34)

Hence, $e_l = E_l = Z_{ll}$ ; $E_p = Z_{pl}$ ;

Also, $e_l = E_p - E_q - v_{pl}$ ;

So that $Z_{ll} = Z_{pl} - Z_{ql} - v_{pl}$ $\forall$ i=1,2,…i.…p,...q,….m, ≠l (35)

Since now the current in the added element is $i_{pl} = -I_l = -1.0$, we have from (29)

$$i_{pl} = y_{pl,pl}v_{pl} + \bar{y}_{pl,rs}\bar{v}_{rs} = -1$$

Solving, $v_{pl} = -1 + \frac{\bar{y}_{pl,rs}\bar{v}_{rs}}{y_{pl,pl}}$

$$v_{pl} = -1 + \frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \qquad (36)$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $y_{pl,pl} = y_{pq,pq}$ (37)

Using (34), (36) and (37) in (35), we get

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \qquad (38)$$

**Special Cases Contd….**

*The following special cases of analysis concerning $Z_{BUS}$ building can be considered with respect to the addition of link to a p-network.*

**Case (c)**: If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then $E_p$=0. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1,2....m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \qquad (39)$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

**Case (d)**: If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is not the reference node, then

$Z_{li} = Z_{pi} - Z_{qi}$

$$Z_{ll} = Z_{pl} - Z_{ql} - z_{pq,pq}$$
$$= Z_{pp} + Z_{qq} - 2\,Z_{pq} + z_{pq,pq} \qquad (40)$$

## MODIFICATION OF $Z_{BUS}$ FOR NETWORK CHANGES

An element which is not coupled to any other element can be removed easily. The $Z_{bus}$ is modified as explained in sections above, by adding in parallel with the element (to be removed), a link whose impedance is equal to the negative of the impedance of the element to be removed. Similarly, the impedance value of an element which is not coupled to any other element can be changed easily. The $Z_{bus}$ is modified again as explained in sections above, by adding in parallel with the element (whose impedance is to be changed), a link element of impedance value chosen such that the parallel equivalent impedance is equal to the desired value of impedance. When mutually coupled elements are removed, the $Z_{bus}$ is modified by introducing appropriate changes in the bus currents of the original network to reflect the changes introduced due to the removal of the elements.

**Examples on $Z_{BUS}$ building**

**Example 1:** *For the positive sequence network data shown in table below, obtain $Z_{BUS}$ by building procedure.*

| Sl. No. | p-q (nodes) | Pos. seq. reactance in pu |
|---|---|---|
| 1 | 0-1 | 0.25 |
| 2 | 0-3 | 0.20 |
| 3 | 1-2 | 0.08 |
| 4 | 2-3 | 0.06 |

**Solution:**
The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.

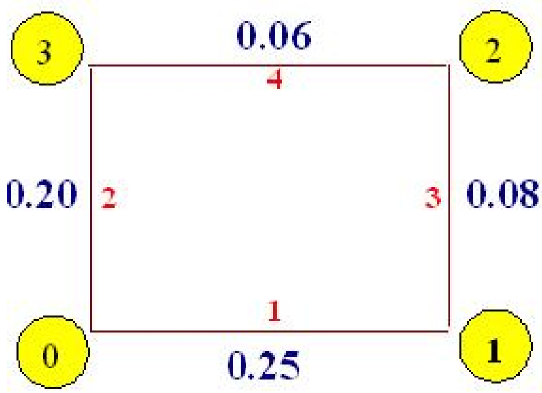
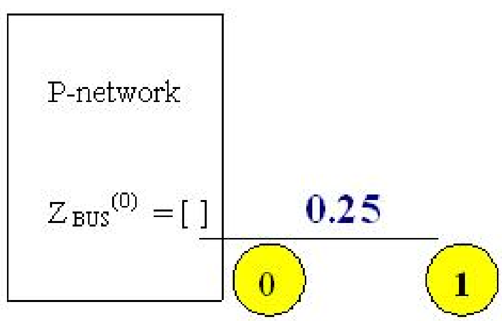



**Fig. E1: Example System**

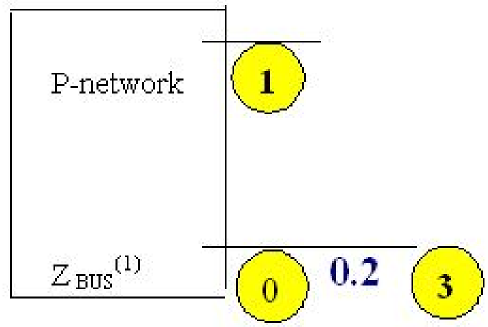
Consider building $Z_{BUS}$ as per the various stages of building through the consideration of the corresponding partial networks as under:

**Step-1**: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;




$$Z_{BUS}^{(1)} = \begin{array}{c|c|} & 1 \\ 1 & 0.25 \end{array}$$

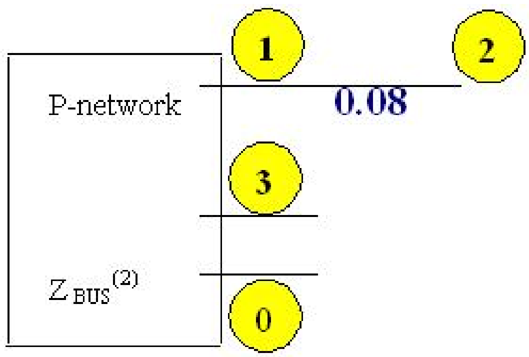
**Step-2**: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;




$Z_{BUS}^{(2)}$ =

| | 1 | 3 |
|---|---|---|
| 1 | 0.25 | 0 |
| 3 | 0 | 0.2 |

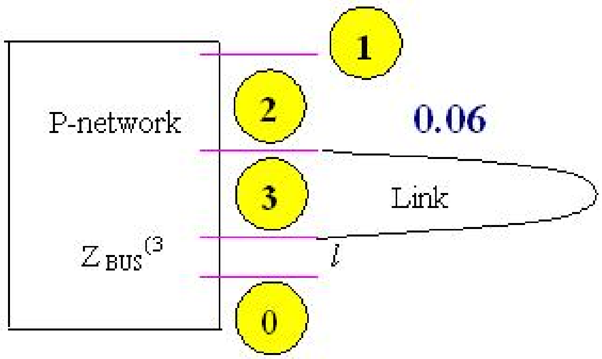
**Step-3**: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;




$Z_{BUS}^{(3)}$ =

| | 1 | 3 | 2 |
|---|---|---|---|
| 1 | 0.25 | 0 | 0.25 |
| 3 | 0 | 0.2 | 0 |
| 2 | 0.25 | 0 | 0.33 |

**Step-4**: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;




$Z_{BUS}^{(4)}$ =

| | 1 | 3 | 2 | l |
|---|---|---|---|---|
| 1 | 0.25 | 0 | 0.25 | 0.25 |
| 3 | 0 | 0.2 | 0 | -0.2 |
| 2 | 0.25 | 0 | 0.33 | 0.33 |
| l | 0.25 | -0.2 | 0.33 | 0.59 |

The fictitious node *l* is eliminated further to arrive at the final impedance matrix as under:

$Z_{BUS}^{(final)}$ =

| | 1 | 3 | 2 |
|---|---|---|---|
| 1 | 0.1441 | 0.0847 | 0.1100 |
| 3 | 0.0847 | 0.1322 | 0.1120 |
| 2 | 0.1100 | 0.1120 | 0.1454 |

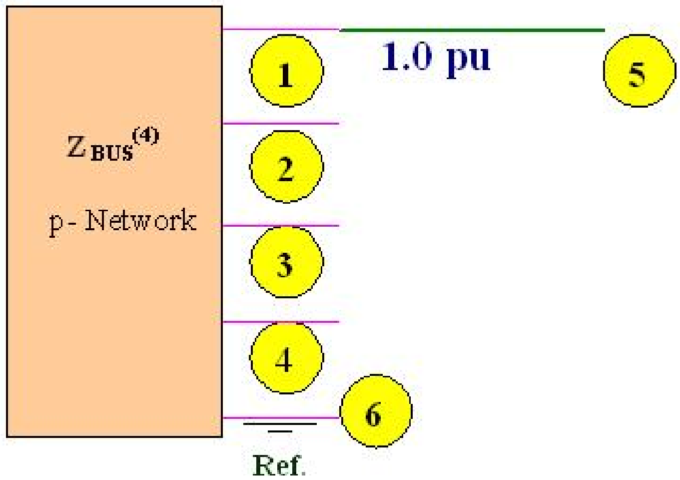
**Example 2:** *The $Z_{BUS}$ for a 6-node network with bus-6 as ref. is as given below. Assuming the values as pu reactances, find the topology of the network and the parameter values of the elements involved. Assume that there is no mutual coupling of any pair of elements.*

$Z_{BUS}$ =

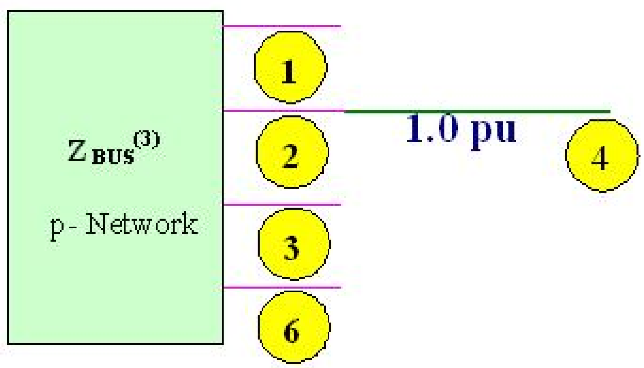
| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 2 | 0 | 0 | 0 | 2 |
| 2 | 0 | 2 | 0 | 2 | 0 |
| 3 | 0 | 0 | 2 | 0 | 0 |
| 4 | 0 | 2 | 0 | 3 | 0 |
| 5 | 2 | 0 | 0 | 0 | 3 |

**Solution:**

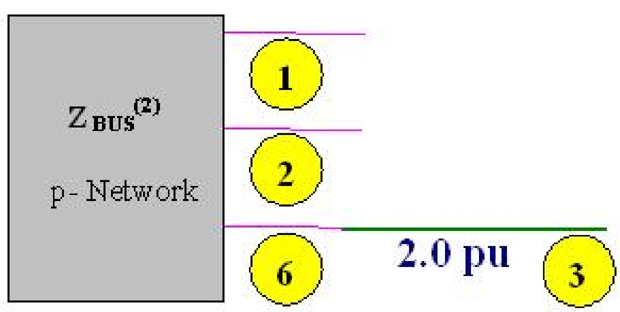
The specified matrix is so structured that by its inspection, we can obtain the network by backward analysis through the various stages of $Z_{BUS}$ building and p-networks as under:



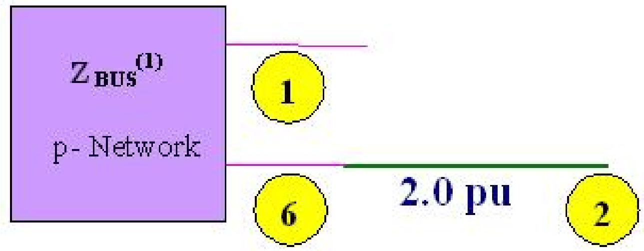
$Z_{BUS}^{(4)}$
p- Network
1
2
3
4
6
1.0 pu
5
Ref.

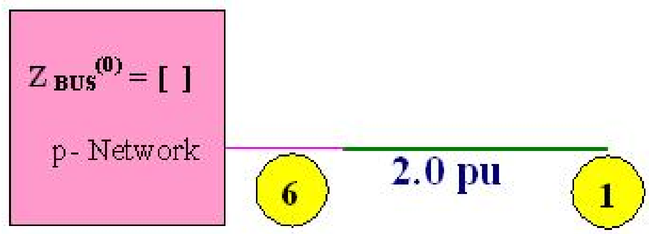



$Z_{BUS}^{(3)}$
p- Network
1
2
3
6
1.0 pu
4

$Z_{BUS}^{(2)}$
p- Network
1
2
6
2.0 pu
3




Thus the final network is with 6 nodes and 5 elements connected as follows with the impedance values of elements as indicated.

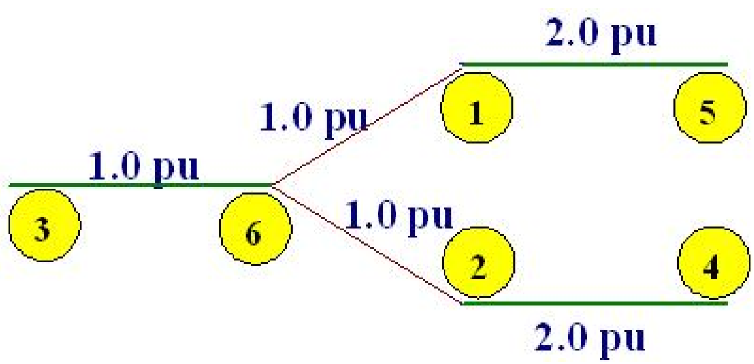
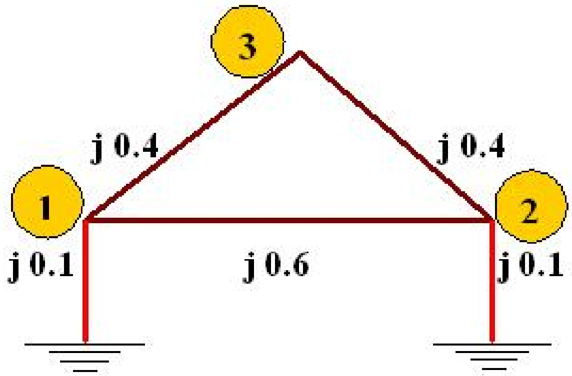


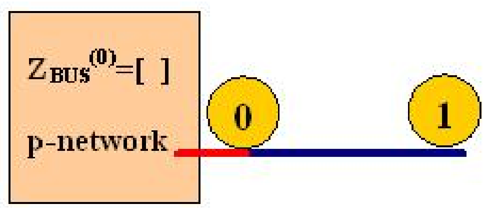

**Fig. E2: Resultant network of example-2**

**Example 3:** *Construct the bus impedance matrix for the system shown in the figure below by building procedure. Show the partial networks at each stage of building the matrix. Hence arrive at the bus admittance matrix of the system. How can this result be verified in practice?*



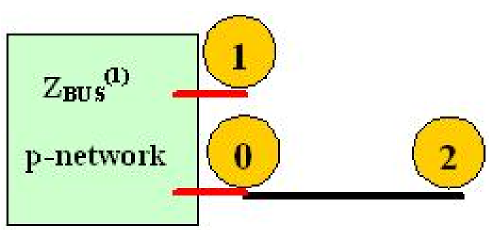

**Solution:** The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

**Step1:** Add branch 1 between node 1 and reference node. (q =1, p = 0)



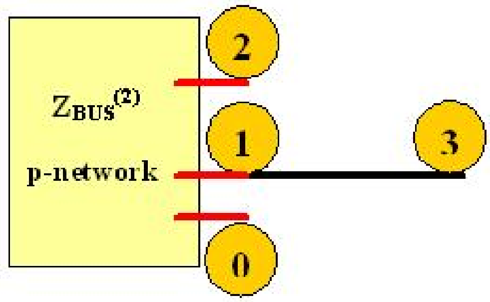

$$Z_{bus}^{(1)} = \begin{matrix} & 1 \\ 1 & [j0.1] \end{matrix}$$

**Step2:** Add branch 2, between node 2 and reference node. (q = 2, p = 0).



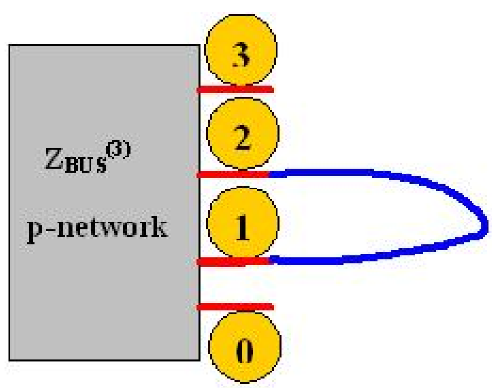
$$Z_{Bus} = \begin{matrix} & 1 & 2 \\ 1 & j0.1 & 0 \\ 2 & 0 & j0.15 \end{matrix}$$

**Step3:** Add branch 3, between node 1 and node 3 (p = 1, q = 3)




$$Z_{bus} = \begin{matrix} & 1 & 2 & 3 \\ 1 & j0.1 & 0 & j0.1 \\ 2 & 0 & j0.15 & 0 \\ 3 & j0.1 & 0 & j0.5 \end{matrix}$$

**Step 4:** Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



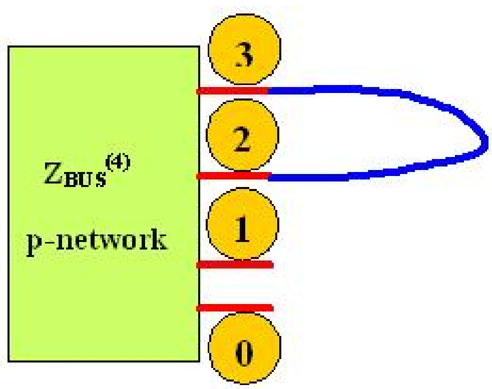
$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} j0.1 & 0 & j0.1 & j0.1 \\ 0 & j0.15 & 0 & -j0.15 \\ j0.1 & 0 & j0.5 & j0.1 \\ j0.1 & -j0.15 & j0.1 & j0.85 \end{bmatrix} \end{array}$$

Now the extra node-*l* has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\, Y_{nj} \,/\, Y_{nn}} \qquad \forall\ i,j = 1,2,3.$$

$$Z_{bus} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix} \end{array}$$

**Step 5:** Add link between node 2 and node 3 (p = 2, q=3)



$$Z_{l1} = Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058$$
$$Z_{l2} = Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588$$
$$Z_{l3} = Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058$$
$$Z_{ll} = Z_{2l} - Z_{3l} + z_{23,23}$$
$$= j0.10588 - (-j0.47058) + j0.4 = j0.97646$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{array}$$

Node *l* is eliminated as shown in the previous step:

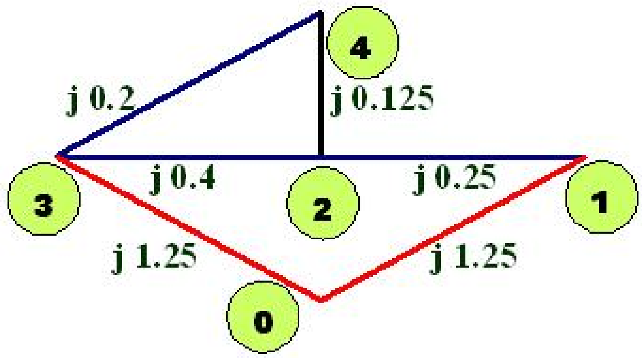
$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{array}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under:

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{array}$$

As a check, it can be observed that the bus admittance matrix, $Y_{BUS}$ can also be obtained by the rule of inspection to arrive at the same answer.

**Example 4:** Form the bus impedance matrix for the network shown below.




**Solution:**

Add the elements in the sequence, 0-1, 1-2, 2-3, 0-3, 3-4, 2-4, as per the various steps of building the matrix as under:

**Step1:** Add element 1, which is a branch between node-1 and reference node.

$$Z_{bus} = \begin{array}{c} \\ 1 \end{array}\begin{array}{c} 1 \\ \left[ j1.25 \right] \end{array}$$

**Step2:** Add element 2, which is a branch between nodes 1 and 2.

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} \quad 1 & \quad\quad 2 \end{array} \\ \begin{bmatrix} j1.25 & j1.25 \\ j1.25 & j1.5 \end{bmatrix} \end{array}$$

**Step3:** Add element 3, which is a branch between nodes 2 and 3

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} \quad 1 & \quad\quad 2 & \quad\quad 3 \end{array} \\ \begin{bmatrix} j1.25 & j1.25 & j1.25 \\ j1.25 & j1.5 & j1.5 \\ j1.25 & j1.5 & j1.9 \end{bmatrix} \end{array}$$

**Step4:** Add element 4, which is a link from node 3 to reference node.

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} j1.25 & j1.25 & j1.25 & j1.25 \\ j1.25 & j1.5 & j1.5 & j1.5 \\ j1.25 & j1.5 & j1.9 & j1.9 \\ j1.25 & j1.5 & j1.9 & j3.15 \end{bmatrix} \end{array}$$

Eliminating node *l*,

$$Z_{bus} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 \\ j0.65476 & j0.78571 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 \end{bmatrix} \end{array}$$

**Step5:** Add element 5, a branch between nodes 3 and 4.

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.75397 & j0.65476 & j0.49603 & j0.49603 \\ j0.65476 & j0.65476 & j0.59524 & j0.59524 \\ j0.49603 & j0.59524 & j0.75397 & j0.75397 \\ j0.49603 & j0.59524 & j0.75397 & j0.95397 \end{bmatrix} \end{array}$$

**Step 6:** Add element 6, a link between nodes 2 & 4.

$$Z_{bus} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ l \end{array}\begin{bmatrix} j0.75397 & j0.65476 & j0.49603 & j0.49603 & j0.15873 \\ j0.65476 & j0.65476 & j0.59524 & j0.59524 & j0.19047 \\ j0.49603 & j0.59524 & j0.75397 & j0.75397 & -j0.15873 \\ j0.49603 & j0.59524 & j0.75397 & j0.95397 & -j0.35873 \\ j0.15873 & j0.19047 & -j0.15873 & -j0.35873 & j0.67421 \end{bmatrix}$$

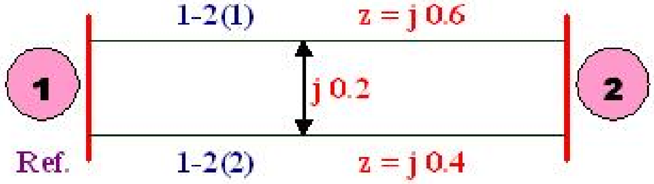
Eliminating node *l* we get the required bus impedance , matrix

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.7166 & j0.6099 & j0.5334 & j0.5805 \\ j0.6099 & j0.7319 & j0.6401 & j0.6966 \\ j0.5334 & j0.6401 & j0.7166 & j0.6695 \\ j0.5805 & j0.6966 & j0.6695 & j0.7631 \end{bmatrix} \end{array}$$

**Example 5:** Form the bus impedance matrix for the network data given below.

| Element | Self Impedance | | Mutual Impedance | |
|---|---|---|---|---|
| | Bus p-q | $z_{pq,\,pq}$ (pu) | Bus r-s | $z_{pq,\,rs}$ (pu) |
| 1 | 1 – 2(1) | j0.6 | | |
| 2 | 1 – 2(2) | j0.4 | 1 – 2(1) | j0.2 |

**Solution:**
Let bus-1 be the reference. Add the elements in the sequence 1-2(1), 1-2(2). Here, in the step-2, there is mutual coupling between the pair of elements involved.




**Step1:** Add element 1 from bus 1 to 2, element 1-2(1). ( p=1, q=2, p is the reference node)

$$Z_{bus} = 2\ \overset{2}{[j0.6]}$$

**Step2:** Add element 2, element 1-2(2), which is a link from bus1 to 2, mutually coupled with element 1, 1-2(1).

$$Z_{bus} = \begin{array}{c} \\ 2 \\ l \end{array}\begin{array}{c} \begin{array}{cc} 2 & l \end{array} \\ \begin{bmatrix} j0.6 & Z_{2l} \\ Z_{l2} & Z_{ll} \end{bmatrix} \end{array}$$

Where,

$$Z_{2l} = Z_{l2} = -Z_{22} + \frac{y_{12(2),12(1)}\left(Z_{12} - Z_{22}\right)}{y_{12(2),12(2)}}$$

$$Z_{12} = Z_{1l} = 0 \text{ (as bus 1 is reference)}$$

Consider the primitive impedance matrix for the two elements given by

$$[z] = \begin{array}{c} \\ 1-2(1) \\ 1-2(2) \end{array} \begin{array}{c} \begin{array}{cc} \text{1-2(1)} & \text{1-2(2)} \end{array} \\ \begin{bmatrix} j0.6 & j0.2 \\ j0.2 & j0.4 \end{bmatrix} \end{array}$$

Thus the primitive admittance matrix is obtained by taking the inverse of [z] as

$$[y] = \begin{array}{c} \\ 1-2(1) \\ 1-2(2) \end{array} \begin{array}{c} \begin{array}{cc} \text{1-2(1)} & \text{1-2(2)} \end{array} \\ \begin{bmatrix} -j2.0 & j1.0 \\ j1.0 & -j3.0 \end{bmatrix} \end{array}$$

Thus,

$y_{12(1),12(2)} = j1.0; \quad y_{12(2),12(2)} = -j3.0$

So that we have,

$$Z_{2l} = Z_{l2} = -j0.6 + \frac{(j1.0)(-j0.6)}{-j3.0} = -j0.4$$

$$Z_{ll} = -Z_{2l} + \frac{1 + y_{12(2),12(1)}(Z_{1l} - Z_{2l})}{y_{12(2),12(2)}} = j0.4 + \frac{1 + (j1.0)(j0.4)}{-j3.0} = j0.6$$

$$Z_{bus} = \begin{array}{c} \\ 2 \\ l \end{array} \begin{array}{c} \begin{array}{cc} 2 & l \end{array} \\ \begin{bmatrix} j0.6 & -j0.4 \\ -j0.4 & j0.6 \end{bmatrix} \end{array}$$

Thus, the network matrix corresponding to the 2-node, 1-bus network given, is obtained after eliminating the extra node-l as a single element matrix, as under:

$$Z_{bus} = 2 \begin{array}{c} 2 \\ [\,j0.3333\,] \end{array}$$

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# ECONOMIC OPERATION OF POWER SYSTEMS

## INTRODUCTION

One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system. Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants.

The operation economics can again be subdivided into two parts.

i) Problem of *economic dispatch*, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.

ii) Problem of *optimal power flow*, which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.

During operation of the plant, a generator may be in one of the following states:

i) Base supply without regulation: the output is a constant.

ii) Base supply with regulation: output power is regulated based on system load.

iii) Automatic non-economic regulation: output level changes around a base setting as area control error changes.

iv) Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting.

Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons. The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.

Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

# PERFORMANCE CURVES

## INPUT-OUTPUT CURVE

This is the fundamental curve for a thermal plant and is a plot of the input in British thermal units (Btu) per hour versus the power output of the plant in MW as shown in Fig 1.

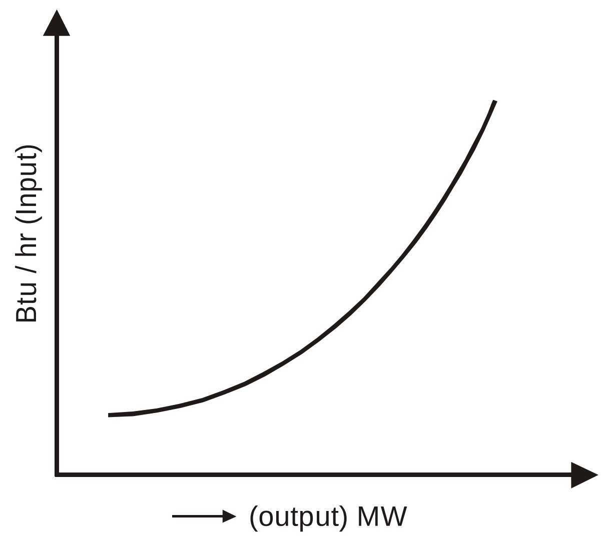



**Fig 1: Input – output curve**

## HEAT RATE CURVE

The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input – output curve at any point. The reciprocal of heat – rate is called fuel – efficiency. The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in Fig .2

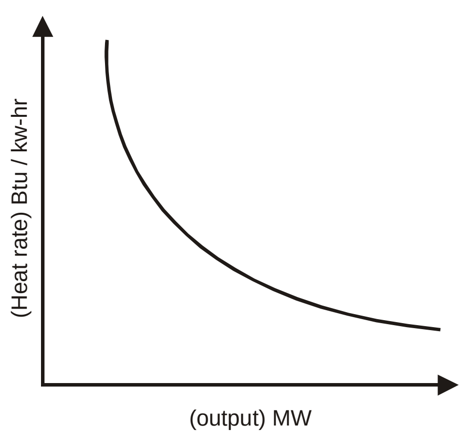


**Fig .2 Heat rate curve.**

## INCREMENTAL FUEL RATE CURVE

The incremental fuel rate is equal to a small change in input divided by the corresponding change in output.

$$\text{Incremental fuel rate} = \frac{\Delta Input}{\Delta Output}$$

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig 3

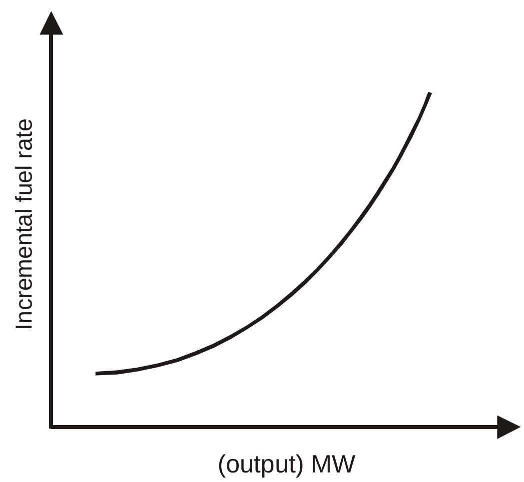



**Fig 3: Incremental fuel rate curve**

## Incremental cost curve

The incremental cost is the product of incremental fuel rate and fuel cost (Rs / Btu or $ / Btu). The curve in shown in Fig. 4. The unit of the incremental fuel cost is Rs / MWh or $ /MWh.

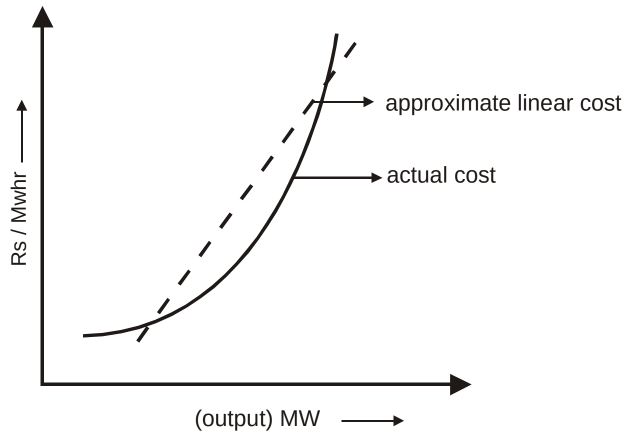



**Fig 4: Incremental cost curve**

In general, the fuel cost $F_i$ for a plant, is approximated as a quadratic function of the generated output $P_{Gi}$.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \text{ Rs / h}$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad \text{Rs / MWh}$$

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labour, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve. While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between $P_{Gmin}$, the minimum loading limit, below which it is technically infeasible to operate a unit and $P_{Gmax}$, which is the maximum output limit.

## ECONOMIC GENERATION SCHEDULING NEGLECTING LOSSES AND GENERATOR LIMITS

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand $P_D$.

Consider a system with $\mathbf{n}_g$ number of generating plants supplying the total demand $P_D$. If $F_i$ is the cost of plant ***i*** in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

where $F_T$ = total cost.
$P_{Gi}$ = generation of plant i.
$P_D$ = total demand.

This is a constrained optimization problem, which can be solved by Lagrange's method.

## LAGRANGE METHOD FOR SOLUTION OF ECONOMIC SCHEDULE

The problem is restated below:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$P_D = \sum_{i=1}^{n_g} P_{Gi} = 0$$

The augmented cost function is given by

$$£ = F_T + a\left( P_D - \sum_{i=1}^{n_g} P_{Gi} \right)$$

The minimum is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = 0 \text{ and } \frac{\partial £}{\partial a} = 0$$

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - a = 0$$

$$\frac{\partial £}{\partial a} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The second equation is simply the original constraint of the problem. The cost of a plant $F_i$ depends only on its own output $P_{Gi}$, hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda \quad ; \qquad i = 1\ldots\ldots n_g$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1\ldots\ldots n_g$$

The above equation is called the co-ordination equation. Simply stated, *for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand.* From  we have

$$P_{Gi} = \frac{\lambda - b_i}{2c_i}$$

We know in a loss less system

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Substituting (8.16) we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of $\lambda$ is obtained from (8.17) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}}$$

It can be seen that $\lambda$ is dependent on the demand and the coefficients of the cost function.

**Example 1.**

The fuel costs of two units are given by

$F_1 = 1.5 + 20\, P_{G1} + 0.1\, P_{G1}^2$ Rs/h

$F_2 = 1.9 + 30\, P_{G2} + 0.1\, P_{G2}^2$ Rs/h

$P_{G1}$, $P_{G2}$ are in MW. Find the optimal schedule neglecting losses, when the demand is 200 MW.

**Solution:**

$$\frac{dF_1}{dP_{G1}} = 20 + 0.2 P_{G1} \quad \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 30 + 0.2 P_{G2} \quad \text{Rs / MWh}$$

$$P_D = P_{G1} + P_{G2} = 200 \text{ MW}$$

For economic schedule

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \lambda$$

$$20 + 0.2\, P_{G1} = 30 + 0.2\,(200 - P_{G1})$$

Solving we get, $P_{G1} = 125$ MW

$P_{G2} = 75$ MW

$$\lambda = 20 + 0.2\,(125) = 45 \text{ Rs / MWh}$$

**Example 2**

The fuel cost in \$ / h for two 800 MW plants is given by

$F_1 = 400 + 6.0\, P_{G1} + 0.004\, P_{G1}^2$

$F_2 = 500 + b_2\, P_{G2} + c_2\, P_{G2}^2$

where $P_{G1}$, $P_{G2}$ are in MW

(a) The incremental cost of power, $\lambda$ is \$8 / MWh when total demand is 550MW. Determine optimal generation schedule neglecting losses.

(b) The incremental cost of power is \$10/MWh when total demand is 1300 MW. Determine optimal schedule neglecting losses.

(c) From (a) and (b) find the coefficients $b_2$ and $c_2$.

**Solution:**

a) $$P_{G1} = \frac{\lambda - b_1}{2c_1} = \frac{8.0 - 6.0}{2 \times 0.004} = 250 \text{ MW}$$

$$P_{G2} = P_D - P_{G1} = 550 - 250 = 300 \text{ MW}$$

b) $P_{G1} = \frac{\text{a} - b_1}{2C_1} = \frac{10 - 6}{2 \times 0.004} = 500 \text{ MW}$

$P_{G2} = P_D - P_{G1} = 1300 - 500 = 800 \text{ MW}$

c) $P_{G2} = \frac{\text{a} - b_2}{2c_2}$

From (a) $300 = \frac{8.0 - b_2}{2c_2}$

From (b) $800 = \frac{10.0 - b_2}{2c_2}$

Solving we get $b_2 = 6.8$
$c_2 = 0.002$

## ECONOMIC SCHEDULE INCLUDING LIMITS ON GENERATOR (NEGLECTING LOSSES)

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi\ (min)} \leq P_{Gi} \leq P_{Gi\ (max)}\ ;\ i = 1, \ldots\ldots\ldots n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

**Example 3**

Incremental fuel costs in \$ / MWh for two units are given below:

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 2.0 \ \$ / \text{MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.012P_{G2} + 1.6 \text{ \$ / MWh}$$

The limits on the plants are $P_{min}$ = 20 MW, $P_{max}$ = 125 MW. Obtain the optimal schedule if the load varies from 50 – 250 MW.

**Solution:**

The incremental fuel costs of the two plants are evaluated at their lower limits and upper limits of generation.

At $P_{G\,(min)}$ = 20 MW.

$$a_{1(min)} = \frac{dF_1}{dP_{G1}} = 0.01\text{x } 20+2.0 = 2.2\text{\$ / MWh}$$

$$a_{2(min)} = \frac{dF_2}{dP_{G2}} = 0.012 \text{ x } 20 + 1.6 = 1.84 \text{ \$ / MWh}$$

At $P_{G\,(Max)}$ =125 Mw

$\lambda_{1(max)}$ = 0.01 x 125 + 2.0 = 3.25 \$ / MWh

$\lambda_{2(max)}$ = 0.012 x 125 + 1.6 = 3.1 \$ / MWh

Now at light loads unit 1 has a higher incremental cost and hence will operate at its lower limit of 20 MW. Initially, additional load is taken up by unit 2, till such time its incremental fuel cost becomes equal to 2.2\$ / MWh at $P_{G2}$ = 50 MW. Beyond this, the two units are operated with equal incremental fuel costs. The contribution of each unit to meet the demand is obtained by assuming different values of $\lambda$; When $\lambda$ = 3.1 \$ / MWh, unit 2 operates at its upper limit. Further loads are taken up by unit 1. The computations are show in Table

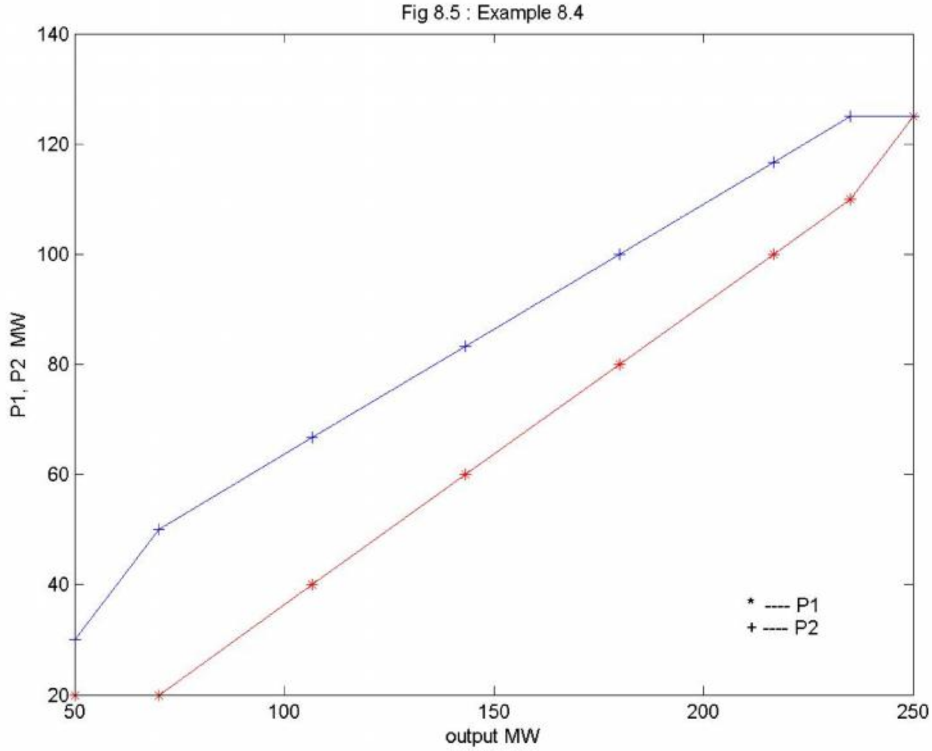
**Table Plant output and output of the two units**

| $\frac{dF_1}{dP_{G1}}$ \$/MWh | $\frac{dF_2}{dP_{G2}}$ \$/MWh | Plant $\lambda$ \$/MWh | $P_{G1}$ MW | $P_{G2}$ MW | Plant Output MW |
|---|---|---|---|---|---|
| 2.2 | 1.96 | 1.96 | $20^+$ | 30 | 50 |
| 2.2 | 2.2 | 2.2 | $20^+$ | 50 | 70 |
| 2.4 | 2.4 | 2.4 | 40 | 66.7 | 106.7 |
| 2.6 | 2.6 | 2.6 | 60 | 83.3 | 143.3 |
| 2.8 | 2.8 | 2.8 | 80 | 100 | 180 |
| 3.0 | 3.0 | 3.0 | 100 | 116.7 | 216.7 |
| 3.1 | 3.1 | 3.1 | 110 | 125* | 235 |

| | | | | | |
|---|---|---|---|---|---|
| 3.25 | 3.1 | 3.25 | 125* | 125* | 250 |

For a particular value of $\lambda$, $P_{G1}$ and $P_{G2}$ are calculated using (8.16). Fig 8.5 Shows plot of each unit output versus the total plant output.

Fig 8.5 : Example 8.4




For any particular load, the schedule for each unit for economic dispatch can be obtained

.

**Example 4.**

In example 3, what is the saving in fuel cost for the economic schedule compared to the case where the load is shared equally. The load is 180 MW.

**Solution:**

From Table it is seen that for a load of 180 MW, the economic schedule is $P_{G1}$ = 80 MW and $P_{G2}$ = 100 MW. When load is shared equally $P_{G1}$ = $P_{G2}$ = 90 MW. Hence, the generation of unit 1 increases from 80 MW to 90 MW and that of unit 2 decreases from 100 MW to 90 MW, when the load is shared equally. There is an increase in cost of unit 1 since $P_{G1}$ increases and decrease in cost of unit 2 since $P_{G2}$ decreases.

Increase in cost of unit 1 $= \int_{80}^{90} \left( \frac{dF_1}{dP_{G1}} \right) dP_{G1}$

$$= \int_{80}^{90} (0.01 P_{G1} + 2.0) dP_{G1} = 28.5 \, \$ / \text{h}$$

Decrease in cost of unit 2 $= \int_{100}^{90} \left( \frac{dF_2}{dP_{G2}} \right) dP_{G2}$

$$= \int_{100}^{90} (0.012 P_{G2} + 1.6) dP_{G2} = -27.4 \, \$ / \text{h}$$

Total increase in cost if load is shared equally = 28.5 – 27.4 = 1.1 $ / h

Hence the saving in fuel cost is 1.1 $ / h if coordinated economic schedule is used.

## ECONOMIC DISPATCH INCLUDING TRANSMISSION LOSSES

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation. The mathematical formulation is now stated as

Minimize $F_T = \sum_{i=1}^{n_g} F_i$

Such That $\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$

where $P_L$ is the total loss.

The Lagrange function is now written as

$$£ = F_T - a \left( \sum_{i=1}^{n_g} P_{Gi} - P_D - P_L \right) = 0$$

The minimum point is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - a \left( 1 - \frac{\partial P_L}{\partial P_{Gi}} \right) = 0 ; \; i = 1 \ldots\ldots n_g$$

$$\frac{\partial £}{\partial a} = \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L = 0 \qquad \text{(Same as the constraint)}$$

Since $\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$, (8.27) can be written as

$$\frac{dF_i}{dP_{Gi}} + a \frac{\partial P_L}{\partial P_{Gi}} = a$$

$$a = \frac{dF_i}{dP_{Gi}} \left( \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} \right)$$

The term $\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}$ is called the penalty factor of plant ***i***, $\mathbf{L_i}$. The coordination equations including losses are given by

$$a = \frac{dF_i}{dP_{Gi}} L_i \text{ ; } i = 1 \ldots\ldots n_g$$

*The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered.*

A rigorous general expression for the loss $P_L$ is given by

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn} + \Sigma_n P_{Gn} B_{no} + B_{oo}$$

where $B_{mn}$, $B_{no}$ , $B_{oo}$ called loss – coefficients , depend on the load composition. The assumption here is that the load varies linearly between maximum and minimum values.

A simpler expression is

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn}$$

The expression assumes that all load currents vary together as a constant complex fraction of the total load current. Experiences with large systems has shown that the loss of accuracy is not significant if this approximation is used.

An average set of loss coefficients may be used over the complete daily cycle in the coordination of incremental production costs and incremental transmission losses. In general, $B_{mn} = B_{nm}$ and can be expanded for a two plant system as

$$P_L = B_{11} P_{G1} + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$$

**Example 5**

A generator is supplying a load. An incremental change in load of 4 MW requires generation to be increased by 6 MW. The incremental cost at the plant bus is Rs 30 / MWh. What is the incremental cost at the receiving end?

**Solution:**

$$\frac{dF_1}{dP_{G1}} = 30$$

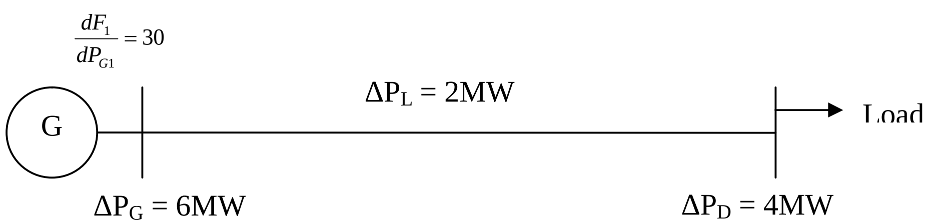



**Fig ; One line diagram of example 5**

$\Delta P_L = \Delta P_G - \Delta P_D = 2$ MW

$\lambda$ at receiving end is given by

$$\lambda = \frac{dF_1}{dP_{G1}} \times \frac{\Delta P_G}{\Delta P_D} = 30 \times \frac{6}{4} = 45 \text{ Rs / MWh}$$

or $$\lambda = \frac{dF_1}{dP_{G1}} \times \frac{1}{1 - \frac{\Delta P_L}{\Delta P_G}} = 30 \times \frac{1}{1 - \frac{2}{6}} = 45 \text{ Rs / MWh}$$

**Example 6**

In a system with two plants, the incremental fuel costs are given by

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 20 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.015P_{G2} + 22.5 \text{ Rs / MWh}$$

The system is running under optimal schedule with $P_{G1} = P_{G2} = 100$ MW.

If $\frac{\partial P_L}{\partial P_{G2}} = 0.2$, find the plant penalty factors and $\frac{\partial P_L}{\partial P_{G1}}$.

**Solution:**

For economic schedule,

$$\frac{dF_i}{dP_{Gi}} L_i = \lambda \; ; \qquad L_i = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_{Gi}}} \; ;$$

For plant 2, $P_{G2} = 100$ MW

$$\therefore \left(0.015 \times 100 + 22.5\right) \frac{1}{1 - 0.2} = \lambda .$$

Solving, $\lambda = 30 \, \text{Rs / MWh}$

$$L_2 = \frac{1}{1 - 0.2} = 1.25$$

$$\frac{dF_1}{dP_{G1}} L_1 = \lambda \Rightarrow (0.01\text{x}100+20)\, L_1 = 30$$

$L_1 = 1.428$

$$L_1 = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_{G1}}}$$

$$1.428 = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_{G1}}} \; ; \text{Solving} \quad \frac{\partial P_L}{\partial P_{G1}} = 0.3$$

**Example 7**

A two bus system is shown in Fig. 8.8 If 100 MW is transmitted from plant 1 to the load, a loss of 10 MW is incurred. System incremental cost is Rs 30 / MWh. Find $P_{G1}$, $P_{G2}$ and power received by load if

$$\frac{dF_1}{dP_{G1}} = 0.02 P_{G1} + 16.0 \ \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.04 P_{G2} + 20.0 \ \text{Rs / MWh}$$

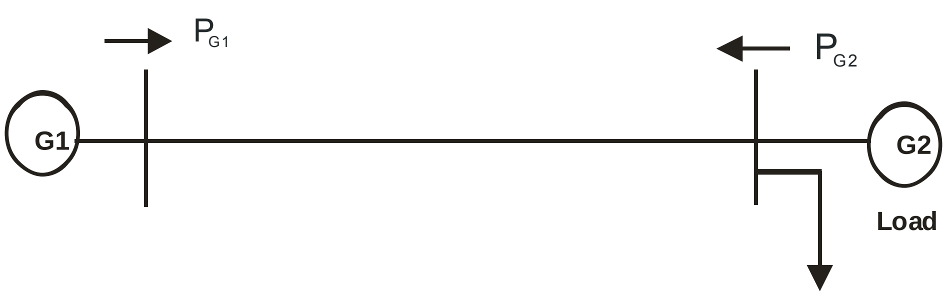


**Fig One line diagram of example 7**

**Solution;**

Since the load is connected at bus 2 , no loss is incurred when plant two supplies the load.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig a

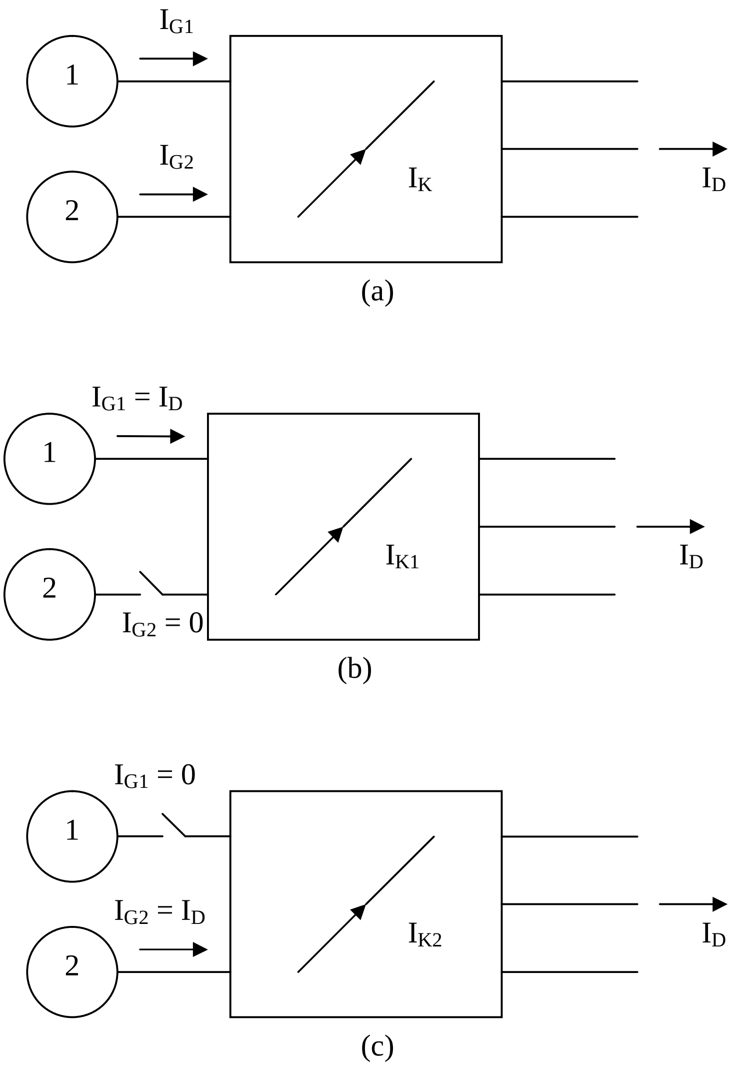



Fig  Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be $I_{K1}$. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current $I_{D,}$ as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

$N_{K1}$ and $N_{K2}$ are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of $I_D$. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1}\, I_{G1} + N_{K2}\, I_{G2}$$

where $I_{G1}$, $I_{G2}$ are the currents supplied by plants 1 and 2 respectively, to meet the demand $I_D$. Because of the assumptions made, $I_{K1}$ and $I_D$ have same phase angle, as do $I_{K2}$ and $I_D$. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}|\angle\delta_1 \text{ and } I_{G2} = |I_{G2}|\angle\delta_2 .$$

where $\delta_1$ and $\delta_2$ are phase angles of $I_{G1}$ and $I_{G2}$ with respect to a common reference. We can write

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\delta_1 + N_{K2}|I_{G2}|\cos\delta_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\delta_1 + N_{K2}|I_{G2}|\sin\delta_2\right)^2$$

$$= N_{K1}^2|I_{G1}|^2\left[\cos^2\delta_1 + \sin^2\delta_1\right] + N_{K2}^2|I_{G2}|^2\left[\cos^2\delta_2 + \sin^2\delta_2\right] + 2\left[N_{K1}|I_{G1}|\cos\delta_1 N_{K2}|I_{G2}|\cos\delta_2 + N_{K1}|I_{G1}|\sin\delta_1 N_{K2}|I_{G2}|\sin\delta_2\right]$$

$$= N_{K1}^2|I_{G1}|^2 + N_{K2}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\delta_1 - \delta_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where $P_{G1}$, $P_{G2}$ are three phase real power outputs of plant1 and plant 2; $V_1$, $V_2$ are the line to line bus voltages of the plants and $\phi_1, \phi_2$ are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and $R_K$ is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\delta_1 - \delta_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K + \frac{P_{G2}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^2 R_K$$

$$P_L = P_{G1}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K$$

$$B_{12} = \frac{\cos(\delta_1 - \delta_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}{}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit $MW^{-1}$.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}{}^2 + \ldots\ldots + \frac{P_{Gn}{}^2}{|V_n|^2 (\cos\phi_n)^2} \sum_K N_{Kn}{}^2 R_K$$
$$+ 2 \sum_{\substack{p,q=1 \\ p \neq q}}^{n} \frac{P_{GP} P_{Gq} \cos(\delta_p - \delta_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_k N_{KP} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{Pq} P_{Gq}$$

$$B_{Pq} = \frac{\cos(\delta_p - \delta_q)}{|V_p||V_q|\cos\phi_P \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

**Example 8**

Calculate the loss coefficients in pu and $MW^{-1}$ on a base of 50MVA for the network of Fig below. Corresponding data is given below.

| | |
|---|---|
| $I_a$ = 1.2 – j 0.4 pu | $Z_a$ = 0.02 + j 0.08 pu |
| $I_b$ = 0.4 - j 0.2 pu | $Z_b$ = 0.08 + j 0.32 pu |
| $I_c$ = 0.8 - j 0.1 pu | $Z_c$ = 0.02 + j 0.08 pu |
| $I_d$ = 0.8 - j 0.2 pu | $Z_d$ = 0.03 + j 0.12 pu |
| $I_e$ = 1.2 - j 0.3 pu | $Z_e$ = 0.03 + j 0.12 pu |

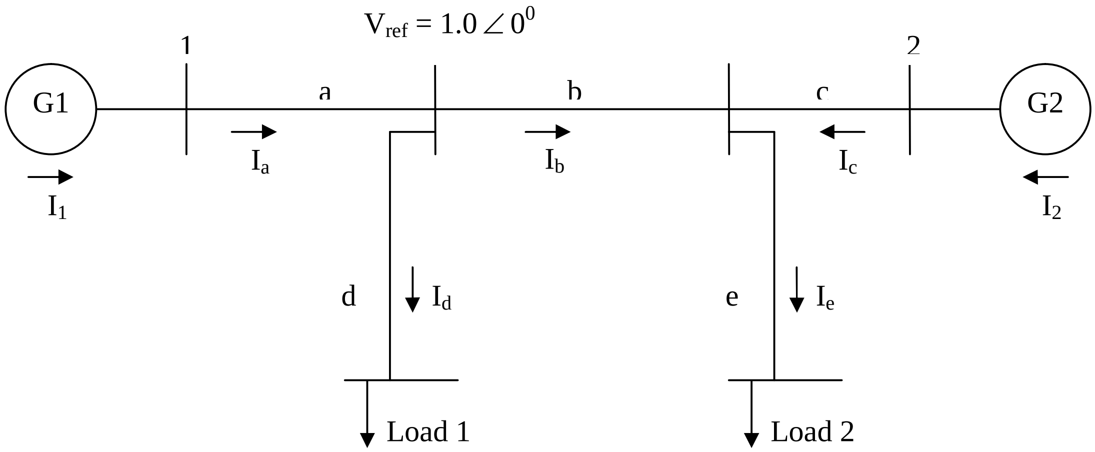



Fig : Example 8

**Solution:**

Total load current

$I_L = I_d + I_e = 2.0 - j\,0.5 = 2.061 \angle -14.03^0 A$

$I_{L1} = I_d = 0.8 - j\,0.2 = 0.8246 \angle -14.03^0$ A

$$\frac{I_{L1}}{I_L} = 0.4; \qquad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$$

If generator 1, supplies the load then $I_1 = I_L$. The current distribution is shown in Fig a.

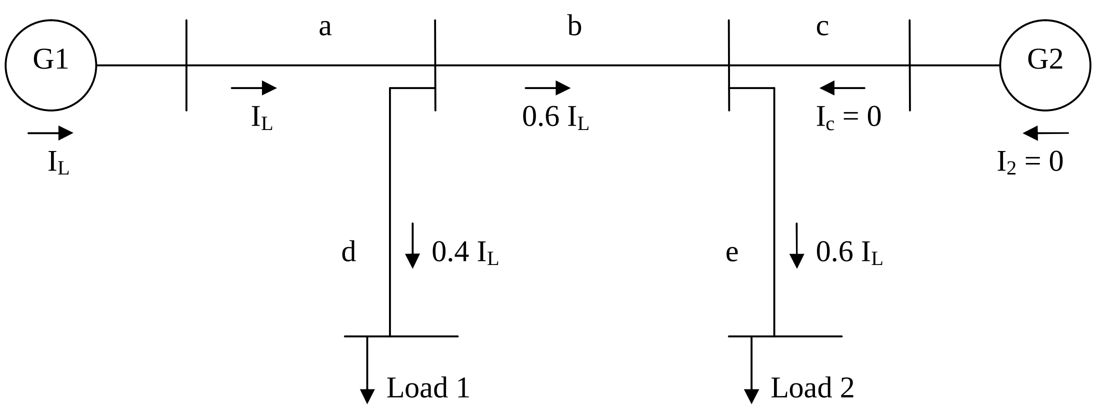



Fig a : Generator 1 supplying the total load

$$N_{a1} = \frac{I_a}{I_L} = 1.0; \;\; N_{b1} = \frac{I_b}{I_L} = 0.6; \;\; N_{C1} = 0; \;\; N_{d1} = 0.4; \;\; N_{e1} = 0.6.$$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

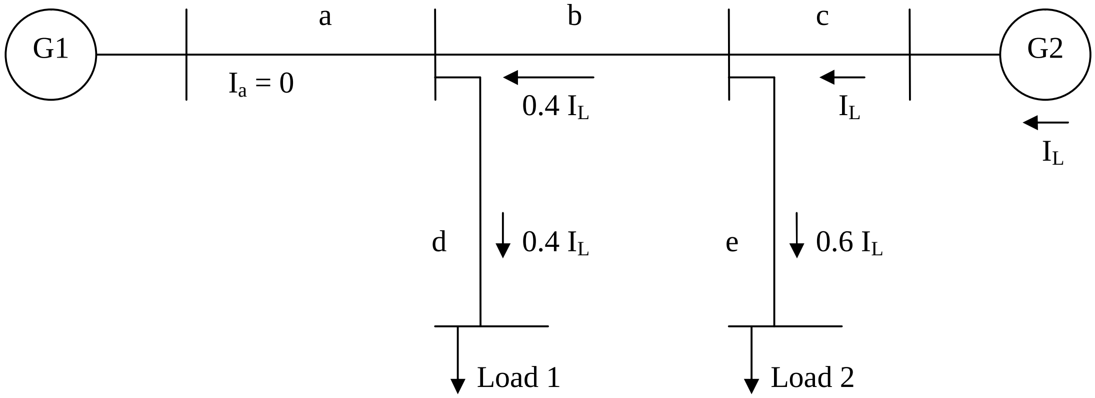



Fig b: Generator 2 supplying the total load

$N_{a2}$ =0; $N_{b2}$ = -0.4; $N_{c2}$ = 1.0; $N_{d2}$ = 0.4; $N_{e2}$ = 0.6

From Fig 8.10, $V_1 = V_{ref} + Z_a I_a$

$$= 1 \angle 0^0 + (1.2 - j\,0.4)\,(0.02 + j0.08)$$

$$= 1.06 \angle 4.78^0 = 1.056 + j\,0.088 \text{ pu.}$$

$V_2 = V_{ref} - I_b\,Z_b + I_c\,Z_c$

$$= 1.0 \angle 0^0 - (0.4 - j\,0.2)\,(0.08 + j\,0.32) + (0.8 - j\,0.1)\,(0.02 + j\,0.08)$$

$$= 0.928 - j\,0.05 = 0.93 \angle \text{-}3.10^0 \text{ pu.}$$

**Current Phase angles**

$$\pm_1 = \text{angle of } I_1(=I_a) = \tan^{-1}\left(\frac{-0.4}{1.2}\right) = -18.43^0$$

$$\pm_2 = \text{angle of } I_2(=I_c) = \tan^{-1}\left(\frac{-0.1}{0.8}\right) = -7.13^0$$

$$\cos(\pm_1 - \pm_2) = 0.98$$

**Power factor angles**

$$\text{¤}_1 = 4.78^0 + 18.43 = 23.21^0; \cos\text{¤}_1 = 0.92$$

$$\text{¤}_2 = 7.13^0 - 3.10^0 = 4.03^0; \cos\text{¤}_2 = 0.998$$

$$B_{11} = \frac{\sum_K N_{K1}{}^2 R_K}{|V_1|^2(\cos\text{¤}_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2(0.920)^2}$$

= 0.0677 pu

$$= 0.0677 \text{ x } \frac{1}{50} = 0.1354 \times 10^{-2}\,\text{MW}^{-1}$$

$$B_{12} = \frac{Cos(\pm_1 - \pm_2)}{|V_1||V_2|(\cos\text{¤}_1)(\cos\text{¤}_2)} \sum_K N_{K1} N_{K2} R_K$$

$$= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)}\left[-0.4\times0.6\times0.08+0.4\times0.4\times0.03+0.6\times0.6\times0.03\right]$$

= - 0.00389 pu

= - 0.0078 x $10^{-2}$ $MW^{-1}$

$$B_{22} = \frac{\sum_{K} N_{K2}{}^{2} R_K}{|V_2|^2\left(\cos\phi_2{}^{2}\right)}$$

$$= \frac{(-0.4)^2\, 0.08+1.0^2\times0.02+0.4^2\times0.03+0.6^2\times0.03}{(0.93)^2(0.998)^2}$$

= 0.056pu $= 0.112\times10^{-2}$ $MW^{-1}$

**HYDRO THERMAL SCHEDULING**

OPTIMAL SCHEDULING OF HYDROTHERMAL SYSTEM

No state or country is endowed with plenty of water sources or abundant coal or nuclear fuel. In states, which have adequate hydro as well as thermal power generation capacities, proper co-ordination to obtain a most economical operating state is essential.

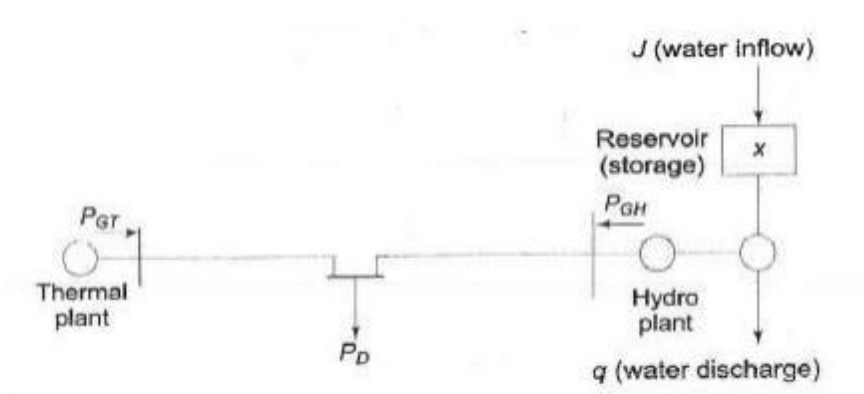
Maximum advantage is to use hydro power so that the coal reserves can be conserved and environmental pollution can be minimized.

However in many hydro systems, the generation of power is an adjunct to control of flood water or the regular scheduled release of water for irrigation. Recreations centers may have developed along the shores of large reservoir so that only small surface water elevation changes are possible.

The whole or a part of the base load can be supplied by the run-off river hydro plants, and the peak or the remaining load is then met by a proper mix of reservoir type hydro plants and thermal plants. Determination of this by a proper mix is the determination of the most economical operating state of a hydro-thermal system.

The hydro-thermal coordination is classified into long term co-ordination and short term coordination. The previous sections have dealt with the problem of optimal scheduling of a power system with thermal plants only. Optimal operating policy in this case can be completely determined at any instant without reference to operation at other times. This, indeed, is the static optimization problem. Operation of a system having both hydro and thermal plants is, however, far more complex as hydro plants have negligible operating cost, but are required to operate under constraints of water available for hydro generation in a given period of time. The problem thus belongs to the realm of dynamic optimization. The problem of minimizing the operating cost of a hydrothermal system can be viewed as one of minimizing the fuel cost of thermal plants under the constraint of water availability (storage and inflow) for hydro generation over a given period of operation.

For the sake of simplicity and understanding, the problem formulation and solution technique are illustrated through a simplified hydrothermal system of Fig. This system consists of one hydro and one thermal plant supplying power to a centralized load and is referred to as a fundamental system. Optimization will be carried out with real power generation as control variable, with transmission loss accounted for by the loss formula. Mathematical Formulation For a certain period of operation T (one year, one month or one day, depending upon the requirement), it is assumed that (i) storage of hydro reservoir at the beginning and the end of the period are specified, and (ii) water inflow to reservoir (after accounting for irrigation use) and load demand on the system are known as functions



of time with complete certainty (deterministic case). The problem is to determine q(t), the water discharge (rate) so as to minimize the cost of thermal generation.

$$C_T = \int_0^T C'(P_{GT}(t))dt \tag{3.1}$$

under the following constraints: (i) Meeting the load demand

$$P_{GT}(t) + P_{GH}(t) - P_L(t) - P_D(t) = 0;\ t\ \varepsilon\ [0,T] \tag{3.2}$$

This is called the power balance equation.
(ii) Water availability

$$X'(T) - X'(0) - \int_0^T J(t)dt + \int_0^T q(t)dt = 0 \tag{3.3}$$

where J(t) is the water inflow (rate), X'(t) water storage, and X‘(0) , X‘ (T) are specified water storages at the beginning and at the end of the optimization interval.
(iii) The hydro generation PGH(t) is a function of hydro discharge and water storage (or head), i.e.

$$P_{GH}(t) = f(X'(t), q(t)) \tag{3.4}$$

The problem can be handled conveniently by discretization. The optimization interval T is subdivided into M subintervals each of time length ΔT. Over each subinterval it is assumed that all the variables remain fixed in value. The problem is now posed as

$$min\ \Delta T \sum_{m=1}^{M} C'(P_{GT}^{m}) = min \sum_{m=1}^{M} C(P_{GT}^{m}) \tag{3.5}$$

under the following constraints:
i)Power balance equation

$$P_{GT}^m + P_{GH}^m - P_L^m - P_D^m = 0 \tag{3.6}$$

where

$P_{GT}^m$= thermal generation in the mth interval

$P_{GH}^m$= hydro generation in the mth interval

$P_L^m$=transmission loss in the mth interval

$= B_{TT}(P_{GT}^m)^2 + 2B_{TH}P_{GH}^m + B_{HH}(P_{GH}^m)^2$

$P_D^m$ = load demand in the mth interval

(ii) Water continuity equation

$$X'^{m} - X'^{(m-1)} - J^{m}\Delta T + q^{m}\Delta T = 0$$

where

$X'^{m}$ = water storage at the end of the mth interval

$J^{m}$ = water inflow (rate) in the mth interval

$q^{m}$ = water discharge (rate) in the mth interval

The above equation can be written as

$$X^{m} - X^{(m-1)} - J^{m} + q^{m} = 0; m = 1,2, \dots, M \qquad (3.7)$$

where $X^{m} = X'^{m}/\Delta T$ = storage in discharge units.

In Eqs. (3.7), $X^{o}$ and $X^{M}$ are the specified storages at the beginning and end of the optimization interval.

(iii) Hydro generation in any subinterval can be expressed as

$$P_{GH}^{m} = h_{o}\{1 + 0.5e(X^{m} + X^{m-1})\}\,(q^{m} - \rho) \qquad (3.8)$$

where

$h_{o} = 9.81 \times 10^{-3} h'_{o}$

$h_{o}$ = basic water head (head corresponding to dead storage)

e = water head correction factor to account for head variation with storage

ρ = non-effective discharge (water discharge needed to run hydro generator at no load).

In the above problem formulation, it is convenient to choose water discharges in all subintervals except one as independent variables, while hydro generations, thermal generations and water storages in all subintervals are treated as dependent variables. The fact, that water discharge in one of the subintervals is a dependent variable, is shown below:

Adding Eq. (3.7) for m = l, 2, ...,M leads to the following equation, known as water availability equation

**Solution Technique**

The problem is solved here using non-linear programming technique in conjunction with the first order gradient method. The Lagrangian $\mathcal{L}$ is formulated by augmenting the cost function of Eq. (3.5) with equality constraints of Eqs. (3.6)-(3.8) through Lagrange multipliers (dual variables) $\lambda_1^m, \lambda_2^m$ *and* $\lambda_3^m$ . Thus,

$$X^M - X^0 - \sum_m J^m + \sum_m q^m = 0 \tag{3.9}$$

Because of this equation, only (M - l) qs can be specified independently and the remaining one can then be determined from this equation and is, therefore, a dependent variable. For convenience, $q^1$ is chosen as a dependent variable, for which we can write

$$q^1 = X^0 - X^M + \sum_m J^m - \sum_{m=2}^{M} q^m \tag{3.10}$$

$$\mathcal{L} = \sum_m \big[C(P_{GT}^m) - \lambda_1^m \left(P_{GT}^m + P_{GH}^m - P_L^m - P_D^m\right) + \lambda_2^m \left(X^m - X^{(m-1)} - J^m + q^m\right) + \lambda_3^m \{P_{GH}^m - h_o\{1 + 0.5e(X^m + X^{m-1})\}\,(q^m - \rho)\}\big] \tag{3.11}$$

The dual variables are obtained by equating to zero the partial derivatives of the Lagrangian with respect to the dependent variables yielding the following equations

$$\frac{\partial \mathcal{L}}{\partial P_{GT}^m} = \frac{dC(P_{GT}^m)}{dP_{GT}^m} - \lambda_1^m \left(1 - \frac{\partial P_L^m}{\partial P_{GT}^m}\right) = 0 \tag{3.12}$$

$$\frac{\partial \mathcal{L}}{\partial P_G^m} = \lambda_3^m - \lambda_1^m \left(1 - \frac{\partial P_L^m}{\partial P_{GH}^m}\right) = 0 \tag{3.13}$$

$$\left(\frac{\partial \mathcal{L}}{\partial X^m}\right)_{\substack{m \neq M \\ \neq 0}} = \lambda_2^m - \lambda_2^{m+1} - \lambda_3^m \big(0.5 h_o e(q^m - \rho)\big) - \lambda_3^{m+1}\big(0.5 h_o e(q^{m+1} - \rho)\big) = 0 \tag{3.14}$$

and using Eq. (3.7) in Eq. (3.11), we get

$$\frac{\partial \mathcal{L}}{\partial q^1} = \lambda_2^1 - \lambda_3^1 \, h_o\big(1 + 0.5\, e\,(2X^0 + J^1 - 2q^1 + \rho)\big) = 0 \tag{3.15}$$

The dual variables for any subinterval may be obtained as follows:

(i) Obtain $\lambda_1^m$ from Eq. (3.12).

(ii) Obtain $\lambda_3^m$ from Eq. (3.13).

(iii) Obtain $\lambda_2^1$ from Eq. (3.15) and other values of $\lambda_2^m$ $(m \neq 1)$ from Eq. (3.14).

The gradient vector is given by the partial derivatives of the Lagrangian with respect to the independent variables. Thus

$$\left(\frac{\partial \mathcal{L}}{\partial q^m}\right)_{m \neq 1} = \lambda_2^m - \lambda_3^m \, h_o\big(1 + 0.5\, e\,(2X^{m-1} + J^m - 2q^m + \rho)\big) \tag{3.16}$$

For optimality the gradient vector should be zero if there are no inequality constraints on the control variables.

# POWER SYSTEM STABILITY

## INTRODUCTION:

Power system stability of modern large inter-connected systems is a major problem for secure operation of the system. Recent major black-outs across the globe caused by system instability, even in very sophisticated and secure systems, illustrate the problems facing secure operation of power systems. Earlier, stability was defined as the ability of a system to return to normal or stable operation after having been subjected to some form of disturbance. This fundamentally refers to the ability of the system to remain in synchronism. However, modern power systems operate under complex interconnections, controls and extremely stressed conditions. Further, with increased automation and use of electronic equipment, the quality of power has gained utmost importance, shifting focus on to concepts of voltage stability, frequency stability, inter-area oscillations etc.

The IEEE/CIGRE Joint Task Force on stability terms and conditions have proposed the following definition in 2004: "*Power System stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded, so that practically the entire system remains intact*".

The Power System is an extremely non-linear and dynamic system, with operating parameters continuously varying. Stability is hence, a function of the initial operating condition and the nature of the disturbance. Power systems are continually subjected to small disturbances in the form of load changes. The system must be in a position to be able to adjust to the changing conditions and operate satisfactorily. The system must also withstand large disturbances, which may even cause structural changes due to isolation of some faulted elements.

A power system may be stable for a particular (large) disturbance and unstable for another disturbance. It is impossible to design a system which is stable under all disturbances. The power system is generally designed to be stable under those disturbances which have a high degree of occurrence. The response to a disturbance is extremely complex and involves practically all the equipment of the power system. For example, a short circuit leading to a line isolation by circuit breakers will cause variations in the power flows, network bus voltages and generators rotor speeds. The voltage variations will actuate the voltage regulators in the system and generator speed variations will actuate the prime mover governors; voltage and frequency variations will affect the system loads. In stable systems, practically all generators and loads remain connected, even though parts of the system may be isolated to preserve bulk operations. On the other hand, an unstable system condition could lead to cascading outages and a shutdown of a major portion of the power system.

## ROTOR ANGLE STABILITY

Rotor angle stability refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. Instability results in some generators accelerating (decelerating) and losing synchronism with other generators. Rotor angle stability depends on the ability of each synchronous machine to maintain equilibrium between electromagnetic torque and mechanical torque. Under steady state, there is equilibrium between the input mechanical torque and output electromagnetic torque of each generator, and its speed remains a constant. Under a disturbance, this equilibrium is upset and the generators accelerate/decelerate according to the mechanics of a rotating body. Rotor angle stability is further categorized as follows:

### Small single (or small disturbance) rotor angle stability

It is the ability of the power system to maintain synchronism under small disturbances. In this case, the system equation can be linearized around the initial operating point and the stability depends only on the operating point and not on the disturbance. Instability may result in

(i) A non oscillatory or a periodic increase of rotor angle

(ii) Increasing amplitude of rotor oscillations due to insufficient damping.

The first form of instability is largely eliminated by modern fast acting voltage regulators and the second form of instability is more common. The time frame of small signal stability is of the order of 10-20 seconds after a disturbance.

**Large-signal rotor angle stability or transient stability**

This refers to the ability of the power system to maintain synchronism under large disturbances, such as short circuit, line outages etc. The system response involves large excursions of the generator rotor angles. Transient stability depends on both the initial operating point and the disturbance parameters like location, type, magnitude etc. Instability is normally in the form of a periodic angular separation. The time frame of interest is 3-5 seconds after disturbance.

The term dynamic stability was earlier used to denote the steady-state stability in the presence of automatic controls (especially excitation controls) as opposed to manual controls. Since all generators are equipped with automatic controllers today, dynamic stability has lost relevance and the Task Force has recommended against its usage.

## MECHANICS OF ROTATORY MOTION

Since a synchronous machine is a rotating body, the laws of mechanics of rotating bodies are applicable to it. In rotation we first define the fundamental quantities. The angle $\theta_m$ is defined, with respect to a circular arc with its center at the vertex of the angle, as the ratio of the arc length *s* to radius *r*.

$$\theta_m = \frac{s}{r} \tag{1}$$

The unit is radian. Angular velocity $\omega_m$ is defined as

$$\omega_m = \frac{d\theta_m}{dt} \tag{2}$$

and angular acceleration as

$$\alpha = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \tag{3}$$

The torque on a body due to a tangential force F at a distance $r$ from axis of rotation is given by

$$T = r\,F \qquad (4)$$

The total torque is the summation of infinitesimal forces, given by

$$T = \int r\,dF \qquad (5)$$

The unit of torque is N-m. When torque is applied to a body, the body experiences angular acceleration. Each particle experiences a tangential acceleration $a = r\sim$ , where r is the distance of the particle from axis of rotation. The tangential force required to accelerate a particle of mass *dm* is

$$dF = a\,dm = r\,\alpha\,dm \qquad (6)$$

The torque required for the particle is

$$dT = r\,dF = r^2\,\alpha\,dm \qquad (7)$$

and that required for the whole body is given by

$$T = \alpha \int r^2 dm = I\,\alpha \qquad (8)$$

Here

$$I = \int r^2 dm \qquad (9)$$

is called the moment of inertia of the body. The unit is Kg – $m^2$. If the torque is assumed to be the result of a number of tangential forces F, which act at different points of the body

$$T = \sum r\,F$$

Now each force acts through a distance

$$ds = r\,d\theta_m$$

The work done is $\sum F \,.\, ds$

$$dW = \sum F\,r\,d\theta_m = d\theta_m\,T$$

$$W = \int T\,d\theta_m \qquad (10)$$

and $$T = \frac{d\,W}{d\bar{}\,{}_m} \qquad (11)$$

Thus the unit of torque may also be Joule per radian.

The power is defined as rate of doing work. Using (11)

$$P = \frac{dW}{dt} = \frac{T\,d\theta_m}{dt} = T\,\omega_m \tag{12}$$

The angular momentum M is defined as

$$M = I\,\omega_m \tag{13}$$

and the kinetic energy is given by

$$KE = \frac{1}{2} I\,\omega_m^2 = \frac{1}{2} M\,\omega_m \tag{14}$$

From (14) we can see that the unit of M is seen to be J-sec/rad.

**SWING EQUATION:**

From (8)

$$I\alpha = T$$

or

$$\frac{I\,d^2\theta_m}{dt^2} = T \tag{15}$$

Here T is the net torque of all torques acting on the machine, which includes the shaft torque (due to prime mover of a generator or load on a motor), torque due to rotational losses (friction, windage and core loss) and electromagnetic torque.

Let $T_m$ = shaft torque or mechanical torque corrected for rotational losses

$T_e$ = Electromagnetic or electrical torque

For a generator $T_m$ tends to accelerate the rotor in positive direction of rotation and for a motor retards the rotor.

The accelerating torque for a generator

$$T_a = T_m - T_e \tag{16}$$

Under steady-state operation of the generator, $T_m$ is equal to $T_e$ and the accelerating torque is zero. There is no acceleration or deceleration of the rotor masses and the machines run at a constant synchronous speed. In the stability analysis in the following sections, $T_m$ is assumed to be a constant since the prime movers (steam turbines or hydro turbines) do no act during the short time period in which rotor dynamics are of interest in the stability studies.

Now (15) has to be solved to determine $\theta_m$ as a function of time. Since $\theta_m$ is measured with respect to a stationary reference axis on the stator, it is the measure of the absolute rotor angle and increases continuously with time even at constant synchronous speed. Since machine acceleration /deceleration is always measured relative to synchronous speed, the rotor angle is measured with respect to a synchronously rotating reference axis. Let

$$\delta_m = \theta_m - \omega_{sm} t \qquad (17)$$

where $\omega_{sm}$ is the synchronous speed in mechanical rad/s and $\delta_m$ is the angular displacement in mechanical radians.

Taking the derivative of (17) we get

$$\frac{d\delta_m}{dt} = \frac{d\theta_m}{dt} - \omega_{sm}$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d^2\theta_m}{dt^2} \qquad (18)$$

Substituting (18) in (15) we get

$$I\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \ \text{N-m} \qquad (19)$$

Multiplying by $\omega_m$ on both sides we get

$$\omega_m I\frac{d^2\delta_m}{dt^2} = \omega_m (T_m - T_e) \ \text{N-m} \qquad (20)$$

From (12) and (13), we can write

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_a \quad W \qquad (21)$$

where M is the angular momentum, also called inertia constant

$P_m$ = shaft power input less rotational losses

$P_e$ = Electrical power output corrected for losses

$P_a$ = acceleration power

M depends on the angular velocity $\mu_m$, and hence is strictly not a constant, because $\mu_m$ deviates from the synchronous speed during and after a disturbance. However, under stable conditions $\mu_m$ does not vary considerably and M can be treated as a constant. (21) is called the *"Swing equation"*. The constant M depends on the rating of the machine and varies widely with the size and type of the machine. Another constant called H constant (also referred to as inertia constant) is defined as

$$H = \frac{\begin{array}{c} stored\ kinetic\ energy\ in\ mega\ \ joules \\ at\ sychronous\ speed \end{array}}{Machine\ rating\ in\ MVA}\ MJ\,/\,MVA \qquad (22)$$

H falls within a narrow range and typical values are given in Table 9.1.

If the rating of the machine is G MVA, from (22) the stored kinetic energy is GH Mega Joules. From (14)

$$GH = \frac{1}{2} M \mu_{sm} \quad \text{MJ} \qquad (23)$$

or

$$M = \frac{2GH}{\mu_{sm}} \quad \text{MJ-s/mech rad} \qquad (24)$$

The swing equation (21) is written as

$$\frac{2H}{\mu_{sm}} \frac{d^2 \text{¢}_m}{d t^2} = \frac{P_a}{G} = \frac{P_{m\,-}P_e}{G} \qquad (25)$$

In (.25) $\text{¢}_m$ is expressed in mechanical radians and $\mu_{sm}$ in mechanical radians per second (the subscript *m* indicates mechanical units). If ¢ and μ have consistent units then mec

$$\frac{2H}{\mu_s} \frac{d^2 \text{¢}}{dt^2} = P_a = P_m - P_e \quad \text{pu} \qquad (26)$$

Here $\mu_s$ is the synchronous speed in electrical rad/s ($\mu_s = \left(\frac{p}{2}\right)\mu_{sm}$) and $P_a$ is acceleration power in per unit on same base as H. For a system with an electrical frequency $f$ Hz, (26) becomes

$$\frac{H}{\text{®}\ f} \frac{d^2 \text{¢}}{dt^2} = P_a = P_m - P_e \quad \text{pu} \qquad (27)$$

when ¢ is in electrical radians and

$$\frac{H}{180\,f}\,\frac{d^2 \text{¢}}{dt^2} = P_a = P_m - P_e \quad \text{pu} \qquad (28)$$

when ¢ is in electrical degrees.

(27) and (28) also represent the swing equation. It can be seen that the swing equation is a second order differential equation which can be written as two first order differential equations:

$$\frac{2H}{\mu_s}\,\frac{d\mu}{dt} = P_m - P_e \quad \text{pu} \qquad (29)$$

$$\frac{d\text{¢}}{dt} = \mu - \mu_s \qquad (30)$$

in which $\mu$, $\mu_s$ and ¢ are in electrical units. In deriving the swing equation, damping has been neglected.

**Table 1 : H constants of synchronous machines**

| Type of machine | H (MJ/MVA) |
|---|---|
| Turbine generator condensing 1800 rpm | 9 – 6 |
| 3600 rpm | 7 – 4 |
| Non condensing 3600 rpm | 4 – 3 |
| Water wheel generator | |
| Slow speed < 200 rpm | 2 – 3 |
| High speed > 200 rpm | 2 – 4 |
| Synchronous condenser | |
| Large | 1.25 } 25% less for hydrogen cooled |
| Small | 1.0 } 25% less for hydrogen cooled |
| Synchronous motor with load varying from 1.0 to 5.0 | 2.0 |

In defining the inertia constant H, the MVA base used is the rating of the machine. In a multi machine system, swing equation has to be solved for each machine, in which case, a common MVA base for the system has to chosen. The constant H of each machine must be consistent with the system base.

Let

$G_{mach}$ = Machine MVA rating (base)

$G_{system}$ = System MVA base

In (9.25), H is computed on the machine rating $G = G_{mach}$.

Multiplying (9.25) by $\dfrac{G_{mach}}{G_{system}}$ on both sides we get

$$\left(\frac{G_{mach}}{G_{system}}\right)\frac{2H}{\mu_{sm}}\frac{d^2\text{¢}_m}{dt^2} = \frac{P_m - P_e}{G_{mach}}\left(\frac{G_{mach}}{G_{system}}\right) \tag{31}$$

$$\frac{2H_{system}}{\mu_{sm}}\frac{d^2\text{¢}_m}{dt^2} = P_m - P_e \quad \text{pu (on system base)}$$

where $H_{\text{system}} = H\dfrac{G_{mach}}{G_{system}}$ (32)

In the stability analysis of a multi machine system, computation is considerably reduced if the number of swing equations to be solved is reduced. Machines within a plant normally swing together after a disturbance. Such machines are called coherent machines and can be replaced by a single equivalent machine, whose dynamics reflects the dynamics of the plant.

**<u>Example 1:</u>**

A 50Hz, 4 pole turbo alternator rated 150 MVA, 11 kV has an inertia constant of 9 MJ / MVA. Find the (a) stored energy at synchronous speed (b) the rotor acceleration if the input mechanical power is raised to 100 MW when the electrical load is 75 MW, (c) the speed at the end of 10 cycles if acceleration is assumed constant at the initial value.

**<u>Solution</u>**:

(a) Stored energy = GH = 150 × 9 = 1350 MJ

(b) $P_a = P_m - P_e$ = 100 – 75 = 25 MW

$$M = \frac{GH}{180f} = \frac{1350}{180 \times 50} = 0.15 \text{ MJ – s /°e}$$

$$0.15\,\frac{d^2\text{¢}}{dt^2} = 25$$

Acceleration $\sim = \frac{d^2 ¢}{dt^2} = \frac{25}{0.15} = 166.6\ ^{o}e/s^2$

$= 166.6 \times \frac{2}{P}\ ^{o}m/s^2$

$= 166.6 \times \frac{2}{P} \times \frac{1}{360}\ rps/s$

$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \times 60$ rpm/s

$= 13.88$ rpm/s

* Note $^{o}e$ = electrical degree; $^{o}m$ = mechanical degree; P=number of poles.

(c) 10 cycles = $\frac{10}{50} = 0.2$ s

$N_S$ = Synchronous speed = $\frac{120 \times 50}{4} = 1500$ rpm

Rotor speed at end of 10 cycles = $N_S + \alpha \times 0.2$

$= 1500 + 13.88 \times 0.2 = 1502.776$ rpm

**Example 2**:

Two 50 Hz generating units operate in parallel within the same plant, with the following ratings:

Unit 1: 500 MVA, 0.8 pf, 13.2 kV, 3600 rpm: H = 4 MJ/MVA

Unit 2: 1000 MVA, 0.9 pf, 13.8 kV, 1800 rpm: H = 5 MJ/MVA

Calculate the equivalent H constant on a base of 100 MVA.

**Solution**:

$$H_{1system} = H_{1mach} \times \frac{G_{1mach}}{G_{system}}$$

$$= 4 \times \frac{500}{100} = 20 \text{ MJ/MVA}$$

$$H_{2system} = H_{2mach} \times \frac{G_{2mach}}{G_{system}}$$

$$= 5 \times \frac{1000}{100} = 50 \text{ MJ/MVA}$$

$$H_{eq} = H_1 + H_2 = 20 + 50 = 70 \text{ MJ/MVA}$$

This is the equivalent inertia constant on a base of 100 MVA and can be used when the two machines swing coherently.

## POWER–ANGLE EQUATION:

In solving the swing equation, certain assumptions are normally made

(i) Mechanical power input $P_m$ is a constant during the period of interest, immediately after the disturbance

(ii) Rotor speed changes are insignificant.

(iii) Effect of voltage regulating loop during the transient is neglected i.e the excitation is assumed to be a constant.

As discussed in section 9.4, the power–angle relationship plays a vital role in the solution of the swing equation.

## POWER–ANGLE EQUATION FOR A NON–SALIENT POLE MACHINE:

The simplest model for the synchronous generator is that of a constant voltage behind an impedance. This model is called the classical model and can be used for cylindrical rotor (non–salient pole) machines. Practically all high–speed turbo alternators are of cylindrical rotor construction, where the physical air gap around the periphery of the rotor is uniform. This type of generator has approximately equal magnetic reluctance, regardless of the angular position of the rotor, with respect to the armature mmf.

**r**

The power output of the generator is given by the real part of $E_g I_a^*$.

$$I_a = \frac{E_g \angle \text{¢} - V_t \angle 0^\circ}{R_a + jx_d} \tag{38}$$

Neglecting $R_a$, $I_a = \dfrac{E_g \angle \text{¢} - V_t \angle 0^\circ}{j x_d}$

$$P = \mathcal{R} \left\{ \left(E_g \angle \text{¢}\right) \left( \frac{E_g \angle 90^\circ - \text{¢}}{x_d} - \frac{V_t \angle 90^\circ}{x_d} \right)^* \right\}$$

$$= \frac{E_g^{\,2}\cos 90^\circ}{x_d} - \frac{E_g V_t \cos(90^\circ + \delta)}{x_d}$$

$$= \frac{E_g V_t \sin\delta}{x_d} \qquad (39)$$

(Note- $\mathcal{R}$ stands for real part of)

The maximum power that can be transferred for a particular excitation is given by $\frac{E_g V_t}{x_d}$ at δ = 90°.

**POWER ANGLE EQUATION FOR A SALIENT POLE MACHINE**:

Here because of the salient poles, the reluctance of the magnetic circuit in which flows the flux produced by an armature mmf in line with the quadrature axis is higher than that of the magnetic circuit in which flows the flux produced by the armature mmf in line with the direct axis. These two components of armature mmf are proportional to the corresponding components of armature current. The component of armature current producing an mmf acting in line with direct axis is called the direct component, $I_d$. The component of armature current producing an mmf acting in line with the quadrature axis is called the quadrature axis component, $I_q$.

Power output $P = V_t I_a \cos\phi$

$$= E_d I_d + E_q I_q \qquad (40)$$

$$E_d = V_t \sin\delta \qquad (41a)$$

$$E_q = V_t \cos\delta \qquad (41b)$$

$$I_d = \frac{E_g - E_q}{x_d} = I_a \sin(\delta + \phi) \qquad (41c)$$

$$I_q = \frac{E_d}{x_q} = I_a \cos(\delta + \phi) \qquad (41d)$$

Substituting (9.41c) and (9.41d) in (9.40), we obtain

$$P = \frac{E_g V_t \sin\delta}{x_d} + \frac{V_t^{\,2}(x_d - x_q)\sin 2\delta}{2 x_d x_q} \qquad (42)$$

(9.42) gives the steady state power angle relationship for a salient pole machine. The second term does not depend on the excitation and is called the reluctance power component. This component makes the maximum power greater than in the classical model. However, the angle at which the maximum power occurs is less than $90^o$.

**TRANSIENT STABILITY**:

As defined earlier, transient stability is the ability of the system to remain stable under large disturbances like short circuits, line outages, generation or load loss etc. The evaluation of the transient stability is required offline for planning, design etc. and online for load management, emergency control and security assessment. Transient stability analysis deals with actual solution of the nonlinear differential equations describing the dynamics of the machines and their controls and interfacing it with the algebraic equations describing the interconnections through the transmission network.

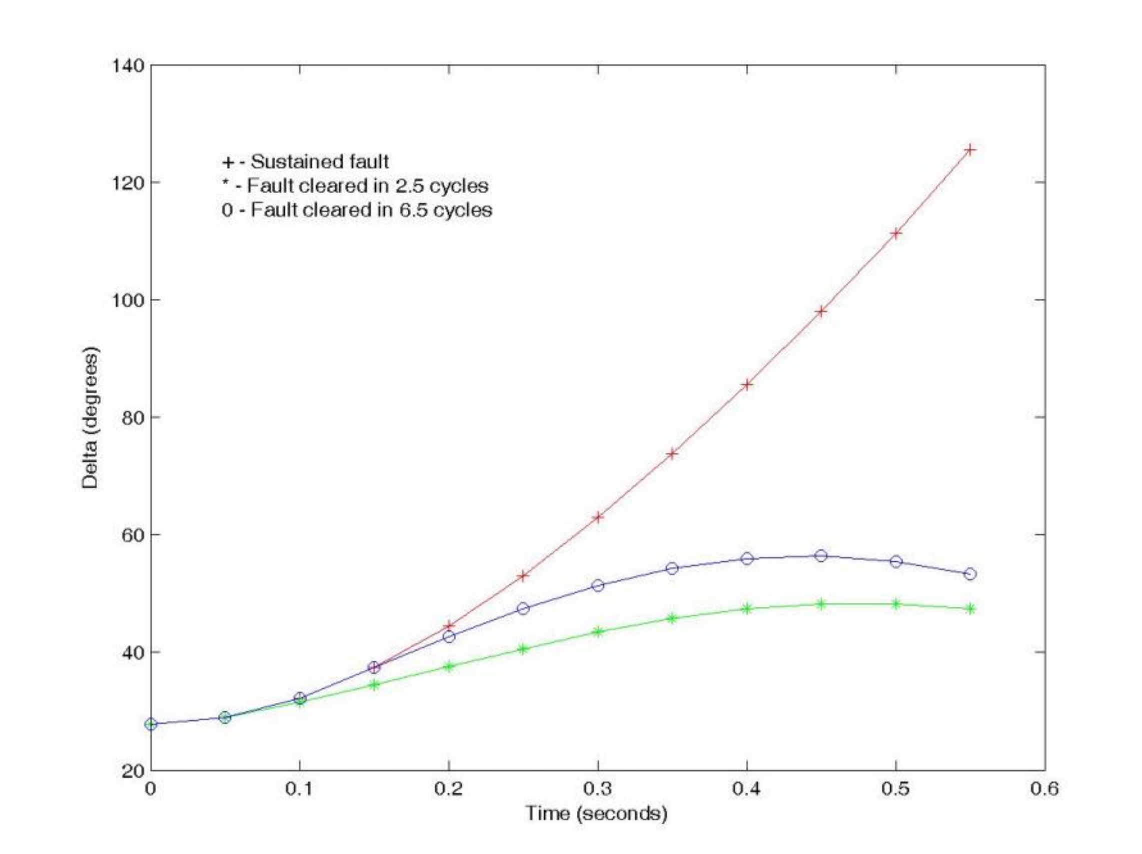
Since the disturbance is large, linearized analysis of the swing equation (which describes the rotor dynamics) is not possible. Further, the fault may cause structural changes in the network, because of which the power angle curve prior to fault, during the fault and post fault may be different. Due to these reasons, a general stability criteria for transient stability cannot be established, as was done in the case of steady state stability (namely $P_S > 0$). Stability can be established, for a given fault, by actual solution of the swing equation. The time taken for the fault to be cleared (by the circuit breakers) is called the *clearing time*. If the fault is cleared fast enough, the probability of the system remaining stable after the clearance is more. If the fault persists for a longer time, likelihood of instability is increased. *Critical clearing time* is the maximum time available for clearing the fault, before the system loses stability. Modern circuit breakers are equipped with auto reclosure facility, wherein the breaker automatically recloses after two sequential openings. If the fault still persists, the breakers open permanently. Since most faults are transient, the first reclosure is in general successful. Hence, transient stability has been greatly enhanced by auto closure breakers.

Some common assumptions made during transient stability studies are as follows:

1. Transmission line and synchronous machine resistances are neglected. Since resistance introduces a damping term in the swing equation, this gives pessimistic results.

2. Effect of damper windings is neglected which again gives pessimistic results.
3. Variations in rotor speed are neglected.
4. Mechanical input to the generator is assumed constant. The governor control loop is neglected. This also leads to pessimistic results.
5. The generator is modeled as a constant voltage source behind a transient reactance, neglecting the voltage regulator action.
6. Loads are modeled as constant admittances and absorbed into the bus admittance matrix.

The above assumptions, vastly simplify the equations. A digital computer program for transient stability analysis can easily include more detailed generator models and effect of controls, the discussion of which is beyond the scope of present treatment. Studies on the transient stability of an SMIB system, can shed light on some important aspects of stability of larger systems. The figure below shows an example of how the clearing time has an effect on the swing curve of the machine.




**Modified Euler's method**:

Euler's method is one of the easiest methods to program for solution of differential equations using a digital computer . It uses the Taylor's series expansion, discarding all second–order and higher–order terms. Modified Euler's algorithm uses the derivatives at the beginning of a time step, to predict the values of the dependent variables at the end of the step ($t_1 = t_0 + \Delta t$). Using the predicted values, the derivatives at the end of the interval are computed. The average of the two derivatives is used in updating the variables. Consider two simultaneous differential equations:

$$\frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values $x_{0,}$ $y_0$, $t_0$ at the beginning of a time step and a step size *h* we solve as follows:

Let

$$D_x = f_x(x_0,y_0,t_0) = \left.\frac{dx}{dt}\right|_0$$

$$D_y = f_y(x_0,y_0,t_0) = \left.\frac{dy}{dt}\right|_0$$

$$\left.\begin{array}{l} x^P = x_0 + D_x\, h \\ y^P = y_0 + D_y\, h \end{array}\right\} \quad \text{Predicted values}$$

$$D_{xP} = \left.\frac{dx}{dt}\right|_P = f_x(x^P,y^P,t_1)$$

$$D_{yP} = \left.\frac{dy}{dt}\right|_P = f_y(x^P,y^P,t_1)$$

$$x_1 = x_o + \left(\frac{D_x + D_{xP}}{2}\right)h$$

$$y_1 = y_o + \left(\frac{D_y + D_{yP}}{2}\right)h$$

$x_1$ and $y_1$ are used in the next iteration. To solve the swing equation by Modified Euler's method, it is written as two first order differential equations:

$$\frac{d¢}{dt} = μ$$

$$\frac{dμ}{dt} = \frac{P_a}{M} = \frac{P_m - P_{\max}\sin ¢}{M}$$

Starting from an initial value $\delta_o$, $\omega_o$ at the beginning of any time step, and choosing a step size Δt s, the equations to be solved in modified Euler's are as follows:

$$\left.\frac{d¢}{dt}\right|_0 = D_1 = \omega_o$$

$$\left.\frac{dμ}{dt}\right|_0 = D_2 = \frac{P_m - P_{\max}\sin ¢_0}{M}$$

$$\delta^P = \delta_0 + D_1\,\Delta t$$

$$\omega^P = \omega_0 + D_2\,\Delta t$$

$$\left.\frac{d¢}{dt}\right|_{P} = D_{1P} = \omega^{P}$$

$$\left.\frac{d\mu}{dt}\right|_{P} = D_{2P} = \frac{P_m - P_{\max}\sin ¢^{P}}{M}$$

$$\delta_1 = \delta_0 + \left(\frac{D_1 + D_{1P}}{2}\right)\Delta t$$

$$\omega_1 = \omega_0 + \left(\frac{D_2 + D_{2P}}{2}\right)\Delta t$$

$\delta_1$ and $\omega_1$ are used as initial values for the successive time step. Numerical errors are introduced because of discarding higher–order terms in Taylor's expansion. Errors can be decreased by choosing smaller values of step size. Too small a step size, will increase computation, which can lead to large errors due to rounding off. The Runge- Kutta method which uses higher–order terms is more popular.

Example :A 50 Hz, synchronous generator having inertia constant H = 5.2 MJ/MVA and $x_d'$ = 0.3 pu is connected to an infinite bus through a double circuit line as shown in Fig. 9.21. The reactance of the connecting HT transformer is 0.2 pu and reactance of each line is 0.4 pu. $|E_g|$ = 1.2 pu and $|V|$ = 1.0 pu and $P_e$ = 0.8 pu. Obtain the swing curve using modified Eulers method for a three phase fault occurs at the middle of one of the transmission lines and is cleared by isolating the faulted line.

**Solution**:

Before fault transfer reactance between generator and infinite bus

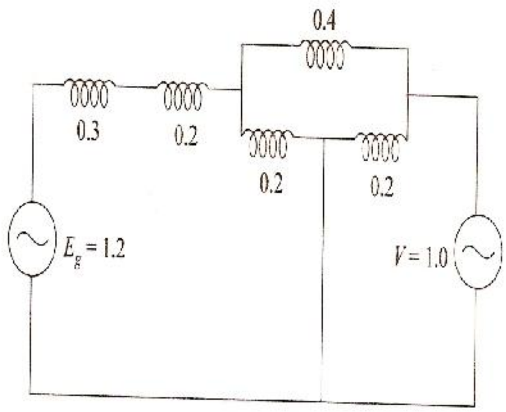
$X_I = 0.3 + 0.2 + \frac{0.4}{2} = 0.7$ pu

$P_{max\ I} = \frac{1.2 \times 1.0}{0.7} = 1.714$ pu.

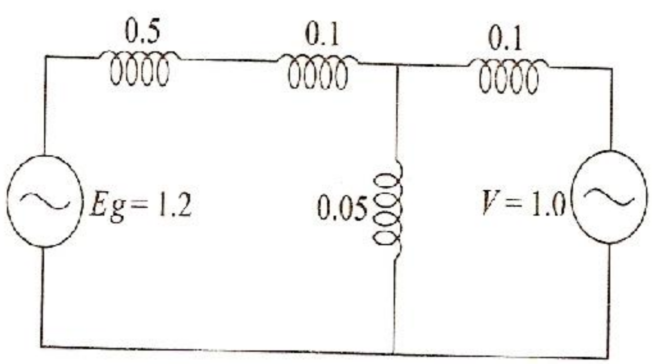
Initial $P_e = 0.8$ pu $= P_m$

Initial operating angle $\delta_o = \sin^{-1} \frac{0.8}{1.714} = 27.82^o = 0.485$ rad.

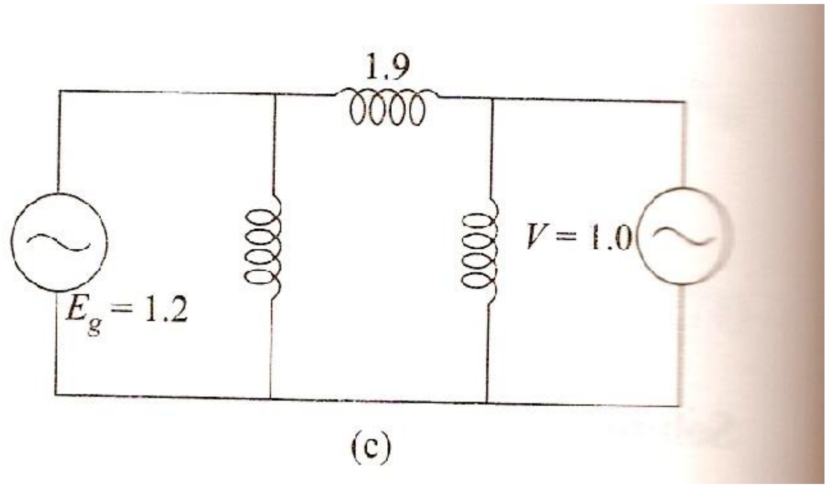
When fault occurs at middle of one of the transmission lines, the network and its reduction is as shown in Fig a to Fig c.



(a)



(b)




(c)

The transfer reactance is 1.9 pu.

$P_{\max II} = \dfrac{1.2 \times 1.0}{1.9} = 0.63$ pu

Since there is no outage, $P_{\max III} = P_{\max I} = 1.714$

$$\delta_{\max} = \text{®} - \sin^{-1}\left(\frac{P_m}{P_{\max III}}\right) = \text{®} - \sin^{-1}\left(\frac{0.8}{1.714}\right) = 2.656 \text{ rad}$$

$$\cos\delta_{cr} = \frac{P_m\,(\text{¢}_{\max} - \text{¢}_o) - P_{\max II}\cos\text{¢}_o + P_{\max III}\cos\text{¢}_{\max}}{P_{\max III} - P_{\max II}}$$

$$= \frac{0.8(2.656 - 0.485) - 0.63\cos(0.485) + 1.714\cos(2.656)}{1.714 - 0.63}$$

$$= \frac{1.7368 - 0.5573 - 1.5158}{1.084} = -0.3102$$

$$\delta_{cr} = \cos^{-1}(-0.3102) = 1.886 \text{ rad} = 108.07^o$$

**with line outage**

$$X_{III} = 0.3 + 0.2 + 0.4 = 0.9 \text{ pu}$$

$$P_{\max III} = \frac{1.2 \times 1.0}{0.9} = 1.333 \text{ pu}$$

$$\delta_{\max} = \text{®} - \sin^{-1}\frac{0.8}{1.333} = 2.498 \text{ rad}$$

**Modified Eulers method**

$\delta_o = 27.82^o = 0.485$ rad

$\omega_o = 0.0$ rad / sec ( at $t = 0^0$)

Choosing a step size of 0.05 s, the swing is computed. Table a gives the values of the derivatives and predicted values. Table b gives the initial values $\delta_o$, $\omega_o$ and the values at the end of the interval $\delta_1$, $\omega_1$. Calculations are illustrated for the time step t = 0.2 s.

$\delta_o = 0.761$

$\omega_o = 2.072$

$P_m = 0.8$

$$M = \left[\frac{5.2}{\Pi \times 50}\right] = 0.0331 \text{ s}^2 / \text{rad}$$

$P_{max}$ (after fault clearance) = 1.333 pu

$D_1 = 2.072$

$$D_2 = \frac{0.8 - 1.333 \sin(0.761)}{0.0331} = -3.604$$

$\delta^P = 0.761 + (2.072 \times 0.05) = 0.865$

$\omega^P = 2.072 + (-3.604 \times 0.05) = 1.892$

$D_{1P} = 1.892$

$$D_{2P} = \frac{0.8 - 1.333 \sin(0.865)}{0.0331} = -6.482$$

$$\delta_1 = 0.761 + \left(\frac{2.072 + 1.892}{2}\right) 0.05 = 0.860$$

$$\omega_1 = 2.072 + \left(\frac{-3.604 - 6.482}{2}\right) 0.05 = 1.82$$

$\delta_1$, $\omega_1$ are used as initial values in next time step.

Table a : Calculation of derivatives in modified Euler's method

| t | $D_1$ | $D_2$ | $\delta^P$ | $\omega^P$ | $D_{1P}$ | $D_{2P}$ |
|---|---|---|---|---|---|---|
| $0^+$ | 0.0 | 15.296 | 0.485 | 0.765 | 0.765 | 15.296 |
| 0.05 | 0.765 | 14.977 | 0.542 | 1.514 | 1.514 | 14.350 |
| 0.10 | 1.498 | 14.043 | 0.636 | 2.200 | 2.200 | 12.860 |
| 0.15 | 2.17 | - 0.299 | 0.761 | 2.155 | 2.155 | - 3.600 |
| 0.20 | 2.072 | - 3.604 | 0.865 | 1.892 | 1.892 | - 6.482 |
| 0.25 | 1.820 | - 6.350 | 0.951 | 1.502 | 1.502 | - 8.612 |
| 0.30 | 1.446 | - 8.424 | 1.015 | 1.025 | 1.025 | - 10.041 |
| 0.35 | 0.984 | - 9.827 | 1.054 | 0.493 | 0.493 | - 10.843 |
| 0.40 | 0.467 | - 10.602 | 1.065 | - 0.063 | - 0.063 | - 11.060 |
| 0.45 | - 0.074 | - 10.803 | 1.048 | - 0.614 | - 0.614 | - 10.720 |
| 0.50 | - 0.612 | - 10.46 | 1.004 | - 1.135 | - 1.135 | - 9.800 |

Table b : calculations of $\delta_o$, $\omega_o$ and $\delta_1$, $\omega_1$ in modified Euler's method

| T | $P_{max}^{(pu)}$ | $\delta_o$ rad | $\omega_o$ rad / sec | $\delta_1$ rad | $\omega_1$ rad / sec | $\delta_1$ deg |
|---|---|---|---|---|---|---|
| $0^-$ | 1.714 | 0.485 | 0.0 | – | – | - |
| $0^+$ | 0.630 | 0.485 | 0.0 | 0.504 | 0.765 | 28.87 |
| 0.05 | 0.630 | 0.504 | 0.765 | 0.561 | 1.498 | 32.14 |
| 0.10 | 0.630 | 0.561 | 1.498 | 0.653 | 2.170 | 37.41 |
| 0.15 | 1.333 | 0.653 | 2.170 | 0.761 | 2.072 | 43.60 |
| 0.20 | 1.333 | 0.761 | 2.072 | 0.860 | 1.820 | 49.27 |
| 0.25 | 1.333 | 0.860 | 1.820 | 0.943 | 1.446 | 54.03 |
| 0.30 | 1.333 | 0.943 | 1.446 | 1.005 | 0.984 | 57.58 |
| 0.35 | 1.333 | 1.005 | 0.984 | 1.042 | 0.467 | 59.70 |
| 0.40 | 1.333 | 1.042 | 0.467 | 1.052 | - 0.074 | 60.27 |
| 0.45 | 1.333 | 1.052 | - 0.074 | 1.035 | - 0.612 | 59.30 |
| 0.50 | 1.333 | 1.035 | - 0.612 | 0.991 | - 1.118 | 56.78 |

**Runge - Kutta method**

In Range - Kutta method, the changes in dependent variables are calculated from a given set of formulae, derived by using an approximation, to replace a truncated Taylor's series expansion. The formulae for the Runge - Kutta fourth order approximation, for solution of two simultaneous differential equations are given below;

$$\text{Given } \frac{dx}{dt} = f_x (x, y, t)$$

$$\frac{dy}{dt} = f_y (x, y, t)$$

Starting from initial values $x_0$, $y_0$, $t_0$ and step size *h*, the updated values are

$$x_1 = x_0 + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6} (l_1 + 2l_2 + 2l_3 + l_4)$$

where $k_1 = f_x (x_0, y_0 ,t_0)\, h$

$$k_2 = f_x \left( x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2} \right) h$$

$$k_3 = f_x \left( x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2} \right) h$$

$$k_4 = f_x (x_0 + k_3, y_0 + l_3, t_0 + h)\, h$$

$$l_1 = f_y (x_0, y_0, t_0)\, h$$

$$l_2 = f_y \left( x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2} \right) h$$

$$l_3 = f_y \left( x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2} \right) h$$

$$l_4 = f_y (x_0 + k_3, y_0 + l_3, t_0 + h)\, h$$

The two first order differential equations to be solved to obtain solution for the swing equation are:

$$\frac{d¢}{dt} = \omega$$

$$\frac{d\mu}{dt} = \frac{P_a}{M} = \frac{P_m - P_{max}\sin\text{¢}}{M}$$

Starting from initial value $\delta_0$, $\omega_0$, $t_0$ and a step size of $\Delta t$ the formulae are as follows

$$k_1 = \omega_0 \; \Delta t$$

$$l_1 = \left[\frac{P_m - P_{max}\sin\text{¢}_0}{M}\right] \Delta t$$

$$k_2 = \left(\mu_0 + \frac{l_1}{2}\right) \Delta t$$

$$l_2 = \left[\frac{P_m - P_{max}\sin\left(\text{¢}_0 + \frac{k_1}{2}\right)}{M}\right] \Delta t$$

$$k_3 = \left(\mu_0 + \frac{l_2}{2}\right) \Delta t$$

$$l_3 = \left[\frac{P_m - P_{max}\sin\left(\text{¢}_0 + \frac{k_2}{2}\right)}{M}\right] \Delta t$$

$$k_4 = (\omega_0 + l_3)\, \Delta t$$

$$l_4 = \left[\frac{P_m - P_{max}\sin\left(\text{¢}_0 + k_3\right)}{M}\right] \Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4]$$

$$\omega_1 = \omega_0 + \frac{1}{6}\,[l_1 + 2l_2 + 2l_3 + l_4]$$

**Example**

Obtain the swing curve for previous example using Runge - Kutta method.

**Solution**:

$\delta_0 = 27.82^0 = 0.485$ rad.

$\omega_0 = 0.0$ rad / sec. ( at $t = 0^-$)

Choosing a step size of 0.05 s, the coefficient $k_1$, $k_2$, $k_3$, $k_4$ and $l_1$, $l_2$, $l_3$, and $l_4$ are calculated for each time step. The values of δ and ω are then updated. Table a gives the coefficient for different time steps. Table b gives the starting values $\delta_0$, $\omega_0$ for a time step and the updated values $\delta_1$, $\omega_1$ obtained by Runge - Kutta method. The updated values are used as initial values for the next time step and process continued. Calculations are illustrated for the time step t = 0.2 s.

$\delta_0 = 0.756$

$M = 0.0331\ s^2 / rad$

$\omega_0 = 2.067$

$P_m = 0.8$

$P_{max} = 1.333$ (after fault is cleared)

$k_1 = 2.067 \times 0.05 = 0.103$

$$l_1 = \left[\frac{0.8 - 1.333 \sin(0.756)}{0.0331}\right] \times 0.05 = -0.173$$

$$k_2 = \left[2.067 - \frac{0.173}{2}\right] 0.05 = 0.099$$

$$l_2 = \left[\frac{0.8 - 1.333 \sin\left(0.756 + \frac{0.103}{2}\right)}{0.0331}\right] \times 0.05 = -0.\,246$$

$$k_3 = \left[2.067 - \frac{0.246}{2}\right] 0.05 = 0.097$$

$$l_3 = \left[\frac{0.8 - 1.333 \sin\left(0.756 + \frac{0.099}{2}\right)}{0.0331}\right] \times 0.05 = -0.\,244$$

$$k_4 = (2.067 - 0.244)\ 0.05 = 0.091$$

$$l_4 = \left[\frac{0.8 - 1.333 \sin(0.756 + 0.097)}{0.0331}\right] \times 0.05 = -0.\,308$$

$$\delta_1 = 0.756 + \frac{1}{6}\ [0.103 + 2 \times 0.099 + 2 \times 0.097 + 0.091] = 0.854$$

$$\omega_1 = 2.067 + \frac{1}{6}\,[-0.173 + 2 \times -0.246 + 2 \times -0.244 - 0.308] = 1.823$$

Now δ = 0.854 and ω = 1.823 are used as initial values for the next time step. The computations have been rounded off to three digits. Greater accuracy is obtained by reducing the step size.

Table a : Coefficients in Runge - Kutta method

| T | $k_1$ | $l_1$ | $k_2$ | $l_2$ | $k_3$ | $l_3$ | $K_4$ | $l_4$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.764 | 0.019 | 0.764 | 0.019 | 0.757 | 0.038 | 0.749 |
| 0.05 | 0.031 | 0.749 | 0.056 | 0.736 | 0.056 | 0.736 | 0.075 | 0.703 |
| 0.10 | 0.075 | 0.704 | 0.092 | 0.674 | 0.091 | 0.667 | 0.108 | 0.632 |
| 0.15 | 0.108 | – 0.010 | 0.108 | – 0.094 | 0.106 | – 0.095 | 0.103 | – 0.173 |
| 0.20 | 0.103 | – 0.173 | 0.099 | – 0.246 | 0.097 | – 0.244 | 0.091 | – 0.308 |
| 8.25 | 0.091 | – 0.309 | 0.083 | – 0.368 | 0.082 | – 0.363 | 0.073 | – 0.413 |
| 0.30 | 0.073 | – 0.413 | 0.063 | – 0.455 | 0.061 | – 0.450 | 0.050 | – 0.480 |
| 0.35 | 0.050 | – 0.483 | 0.038 | – 0.510 | 0.037 | – 0.504 | 0.025 | – 0.523 |
| 0.40 | 0.025 | – 0.523 | 0.012 | – 0.536 | 0.011 | – 0.529 | – 0.001 | – 0.534 |
| 0.45 | – 0. 001 | – 0.534 | – 0.015 | – 0.533 | – 0.015 | – 0.526 | – 0.027 | – 0.519 |
| 0.50 | – 0.028 | – 0.519 | – 0.040 | – 0.504 | – 0.040 | – 0.498 | – 0.053 | – 0.476 |

Table b: δ, ω computations by Runge - Kutta method

| t (sec) | $P_{max}$ (pu) | $\delta_0$ (rad) | $\omega_0$ rad/sec | $\delta_1$ rad | $\omega_1$ rad/sec | $\delta_1$ deg |
|---|---|---|---|---|---|---|
| $0^-$ | 1.714 | 0.485 | 0.0 | | | |
| $0^+$ | 0.630 | 0.485 | 0.0 | 0.504 | 0.759 | 28.87 |
| 0.05 | 0.630 | 0.504 | 0.756 | 0.559 | 1.492 | 32.03 |
| 0.10 | 0.630 | 0.559 | 1.492 | 0.650 | 2.161 | 37.24 |
| 0.15 | 1.333 | 0.650 | 2.161 | 0.756 | 2.067 | 43.32 |
| 0.20 | 1.333 | 0.756 | 2.067 | 0.854 | 1.823 | 48.93 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.25 | 1.333 | 0.854 | 1.823 | 0.936 | 1.459 | 53.63 |
| 0.30 | 1.333 | 0.936 | 1.459 | 0.998 | 1.008 | 57.18 |
| 0.35 | 1.333 | 0.998 | 1.008 | 1.035 | 0.502 | 59.30 |
| 0.40 | 1.333 | 1.035 | 0.502 | 1.046 | – 0.029 | 59.93 |
| 0.45 | 1.333 | 1.046 | – 0.029 | 1.031 | – 0.557 | 59.07 |
| 0.50 | 1.333 | 1.031 | – 0.557 | 0.990 | – 1.057 | 56.72 |

**Note:** $\delta_0$, $\omega_0$ indicate values at beginning of interval and $\delta_1$, $\omega_1$ at end of interval. The fault is cleared at 0.125 seconds. $\therefore$ $P_{max}$ = 0.63 at t = 0.1 sec and $P_{max}$ = 1.333 at t = 0.15 sec, since fault is already cleared at that time. The swing curves obtained from modified Euler's method and Runge - Kutta method are shown in Fig. It can be seen that the two methods yield very close results.

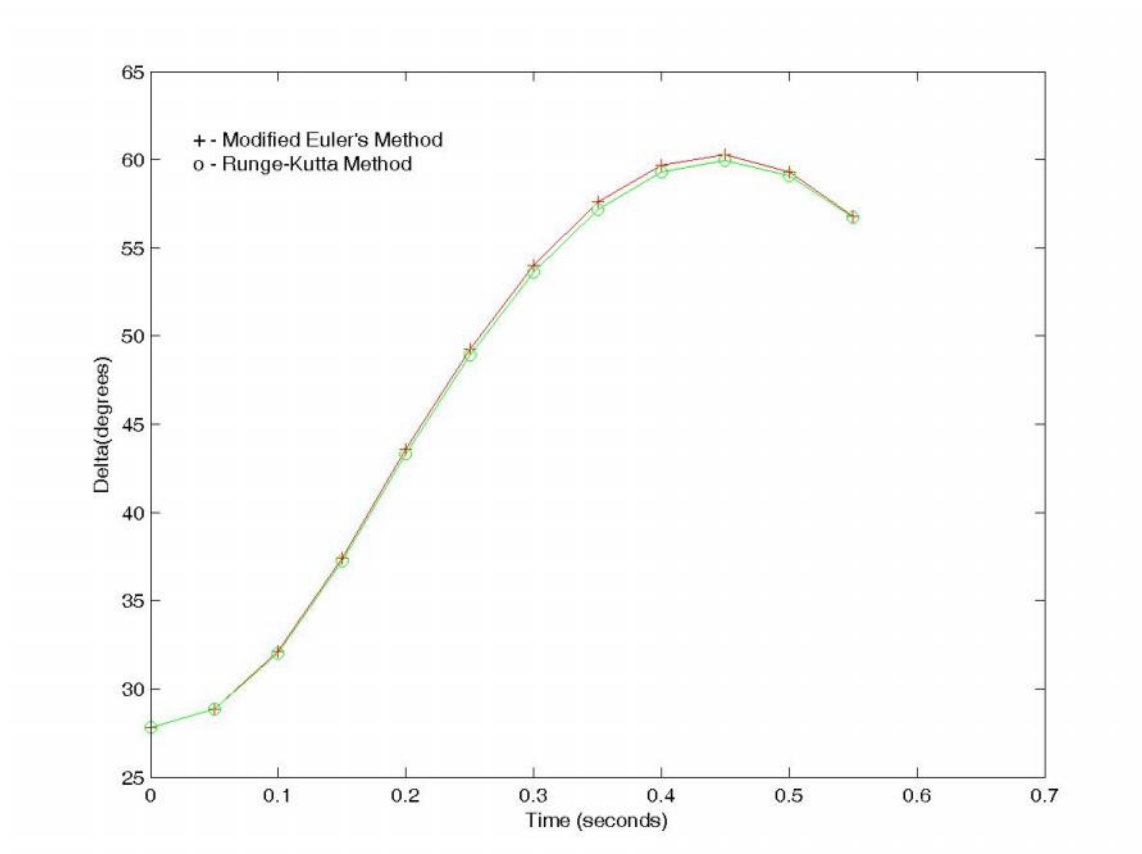



Fig: : Swing curves with Modified Euler' and Runge-Kutta methods

**Milne's Predictor Corrector method:**

The Milne's formulae for solving two simultaneous differential equations are given below.

Consider $\frac{dx}{dt} = f_x\,(x, y, t)$

$$\frac{dy}{dt} = f_y\,(x, y, t)$$

With values of x and y known for four consecutive previous times, the predicted value for n + 1$^{th}$ time step is given by

$$x_{n+1}^{P} = x_{n-3} + \frac{4h}{3}\left[2x'_{n-2} - x'_{n-1} + 2x'_{n}\right]$$

$$y_{n+1}^{P} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

Where $x'$ and $y'$ are derivatives at the corresponding time step. The corrected values are

$$x_{n+1} = x_{n-1} + \frac{h}{3}\left[x'_{n-1} + 4x'_{n} + x'_{n+1}\right]$$

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_{n} + y'_{n+1}\right]$$

where $x'_{n+1} = f_x\left(x_{n+1}^{P},\, y_{n+1}^{P},\, t_{n+1}\right)$

$$y'_{n+1} = f_y\left(x_{n+1}^{P},\, y_{n+1}^{P},\, t_{n+1}\right)$$

To start the computations we need four initial values which may be obtained by modified Euler's method, Runge - Kutta method or any other numerical method which is self starting, before applying Milne's method. The method is applied to the solution of swing equation as follows:

Define $¢'_{n} = \left.\frac{d¢}{dt}\right|_{n} = μ_{n}$

$$μ'_{n} = \left.\frac{dμ}{dt}\right|_{n} = \frac{P_m - P_{\max}\sin ¢_{n}}{M}$$

$$¢_{n+1}^{P} = ¢_{n-3} + \frac{4\Delta t}{3}\left[2¢'_{n-2} - ¢'_{n-1} + 2¢'_{n}\right]$$

$$\mu_{n+1}^{P} = \mu_{n-3} + \frac{4\Delta t}{3}\left[2\mu'_{n-2} - \mu'_{n-1} + 2\mu'_{n}\right]$$

$$\delta_{n+1} = \delta_{n-1} + \frac{\Delta t}{3}\left[¢'_{n-1} + 4¢'_{n} + ¢'_{n+1}\right]$$

$$\omega_{n+1} = \omega_{n-1} + \frac{\Delta t}{3}\left[\mu'_{n-1} + 4\mu'_{n} + \mu'_{n+1}\right]$$

where $¢'_{n+1} = \mu_{n+1}^{P}$

$$\mu'_{n+1} = \frac{P_m - P_{\max}\sin ¢_{n+1}^{P}}{M}$$

**Example**

Solve example using Milne's method.

**Solution**:

To start the process, we take the first four computations from Range Kutta method

| | | |
|---|---|---|
| t = 0.0 s | $\delta_1 = 0.504$ | $\omega_1 = 0.759$ |
| t = 0.05 s | $\delta_2 = 0.559$ | $\omega_2 = 1.492$ |
| t = 0.10 s | $\delta_3 = 0.650$ | $\omega_3 = 2.161$ |
| t = 0.15 s | $\delta_4 = 0.756$ | $\omega_4 = 2.067$ |

The corresponding derivatives are calculated using the formulae for $¢'_n$ and $\mu'_n$. We get

$¢'_1 = 0.759$ $\quad \mu'_1 = 14.97$

$¢'_2 = 1.492$ $\quad \mu'_2 = 14.075$

$¢'_3 = 2.161$ $\quad \mu'_3 = 12.65$

$¢'_4 = 2.067$ $\quad \mu'_4 = -3.46$

We now compute $\delta_5$ and $\omega_5$, at the next time step i.e t = 0.2 s.

$$¢_5^{P} = ¢_1 + \frac{4\,\Delta\,t}{3}\left[2¢'_2 - ¢'_3 + 2¢'_4\right]$$

$$= 0.504 + \frac{4 \times 0.05}{3}\left[2 \times 1.492 - 2.161 + 2 \times 2.067\right] = 0.834$$

$$\mu_5^{P} = \mu_1 + \frac{4\,\Delta\,t}{3}\left[2\mu'_2 - \mu'_3 + 2\mu'_4\right]$$

$$= 0.759 + \frac{4 \times 0.05}{3}\left[2 \times 14.075 - 12.65 + 2 \times (-3.46)\right] = 1.331$$

$$¢'_5 = 1.331$$

$$\mu'_5 = \frac{0.8 - 1.333 \sin(0.834)}{0.0331} = -5.657$$

$$\delta_5 = \delta_3 + \frac{\Delta t}{3}\left[¢'_3 + 4¢'_4 + ¢'_5\right]$$

$$= 0.65 + \frac{0.05}{3}\left[2.161 + 4 \times 2.067 + 1.331\right] = 0.846$$

$$\omega_5 = \omega_3 + \frac{\Delta t}{3}\left[\mu'_3 + 4\mu'_4 + \mu'_5\right]$$

$$= 2.161 + \frac{0.05}{3}\left[12.65 - 4 \times 3.46 - 5.657\right] = 2.047$$

$$¢'_5 = \omega_5 = 2.047$$

$$\mu'_5 = \frac{0.8 - 1.333 \sin(0.846)}{0.0331} = -5.98$$

The computations are continued for the next time step in a similar manner.

## MULTI MACHINE TRANSIENT STABILITY ANALYSIS

A typical modern power system consists of a few thousands of nodes with heavy interconnections. Computation simplification and memory reduction have been two major issues in the development of mathematical models and algorithms for digital computation of transient stability. In its simplest form, the problem of a multi machine power system under going a disturbance can be mathematically stated as follows:

$$\dot{x}(t) = f_I(x(t)) \qquad -\infty \le t \le 0$$

$$\dot{x}(t) = f_{II}(x(t)) \qquad 0 < t \le t_{ce}$$

$$\dot{x}(t) = f_{III}(x(t)) \qquad t_{ce} < t < \infty$$

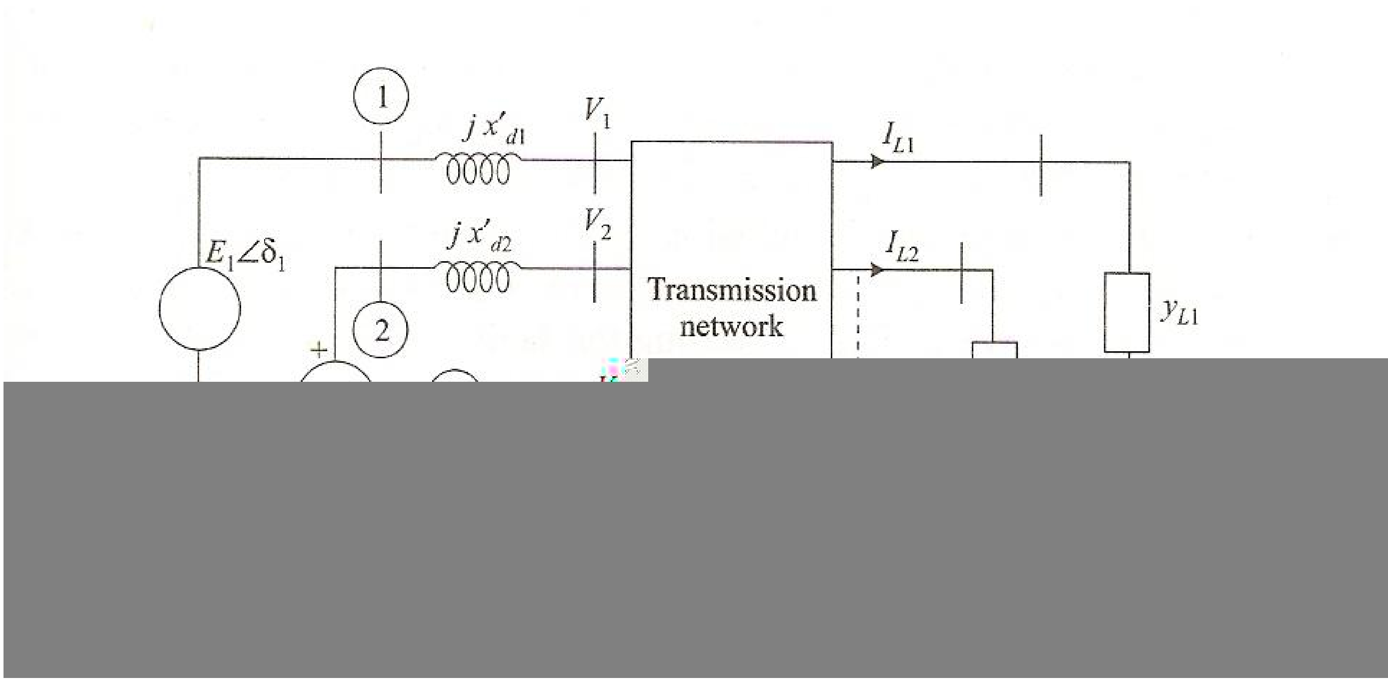
$x(t)$ is the vector of state variables to describe the differential equations governing the generator rotor dynamics, dynamics of flux decay and associated generator controller dynamics (like excitation control, PSS, governor control etc). The function $f_I$ describes the dynamics prior to the fault. Since the system is assumed to be in steady state, all the state variable are constant. If the fault occurs at t = 0, $f_{II}$ describes the dynamics during fault, till the fault is cleared at time $t_{cl}$. The post–fault dynamics is governed by $f_{III}$. The state of the system $x_{cl}$ at the end of the fault-on period (at $t = t_{cl}$) provides the initial condition for the post–fault network described which determines whether a system is stable or not after the fault is cleared. Some methods are presented in the following sections to evaluate multi machine transient stability. However, a detailed exposition is beyond the scope of the present book.

## REDUCED ORDER MODEL

This is the simplest model used in stability analysis and requires minimum data. The following assumptions are made:

- Mechanical power input to each synchronous machine is assumed to be constant.
- Damping is neglected.
- Synchronous machines are modeled as constant voltage sources behind transient reactance.
- Loads are represented as constant impedances.

With these assumptions, the multi machine system is represented as in Fig. 9.26.




**<u>Fig 9.26 Multi machine system</u>**

Nodes 1, 2 …… n are introduced in the model and are called internal nodes (the terminal node is the external node connected to the transmission network). The swing equations are formed for the various generators using the following steps:

**<u>Step 1</u>:** All system data is converted to a common base.

**<u>Step 2</u>:** A prefault load flow is performed, to determine the prefault steady state voltages, at all the external buses. Using the prefault voltages, the loads are converted into equivalent shunt admittance, connected between the respective bus and the reference node. If the complex load at bus *i* is given by

$$S_i = P_{Li} + Q_{Li}$$

the equivalent admittance is given by

$$\mathrm{Y}_{\mathrm{Li}} = \frac{S^*_{Li}}{|V_{Li}|^2} = \frac{P_{Li} - jQ_{Li}}{|V_{Li}|^2}$$

**<u>Step 3</u>:** The internal voltages are calculated from the terminal voltages, using

$$|E_i| \angle ¢'_i \;\; = |V_i| + \; j\, x'_{d\,i}\; I_i$$

$$= \;\; |V_i| + \; jx'_{di} \frac{S^*_{G\,i}}{|V_i|}$$

$$= |V_i| + \; j\, x'_{d\,i} \; \frac{(P_{G\,i} \; - \; j\, Q_{G\,i})}{|V_i|}$$

$¢'_i$ is the angle of $E_i$ with respect to $V_i$. If the angle of $V_i$ is $\beta_i$, then the angle of $E_i$, with respect to common reference is given by $¢_i = ¢'_i \;\; + \quad {}_i$. $P_{Gi}$ and $Q_{Gi}$ are obtained from load flow solution.

**<u>Step4</u>:** The bus admittance matrix $Y_{bus}$ formed to run the load flow is modified to include the following.

(i) The equivalent shunt load admittance given by, connected between the respective load bus and the reference node.

(ii) Additional nodes are introduced to represent the generator internal nodes. Appropriate values of admittances corresponding to $x'_d$ , connected between the internal nodes and terminal nodes are used to update the $Y_{bus}$.

(iii) $Y_{bus}$ corresponding to the faulted network is formed. Generally transient stability analysis is performed, considering three phase faults, since they are the most severe. The $Y_{bus}$ during the fault is obtained by setting the elements of the row and column corresponding to the faulted bus to zero.

(iv) $Y_{bus}$ corresponding to the post–fault network is obtained, taking into account line outages if any. If the structure of the network does not change, the $Y_{bus}$ of the post-fault network is same as the prefault network.

**<u>Step 5</u>:** The admittance form of the network equations is

$$I = Y_{bus}\, V$$

Since loads are all converted into passive admittances, current injections are present only at the *n* generator internal nodes. The injections at all other nodes are zero. Therefore, the current vector I can be partitioned as

$$I = \begin{bmatrix} I_n \\ 0 \end{bmatrix}$$

where $I_n$ is the vector of current injections corresponding to the *n* generator internal nodes. $Y_{bus}$ and V are also partitioned appropriately, so that

$$\left[\frac{I_n}{0}\right] = \left[\begin{array}{c|c} \frac{Y_1}{Y_3} & \frac{Y_2}{Y_4} \end{array}\right]\left[\frac{E_n}{V_t}\right]$$

where $E_n$ is the vector of internal emfs of the generators and $V_t$ is the vector of external bus voltages. From (9.91) we can write

$$I_n = Y_1 E_n + Y_2 V_t$$

$$0 = Y_3 E_n + Y_4 V_t$$

we get

$$V_t = -Y_4^{-1} Y_3 E_n$$

$$I_n = \left(Y_1 - Y_2\ Y_4^{-1}\ Y_3\right)\ E_n = \hat{Y}\ E_n$$

where $\hat{Y} = Y_1 - Y_2\ Y_4^{-1}\ Y_3$ is called the reduced admittance matrix and has dimension $n \times n$. $\hat{Y}$ gives the relationship between the injected currents and the internal generator voltages. It is to be noted we have eliminated all nodes except the *n* internal nodes.

**<u>Step 6</u>:** The electric power output of the generators are given by

$$P_{Gi} = \mathcal{R}\,[E_i\ I_i^*.]$$

Substituting for $I_i$ from (9.94) we get

$$P_{Gi} = |E_i|^2\ \hat{G}_{ii} + \sum_{j=1 \neq i}^{n} |E_i|\,|E_j|\left(\hat{B}_{ij}\ \sin\,(¢_i - ¢_j) + \hat{G}_{ij}\cos\,(¢_i - ¢_j)\right)$$

(This equation is derived in chapter on load flows)

**<u>Step 7</u>:** The rotor dynamics representing the swing is now given by

$$M_i\ \frac{d^2 ¢_i}{dt^2} = P_{Mi} - P_{Gi} \qquad i = 1\ldots\ldots\ldots..n$$

The mechanical power $P_{Mi}$ is equal to the pre-fault electrical power output, obtained from pre-fault load flow solution.

**<u>Step 8</u>:** The *n* second order differential equations can be decomposed into *2n* first order differential equations which can be solved by any numerical method .

Though reduced order models, also called classical models, require less computation and memory, their results are not reliable. Further, the interconnections of the physical network of the system is lost.

**FACTORS AFFECTING TRANSIENT STABILITY:**

The relative swing of a machine and the critical clearing time are a measure of the stability of a generating unit. From the swing equation, it is obvious that the generating units with smaller H, have larger angular swings at any time interval. The maximum power transfer $P_{max} = \frac{E_g V}{x'_d}$, where V is the terminal voltage of the generators. Therefore an increase in $x'_d$ , would reduce $P_{max}$. Hence, to transfer a given power $P_e$, the angle δ would increase since $P_e = P_{max} \sin \delta$, for a machine with larger $x'_d$ . This would reduce the critical clearing time, thus, increasing the probability of losing stability.

Generating units of present day have lower values of H, due to advanced cooling techniques, which have made it possible to increase the rating of the machines without significant increase in the size. Modern control schemes like generator excitation control, Turbine valve control, single-pole operation of circuit breakers and fast-acting circuit breakers with auto re-closure facility have helped in enhancing overall system stability. Factors which can improve transient stability are

(i) Reduction of transfer reactance by using parallel lines.

(ii) Reducing transmission line reactance by reducing conductor spacing and increasing conductor diameter, by using hollow cores.

(iii) Use of bundled conductors.

(iv) Series compensation of the transmission lines with series capacitors. This also increases the steady state stability limit. However it can lead to problem of sub-synchronous resonance.

(v) Since most faults are transient, fast acting circuit breakers with rapid re-closure facility can aid stability.

(vi) The most common type of fault being the single-line-to-ground fault, selective single pole opening and re-closing can improve stability.

(vii) Use of braking resistors at generator buses. During a fault, there is a sudden decrease in electric power output of generator. A large resistor, connected at the generator bus, would partially compensate for the load loss and help in decreasing the acceleration of the generator. The braking resistors are switched during a fault through circuit breakers and remain for a few cycles after fault is cleared till system voltage is restored.

(viii) Short circuit current limiters, which can be used to increase transfer impedance during fault, there by reducing short circuit currents.

(ix) A recent method is fast valving of the turbine where in the mechanical power is lowered quickly during the fault, and restored once fault is cleared.