Thursday
30.7.15.

Network Analysis (Eelectric Circuit Analysis)

NA:- "Network analysis means to find the current through or voltage across any branch of the network by using fundamental laws and various simplification techniques."

Network:- "The combination of elements such as different electrical parameters or different electrical elements (resistors, capacitors, inductors) along with various sources of energy to give rise to complicated electrical circuit, generally refered as Networks."

Network element:- "Any individual circuit element with two terminals which can be connected to other circuit element is called network element."

Network elements can be either active elements or passive elements.

Active elements:- "elements which supplied power or energy to the network." Active elements possess energy of their own & can impart it to other elements of the circuit. Voltage source & current source are the example of active elements.

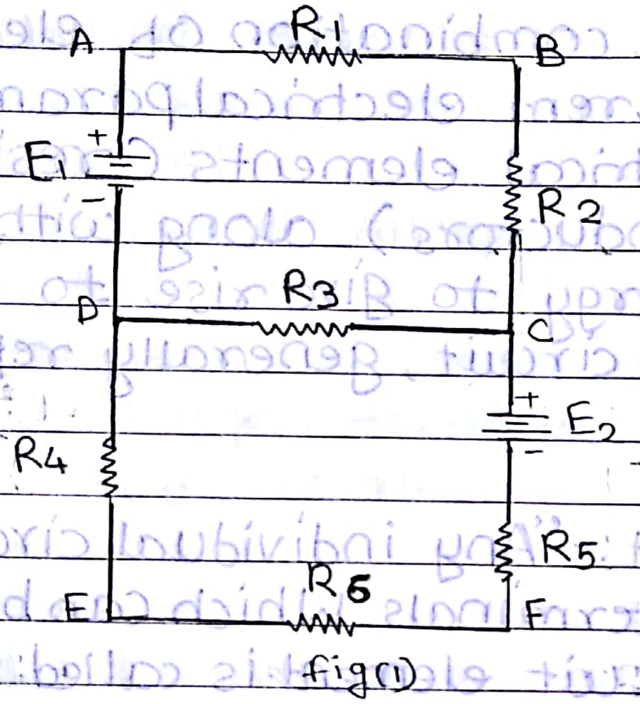
Passive elements:- "Passive elements are the elements which either store energy, dissipate energy in the form of heat."

Resistor, inductor, capacitor are the three basic passive elements.

Inductors & capacitors can store energy & resistors dissipate energy in the form of heat.

Branch:- "A part of the network which connects the various points of the network with one another is called branch.

In the fig. below AB, BC, CD, DA, DE, EF, are the various branches. A branch may consist more than one element.



fig(1)

Junction point:- A point where 3 or more branches meet is called junction point. Point D & C are the junction points in the network.

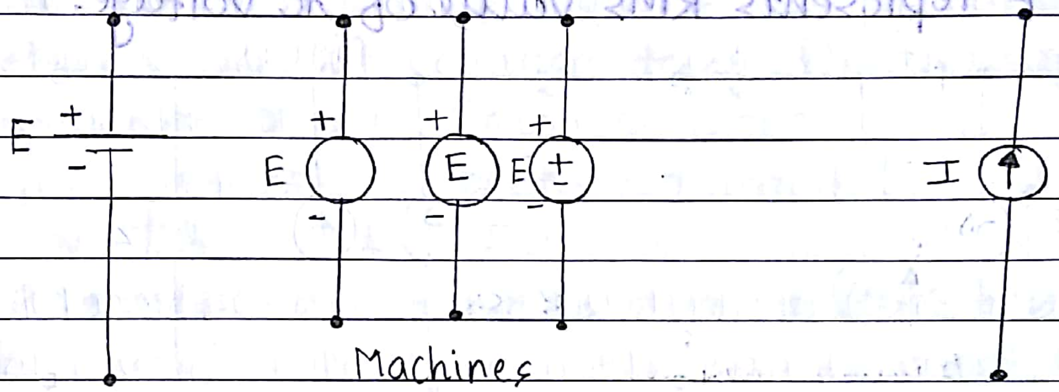
Node:- "A point at which 2 or more elements joined together is called Node. The junction points are also the nodes of the network"

In the network shown in fig A, B, C, D, E, F are the nodes of the network.

Ideal voltage source:- "An ideal voltage source is one which delivers energy to the load at the constant terminal voltage irrespective of the current drawn by the load." It's internal resistance is zero.

Ideal current source:- "An ideal current source is one which delivers energy with a constant current to the load irrespective of the terminal voltage across the load." It's internal resistance is zero.

Ideal DC voltage sources & ideal dc current sources are represented symbolically as shown in fig below.



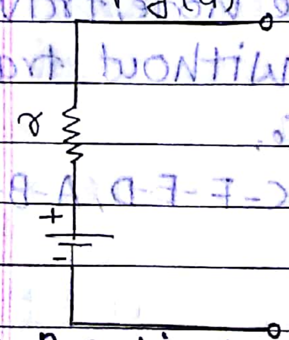
Battery fig(2) fig(3)

a) Various methods of representing ideal DC voltage source

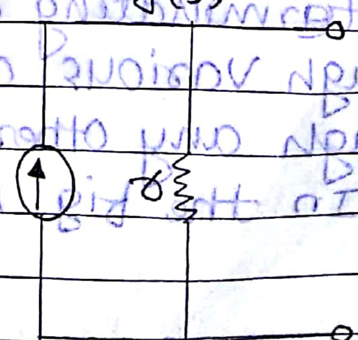
b) Method of representing ideal dc current source.

An ideal source does not exactly represent any physical device. A practical source always possess a very small value of internal resistance 'r'. The symbolic representations of practical voltage source & practical current source are shown in fig below.

fig(4)



fig(5)



a) Practical voltage source

b) practical current source

DATE: / /

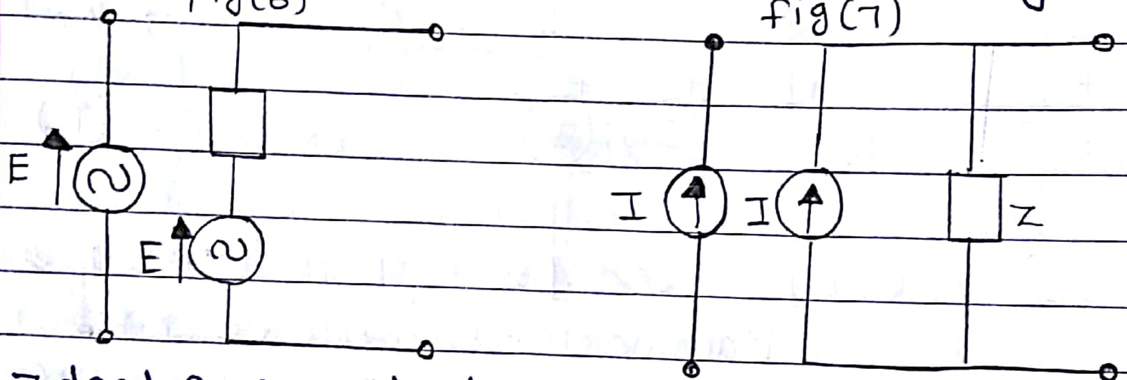
The internal resistance of a voltage source is always connected in series with it. and for a current Source it is always connected in parallel with it.

An ideal Ac voltage source & practical voltage source are shown in the fig. Where z is the terminal impedance of the practical voltage source. $Z = R + jx$ or $Z = V/I$ x - reactance

E represents RMS value of Ac voltage.

fig (6)

fig (7)



ⓐ Ideal & practical ac voltage source.

Z - internal impedance
I - rms value of current
ⓑ Ideal & practical ac current source

Monday
Date
3-8-15

Mesh or loop:- It is a set of branches forming closed path in a network such a way that one branch is removed then remaining branches do not form a closed path.

A loop is also can be defined as a closed path which originates from particular node, terminating at the same node, travelling through various other nodes, without travelling through any other node twice.

In the fig A-B-G-D-A, D-C-F-E-D, A-B-C-F-E-D-A

Classification of electrical Network:-

i. Linear network:- A ckt or network whose parameters i.e. elements like resistance, inductances, capacitances are always constant irrespective of change in time, voltage, temp is known as linear network.

Ohm's law is applicable.

ii Non linear network:- A ckt whose parameters changed their values will change temp, time, voltage etc. is known as non linear network.

Ohm's law is not applicable.

iii Bilateral:- A ckt whose characteristics, behavior is same irrespective of direction of current through various element of it is called bilateral network.

ex. Transmission line, ckt consist of only resistance.

iv. Unilateral network:- A ckt whose operation, behavior is dependent of a direction of current through various element called unilateral network.

v. Active network:- A ckt which contains atleast one source of energy is called active network. an energy source may be voltage or current source.

vi Passive network:- A ckt which contains no energy source called passive network.

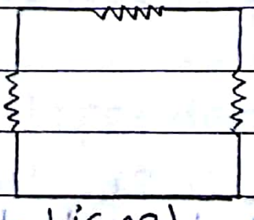


fig (8)

vii Lumped Network:- A network in which all the network element are physically separable called lumped network. Most of the electrical networks are lumped in nature which contains R, C, L voltage source.

viii Distributed network:- A network in which a ckt element resistance, inductance cannot be physically separable. for analysis purpose called distributed network.

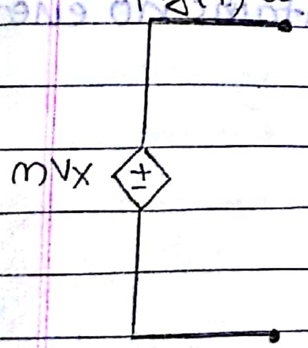
~~Transformation~~:-

Independent & dependent sources:- All the sources distributed in previous section/day fig (a) & (b) are independent sources.

As the voltage of voltage source is completely independent of current & the current of current source completely independant of voltage source but their are special kinds of sources in which the source voltg or source current depends upon current or voltage else where in the ckt. such sources are called dependent sources or controlled source.

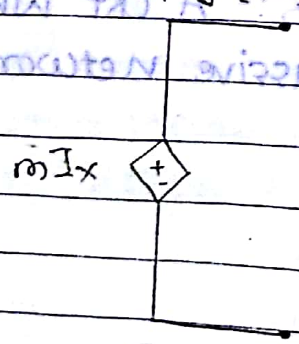
Diamond symbol is used to represent dependent sources & circle symbol used to represent independent source. fig. shows dependant voltage & current sources.

fig (1) a



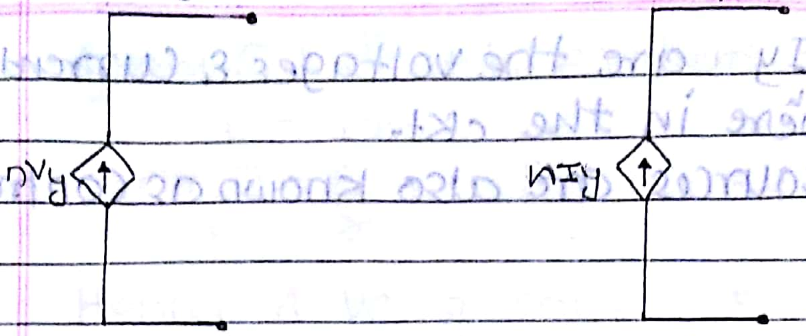
Voltage dependent voltage source

fig (1) b.



Current dependent voltage source

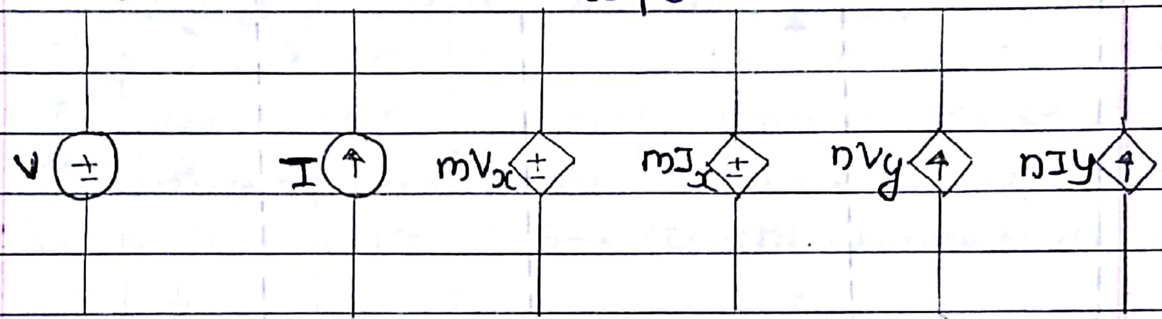
fig (2) a. fig (2) b



voltage dependent current source — current dependent current source

Tuesday 4-8-15

Ideal sources ; dependent sources



fig(3)a m & n are the const. fig(3)b ii)

Dependent sources are those where value of source depends on voltage or current elsewhere in ckt

Voltage dependent voltg source:- It produces a voltage as a function of voltages elsewhere in the given ckt

Current dependent current source:- It produces a current as a function of currents elsewhere in given ckt.

Current dependent voltage source:- It produces a a voltg as a function of currents elsewhere in given ckt

Voltg dependent current source:- It produces a current as a function of voltage elsewhere in given ckt.

V_x, I_x, V_y, I_y are the voltages & current resp. present elsewhere in the ckt.

Dependent sources are also known as controlled source.

* Source transformation:- 2 sources are said to be identical when they produce identical terminal voltage V_L & load current I_L

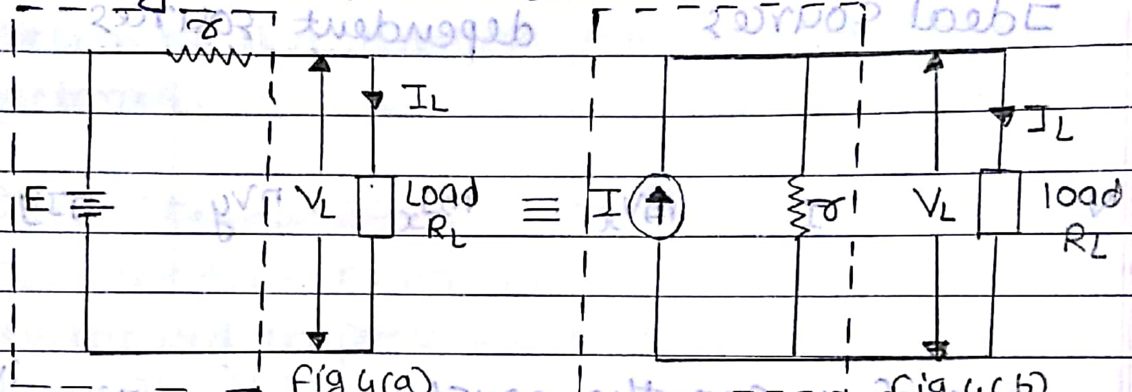


fig 4(a) fig 4(b)

The ckt 4(a) & 4(b) represents practical voltg source & practical current source resp. with load R_L connected to both the sources. The terminal voltage V_L & load current I_L across the terminals are same. Hence the practical voltage source shown in dotted box in fig. 4(a) is equivalent to practical current source shown in dotted box shown in fig(b).

The 2 equivalent sources should also provide the same open ckt voltage & short circuit current. from fig 4(a) $I_L = \frac{E}{r+R_L}$ ①

from fig 4 (b) $I_L = \frac{I \cdot r}{r+R_L}$ ② ∵ current division formula

$$\boxed{\text{current division formula} = \text{main current} \cdot \frac{\text{other branch resistance}}{\text{Total resistance}}}$$

from ① & ② it is evident that

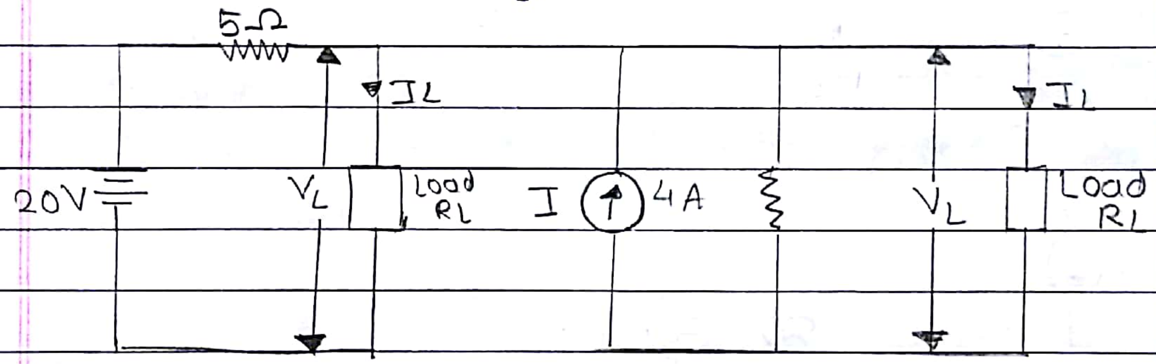
$E = I r$

or $\boxed{I = \frac{E}{r}}$ ——— ③

Hence A voltg source 'E' in series with its internal resistance 'r' can be converted into a current source $I = \frac{E}{r}$, with its internal resistance connected in $\parallel^{el}$ with it.

$\parallel^{rly}$ A current source I in $\parallel^{el}$ with its internal resistance 'r' can be converted into voltage source $E = Ir$, in series with its internal resistance 'r'.

ex. Transform the voltg. source of 20V with the internal resistance of 5Ω to the current source.



$I = \frac{E}{R} = \frac{20}{5} = 4A$

Ideal voltg. sources connected in series:- If 2 voltg sources are in series then the equivalent is drawn based on the polarities of the 2 sources

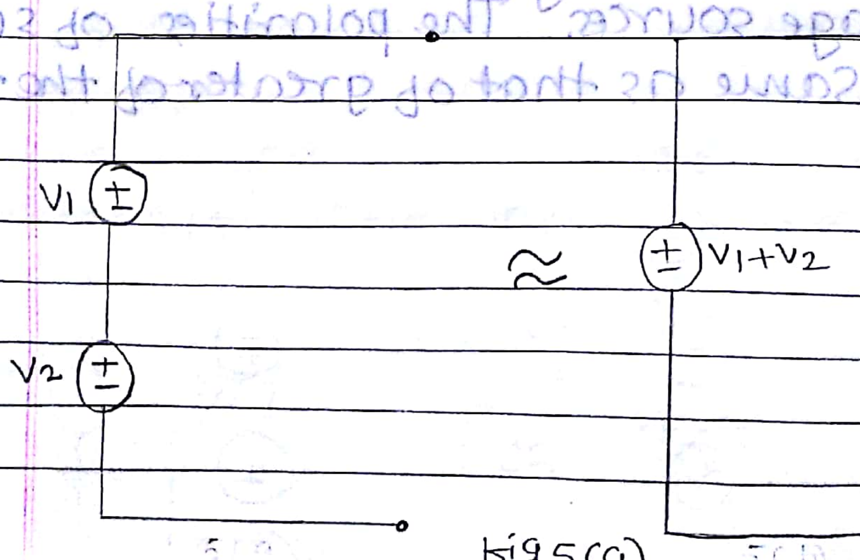


fig 5(a)

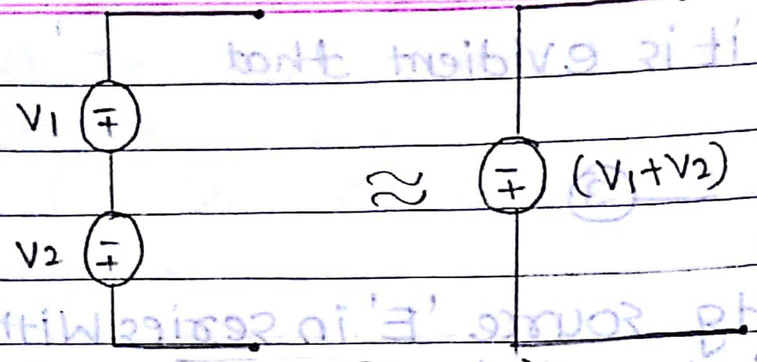


fig 5(b)

Thus the polarities of 2 sources are same then equivalent single source is the add^n of 2 sources with polarities same as that of the 2 sources.

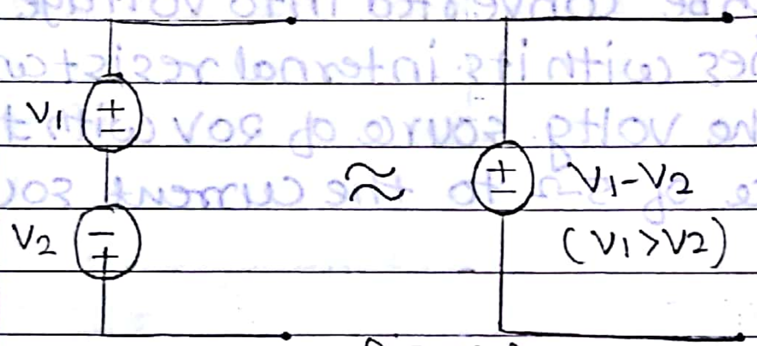


fig 5(c)

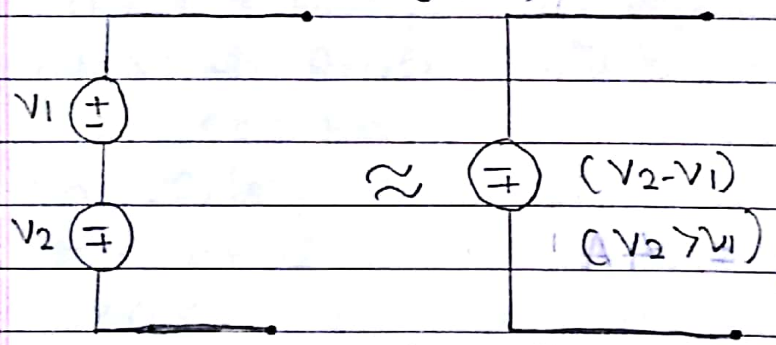
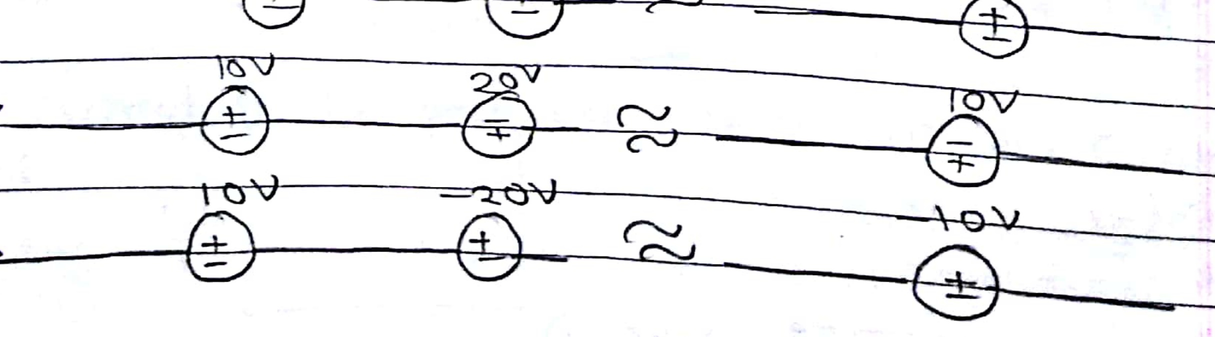
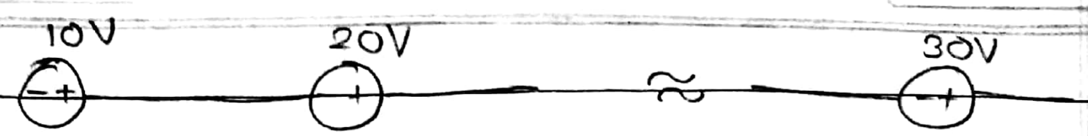


fig 5(d)

Thus if polarities of 2 sources are different then the equivalent single source is the difference bet^n 2 voltage sources. The polarities of such source is same as that of greater of the two sources.

ex.1)
2)
3)



4. 

Practical voltage sources connected in series :-

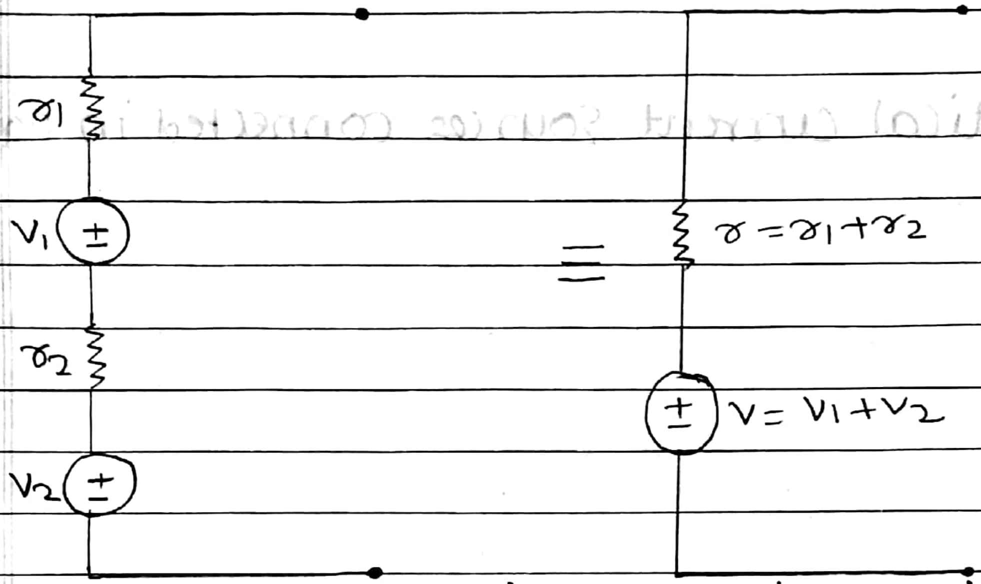
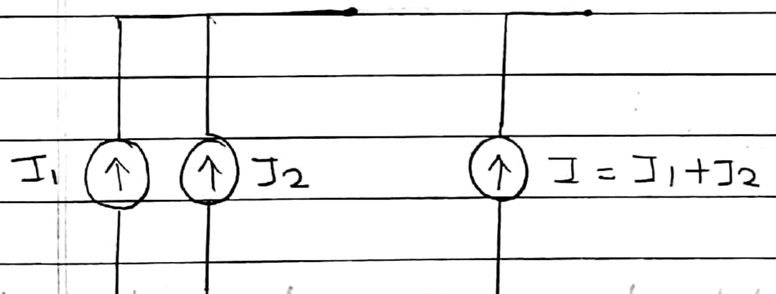
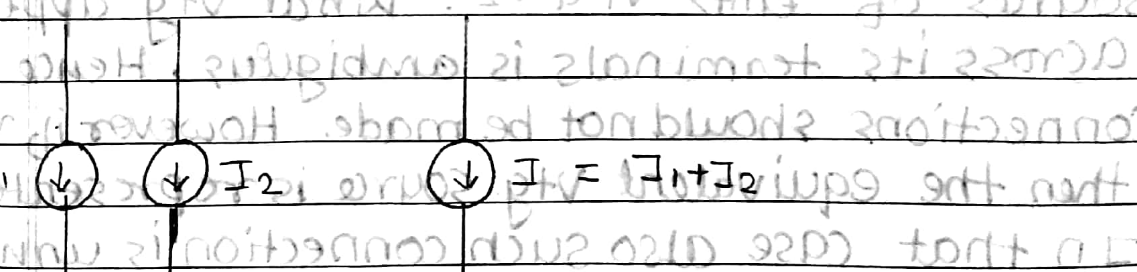
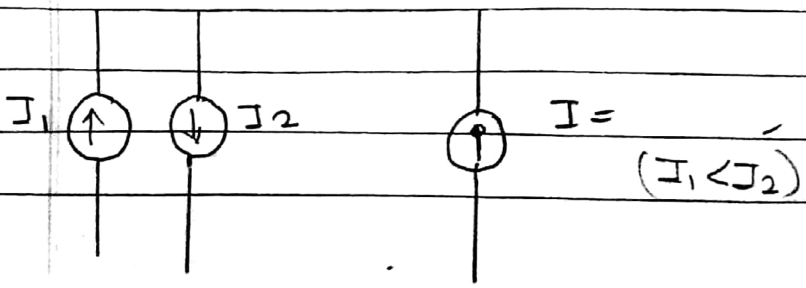


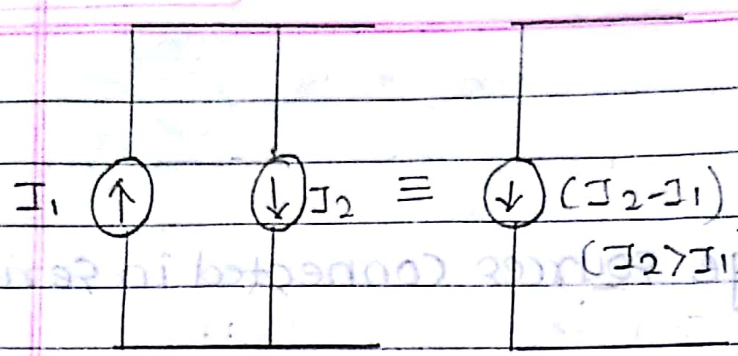
fig (6) — equivalent single practical vtg. source

Ideal current sources connected in parallel

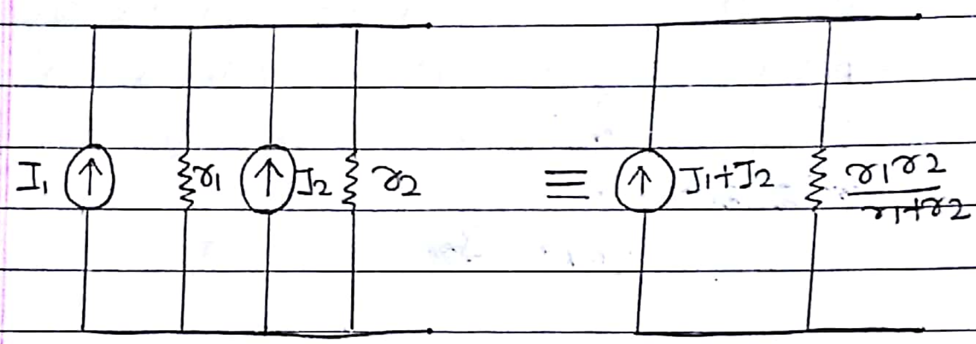






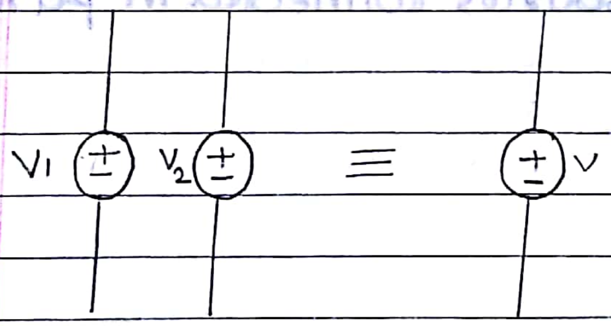


Practical current sources connected in parallel:-



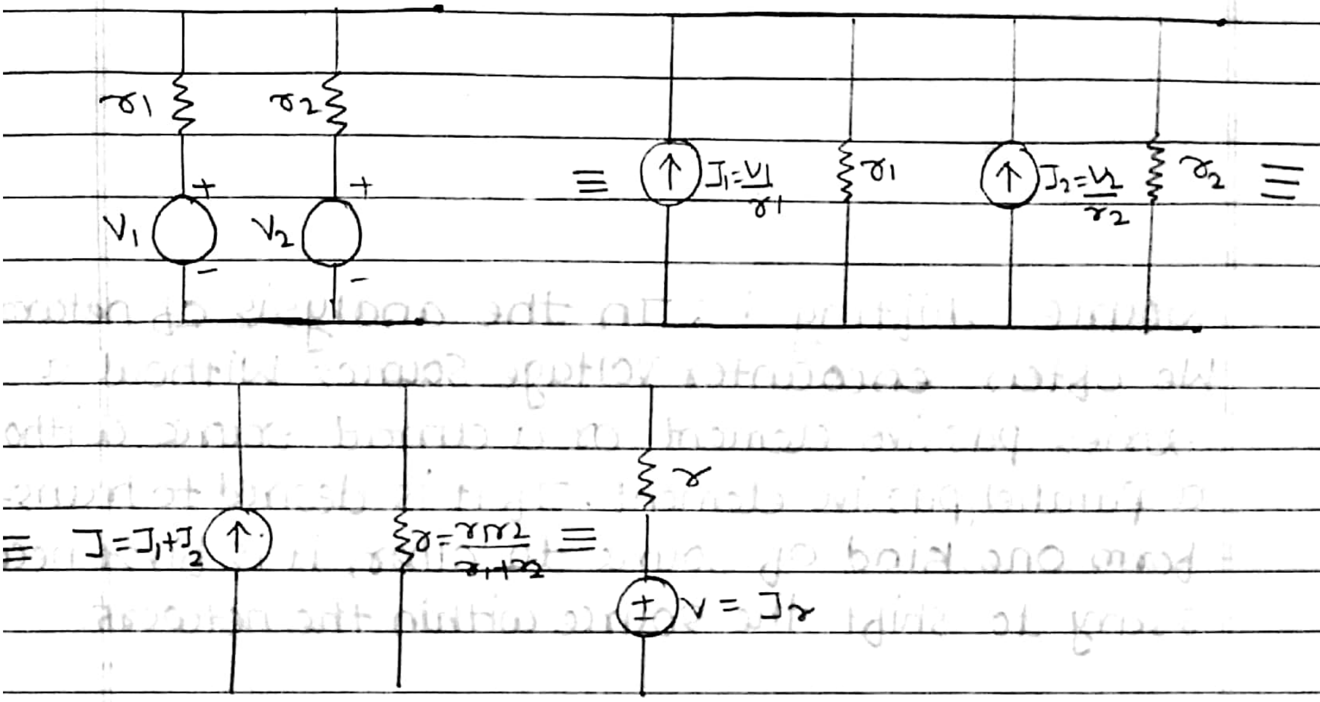
Ideal voltage sources connected in parallel:-

$V = V_1 = V_2$

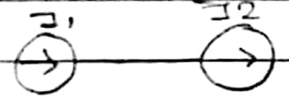


The ckt above represents 2 ideal vtg. sources of emfs V_1 & V_2. What vtg appears across its terminals is ambiguous. Hence such connections should not be made. However if $V_1 = V_2 = V$, then the equivalent vtg. source is represented by V. In that case also such connection is unnecessary as only one vtg source serves the purpose.

Practical voltage source connected in parallel:-

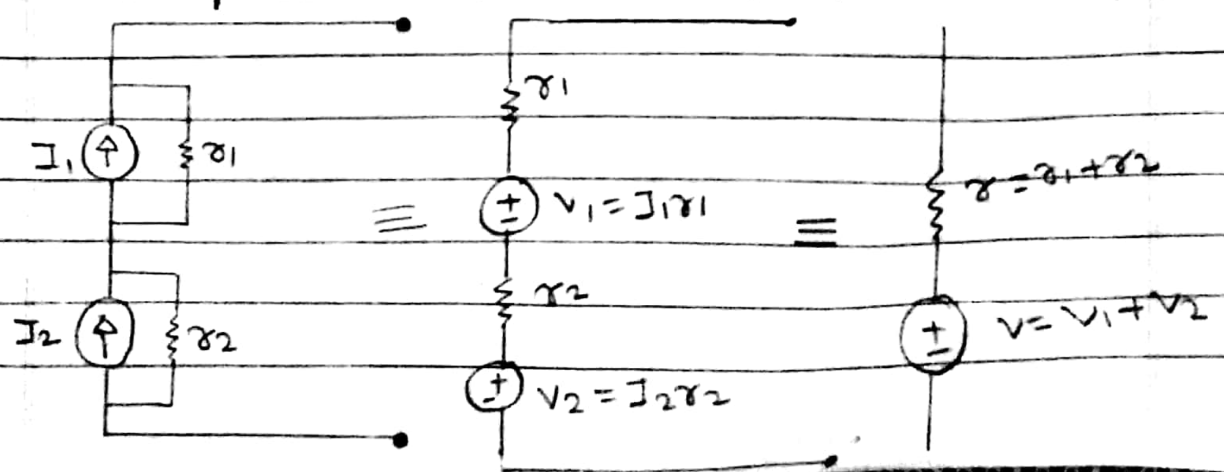


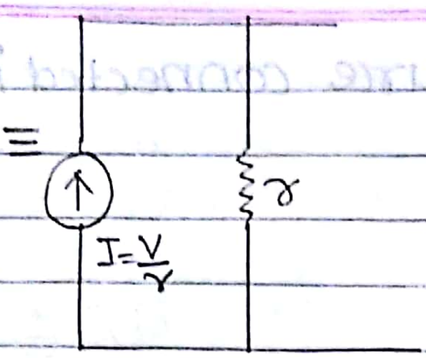
Two current sources connected in series :-
When two identical current sources are connected in series as shown in the fig. below.



What current
Hence such connection is not permisible.
However if $J_1 = J_2 = I$, then current in the line is I. but such a connection is not necessary as only one current source serve the purpose.

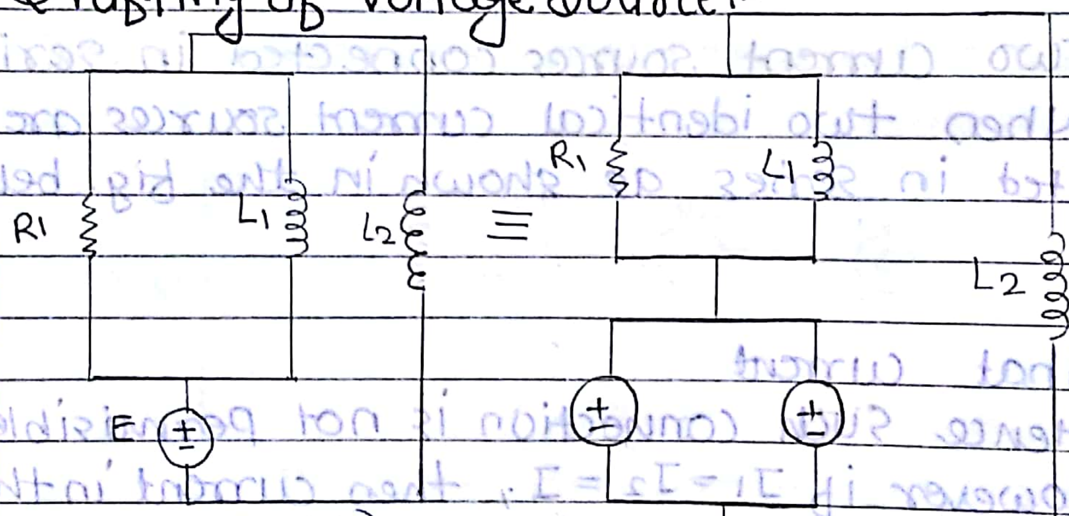
Two practical current sources connected in series:-





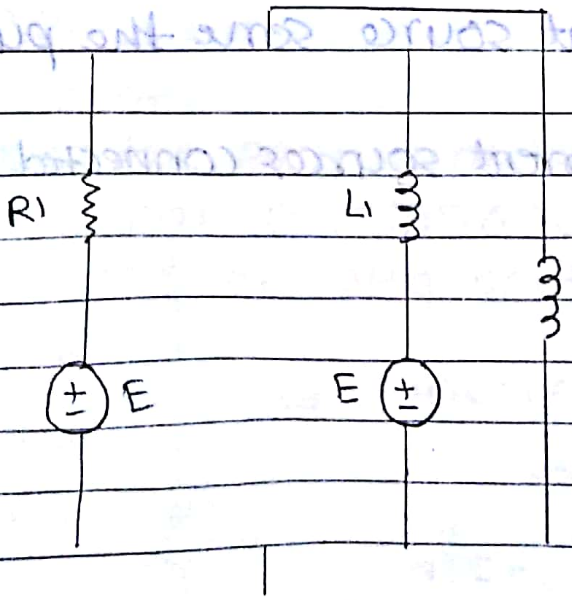
Source Shifting :- In the analysis of network We often encounter voltage sources without a series passive element or a current source without a parallel passive element. If it is desired to transform one kind of source to other, it is first necessary to shift the source within the network.

Shifting of voltage Source:-



12(a)

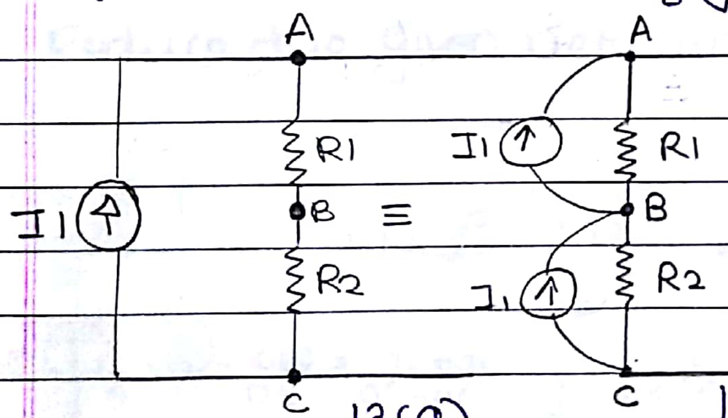
12(b)



12(c)

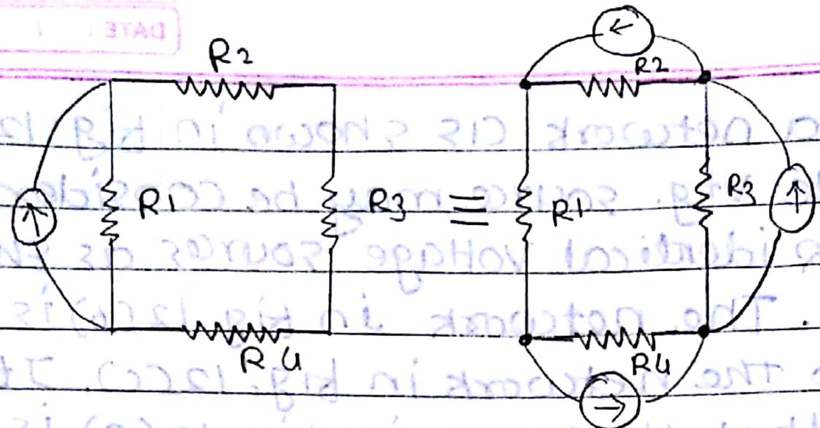
consider a network as shown in fig 12(a). The single vtg. source may be consider equivalent to 2 identical voltage sources as shown in fig 12(b). The network in fig 12(b) is equivalent to the network in fig. 12(c). It is observed that the source in fig 12(a) is pushed through the node in obtaining an equivalent network as shown in fig 12(c) in which the current through the various elements of the network remains unchanged after the transformer.

Shifting of current source:-
consider a network as shown in fig. 13(a) containing a single current source whose equivalent ckt shown in fig (b)



13(a) 13(b)

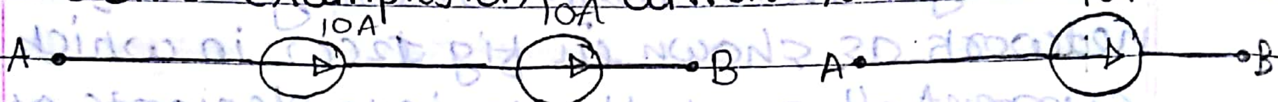
In fig. 13(b) the current I_1 enters the node B and leaves the node B. while the currents at ~~load~~ node A & C remains same as in fig 13(a)
The general principle of current source shifting is to maintain the same currents at all the nodes of the network after shifting.



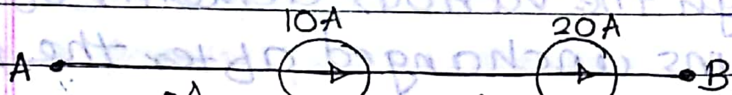
Date 6-8-15

Some examples on current source

1)

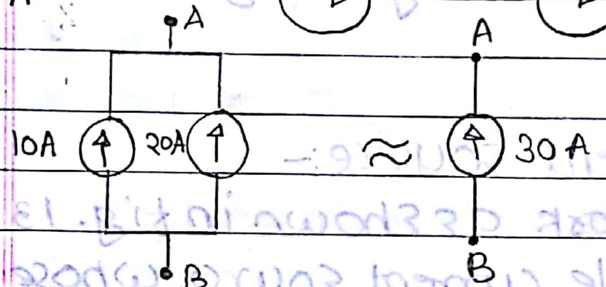


2)

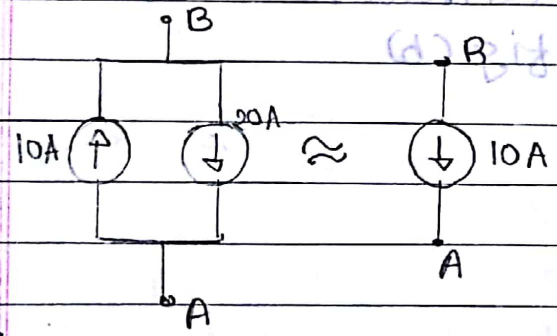


Not permissible.

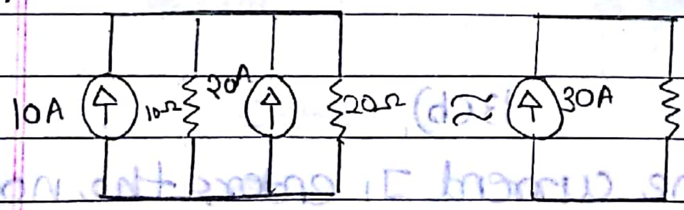
3)



4)



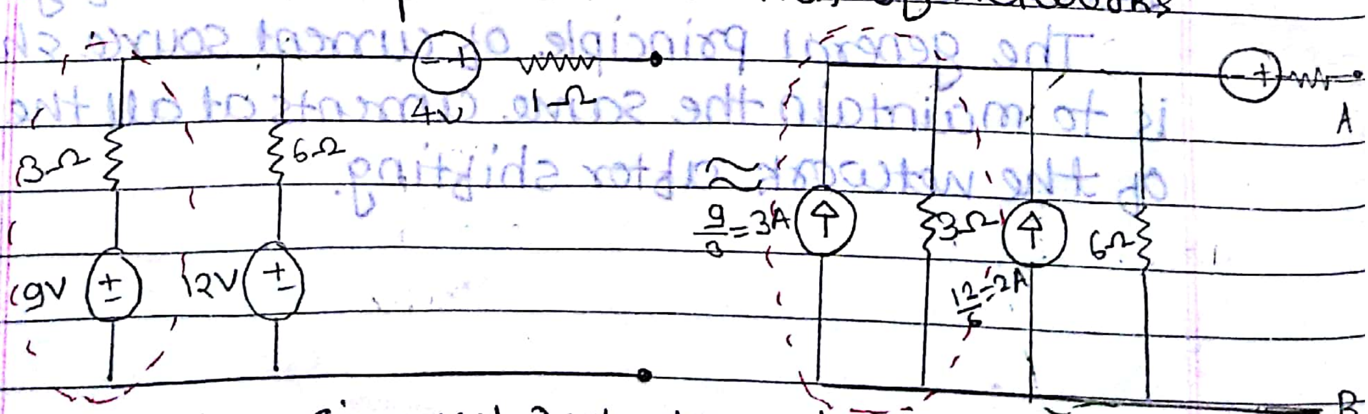
5)



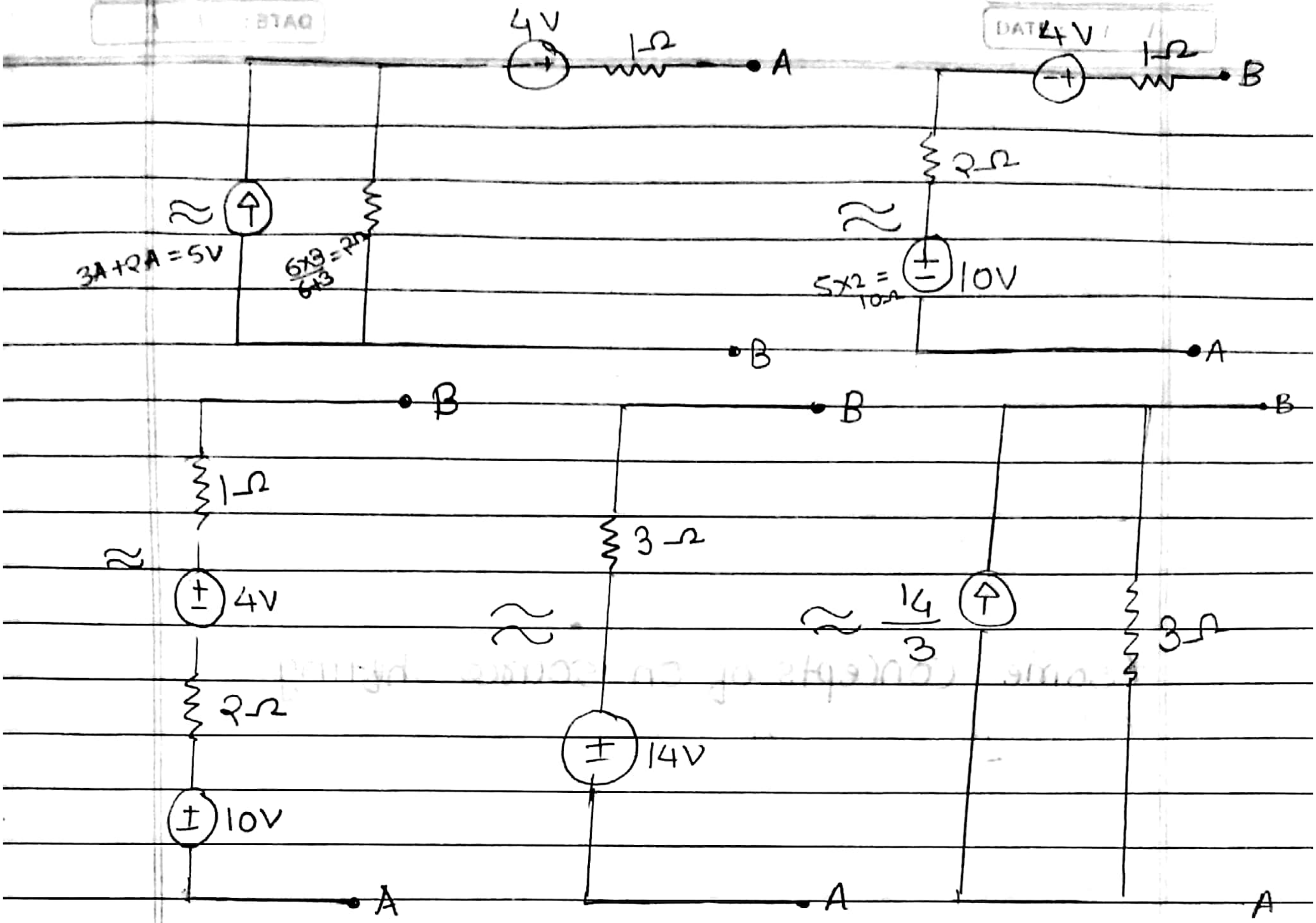
$\frac{10*20}{10+20} = \frac{200}{30} = 6.6$

Some examples on reduction of networks

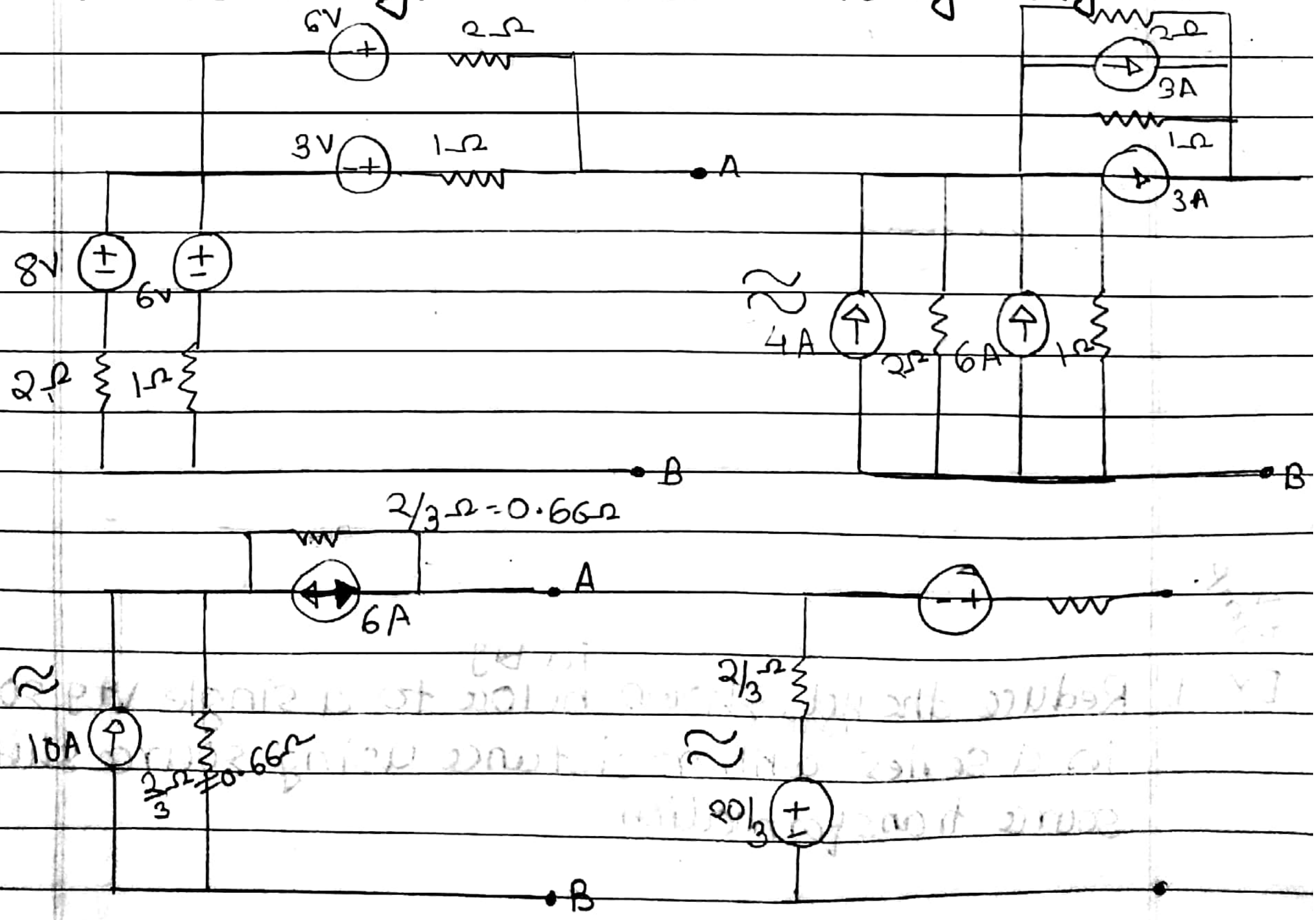
1)

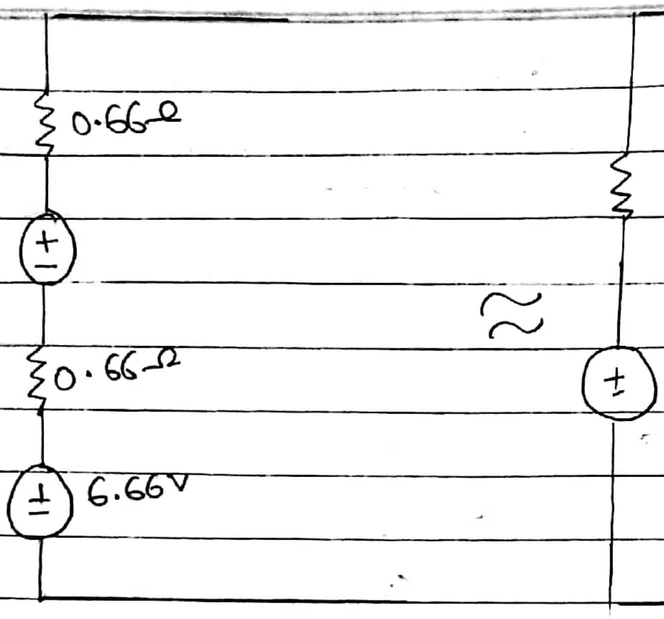


Reduce the given network to a single current source



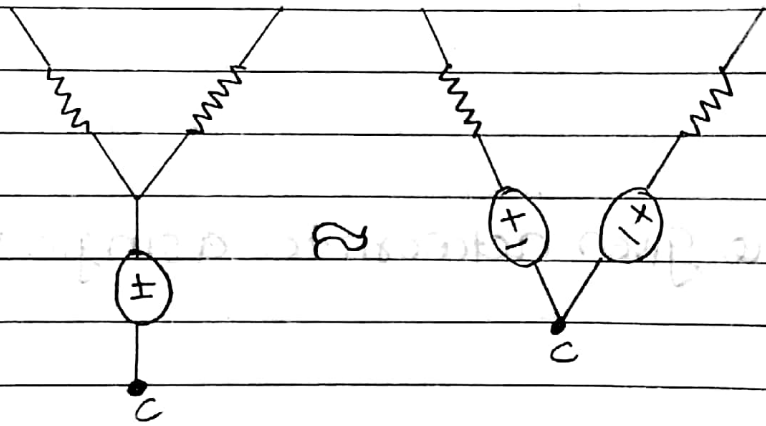
2. Reduce the given network to a single vtg source



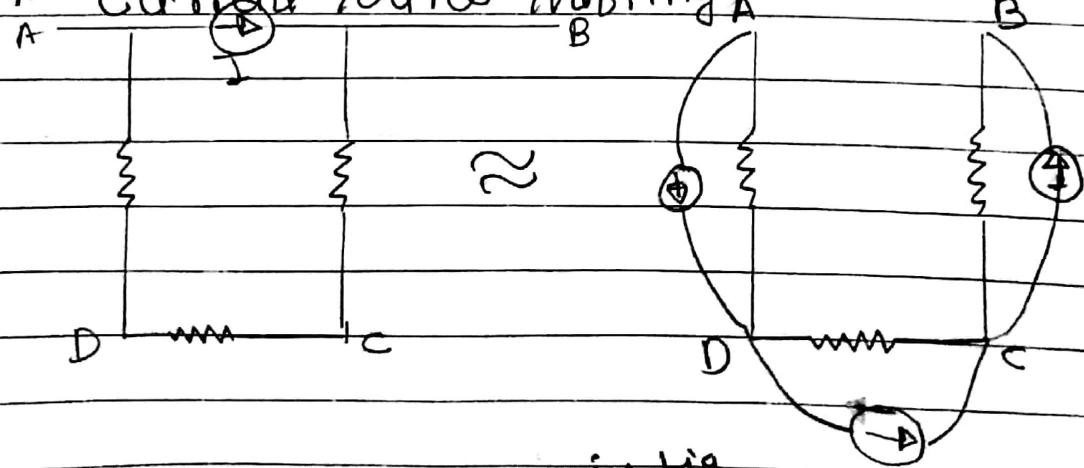


* Some concepts ~~of~~ on source shifting:-

* Voltage source shifting:-

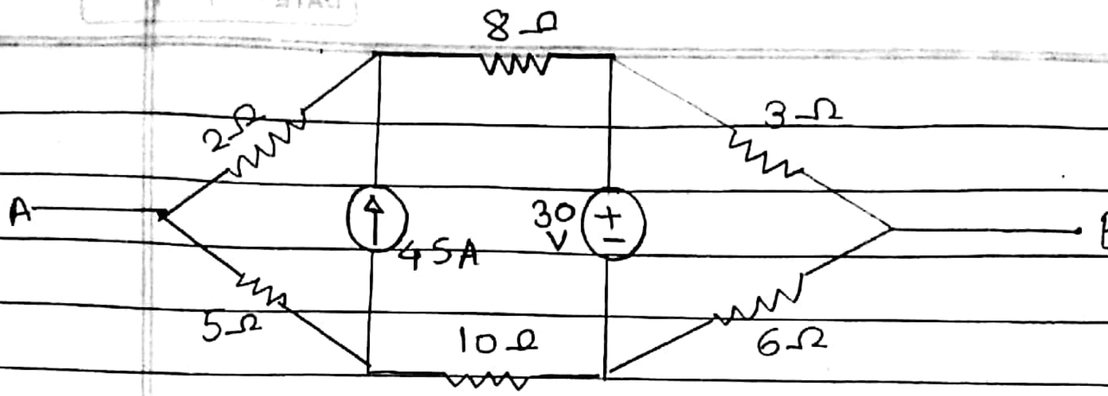


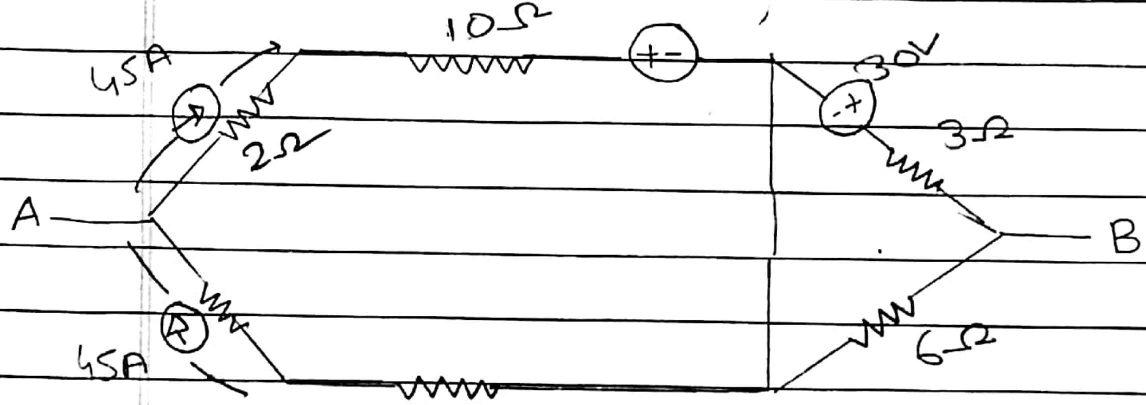
* Current source shifting:-

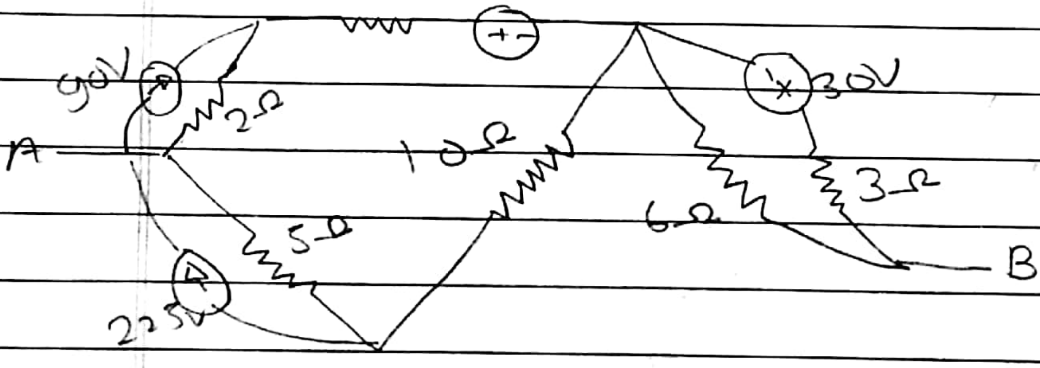


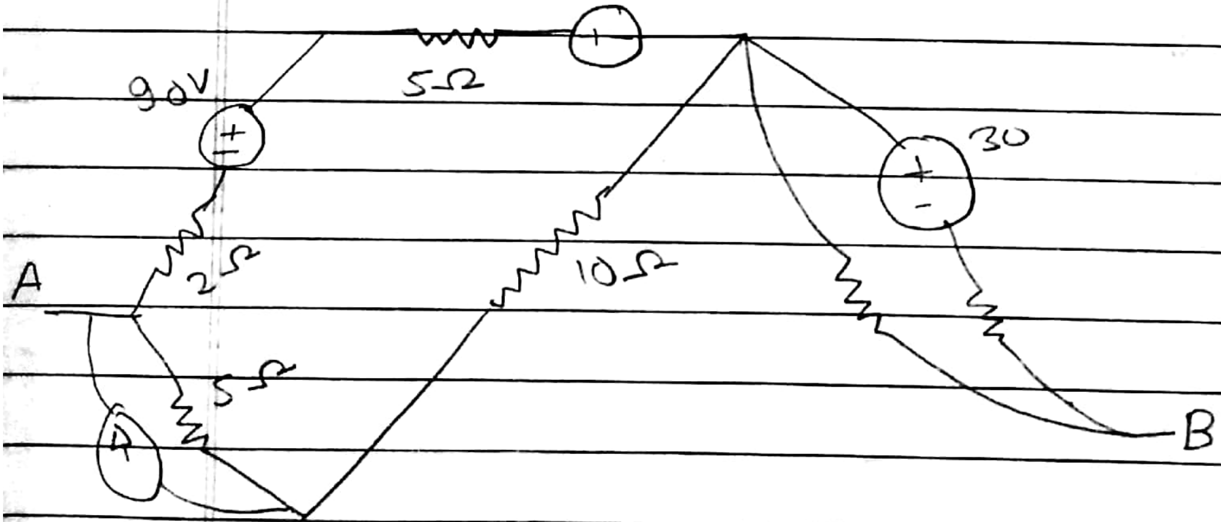
KV Imp

Ex: 1. Reduce the network shown in fig. below to a single vtg source in a series with resistance using source shift & source transformation


8Ω
2Ω
3Ω
A
45A
30 V
B
5Ω
10Ω
6Ω

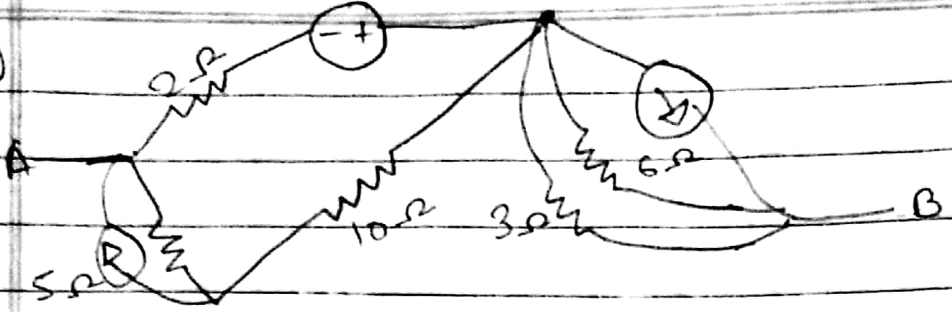

10Ω
45A
30V
2Ω
3Ω
A
B
6Ω
45A


90V
2Ω
30V
A
10Ω
3Ω
6Ω
5Ω
B
225V

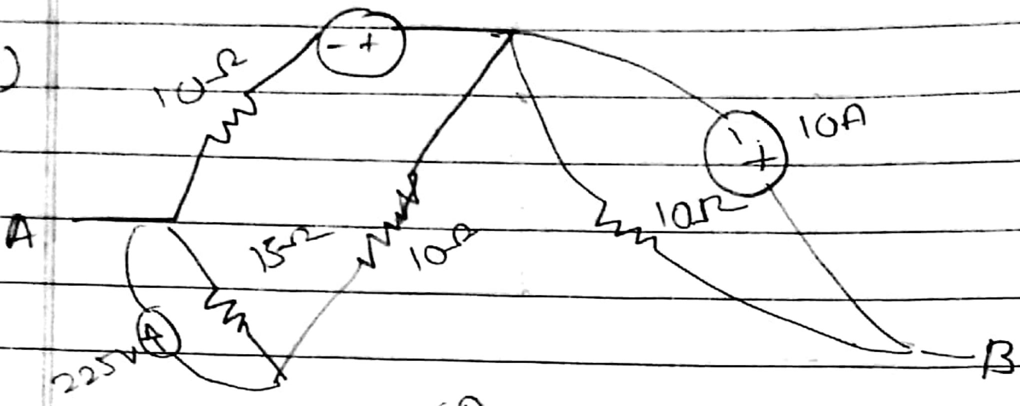

5Ω
90V
30
2Ω
10Ω
A
5Ω
B

DATE

6)



7)



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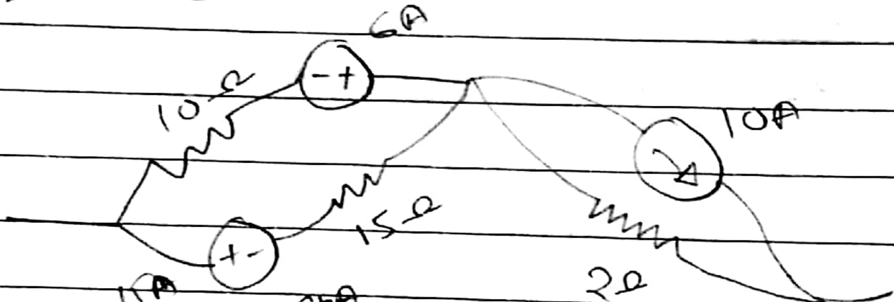

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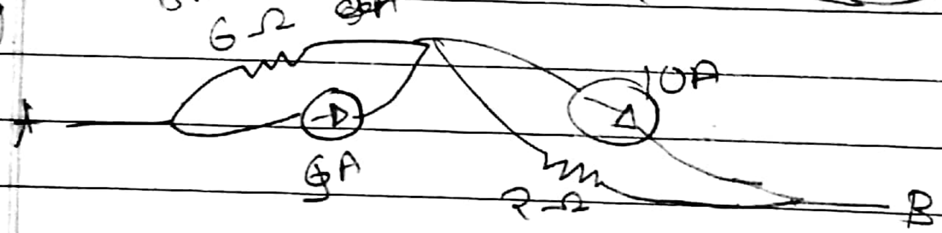

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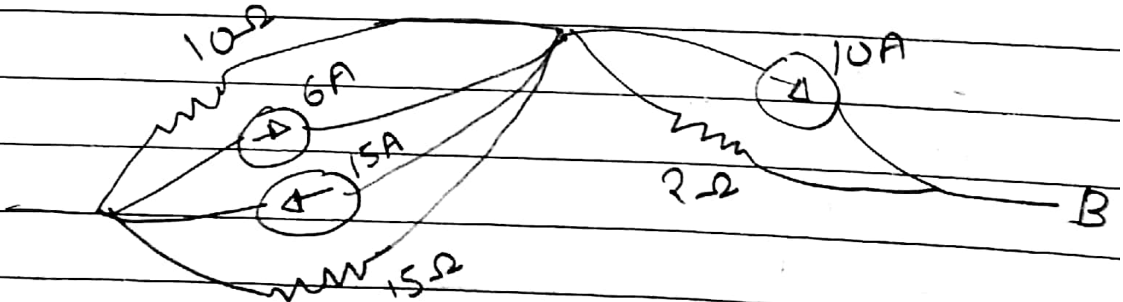

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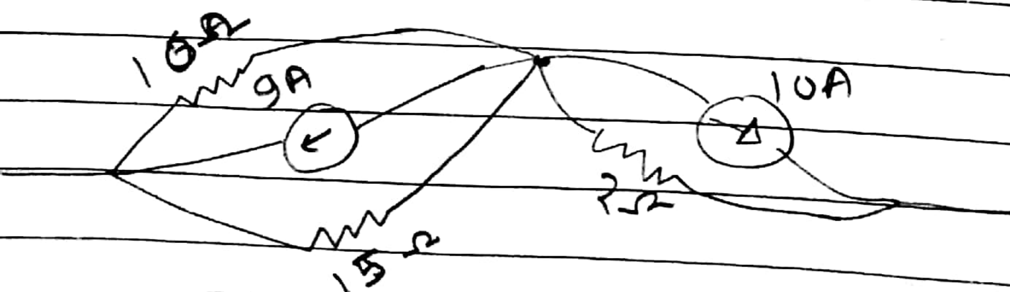

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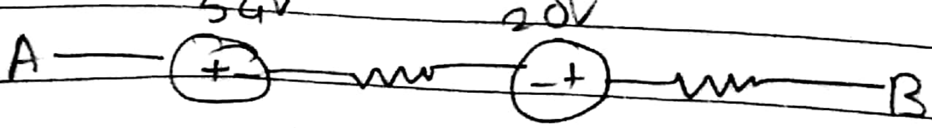

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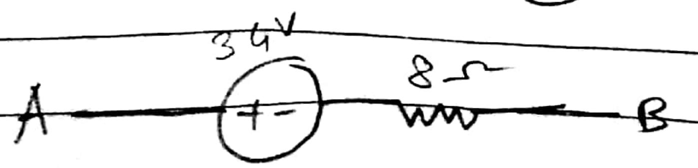
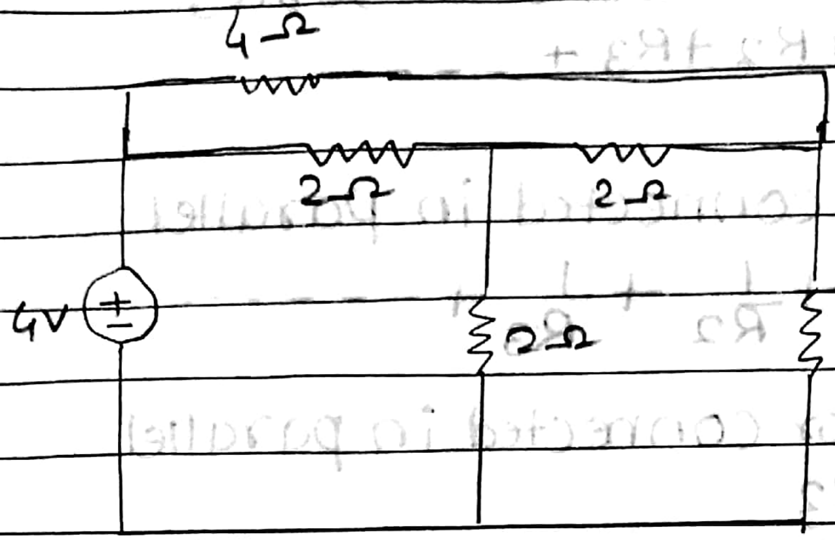
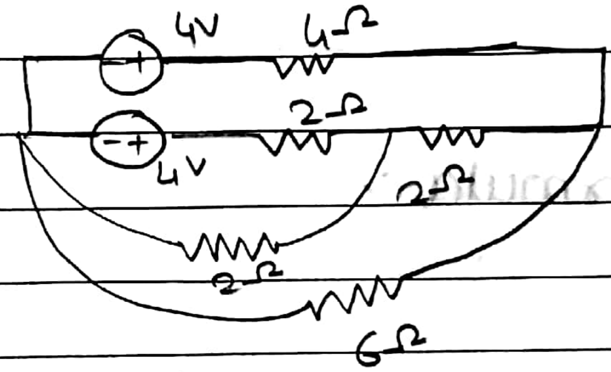


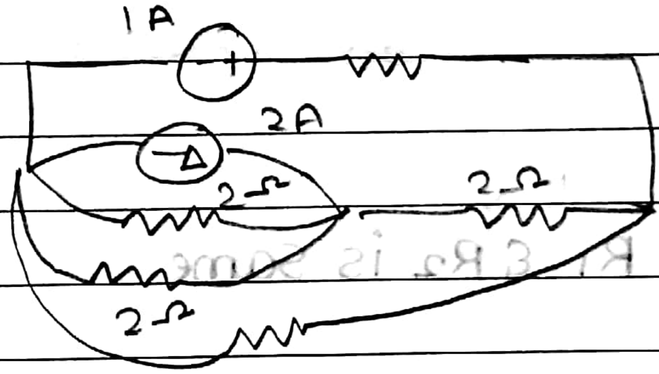
1)

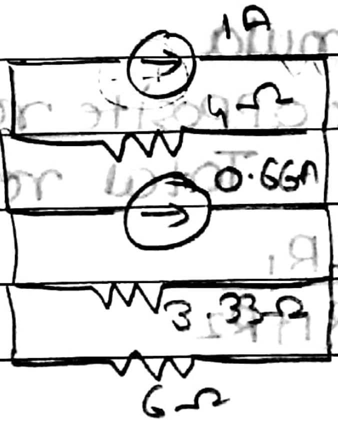

4Ω
2Ω
2Ω
4V
2Ω
6Ω

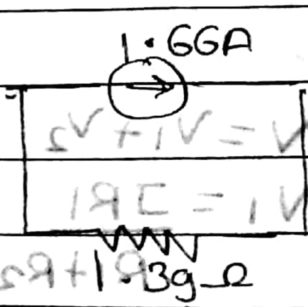
2)

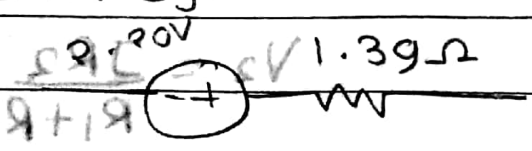

4V
4Ω
2Ω
4V
2Ω
2Ω
6Ω

3)


1A
2A
2Ω
2Ω
2Ω


1A
4Ω
0.66A
3.33Ω
6Ω


1.66A
1.39Ω


2.0V
1.39Ω

Saturday
8-8-2015

Some Formulae

1. Resistances connected in series

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

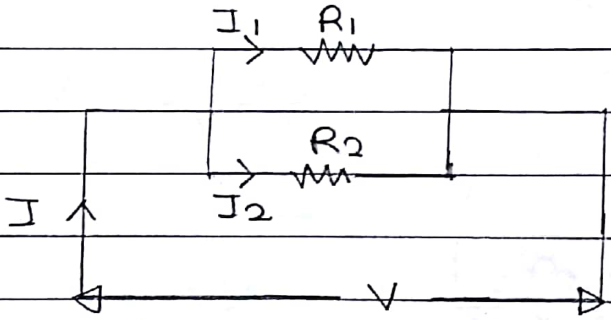
2. Resistances connected in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Two resistor connected in parallel

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

3. Current division formula :-

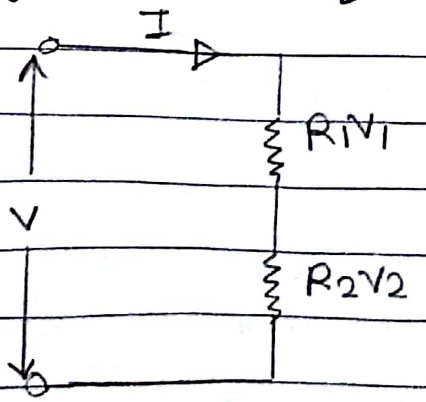


* voltage across R_1 & R_2 is same.
* $I = I_1 + I_2$
* By current division formula

$$I_1 = \frac{\text{main current} \times \text{opposite resistance}}{\text{Total resistance}}$$

$$I_1 = \frac{IR_2}{R_1 + R_2}, \quad I_2 = \frac{IR_1}{R_1 + R_2}$$

4 Voltage division formula:-



$$V = V_1 + V_2$$

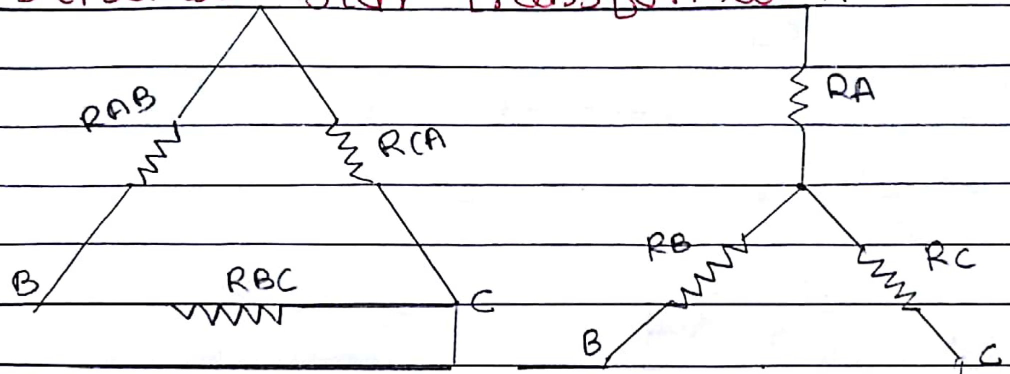
$$V_1 = \frac{IR_1}{R_1 + R_2}$$

$$V_2 = \frac{IR_2}{R_1 + R_2}$$

VV Imp

Delta to Star (Δ-Y) & Star to delta (Y-Δ) Transformation:-

i) Delta to Star transformation



Let RAB, RBC and RCA are the three resistance connected in delta as shown in fig. RA, RB, RC are the three equivalent resistances connected in star as shown in fig.

$$R_A + R_B = \frac{R_{AB}(R_{BC}+R_{CA})}{R_{AB}+R_{BC}+R_{CA}} = \frac{R_{AB}(R_{BC}+R_{CA})}{\Sigma R_{AB}} \quad (1)$$

$$R_B + R_C = \frac{R_{BC}(R_{CA}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}} = \frac{R_{BC}(R_{CA}+R_{AB})}{\Sigma R_{AB}} \quad (2)$$

$$R_C + R_A = \frac{R_{CA}(R_{AB}+R_{BC})}{R_{AB}+R_{BC}+R_{CA}} = \frac{R_{CA}(R_{AB}+R_{BC})}{\Sigma R_{AB}} \quad (3)$$

eqn (1) − eqn (2)

$$R_A - R_C = \frac{R_{AB}R_{CA} - R_{BC}R_{CA}}{\Sigma R_{AB}} \quad (4)$$

eqn (3) + eqn (4)

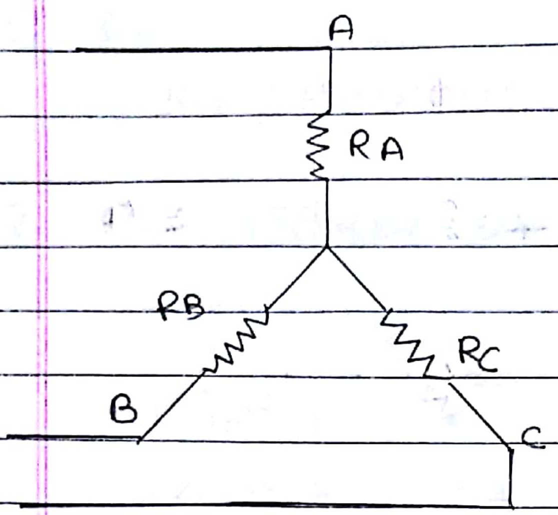
$$2R_A = \frac{2R_{AB}R_{CA}}{\Sigma R_{AB}}$$

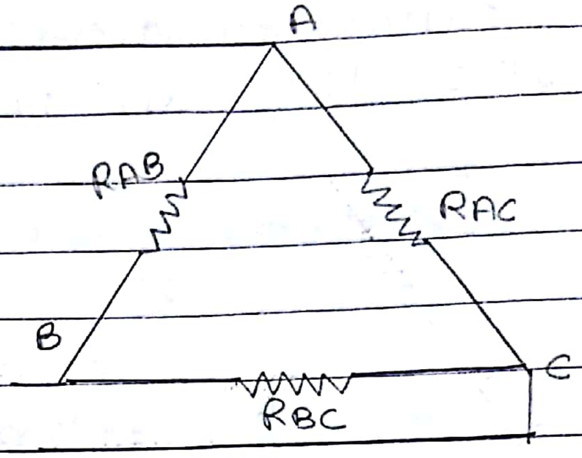
$$\Rightarrow \boxed{R_A = \frac{R_{AB}R_{CA}}{\Sigma R_{AB}}} \quad (5)$$

$$\Rightarrow \boxed{R_B = \frac{R_{BC}R_{AB}}{\Sigma R_{AB}}} \quad (6)$$

$$\Rightarrow \boxed{R_C = \frac{R_{BC}R_{CA}}{R_{AB}}} \quad (7)$$

ii) Star to Delta transformation:-





from eqⁿ ⑤, ⑥, ⑦ we can write

$$R_AR_B + R_BR_C + R_CR_A = \frac{R_{AB}R_{BC}R_{CA}(R_{AB}+R_{BC}+R_{CA})}{(\Sigma R_{AB})^2}$$

$$= \frac{R_{AB}R_{CA}}{\Sigma R_{AB}} \times \frac{R_{BC}(R_{AB}+R_{BC}+R_{CA})}{\Sigma R_{AB}}$$

$$= R_A \times \frac{R_{BC}(R_{AB}+R_{BC}+R_{CA})}{\Sigma R_{AB}}$$

$$= R_{BC} \times R_A \frac{\Sigma R_{AB}}{\Sigma R_{AB}}$$

$$R_AR_B + R_BR_C + R_CR_A = R_A \cdot R_{BC}$$

$$\therefore R_{BC} = \frac{R_AR_B + R_BR_C + R_CR_A}{R_A}$$

$$\boxed{R_{BC} = R_B + R_C + \frac{R_BR_C}{R_A}}$$

||ly

$$\boxed{R_{AC} = R_A + R_C + \frac{R_AR_C}{R_B}}$$

$$\boxed{R_{AB} = R_A + R_B + \frac{R_AR_B}{R_C}}$$

Delta connections:- Delta connected networks.

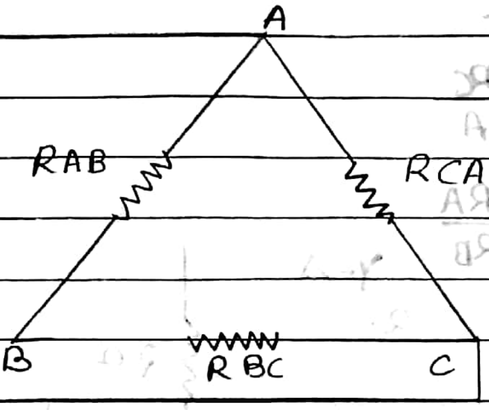


fig (14.a)

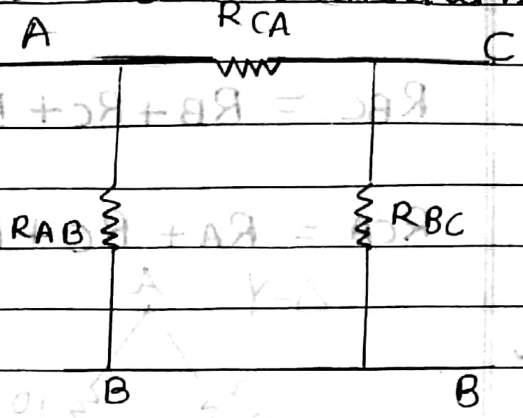


fig (14.b) (π) connection

Star connected networks

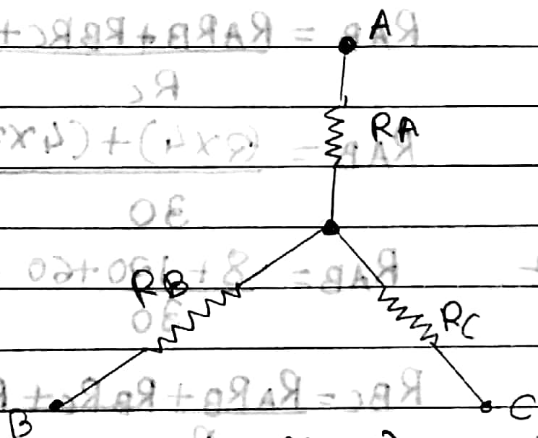


fig (15.a)

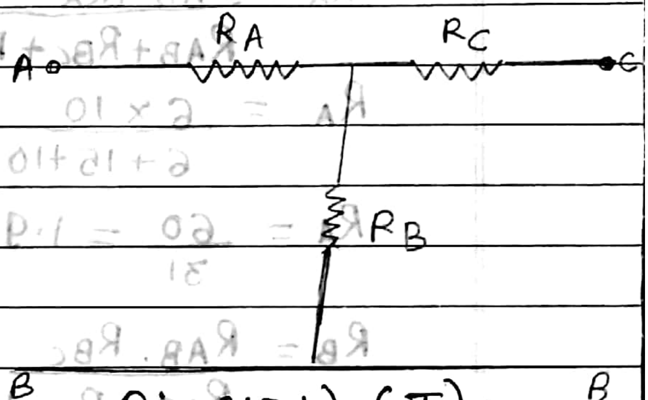
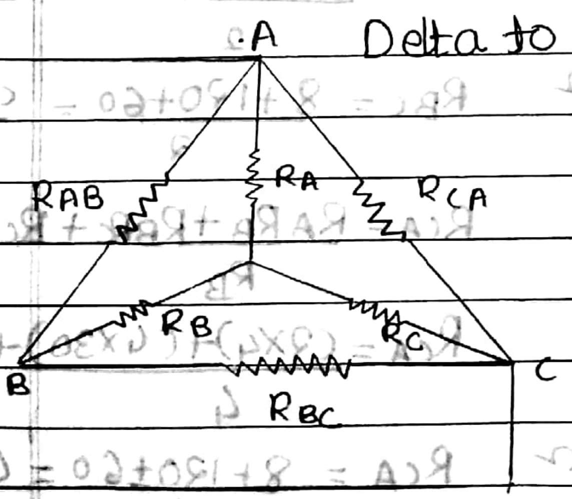


fig (15.b) (T) connection

Delta to star



$$R_A = \frac{R_{AB} \cdot R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \quad (8)$$

$$R_B = \frac{R_{AB} \cdot R_{BC}}{R_{AB}+R_{BC}+R_{CA}} \quad (9)$$

$$R_C = \frac{R_{BC} \cdot R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \quad (10)$$

Star to delta

$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} \quad (11)$$

$$R_{BC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A} \quad (12)$$

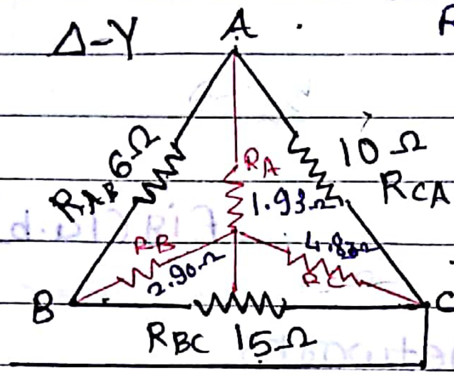
$$R_{CA} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B} \quad (13)$$

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{CA} = R_A + R_C + \frac{R_C R_A}{R_B}$$

1\. Δ-Y



$$R_A = \frac{R_{AB} \cdot R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A = \frac{6 \times 10}{6 + 15 + 10}$$

$$R_A = \frac{60}{31} = 1.93\,\Omega$$

$$R_B = \frac{R_{AB} \cdot R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

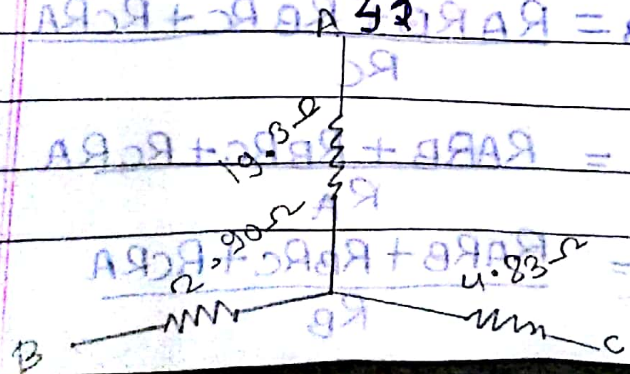
$$R_B = \frac{6 \times 15}{6 + 15 + 10}$$

$$R_B = \frac{90}{31} = 2.90\,\Omega$$

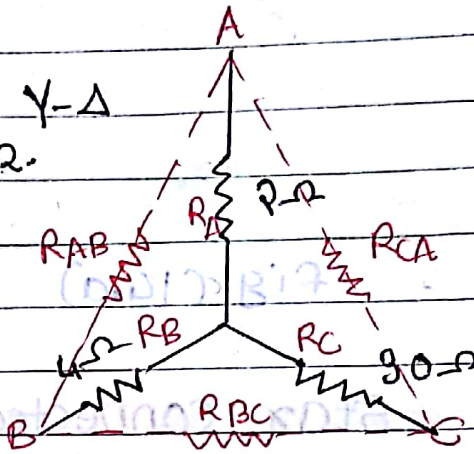
$$R_C = \frac{R_{BC} \cdot R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{15 \times 10}{6 + 15 + 10}$$

$$R_C = \frac{150}{31} = 4.83\,\Omega$$



2\. Y-Δ



$$R_{AB} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_{AB} = \frac{(2 \times 4) + (4 \times 30) + (2 \times 30)}{30}$$

$$R_{AB} = \frac{8 + 120 + 60}{30} = 6.26\,\Omega$$

$$R_{BC} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

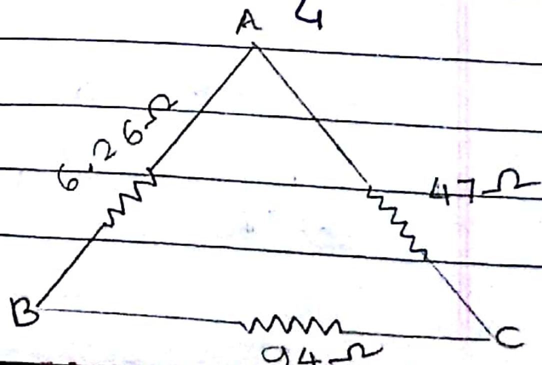
$$R_{BC} = \frac{(2 \times 4) + (4 \times 30) + (2 \times 30)}{2}$$

$$R_{BC} = \frac{8 + 120 + 60}{2} = 94\,\Omega$$

$$R_{CA} = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_{CA} = \frac{(2 \times 4) + (4 \times 30) + (2 \times 30)}{4}$$

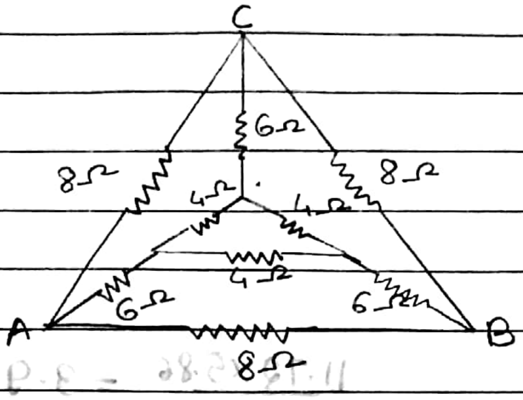
$$R_{CA} = \frac{8 + 120 + 60}{4} = 47\,\Omega$$



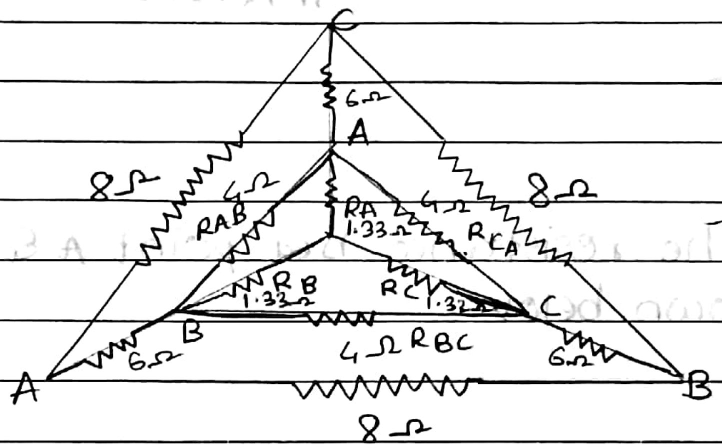
Monday
10-8-15

PAGE NO.
DATE: / /

3. In the given network shown below find the resistance betⁿ points A & B.



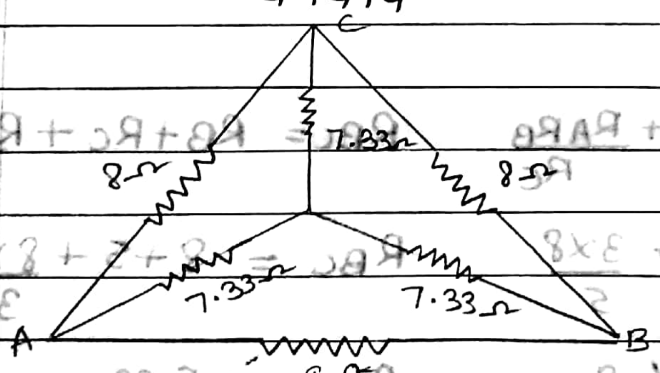
Solⁿ



$$R_A = \frac{4 \times 4}{4+4+4} = 1.33\,\Omega$$

$$R_B = \frac{4 \times 4}{4+4+4} = 1.33\,\Omega$$

$$R_C = \frac{4 \times 4}{4+4+4} = 1.33\,\Omega$$



$6 + 1.33 = 7.33\,\Omega$

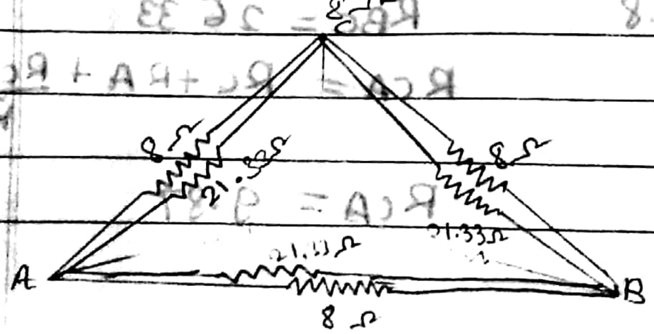
$6 + 1.33 = 7.33\,\Omega$

$6 + 1.33 = 7.33\,\Omega$

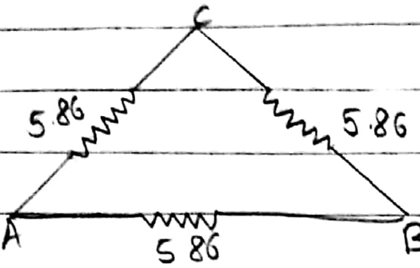
$$R_{AB} = 7.33 + 7.33 + \frac{(7.33 \times 7.33)}{7.33}$$

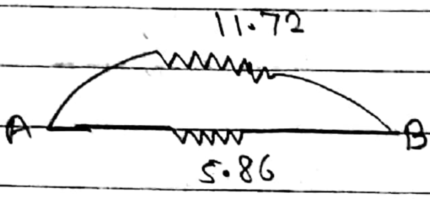
$$R_{AB} = 14.66 + 7.33$$

$$R_{AB} = 21.99 = R_{BC} = R_{CA}$$

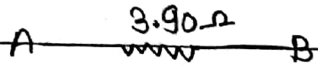


$$\frac{21.99 \times 8}{21.99+8} = 5.86\,\Omega$$

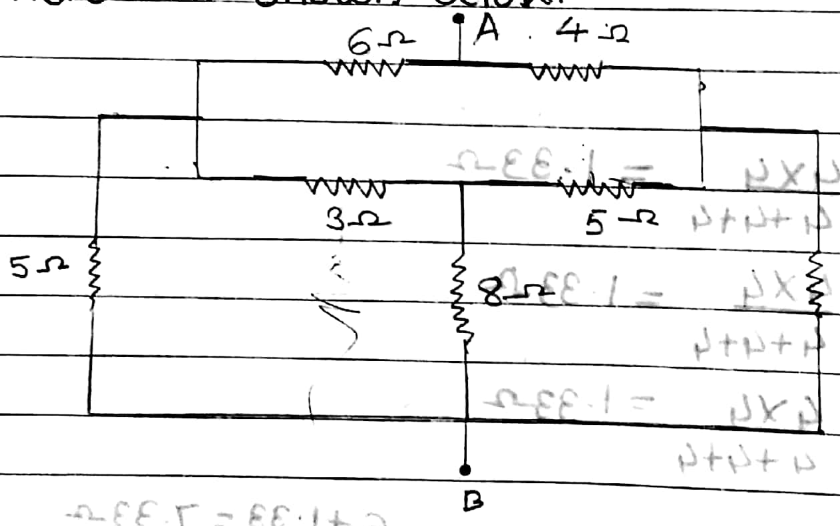


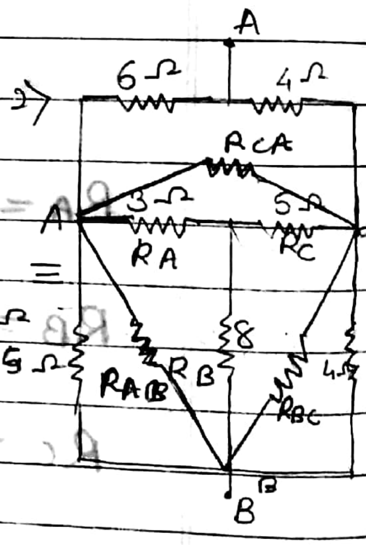


$$\frac{11.72 \times 5.86}{11.72+5.86} = 3.90\,\Omega$$



4 Determine the resistance betn point A & B in a network shown below.





$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$= 3 + 8 + \frac{3 \times 8}{5}$$

$$= 3 + 8 + 4.8$$

$$R_{AB} = 15.8$$

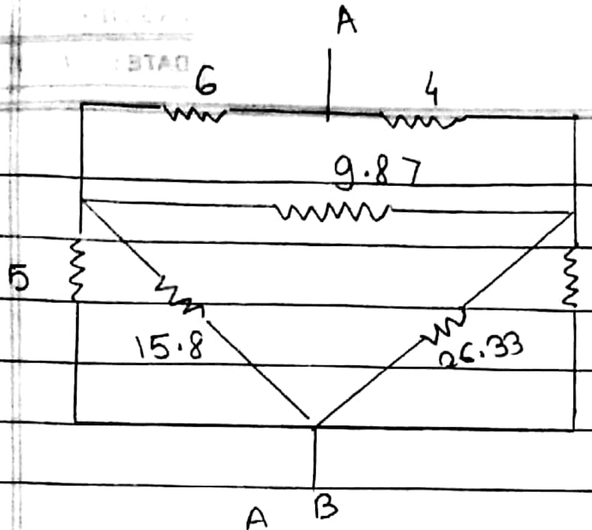
$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{BC} = 8 + 5 + \frac{8 \times 5}{3}$$

$$R_{BC} = 26.33$$

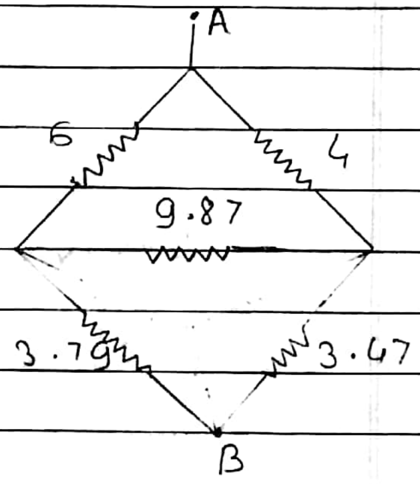
$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

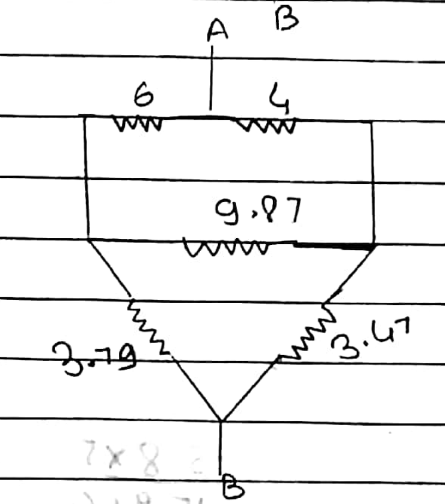
$$R_{CA} = 9.87$$

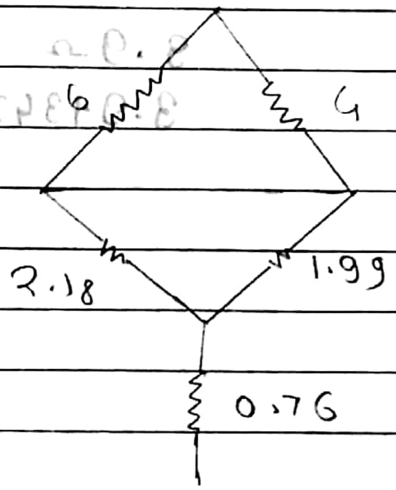


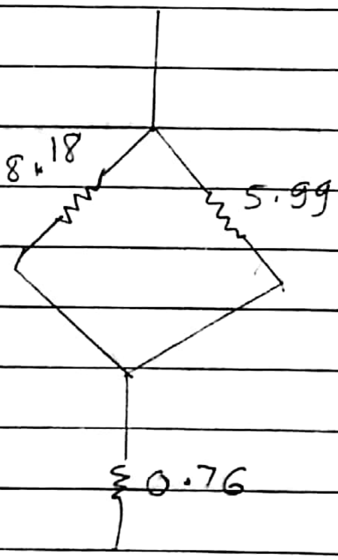
$$\frac{5\times15.8}{5+15.8} = 3.79$$

$$\frac{26.33\times4}{26.33+4} = 3.47$$









$$R_A = \frac{R_{AB}\cdot R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

$$= \frac{37.40}{17.13}$$

$R_A = 2.18\,\Omega$

$$R_C = \frac{R_{BC}\times R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

$$= \frac{34.24}{17.13}$$

$R_C = 1.99\,\Omega$

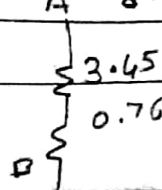
$$R_B = \frac{R_{BC}\times R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

$$= \frac{\cancel{34.24}\ 3.79\times3.47}{17.13}$$

$= 0.76$

8.18 & 5.99 are ||el

$$\frac{8.18\times5.99}{8.18+5.99} = 3.45$$

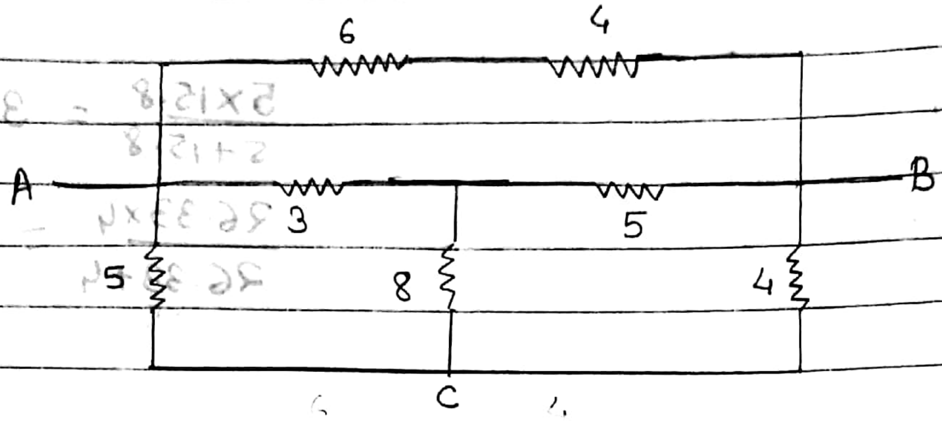


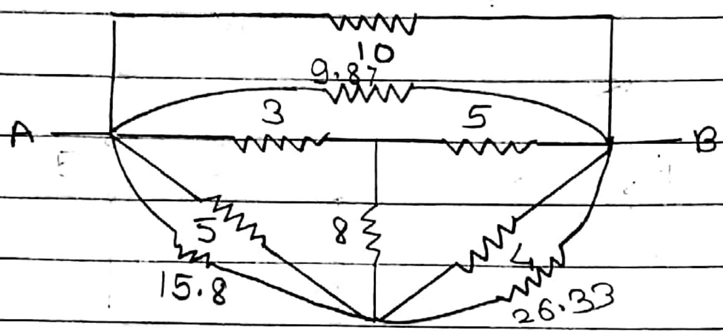
$AB = 3.45 + 0.76$

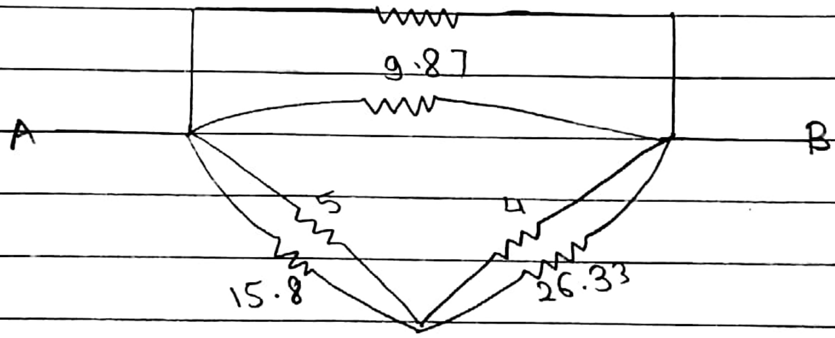
$AB = 4.21\,\Omega$

Tuesday
11.08.15

5.



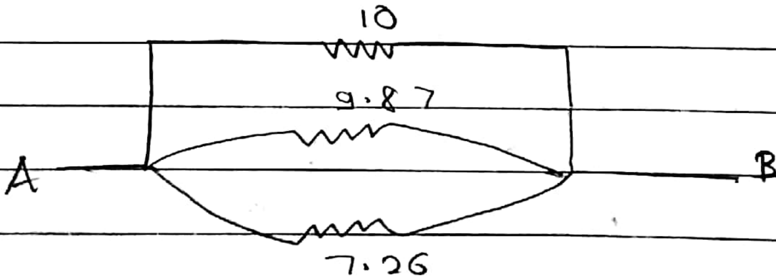


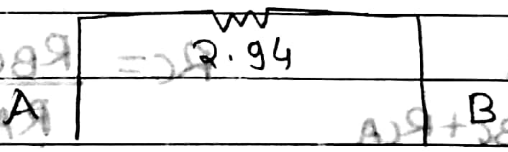


$$\frac{15.8 \times 5}{15.8+4} \quad \frac{26.33 \times 4}{26.33+4}$$

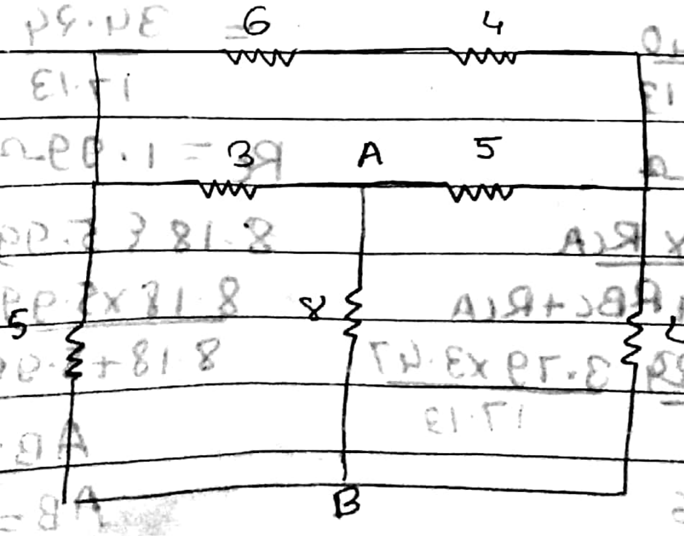
$3.9\,\Omega$ $3.47\,\Omega$

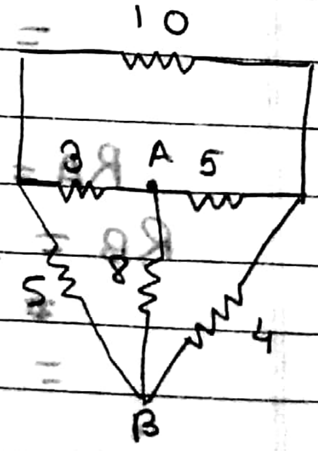
$3.9 + 3.47 = 7.3$

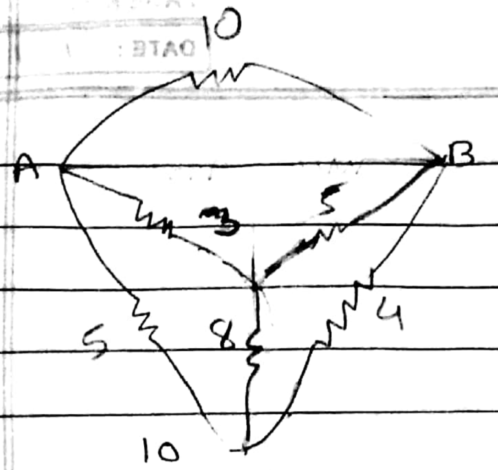




6.





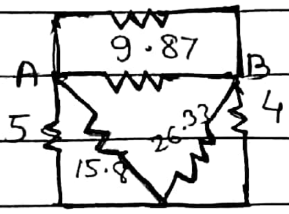


$$R_A = \frac{R_{AB} \cdot R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

$$R_{AB} = 3+5+\frac{3\times5}{8} = 9.87\,\Omega$$

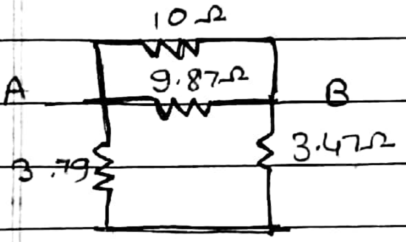
$$R_{BC} = 5+8+\frac{5\times8}{3} = 26.33\,\Omega$$

$$R_{AC} = 3+8+\frac{3\times8}{5} = 15.8\,\Omega$$



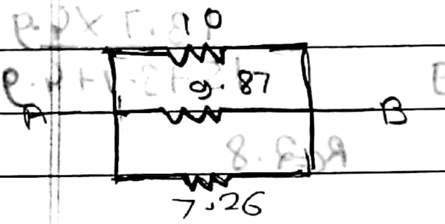
$$R_1 = 3.79\,\Omega$$

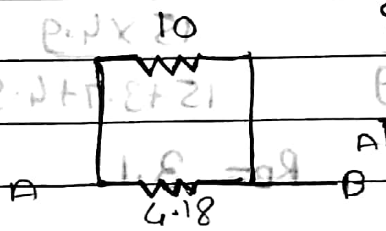
$$R_2 = \frac{4\times26.33}{4+26.33} = 3.472\,\Omega$$

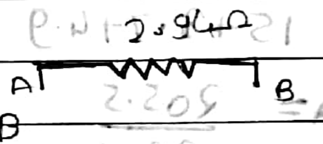


$$R = 3.79+3.47 = 7.26\,\Omega$$

$$R = \frac{9.87\times7.26}{9.87+7.26} = 4.18\,\Omega$$



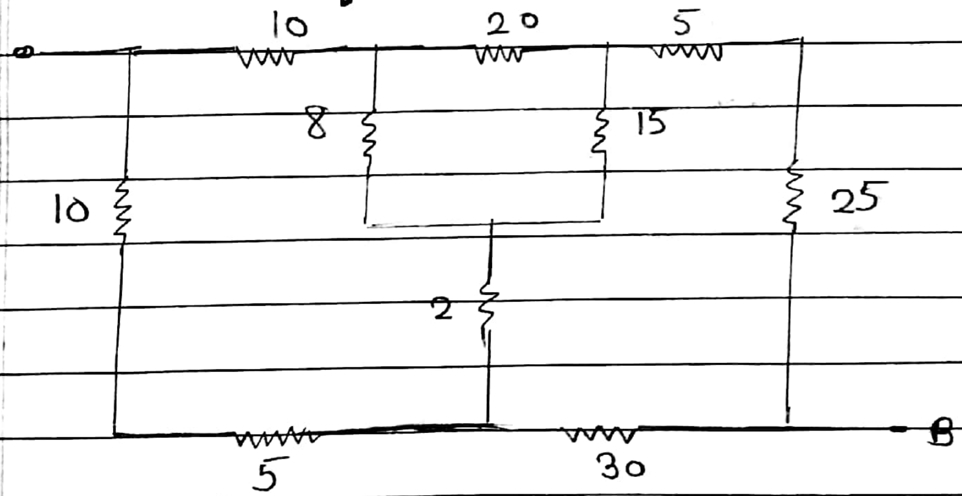


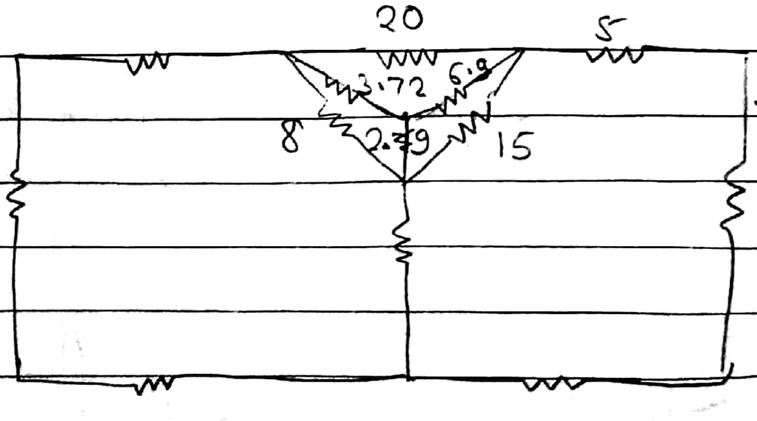


$$R = \frac{4.18\times10}{4.18+10}$$

$$R = 2.94\,\Omega$$

7. Find the equivalent resistance betⁿ A & B.

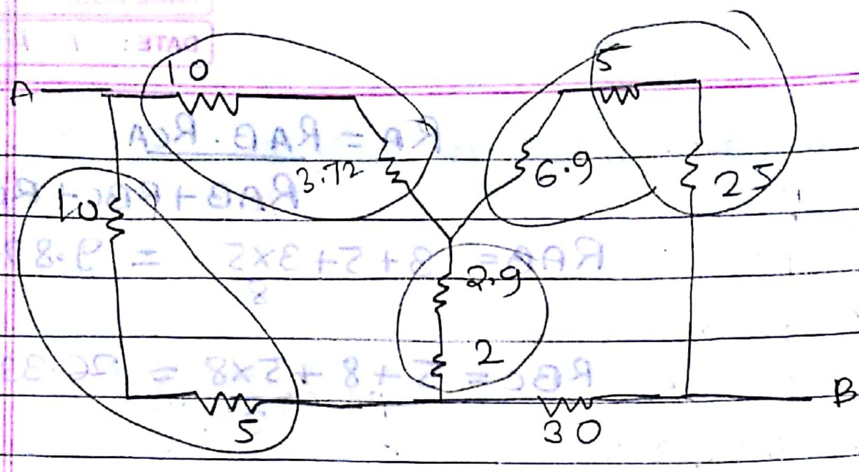


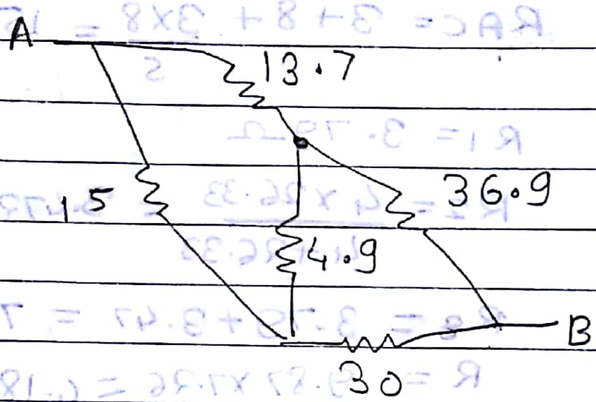


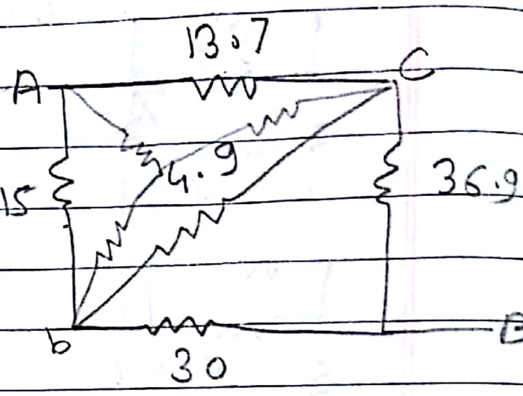
$$= \frac{20\times8}{15+8+20} = 3.72$$

$$= \frac{20\times15}{15+8+20} = 6.9$$

$$= \frac{8\times15}{15+8+20} = 2.79$$



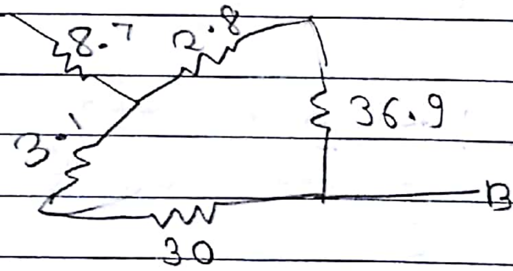


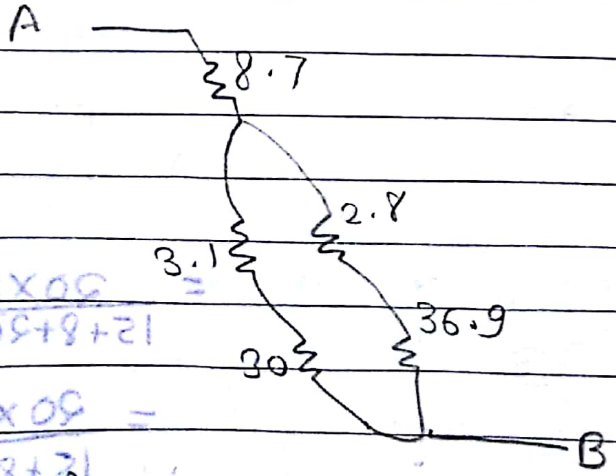


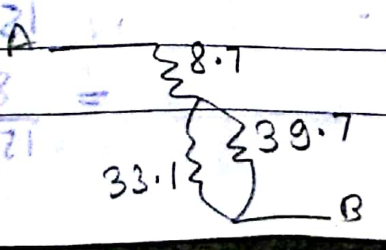
$$\frac{13.7 \times 15}{15+3.7+4.9} \qquad \frac{15 \times 4.9}{15+3.7+4.9} \qquad \frac{13.7 \times 4.9}{15+3.7+4.9}$$

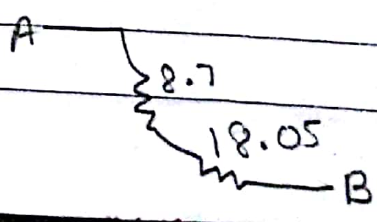
$R_A = \dfrac{205.5}{23.6}$ $\quad R_B = 3.1$ $\quad R_C = 2.8$

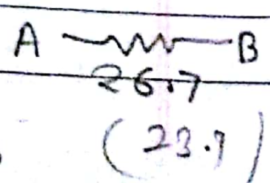
$R_A = 8.7$



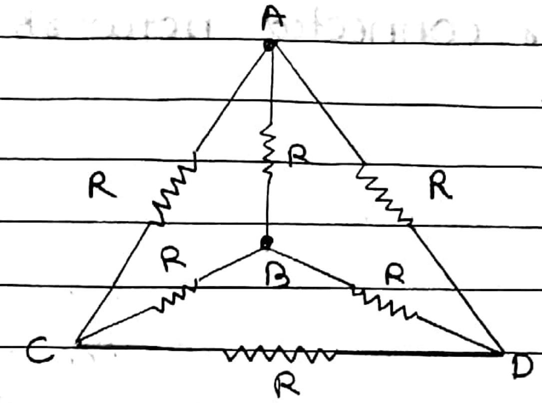


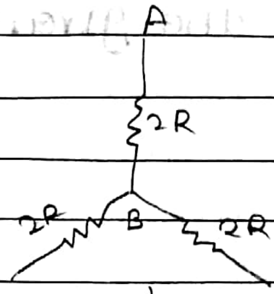




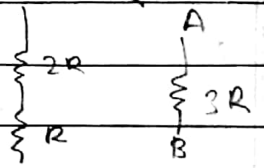


8. Find the resistance bet^n AB of the network.

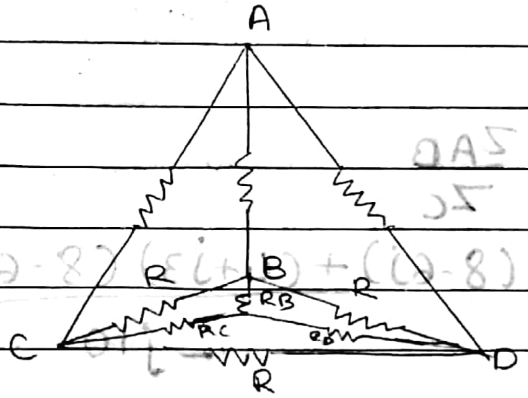




$\frac{2R\times 2R}{2R+2R} = \frac{4R^2}{4R}$

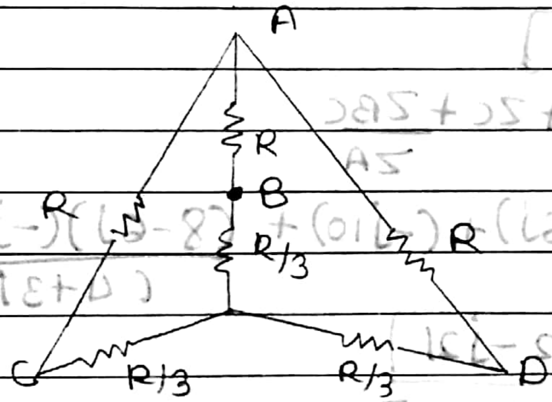


$\frac{2R\times R}{2R+R} = \frac{2R^2}{3R}$



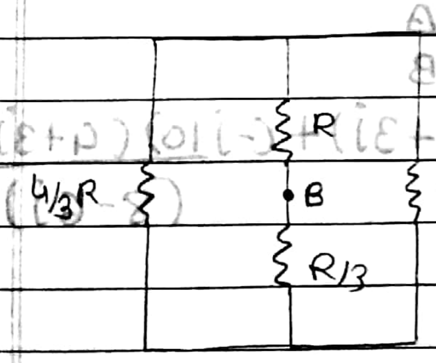
$R_B = \frac{R\times R}{R+R+R} = \frac{R^2}{3R} = \frac{R}{3}$.

$R_C = R/3$, $R_D = \frac{R}{3}$

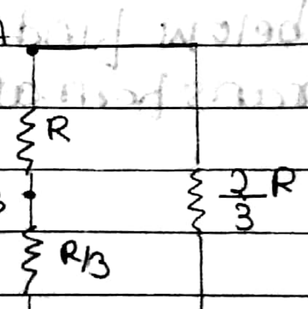


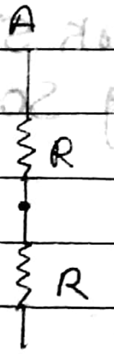
R & $\frac{R}{3}$ in series.

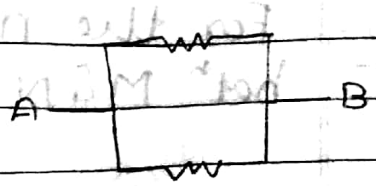
$R + \frac{R}{3} = \frac{4}{3}R$.



$\frac{4R}{3} \parallel \frac{4R}{3} \Rightarrow \frac{2}{3}R$.







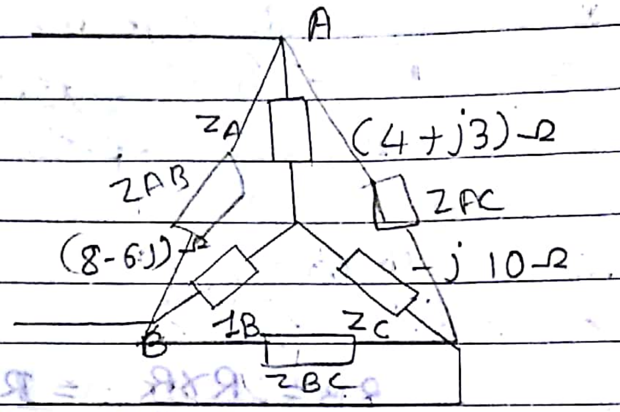
$\frac{R\times R}{R+R} = \frac{R^2}{2R}$

$\therefore R/2$

$\frac{2}{3}R$ & $\frac{R}{3}$ in series. $\Rightarrow \frac{2}{3}R + R/3 = R$

DATE:

9. Obtain the delta connected equivalent network for the given star connected network shown below



$$Z_{AB} = Z_A + Z_B + \frac{Z_A Z_B}{Z_C}$$

$$= (4+j3) + (8-6j) + \frac{(4+j3)(8-6j)}{-j10}$$

$$\boxed{Z_{AB} = 12 + 2j}$$

$$Z_{BC} = Z_B + Z_C + \frac{Z_B Z_C}{Z_A}$$

$$= (8-6j) + (-j10) + \frac{(8-6j)(-j10)}{(4+3j)}$$

$$\boxed{Z_{BC} = -11.2 - j21}$$

$$Z_{CA} = Z_C + Z_A + \frac{Z_C Z_A}{Z_B}$$

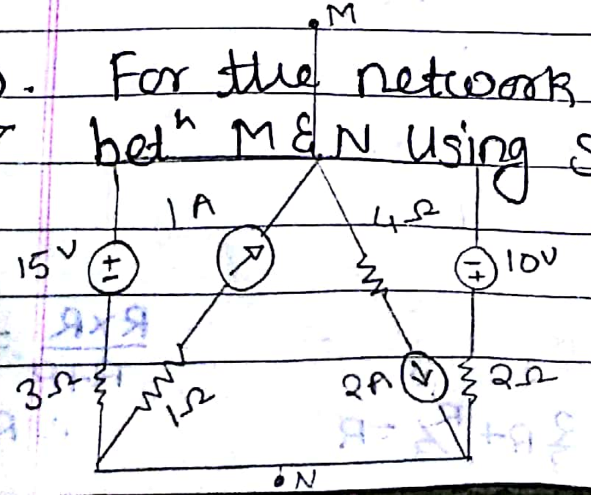
$$= (-j10) + (4+3j) + \frac{(-j10)(4+3j)}{(8-6j)}$$

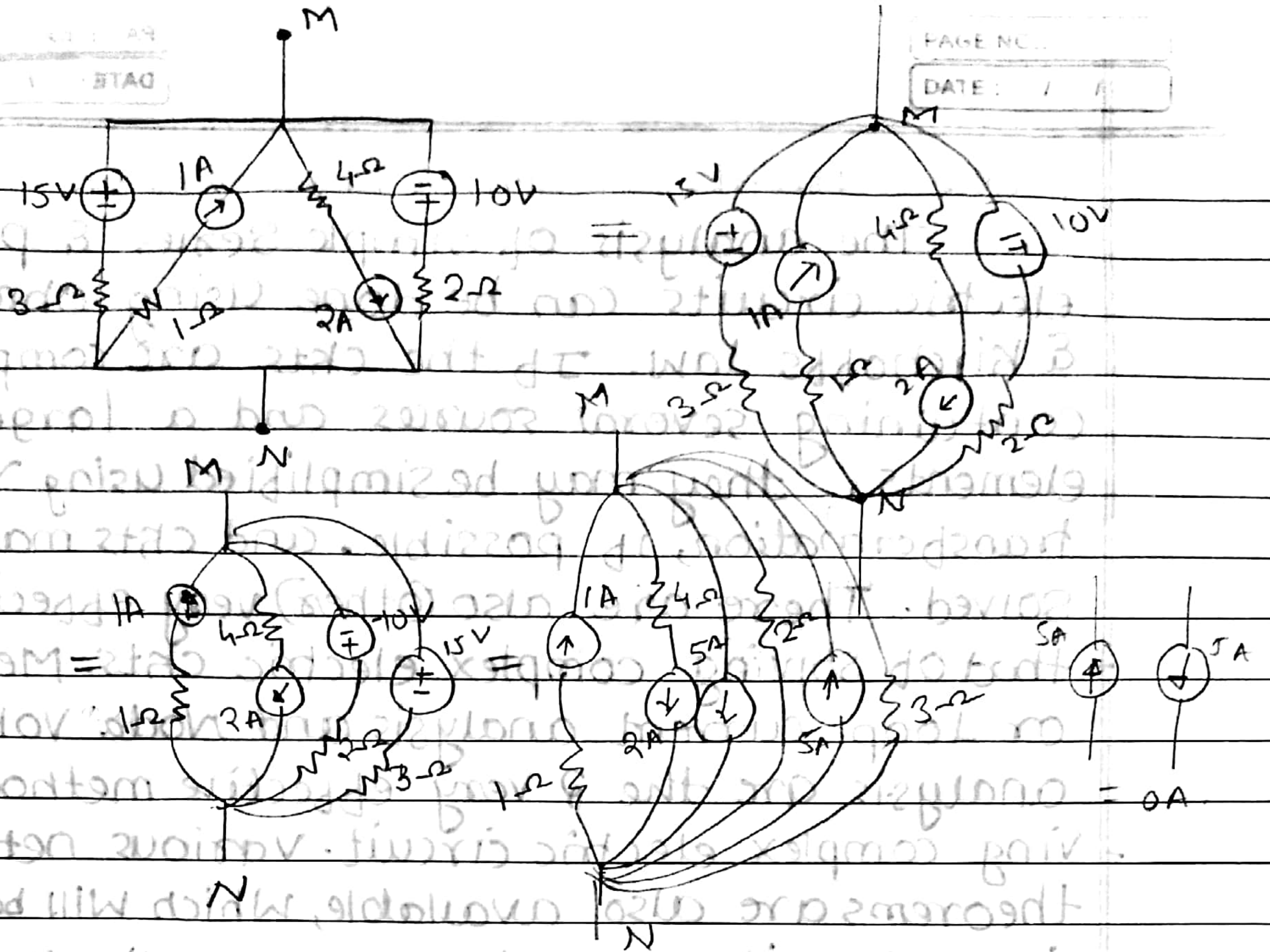
Thursday 13-8-15

$$\boxed{Z_{CA} = 8.8 - j1.4}$$

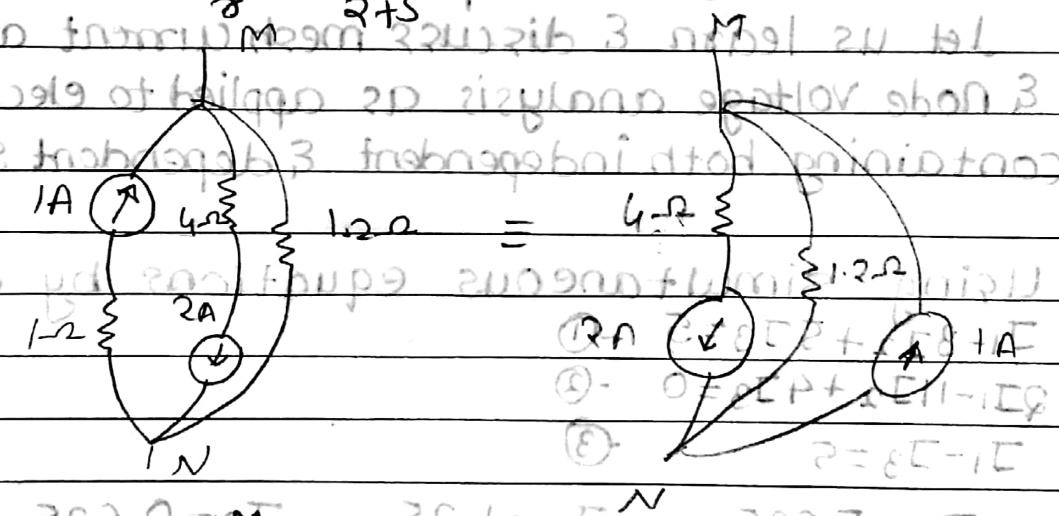
10. For the network shown below find the P.d. betn M & N using source transformation.

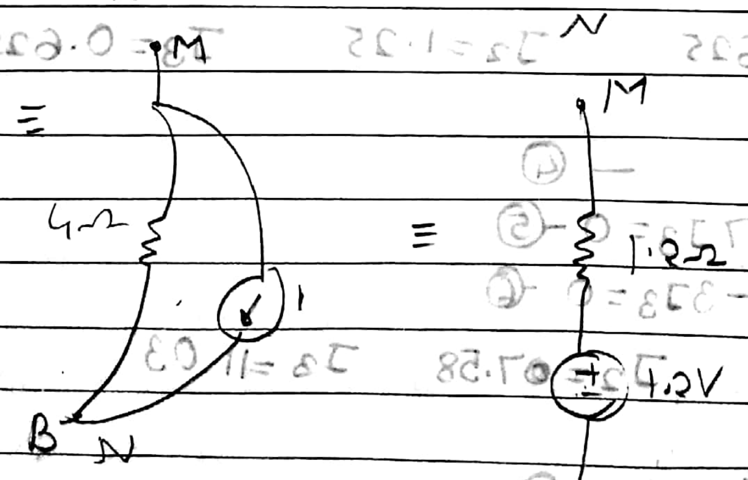
Dec-14





$$I = \frac{E}{r} = \frac{2\times 3}{2+5} = 1.2\Omega$$





When resistance is series with current source neglect resistance.

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* Mesh current Analysis & Node Voltage Analysis:-

The analysis of simple series & parallel electric circuits can be done using Ohms law & Kirchoffs law. If the ckts are complex, containing several sources and a large No. of elements, they may be simplified using Y-Δ transformation, if possible. and ckts may be solved. There are also (other) very effective method of solving complex electric ckts. Mesh current or loop current analysis and Node voltage analysis are the 2 very effective method of solving complex electric circuit. Various network theorems are also available, which will be discussed in next units. are also very effective alternate methods to solve complex electric circuits.

Let us learn & discuss mesh current analysis & node voltage analysis as applied to electric ckts containing both independent & dependent sources.

Monday 17-8-15

Using simultaneous equations by calculator

1) $I_1 - 3I_2 + 5I_3 = 5$ - (1)

$2I_1 - 11I_2 + 4I_3 = 0$ - (2)

$I_1 - I_3 = 5$ (3)

$\boxed{I_1 = 5.625}$ $\boxed{I_2 = 1.25}$ $\boxed{I_3 = 0.625}$

2) $I_1 = 10$ — (4)

$10I_1 - 3I_2 - 7I_3 = 0$ - (5)

$-2I_1 + 7I_2 - 3I_3 = 0$ - (6)

$\boxed{I_1 = 10}$ $\boxed{I_2 = 07.58}$ $\boxed{I_3 = 11.03}$

3) $3i_1 + i_2 - 2i_3 = 1$ - (7)

$i_1 + 6i_2 + 3i_3 = 0$ - (8)

$-2i_1 + 3i_2 + 6i_3 = 6$ - (9) $\boxed{I_1 = 3}$ $\boxed{I_2 = -2}$ $\boxed{I_3 = 3}$

4) $-i_1-4i_2+4i_3=7$ — (10)
$i_1+6i_2-3i_3=0$ — (11)
$i_1+i_3=-7$ — (12)
$\boxed{i_1=-9}$, $\boxed{i_2=2.5}$, $\boxed{i_3=2}$

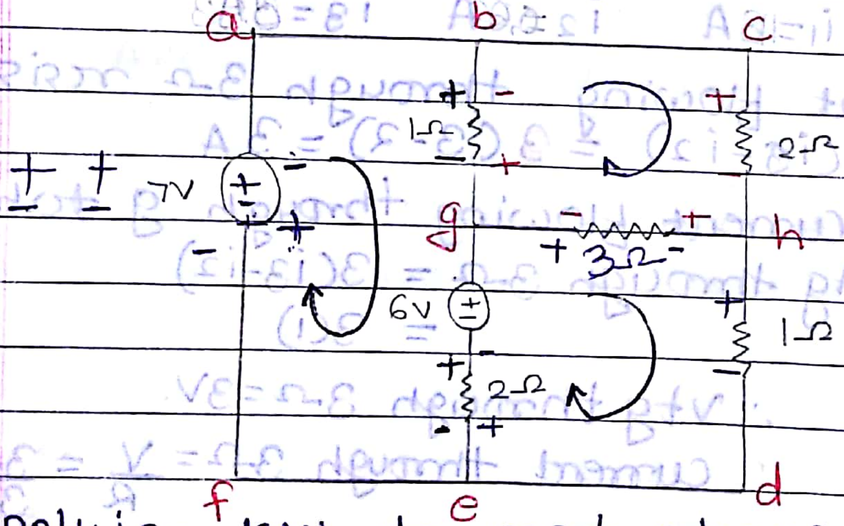
5) $-6V_1+V_2+5V_3=17$ — (13)
$V_1-6V_2+2V_3=-3$ — (14)
$5V_1+2V_2-11V_3=-25$ — (15)
$\boxed{V_1=-1}$, $\boxed{V_2=1}$, $\boxed{V_3=2}$

6) $-7V_1+4V_3=11$ — (16)
$6V_2-5V_3=-17$ — (17)
$V_3=22$ — (18)
$\boxed{V_1=11}$ $\boxed{V_2=15.5}$ $\boxed{V_3=22}$

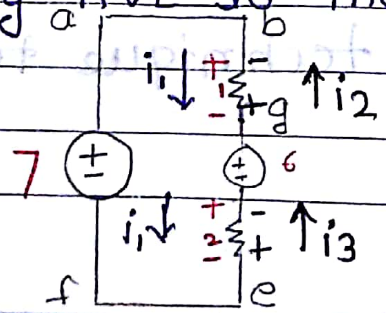
* Mesh analysis problems:- Applying KVL & writing KVL equations
or (loop analysis problems)

Type-1 problems

1. Use mesh analysis to find current flowing through 3Ω resistor in the following ckt.

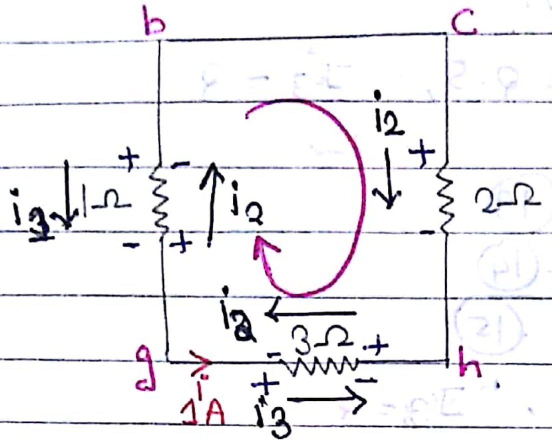


Applying KVL to mesh abgef



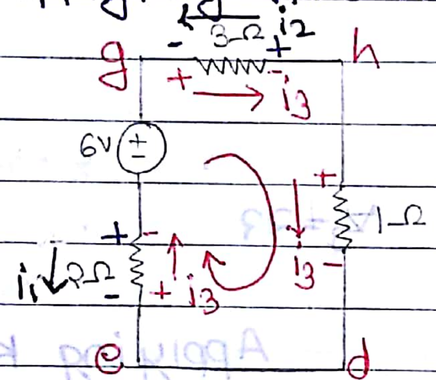
$-1(i_1-i_2)-6-2(i_1-i_3)+7=0$
$\Rightarrow -i_1+i_2-6-2i_1+2i_3+7=0$
$\Rightarrow -3i_1+i_2+2i_3+1=0$
$\Rightarrow \boxed{3i_1-i_2-2i_3=1}$ — (1)

Applying KVL to mesh bchg



$$-2i_2 - 3(i_2 - i_3) - 1(i_2 - i_1) = 0$$
$$-2i_2 - 3i_2 + 3i_3 - i_2 + i_1 = 0$$
$$\boxed{i_1 - 6i_2 + 3i_3 = 0} \quad -(2)$$

Applying KVL to mesh ghde



$$-3(i_3 - i_2) - 1(i_3) - 2(i_3 - i_1) + 6 = 0$$
$$-3i_3 + 3i_2 - i_3 - 2i_3 + 2i_1 + 6 = 0$$
$$\boxed{+2i_1 + 3i_2 - 6i_3 = -6} \quad -(3)$$

Solving 3 simultaneous eq^n

$$3i_1 - i_2 - 2i_3 = 1$$
$$i_1 - 6i_2 + 3i_3 = 0$$
$$+2i_1 + 3i_2 - 6i_3 = -6$$

$\therefore$ $\boxed{i_1 = 3A}$ $\boxed{i_2 \; 2A}$ $\boxed{i_3 = 3A}$

{ current flowing through 3Ω resistance
$3(i_3 - i_2) = 3(3-2) = 3A$

1A current flowing through g to h.

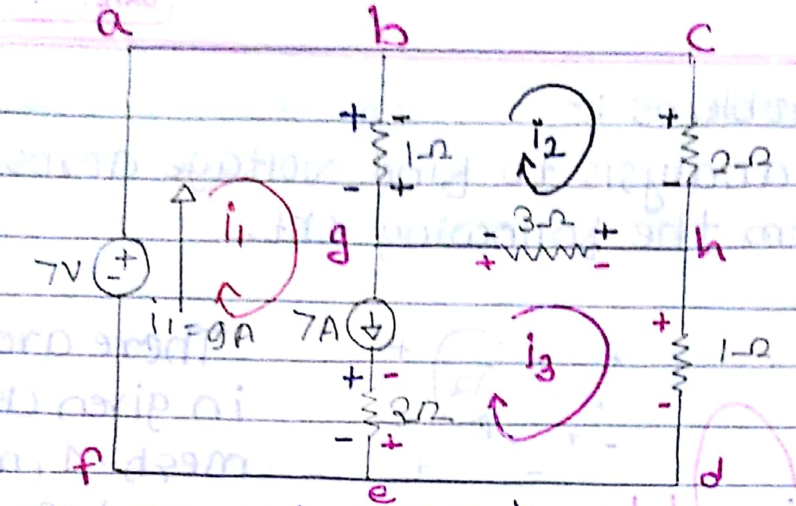
Vtg through $3\Omega = 3(i_3 - i_2)$
$= 3(1)$

$\therefore$ Vtg through $3\Omega = 3V$.

$\therefore$ current through $3\Omega = \frac{V}{R} = \frac{3V}{3\Omega} = 1A$.

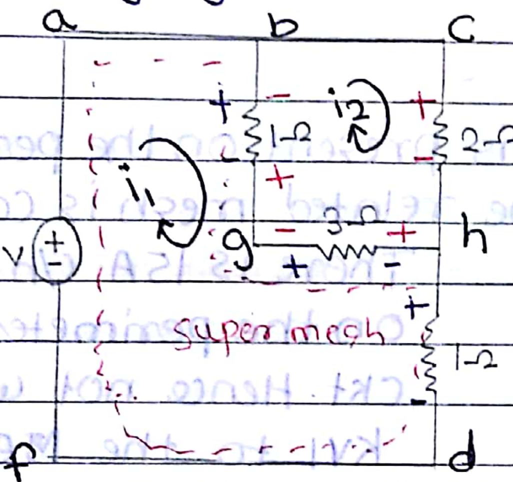
TYPE-2 Problems.

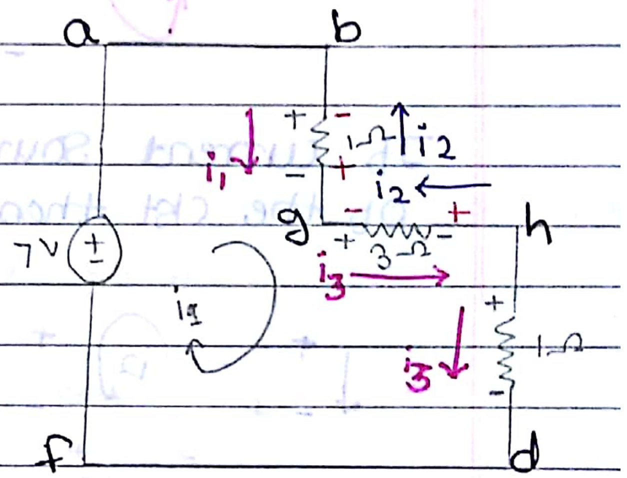
2. Use mesh analysing technique to find current in 7V voltage source.



Solⁿ: If current source is common with any 2 meshes remove current & ~~vtg~~ Resistance source & form supermesh.

Applying KVL for super mesh



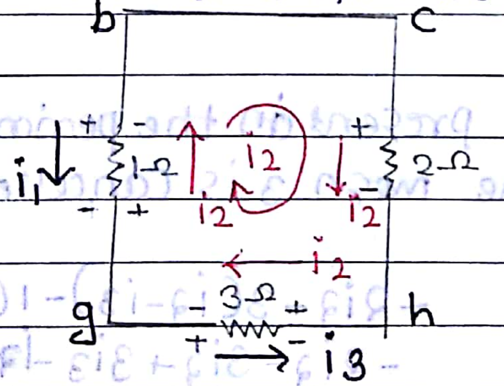


$$-1(i_1-i_2)-3(i_3-i_2)-1(i_3)+7=0$$

$$-i_1+i_2-3i_3+3i_2-i_3+7=0$$

$$\boxed{-i_1+4i_2-4i_3=-7} \quad (1)$$

Applying KVL to mesh-2

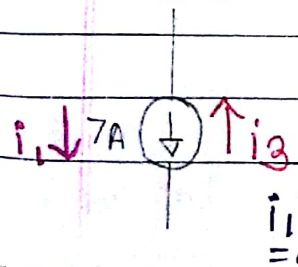


$$-2(i_2)-3(i_2-i_3)-1(i_2-i_1)=0$$

$$-2i_2-3i_2+3i_3-i_2+i_1=0$$

$$\boxed{i_1-6i_2+3i_3=0} - (2)$$

Writing eqⁿ for 7A current source



$$\boxed{i_1-i_3=7} - (3)$$

$$\boxed{i_1=9A} \quad \boxed{i_2=2.5A} \quad \boxed{i_3=2A}$$

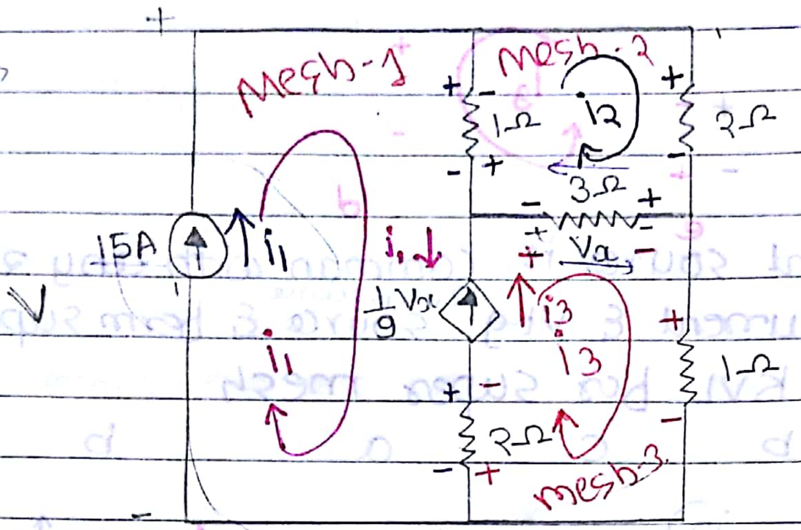
i_1 is current flowing from f to g in 7V vtg. source. $=9A$.

Date
18-8-15

Type-3 problems :-

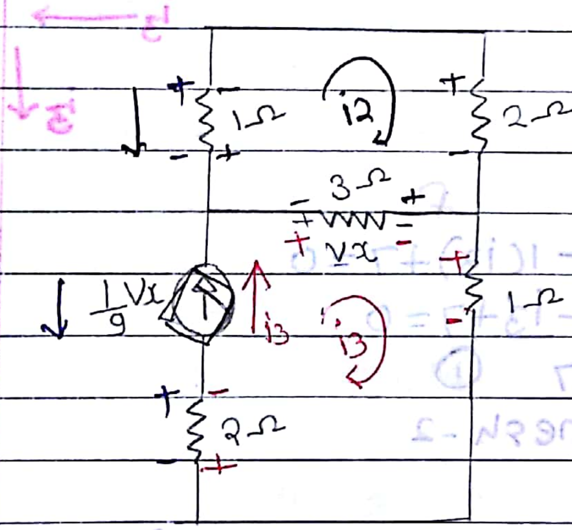
3. Use mesh analysis to find voltage across 15A source from the following ckt.

mixture of types 2, 3



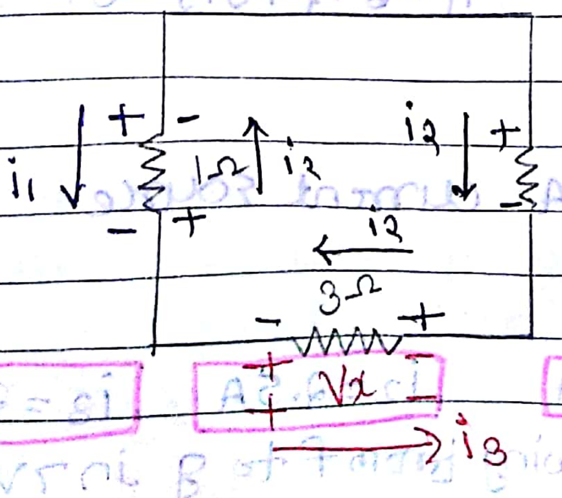
There are 3 meshes in given ckt. mesh-1, mesh-2, mesh-3.

If current source is present on the perimeter of the ckt then the related mesh is cancelled.



There is 15A c/n source on the perimeter of given ckt. Hence not writing KVL to the Mesh-1. Not writing KVL to mesh 3 but writing current eqⁿ

Here current source is present on the perimeter of the ckt. hence the mesh 3 is cancelled.



$$-2i_2-3(i_2-i_3)-1(i_2-i_1)=0$$

$$-2i_2-3i_2+3i_3-i_2+i_1=0$$

$$\boxed{i_1-6i_2+3i_3=0} \quad -(1)$$

Writing eq^n for 15A c/n source

$$\boxed{i_1 = 15} \quad -(2) \qquad I_1 + 0 + 0 = 15$$

writing eq^n $\frac{1}{9}V_x$ c/n source.

$$(i_3 - i_1) = \frac{1}{9} V_x \qquad V = IR$$

$$= \frac{1}{9} \times 3 (i_3 - i_2) \qquad V_x = (i_3 - i_2)R$$

$$= \frac{}{3} \qquad V_x = (i_3 - i_2)3$$

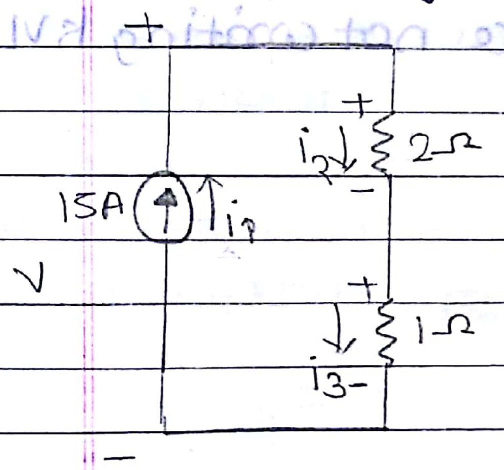
$$i_3 - i_1 = \frac{1}{3} i_3 - \frac{1}{3} i_2$$

$$i_3 - \frac{1}{3} i_3 - i_1 + \frac{1}{3} i_2 = 0$$

$$\boxed{-i_1 + \frac{1}{3} i_2 + \frac{2}{3} i_3 = 0} \quad -(3)$$

$$\therefore \boxed{i_1 = 15A} \quad \boxed{i_2 = 11A} \quad \boxed{i_3 = 17A}$$

For finding V consider a closed path where single current can flow.



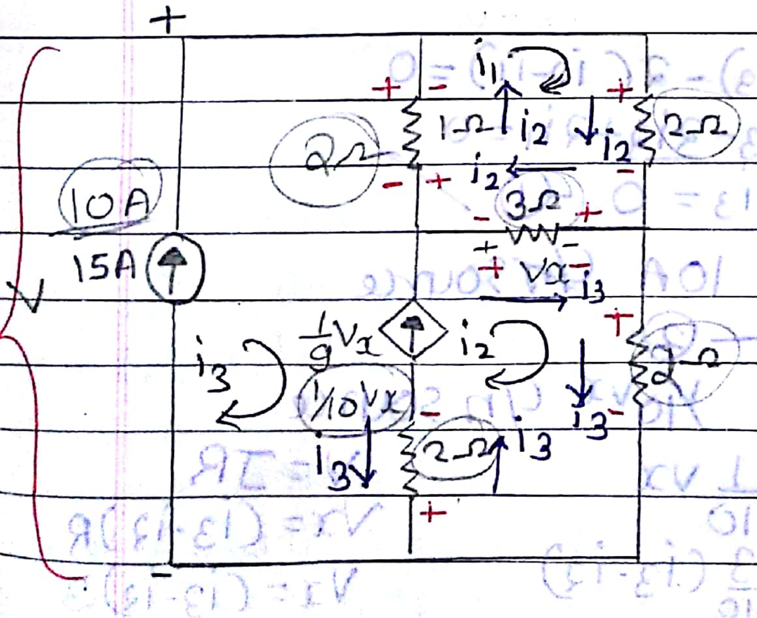
$$V - 2i_2 - i_3 = 0$$

$$\therefore 2i_2 + i_3 = V$$

$$\therefore (2 \times 11) + 17 = V$$

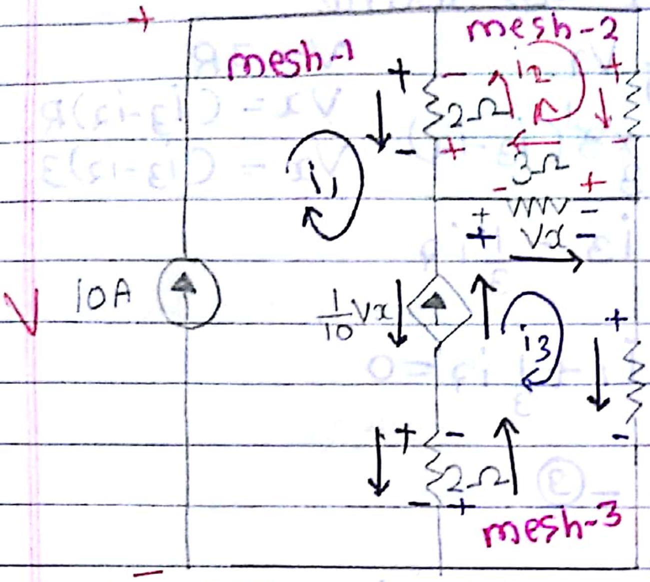
$$\therefore V = 22 + 17$$

$$\boxed{\therefore V = 39V}$$

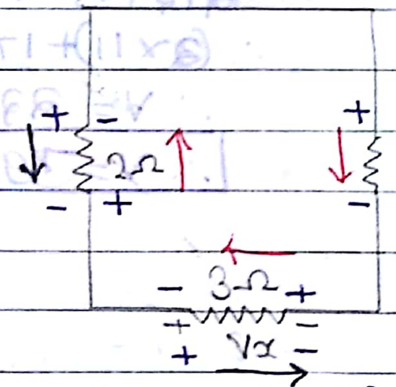


There is 15A current source on the perimeter of given ckt. Hence not writing KVL to the mesh-1.
& $\frac{1}{9}V_x$ is current source is

4. Use mesh analysis to find vtg. across 10A source from the following ckt.



There is 10A C/n source on the perimeter of given ckt. Hence not writing KVL to mesh-1
There is $\frac{1}{10}$ vx C/n source on the perimeter of mesh 3. hence not writing KVL to mesh.3.



$$-2i_2-3(i_2-i_3)-2(i_2-i_1)=0$$
$$-2i_2-3i_2+3i_3-2i_2+2i_1=0$$
$$\boxed{2i_1-7i_2+3i_3=0} \quad (1)$$

Writing eqn for 10A C/n source

$$\boxed{i_1=10A} \quad (2)$$

writing eqn for $\frac{1}{10}vx$ C/n source

$$i_3-i_1=\frac{1}{10}vx$$
$$i_3-i_1=\frac{3}{10}(i_3-i_2)$$

$V=IR$

$Vx=(i_3-i_2)R$

$Vx=(i_3-i_2)3$

$i_3 - i_1 = \frac{3}{10} i_3 - \frac{3}{10} i_2$

$i_3 - \frac{3}{10} i_3 - i_1 + \frac{3}{10} i_2 = 0$

$\boxed{-i_1 + \frac{3}{10} i_2 + \frac{7}{10} i_3 = 0 \quad -③}$ $\Rightarrow$ $\boxed{-i_1 + 0.3 i_2 + 0.7 i_3 = 0}$ $-②$

$\boxed{i_1 = 10A}$ $\boxed{i_2 = 7.5A}$ $\boxed{i_3 = 11.03A}$

For finding V consider a closed path where single current can flow.

$V - 2i_2 - 2i_3 = 0$

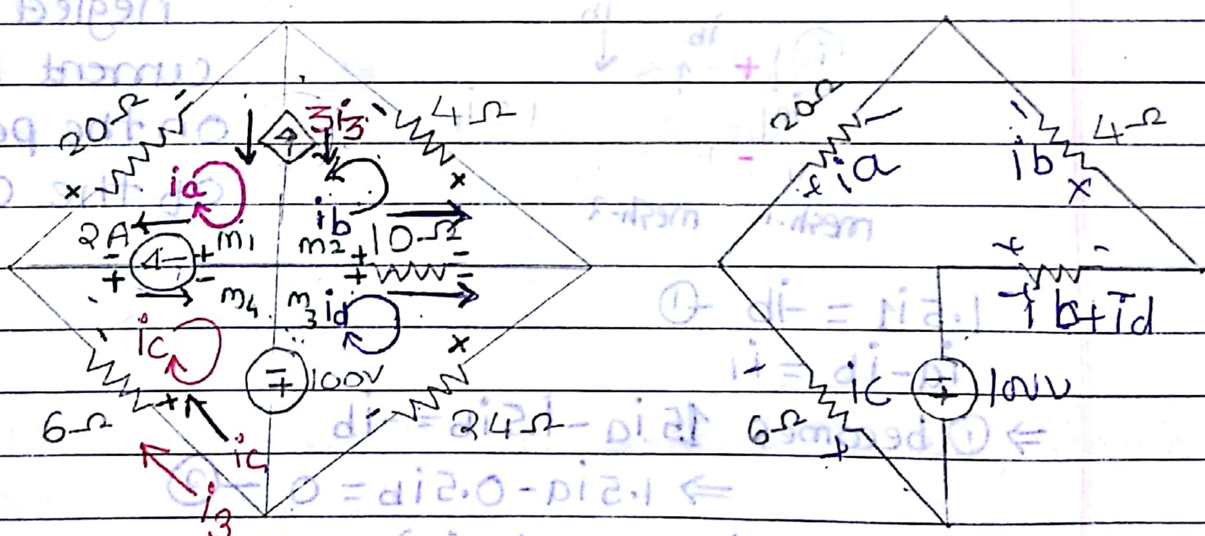
$\therefore 2i_2 + 2i_3 = V$

$\therefore 2(7.5) + 2(11.03) = V$

$15 + 22.06 = V$

$\boxed{\therefore V = 37.06V}$

5 Use mesh analysis to find the current through 4Ω resistance in following ckt.



$3i_3 = 3i_c$ In betn m_1 & m_4 common current

$i_a - i_c = 2 \quad -①$ 2A is there so remove it.

$i_a + i_b = -3i_c$ 11^{rly} in betn m_1 & m_2 $3i_3$ A is

$i_a + i_b + 3i_c = 0 \quad -②$ current source hence remove it.

$-20i_a + 4i_b + 10i_b - 6i_c + 10i_d = -100$

$-20i_a + 14i_b - 6i_c + 10i_d = -100$

$ia - ic = 2$ —① $\Rightarrow$ $ia = 2 + ic$ —④

$ia + ib + 3ic = 0$ —②

$-20ia + 14ib - 6ic + 10id = -100$ —③

2-12A
-2-48A
0-12 06A
-2.211A

② $\Rightarrow 2 + ic + ib + 3ic = 0$

$\Rightarrow ib + 4ic = -2$ —⑤

③ $\Rightarrow -20(2 + ic) + 14ib - 6ic + 10id = -100$

$-40 - 20ic + 14ib - 6ic + 10id = -100 + 40$

$14ib - 26ic + 10id = -60$ —⑥ $-100+40$

$-10(ib + id) - 24id = 100$

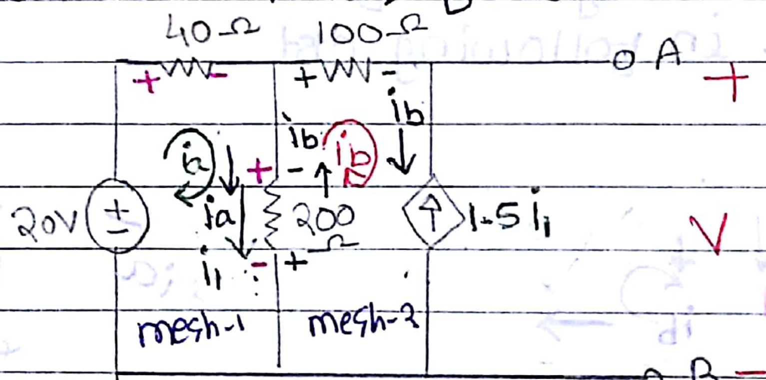
$-10ib - 34id = 100$ —⑦

Solving 5, 6, 7, $\boxed{ib = -2.48A}$ $ia = 2 + 0.12$

$\boxed{ic = 0.12A}$ $\boxed{ia = 2.12A}$

$\boxed{id = -2.2A}$

6 Use mesh analysis to find vtg across the terminals A & B for the ckt given below.



neglect the 1.5i1 current bcoz it is on the perimeter of the ckt

$1.5 i_1 = -ib$ —①

$ia - ib = i_1$

$\Rightarrow$ ① becomes $1.5 ia - 1.5 ib = -ib$

$\Rightarrow 1.5ia - 0.5ib = 0$ —②

$20 - 40ia - 200(ia - ib) = 0$

$20 - 40ia - 200ia + 200ib = 0$

$-240ia + 200ib = -20$ —③

$\boxed{ia = -0.05A}$

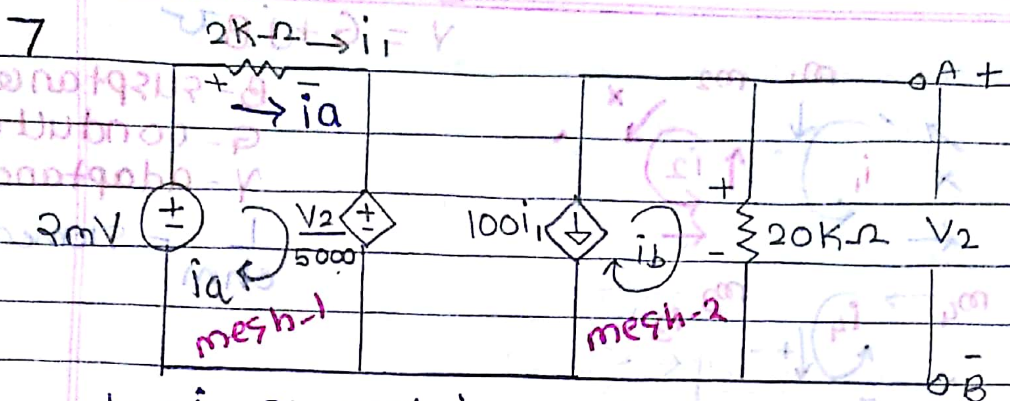
$\boxed{ib = -0.166A}$

$+20 - 40ia - 100ib - V = 0$

$20 - 40(-0.05) - 100(-0.166) = V$

$\Rightarrow 20 + 2 + 16.6 = V$ $\Rightarrow \boxed{V = 38.6V}$

7



$100i_1$ current is on the perimeter of the ckt. Hence it is neglecting. & writing eqⁿ for $100i_1$.

i.e $i_b = -100i_1$ —①

but for mesh① $i_a = i_1$

∴ ① ⇒ $i_b = -100i_a$

⇒ $100i_a + i_b = 0$ —② ⇒ $100i_a - 100i_a = 0$ ⇒

$-2000i_a - \frac{V_2}{5000} + 20\times10^{-3} = 0$

$-2000i_a - \frac{V_2}{5000} + 0.02 = 0$

$-2000i_a - \frac{V_2}{5000} = -0.02$

$-\frac{V_2}{5000} = -0.02$

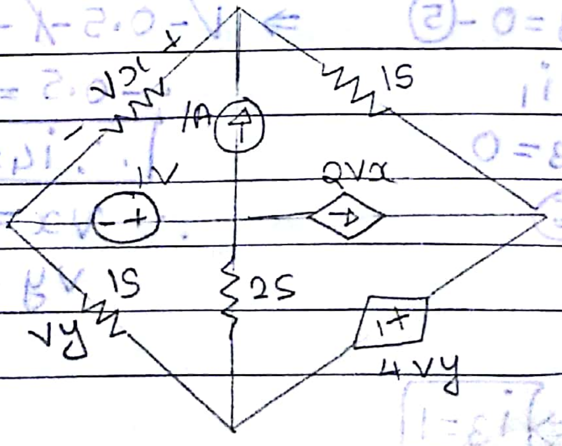
∴ $-V_2 = 5000\times(-0.02)$

$-V_2 = -100$

$\boxed{\therefore V_2 = 100V}$

Date 20-8-15

7. Use mesh analysis to find V_x & V_y in the ckt shown below.



$\frac{1}{Z} = Y$ mho or seimen

$Z = R + jX\ \Omega$

$Y = G + jB\ ℧$

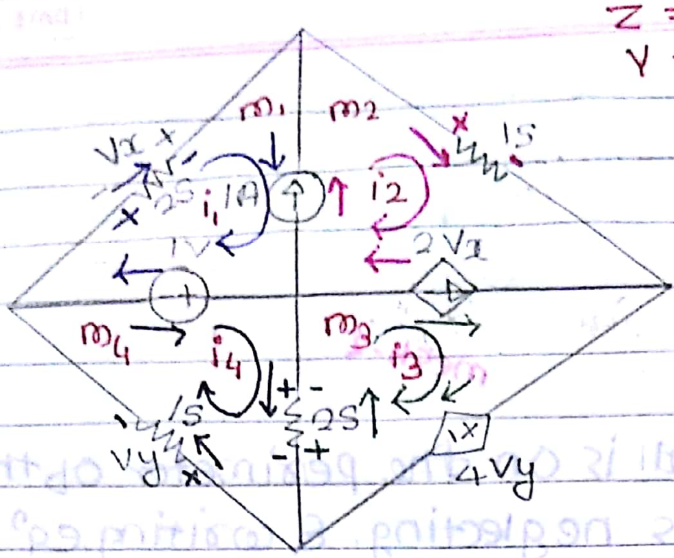
B - susptance

G - conductance

Y - adaptance

$\frac{1}{ohm}$ = siemen

Sol^n



1A current source is common in bet^n m1 & m2 and 2Vx current source is common in bet^n m2 & m3 hence neglect the 1A & 2Vx & writing eq^n for 1A & 2Vx.

Converting siemen into ohm

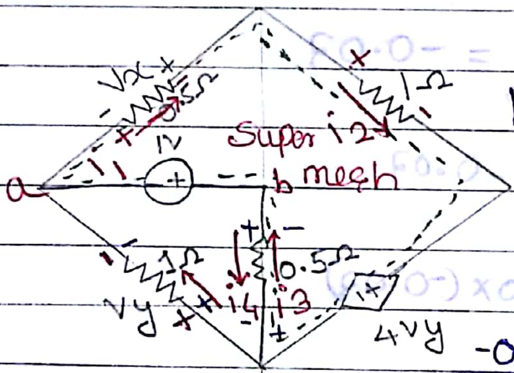
i.e $1S = 1/1 = 1\Omega$

$2S = 1/2 = 0.5\Omega$

$i_2 - i_1 = 1A$ — (1)

$i_3 - i_2 = 2Vx$ — (2)

but $Vx = IR = -0.5I_1$ — (3)



Applying KVL to supermesh

$-i_2 - 4Vy - 0.5(i_3 - i_4) - 1 - 0.5i_1 = 0$

$-i_2 - 4i_4 - 0.5i_3 + 0.5i_4 - 1 - 0.5i_1 = 0$

$-0.5i_1 - i_2 - 0.5i_3 - 3.5i_4 = 1$ — (4)

Applying KVL to abca

$1 - 0.5(i_4 - i_3) - i_1 = 0$

$1 - 0.5i_4 + 0.5i_3 - i_1 = 0$

$i_1 - 0.5i_3 + 0.5i_4 = 1$ — (7)

$\Rightarrow \cancel{1} - 0.5 - \cancel{1} + 0.5i_4 = 0$

$\therefore -0.5 = -0.5i_4$

$\boxed{\therefore i_4 = 1}$

$\therefore Vx = -0.5V$

$Vy = 1V$

$i_2 - i_1 = 1A$

$i_3 - i_2 = 2Vx$

but $Vx = -0.5I_1$

$\therefore i_3 - i_2 = (0.5I_1 \times 2)$

$\therefore i_1 - i_2 + i_3 = 0$ — (5)

from (1) $i_2 = 1 + i_1$

(5) $\Rightarrow \cancel{i_1} - 1 - \cancel{i_1} + i_3 = 0$

$\boxed{i_3 = 1}$ — (6)

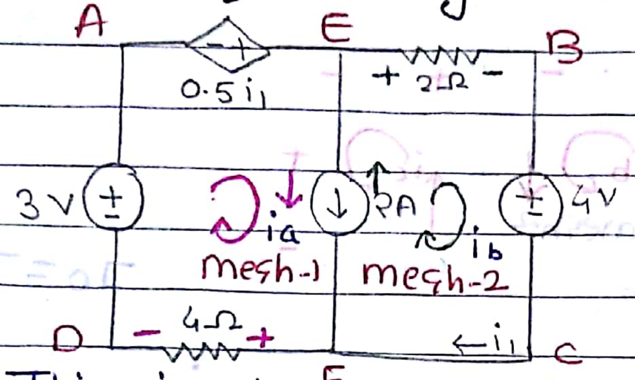
$\therefore i_2 = 1 + 1$

$\therefore \boxed{i_2 = 2}$

(5) $\Rightarrow 1 - 2 = -i_3 \Rightarrow \boxed{i_3 = 1}$

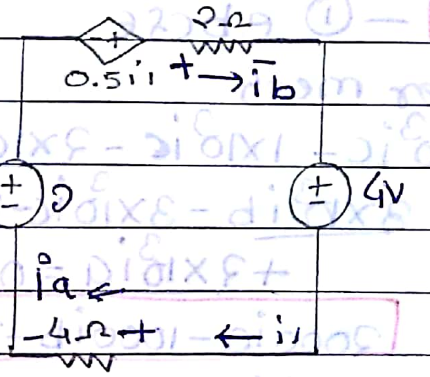
60.G.R

8. For the following ckt determine current i_1.



Here $i_1 = i_b$

This is type-2 problem. neglect 2A current as it is common for both the meshes & writing eq^n for current source i.e $\boxed{i_a - i_b = 2}$ —①



Writing KVL eq^n for supermesh

$0.5i_1 - 2i_b - 4 - 4i_a + 3 = 0$

Here $i_1 = i_b$

$\therefore 0.5i_b - 2i_b - 4i_a - 1 = 0$

$-1.5i_b - 4i_a - 1 = 0$

$\therefore \boxed{-4i_a - 1.5i_b = 1}$ —②

$\boxed{i_a = 0.36A}$

$\boxed{i_b = -1.63A}$

Solving ① & ② by cramers rule

$\therefore$ current through $i_1 = i_b = -1.63A$.

$$\Delta = \begin{bmatrix} 1 & -1 \\ 4 & 1.5 \end{bmatrix}$$

replacing R_1 by d_1 & d_2

$$\Delta_1 = \begin{bmatrix} 2 & -1 \\ -1 & 1.5 \end{bmatrix} \qquad \Delta_2 = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix}$$

$$i_a = \frac{\Delta_1}{\Delta} = \begin{bmatrix} 2 & -1 \\ -1 & 1.5 \end{bmatrix} \div \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix}$$

$$= (3-1) \div (1.5+4)$$

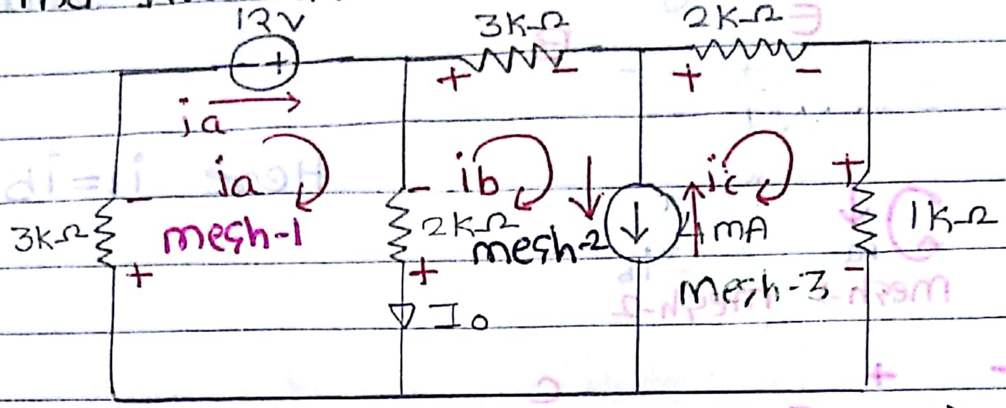
$\therefore \boxed{i_a = 0.36A}$

$$i_b = \frac{\Delta_2}{\Delta} = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix} \div \begin{bmatrix} 1 & -1 \\ 4 & 1.5 \end{bmatrix}$$

$\therefore \boxed{i_b = -1.63A}$

9. Find the current I_o in a network shown below.

R.



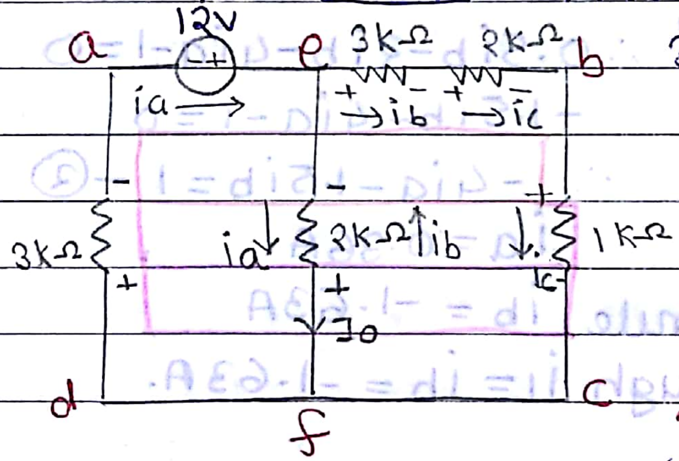
$I_o = I_b$

In the given n/w 4mA current is common bet mesh 2 & mesh3 Hence it is neglecting & writing eqn for 4mA current source.

i.e $\boxed{i_b - i_c = 4mA}$ — ① ebcfe

Writing KVL for super mesh

$12 - 3\times10^3 i_b - 2\times10^3 i_c - 1\times10^3 i_c - 2\times10^3(i_b - i_a) = 0$



$3\times10^3 i_b - 3\times10^3 i_c - 2\times10^3 i_b + 2\times10^3 i_a = 0$

$\boxed{2000 i_a - 1000 i_b - 3000 i_c = 0}$ — ②

Writing KVL to aefda

$12 - 2000(i_a - i_b) - 3000 i_a = 0$

$12 - 2000 i_a + 2000 i_b - 3000 i_a = 0$

$\therefore \boxed{-5000 i_a + 2000 i_b = 12}$ — ③

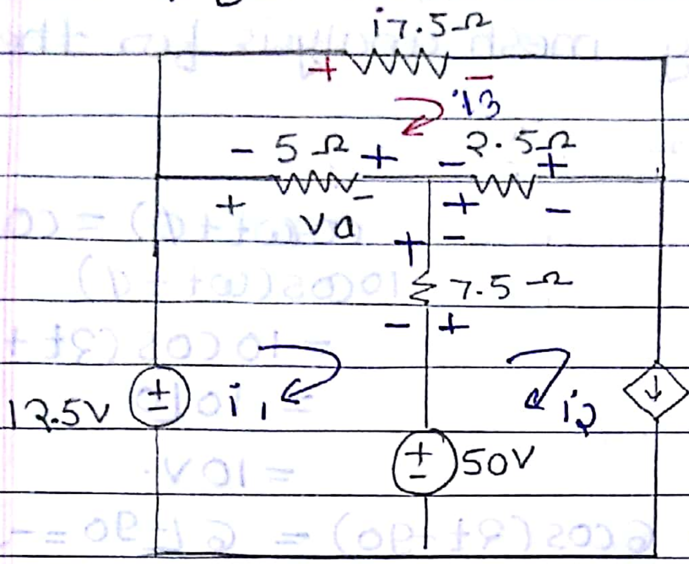
I_o is flowing bet mesh 1 - & 2

$\therefore$ Hence $I_o = I_a - I_b$

$= 3.33 - 2.33$

$\boxed{I_o = 1mA}$

10. Use mesh analysis for the ckt given below. and find out V_a.



The current source 0.2Va is present on perimeter of the ckt then the mesh-2 is cancelled. Writing the eq^n for current source.

$i_2 = 0.2V_a$ $\quad V_a = 5(i_1 - i_3)$

$i_2 = 0.2(5i_1 - 5i_3)$ $\quad V_a = 5i_1 - 5i_3$

$i_2 = i_1 - i_3$

$i_1 - i_2 - i_3 = 0$ — ①

Writing the KVL for mesh-①

$12.5 - 5(i_1 - i_3) - 7.5(i_1 - i_2) - 50 = 0$

$12.5 - 5i_1 + 5i_3 - 7.5i_1 + 7.5i_2 - 50 = 0$

$12.5i_1 - 7.5i_2 - 5i_3 = 75$ — ②

Writing KVL for Mesh-3

$-5(i_3 - i_1) - 17.5i_3 - 2.5(i_3 - i_2) = 0$

$-5i_3 + 5i_1 - 17.5i_3 - 2.5i_3 + 2.5i_2 = 0$

$5i_1 + 2.5i_2 - 25i_3 = 0$

$\therefore i_1 = 13.2A \quad i_2 = 9.6A \quad i_3 = 3.6A$

$V_a = 5i_1 - 5i_3$

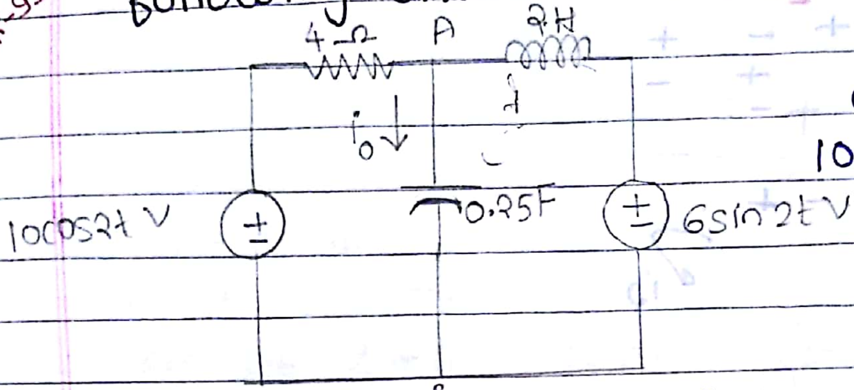
$\boxed{\therefore V_a = 48V}$

Date
24-8-15

V.V Imp **Type-4 :- Problems on mesh analysis**

1. Solve for I_o using mesh analysis for the following ckt.

G.R-95



$\cos(\omega t+\phi) = \cos\phi$

$10\cos(\omega t+\phi)$

$= 10\cos(2t+0)$

$= 10\angle 0$

$= 10V.$

$6\sin 2t = 6\cos(2t-90) = 6\angle -90 = -j6V$

The first step in analysis is to draw the phasor ckt equivalent where we are converting the ckt from time domain to frequency domain.

Here $\omega = 2$

$10\cos 2t = 10\angle 0V$ $\because 10\cos(2t+\phi) = 10\angle\phi$

$6\sin 2t = 6\cos(2t-90)$

$6\sin 2t = 6\angle -90V$ $\because 6\sin 2t = 6\cos(2t-90)$

$L = 2H$

$jX_L = j\omega L$

$= j \times 2 \times 2$

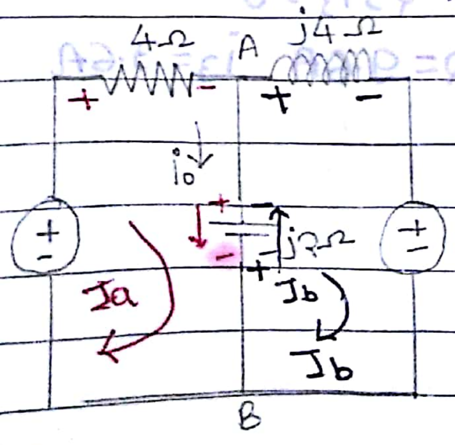
$\therefore jX_L = j4\Omega$

Here $\cos 2t$ & $\sin 2t$ are $\cos\omega t$ & $\sin\omega t$ $\Rightarrow \omega = 2$

$C = 0.25F$

$-jX_c = \frac{-j}{\omega C} = \frac{-j}{2\times 0.25} = -j2\Omega$

$(10+0j) = 10\angle 0$



$6\angle -90V = (0-6j)V$

Applying KVL for mesh-1.

$$10\angle 0 - 4I_a - (0-j2)(I_a - I_b) = 0$$
$$10 - 4I_a + j2I_a - j2I_b = 0$$
$$10 - I_a(4-j2) - j2I_b = 0$$
$$\therefore I_a(4-j2) + j2I_b = 10 \quad \text{(1)}$$

Applying KVL for mesh-2

$$-6\angle -90 - (-j2)(I_b - I_a) - j4I_b = 0$$
$$-(-6j) + j2I_b - j2I_a - j4I_b = 0$$
$$-j2I_a + I_b(j2 - j4) + 6j = 0$$
$$-j2I_a - j2I_b + j6 = 0$$
$$\therefore j2I_a + j2I_b = j6$$
$$\therefore \cancel{j}I_a + \cancel{j}I_b = \cancel{j}3 \quad \text{(2)} \quad I_a + I_b = 3$$

Solving eqⁿ (1) & (2) by cramers rule

$$\Delta = \begin{bmatrix} 4-j2 & +j2 \\ 1 & 1 \end{bmatrix} \Rightarrow \Delta = \cancel{4-2j-2j}$$
$$4-2j-2j = 4-4j$$

$$\Delta_1 = \begin{bmatrix} 10 & +2j \\ 3 & 1 \end{bmatrix}$$

$$\Delta_1 = 10 - 6j$$

$$\Delta_2 = \begin{bmatrix} 4-j2 & 10 \\ 1 & 3 \end{bmatrix}$$

$$\Delta_2 = 12 - 6j - 10 \Rightarrow 2 - 6j = \Delta_2$$

$$I_a = \frac{\Delta_1}{\Delta} = \frac{10-6j}{4-4j} \Rightarrow (2+0.5j)$$

$$I_b = \frac{2-6j}{4-4j} \Rightarrow (1-0.5j)$$

$$\Rightarrow \boxed{I_a = (2+0.5j)A}$$
$$\Rightarrow \boxed{I_b = (1-0.5j)}$$

I_o is current flowing from A to B.

Here the direction of I_o & I_a are same

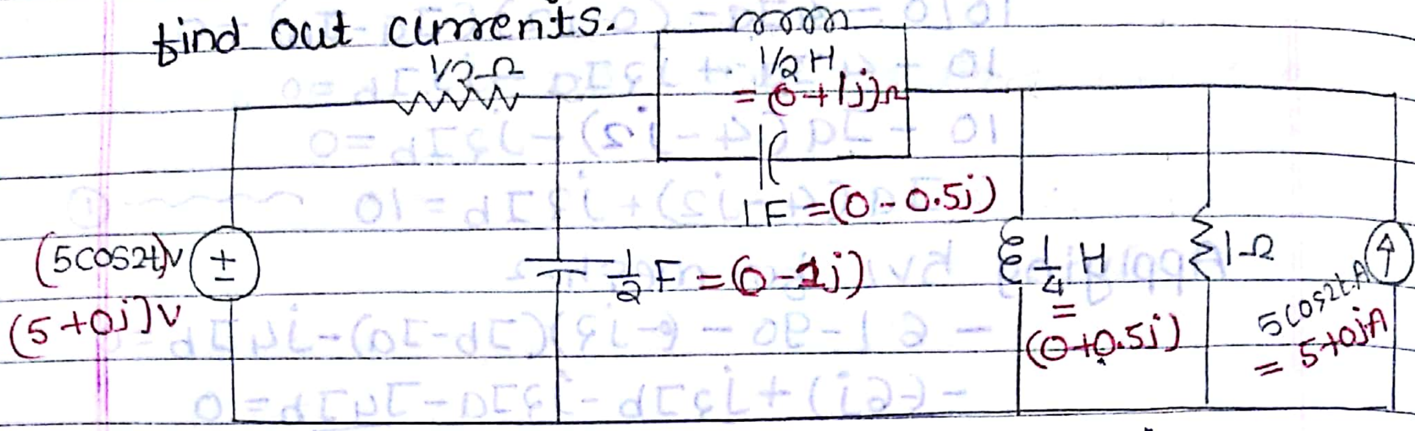
Hence $I_o = I_a - I_b$

$I_o = (2+0.5j) - (1-0.5j)$ [Hence the steady state value of $I_o = (1+1j)A$]

$I_o = (1+1j)\,A$

Date
25-8-15

2 Use mesh analysis for the following ckt and find out currents.



$5\cos 2t = 5\cos(2t+0) = 5\angle 0 = 5+0jV.$

$5\cos 2t = 5\cos(2t+0) = 5\angle 0 = 5+0jA.$

Here $\omega = 2$ and $L = 1/2\,H$.

$\therefore jX_L = j\omega L$

$\therefore jX_L = j \times 2 \times 1/2$

$\therefore jX_L = (0+1j)\,\Omega$

Here $\omega = 2$ and $L = 1/4\,H$

$\therefore jX_L = j\omega L$

$\therefore jX_L = j \times \cancel{2} \times 1/\cancel{4}_2$

$\therefore jX_L = j \times 1/2$

$\therefore jX_L = (0+0.5j)\,\Omega$

Here $C = 1F$ and $1/2\,F$.

$\therefore -jX_C = \frac{-j}{\omega C}$ & $-jX_C = \frac{-j}{\omega C}$

$\therefore -jX_C = \frac{-j}{2\times 1}$ & $-jX_C = \frac{-j}{\cancel{2}\times 1/\cancel{2}_1}$

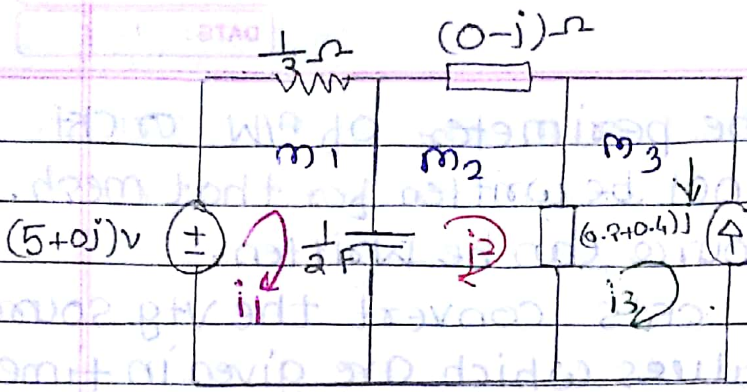
$\therefore -jX_C = (0-0.5j)$ & $-jX_C = (0-j)$

In fig. $\frac{1}{2}H$ & $1F$ are parallel.

Hence $\frac{(0+1j)\times(0-0.5j)}{(0+1j)+(0-0.5j)} = (0-1j)$

And $1/4\,H$ & $1\,\Omega$ are also parallel

Hence $\frac{(0+0.5j)\cancel{(5+0j)}\times 1}{(0+0.5j)\cancel{+(5+0j)}+1} = 0.2+0.4j$



$(5+0j)$ is present on the perimeter of ckt. so it is neglected.
writing eq^n for it i.e $i_3 = (5+0j)$ —— (1)
writing KVL for mesh (1) $\Rightarrow (5+0j) - 0.5 i_1 - (0-j)(i_1 - i_2) = 0$
$\therefore 5 - 0.5 i_1 + j i_1 - j i_2 = 0 \Rightarrow -(j - 0.5) i_1 + j i_2 = 5$ —— (2)
writing KVL for mesh (2) $-(0-j)(i_2 - i_1) - (0-j) i_2 - (0.2+0.4j)(i_2 - i_3) = 0$
$\therefore -(0+j) i_1 + (-j + j - 0.2 - 0.4j) i_2 + (0.2+0.4j) i_3 = 0$
$\therefore -(0+j) i_1 + (-0.2 - 0.4j) i_2 + (0.2+0.4j) i_3 = 0$
Solving (2) & (3) & (1) we get
$i_1 = (1.66 + 3.33j)$ A $\quad i_3 = (-1.66 + 0j)$ A
$i_2 = (1.66 - 2j)$ A

1. Write a procedure for mesh analysis technique

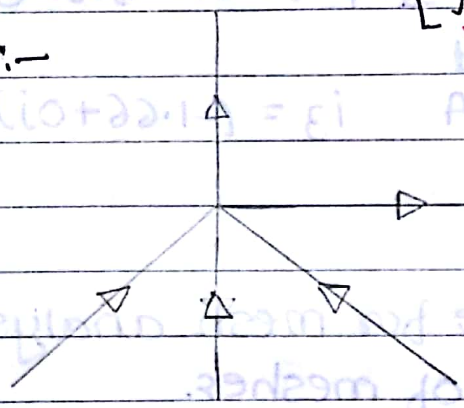
1. Identify the no. of meshes.
2. Assign mesh current to each meshes in arbitrary direction. (clockwise or anticlockwise) usually prefare clockwise direction.
3. If the ckt or n/w contains only vtg. sources (independent or dependent) & all other components with a vtg drop then write KVL eq^n to each mesh. and solve them for the mesh currents.
4. If the ckt or n/w contains current sources (independent or dependent) bet^n the 2 meshes then remove that Entire branch of c/n source. and form a supermesh. write KVL eq^n to supermesh & remaining meshes & write eq^n for the c/n source. Solve all the eq^{ns} for mesh c/n.
5. If the c/n source (dependent or independent)

is existing on the perimeter of n/w or ckt. KVL eqn should not be written for that mesh. A eqn for c/n source can be written.

6. for alternating ckts convert the vtg. source & c/n source values which are given in time domain to the frequency domain & all other components such as inductance, capacitance given in Henry & farad convert into inductive reactance jX_L in Ω & $-jX_C$ in Ω respectively.

Kirchoff's current law applied to n/w. [Node voltage analysis]

KCL:-



Statement:- "The total c/n flowing towards junction the node is equal to the total c/n flowing away from that node."

Or "The algebric sum of all the c/n meeting at the node point is always zero."

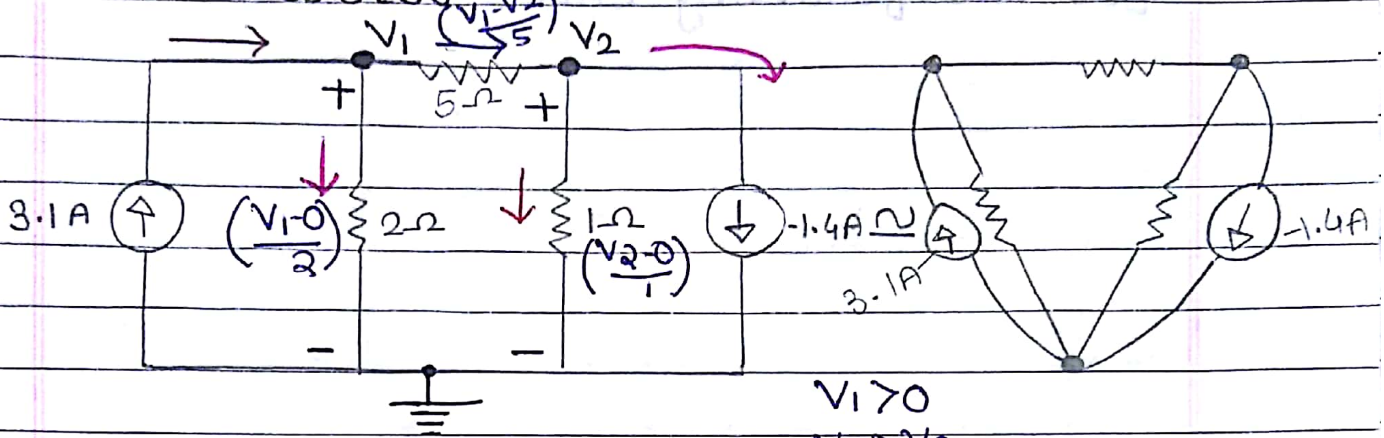
i.e $\boxed{\Sigma I \text{ at a node} = 0}$

Node:- "It is a point where two or more than two ckt elements are connected."

Node voltage:- "It is a vtg. at node wrt some reference node". In node analysis node vtg. is taken as a variable.

Type-5 Examples

1: Use node analysis to find voltage across 5Ω resistor.



$V_1 > 0$

$V_1 > V_2$

$V_2 > 0$

(Reference node should have more common points)

KCL at node V_1

$$3.1 - \frac{V_1}{2} - \frac{(V_1-V_2)}{5} = 0$$

$$3.1 - 0.5V_1 - 0.2V_1 + 0.2V_2 = 0$$

$$3.1 - 0.7V_1 + 0.2V_2 = 0$$

$$\therefore -0.7V_1 + 0.2V_2 = -3.1$$

$$\therefore 0.7V_1 - 0.2V_2 = 3.1 \quad -(1)$$

KCL at node V_2

$$\frac{V_1-V_2}{5} - V_2 + 1.4 = 0$$

$$0.2V_1 - 0.2V_2 - V_2 = -1.4$$

$$0.2V_1 - 1.02V_2 = -1.4 \quad -(2)$$

$$\therefore \boxed{V_1 = 5.1V}$$

$$\therefore \boxed{V_2 = 2.3V}$$

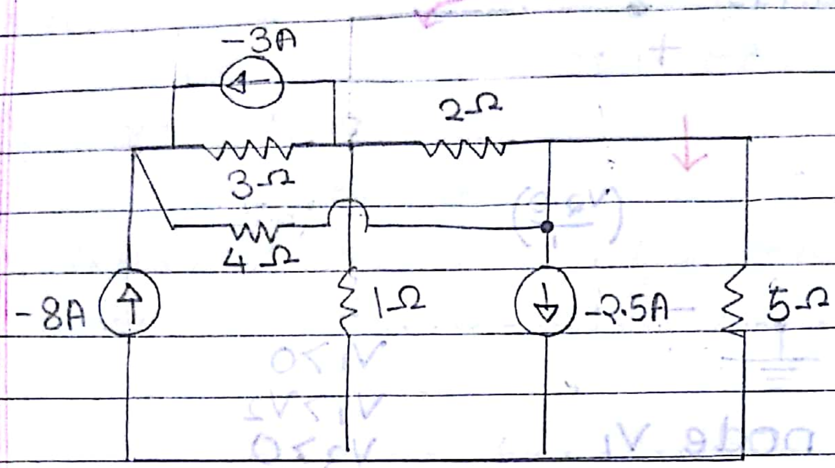
Voltage across 5Ω resistor is

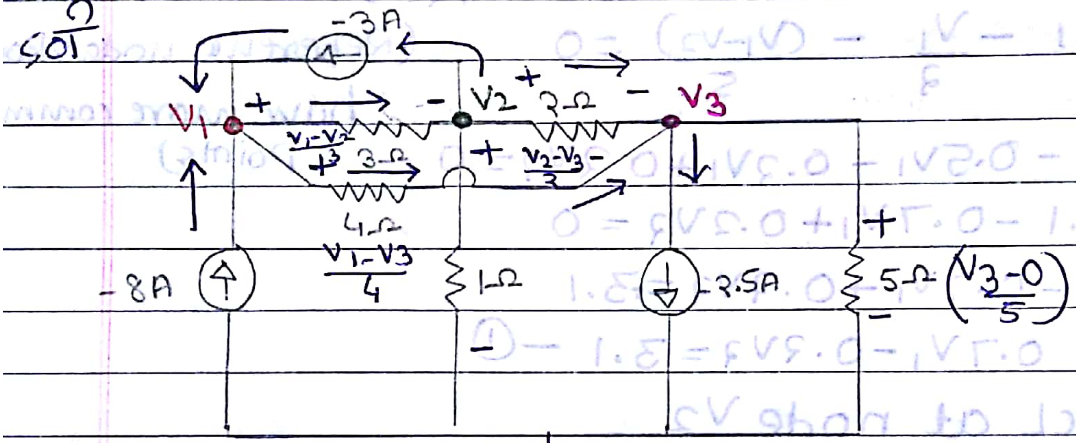
$$\left(\frac{V_1-V_2}{\cancel{5}}\right) \times \cancel{5}$$

$$= 5.1 - 2.3$$

$$\boxed{V = 2.8V}$$

2. Use node analysis to find c/n in 4Ω resistor for the following ckt.





0 reference

KCL at V_1

$$-8 - \frac{(V_1-V_3)}{4} - \frac{(V_1-V_2)}{3} - 3 = 0 \quad (1)$$

KCL at V_2

$$-(-3) + \frac{(V_1-V_2)}{3} - V_2 - \frac{(V_2-V_3)}{2} = 0 \quad (2)$$

KCL at V_3

$$\frac{(V_2-V_3)}{2} + \frac{(V_1-V_3)}{4} - (-2.5) - \frac{V_3}{5} = 0 \quad (3)$$

$$(1) \Rightarrow -96 - 3V_1 + 3V_3 - 4V_1 + 4V_2 - 36 = 0 \quad (4)$$

$$18 + 2V_1 - 2V_2 - 6V_2 - 3V_2 + 3V_3 = 0 \quad (5)$$

$$10V_2 - 10V_3 + 5V_1 - 5V_3 + 50 - 4V_3 = 0 \quad (6)$$

$$(4) \Rightarrow -7V_1 + 4V_2 + 3V_3 - 132 = 0 \Rightarrow -7V_1 + 4V_2 + 3V_3 = 132$$

$$(5) \Rightarrow 2V_1 - 11V_2 + 3V_3 = -18$$

$$(6) \Rightarrow 5V_1 + 10V_2 - 19V_3 = -50$$

$\boxed{V_1 = -23.86V}$ $\boxed{V_2 = -4.31V}$ $\boxed{V_3 = -5.9V}$

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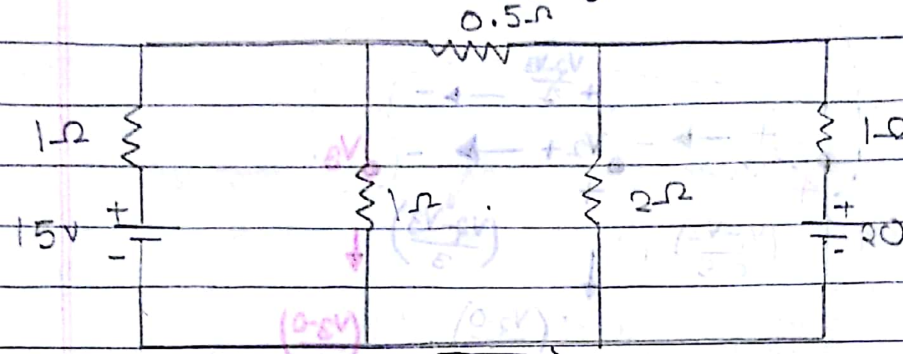
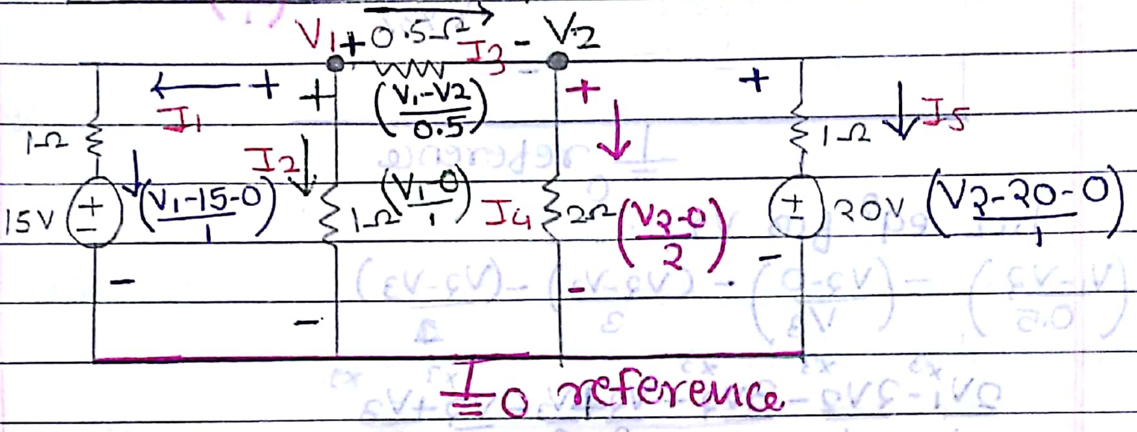
Type-6 examples

3. Find the c/n through each resistor of the circuit shown in fig. below using Nodal analysis



Sol^n



Apply KCL at Node V_1

$$-\left(\frac{V_1-15}{1}\right)-\left(\frac{V_1-0}{1}\right)-\left(\frac{V_1-V_2}{0.5}\right)=0$$

$$-V_1+15-V_1-2V_1+2V_2=0$$

$$-4V_1+2V_2=-15 \quad \text{(1)}$$

Apply KCL at V_2 ⇒ $\left(\frac{V_1-V_2}{0.5}\right)-\left(\frac{V_2-0}{2}\right)-\left(\frac{V_2-20-0}{1}\right)=0$

$$2V_1-2V_2-\frac{1}{2}V_2-V_2+20=0$$

$$4V_1-4V_2-V_2-2V_2+40=0$$

$$4V_1-7V_2=-40 \quad \text{(2)}$$

Apply

$$\boxed{V_1=9.25V}$$

$$\boxed{V_2=11V}$$

I_1 = Current through 1Ω $= V_1-15 = 9.25-15 = 5.75A$

I_2 = current through 1Ω $= V_1-0 = 9.25-0 = 9.25A$

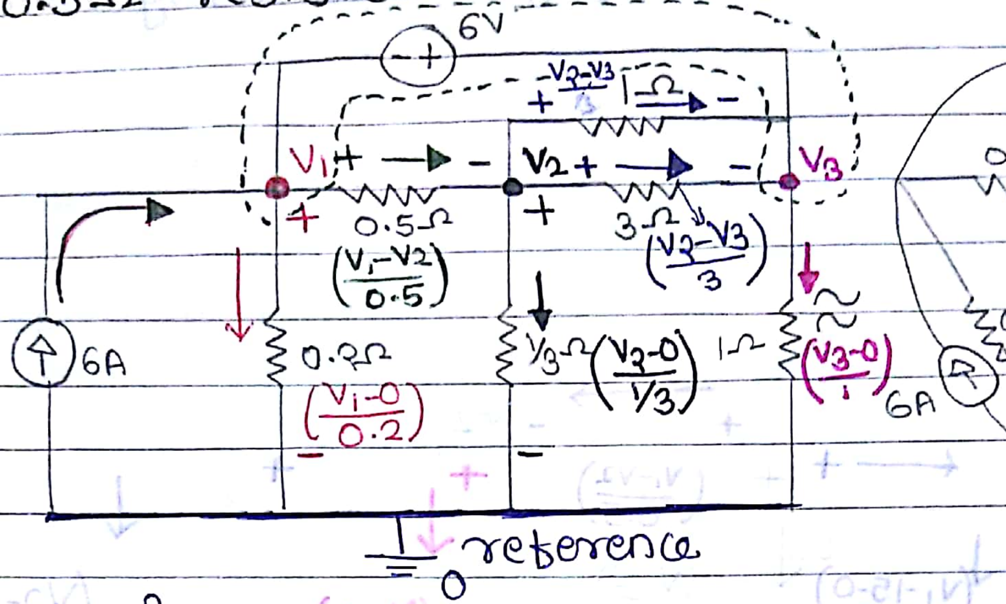
I_3 = Current through 0.5Ω $= \frac{(V_1-V_2)}{0.5} = \frac{9.25-11}{0.5} = -3.5A$

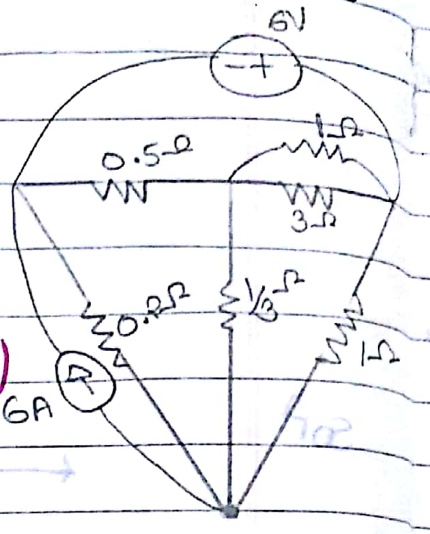
I_4 = Current through 2Ω $= \frac{V_2}{2} = \frac{11}{2} = 5.5A$

I_5 = Current through 1Ω $= V_2-20 = 11-20 = -9A$

Type-7 Examples :-

4 Use nodel analysis to find current through $0.5\,\Omega$ resistor in the ckt shown below.





KCL eq^n for V_2

$$\left(\frac{V_1-V_2}{0.5}\right)-\left(\frac{V_2-0}{1/3}\right)-\frac{(V_2-V_3)}{3}-\frac{(V_2-V_3)}{1}$$

$$\frac{2V_1}{1}-\frac{2V_2}{1}-\frac{3V_2}{1}-\frac{V_2}{3}+\frac{V_3}{3}-\frac{V_2}{1}+\frac{V_3}{1}$$

$$6V_1-6V_2-9V_2-V_2+V_3-3V_2+3V_3=0$$

$$6V_1-19V_2+4V_3=0 \quad (1)$$

writing eq^n for Vtg. source

$$V_1-V_3=-6V. \quad (2)$$

writing eq^n super node

$$6-\frac{V_1}{0.2}-\frac{(V_1-V_2)}{0.5}+\frac{(V_2-V_3)}{1}+\frac{(V_2-V_3)}{3}-\frac{V_3}{1}=0$$

$$6-5V_1-2V_1+2V_2+V_2-V_3+\frac{(V_2-V_3)}{3}-V_3=0$$

$$18-15V_1-6V_1+6V_2+3V_2-3V_3+V_2-V_3-3V_3=0$$

$$-21V_1+10V_2-7V_3-18=0$$

$$21V_1-10V_2+7V_3=18 \quad (3)$$

$\therefore$ $V_1=-0.5V$

$V_2=1V$

$V_3=5.5V$

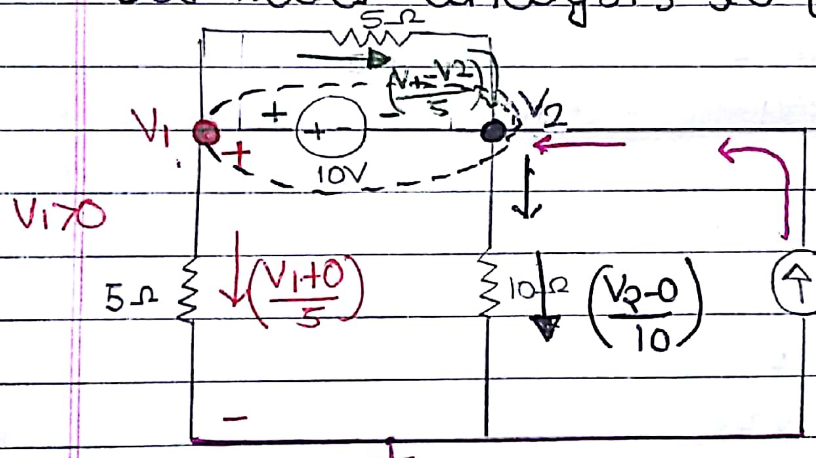
current through 0.5 resistor is

$$I_{5\Omega} = \frac{V_1 - V_2}{0.5}$$

$$I_{5\Omega} = \frac{-0.5-1}{0.5}$$

$$\boxed{I = -3A}$$

5. Use nodal analysis to find current through 10Ω resistance



Sol^n 10V vtg. source is in bet^n node V_1 & V_2. Hence super node V_1 & V_2 is formed. Hence writing one eq^n for vtg. source eq^n and also writing KCL to supernode (V_1, V_2). as two nodes are present we get 2 eq^{ns}.

Writing eq^n for voltage source

$$V_1 - V_2 = 10V \quad \text{(1)}$$

Applying KCL to Supernode (V_1, V_2)

$$-\cancel{\left(\frac{V_1-V_2}{5}\right)} - \frac{V_1}{5} + \cancel{\left(\frac{V_1-V_2}{5}\right)} - \frac{V_2}{10} + \frac{1}{1} = 0$$

$$-2V_1 - V_2 = -10 \quad \text{(2)}$$

$$\boxed{V_1 = 6.66V}$$

$$\boxed{V_2 = -3.33V}$$

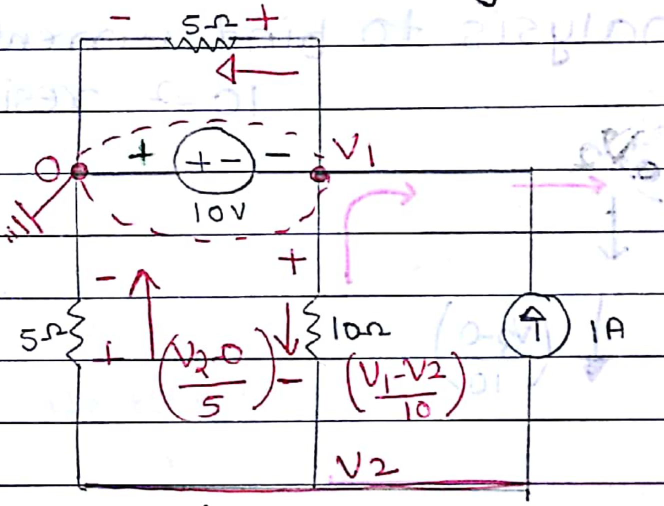
$$I_{10\Omega} = \frac{V_2}{10} = \frac{-3.33}{10}$$

$$\boxed{\therefore I_{10\Omega} = -0.33A}$$

Note :- Assume there are four nodes & a reference node.
$V_1, V_2, V_3, V_4, 0$

Super node (V_1, V_2) write a KCL, writing simple eqⁿ for vtg. source
Super node (V_3, V_4) " " " " " " " " " " "
Super node (V_2, V_3) " " " " " " " " " " "
Super node (V_1, V_3)
Super node $(V_2, 0)$ Not writing KCL but writing eqⁿ for vtg. source
Super node $(V_3, 0)$ " " " " " " " " " " "
Super node $(V_1, 0)$ " " " " " " " " " " "
Super node $(V_4, 0)$

6. Find c/n $(V_1 - 0)/5$ through 10Ω resistance.



$$-\frac{V_2}{5} + \left(\frac{V_1 - V_2}{10}\right) - 1 = 0$$

$$-2V_2 + V_1 - V_2 - 10 = 0$$

$$\therefore V_1 - 3V_2 = 10 \quad \text{(1)}$$

$$0 - V_1 = 10V$$

$$\therefore V_1 = -10V \quad \text{(2)}$$

$$\therefore V_1 = -10V$$

$$V_2 = -6.66V.$$

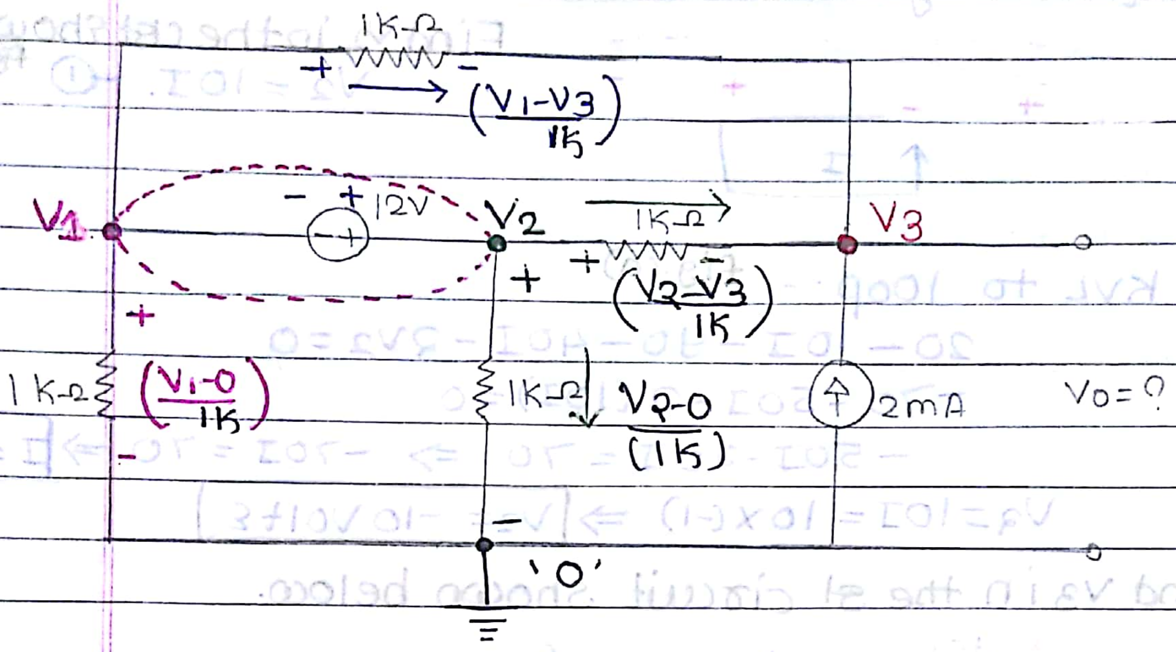
$$\therefore I_{10\Omega} = \frac{-10 + 6.66}{10} = -0.33A$$

$$\boxed{I_{10} = -0.33A}$$

28.8.15

Date 29-8-15

7. Use node analysis to find V_o in the following circuit.



1> Supernode (V_1, V_2) – Apply KCL

2> Voltage source 12V – write simple eqⁿ

3> Node V_3 – Apply KCL

Apply KCL at Supernode V_1, V_2

$$-\left(\frac{V_1-V_3}{1K}\right)-\left(\frac{V_1-0}{1K}\right)-\left(\frac{V_2-V_3}{1K}\right)-\left(\frac{V_2-0}{1K}\right)=0$$

$$\frac{-V_1+V_3-V_1-V_2+V_3-V_2}{1K}=0$$

$$-2V_1-2V_2+2V_3=0$$

$$-V_1-V_2+V_3=0 \sim\sim (1)$$

Writing a eqⁿ for 12V source {consider polarities of V_1 & V_2 as per the value of vtg source}

$$V_2-V_1=12 \sim\sim (2)$$

Apply KCL at node V_3

$$\left(\frac{V_1-V_3}{1K}\right)+\left(\frac{V_2-V_3}{1K}\right)+2m=0$$

$$\frac{V_1-V_3+V_2-V_3+2\times10^{-3}\times10^{3}}{1K}$$

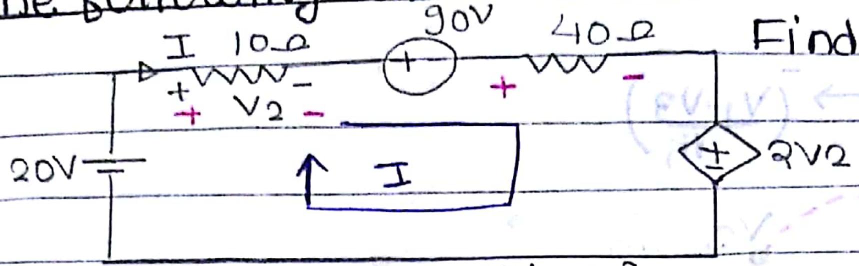
$$\therefore V_1+V_2-2V_3=-2$$

$\boxed{V_o=2V=V_3}$

$\boxed{\therefore V_1=-5V}$ $\boxed{V_2=7V}$ $\boxed{V_3=2V}$

8. Use analysis to find vtg. across 15A for the following ckt.

9. Find V_2 in the ckt shown in fig.a



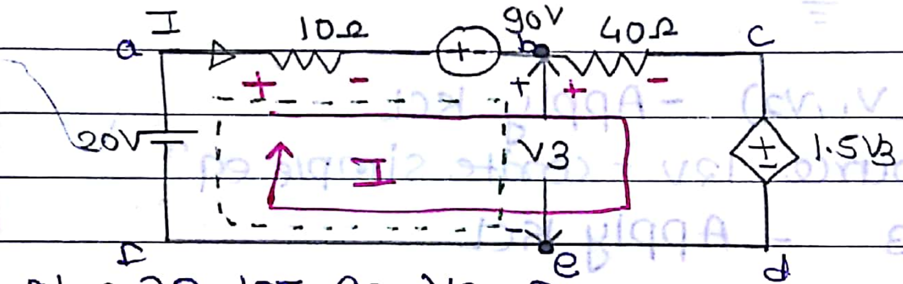
$V_2 = 10I$. —①

fig.(a)

KVL to loop:-

$$20 - 10I - 90 - 40I - 2V_2 = 0$$
$$-70 - 50I - 2(10I) = 0$$
$$-50I - 20I = 70 \Rightarrow -70I = 70 \Rightarrow \boxed{I = -1A}$$
$$V_2 = 10I = 10 \times (-1) \Rightarrow \boxed{V_2 = -10\text{ Volts}}$$

10. Find V_3 in the st circuit shown below.



KVL to abef: $20 - 10I - 90 - V_3 = 0$

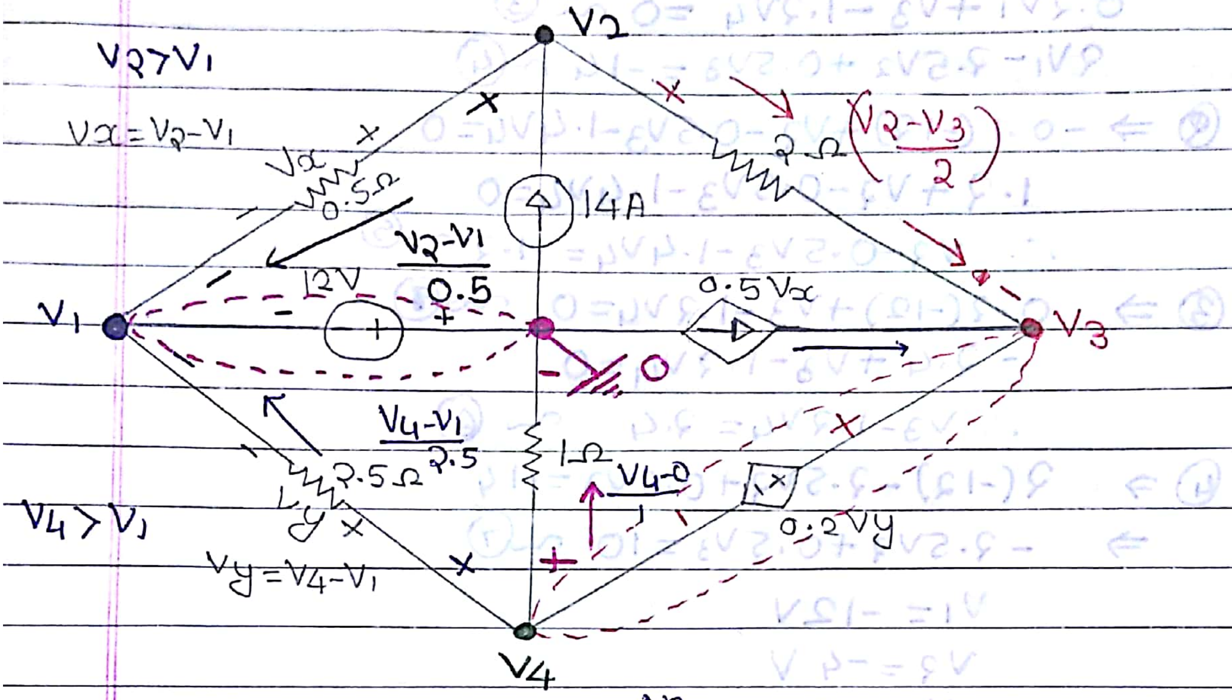
$$-70 - 10I - V_3 = 0 \Rightarrow \boxed{V_3 = -10I - 70}$$

KVL to abcdef:- $20 - 10I - 90 - 40I - 1.5(-10I - 70) = 0$

$$-70 - 50I + 15I + 105 = 0 \Rightarrow 35 = 35I \Rightarrow \boxed{I = 1}$$
$$V_3 = -10I - 70 \Rightarrow V_3 = -10 - 70 \Rightarrow \boxed{V_3 = -80V.}$$

$$10 + 3j = \sqrt{10^2 + 3^2} = \sqrt{109} = 10.44 \text{ — magnitude}$$
$$\text{angle} = \tan^{-1}(3/4) = 16.69$$
$$10 + 3j = 10.44 \angle 16.69$$
$$\frac{(10+3j)}{(10\angle 50)} = \frac{10.44\angle 16.69}{10\angle 50} = 1.044 \angle -33.31$$

9. Use nodel analysis to find the c/n through independent vtg. source for the folowing ckt.



Supernode $(V_1, 0)$ ~ NO KCL Supernode (V_3, V_4) ~ KCL

voltage source 12V ~ eq^n Voltage source $0.2V_y$ ~ eq^n

Node V_2 ~ KCL

writing eq^n for 12V

$0 - V_1 = 12$

$\Rightarrow V_1 = -12$ ~ (1)

writing KCL at V_3 & V_4

$$0.5V_x + \left(\frac{V_2-V_3}{2}\right) - \left(\frac{V_4-0}{1}\right) - \left(\frac{V_4-V_1}{2.5}\right) = 0$$

$$0.5(V_2-V_1) + \frac{(V_2-V_3)}{2} - V_4 - \left(\frac{V_4-V_1}{2.5}\right) = 0$$

$$0.5V_2 - 0.5V_1 + 0.5V_2 - 0.5V_3 - V_4 - 0.4V_4 + 0.4V_1 = 0$$

$$1V_2 - 0.1V_1 - 0.5V_3 - 1.04V_4 = 0$$

$$-0.1V_1 + V_2 - 0.5V_3 - 1.04V_4 = 0 \quad \sim (2)$$

writing KCL at V_2

$V_3 - V_4 = 0.2V_y$ ~ (3)

$$-\left(\frac{V_2-V_1}{0.5}\right) - \left(\frac{V_2-V_3}{2}\right) + 14 = 0$$

$\therefore V_3 - V_4 = 0.2(V_4 - V_1)$

$\therefore V_3 - 1.2V_4 + 0.2V_1 = 0$ ~ (3)

$$-2V_2 + 2V_1 - 0.5V_2 + 0.5V_3 + 14 = 0$$

$$2V_1 - 2.5V_2 + 0.5V_3 = -14 \quad \sim (4)$$

$V_1 = -12 \sim (1)$

$-0.1V_1 + V_2 - 0.5V_3 - 1.4V_4 = 0 \sim (2)$

$0.2V_1 + V_3 - 1.2V_4 = 0 \sim (3)$

$2V_1 - 2.5V_2 + 0.5V_3 = -14 \sim (4)$

$(2) \Rightarrow -0.1(-12) + V_2 - 0.5V_3 - 1.4V_4 = 0$

$1.2 + V_2 - 0.5V_3 - 1.4V_4 = 0$

$\therefore V_2 - 0.5V_3 - 1.4V_4 = -1.2 \sim (5)$

$(3) \Rightarrow 0.2(-12) + V_3 - 1.2V_4 = 0 \sim (8)$

$-2.4 + V_3 - 1.2V_4 = 0$

$\therefore V_3 - 1.2V_4 = 2.4 \sim (6)$

$(4) \Rightarrow 2(-12) - 2.5V_2 + 0.5V_3 = -14$

$\Rightarrow -2.5V_2 + 0.5V_3 = 10 \sim (7)$

$V_1 = -12V$

$V_2 = -4V$

$V_3 = 0V$

$V_4 = -2V$

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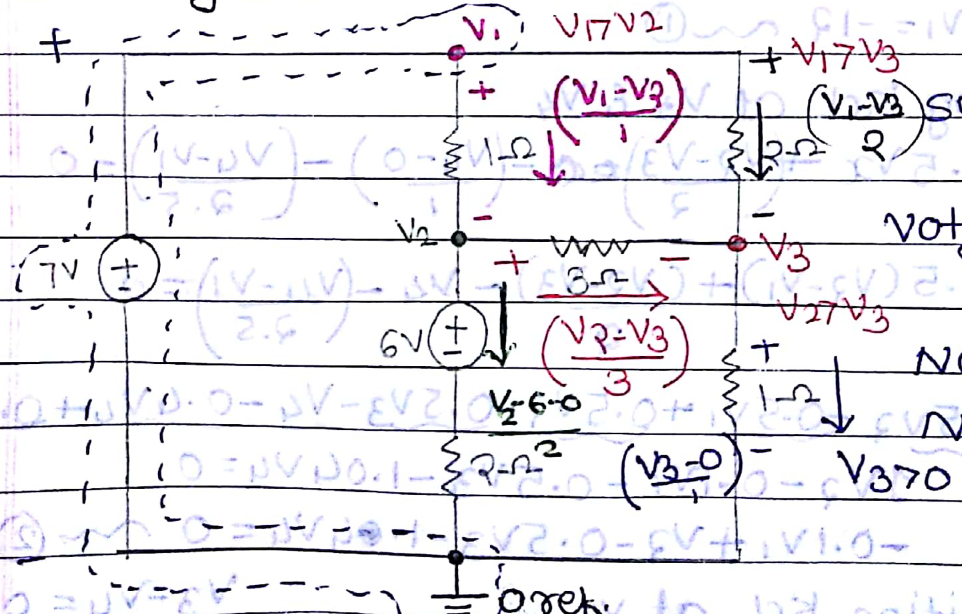
10. Use nodal analysis to determine the c/n flowing through 3Ω resistor.



supernode $(V_1, 0)$
↳ No KCL

Voltg. source 7V
↳ simple eq^n

Node $V_2 \rightarrow$ KCL

Node $V_3 \rightarrow$ KCL

Writing eq^n → 7V vtg. source

$V_1 - 0 = 7$

$\boxed{\therefore V_1 = 7} \sim (1)$

KCL at node V_2.

$$\left(\frac{V_1-V_2}{1}\right)-\left(\frac{V_2-V_3}{3}\right)-\left(\frac{V_2-6-0}{2}\right)=0$$

$$V_1-V_2-0.33V_2+0.33V_3-0.5V_2+3=0$$

$$V_1-1.83V_2+0.33V_3=-3$$

$$\therefore 7-1.83V_2+0.33V_3+3=0$$

$$\therefore -1.83V_2+0.33V_3=-10 \quad \text{②}$$

KCL at node V_3

$$\left(\frac{V_1-V_3}{2}\right)+\left(\frac{V_2-V_3}{3}\right)-\left(\frac{V_3-0}{1}\right)=0$$

$$0.5V_1-0.5V_3+0.33V_2-0.33V_3-V_3=0$$

$$(0.5\times7)+0.33V_2-1.83V_3=0$$

$$\therefore +0.33V_2-1.83V_3=-3.5 \quad \text{③}$$

$V_1=7V$

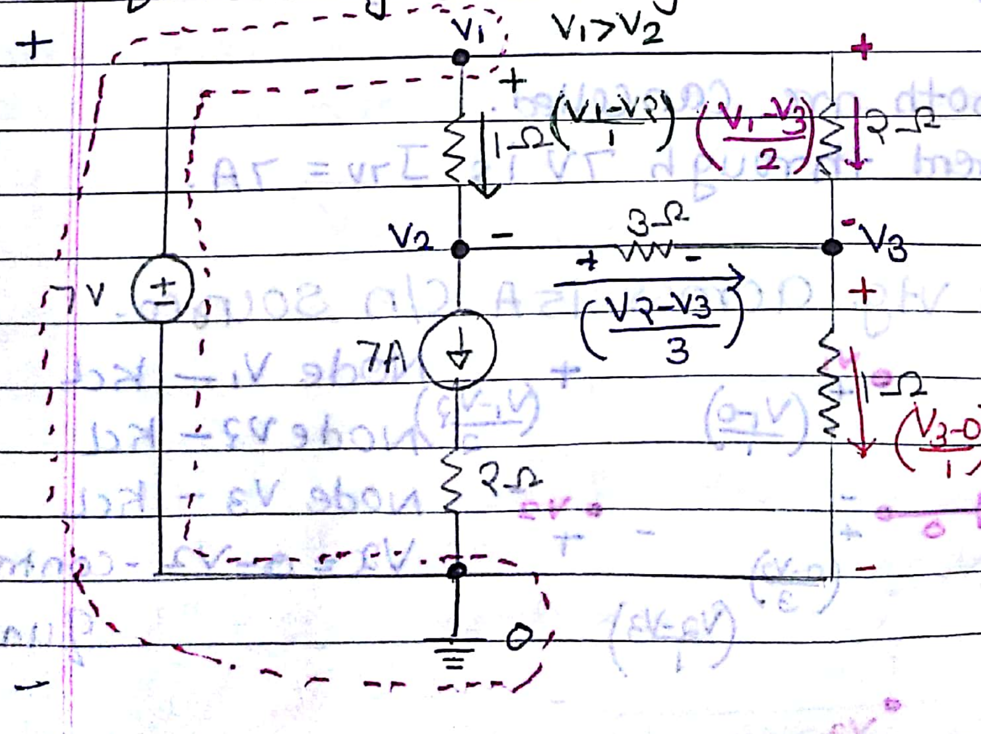
$V_2=6V$

$V_3=3V$

$$I_{3\Omega}=\frac{V_2-V_3}{3}=\frac{6-3}{3}$$

$$\boxed{\therefore I_{3\Omega}=1A}$$

11. Find the c/n through 7V Vtg. source in the following ckt using Nodal analysis.



Supernode $(V_1, 0)$ → No KCL

Vtg. source 7V → Simple eq^n

Node V_2 → KCL

Node V_3 – KCL

$V_1 - 0 = 7$

$\therefore V_1 = 7$ ——— (1)

7A C/n source is connected to V_2 directly

$\therefore$ Hence $-7 + \left(\frac{V_1 - V_2}{1}\right) - \left(\frac{V_2 - V_3}{3}\right) = 0$

$-7 + V_1 - V_2 - 0.33V_2 + 0.33V_3 = 0$

$-\cancel{7} + \cancel{7} - V_2 - 0.33V_2 + 0.33V_3 = 0$

$\therefore -1.33V_2 + 0.33V_3 = 0$ ——— (2)

KCL at node V_3.

$\left(\frac{V_1 - V_3}{2}\right) + \left(\frac{V_2 - V_3}{3}\right) - \frac{(V_3 - 0)}{1} = 0$

$0.5V_1 - 0.5V_3 + 0.33V_2 - 0.33V_3 - V_3 = 0$

$(0.5 \times 7) - 0.5V_3 - 0.33V_3 - V_3 + 0.33V_2 = 0$

$3.5 - 1.83V_3 + 0.33V_2 = 0$

$\therefore 0.33V_2 - 1.83V_3 = -3.5$ ——— (3)

$\therefore V_1 = 7V$

$\therefore V_2 = 0.496V$

$\therefore V_3 = 2.002V$

At node V_2:-

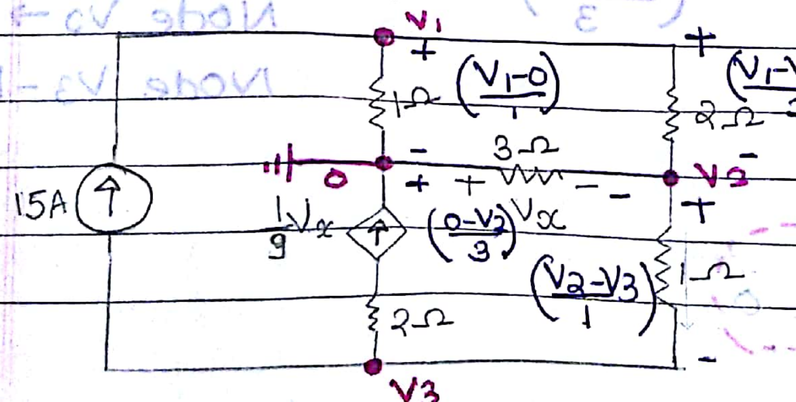
$\left(\frac{V_1 - V_2}{1}\right) = 0.50$ is C/n entering

& $\left(\frac{V_2 - V_3}{1}\right) = -0.50$ is C/n leaving ~~them~~

Hence both are cancelled.

& current through 7V is $I_{7V} = 7A$.

12. Find the vtg. across 15A C/n source.



Node V_1 – KCL

Node V_2 – KCL

Node V_3 – KCL

$V_x = 0 - V_2$ - controling quantity

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KCL at node V_1

$$15 - \left(\frac{V_1 - 0}{1}\right) - \left(\frac{V_1 - V_2}{2}\right) = 0$$

$$15 - V_1 - 0.5V_1 + 0.5V_2 = 0$$

$$-1.5V_1 + 0.5V_2 = -15 \sim (1)$$

KCL at node V_2

$$\left(\frac{V_1 - V_2}{2}\right) + \left(\frac{0 - V_2}{3}\right) - \left(\frac{V_2 - V_3}{1}\right) = 0$$

$$0.5V_1 - 0.5V_2 - 0.33V_2 - V_2 + V_3 = 0$$

$$0.5V_1 - 1.83V_2 + V_3 = 0 \sim (2)$$

KCL at node V_3

$$-15 - \frac{1}{9}V_x + \left(\frac{V_2 - V_3}{1}\right) = 0$$

$$-15 - \frac{1}{9}(-V_2) + V_2 - V_3 = 0$$

$$\cancel{-15 - 0.11V_2 + (V_2) - V_3 = 0}$$

$$\cancel{-15 - 0.89V_2 - V_3 = 0}$$

$$\cancel{\therefore -0.89V_2 - V_3 = 15 \sim (3)}$$

$$-15 + 1.11V_2 - V_3 = 0$$

$$\therefore 1.11V_2 - V_3 = 15$$

$V_1 = 4V$

$V_2 = -18V$

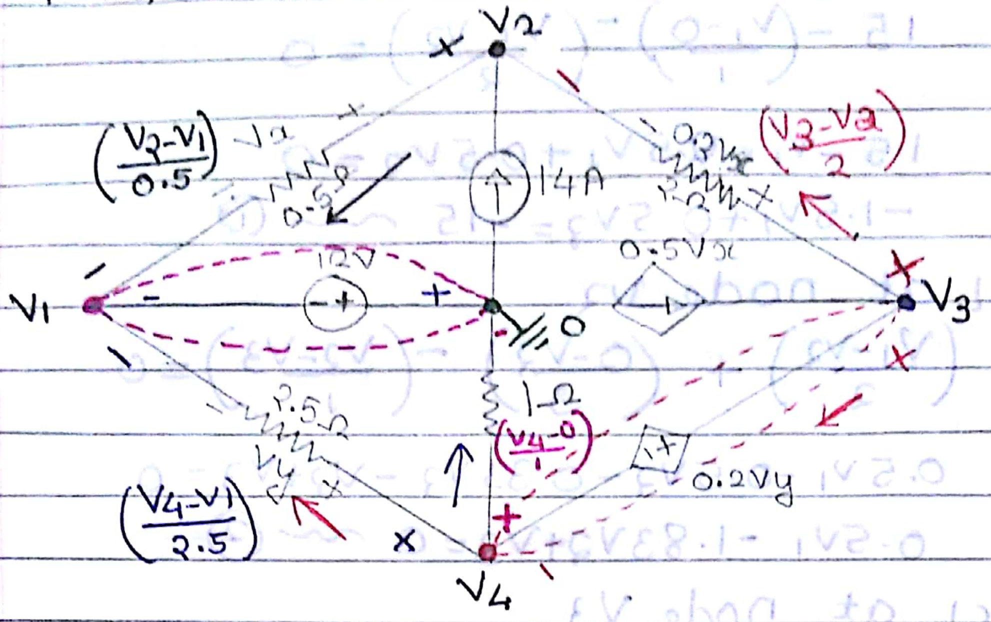
$V_3 = -35V$

$$V_{15A} = V_1 - V_3$$

$$= 4 - (-35)$$

$$\boxed{V_{15A} = 39\ V.}$$

13. Find V_1, V_2, V_3, V_4 in the following ckt



Supernode $(V_1, 0)$ ~ No KCl

Supernode (V_3, V_4) ~ Writing KCL.

writing eq^n to vtg source ~ 12V

writing eq^n to $0.2V_y$.

Node V_2 ~ KCl

controling quantity

Writing eq^n for 12V

$V_3 - V_2 = 0.2V_x$

$0 - V_1 = 12$

$\boxed{\therefore V_1 = -12V}$ ~ ①

Writing KCL at V_3 & V_4

$$0.5V_x - \left(\frac{V_3-V_2}{2}\right) - \left(\frac{V_4-0}{1}\right) - \left(\frac{V_4-V_1}{2.5}\right) = 0$$

$$0.5(V_2-V_3) - 0.5V_2 + 0.5V_3 - V_4 - 0.4V_1 + 0.4V_4 = 0$$

$$\cancel{0.5V_2} - \cancel{0.5V_3} - \cancel{0.5V_2} + \cancel{0.5V_3} - V_4 - 0.4V_1 + 0.4V_4 = 0$$

writing KCL at V_2

$$\left(\frac{V_2-V_3}{2}\right) - \left(\frac{V_2-V_1}{0.5}\right) + 14 = 0$$

$$0.5V_2 - 0.5V_3 - 2V_2 + 2V_1 + 14 = 0$$

$$2V_1 - 1.5V_2 - 0.5V_3 = -14$$

$$2(-12) - 1.5V_2 - 0.5(2.4) = -14$$

$$-24 + 14 - 1.2 - 1.5V_2 = 0 \Rightarrow -1.5V_2 = 11.2$$

$$\therefore V_2 = -7.46V$$

$V_3 - V_4 = 0.2 V_y$

$V_3 - V_4 = 0.2(V_4 - V_1)$

$V_3 - V_4 = 0.2V_4 - 0.2V_1$

$0.2V_1 + V_3 - 1.2V_4 = 0$

$\therefore 0.2(-12) + V_3 - (1.2 \times 0) = 0$

$\therefore -2.4 + V_3 = 0$

$\therefore V_3 = 2.4V$

$V_1 = -12V$

$V_2 = -7.46V$

$V_3 = 2.4V$

$V_4 = 0V$

Writing KCL at V_3 & V_4

$0.5 V_x - \frac{(V_3 - V_2)}{2} - \frac{(V_4 - 0)}{1} - \left(\frac{V_4 - V_1}{2.5}\right)$

$0.5(V_2 - V_1) - 0.5V_3 + 0.5V_2 - V_4 - 0.4V_4 + 0.4V_1 = 0$

$0.5V_2 - 0.5V_1 - 0.5V_3 + 0.5V_2 - V_4 - 0.4V_4 + 0.4V_1 = 0$

$-0.1V_1 + V_2 - 0.5V_3 - 1.4V_4 = 0$

$-1.2 + V_2 - 0.5V_3 - 1.4V_4 = 0$

$\therefore V_2 - 0.5V_3 - 1.4V_4 = 1.2$ ⟶ (2)

Writing KCL at node V_2

$14 = \frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} = \frac{4V_2 - 4V_1 + V_2 - V_3}{2}$

i.e $28 = 4V_2 + 48 + V_2 - V_3$

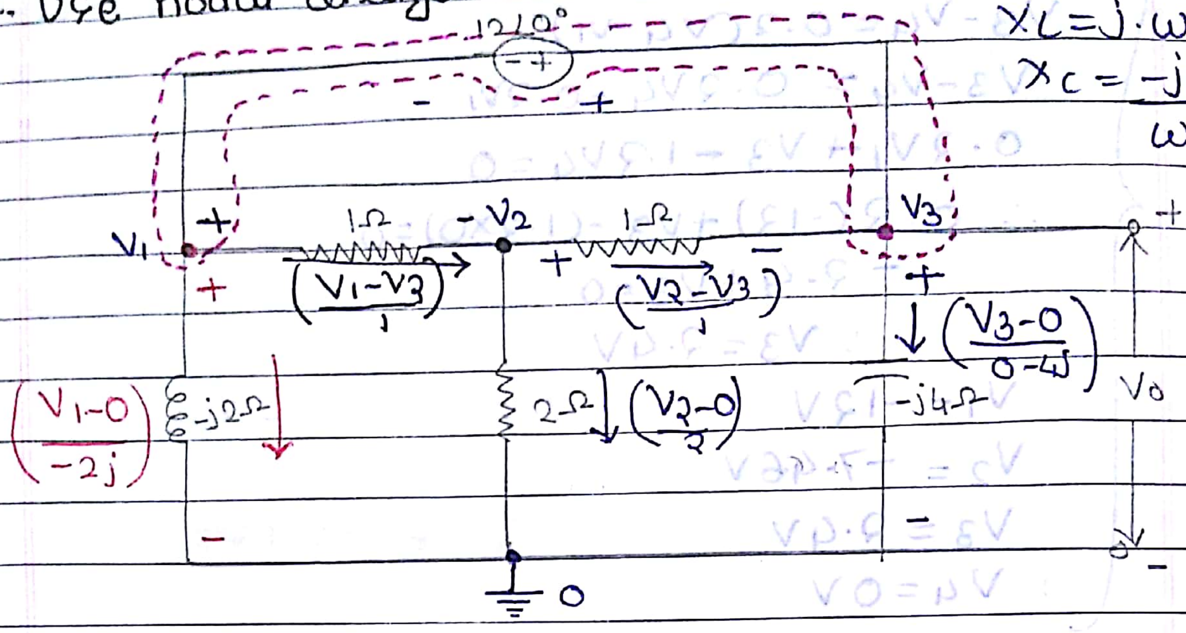
$\therefore V_3 - 5V_2 = 20$ ⟶ (3)

$V_1 = -12V \therefore V_2 = -4V, V_3 = 0V, V_4 = -2V$

14. Use nodal analysis and find V_0.

$X_L = j \cdot \omega \cdot L$

$X_C = \frac{-j}{\omega \cdot C}$



Super node $(V_1, V_3) \rightarrow$ writing KCL.

Voltage source $12\angle 0 \rightarrow$ simple eqn.

Node V_2:- Applying KCL.

K.C.L at supernode V_3

$$-\left(\frac{V_1-0}{-2j}\right) - \left(\frac{V_1-V_2}{1}\right) + \left(\frac{V_2-V_3}{1}\right) - \left(\frac{V_3-0}{0-4j}\right) = 0$$

$$\frac{V_1}{2j} - V_1 + V_2 + V_2 - V_3 + \frac{V_3}{4j} = 0$$

$$0.5j\,V_1 - V_1 + 2V_2 - V_3 + 0.25j\,V_3 = 0$$

$$V_1(0.5j-1) + 2V_2 - V_3(1-0.25j) = 0 \quad (*)$$

$$\cancel{(0.5j)}\; V_1(-1+0.5j) + 2V_2 - V_3(1-0.25j) = 0 \quad (1)$$

simple eqn.

$$-V_1 + V_3 = 12\angle 0 \quad (2)$$

$$\Rightarrow V_1 = V_3 - 12\angle 0°$$

Node V_2

$$\left(\frac{V_1-V_2}{1}\right) - \left(\frac{V_2-V_3}{1}\right) - \left(\frac{V_2-0}{2}\right) = 0$$

$$V_1 - V_2 - V_2 + V_3 - 0.5V_2 = 0$$

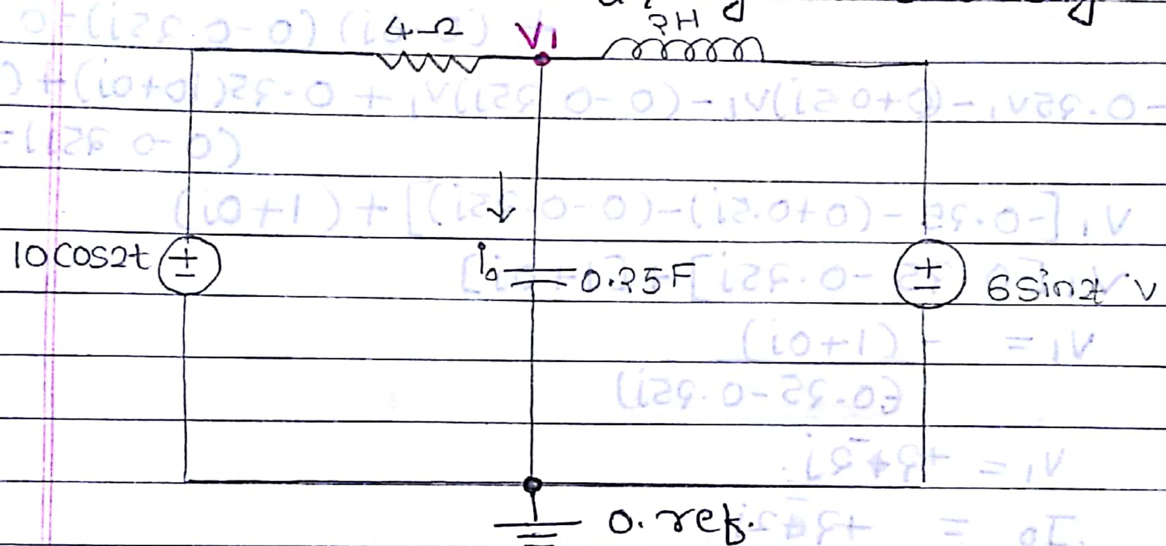
$$V_1 - 2.5V_2 + V_3 = 0 \quad (3)$$

$$\Delta = \begin{bmatrix} (-1+0.5j)j & 2 & -(1-0.25j) \\ -1 & 0 & 1 \\ 1 & -2.5 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 12\angle 0 \\ 0 \end{bmatrix}$$

$$\therefore \Delta = (-1-0.5j) \times 2.5 - 2(-1-1) + -\cancel{1}(1-0.25j)(2.5) = 0$$

Date
3-9-15

15. find the I_o using node analysis.



given $\omega = 2$ rad/sec $\qquad C = 0.25F$.

$10\cos 2t = 10\cos(2t + 0)$ $\qquad -jX_C = \frac{-j}{\omega C}$

$\boxed{10\cos t = 10\angle 0 V.}$

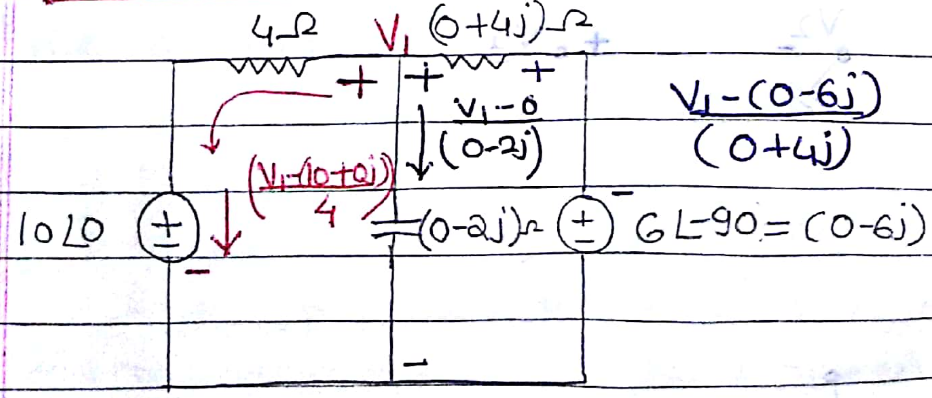
$jX_L = j\omega L$ $\qquad -jX_C = \frac{-j}{2 \times 0.25}$

$\therefore jX_L = j \times 2 \times 2$

$\boxed{\therefore jX_L = (0+4j)\Omega}$ $\qquad \boxed{-jX_C = \frac{-j}{0.5} = (0-2j)}$

$6\sin 2t = 6\cos(2t - 90)$

$\boxed{6\sin 2t = 6\angle -90.}$



Applying KCL at node V_1

$$-\left\{\frac{V_1-(10+0j)}{4}\right\}-\left\{\frac{V_1-0}{(0-2j)}\right\}-\left\{\frac{V_1-(0-6j)}{(0+4j)}\right\}=0$$

$$\frac{-V_1+(10+0j)}{4}-\frac{V_1}{(0-2j)}-\frac{V_1+(0-6j)}{(0+4j)}=0$$

$$-0.25V_1+0.25(10+0j)-(0+0.5j)V_1-(0-0.25j)V_1+(0-6j)(0-0.25j)=0$$

$$-0.25V_1+0.25(10+0j)-(0+0.5j)V_1-(0-0.25j)V_1+(0-6j)(0-0.25j)=0$$

$$-0.25V_1-(0+0.5j)V_1-(0-0.25j)V_1+0.25(10+0j)+(0-6j)(0-0.25j)=0$$

$$V_1[-0.25-(0+0.5j)-(0-0.25j)]+(1+0j)$$

$$V_1[-0.25-0.25j]+[1+0j]$$

$$V_1=\frac{-(1+0j)}{(-0.25-0.25j)}$$

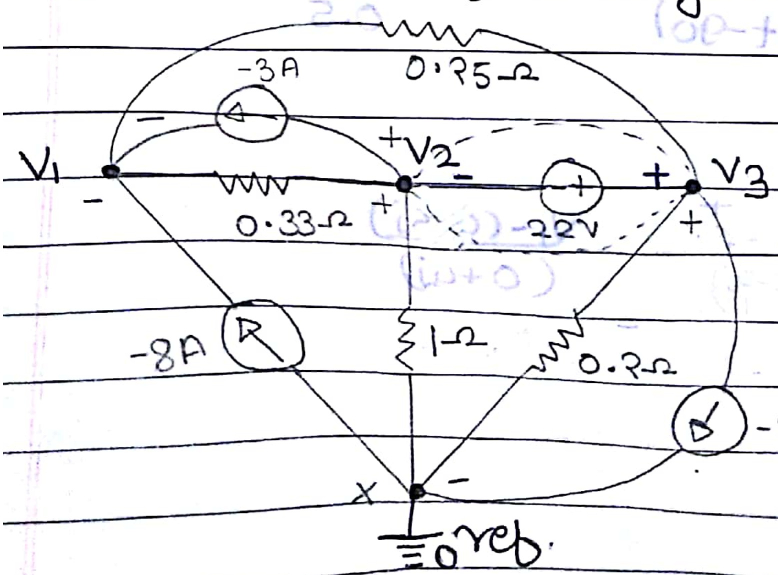
$$V_1=+2+2j.$$

$$I_0=\frac{+2+2j}{-2j}$$

$$i_0=(1+1j)\,A$$

$$i_0=1.4\angle 45^\circ A.$$

16. Use node analysis to find c/n through 0.2 Ω resistor in following ckt. 32.5A.

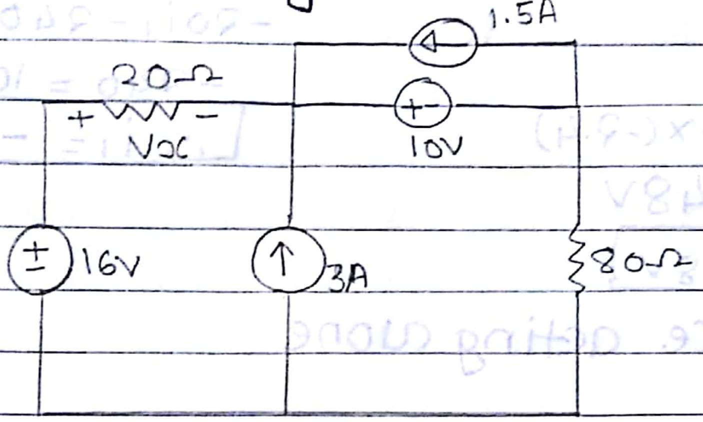


$V_3-V_2=22V$ ①

Network Theorems - I

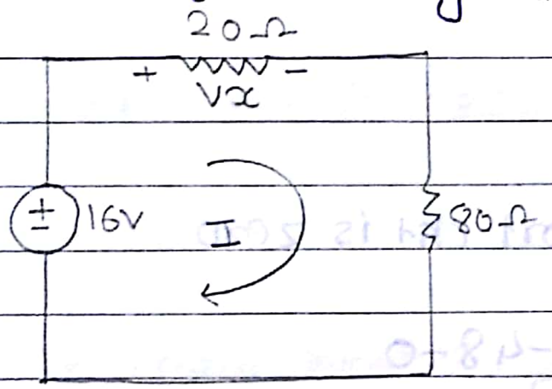
Superposition Theorem:-

1. Find the components of V_x caused by each source acting alone in the ckt shown below.



c/n source - Open
vtg. source - short

Ans. ① 16V source acting alone



Applying KVL to the mesh

$16 - 20I - 80I = 0$

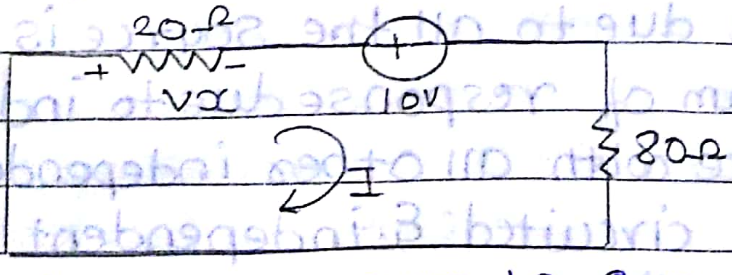
$\therefore 16 = 100I$

$\boxed{\therefore I = 0.16}$

$\therefore V_x = 0.16 \times 20$

$\boxed{\therefore V_x = 3.2V}$

② 10V source acting alone

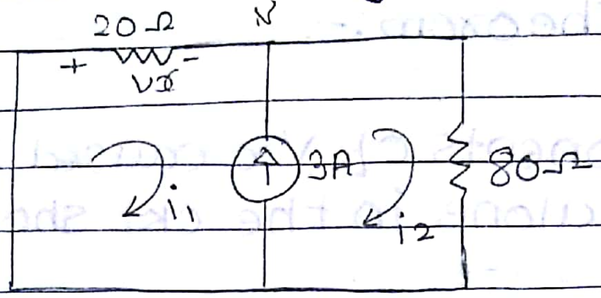


Applying KVL $-20I - 10 - 80I = 0 \Rightarrow -10 = 100I$

$\Rightarrow I = -0.1$

$\Rightarrow \boxed{V_x = -2V}$

③ 3A source acting alone



$i_2 - i_1 = 3$

$\therefore i_2 = 3 + i_1$ — ①

$-20i_1 - 80i_2 = 0$

$-20i_1 - 80(3+i_1) = 0$

$-20i_1 - 240 - 80i_1 = 0$

$-240 = 100i_1$

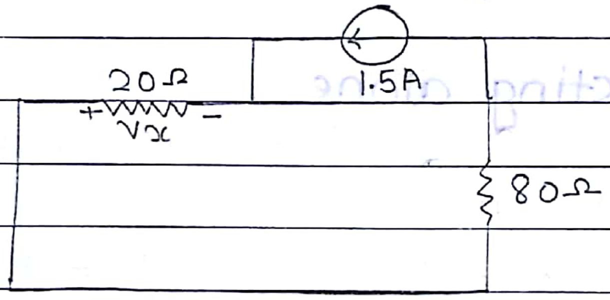
$\boxed{\therefore i_1 = -2.4A}$

$Vx = 20 \times (-2.4)$

$Vx = -48V$

$\boxed{Vx = -48V}$

④ 1.5A source acting alone



C/n through short ckt is zero

$\therefore \boxed{Vx = 0}$

$\therefore Vx = 3.2 - 2 - 48 - 0$

$\boxed{\therefore Vx = -46.8V}$

(2. Using superposition theorem find)

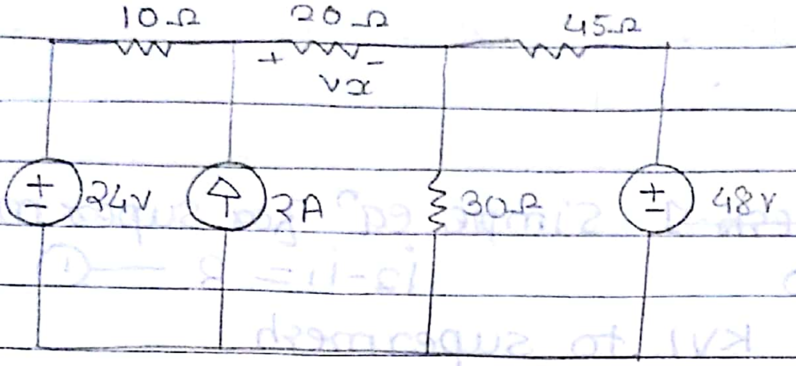
Date 22-09-15

1. Superposition Theorem:-

more than one

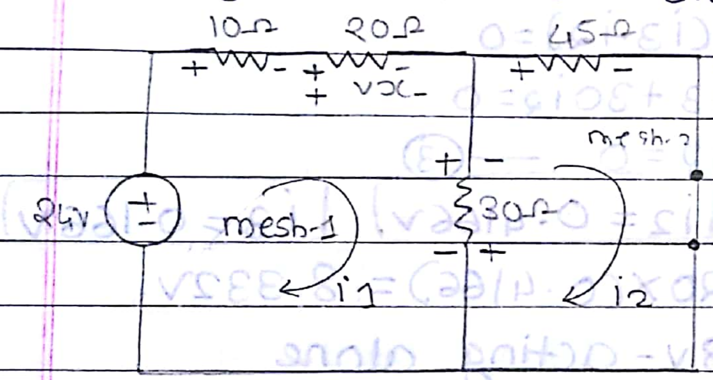
Statement:- "In any bilateral network containing independent sources (Independent voltge & C/n sources) The response due to all the source is equals to algebraic sum of response due to individual independent source with all other independent voltage source short circuited & independent c/n sources open circuited."

1 Use superposition theorem to find value of V_x in the circuit shown below.



Voltage source 24V
current source 2A
Voltage source 48 V

i) voltage source 24V acting alone
Current source 2A - open circuit
Voltage source 48V - Short circuited



KVL to mesh 1

$$24-10i_1-20i_1-30(i_1-i_2)=0$$
$$24-30i_1-30i_1+30i_2=0$$
$$24-60i_1+30i_2=0$$
$$\therefore -60i_1+30i_2=-24 \quad ---- \quad (1)$$

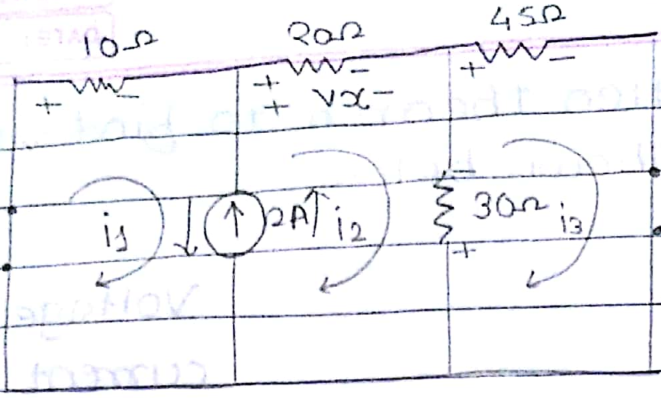
KVL to mesh -2

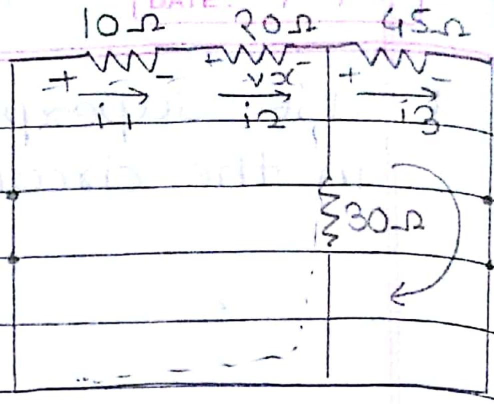
$$-45i_2-30(i_2-i_1)$$
$$-45i_2-30i_2+30i_1=0$$
$$-75i_2+30i_1=0$$

$\boxed{i_1=0.5A}$ $\boxed{i_2=0.2A}$

$V_x = 20\times0.5 = \boxed{10\text{Volts.}}$

ii) current source 2A-acting alone
Voltage source 24V-Short circuit
Voltage source 48V-Short circuit





~~KVL for mesh-1~~ Simple eq^n for super mesh

~~$-10i_1-2=0$~~ $i_2-i_1=2$ —— (1)

$\therefore$ ~~$-10i_1=$~~ KVL to supermesh

$-10i_1-20i_2-30(i_2-i_3)=0$

$-10i_1-20i_2-30i_2+30i_3=0$

$-10i_1-50i_2-30i_3=0$ —— (2)

KVL to mesh-3

$-45i_3-30(i_3-i_2)=0$

$-45i_3-30i_3+30i_2=0$

$-75i_3+30i_2=0$ —— (3)

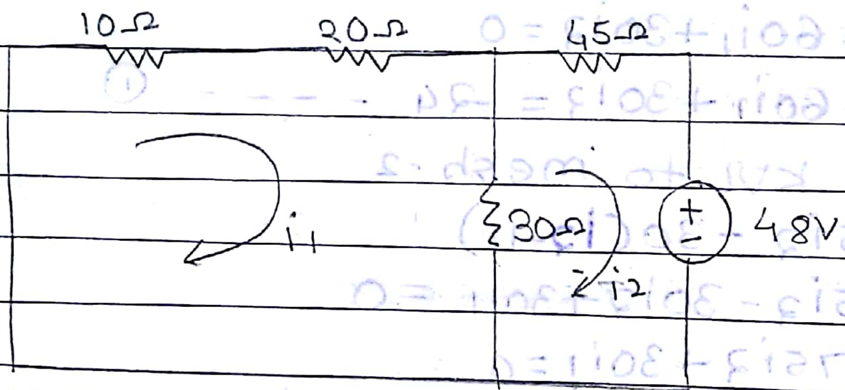
$\boxed{i_1=-1.5833V}$ $\boxed{i_2=0.4166V}$ $\boxed{i_3=0.166V}$

$\therefore V_x = 20i_2 = 20\times(0.4166) = \boxed{8.332V.}$

iii) Voltage source 48V - acting alone

current source 2A - open circuit

voltage source 24V - short circuit.



KVL to mesh-1

$-10i_1-20i_1-30(i_1-i_2)=0$

$-10i_1-20i_1-30i_1+30i_2=0$

$-60i_1+30i_2=0$ —— (1)

KVL to mesh-2

$-45i_2-24-30(i_2-i_1$

$-45i_2-30i_2+30i_1=24$

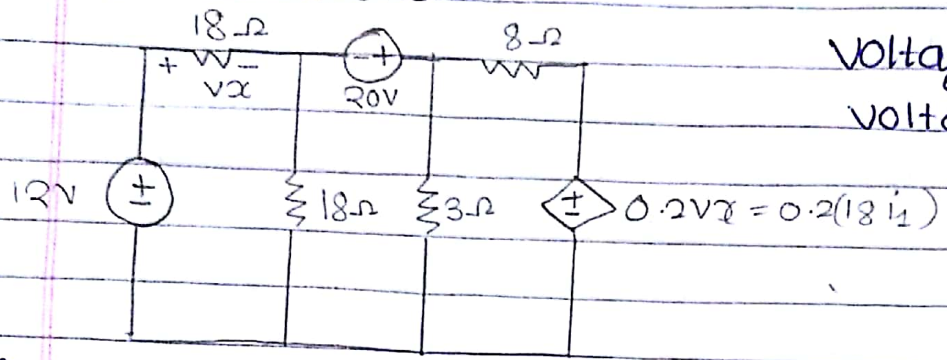
$-75i_2+30i_1=24$ (2)

$\therefore \boxed{i_1=-0.2V}$ $\boxed{i_2=-0.4V}$

$\therefore V_x = 20\times(-0.4) = \boxed{-8 \text{ volts}}$

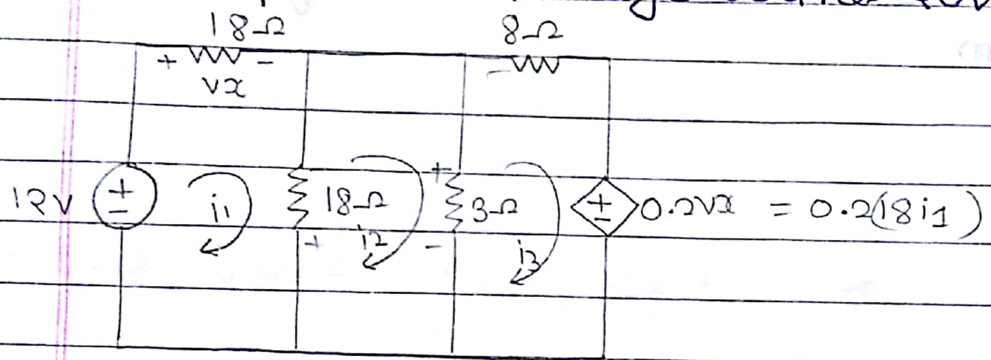
$\therefore V_x = 8.332-8+10 = 10.332V.$

Q Using superposition theorem find the C/n in 8 Ω resistor.



Voltage source 12V.
Voltage source 20V

i) Independent voltage source 12V - Acting alone
Independent voltage source 20V - short circuit.



$-18i_1 - 18(i_1 - i_2) + 12 = 0$

$-18i_1 - 18i_1 + 18i_2 = -12$

$-36i_1 + 18i_2 = -12$ —(1)

$-3(i_2 - i_3) - 18(i_2 - i_1) = 0$

$-3i_2 + 3i_3 - 18i_2 + 18i_1 = 0$

$18i_1 - 21i_2 + 3i_3 = 0$ —(2)

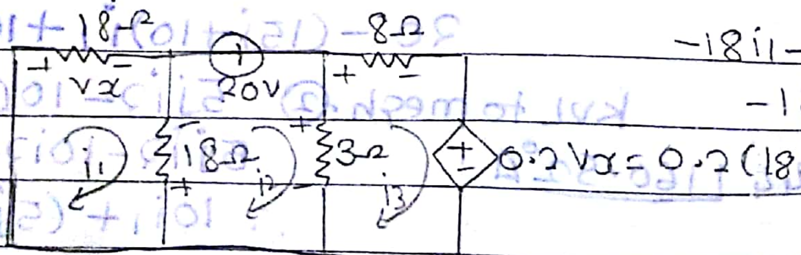
$-8i_3 - 3(i_3 - i_2) - 0.2(18i_1) = 0$

$-8i_3 - 3i_3 + 3i_2 - 3.6i_1 = 0$

$-11i_3 + 3i_2 - 3.6i_1 = 0$ —(3)

$\boxed{i_1 = 0.576A}$ $\boxed{i_2 = 0.4859A}$ $\boxed{i_3 = -0.056A}$

ii) Independent Voltage source 20V - acting alone
Independent Voltage source 12V - short circuit



$-18i_1 - 18(i_1 - i_2) = 0$

$-18i_1 - 18i_1 + 18i_2 = 0$

$-36i_1 + 18i_2 = 0$ —(1)

$20 - 3(i_2 - i_3) - 18(i_2 - i_1) = 0$

$-3i_2 + 3i_3 - 18i_2 + 18i_1 = -20$

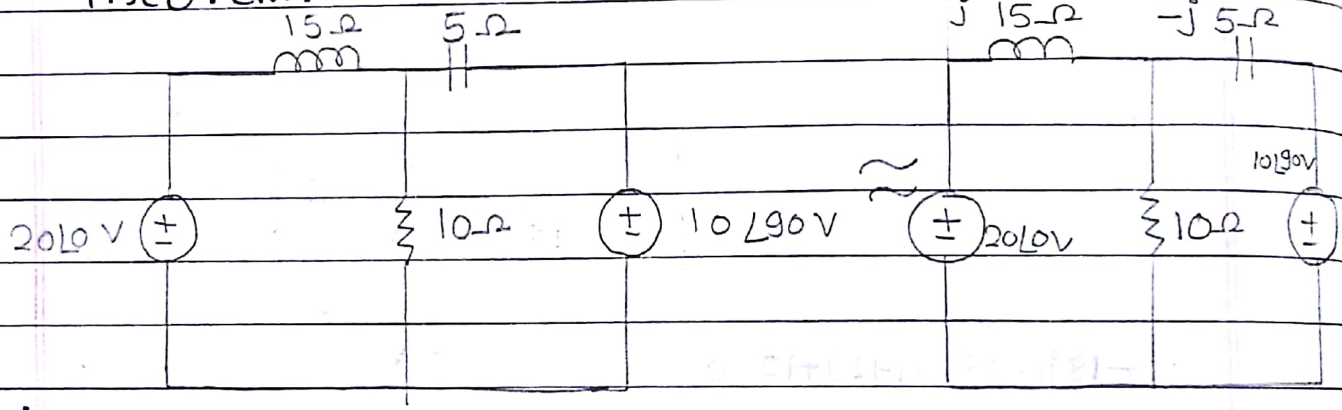
$18i_1 - 21i_2 + 3i_3 = -20$ —(2)

$-8i_3 - 3.6i_1 - 3i_3 + 3i_2 = 0$

$-3.6i_1 + 3i_2 - 11i_3 = 0$ —(3)

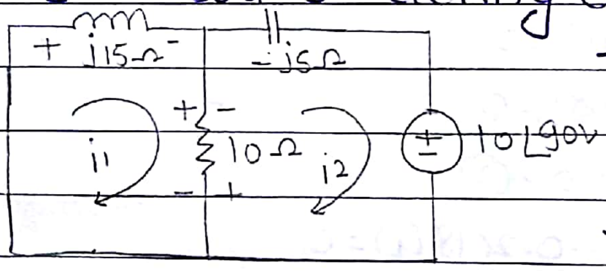
$i_1 = 0.856$
$i_2 = 1.713$
$= 0.186$

3. Determine the current through 10 Ω resistor of the network shown below using superposition Theorem.



Sol[n] i) 20∠0V source - short
10∠90° v source - acting along

ANS. 1.44∠160.52



$-(+15j i_1) - 10(i_1 - i_2) = 0$

$-15j i_1 - 10 i_1 + 10 i_2 = 0$

$-(15j+10)i_1 + 10 i_2 = 0$ — ①

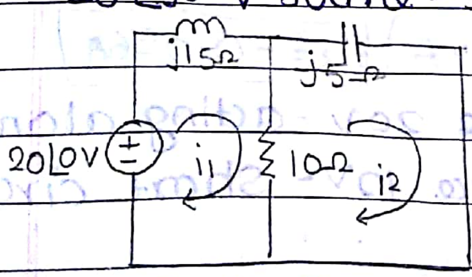
$-(-5j i_2) - 10\angle 90 - 10(i_2 - i_1) = 0$

$5j i_2 - 10\angle 90 - 10 i_2 + 10 i_1 = 0$

$\therefore 10 i_1 + (5j-10) i_2 - 10\angle 90 = 0$

$\therefore 10 i_1 + (5j-10) i_2 = -10j$ — ②

ii) 20∠0V Source - acting alone
10∠90° V source - short



KVL:- $20\angle 0 - j15 i_1 - 10(i_1 - i_2) = 0$

$20 - j15 i_1 - 10 i_1 + 10 i_2 = 0$

$20 - (15j+10) i_1 + 10 i_2 = 0$ — ③

KVL to mesh ② $5j i_2 - 10(i_2 - i_1) = 0$

$5j i_2 - 10 i_2 + 10 i_1 = 0$

$\therefore 10 i_1 + (5j-10) i_2 = 0$ — ④

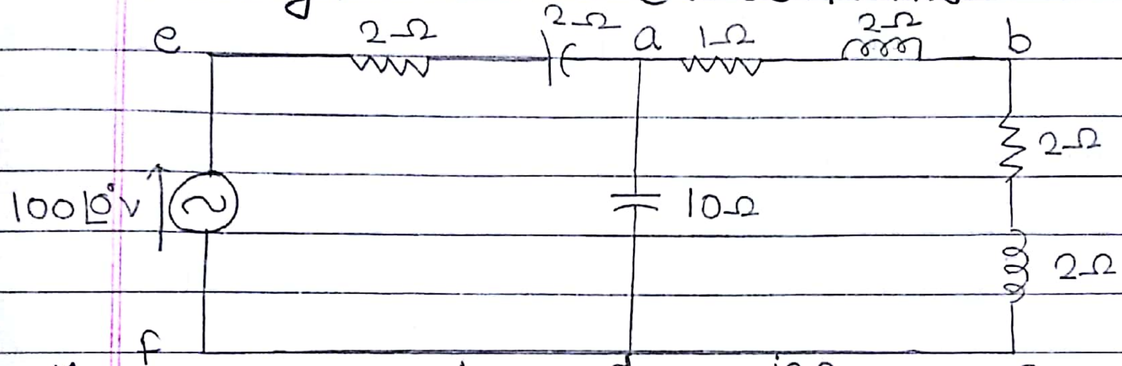
solving ①, ②, ③, ④
we get - $\boxed{I_{10\Omega} = 1.44\angle 160.52^\circ A}$

$25+25i \div (0.37+0.19i)$
$= 80.92$
$+ 26.01i$

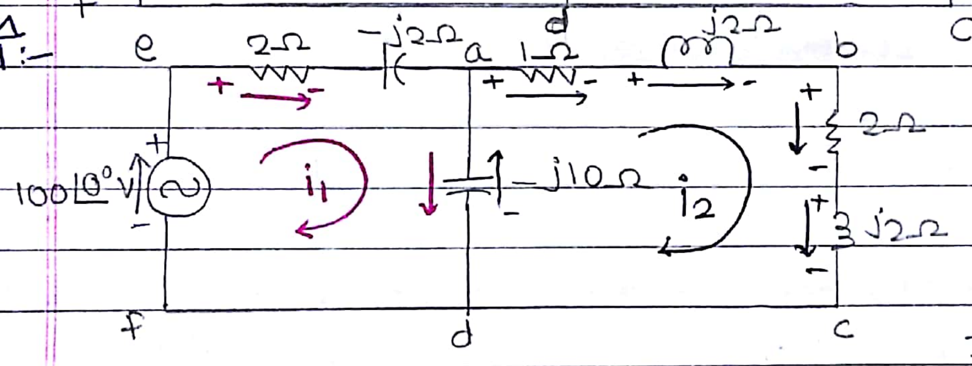
2. Reciprocity Theorem :-

Statement:- "If any linear, bilateral network containing only one independent source, the ratio of exicitation to response remains constant when their positions are interchanged."

1. Verify reciprocity thm. by finding a c/n through the branch bc in the ckt shown below.



Sol:-



$I = 16.86 \angle -35.3 \; A$

mesh-1 $\quad -2i_1 - j2i_1 + j10(i_1 - i_2) + 100\angle 0 = 0$

$-2i_1 - j2i_1 + 10ji_1 - j10i_2 = -100$

$-2i_1 - (j2 - 10j)i_1 - j10i_2 = -100 \quad (1)$

$-2i_1 + 8ji_1$
$(-2+8j)i_1$

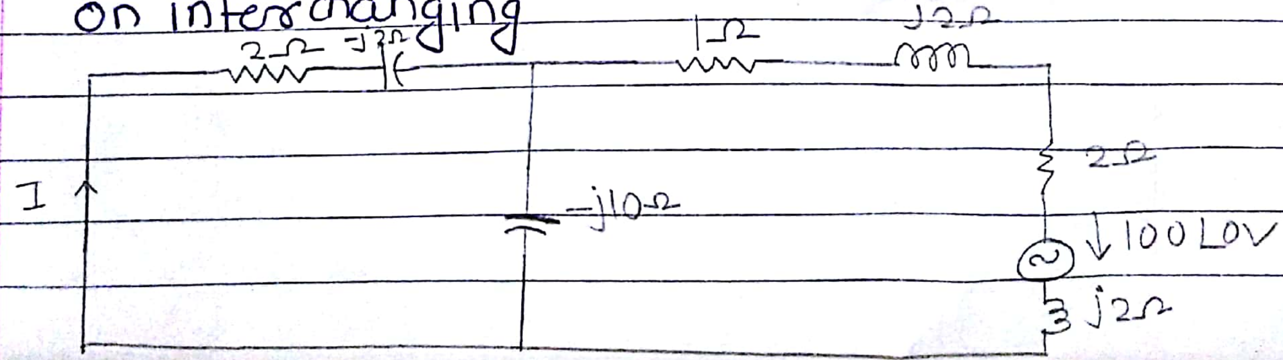
mesh-2 $\quad -i_2 - j2i_2 - 2i_2 - j2i_2 + j10(i_2 - i_1) = 0$

$-i_2 + (-j2 - 2 - j2 + 10j)i_2 - 10ji_1 = 0$

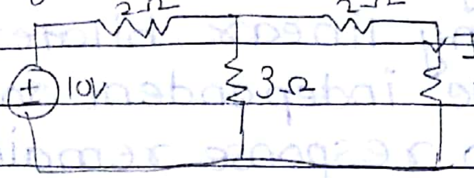
$\therefore (-3+6j)i_2 - 10ji_1 = 0 \quad (2)$

Solving (1) & (2) $I = 16.89 \angle -35.31° A.$

on interchanging



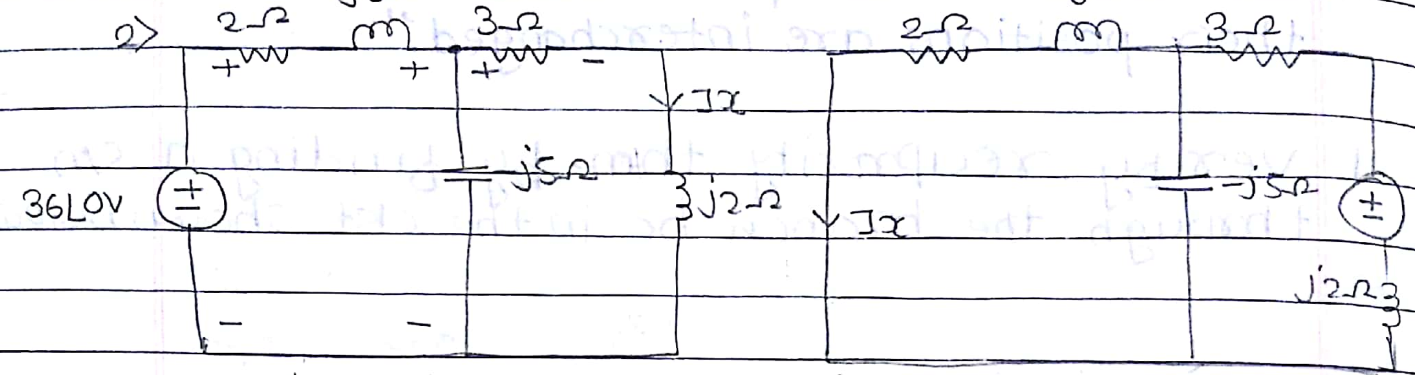
1) verify R.T. for Vtg V & c/n I in network shown below



$-5i_1+3i_2=-10$
$3i_1-7i_2=0$
j3Ω

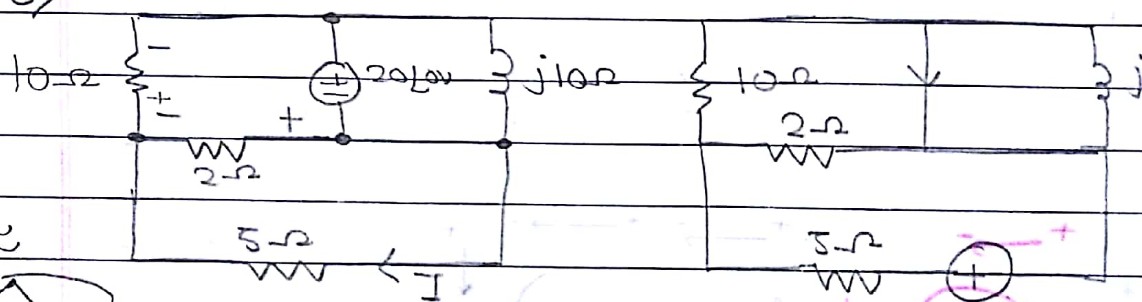
$-5i_1+3i_2=0$
$3i_1-7i_2=10$
j3Ω

2)



Ans $I_x = 6.49 \angle -64.35$ A

3)



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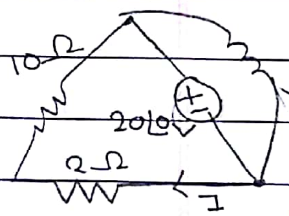
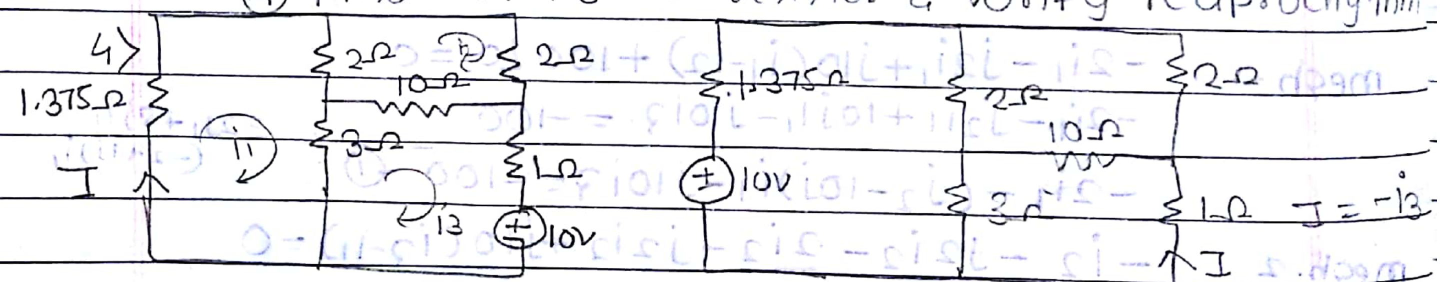


④ Find c/n 1.375Ω resistor & verify reciprocity thm

4)



$I = i_1$

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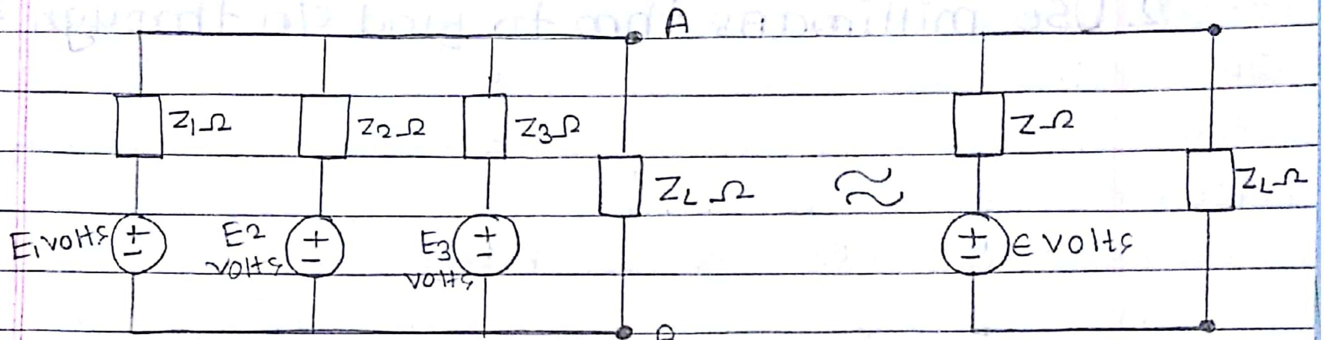
Milliman's Theorem:-

Statement:- "Several voltage sources $E_1, E_2, E_3 \ldots E_n$ with their internal impedances $Z_1, Z_2, Z_3 \ldots Z_n$ connected in parallel may replaced by a single voltage source E with the internal impedance Z." consider the circuit below."



$$E = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3} = \frac{\Sigma EY}{\Sigma Y} \quad (2)$$

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{1/Z_1 + 1/Z_2 + 1/Z_3} \quad (1)$$

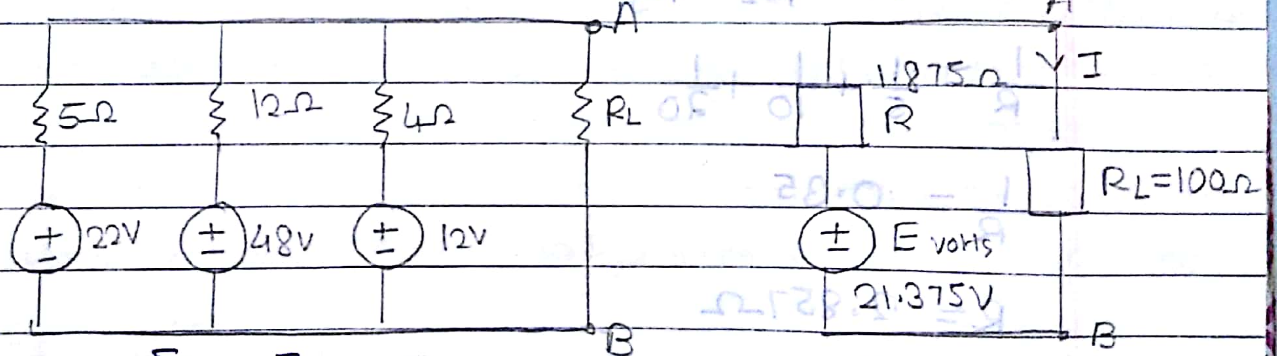
$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \quad (3)\ Z \text{ in } \Omega \text{ or ohm}$$

$\therefore Y = 1/Z$ ℧ or mho

or $Y = Y_1 + Y_2 + Y_3$ — (4) Y in ℧ or mho

$$I = \frac{Z + Z_L}{E}$$

1. Use millimans thm & find c/n through load resistance $R_L = 100\,\Omega$ shown in ckt below.



$$E = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}}{1/R_1 + 1/R_2 + 1/R_3}$$

$$E = \frac{22/5 + 48/12 + 12/4}{1/5 + 1/12 + 1/4}$$

$$E = \frac{4.4 + 4 + 3}{0.2 + 0.083 + 0.25}$$

$$\boxed{\therefore E = 21.38V}$$

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{12} + \frac{1}{4}$$

$$\therefore \frac{1}{R} = 0.533$$

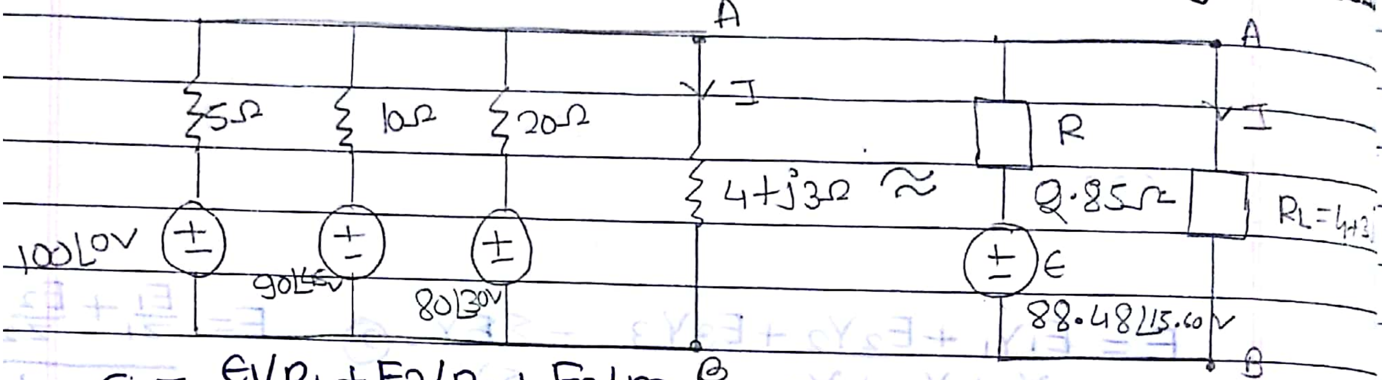
$$\boxed{\therefore R = 1.875\Omega}$$

$$I = \frac{E}{R+R_L}$$

$$= \frac{21.38}{1.875+100}$$

$$\boxed{I = 0.209A}$$

2. Use millimans thm to find c/n through 4+j3Ω



$$E = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3}$$

$$E = \frac{100\angle 0/5 + 90\angle 45/10 + 80\angle 30/20}{1/5 + 1/10 + 1/20} = \frac{20 + 6.36 + 3.464}{0.35}$$

$$E = 85.21$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20}$$

$$\frac{1}{R} = 0.35$$

$$R = 2.857\Omega$$

$$I = \frac{E}{R+R_L}$$

$$I = \frac{85.21}{2.857+(4+3j)}$$

$$I = 10.43 - 4.56i$$

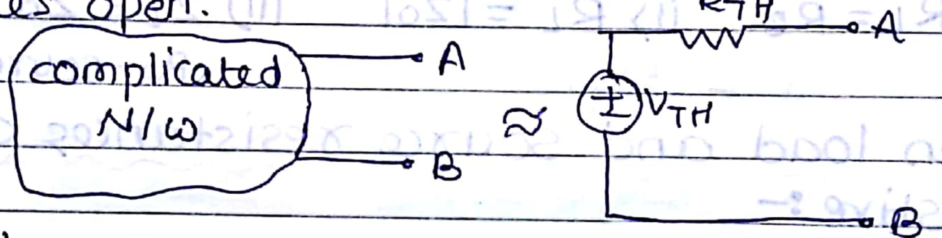
26-09-15

Network Theorems - 2

Thevenin Theorem:-

Statement:- Any linear bilateral N/w with two terminals A and B can be reduced to a simpler circuit consisting of single independent vtg. source (V_{TH}) and resistance (R_{TH}) (or impedance Z_{TH} in AC circuits) in series with it.

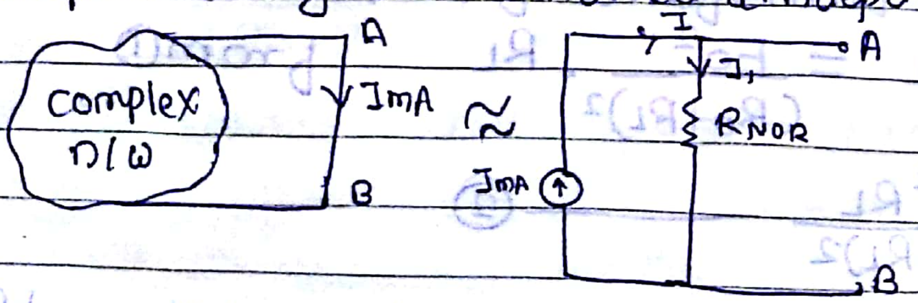
The value of independent vtg. source is the vtg. a/c terminals A & B. and the values of resistance (or impedance or ac ckt) is equivalent resistance (or impedance or AC ckt) of the circuit seen from the terminals A&B with all independent vtg. sources shorted & independent c/n sources open.



Norton Theorem:-

tatement:- Any linear bilateral n/w with two terminals A & B can be reduced to a simpler ckt consisting of single independent c/n source (V_{TH}) & resistance (R_{TH}) (or impedance Z_{TH} in ac ckts) in $\parallel^{el}$ with it.

* The value of independent c/n source is the c/n through terminals A & B & the value of resistance (or impedance on ac ckt) is equivalent resistance (or impedance in ac ckt) of the ckt seen from the terminals A&B with all independent vtg. source shorted & independent c/n sources open.



$\therefore V_A = 94.58\angle 58.98$

$E_0 = V_A - 0$

$\therefore E_0 = 94.58\angle 58.98\,V$

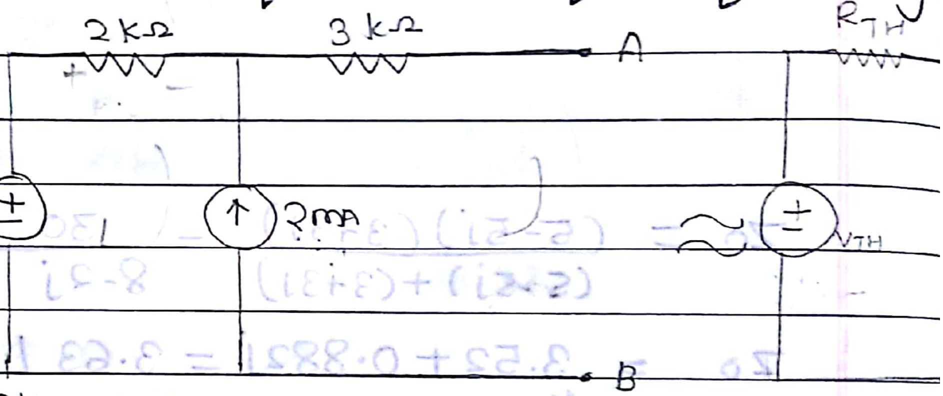
$\therefore P_{max} = \dfrac{E_0^2}{4R_L}$

$= \dfrac{(94.58)^2 \angle 58.98}{4\times 3.52}$ don't consider angle

$P_{max} = 634.32W$

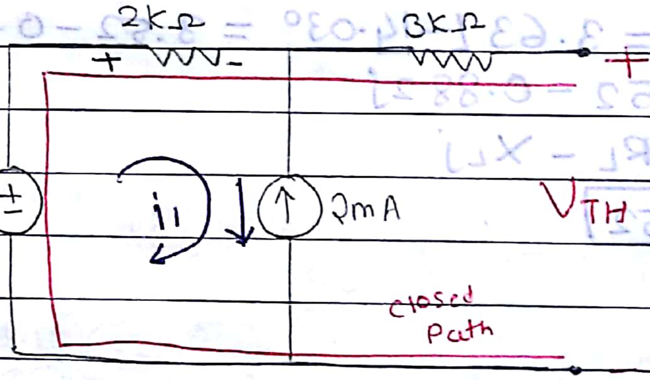
$\boxed{P_{max} = 614.71W}$

3. Find the Thevenin equivalent of the following Ckt.



Sol^n

To find V_{TH}:



C/n source is on perimeter. writing eq^n for that i.e $\boxed{i_1 = -2mA}$ —

3kΩ doesn't effect V_{TH} ∵ no c/n is flowing

$\cancel{-2i_1 - i_1 + 4 = 0}$

$\cancel{\therefore -2k(-2m) - 2m + 4 = 0}$

$\cancel{= 4m - 2m + 4}$

$\cancel{= 2m + 4}$

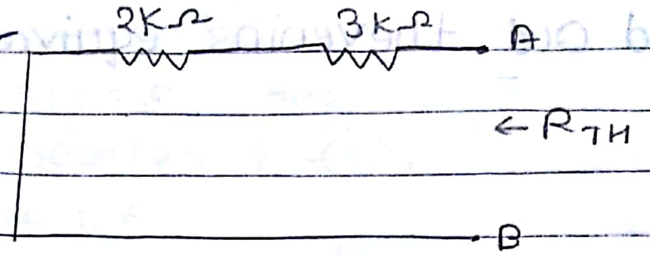
$-2ki_1 + 4 = 0 - V_{TH}$

$-2k(-2m) + 4 = 0 - V_{TH}$

$4 + 4 = 0 - V_{TH}$

$\therefore \boxed{V_{TH} = 8V}$

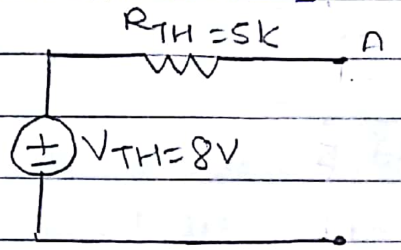
To find R_{TH}:-



$R_{TH} = 2K + 3K = 5K$

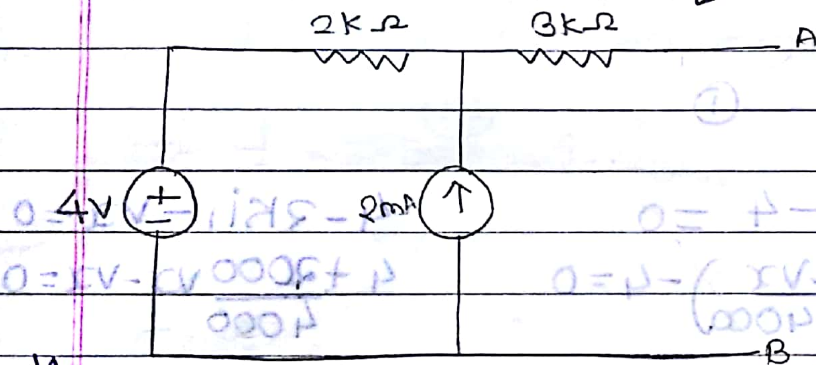
$\therefore R_{TH} = 5K\Omega$

Thevenins equivalent

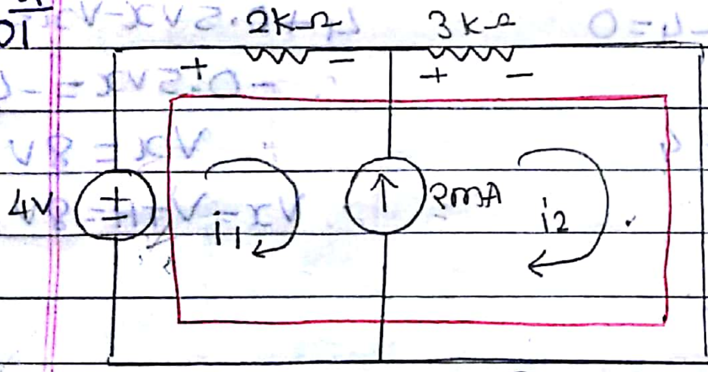


I =

4. Find the Nortons equivalent of the following



Sol



$i_2 - i_1 = 2m$

$-2k(i_1) - 3k(i_2) + 4 = 0$

$4k - 6ki_2 + 4 = 0$

$4004 = 6000 i_2$

$\therefore i_2 =$

$i_2 - i_1 = 2m$ — ①

$4 - 2ki_1 - 3ki_2 = 0$

$\therefore -2ki_1 - 3ki_2 = -4$ — ②

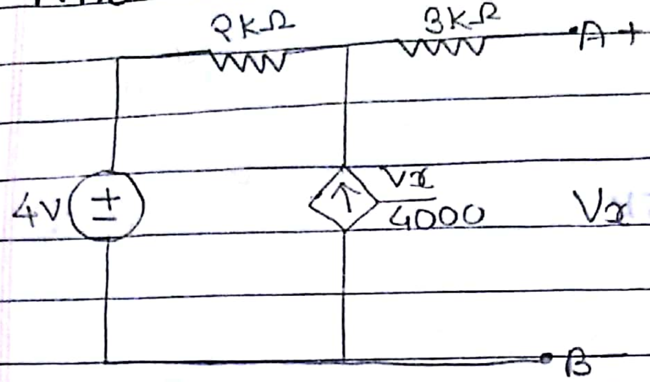
$\therefore i_1 = -0.4mA$

$\therefore i_2 = 1.6mA$

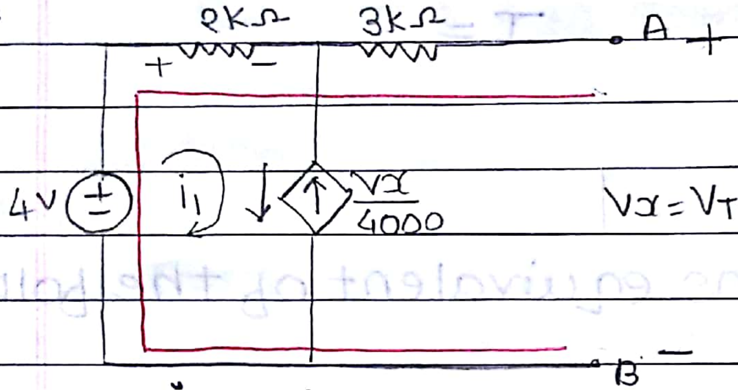
Nortons equivalent $= i_2 = 1.6mA$

ype-2 problems

5. Find out thevenins equivalent circuit.



Sol^n.



$Vx = V_{TH} = -2.66V$

$$i_1 = \frac{-Vx}{4000} \quad \text{—— (1)}$$

$$\cancel{-Vx + 2Ki_1 - 4 = 0}$$

$$\cancel{-Vx + 2000\left(\frac{-Vx}{4000}\right) - 4 = 0}$$

$$\cancel{-Vx - \frac{2000}{4000}Vx - 4 = 0}$$

$$\cancel{-Vx - 0.5Vx = 4}$$

$$\cancel{-1.5Vx = 4}$$

$$\cancel{\therefore Vx = -\frac{4}{1.5}}$$

$$\cancel{\boxed{\therefore Vx = -2.66V}}$$

$$4 - 2Ki_1 - Vx = 0$$

$$4 + \frac{2000}{4000}Vx - Vx = 0$$

$$4 + 0.5Vx - Vx = 0$$

$$\therefore -0.5Vx = -4$$

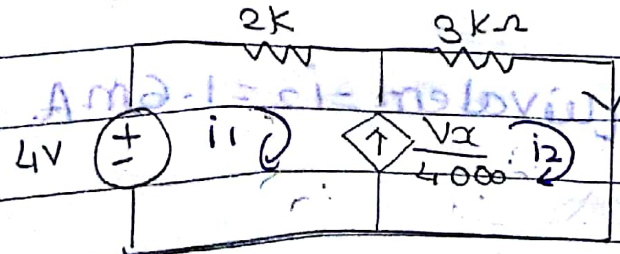
$$\boxed{\therefore Vx = 8V}$$

$$\boxed{\therefore Vx = V_{TH} = 8V}$$

$\therefore V_{TH} = Vx = -2.66V$

To find R_{TH}: we have $R_{TH} = \frac{V_{TH}}{I_{NOR}}$

To find I_{NOR}:



$$i_2 - i_1 = \frac{Vx}{4000}$$

$$\therefore i_2 - i_1 = \frac{8}{4000}$$

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$\therefore\ i_2 - i_1 = 2mA$. —— (1)

$4 - 2ki_1 - 3ki_2 = 0$

$\therefore\ -2000i_1 - 3000i_2 = -4$ —— (2)

$\therefore\ \boxed{i_1 = -0.4mA}$

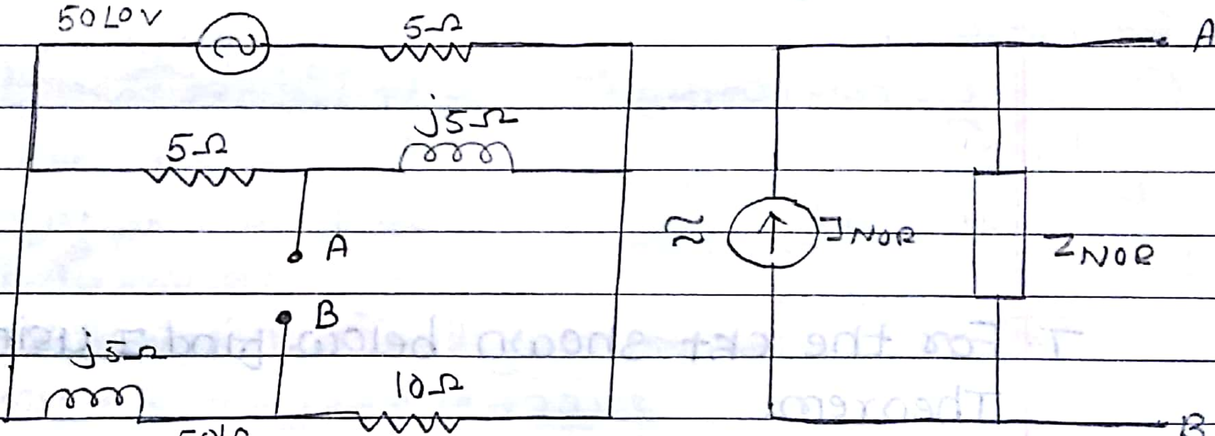
$\boxed{i_2 = 1.6mA}$ $\boxed{I_{NOR} = i_2 = 1.6mA}$

$\therefore\ R_{TH} = \frac{V_{TH}}{I_{NOR}}$

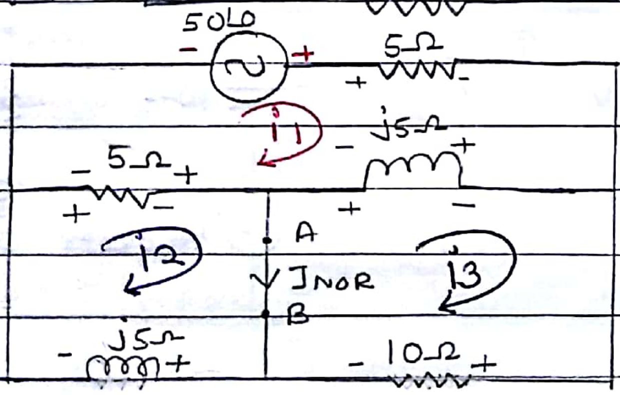
$= \frac{8}{1.6 \times 10^{-3}}$

$\boxed{R_{TH} = 5k\Omega}$

6. Obtain norton equivalent at the terminals AB of the ckt shown below



Sol.



$50\angle 0 - 5i_1 - 5j(i_1 - i_3) - 5(i_1 - i_3)$

$50 - 5i_1 - 5ji_1 + 5ji_3 - 5i_1$

$+ 5i_3 = 0$

$50 - 10i_1 - 5ji_1 + 5ji_3 + 5i_3 = 0$

$50 - i_1(10 + 5j) + i_3(5 + 5j) = 0$

$\therefore\ -i_1(10+5j) + i_3(5+5j) = -50$ —— (1)

$-5(i_2 - i_1) - 5ji_2 = 0$

$-5i_2 + 5i_1 - 5ji_2 = 0$

$5i_1 - i_2(5+5j) = 0$ —— (2)

$-5j(i_3 - i_1) - 10i_3 = 0$

$-5ji_3 + 5ji_1 - 10i_3 = 0$

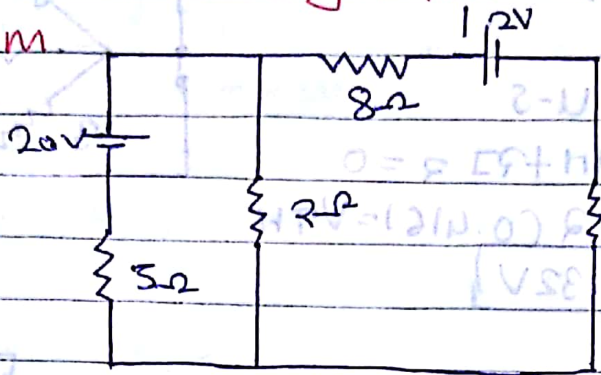
$5ji_1 - i_3(10+5j) = 0$ —— (3)

$$\Delta = \begin{vmatrix} -(10+5j) & 0 & (5+5j) \\ 5 & -(5+5j) & 0 \\ 5j & 0 & (10+5j) \end{vmatrix} = \begin{bmatrix} -50 \\ 0 \\ 0 \end{bmatrix}$$

$\Delta = (-10 - 5j)$

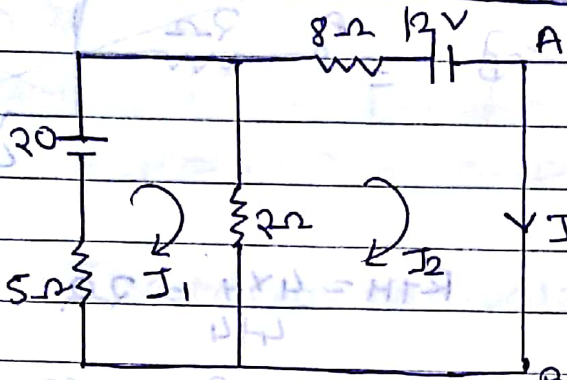
⑨

1. Find c/n through 10Ω resistance using Nortons thm.



$R_{TH} = \frac{V_{TH}}{I_N}$

Sol:



$I_N = I_2$

$-5I_1 + 20 - 2(I_1 - I_2) = 0$

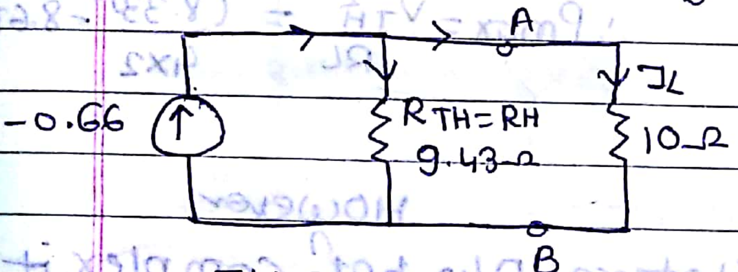
$+7I_1 - 2I_2 = 20$ ~ ①

$2(I_1 - I_2) - 8I_2 - 12 = 0$

$2I_1 - 10I_2 = 12$ ~ ②

$I_1 = 2.66A \quad I_2 = 0.66A$

$\therefore I_N = I_2 = -0.66A$



$I_L = I_N \times \frac{R_{TH}}{R_{TH} + R_L}$

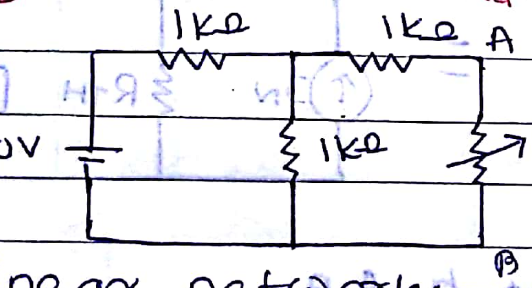
$5 \| 2 + 8$

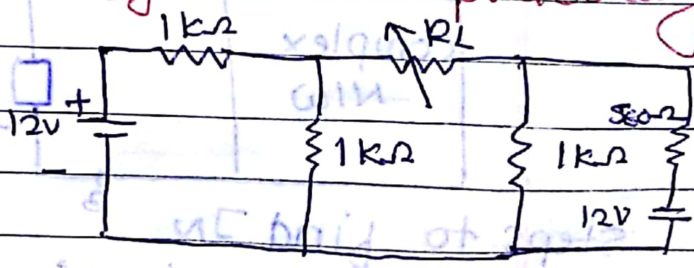
$R_{TH} = \frac{10}{7} + 8 = 9.4286$

$I_L = 0.66 \times \frac{9.43}{(9.43 + 10)}$

$\therefore I_L = -0.33A$

2 Determine R_L so that max. power is transferred from source to load & verify the source practically.





i) linear network:- A N/W whose parameters doesnot change due to change in

ii) Bilateral - A ckt whose characteris doesnot change due to change of direction of vtg & c/n

ex transmission lines

The rel^n bet^n vtg & c/n is a linear curve. bilateral means rel^n bet^n vtg & c/n doesn't change in both direction in N/W.

$\therefore I_1 = 1.2A.$

$I_2 = 0.416A.$

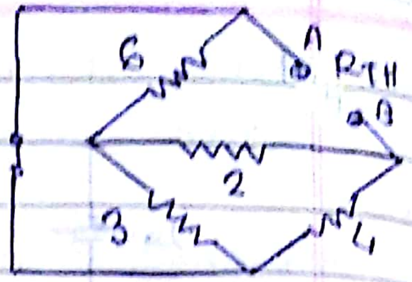
KVL to s-r-u-s

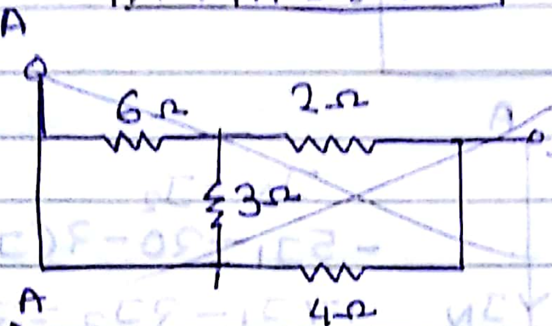
$$6I_1 - V_{TH} + 2I_2 = 0$$

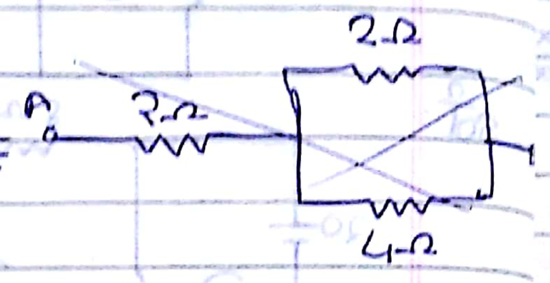
$$6(1.2) + 2(0.416) = V_{TH}$$

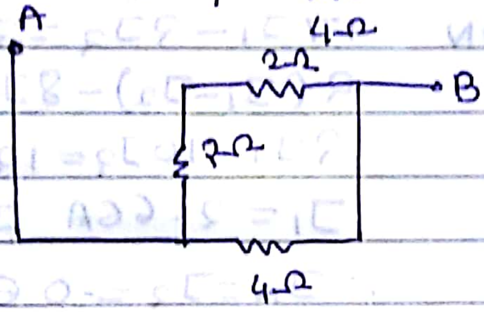
$$\boxed{\therefore V_{TH} = 8.32V}$$

→ Next:-







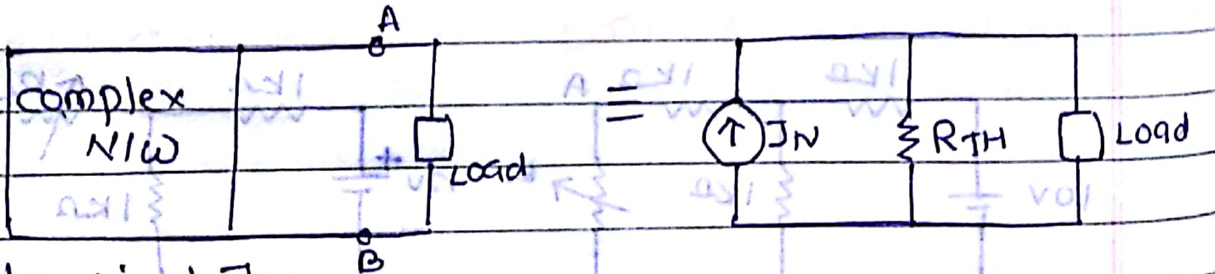


$$R_{TH} = \frac{4\times4}{4+4} = 2\Omega$$

$$\therefore P_{max} = \frac{V_{TH}^2}{R_L} = \frac{(8.33)^2}{4\times2} = 8.67W$$

Nortons theorem:—

Statement:- Any linear bilateral n/w however complex it may be, it can be replaced by an equivalent ckt with c/n source I_N is parallel with the resistance R_{TH}, connec-ted to the load.



Steps to find I_N

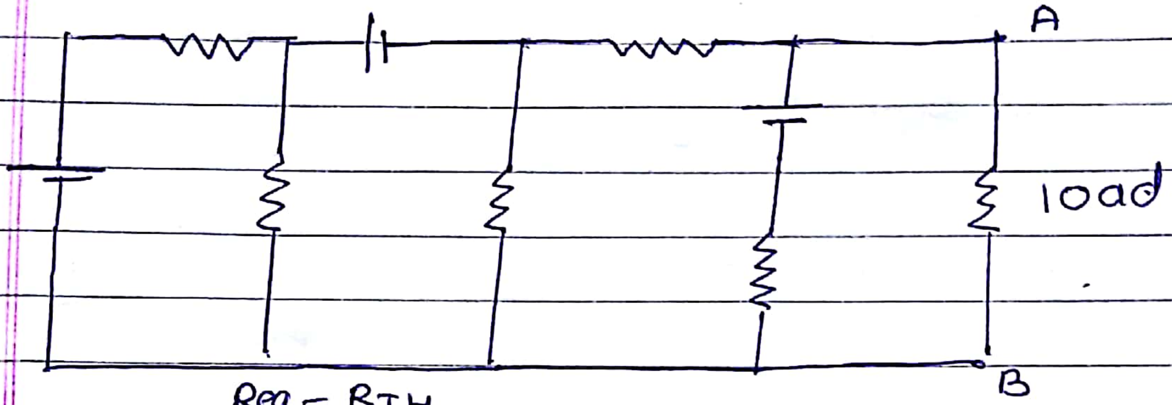
1. Remove load & indicate by A, B
2. Short ckt the terminals A & B
3. Find the short ckt I_N Bet^n A & B

①

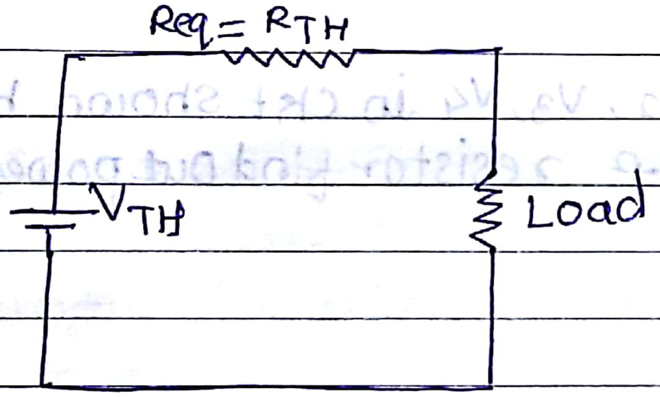
Network Theorem - 2

1) Thevenins thm:-

Statement:- Any linear bilateral n/w however complex it may be connected to a load can be replaced by a voltage source V_{TH} & an equivalent resistance seen from the load terminals connected across the same load terminals.



$\approx$



$V_{TH} \rightarrow$ Thevenins vtg.
$R_{TH} \rightarrow$ Thevenins resist.

To obtain V_{TH}:-

1. First remove the load & indicate terminals as AB.
2. Obtain the vtg. a/c AB (is open ckt. vtg.)
3. This open ckt. is considered as V_{TH}.

To obtain R_{TH}:-

1. Remove the load an indicate terminals as AB.
2. make vtg. source zero by short circuiting & current source zero by open ckting.
3. Simplify the resistances to obtain R_{AB}
4. The R_{AB} is equivalent resistance of the ckt. i.e R_{TH}.

* **Maximum power transfer theorem:—**

Statement:— "In any linear bilateral network, maximum power is transferred from source to load when

i) The load resistance (R_L) is equal to the source resistance (R_0)

ii) The load resistance (R_L) is equal to the magnitude of source impedance ($|Z_0|$)

iii) The load impedance (Z_L) is equal to complex conjugate of source impedance (Z_0^*).

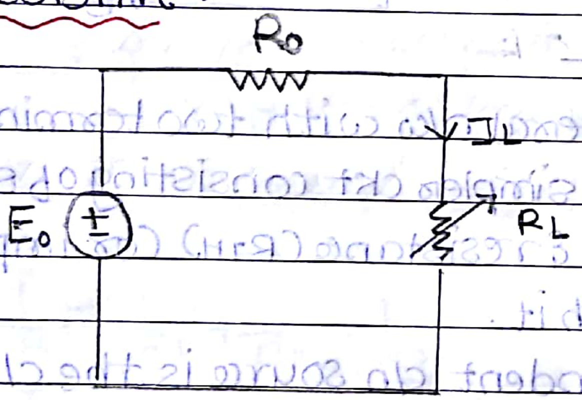
Maximum power transferred when,

i) $R_L = R_0$ ii) $R_L = |Z_0|$ iii) $Z_L = Z_0^*$

Proof:-

R_0 - source resistance

i) When load and source resistances are purely resistive:-



From above circuit

$$I_L = \frac{E_0}{R_0 + R_L} \quad (1)$$

The power transferred from source to load is given by

$$P = I_L^2 R_L = \frac{E_0^2}{(R_0+R_L)^2} \cdot R_L \quad \text{from (1)}$$

$$P = \frac{E_0^2 R_L}{(R_0+R_L)^2} \quad (2)$$

Power transferred is maximum when $\frac{dP}{dR_L} = 0$

$$\frac{dP}{dR_L} = \frac{(R_0+R_L)^2 E_0^2 - E_0^2 R_L \times 2(R_0+R_L)}{(R_0+R_L)^4} = 0$$

linear are those N/w which obeys ohms law.
Bilateral are those in which c/n can flow in both direction

$\therefore (R_0+R_L)^2 E_0^2 - E_0^2 R_L \times 2(R_0+R_L) = 0$

$(R_0+R_L)^2 - 2R_L(R_0+R_L) = 0$ ~~$\therefore$ for max power transfer $R_L = R_0$~~

or $\boxed{R_L = R_0}$

Hence maximum power transferred to load when $R_L = R_0$ under this condition

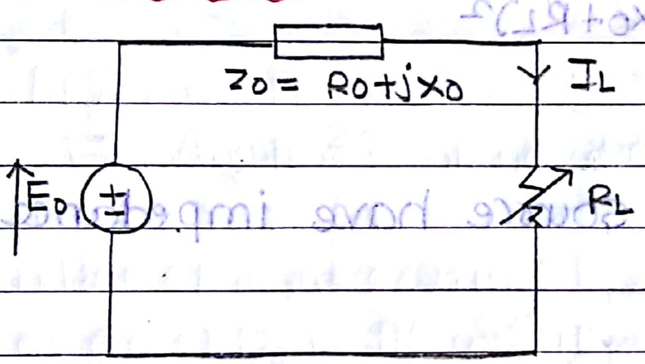
$$I_L = \frac{E_0}{R_0+R_L} = \frac{E_0}{2R_L} \quad \text{--- (3)} \quad \because R_L = R_0$$

The max. power transferred is given by

$$P_{max} = I_L^2 R_L = \frac{E_0^2}{4R_L^2} \times R_L = \frac{E_0^2}{4R_L}$$

$$\boxed{\therefore P_{max} = \frac{E_0^2}{4R_L}} \quad \text{--- (4)}$$

ii) When load is purely resistive & source has impedance :-



let $Z_0 = R_0 + jX_0$ be the internal impedance of the source E_0 as shown in above fig.

$$I_L = \frac{E_0}{Z_0+R_L} = \frac{E_0}{R_0+jX_0+R_L} = \frac{E_0}{(R_0+R_L)+jX_0}$$

$$I_L = \frac{E_0}{\sqrt{(R_0+R_L)^2+X_0^2}} \quad \text{--- (5)}$$

The power consumed by the load is given by

$$P = I_L^2 R_L$$

$$P = \frac{E_0^2}{\left(\sqrt{(R_0+R_L)^2+X_0^2}\right)^2} \cdot R_L$$

$$P = \frac{E_0^2 \cdot R_L}{(R_0+R_L)^2+X_0^2} \quad \text{--- (6)}$$

*-conjugate

a+jb its conjugate is a-jb

10∠45 its conjugate is 10∠-45

Diff. ⑥ w.r.t. R_L

$$\frac{dP}{dR_L} = \frac{\left[(R_o+R_L)^2+X_o^2\right]E_o^2 - E_o^2R_L\left(2.(R_o+R_L)\right)}{\left[(R_o+R_L)^2+X_o^2\right]^2} = 0$$

$$\left[(R_o+R_L)^2+X_o^2\right] - R_L\,2(R_o+R_L) = 0$$

$$R_o^2+R_L^2+\cancel{2R_oR_L}+X_o^2-\cancel{2R_oR_L}-2R_L^2 = 0$$

$$R_o^2+X_o^2-R_L^2=0 \Rightarrow R_L^2 = R_o^2+X_o^2$$

$$R_L^2 = R_o^2+X_o^2$$

$$\therefore R_L = \sqrt{R_o^2+X_o^2}$$

$$\boxed{\therefore R_L = |Z_o|}$$

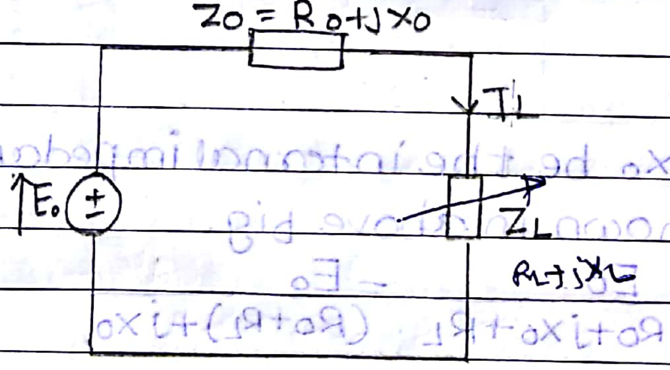
Thus power transferred from source to load is maximum when $R_L = \sqrt{R_o^2+X_o^2}$ or $R_L = |Z_o|$.

So $I_L = \dfrac{E_o}{R_o+jX_o+R_L}$ ⑦

and $P_{max} = I_L^2 R_L$

$$P_{max} = \frac{E_o^2}{(R_o+jX_o+R_L)^2}.R_L$$

iii) When both load & source have impedance



$$I_L = \frac{E_o}{(R_L+R_o)+j(X_L+X_o)}$$

$$= \frac{E_o}{\sqrt{(R_o+R_L)^2+(X_L+X_o)^2}}$$

$$P = I_L^2R_L = \frac{E_o^2}{(R_o+R_L)^2+(X_L+X_o)^2}.R_L$$ ①

Power transferred is maximum when $X_L = -X_o$

$$\therefore P_1 = \frac{E_o^2}{(R_o+R_L)^2}.R_L$$

The power transferred is further maximum when $\frac{dP_1}{dR_L}=0$ ② i.e $R_L = R_o$ proved earlier in (case I)

Dependent source can't be open or closed

$Z_0 = R\ 3+4j$

PAGE NO.: $Z_L = 3-4j$

DATE: / /

From Conditions ① and ② it is evident that the power transferred is maximum when

$R_L + jX_L = R_0 - jX_0$

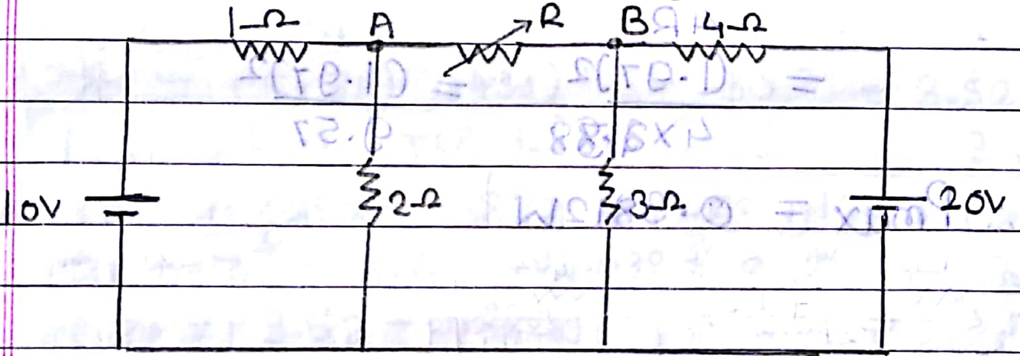
i.e when $Z_L = Z_0^*$

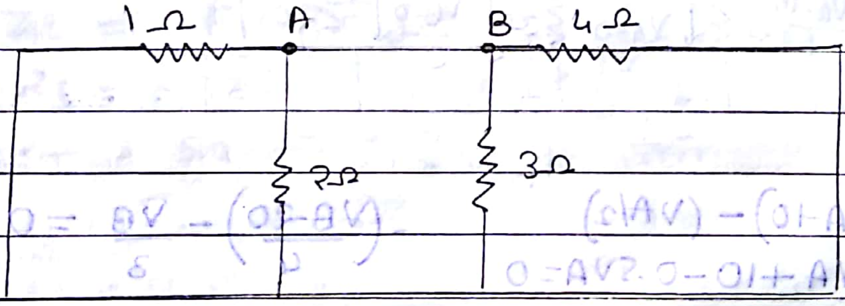
~~The maximum power transferred is~~ $P_m = \frac{E_0^2}{4R_L}$

$\left(\because I_L = \frac{E_0}{2R_L} \right)$

$$P_m = I_L^2 R_L = \left(\frac{E_0}{2R_L}\right)^2 \cdot R_L = \frac{E_0^2}{4R_L^2} \cdot R_L = \frac{E_0^2}{4R_L}$$

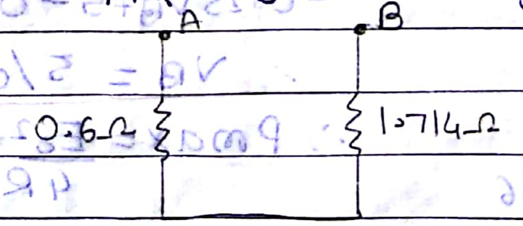
1. Find the value of 'R' for which power transferred across AB of the circuit shown in fig below is max. & the max. power transferred.



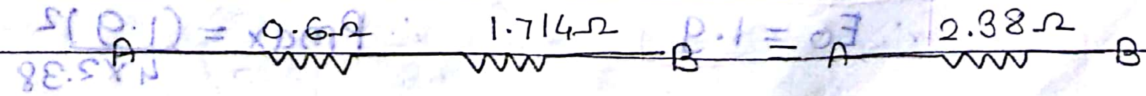


find R_0:- $1\Omega \parallel 2\Omega$ $\qquad$ $3\Omega \parallel 4\Omega$

$\frac{1 \times 2}{1+2} = \frac{2}{3} = 0.6\Omega$ $\qquad$ $\frac{3 \times 4}{3+4} = \frac{12}{7} = 1.714\Omega$

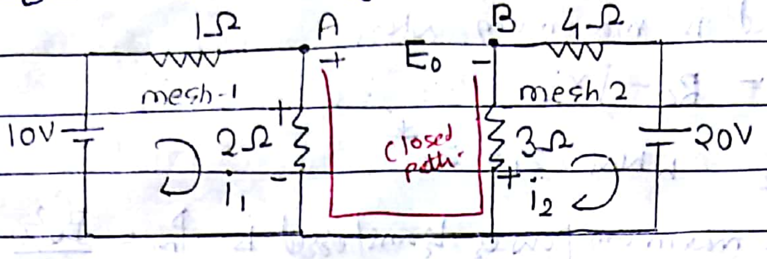


A & B are in series



$\therefore R_0 = 2.38\Omega$

To find E_0 :- vtg. across AB is E_0



$+10-1I_1-2I_1=0$ $\qquad$ $-4I_2-20-3I_2=0$

$-10-3I_1=0$ $\qquad$ $-20-7I_2=0$

$\therefore -10=3I_1$ $\qquad$ $-7I_2=20$

$\boxed{\therefore I_1=3.3V}$ $\qquad$ $\boxed{\therefore I=-2.857}$

$E_0-2i_1-3i_2=0$

$\therefore E_0=2(3.3)+3(-2.857)$

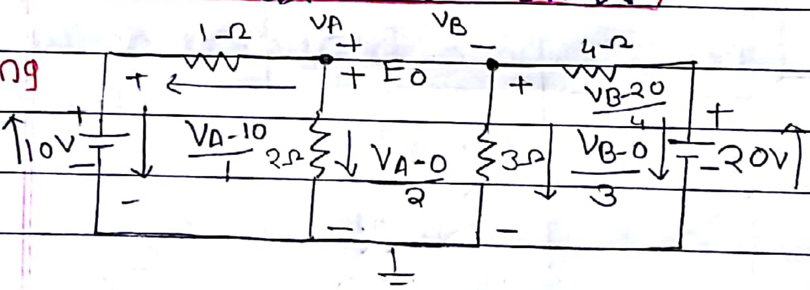
$\boxed{E_0=-1.97V}$

$\therefore P_{max}=\dfrac{E_0^2}{4R}$

$=\dfrac{(1.97)^2}{4\times 2.88}$ $\qquad$ $=\dfrac{(1.97)^2}{9.57}$

$\boxed{\therefore P_{max}=0.3812W}$

or Applying KCL



$-(VA-10)-(VA/2)$ $\qquad$ $-\left(\dfrac{VB-20}{4}\right)-\dfrac{VB}{3}=0$

$\therefore -VA+10-0.5VA=0$

$\therefore 1.5VA=10$ $\qquad$ $-\dfrac{VB+20}{4}-\dfrac{VB}{3}=0$

$\therefore VA=10/1.5$

$\boxed{\therefore VA=6.66}$ $\qquad$ $-0.25VB+5-0.33VB=0$

$\boxed{\therefore VB=5/0.58=8.57}$

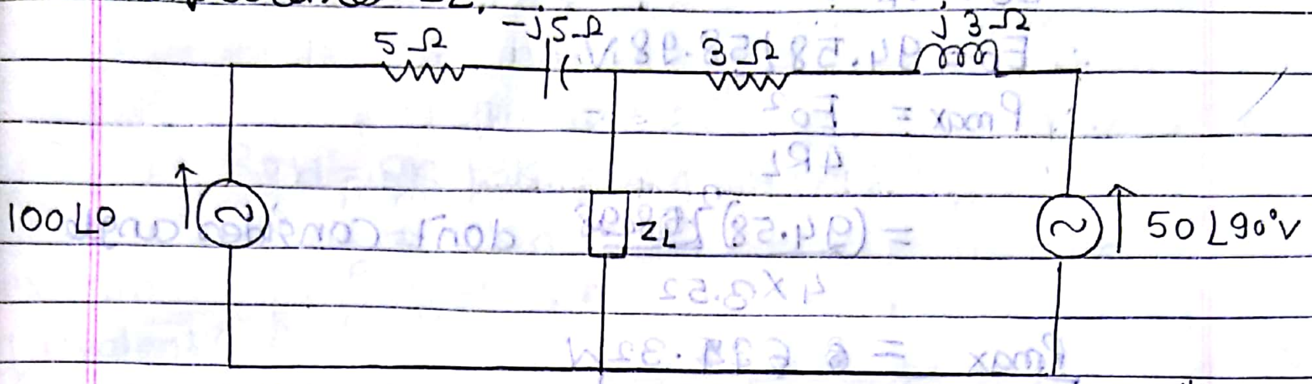
$\therefore E_0=VB-VA$ $\qquad$ $\therefore P_{max}=\dfrac{E_0^2}{4R}$

$\therefore E_0=8.57-6.66$

$\boxed{\therefore E_0=1.9}$ $\qquad$ $\therefore P_{max}=\dfrac{(1.9)^2}{4\times 2.38}$

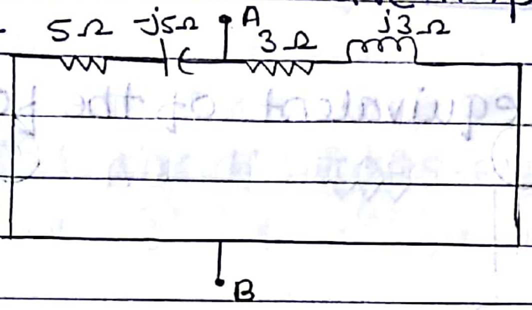
$\boxed{P_{max}=0.3812W}$

2 Find the maximum power transferred to load impedance Z_L.



load & source are impedance. ∴ $Z_L = Z_0^*$

To find Z_0:- and R_L



$$Z_0 = \frac{(5-5j)(3+3j)}{(5-5j)+(3+3j)} = \frac{30}{8-2j} = 3.52+0.882j = 3.63\angle 14.03$$

$$Z_0 = 3.52+0.882j = 3.63\angle 14.03$$

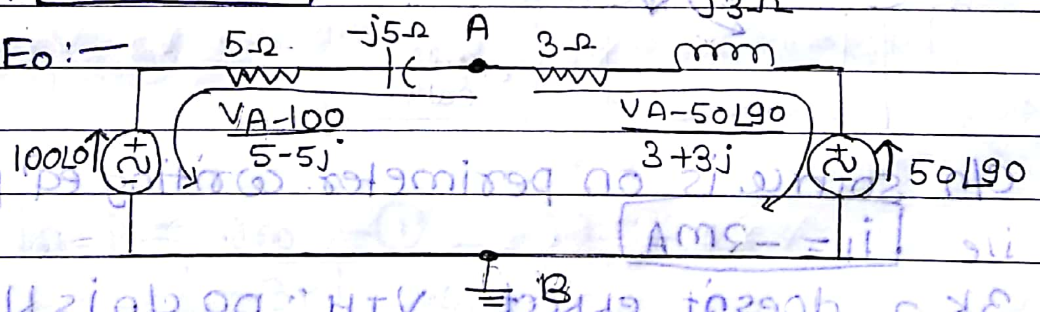
$$Z_L = Z_0^* = 3.63\angle -14.03^\circ = 3.52-0.88j$$

$$\therefore Z_L = 3.52-0.882j$$

$$\therefore Z_L = R_L - X_L j$$

$$\therefore \boxed{R_L = 3.52}$$

To find E_0:—



$$-\frac{(V_A-100)}{5-5j} - \frac{(V_A-50\angle 90)}{3+3j} = 0$$

$$\frac{V_A}{5-5j} - \frac{100}{5-5j} + \frac{V_A}{3+3j} - \frac{50\angle 90}{3+3j} = 0$$

$$V_A\left(\frac{1}{5-5j} + \frac{1}{3+3j}\right) - \frac{100}{5-5j} - \frac{50\angle 90}{3+3j} = 0$$

$$V_A(0.2-0.2j) = 18.33+18.33j$$

$\therefore V_A = 94.58 \angle 58.98$

$E_0 = V_A - 0$

$\therefore E_0 = 94.58 \angle 58.98 \, V$

$\therefore P_{max} = \dfrac{E_0^2}{4R_L}$

$= \dfrac{(94.58)^2 \cancel{\angle 58.98}}{4 \times 3.52}$ don't consider angle

$P_{max} = \cancel{8}\, 6\cancel{34}.32 W$

$\boxed{P_{max} = 614.71 W}$

Resonant circuits

DATE: 3/18/15

Introduction:- Resonance is the phenomenon which occurs in AC ckts containing all the 3 elements R, L & C. In many of the electrical ckts resonance is very important phenomenon. The study of resonance is very useful, particularly in the field of communications.

ex: The radio receiver has ability to select certain desired frequency, transmitted by station. The radio receiver rejects all other unwanted frequencies transmitted by other stations. Such selection of required frequency & rejection of unwanted frequency is really based on the principle of resonance. i.e. when the ckt is under series resonance.

Resonance in series ckts is referred as series resonance or simply resonance. ||ly resonance in ||el ckts is referred as parallel or anti resonance.

When the ckt is under series resonance the current is maximum for the frequency fr known as resonant frequency, decreasing to the left and right of this frequency as shown in fig.(1)

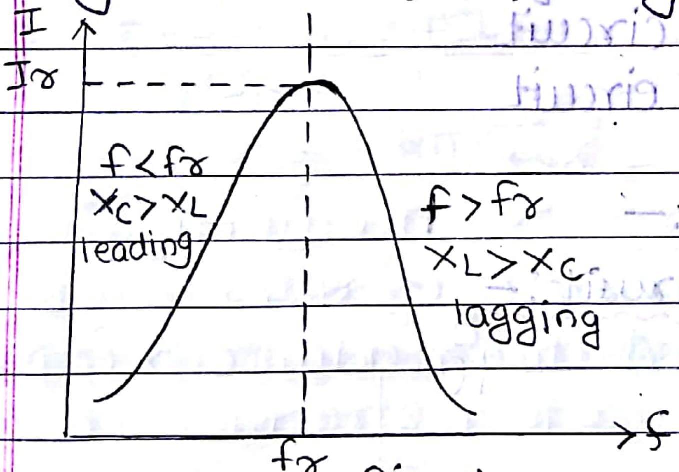


fig.(1)

In general, resonance is defined as phenomenon in which applied vtg & resulting c/n are in phase. The radio or television receiver has a response curve for each broadcasting station as shown in fig (1) above. The receiver is tuned to this frequency to obtain signals from that particular signal. A resonant ckt must have an inductance & capacitance. the resistance will be always present either due to lack of ideal elements or due to the presence of resistive elements itself.

When resonance occurs at any instant, the energy absorbed by one reactive element is exactly equal to the energy released by another reactive element, within the system.

The total apparent power is ~~on~~ simply the average power dissipated by the resistive elements. The average power absorbed by the system will also be maximum at resonance.

The resonant condition in ac circuits may be achieved by varying the frequency of supply keeping the N/W elements constant or by varying 'L' or 'C', keeping the frequency constant.

There are two types of resonant circuits
1. Series resonant circuit
2. Parallel resonant circuit

* Series resonance :-

1. series resonance circuit :-

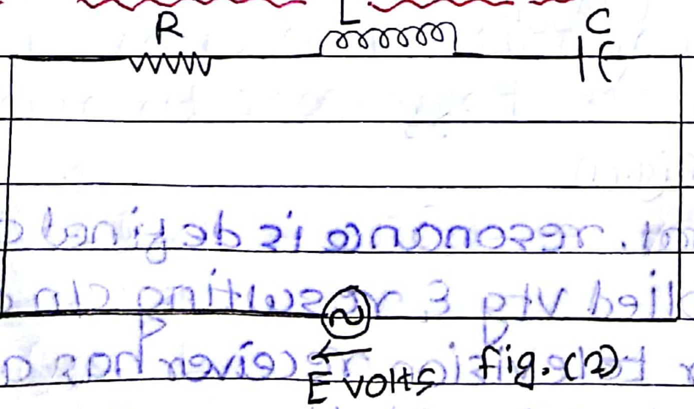


fig.(2)

A series resonant circuit consists of ~~an inductive coil~~ a resistance of 'R' in Ω, an inductance 'L' in Henry in series with capacitance 'c' in farad connected across alternating voltage 'v' whose frequency can be varied as shown in fig(2).

The impedance of the circuit is given by

$$Z = R + jX_L - jX_C$$

$$Z = R + j(X_L - X_C)$$

W.K.T. $X_L = 2\pi f L$ & $X_C = 1/2\pi f C$

by varying the frequency of supply, X_L is made equal to X_C then $Z = R$. The current is in phase with voltage. The power factor of this circuit is unity. Under these conditions the ckt is said to be in series resonance.

let f_r be the frequency at which X_L becomes X_C i.e $X_L = X_C$ then $2\pi f_r L = \frac{1}{2\pi f_r C}$

or $2\pi f_r L \times 2\pi f_r C = 1$

$\therefore \quad 4\pi^2 f_r^2 LC = 1$

$\therefore \quad f_r^2 = \frac{1}{4\pi^2 LC}$

$$\therefore \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \quad (1)$$

f_r is known as resonant frequency.

At resonance the current through the ckt is maximum and is given by

$$\boxed{I_r = \frac{E}{R} = I_m} \quad (2)$$

For the values of frequency less than f_r, $X_C > X_L$. & for the frequency more than f_r, $X_L > X_C$. these conditions are indicated on fig(1)

Date 5-10-15

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* Quality factor :- (Q_s) :- During series resonance vtges across reactive elements i.e inductance & capacitance is increased to many times more than applied voltage itself.

At resonance, $I_r = \frac{E}{R} = I_m$ — ②

The vtg. across the inductance 'L' is

$$V_L = I_m X_L = \frac{E}{R} X_L = \frac{E}{R} \omega_r L = \frac{E}{R} X_{Lr} = \frac{X_{Lr} E}{R}$$

$$V_L = Q_s E \quad \text{— ③} \quad \text{where } Q_s = \frac{X_{Lr}}{R} = \frac{\omega_r L}{R} \quad \text{— ④}$$

where Q_s is known as quality factor of series resonant ckt or simply quality factor of the coil (inductance).

The vtg. across 'c' is

$$V_c = I_m . X_c$$

$$V_c = \frac{E}{R} . \frac{1}{\omega_r C}$$

$$= \frac{1}{\omega_r C R} . E$$

$$V_c = Q_s . E$$

$$\text{where } Q_s = \frac{1}{\omega_r C R} = \frac{X_{cr}}{R} \quad \text{— ⑤}$$

The quality factor of a coil ~~eqⁿ ④ & ⑤ may be~~ in view of eqⁿˢ ④ & ⑤ may be defined as ratio of inductive reactance or capacitive reactance at resonance to the resistance of the coil.

from ④ $$Q_s = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \frac{\cancel{2\pi} L}{R} . \frac{1}{\cancel{2\pi}\sqrt{LC}} \quad \because \text{from ①}$$

$$Q_s = \frac{L}{R\sqrt{LC}}$$

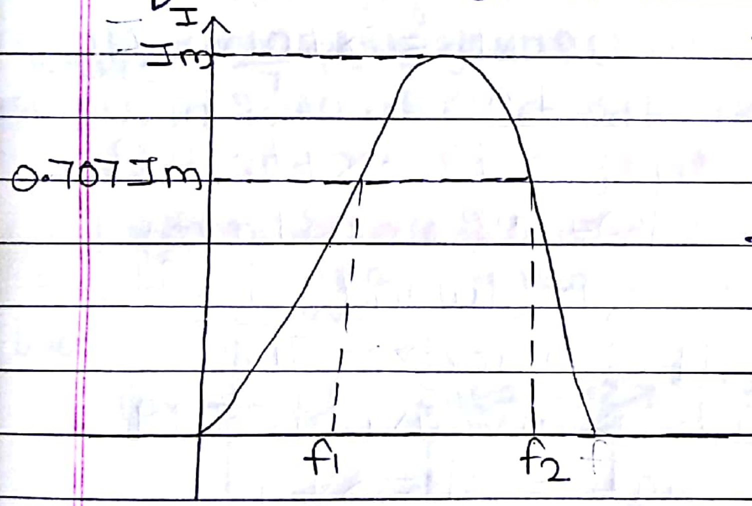
$$\therefore Q_s = \frac{\sqrt{L}}{R\sqrt{C}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\boxed{\therefore Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}} \quad \text{—⑥}$$

from ⑥ we understand that quality factor depends on the resistance of resonant ckt. Higher the resistance smaller will be the value of Q_s.

* Selectivity and Bandwidth:– The fig.(3) represents the frequency response curve of a series resonant ckt i.e variation of I w.r.t. frequency(f) when E is kept constant. The frequencies f_1 & f_2 corresponding to $\frac{I_m}{\sqrt{2}} = 0.707 I_m$ are called Band frequencies or cut-off frequencies or half power frequencies.

"The range of frequencies betn these two cut-off frequencies i.e $(f_2 - f_1)$ is called bandwidth" of resonance circuit.



f_1 & f_2 are also called as half power frequencies because the power delivered by the ckt at these frequencies is half of power delivered by it at the resonance frequencies. This fact can be proved as follows.

Let P_m = maximum power delivered by the ckt
= Power delivered at resonant frequency
$= I_m^2 R$

Power at f_1 or $f_2 = \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{I_m^2 R}{2}$

resonance ckt is always adjusted to select a band of frequencies lying betn f_1 & f_2.
Hence the frequency response curve shown in fig(3) is also known as selectivity curve.

The smaller the bandwidth, higher is the selectivity.

Effect of R on frequency response curve

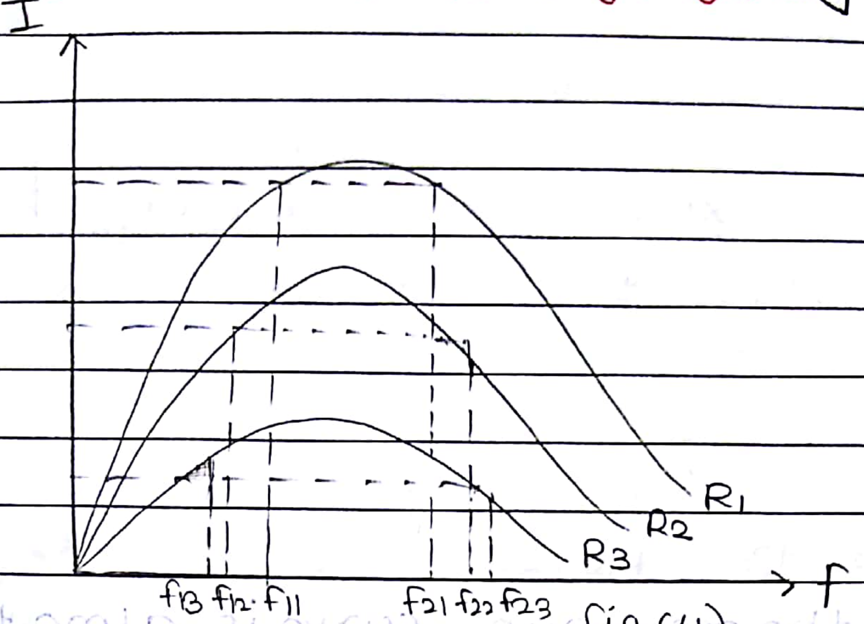
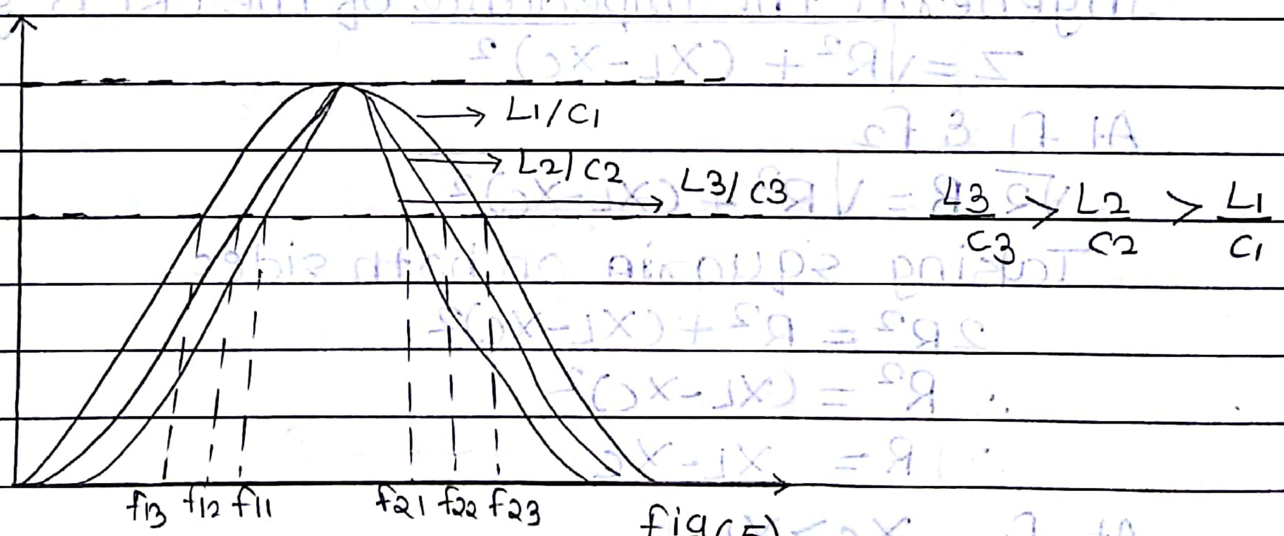


fig.(4)

The shape of curve depends on R, L, C.
If resistance decreased keeping L & C constant then bandwidth decreases, selectivity increases & vice versa.

Effect of L/C on frequency response curve



fig(5)

f_{11} & f_{21} are cutoff frequencies for L_3/C_3
f_{12} & f_{22} are cut off frequencies for L_2/C_2
f_{13} & f_{23} are cut off frequencies for L_1/C_1

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IMP * Expression for f_1 & f_2 or ω_1 & ω_2

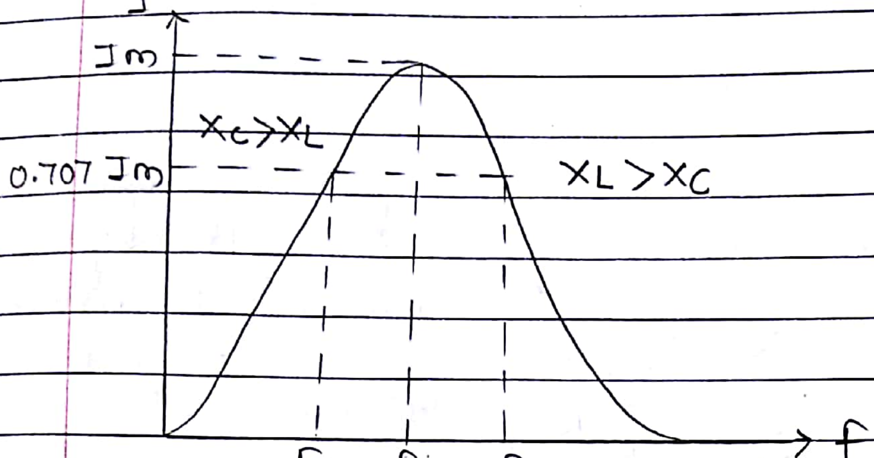

fig.(6)

If $Q_S \geq 10$, the resonant curve is almost symmetrical about resonance frequency. Then f_1 & f_2 are equidistant from f_r as shown in fig.(6) above.

At f_1 & f_2 the current is $I_m/\sqrt{2}$. & hence the impedance is $\sqrt{2}$ times the value of impedance at resonant frequency f_r.

$\therefore$ At f_1 & f_2 $\quad Z = \sqrt{2}R$

In general the impedance of the ckt is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At f_1 & f_2

$$\sqrt{2}\,R = \sqrt{R^2 + (X_L - X_C)^2}$$

Taking square on both sides

$$2R^2 = R^2 + (X_L - X_C)^2$$

$$\therefore R^2 = (X_L - X_C)^2$$

$$\therefore \boxed{R = X_L - X_C} \quad (7)$$

At f_1, $X_C > X_L$

Hence eq^n (7) can be written as

$$\boxed{R = X_C - X_L} \quad (8)$$

The solution of eq^n (8) gives f_1

$$\therefore R = \frac{1}{\omega_1 C} - \omega_1 L$$

$$R = \frac{1 - \omega_1^2 LC}{\omega_1 C}$$

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$\therefore R\omega_1 C = 1 - \omega_1^2 LC$

$\therefore R\omega_1 C - 1 + \omega_1^2 LC = 0$

Dividing by LC

$\therefore \frac{R\omega_1}{L} - \frac{1}{LC} + \omega_1^2 = 0$

$\therefore \boxed{\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0}$ — (9)

general eqⁿ $\boxed{ax^2+bx+c=0}$ — 10

$x = -b \pm \sqrt{b^2-4ac} / 2a$

comparing (9) & (10)

$a = 1$

$b = R/L$

$c = -1/LC$

$\therefore \omega_1 = \frac{-R/L \pm \sqrt{R^2/L^2 + 4/LC}}{-2}$

$\therefore \omega_1 = \frac{-R/L \pm \sqrt{R^2/L^2 + 4/LC}}{2}$

The -ve sign gives -ve values of ω_1 & hence discard.

$$\therefore \boxed{f_1 = \frac{1}{2\pi}\left[-R/2L + \sqrt{(R/2L)^2 + 1/CL}\right]}$$

$$\therefore \boxed{\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$ — (11)

At f_2, $X_L > X_C$

Hence eqⁿ (7) can be written as

$R = X_L - X_C$

The solution of eqⁿ s given f_1

$\therefore R = \omega_2 L - \frac{1}{\omega_2 C}$

$\therefore R = \frac{\omega_2^2 LC - 1}{\omega_2 C}$

$\therefore R\omega_2 C + 1 - \omega_2^2 LC = 0$

$\therefore R\omega_2 C = \omega_2^2 LC - 1$

$\therefore \omega_2^2 LC - R\omega_2 C - 1 = 0$

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$\therefore \omega_2^2 - R/L\,\omega_2 - 1/LC = 0$ (12)

Comparing (12) with (2)

$$\omega_2 = \frac{R/L \pm \sqrt{(R/L)^2 + 4/LC}}{2}$$

$$\omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

−ve sign is discarded.

$$\therefore \boxed{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$ (13)

$$\therefore \boxed{f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]}$$

from (11) & (13) the bandwidth is given by

$$\text{Bandwidth} = f_2 - f_1$$

$$= \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] - \frac{1}{2\pi}\left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$\therefore \boxed{\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}}$$ (14)

or $\boxed{\omega_2 - \omega_1 = \frac{R}{L}}$

But at resonance,

$$Q_s = \frac{X_{Lr}}{R}$$

$$Q_s = \frac{2\pi f_r L}{R}$$

$$Q_s = 2\pi f_r \frac{1}{(R/L)}$$

$$Q_s = 2\pi f_r \cdot \frac{1}{2\pi (f_2 - f_1)}$$ by eqⁿ (14)

$$\therefore \boxed{Q_s = \frac{f_r}{f_2 - f_1}}$$ (15)

∴ $\boxed{\text{Bandwidth} = f_2 - f_1 = \frac{f_r}{Q_s}}$ —— (16)

Eqⁿ (15) may also be written as

$$\boxed{\frac{f_2 - f_1}{f_r} = \frac{1}{Q_s}} \quad \text{—— (17)}$$

sometimes $\frac{f_2-f_1}{f_r}$ is referred as fractional bandwidth

* Relation betⁿ f_r, f_1 & f_2

The impedance of an RLC resonance ckt at f_1 & f_2 are given by

$Z_1 = \sqrt{(R^2 + (X_{C1} - X_{L1})^2)}$ and

$Z_2 = \sqrt{(R^2 + (X_{L2} - X_{C2})^2}$

But $Z_1 = Z_2$

$\sqrt{R^2 + (X_{C1} - X_{L1})^2} = \sqrt{R^2 + (X_{L2} - X_{C2})^2}$

∴ squaring on both sides

$\cancel{R^2} + (X_{C1} - X_{L1})^2 = \cancel{R^2} + (X_{L2} - X_{C2})^2$

∴ $X_{C1} - X_{L1} = X_{L2} - X_{C2}$

i.e $X_{C1} + X_{C2} = X_{L1} + X_{L2}$

i.e $\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_1 L + \omega_2 L$

i.e $\frac{1}{C}\left[\frac{1}{\omega_1} + \frac{1}{\omega_2}\right] = L(\omega_1 + \omega_2) \Rightarrow \frac{1}{C}\left[\frac{\cancel{\omega_2 + \omega_1}}{\omega_1\omega_2}\right] = L(\cancel{\omega_1 + \omega_2})$

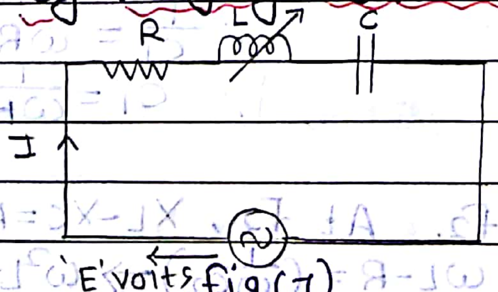
$\omega_1\omega_2 = 1/LC$

∴ $\omega_1 \omega_2 = \frac{1}{LC} = \omega_r^2$

∴ $\omega_r = \sqrt{\omega_1\omega_2}$ or $f_r = \sqrt{f_1 f_2}$

* Resonance by varying circuit elements:- resonant

i) By varying inductance:- consider RLC series ckt. Resonant condition being obtained by varying L as in fig(7)



fig(7)

At resonance,

$X_L = X_C$

$\omega L_r = 1/\omega C$

or $\boxed{L_r = \frac{1}{\omega^2 C}}$ —— (18)

Where L_r is inductance at resonance.

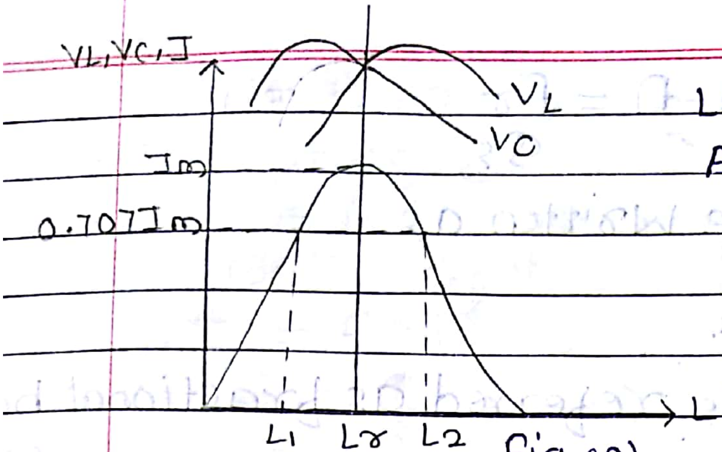


fig (8)

Let L_1 = inductance at F_1

At F_1, $X_C - X_L = R$

$\therefore (1/\omega C) - \omega L_1 = R$

$\therefore (1/\omega C - R) = \omega L_1$

$\therefore \omega L_1 = \frac{1}{\omega C} - R$

$$\boxed{\therefore L_1 = \frac{1}{\omega^2 C} - \frac{R}{\omega}} \quad \text{(19)}$$

let L_2 = inductance at F_2

At F_2, $X_L - X_C = R$ i.e $\omega L_2 - 1/\omega C = R$

$\therefore \quad \omega L_2 = R + 1/\omega C$

$$\boxed{\therefore L_2 = R/\omega + 1/\omega^2 C} \quad \text{(20)}$$

* For frequencies less than f_r, $(X_C > X_L)$ & hence $V_C > V_L$
* at f_r, $V_C = V_L$
* For frequencies more than f_r, $X_L > X_C$ & hence $V_L > V_C$. The variation of voltage is V_L, V_C w.r.t 'L' are shown in fig(8)

ii) By varying capacitance:- consider RLC series circuit. Resonant condition being obtained by varying 'c' as shown in fig.(9)

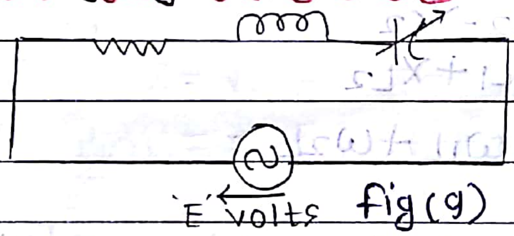


fig (9)

At resonance, $X_C = X_C$ i.e $(1/\omega C_r) = \omega L$

$$\boxed{\therefore C_r = \frac{1}{\omega^2 L}} \quad \text{(21)}$$

where C_r is capacitance at resonance

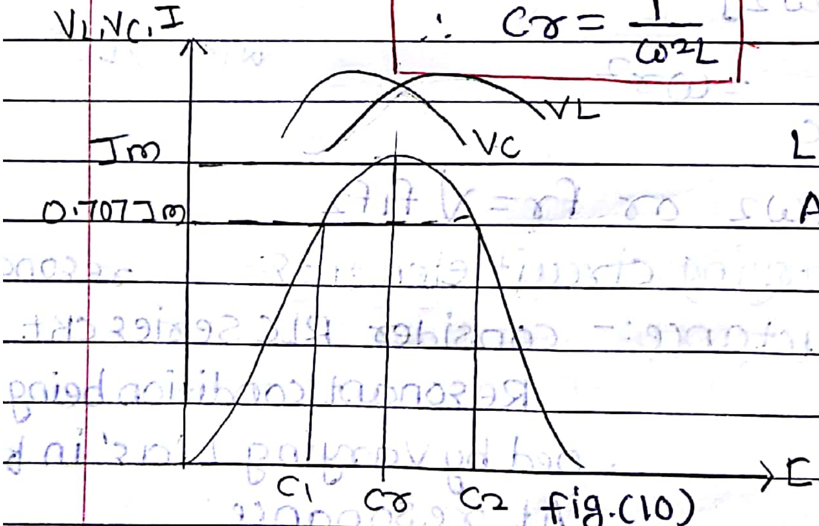


fig.(10)

Let C_1 = capacitance at f_1

At C_1, $X_C - X_L = R$

$\therefore (1/\omega C_1) - \omega L = R$

$\therefore R + \omega L = \frac{1}{\omega C_1}$

$\therefore \frac{1}{C_1} = \omega R + \omega^2 L$

$$\boxed{\therefore C_1 = \frac{1}{\omega R + \omega^2 L}} \quad \text{(22)}$$

let C_2 = capacitance at f_2. At F_2, $X_L - X_C = R$

i.e $\omega L - (1/\omega C_2) = R \Rightarrow \omega L - R = (\frac{1}{\omega C_2}) \Rightarrow \omega^2 L - \omega R = 1/C_2$

$$\boxed{\therefore C_2 = \frac{1}{\omega^2 L - \omega R}} \quad \text{(23)}$$

Variation of voltage across capacitance & inductance w.r.t. capacitance is as shown in fig.(10).

* Variation of V_R, V_L & V_C w.r.t. frequencies:-

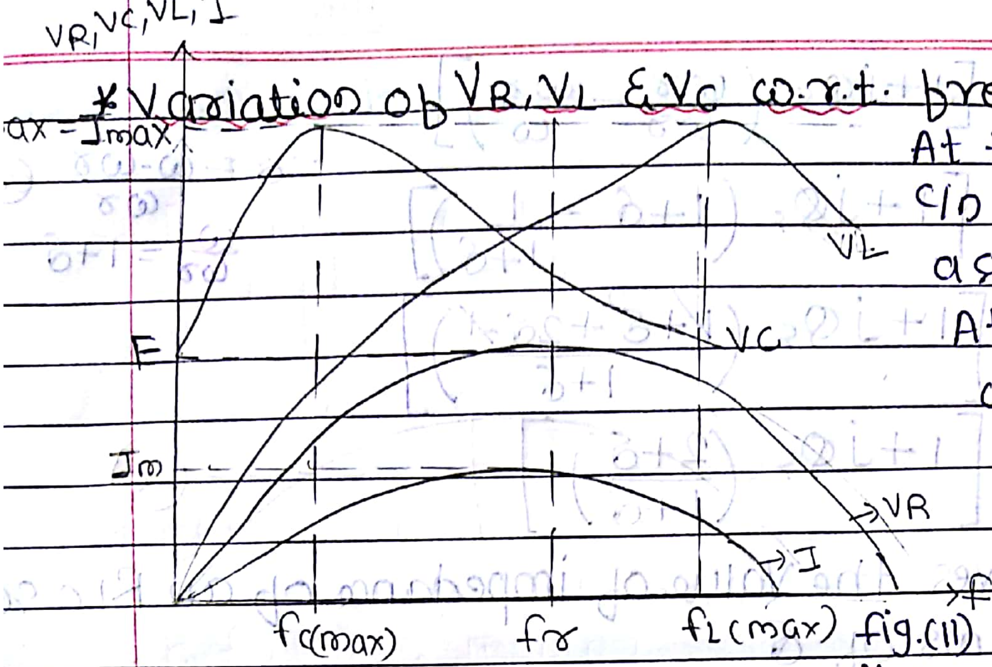


fig.(11)

At $f=0$, X_C becomes ∞, c/n zero, capacitance act as open.

At $f=0$, $V_C = E$

w.k.t $V_C = IX_C$

$V_C = I/\omega C$

$\therefore V_C = \dfrac{I}{2\pi f C}$

Imp **Frequency Deviation (δ):-** "The frequency deviation of an RLC series ckt is defined as the ratio of difference bet^n operating frequency & resonant frequency to the resonant frequency."

$$\delta = \frac{\omega - \omega_r}{\omega_r} = \frac{f - f_r}{f_r}$$ (25)

Where ω = angular frequency operating frequency in rad/sec
ω_r = resonant frequency in rad/sec
f = operating frequency in Hz
f_r = resonant frequency in Hz

The impedance of an RLC series ckt is given by

$$Z = R + jX_L - jX_C$$

$$Z = R + j\omega L - j\frac{1}{\omega C}$$

$$Z = R\left[1 + \frac{j\omega L}{R} - j\frac{1}{\omega CR}\right]$$

$$Z = R\left[1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)\right]$$

$$Z = R\left[1 + j\left(\frac{\omega_r L}{R}\cdot\frac{\omega}{\omega_r} - \frac{1}{\omega_r CR}\cdot\frac{\omega_r}{\omega}\right)\right]$$ $\times$ & $\div$ by ω_r

$$Z = R\left[1 + j\left(Q_s\frac{\omega}{\omega_r} - Q_s\frac{\omega_r}{\omega}\right)\right]$$

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$$Z = R\left[1 + jQ_s\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)\right]$$

$\because \delta = \frac{\omega - \omega_r}{\omega_r}$ (25)

$\frac{\omega}{\omega_r} = 1+\delta$

$$Z = R\left[1 + jQ_s\left(1+\delta - \frac{1}{1+\delta}\right)\right]$$

$$Z = R\left[1 + jQ_s\left(\frac{\cancel{1}+\delta^2+2\delta-\cancel{1}}{1+\delta}\right)\right]$$

$$\boxed{Z = R\left[1 + jQ_s\left(\frac{2+\delta}{1+\delta}\right)\right]} \quad \text{------ (26)}$$

Eq^n (26) gives the value of impedance of an RLC series ckt interms of δ

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If $\omega = \omega_r$ then $\delta = 0$

Hence $Z = R$ which is true at resonance.

At frequencies near resonant frequency δ is very small.

$$\therefore Z = R(1 + jQ_s 2\delta)$$

$$\boxed{Z = R(1 + j2Q_s\delta)} \quad \text{------ (27)}$$

At f_1 & f_2 w.k.T.

$Z = \sqrt{2}R$ To satisfy this condition in eq (27) $Q_s\delta = 0.5$ at f_2 and

$Q_s\delta = -0.5$ at f_1

$\therefore$ At f_2, $Q_s\delta = 0.5$

and At f_1, $Q_s\delta = -0.5$

at f_1 δ is negative. from eq^n (25) we get

$$-\delta = \frac{f_1 - f_r}{f_r}$$

$$\text{or } \boxed{f_1 = f_r(1-\delta)} \quad \text{------ (28)}$$

At f_2 δ is positive.

$$\text{i.e } \boxed{f_2 = f_r(1+\delta)} \quad \text{------ (29)}$$

Problems :-

1. A coil of 5m henry inductance & 10Ω resistance is connected in series with 5μF capacitor. determine frequency at which ckt resonates.

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Expression for $X_C(max)$ & $X_L(max)$:-

$L = 5mh.$

$C = 5\mu F.$

$R = 10\Omega.$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{5m \times 5\mu}}$$

$$=$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore 2\pi\sqrt{LC} = \frac{1}{f_r}$$

$$= \sqrt{LC} = \frac{1}{2\pi f_r}$$

$$\therefore \sqrt{C} = \frac{1}{2\pi f_r \sqrt{L}}$$

$$= \frac{1}{2\pi \times 5k \times \sqrt{5m}}$$

$$=$$

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Find the resonant frequency in a series resonant ckt having an inductance of 50mH & condenser of 5uF. Find the resistance of the ckt if the ckt draws a ch of 10 mH. At resonance with the supply Vtg of 50V. Also find quality factor of the ckt.

2. In a series RLC ckt driven with sinosoidal vtg. source. Determine value of c required to achieve resonance at ckt 5KHz. If value of resistance & inductance are 2 Ω & 1 Henry resp.

Sol^n: given $R = 2\Omega$

$L = 5mH$

$f_r = 5KHz$

We have $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$\therefore \sqrt{LC} = \dfrac{1}{2\pi f_r}$

$\therefore LC = \dfrac{1}{4\pi^2 f_r^2}$

$\therefore C = \dfrac{1}{4\pi^2 f_r^2 L}$ ——— (1)

$= \dfrac{1}{4\pi^2 \times (5K)^2 \times}$

$= 1.0132 \mu F$

3. A series RLC Ckt $R = 10\Omega$, $L = 0.1H$ & $C = 100\mu F$. is connected across 200 V variable frequency source. find

a) The resonant frequency

b) Impedance at this frequency

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c) Voltage drop across inductance & capacitance at this frequency

d) Quality factor

e) band width

Sol^n: given, $R = 10\Omega$

$L = 0.1H$

$C = 100\mu F$

$E = 200V$

a) We have $f_r = \frac{1}{2\pi\sqrt{LC}}$ — ①

$f_r = \frac{1}{2\pi\sqrt{0.1\times100\mu}}$

$f_r = 50.3292$

$\boxed{\therefore f_r = 50.33Hz}$

b) At resonance,

$\boxed{Z = R = 10\Omega}$

c) $V_{Lr} = V_{Cr} = I X_{Lr}$ — ②

but $X_{Lr} = 2\pi f_r L$ — ③

$\therefore X_{Lr} = 2\pi\times50.33\times0.1$

$\therefore X_{Lr} = 31.622$

③ ⇒ $V_{Lr} = V_{Cr} = I X_{Lr}$

$= 20\times31.622$

$\boxed{V_{Lr} = V_{Cr} = 632.45 \text{ Volts}}$

$\because I_r = \frac{V}{R}$

$I_r = \frac{200}{10} = 20A$

d) $Q_s = \frac{X_{Lr}}{R}$ — ④

$= \frac{31.622}{10}$

$\boxed{Q_s = 3.162}$

e) Bandwidth $= \frac{f_r}{Q_s}$ — ⑤

Bandwidth $= \frac{50.33}{3.162}$

$\boxed{\text{Bandwidth} = 15.916}$

4. A coil of resistance 20Ω, inductance 0.2H is connected in series with capacitance across 230 V supply.

i) find the value of capacitance for which resonan-ce occurs at 100Hz

Find ii) The Vtg across capacitor & c/n through capacitor

iii) Quality factor of coil.

(capacitance is also called as condenser)

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* Expression for fcmax & FLmax :-

i) fcmax is the frequency at which Vcmax occurs. fcmax occurs earlier to fr, for which $X_C > X_L$.

$V_C = IX_C$

$$V_C = \frac{E}{Z} \cdot \frac{1}{\omega C}$$

$$V_C = \frac{E}{\sqrt{R^2 + (X_C - X_L)^2}} \cdot \frac{1}{\omega C}$$

Taking square on both sides

$$V_C^2 = \frac{E^2}{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \cdot \frac{1}{\omega^2 C^2}$$

$$V_C^2 = \frac{E^2}{\omega^2 C^2 R^2 + \omega^2 C^2 \left(\frac{1}{\omega^2 C^2} + \omega^2 L^2 - 2\frac{L}{C}\right)}$$

$$V_C^2 = \frac{E^2}{\omega^2 C^2 R^2 + (1 + \omega^4 L^2 C^2 - 2\omega^2 LC)}$$

$$V_C^2 = \frac{E^2}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}$$

Vc is maximum when $\frac{dV_C^2}{d\omega} = 0$

So differentiating V_C^2 w.r.t. ω.

$$\frac{dV_C^2}{d\omega} = \frac{0 - E^2 \{2\omega C^2 R^2 + 2(\omega^2 LC - 1) 2\omega LC\}}{\{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2\}^2} = 0$$

As, $E \neq 0$, $2\omega C^2 R^2 + 4\omega^3 L^2 C^2 - 4\omega LC = 0$

i.e $2\omega C (CR^2 + 2\omega^2 L^2 C - 2L) = 0$

i.e $CR^2 + 2\omega^2 L^2 C - 2L = 0$

$$\therefore \omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\therefore \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\boxed{\therefore f_{cmax} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

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ii) $f_L max$ is the frequency at which $V_L max$ occurs
$f_L max$ occurs after f_r, bcoz which $X_L > X_C$.

$$V_L = I X_L$$

$$V_L = \frac{E}{Z} \cdot \omega L$$

$$V_L = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} \cdot \omega L = \frac{E\omega L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{E\omega^2 LC}{\sqrt{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}}$$

Taking square on both sides

$$V_L^2 = \frac{E^2 \omega^4 L^2 C^2}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}$$

V_L is max. when $\frac{dV_L^2}{d\omega} = 0$

$$\therefore \frac{dV_L^2}{d\omega} = \frac{\{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2\} 4\omega^3 E^2 L^2 C^2 - E^2 \omega^4 L^2 C^2 \{2\omega C^2 R^2 + 2(\omega^2 LC - 1) 2\omega LC\}}{\{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2\}} = 0$$

i.e $\omega^3 E^2 L^2 C^2 [4\{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2\} = \omega \{2\omega C^2 R^2 + 4\omega^3 L^2 C^2 - 4\omega LC\}] = 0$

i.e $2\omega^2 C^2 R^2 - 4\omega^2 LC + 4 = 0$

or $4\omega^2 LC - 2\omega^2 C^2 R^2 = 4$

or $2\omega^2 LC - \omega^2 C^2 R^2 = 2$

$$\therefore \omega^2 = \frac{2}{2LC - C^2 R^2}$$

$$\omega^2 = \frac{1}{LC - \frac{C^2 R^2}{2}}$$

$$\omega = \sqrt{\frac{1}{LC - \frac{R^2 C^2}{2}}}$$

$$\boxed{\therefore f_L max = \frac{1}{2\pi} \sqrt{\frac{1}{LC - \frac{R^2 C^2}{2}}}}$$ (31)

$F_C max$
↓
$V_C max$
↓
$X_C > X_L$
↓
$V_C = I X_C$

$F_L max$
↓
$V_L max$
↓
$X_L > X_C$
↓
$V_L = I X_L$

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* Parallel resonance :-

* Practical parallel resonance circuit :-

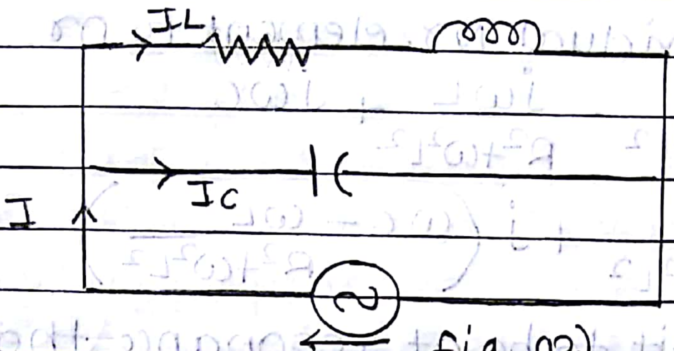

fig. (12)

Practical ‖el resonance ckt is as shown in above fig. It consists of inductive coil of ~~inductance 'L'~~, resistance 'R' and inductance 'L' is placed in ‖el with capacitance 'C' & connected to an alternating supplies of vtg 'E' volts of variable frequency 'f'.
The impedance of the coil is given by 'Z'.

$$Z_L = R + j\omega_L$$

The admitance of the coil is

$$Y_L = \frac{1}{Z_L} = \frac{1}{R+j\omega L} \times \frac{R-j\omega L}{R-j\omega L}$$

$$Y_L = \frac{R-j\omega L}{R^2 - j^2\omega^2 L^2}$$

$$Y_L = \frac{R-j\omega L}{R^2-(-1)\omega^2 L^2} \qquad \because j^2 = -1$$

$$Y_L = \frac{R-j\omega L}{R^2+\omega^2 L^2}$$

$$Z_C = \frac{-j}{\omega C}$$

$$Y_C = \frac{1}{Z_C} = j\omega C \qquad -j = \frac{1}{(1/j)}$$

$$Y_C = \frac{1}{Z_C} = j\omega C$$

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Total admitance of the circuit is

$$Y = Y_L + Y_C$$

$$Y = \frac{R - j\omega L}{R^2+\omega^2L^2} + j\omega C$$

Divide individual Nr. element to Dr.

$$Y = \frac{R}{R^2+\omega^2L^2} - \frac{j\omega L}{R^2+\omega^2L^2} + j\omega C$$

$$Y = \frac{R}{R^2+\omega^2L^2} + j\left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)$$

For the circuit to be at resonance the impedance of the ckt should be purely resistive or admitance must be purely conductive. Hence the imaginary part of the admitance must be zero.

$\therefore$ At resonance

$$\omega_r C - \frac{\omega_r L}{R^2+\omega_r^2L^2} = 0$$

$$\therefore \cancel{\omega_r} C = \frac{\cancel{\omega_r} L}{R^2+\omega_r^2L^2}$$

$$\therefore \boxed{R^2+\omega_r^2L^2 = \frac{L}{C}}$$

$$\therefore \omega_r^2L^2 = \frac{L}{C} - R^2$$

$$\therefore \omega_r^2 = \frac{\frac{L}{C} - R^2}{L^2} = \frac{L/C}{L^2} - \frac{R^2}{L^2}$$

$$\omega_r^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\therefore 2\pi f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$ (32)

where f_r = resonant frequency of practical $\parallel^{el}$ ckt

At resonance admitance of the ckt is purely conductive.

$$Y_r = \frac{R}{R^2 + \omega_r^2 L^2}$$

$$\therefore Y_r = \frac{R}{L/C}$$

$$\therefore \boxed{Y_r = \frac{RC}{L}} \quad (33)$$

Z_r is impedance of practical $\parallel$el ckt at resonance & is known as dynamic resistance.
The c/n at resonance is given by

$$I_r = EY_r$$

$$\boxed{I_r = \frac{ERC}{L}} \quad (34)$$

Parallel Resonant circuit considering the capacitance to have resistance :-

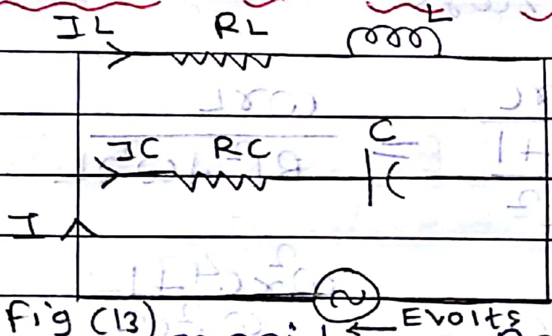


Fig (13) consider a parallel circuit as shown in fig 13.

$$Z_L = R_L + j\omega L$$

$$\text{i.e. } Y_L = \frac{1}{R_L + j\omega L} \times \frac{R_L - j\omega L}{R_L - j\omega L}$$

$$Y_L = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2}$$

|||rly

$$Z_C = R_C - \frac{j}{\omega C}$$

$$\text{i.e. } Y_C = \frac{1}{R_C - \frac{j}{\omega C}} \times \frac{R_C + j\frac{1}{\omega C}}{R_C + j\frac{1}{\omega C}}$$

$$Y_C = \frac{R_C + j\,1/\omega C}{R_C^2 + 1/\omega^2 C^2}$$

Total Admitance Y

$$Y = Y_L + Y_C$$

$$Y = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j\cdot 1/\omega C}{R_C^2 + 1/\omega^2 C^2}$$

$$Y = \frac{R_L}{R_L^2 + \omega^2 L^2} - \frac{j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + 1/\omega^2 C^2} + \frac{j/\omega C}{R_C^2 + 1/\omega^2 C^2}$$

$$Y = \left(\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + 1/\omega^2 C^2}\right) + j\left(\frac{1/\omega C}{R_C^2 + 1/\omega^2 C^2} - \frac{\omega L}{R_L^2 + \omega^2 L^2}\right)$$

At resonance the admitance is purely conductive hence imaginary part of eqⁿ (35) is zero.

i.e $$j\left(\frac{1/\omega_r C}{R_C^2 + 1/\omega_r^2 C^2} - \frac{\omega_r L}{R_L^2 + \omega_r^2 L^2}\right) = 0$$

$$\Rightarrow \frac{1/\omega_r C}{R_C^2 + 1/\omega_r^2 C^2} = \frac{\omega_r L}{R_L^2 + \omega_r^2 L^2}$$

$$\therefore \frac{1/\omega_r C}{\frac{\omega_r^2 C^4 + 1}{\omega_r^2 C^2}} = \frac{\omega_r L}{R_L^2 + \omega_r^2 L}$$

$$\therefore \frac{\frac{1}{\omega_r C}}{\omega_r L} = \frac{\frac{\omega_r^2 C^4 + 1}{\omega_r^2 C^2}}{R_L^2 + \omega_r^2 L}$$

$$\Rightarrow \frac{1}{\omega_r^2 LC} = \frac{\omega_r^2 C^4 + 1}{\omega_r^2 C^2 (R_L^2 + \omega_r^2 L)}$$

$$\Rightarrow 1 = \frac{\cancel{\omega_r^2} L \cancel{C} (\omega_r^2 C^4 + 1)}{\cancel{\omega_r^2} C^{\cancel{2}} (R_L^2 + \omega_r^2 L)}$$

$$\Rightarrow \frac{C}{L} = \frac{\omega_r^2 C^4 + 1}{\omega_r^2 L + R_L^2}$$

$$\Rightarrow \frac{C}{L} = \frac{\cancel{\omega_r^2}}{\cancel{\omega_r^2}}\left[\frac{C^4 + 1/\omega_r^2}{L + R_L^2/\omega_r^2}\right]$$

$$\therefore C\left[L + \frac{R_L^2}{\omega_r^2}\right] = L\left[C^4 + \frac{1}{\omega_r^2}\right]$$

$$\therefore LC + \frac{RL^2C}{\omega^2 r} = LC^4 + \frac{L}{\omega^2 r}$$

$$\therefore LC - LC^4 + \frac{RL^2C - L}{\omega^2 r} = 0$$

$$\therefore L\left(C - C^4 + \frac{RLC - 1}{\omega^2 r}\right) = 0$$

$$\therefore C - C^4 + \frac{RLC - 1}{\omega^2 r} = 0$$

$$\therefore C\left(1 - C^3 + \frac{RL - 1}{\omega^2 r}\right) = 0$$

$$\therefore 1 - C^3 + \frac{RL - 1}{\omega^2 r} = 0$$

$$\therefore \frac{\omega^2 r - \omega^2 r C^3 + RL - 1}{\omega^2 r} = 0$$

$$\therefore \omega^2 r - \omega^2 r C^3 + RL = 1$$

$$\therefore \omega^2 r - \omega^2 r C^3 = 1 - RL$$

$$\therefore \omega^2 r (1 - C^3) = 1 - RL$$

$$\therefore \omega^2 r = \frac{1 - RL}{1 - C^3}$$

$$\omega r = \sqrt{\frac{1/LC\,(RL^2 - L/C)}{(RC^2 - L/C)}} = \frac{1}{\sqrt{LC}} \sqrt{\frac{RL^2 - L/C}{RC^2 - L/C}}$$

$(RC^2 - L/C)$

$$\therefore fr = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{RL^2 - L/C}{RC^2 - L/C}} \quad (36)$$

fig 13

The ckt resonates at all frequencies if $RL^2 = RC^2 = L/C$. eq^n (36) gives resonant frequency for the ||el ckt as shown in fig 13. The admitance at resonance is purely conductive $\therefore$ $Yr = \frac{RL}{RL^2 + \omega^2 L^2} + \frac{RC}{RC^2 + 1/\omega^2 C^2}$ (37)

The Cu at resonance is given by

$$Ir = E\,Yr \Rightarrow Ir = E\left(\frac{RL}{RL^2 + \omega^2 L^2} + \frac{RC}{RC^2 + 1/\omega^2 C^2}\right)$$

The frequency response curve of $||^{el}$ resonant ckt is as shown in fig. 14 below.

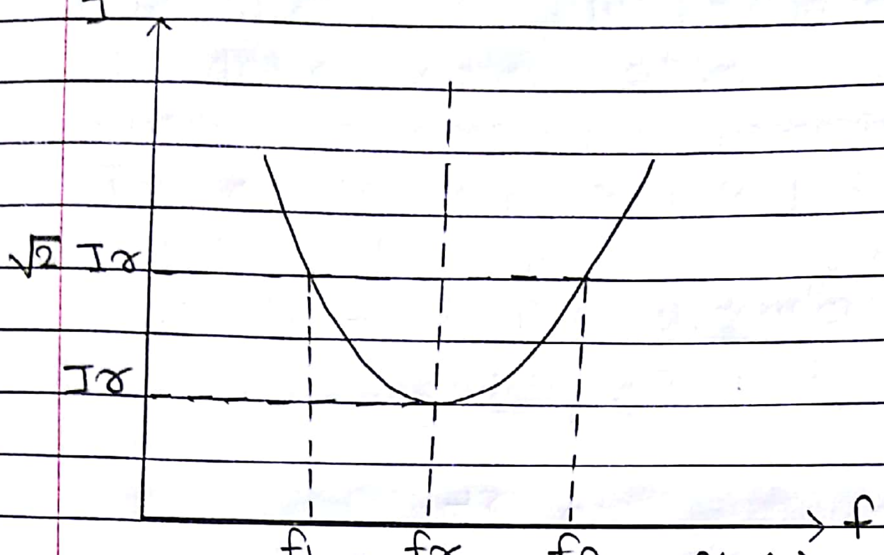


fig.(14)

from fig.(14) we find that c/n is minimum at resonance. Hence the i/p impedance of the ckt is maximum at resonance. Since the c/n at resonance is minimum the parallel ckt at resonance is called as rejector circuit. The half power points or cut off frequencies (f_1, f_2) of the rejecter ckt are given by the points at which c/n is $\sqrt{2}\, I_r$.

~~A g~~

* A general parallel resonant circuit:-

A general $||^{el}$ resonant ckt considering ideal elements R, L & C is shown in fig. 15.

The conductance of R is $G = \frac{1}{R}$

The susceptance of L is $-jB_L = -j\frac{1}{X_L} = \frac{-j}{\omega L}$ $\because X_L = \frac{1}{B_L}$

The susceptance of C is $jB_C = j\frac{1}{X_C} = j\omega C$

The total admittance of the ckt is given by

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

For the circuit to be at resonance

$$\omega_r C - \frac{1}{\omega_r L} = 0$$

$$\therefore \omega_r C = \frac{1}{\omega_r L}$$

$$\therefore \omega_r^2 = \frac{1}{LC}$$

$$\therefore \omega_r = \frac{1}{\sqrt{LC}}$$

Fig.(15)

$$\boxed{\therefore f_r = \frac{1}{2\pi\sqrt{LC}}} \quad (38)$$

* Q Factor for parallel resonant circuit

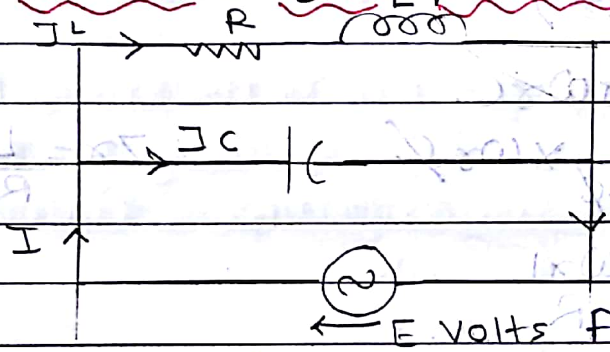


fig.(16).

consider ||el resonant ckt as shown in the fig (16).
The vector diagram for this ckt is as shown in fig (17).

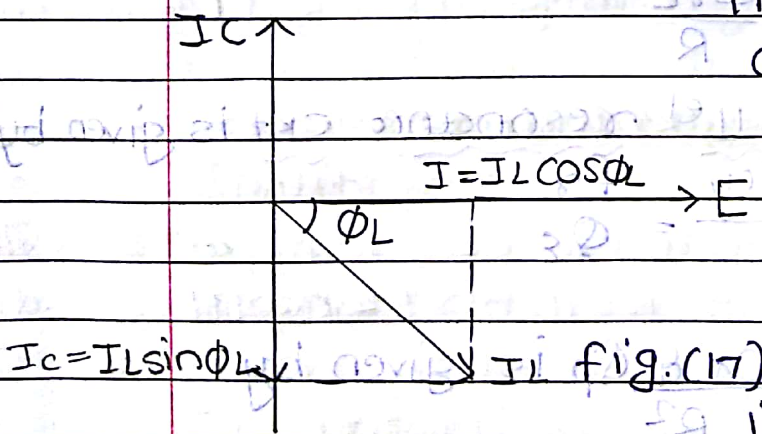


fig.(17)

The c/n IL lags E by an angle ϕ_L.
The ckt is at resonance when reactive component of c/n I is zero. i,e I is in phase with E.

i,e $I = I_L \cos\phi_L$

$I_C = I_L \sin\phi_L$

At resonance only reactive c/n flow through the branches

$I_L \sin\phi_L$ through R-L branch and
I_C through C branch.
These currents will be many times more than the total current at resonance. Hence there is current magnification in a parallel resonant ckt.
The quality factor of a ‖el resonant ckt is defined as the c/n magnification.

$$\therefore Q_p = \frac{\text{c/n through capacitance at resonance}}{\text{Total current at resonance}}$$

$$\therefore Q_p = \frac{I_{cr}}{I_r}$$

$$\therefore Q_p = \frac{(E\omega_r C)}{\left(\frac{E}{Z_r}\right)}$$

$$\therefore Q_p = Z_r \omega_r C.$$

$$\therefore Q_p = \frac{L}{R\cancel{C}} \times \omega_r \cancel{C} \qquad \because Z_r = \frac{L}{RC}$$

$$\therefore Q_p = \frac{\omega_r L}{R}$$

$$\boxed{Q_p = Q_s} \text{ --- } 39$$

Hence the eq^{ns} for quality factor for series resonance ckt & practical ‖el resonance are same.

i.e. $Q_p = Q_s = \frac{\omega_r L}{R}$

The bandwidth of ‖el resonance ckt is given by

$$B.W = \frac{\text{resonant frequency}}{\text{quality factor}} = \frac{f_r}{Q_s}$$

The relation bet^n ω_r & Q_p is given by

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\therefore \omega_r = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}}$$

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{1}{Q_p^2}}} \quad (40) \qquad \because Q_p^2 = \frac{\omega_r L}{R} \times \frac{1}{\omega_r CR}$$

$$Q_p^2 = \frac{L}{CR^2}$$

Relation bet^n Z_r & Q_p :-

$$\boxed{Z_r = R(1 + Q_p)^2} \quad (41)$$

Difference bet^n series and parallel resonance ckt

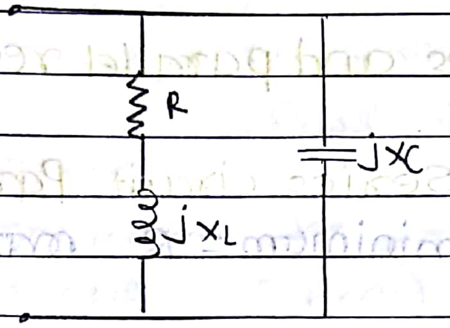
Parameter	Series circuit	Parallel circuit
Impedance resonance (Z_r)	minimum = R	maximum = $\frac{L}{CR}$
current at resonance (I_r)	maximum = $\frac{E}{R}$	minimum = $\frac{ECR}{L}$
Power factor at resonance	unity	unity
Resonant frequency (f_r)	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Quality factor $Q_s = Q_p$	$\frac{\omega_r L}{R}$	$\frac{\omega_r L}{R}$

June-11
1. Explain properties of RLC series ckt. 4m.
2. Find the resonant frequency in a series resonant ckt having an inductance of 50mH & conductance of ~~5~~ μF. Find resistance of the ckt. Draws a c/n of 10mA at resonance with supply vtg of 50V. also find Quality factor. 6m.
3. a) Explain in brief bandwidth & selectivity in series resonant ckt.
b) A series RLC ckt has R=2Ω, L=2mH, C=10μF calculate Q factor, bandwidth, the resonant frequencies & half power frequencies f_1 & f_2 - 10m

Dec-11.

1. Define the terms :-
a) resonance b) Q factor c) half power frequency
d) bandwidth e) selectivity pertaining to a series RLC ckt.

2. obtain an expression for resonance frequency for a ckt shown below.



3. obtain the condition for maximum value of V_L by variation of inductance

June-12

1. Define Quality factor & bandwidth also establish relationship bet^n quality factor & bandwidth in series resonance ckt & there by P.T.
$Q = \frac{f_0}{BW}$ $(f_0 = f_r)$ where f_0 is resonance frequency.

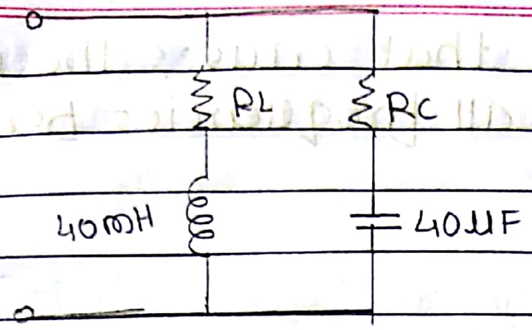
2. A series RLC circuit with $R = 10\,\Omega$, $L = 10\,mH$, $C = 1\,\mu F$ has an applied vtg. of 200V at resonant frequency. calculate resonant frequency f_0, c/o in the ckt at resonant, vtg across the element at resonant also find quality factor & bandwidth

Dec-12

1. Define the following ckt with reference to resonant ckt.
a) resonance b) Q-factor c) selectivity d) bandwidth

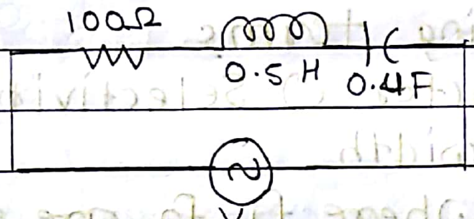
2. A series RLC ckt has $R = 10\,\Omega$, $L = 0.01H$, $C = 0.01\mu F$ it is connected across 10mv supply. calculate
a) f_0 b) Q_0 c) bandwidth d) f_1 & f_2 e) I_0 (I_r)

3. Determine R_L & R_C for which the ckt shown in fig. below resonates at all frequencies



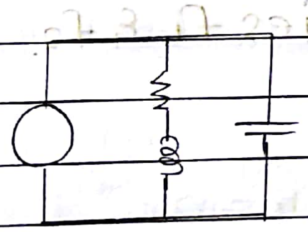
$R_L = R_C = \sqrt{\frac{L}{C}}$

June-13 1. For the series RLC ckt shown in fig. below. Find the resonant frequency, half power frequencies, band width, Quality factor



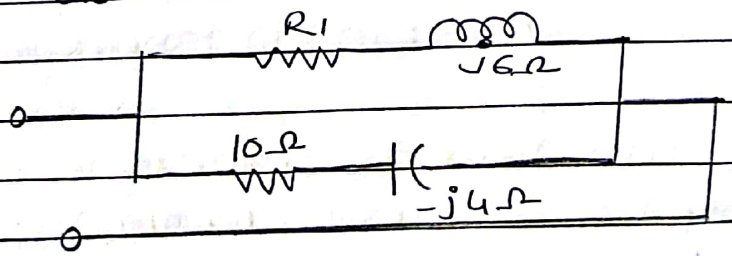
2. Derive expression for fr, Q & B.W. of ‖el resonant ckt with lossless capacitor in ‖el with a coil of resistance R & inductance L.

Ans



June-14 1. A 220V, 100Hz AC source supplies a series RLC ckt with a capacitor & coil, if the coil has 50mΩ resistance & 5mH inductance, find at resonant frequency of 100Hz, what is value of capacitance, also calculate Q factor & half power frequencies of ckt.

2. Find the value of R1 such that the ckt given below is resonant



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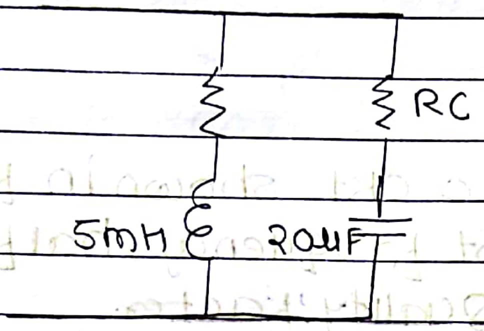
3. Determine R_L & R_C that causes the ckt to be at resonant at all frequencies for the ckt shown below.



June-15

1. Define the following terms
a) resonance b) Q factor c) Selectivity of series RLC ckt d) bandwidth.
2. P.T. $f_0 = \sqrt{f_1 f_2}$ where f_1, f_2 are 2 half power frequencies of resonant ckt
3. A series RLC ckt has $R = 4\Omega$, $L = 1mH$, $C = 10\mu F$ calculate Q factor, bandwidth, resonant frequency, half power frequencies f_1 & f_2.

Transient Behaviour And Initial Conditions.

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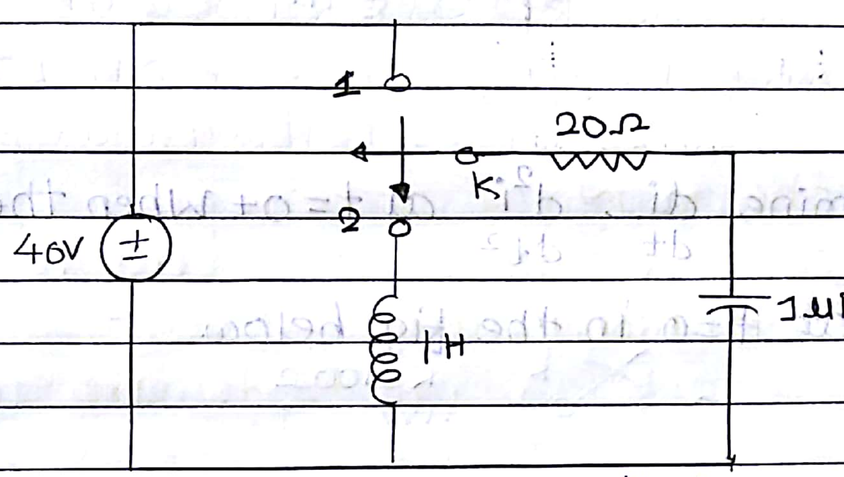
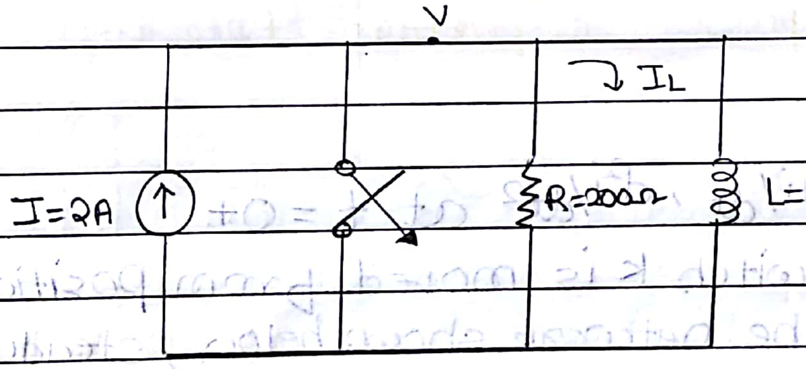
June-11 1. Explain the behavior of R, L, C elements at the time of switchng at $t=0$, $t=0+$, $t=\infty$

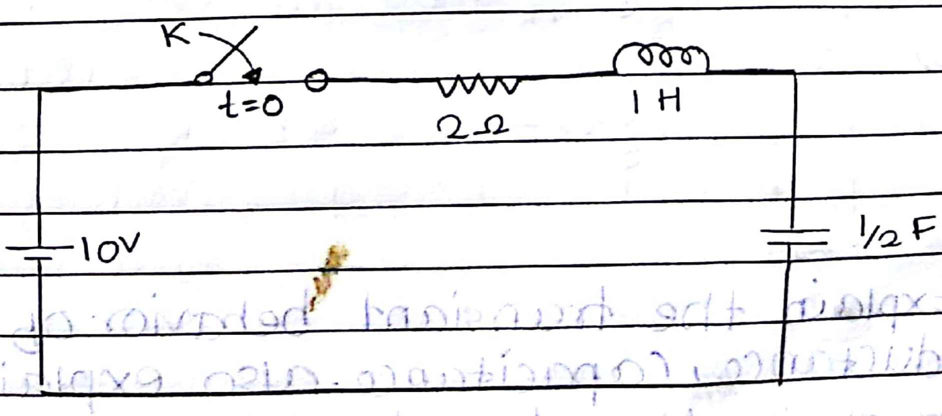
2. Determine i, $\frac{di}{dt}$ and $\frac{di^2}{dt^2}$ at $t=0+$ when the switch K is moved from position 1 to 2 at $t=0$ in network shown below.



3. Determine V, dV/dt and d^2V/dt^2 at $t=0+$ when the switch K is opened at $t=0$ in the fig shown below.



Dec-11 1. In a network shown switch K is closed at $t=0$ with the capacitor uncharged find the values for $i(0+)$, $\frac{di}{dt}(0+)$ at $t=0+$ also find $\frac{d^2i}{dt^2}(0+)$



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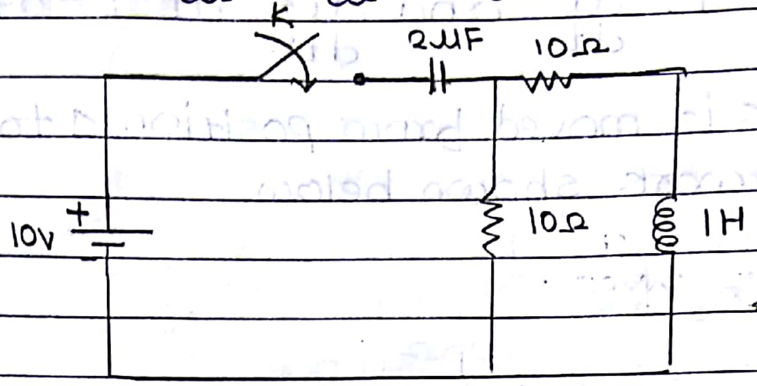
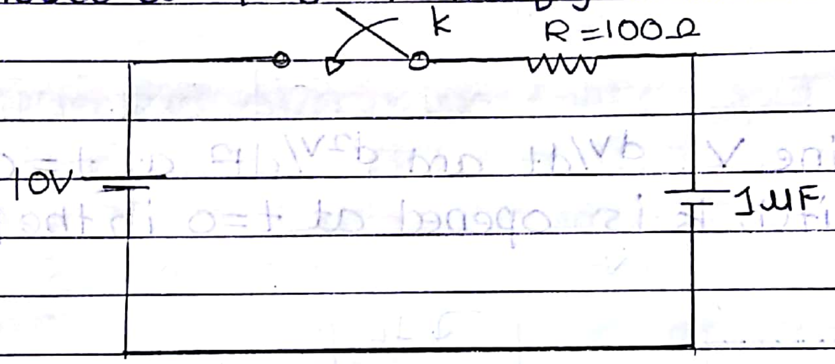
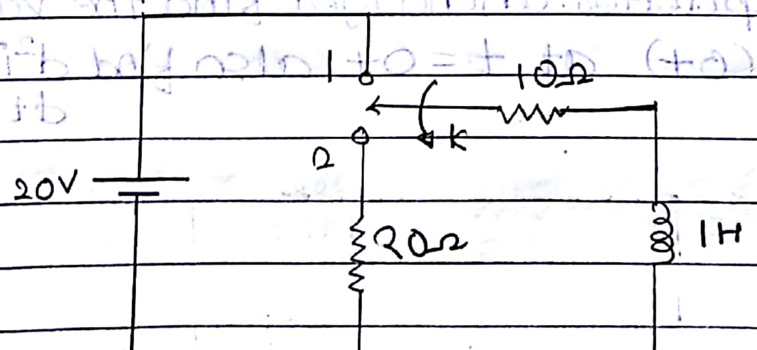
2. In the given ckt switch K is closed at $t=0$. Find $i_1, i_2, \frac{di_1}{dt}, \frac{di_2}{dt^2}, \frac{d^2i_2}{dt^2}$ at $t=0+$



June-12. 1 Determine $\frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t=0+$ when the switch is closed at $t=0$ in the fig. below.

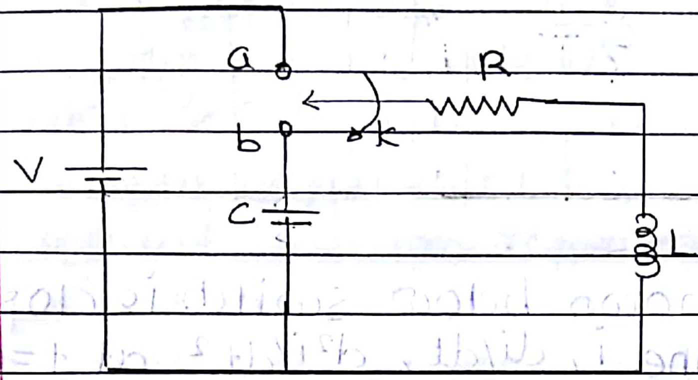


2. Determine di/dt, d^2i/dt^2 at $t=0+$ When the switch K is moved from position 1 to 2 at $t=0$ in the network shown below, steady state having reached before switching



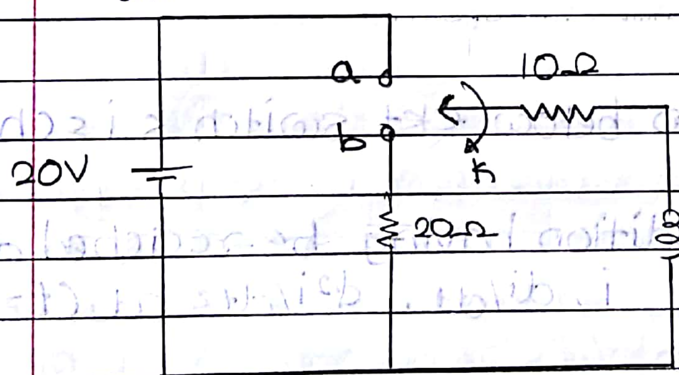
Dec-12 1. Explain the transiant behavior of resistance, inductance, capacitance. also explain procedure for evaluating transiant behavior

2. In the N/w shown below k is changed from position a to b at t=0 solve for i, di/dt & $\frac{d^2i}{dt^2}$ at t=0+. If R=1000Ω, L=1H, C=0.1μF, V=100V. Assume that capacitor is initially uncharged.

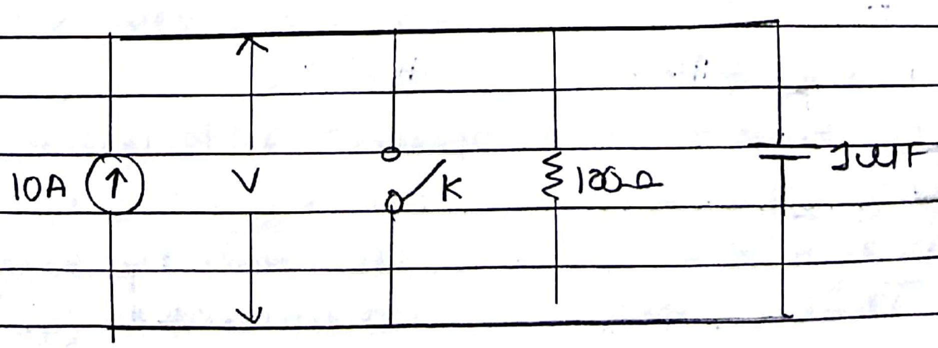


June-13.

1. In the ckt shown below switch k is changed from position a to b at t=0 steady state condition having reached before switching. find i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at t=0+



2. In the ckt shown below switch k is opened at t=0, find the values of v, dv/dt, d²v/dt²



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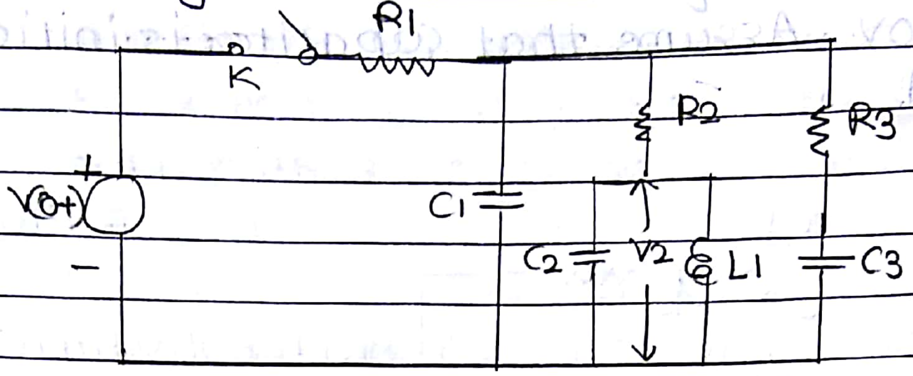
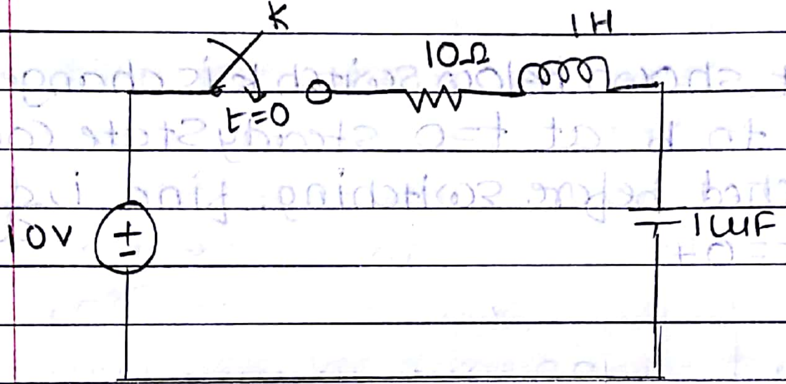
3. In the ckt shown below switch K closed at t=0 find the values of V_1, V_2, V_3 at $t=0+$. The ckt is initially relaxed.

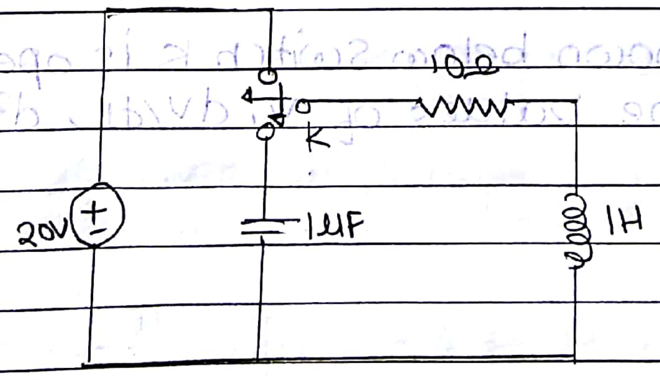


June-14 Dec

1. In the N/w shown below switch is closed at t=0, Determine i, di/dt, d^2i/dt^2 at $t=0+$



2. In the ckt shown below ckt switch K is changed from
steady state condition having be reached at position 1. find i, di/dt, d^2i/dt^2 at ($t=0+$)



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Introduction:- The initial conditions of a N/ω are the conditions preveling in the elements of the N/ω at the instant of closing the switch at $t=0$

In a switching operation $t=0$ is taken as reference. The initial conditions in a N/ω may be the vtg across the various elements, the currents through various elements or charges existing on them at the time of switching operation i.e at $t=0$ Immediately before a switching operation, these quantities are referred as $V(0-)$, $i(0-)$, $q(0-)$ at $t=0-$.

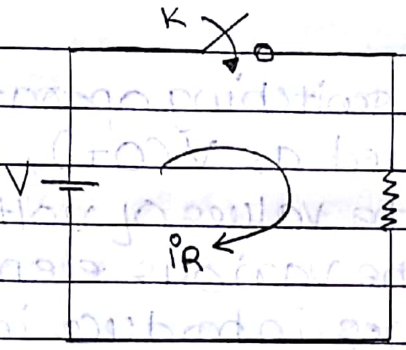
Immediately after the switching operation these quantities are referred as $V(0+)$, $i(0+)$, $q(0+)$ at $t=0+$. Knowing the values of voltages, currents and charges on the various elements at $t=0-$ and the changes introduce immediately after the switching operation i.e at $t=0^+$ additional eq^us can be written, which can be solved simultaneously with the general Differential eq^us, to evaluate the constants. The conditions existing on the various elements of the network at $t=\infty$ are called the final conditions.

The initial conditions of the N/ω depend on the past history of the N/ω. prior to $t=0-$ & the N/ω fluctuare at $t=0+$ after switching they also depend on nature of the elements in the N/ω it is assumed that switching time is 0.

In this chapter we concentrate on finding the change in selected variables in ckt when a switch is thrown open from closed position or vice versa. The time of throwing switch is consider to be $t=0$ & we want to determine value of variable at $t=0-$ & $t=0+$ immediately before & after throwing switch. Thus a switched ckt is an electrical ckt with 1 or more switches that open or closed at time $t=0$. We are very much interested in change in c/n and voltages of energy storing elements after the switch is thrown since these variables along with sources will dictate the ckt behavior for $t>0$.

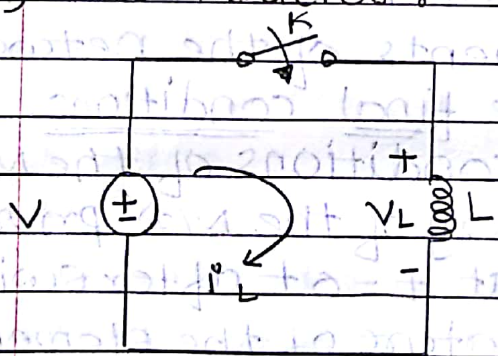
* Initial & final condition in elements:-

(i) The resistor:-



When a vtg. V is applied across resistance 'r' by closing switch the c/n through R is given by $\boxed{i_R = \frac{V}{R}}$ ① This eq^n indicates that c/n through resistor 'R' changes instantaneusly. Hence in a resistor c/n changes instantaneusly & energy is dissipated as heat & it doesnot store any energy.

(ii) The inductor:-



When a vtg. V is applied across inductance 'L' henry the vtg across inductance is given by

$$\boxed{V_L = L\frac{di_L}{dt}} \quad —②$$

If the c/n flowing through inductance is dc then $\frac{di_L}{dt}=0$ hence vtg. across inductor is zero. Hence under steady state conditions the inductor acts as a short circuit.

The c/n through inductance is given by

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L \, dt$$

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$$\therefore i_{L(t)} = \frac{1}{L}\int_{-\infty}^{0-} V_L\,dt + \frac{1}{L}\int_{0-}^{t} V_L.dt \quad \text{—③}$$

Putting $t=0+$ on both sides
The 1st term in RHS eqⁿ ③ represents initial value of c/n through inductor before closing the switch i.e. $i_L(0-)$.
When switch is closed at $t=0$ then eqⁿ ③ can be written as

$$i_{L(0+)} = i_L(0-) + \frac{1}{L}\int_{0-}^{0+} V_L.dt \quad \text{—④}$$

It is assumed that switching operation does not consume any time thus integration from (0-) to (0+) is zero

$$\therefore i_{L(0+)} = i_{L(0-)}$$

Thus current through inductor can't change instantaneously. This means that c/n through inductor before & after switching operation is same.
Hence at $t=0+$ the inductor acts as an open circuit (o.c), If it does not carry any initial c/n if the inductor carrying an initial c/n I_0 before switching operation, then immediately after switching operation i.e at $t=0+$ it acts as c/n source of I_0

Note:- The switch is closed at $t=0$ hence $t=0-$ corresponds to the instant when the switch is just opened. and at $t=0+$ corresponds to instant when switch is just closed.

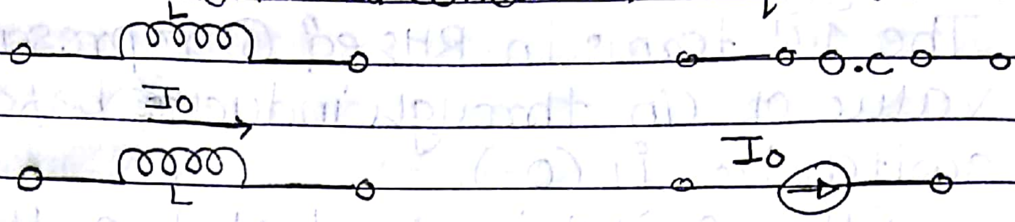
If $i_{L(0-)}=0$ we get $i_{L(0+)}=0$.

This means that at $t=0+$ the inductor will act as an open ckt. irrespective of the vtg across the terminal.

If $i_L(0-) = I_0$ then $i_{L(0+)} = I_0$

In this case at $t=0+$ the inductor can be thought of c/n source I_0 ampere.

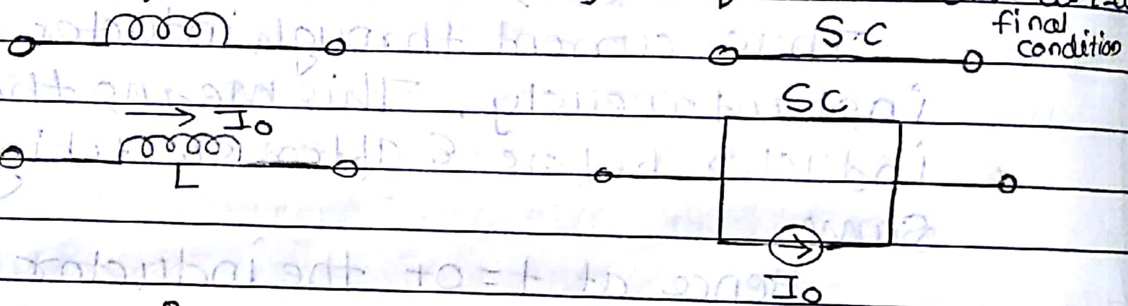
$(i_L = 0-)$ Element (and initial condition) Equivalent ckt at $t=0+$



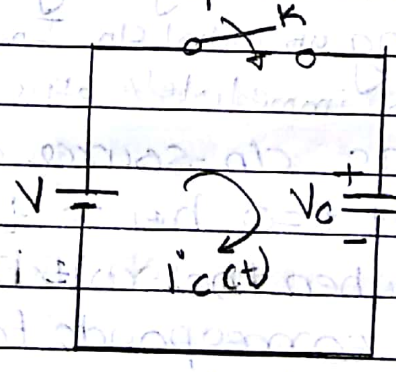
The final condition equivalent circuit of an inductor is derived from basic relationship $V = L \cdot \frac{di_L}{dt}$ under steady state condition

$\frac{di_L}{dt} = 0$ This means $v = 0$. Hence L acts as short at $t = \infty$ (final or steady state)

Element (& initial condition) Equivalent ckt at $t=\infty$



iii) The capacitor :-



The c/n through capacitor is given by

$$\boxed{i_c = C\frac{dv_c}{dt}} \quad (5)$$

If a dc vtg is applied then $\frac{dv_c}{dt} = 0$ and hence $i_c = 0$

Thus for dc quantities capacitor acts as an open ckt. The vtg across capacitance is given by $V_c = \frac{1}{C}\int_{-\infty}^{t} i_c \, dt$

$$\therefore V_C(t) = \frac{1}{C}\int_{-\infty}^{0-} i_C\,dt + \frac{1}{C}\int_{0-}^{t} i_C\,dt \quad \text{---} (6)$$

$\frac{1}{C}\int_{-\infty}^{0-} i_C\,dt = V_C(0-)$ and is constant.

When switch K is closed at $t=0$ the eq^n (6) can be written as, Putting $t=0+$ on both sides

$$V_C(0+) = V_C(0-) + \frac{1}{C}\int_{0-}^{0+} i_C\,dt$$

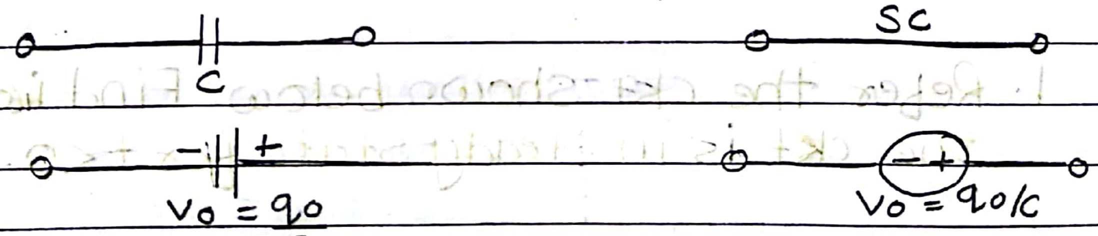
$$\therefore V_C(0-) = V_C(0+)$$

Thus vtg. across capacitor doesnot change instantaneusly hence if capacitor doesnot have any initial charge at $t=0-$ then at $t=0+$ its vtg will be zero. Thus capacitor acts as a short circuit at $t=0+$.

If at $t=0-$ the capacitor has initial vtg of V_0 will charge q_0 then at $t=0+$ it acts as voltage source of V_0.

If $V(0-)=0$ then $V(0+)=0$. This means that capacitor C acts as short circuit conversly $V(0-)=\frac{q_0}{C}$ then $V(0+)=\frac{q_0}{C}$

Element & initial condition   Equivalent ckt at $t=0+$



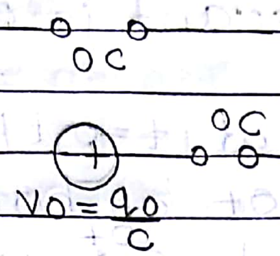
The final condition equivalent ckt is derived as $i = C\frac{dv}{dt}$

Steady state $\frac{dv}{dt} = 0$

at $t=\infty$, $i=0$

This means that at $t=\infty$ or steady state 'c' acts as open ckt

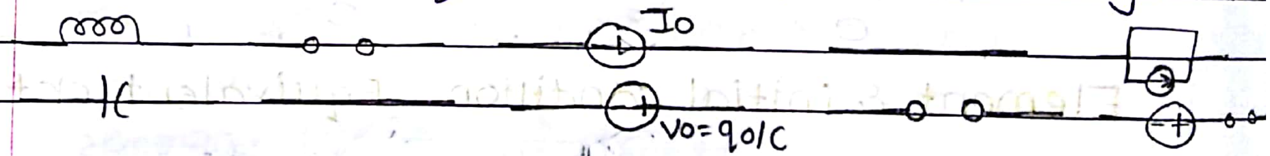
Final condition at $t=\infty$



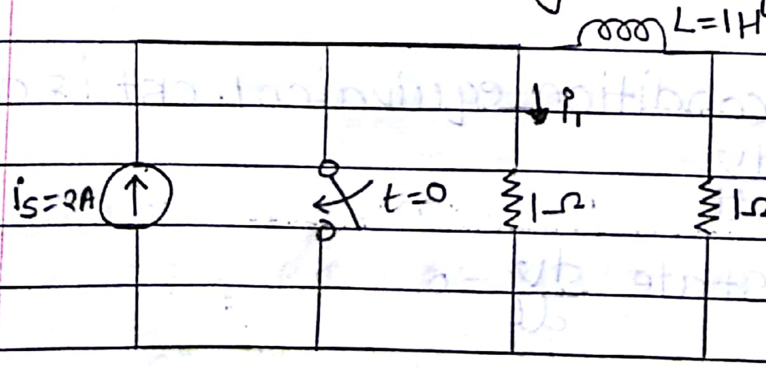
Procedure for evaluating initial conditions:-

There is no unique procedure that must be followed in initial condition. We usually solve for initial values of C/n, vtg & then we solve for derivatives. For finding initial values of C/n & vtgs, an equivalent N/w of the original N/w at $t=0^+$ is constructed according to the following rules.

1. Replace all inductors with short ckt or with C/n sources having value of C/n flowing at $t=0^-$
2. Replace all capacitors with short ckt or with vtg source of value $V_0 = q_0/C$ if there is an initial charge.
3. resistors are left in the N/w without any changes



1. Refer the ckt shown below. Find $i_1(0+)$ & $i_L(0+)$ the ckt is in steady state for $t<0$.



Sol^n

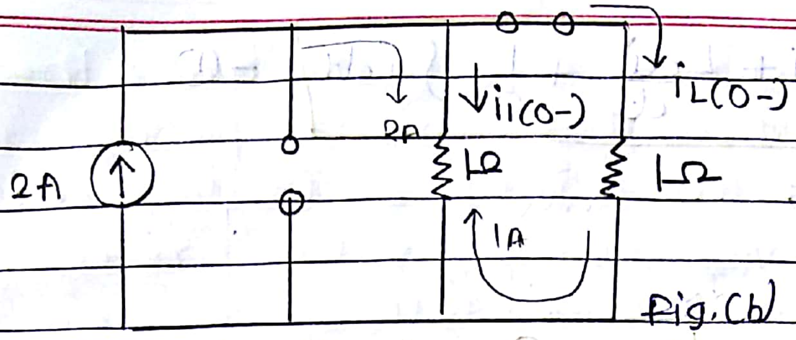


Fig. (b)

At steady state the ckt will be redrawn as shown in fig (b)

By c/n division formula

$$i_L(0-) = \frac{2\times 1}{1+1} = 1A.$$

Since the c/n in an inductor can't change instantaneusly. We have $i_L(0+) = i_L(0-) = 1A$.

$\therefore$ $\boxed{i_L(0+) = 1A}$

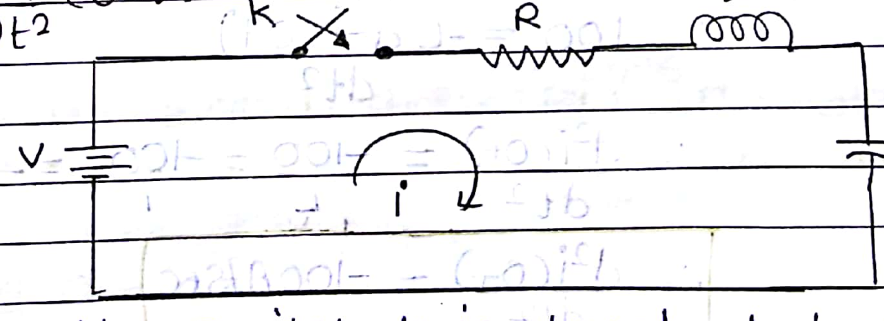
At $t=(0-)$, $i_1(0-) = 2-1 = 1A$

Please note that c/n in resistor can change instantaneusly. since at $t=0^+$ the switch is just closed. Vtg across R_1 equal to zero because of switch being short circuted

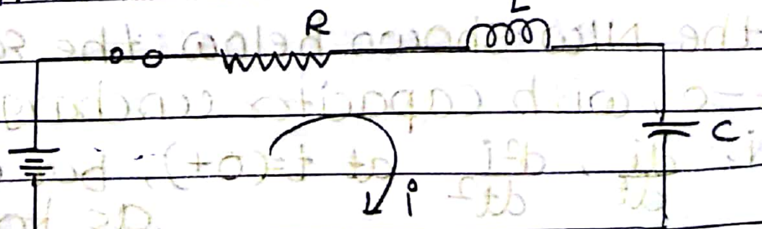
Hence $\boxed{i_1(0+) = 0A}$

Thus c/n in resistor changes abruptly.

2. In the ckt shown in fig below. $V=10V$, $R=10\Omega$, $L=1H$, $C=10\mu F$ & $V_C(0)=0$. find $i(0+)$, $\frac{di}{dt}(0+)$ and $\frac{d^2i}{dt^2}(0+)$



Sol^n When the switch K is closed at $t=0$



KVL to the above ckt $V - Ri - L\frac{di}{dt} - \frac{1}{C}\int i\,dt$

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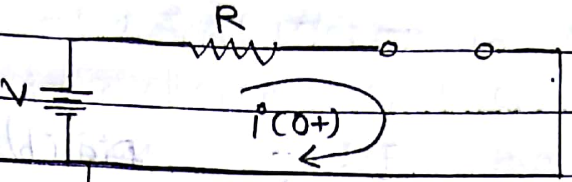
$$\therefore \boxed{V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt} \quad -①$$

At $t = 0+$



$$\boxed{i(0+) = 0} \quad —— ②$$

Substituting ② in ①

$$V = Ri(0+) + L\frac{di(0+)}{dt} + \frac{1}{C}\int i(0+)\,dt$$

$$\therefore V = 0 + L.\frac{di}{dt} + 0$$

$$\therefore \frac{V}{L} = \frac{di(0+)}{dt} = \frac{100}{1H}$$

$$\boxed{\therefore \frac{di}{dt}(0+) = 10\,A/sec}$$

Diff. eq^n ① w.r.t. t.

$$R.\frac{di}{dt} + L.\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$R\frac{di(0+)}{dt} + L.\frac{d^2i(0+)}{dt^2} + \frac{i(0+)}{C}$$

$$\text{i.e } 10 \times 10 + L.\frac{d^2i(0+)}{dt^2} + 0$$

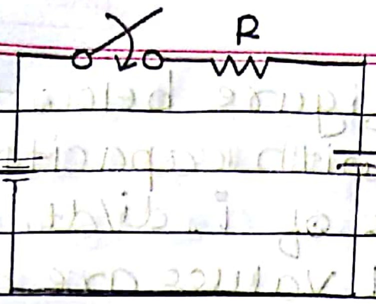
$$\therefore 100 = -L\frac{d^2i(0+)}{dt^2}$$

$$\therefore \frac{d^2i(0+)}{dt^2} = \frac{-100}{L} = \frac{-100}{1} = -100\,A/sec^2$$

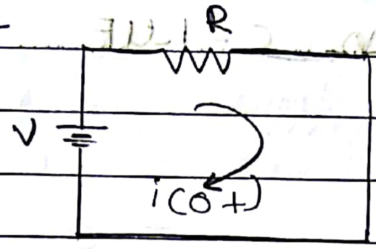
$$\boxed{\therefore \frac{d^2i(0+)}{dt^2} = -100\,A/sec}$$

8. In the N/w shown below the switch K is closed at $t=0$, with capacitor uncharge. Find values of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=(0+)$, for element values as follows

$V = 100V$, $R = 1000\Omega$, $C = 1\mu F$



Sol^n At $t=0+$



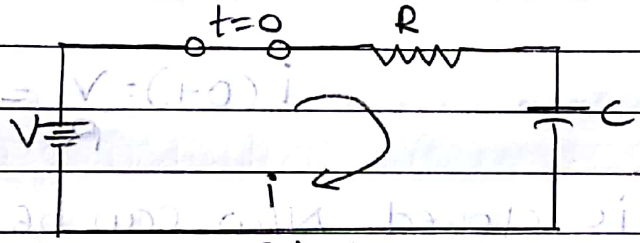
$i(0+) = \frac{V}{R}$

$= \frac{100}{1000}$

$i(0+) = 0.1A$ — (2)

Diff (1) w.r.t. t.

When k is closed at $t=0$



$V = Ri + \frac{1}{C}\int i\,dt$

On diff. we get

$0 = R\frac{di}{dt} + \frac{i}{C}$ —— (3)

$0 = R\frac{di(0+)}{dt} + \frac{i(0+)}{C}$ Substituting (2) & all values

$\therefore 0 = 1000\frac{di(0+)}{dt} + \frac{0.1}{1\mu F}$

$\therefore -1000\frac{di(0+)}{dt} = \frac{0.1}{1\times10^{-6}}$

$\therefore \frac{di(0+)}{dt} = -\frac{0.1}{1\times10^{-6}\times1000}$

$\therefore \frac{di(0+)}{dt} = -100$ A/sec.

Diff (3) w.r.t. t we get

$0 = R$

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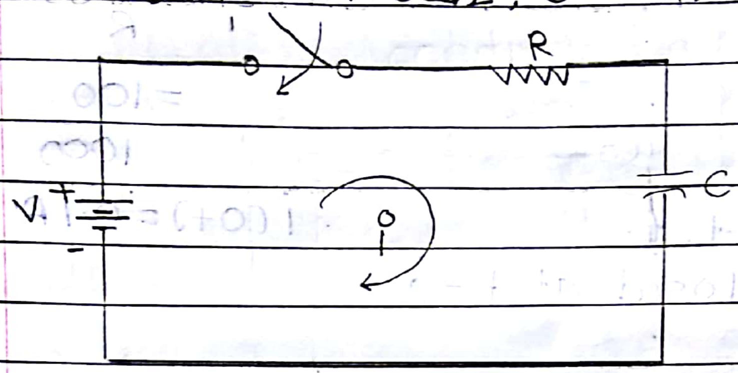
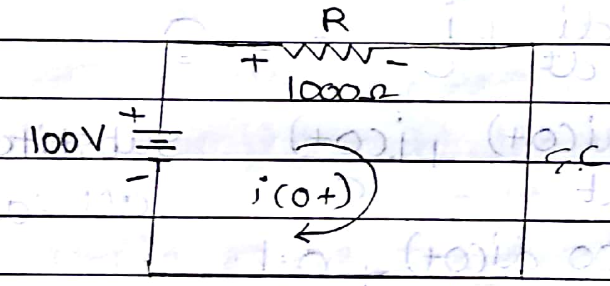
1. In the network of figure below the switch K is closed at $t=0$, with capacitor uncharged, find the values of i, di/dt, d^2i/dt^2 at $t=0+$, for element values are as follows $V=100V$, $R=1000\Omega$, $C=1\mu F$



Sol^n. At $t=0+$ $i(0+) = \frac{V}{R} = \frac{100}{1000} = 0.1A$

If switch K is closed N/w can be draw as follows.

for $t=0+$

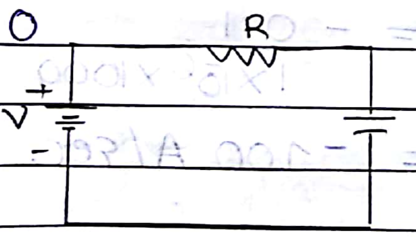


KVL to loop:- $100 - 1000\,i(0+) = 0$

$1000\,i(0+) = 100$

$\therefore \boxed{i(0+) = 0.1A} \quad -①$

At $t=0$



KVL to ckt

$V - Ri - \frac{1}{C}\int i\,dt = 0 \quad ②$

Diff ① w.r.t. t

$R\frac{di}{dt} + \frac{i}{C} = 0$

for $i(0+)$

$R\frac{d}{dt}i(0+) + \frac{i(0+)}{C} = 0 \quad ③$

Initial condition of element at $t = 0^-$ (Just open)	condition of element at $t = 0^+$ (Just closed)	Final or steady state condition of element at $t = \infty$
L No current	O.C open ckt	S.C. short ckt
L $I_0 \rightarrow$	I_0 Amps	I_0 amps
C	S.C	O.C
q_0 C $-$ $+$ $V_0 = q_0/C_0$	$V_0 = q_0/C$	O.C. $V_0 = q_0/C$

from ①

$$1000 \frac{di(0^+)}{dt} + \frac{0.1}{1\times10^{-6}} = 0$$

$$1000 \frac{di(0^+)}{dt} = -10^5$$

$$\boxed{\frac{di(0^+)}{dt} = \frac{-10^5}{10^3} = -10^2 = -100 \text{ A/sec}}$$

Diff ③ w.r.t t

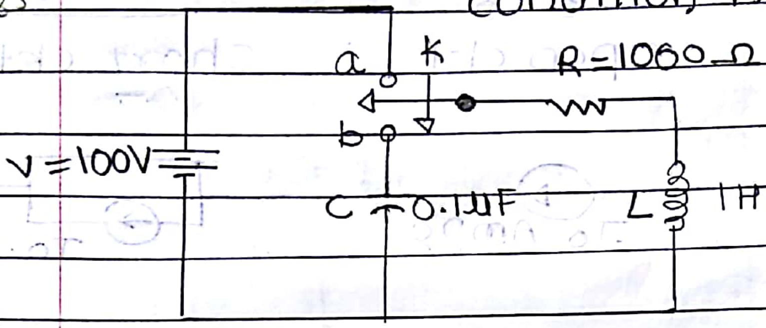
$$R \frac{d^2 i(0^+)}{dt^2} + \frac{1}{C}\frac{di}{dt}(0^+) = 0$$

$$\frac{d^2 i(0^+)}{dt^2} = \frac{-0.1}{1\times10^{-6}} \times \frac{(-100)}{1000}$$

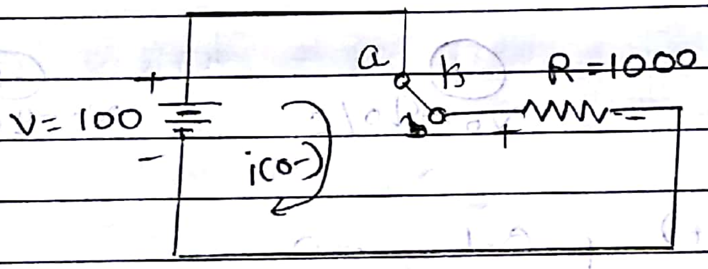
$$\boxed{\frac{d^2 i(0^+)}{dt^2} = \frac{10}{1\times10^{-3}} = 10^5 \text{ A/sec}^2}$$

2. In the N/W shown in fig. below K is changed from position A to B at $t = 0$. Solve for i, di/dt, d^2i/dt^2 at $t = 0+$, if $R = 1000\,\Omega$, $L = 1\,H$, $C = 0.1\,\mu F$, $V = 100\,V$. Assume that capacitor is initially uncharged. Steady state condition having been reached at position A.

PTMC 131/238 GR 285.



At ~~posit~~ 0+ position A.



$100 - 10^3 i(0-) = 0$

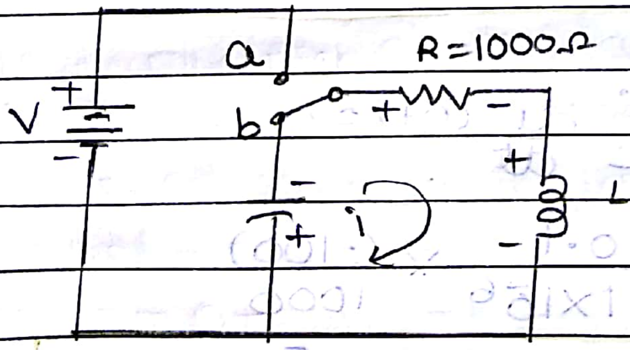
$\therefore i(0-) = 0.1A$ ①

i.e inductor has initial c/n of 0.1A

When K is changed from A to B then

$i(0-) = i(0+) = 0.1A$ — ②

When K is at position B



writing KVL

$-Ri - L\,di/dt - 1/c \int i\,dt = 0$ — ③

Diff. ③ $-R\dfrac{di}{dt} - L\dfrac{d^2i}{dt^2}$

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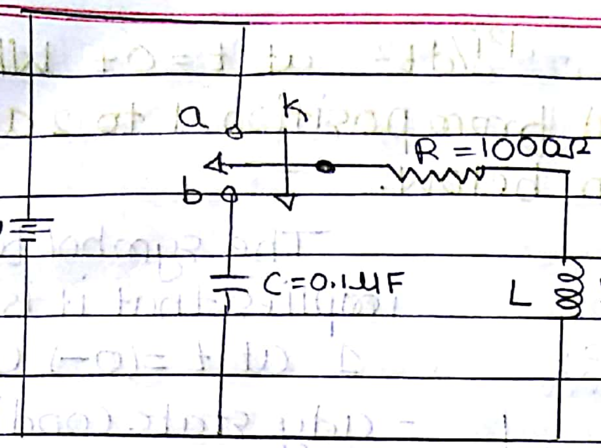
2)



When switch 'k' is at position 'a'.

$$i(0+) = \frac{V}{R} = \frac{100}{1000} = 0.1A$$

$$\boxed{\therefore i(0+) = i(0-) = 0.1A.}$$

i.e the inductor has initial current of 0.1A.

When k changed from a to b $i(0+) = 0.1A$.

When k is at b

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \quad -(1)$$

i.e $Ri(0+) + L\frac{di}{dt}(0+) + \frac{1}{C}\int i(0+)\,dt = 0$

But it is given that

$$\frac{1}{C}\int i(0+)dt = V_C(0+) = 0$$

$$\therefore 1000 \times 0.1 + 1 \times \frac{di}{dt}(0+) + 0 = 0$$

i.e $\boxed{\frac{di}{dt}(0+) = -100\ A/sec.}$

Diff. eq^n (1)

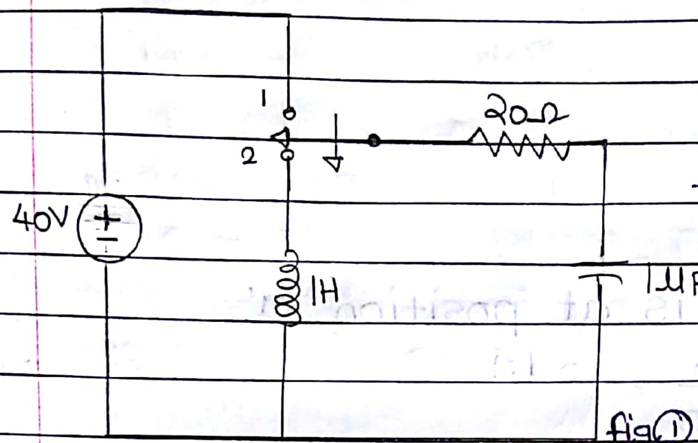
$$R.\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

i.e $R\frac{di}{dt}(0+) + L\frac{d^2i}{dt^2}(0+) + \frac{i(0+)}{C} = 0$

i.e $1000 \times (-100) + 1 \times \frac{d^2i(0+)}{dt^2} + \frac{0.1}{1 \times 10^{-6}} = 0$

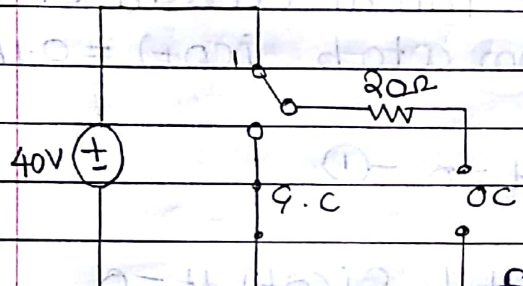
$$\boxed{\therefore \frac{d^2i}{dt^2}(0+) = -9 \times 10^5\ A/sec^2.}$$

3. Determine i, di/dt, d^2i/dt^2 at $t = 0+$ when the switch a is moved from position 1 to 2 at $t=0$ in the N/ω shown below.



The symbol for switch implies that it is at position 1 at $t=(0-)$ under steady state condition inductor acts as S.C & capacitor acts as O.C The N/ω at $t=0-$ is as fig① shown in fig. below

At position 1 ckt is under steady state



fig②

When switch 'k' is at position 'a' (Steady state)

$i(0-) = 0 = i(0+) = 0A.$

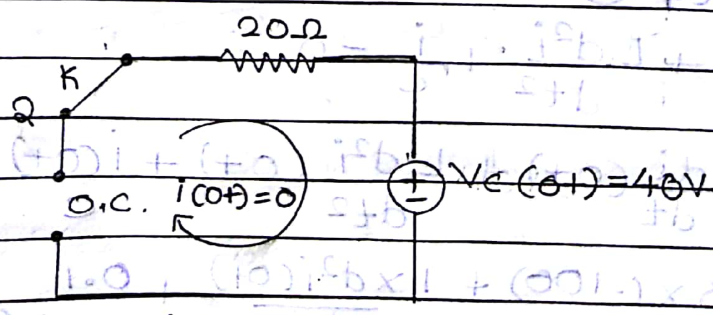
$\boxed{\therefore i(0-) = i(0+) = 0A.}$

W.K.T. voltage across capacitor can't change instantaneusly. This means that $V_c(0+) = V_c(0-) = 40V$

$\boxed{V_c(0+) = V_c(0-) = 40V.}$

For the ckt diagram for $t = 0+$ is as shown in fig below.

at $t = 0+$



fig(3)

Switch is closed at $t=0$

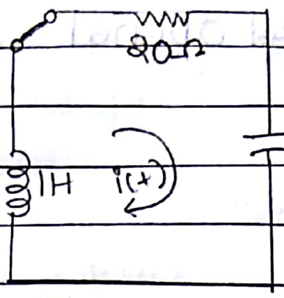


fig. 4.

KVL to the fig ④

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt = 0 \quad ①$$

$$Ri(t) + L\frac{di(t)}{dt} + V_c(t) = 0$$

At $t = 0+$ we get

$$Ri(0+) + L\frac{di(0+)}{dt} + V_c(0+) = 0$$

$$0 + 1\frac{di(0+)}{dt} + 40 = 0$$

$$\therefore \boxed{\frac{di(0+)}{dt} = -40\ A/sec}$$

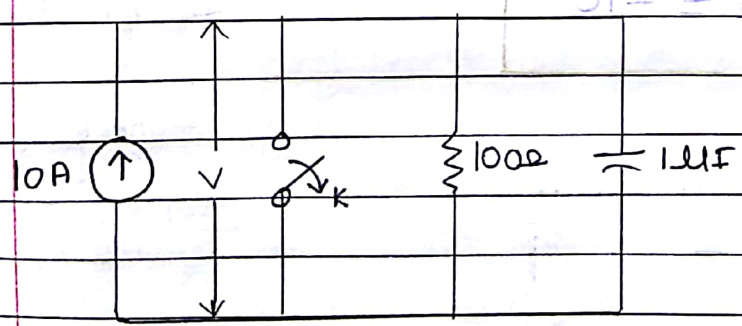
Diff ① w.r.t. t.

$$R.\frac{di}{dt}(0+) + L\frac{d^2i}{dt^2}(0+) + \frac{i(0+)}{C} = 0$$

$$20 \times -40 + 1.\frac{d^2i}{dt^2}(0+) + 0 = 0$$

$$\therefore \boxed{\frac{d^2i(0+)}{dt^2} = 800A/sec^2}$$

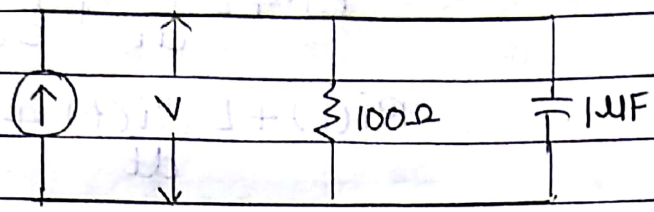
4. In the ckt shown in the fig. below the switch K is open at $t=0$. Find the values of v, dv/dt, d^2v/dt^2 at $t=0+$



Sol^y When the Switch K is closed all the c/n flows thro- -ugh the short circuit. The capacitor is not cha- -rged as it acts as an open circuit.

∴ $\boxed{V_C(0-) = 0 = V_C(0+)}$

When the switch K is ~~closed~~ opened



Writing KCL

$$10 = \frac{V}{100} + C\frac{dV}{dt} \quad \text{—} \ (1)$$

$$\therefore \frac{dV(0+)}{dt} = \left(10 - \frac{V(0+)}{100}\right)/C = \left(10 - \frac{0}{100}\right)/1\times10^{-6}$$

$$\therefore \frac{dV(0+)}{dt} = \left(\frac{10\cancel{0}\cancel{0}}{1\cancel{0}\cancel{0}}\right)\times 10^{6}$$

$$\boxed{\therefore \frac{dV(0+)}{dt} = 10^{7}\ V/sec}$$

Diff (1) w.r.t. t.

$$0 = \frac{1}{100}\frac{dV}{dt} + C\frac{d^2V}{dt^2}$$

$$0 = \frac{1}{100}\frac{dV(0+)}{dt} + C\frac{d^2V(0+)}{dt^2} = 0$$

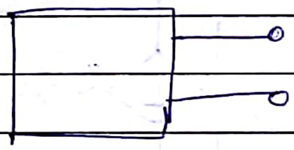
$$0 = \frac{1}{100}\times10^{7} + 1\times10^{-6}\frac{d^2V(0+)}{dt^2} = 0$$

$$-10^{5} = 10^{-6}\frac{d^2V(0+)}{dt^2}$$

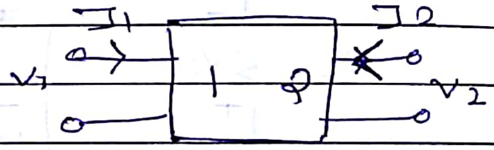
$$\boxed{\therefore \frac{d^2V(0+)}{dt^2} = -10^{11}}$$

Two Port Network Parameters:-

1) 1 port N/W:-

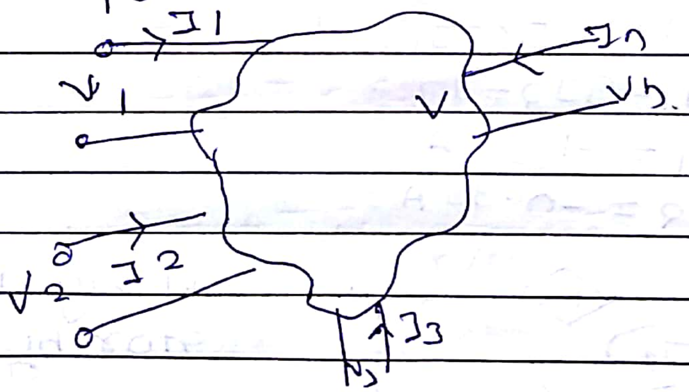


2) 2 port N/W



Place where we can apply something or getting something. called port.

3) multiport N/W.



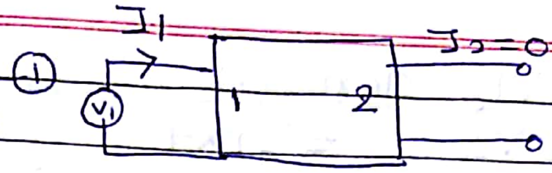
* Two port Network:-

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — ①

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — ②

$Z_{11}, Z_{12}, Z_{21}, Z_{22}$ - Z parameter: O.C. parameter

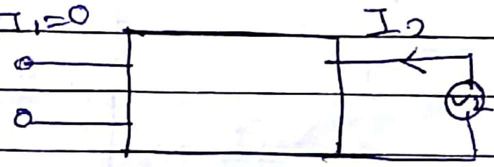
$Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$ — ①



$Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$ — ②

Z_{11} – driving point impedance at port-1

$Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$



$Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0}$

Z_{22} – Driving point impedance at port 2

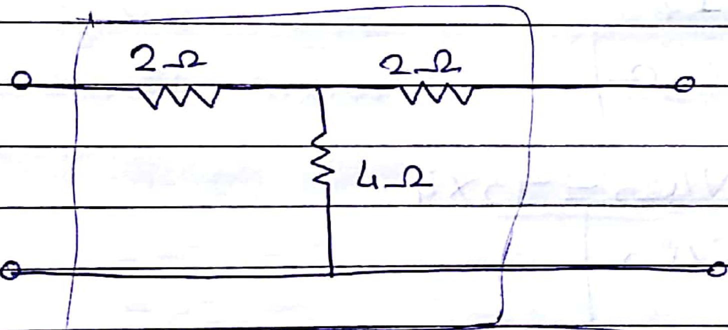
Z_{11} is n/w function of 2 port N/w.

V_1/I_1 is n/w function &

Z_{21} = Transfer impedance

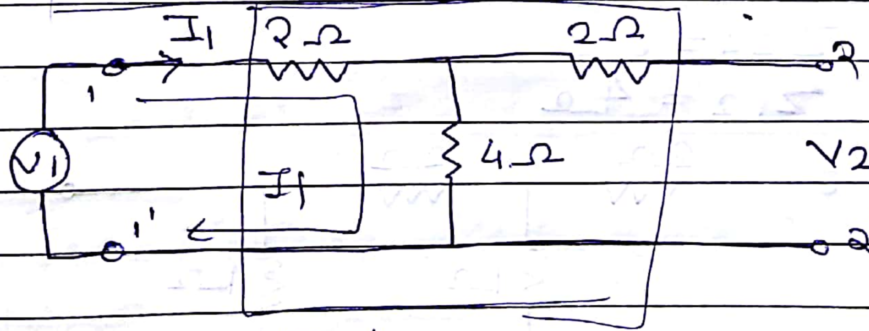
Z_{12} = Transfer impedance

①



Find z parameter for given N/W.

Ans



port 2 is open
$\therefore I_2 = 0.$

$Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$

$V_1 = 2I_1 + 4I_1 \Rightarrow V_1 = 6I_1 \Rightarrow \dfrac{V_1}{I_1} = 6\Omega$

$\therefore Z_{11} = \dfrac{V_1}{I_1} = 6\ \Omega$

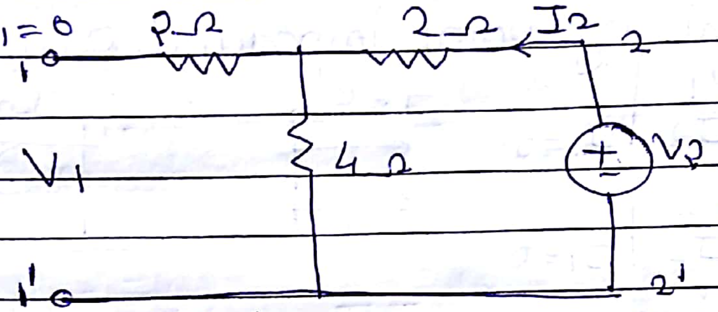
$Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$

$V_2 = V_{4\Omega}$ across $4\,\Omega$.

$V_2 = V_{4\Omega} = I_1 \times 4$

$\frac{V_2}{I_1} = 4\,\Omega$

$\therefore Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 4\,\Omega$



11' is open
$\therefore I_1 = 0$.
exitation is applied bet^n 22'

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_2=0}$

$V_2 = 6 I_2$

$\boxed{\therefore \frac{V_2}{I_2} = 6\,\Omega}$

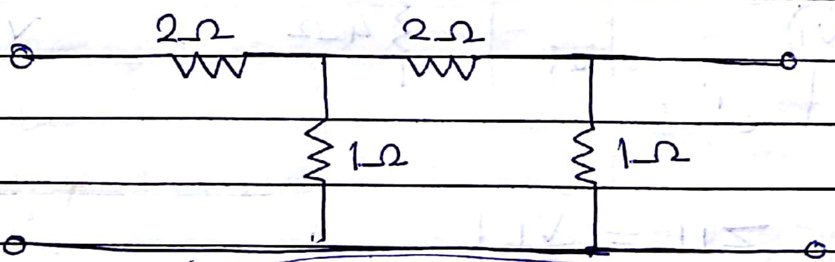
$V_1 = V_{4\Omega} = I_2 \times 4$

$\boxed{\frac{V_1}{I_2} = 4\,\Omega}$

$\therefore Z_{22} = 6\,\Omega$

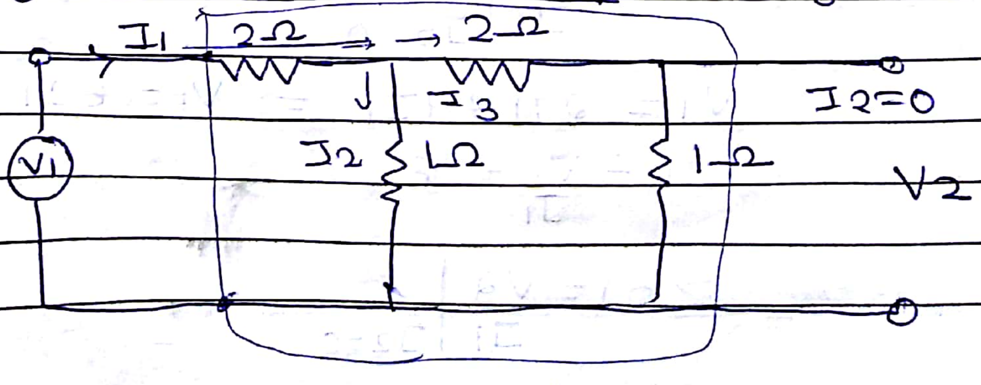
$Z_{12} = 4\,\Omega$

(2)



find Z parameter

Sol^n



i/p Impedance of CRO - 1MΩ

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$

$V_1 = I_1 \times$ impedance betⁿ 1 & 1'

$V_1 = I_1 \times$ Resistance betⁿ 1 & 1'

2Ω series 1Ω = 3Ω

$3\Omega \parallel 1\Omega = \frac{3\times 1}{3+1} = \frac{3}{4}$ is series with 2Ω

$\frac{3}{4}\Omega + 2\Omega =$

$\therefore V_1 = 2.75 I_1$

$\therefore \boxed{\frac{V_1}{I_1}\Big|_{I_2=0} = Z_{11} = 2.75\Omega}$

$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$

$I_3 = \frac{I_1 \times 1}{2+1+2}$ $I_3 = \frac{I_1}{4}$

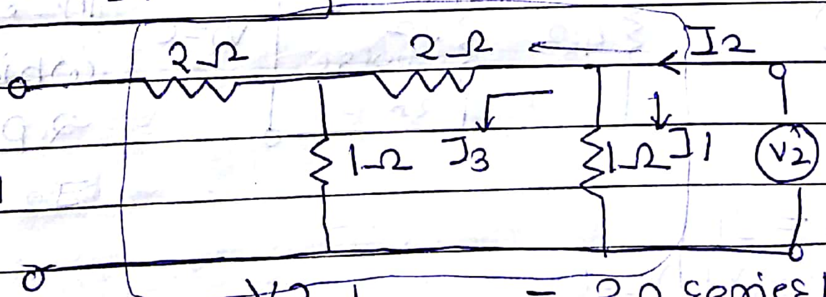
$V_{1\Omega} = I_3 \times 1$

$V_{1\Omega} = \frac{I_1}{4} \times 1$

$\therefore \frac{V_{1\Omega}}{I_1} = \frac{1}{4}$ $V_2 = V_{1\Omega}$

$\therefore V_2 = \frac{I_1}{4}$

$\boxed{\therefore \frac{V_2}{I_1} = \frac{1}{4}} = Z_{21}$



$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$ = 2Ω series 1Ω = 3Ω ∥ 1Ω
= 3/4 Ω

$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$ =

$I_3 = \frac{I_2 \times 1\Omega}{2+1+1} = \frac{I_2}{4} \Rightarrow I_3 = \frac{I_2}{4}$

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$$V_1 = \frac{I_2}{4} \times 1 = \frac{I_2}{4}$$

$$\boxed{\therefore \frac{V_1}{I_2} = \frac{1}{4}\Omega = Z_{12}}$$

* Y Parameter :- (short circuit Admitance)

$I_1 = Y_{11}V_1 + Y_{12}V_2$ — ① $\qquad V+q=0=$ s.c.

$I_2 = Y_{21}V_1 + Y_{22}V_2$ — ②

Y parameters $Y_{11}, Y_{12}, Y_{21}, Y_{22}$

① ⇒ $I_1 = Y_{11}V_1 \big|_{V_2=0}$

$$\boxed{Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}}$$ ℧ or seimens driving point admitance at port 1

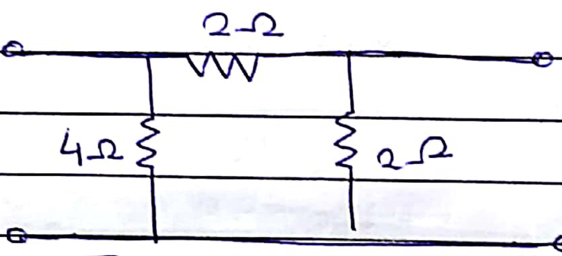
② ⇒ $I_2 = Y_{21}V_1$

$$\boxed{Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}}$$ ℧/seime, transfer admitance

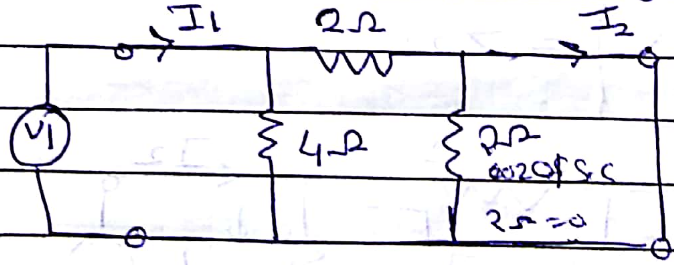
$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ Transfer admitance

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$ driving point admitance

ex: ①



sol^n



I2 is not going in, which is coming out which is against 2 port. Hence I2 is consider as -ve

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$

$2\Omega \| 4\Omega = 2\times4/(2+4) = 1.33$

$\frac{V_1}{I_1} = 1.33$

or $V_1 = I_1 \times 1.33 \Rightarrow \frac{I_1}{V_1} = \frac{1}{1.33} = 0.75$ ℧

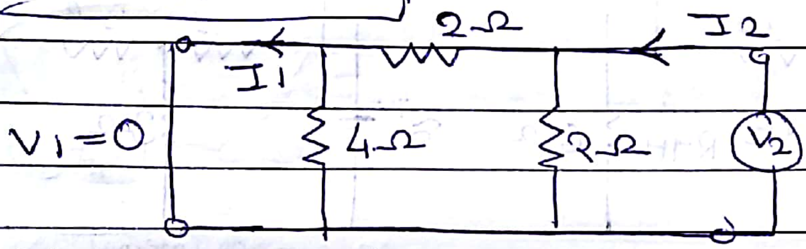
$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ I_2 is coz of I_1

$I_2 = \frac{I_1 \times 4}{2+4} = \frac{4I_1}{6} \doteq 0.66 I_1$

$\boxed{I_2 = 0.66 I_1}$ $\because$ I_2 = $-$ve always for 2 port N/W.

$Y_{21} = \frac{I_2}{V_1} = \frac{0.66 I_1}{1.33 I_1}$

$\boxed{Y_{21} = 0.5 \mho}$



$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$ $2\Omega \parallel 2\Omega = 1\Omega$

$Y_{22} = 1\Omega$

$Y_{21} = -0.5\Omega$

Hybrid parameters:—

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

By S.C. V_2 we are getting $h_{11} = V_1/I_1 \big|_{V_2=0}$

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ driving point impedance at port-1

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ current gain. unitless.

By O.C Port 1 we get

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ vtg gain. unitless

$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$ driving point admitance at port 2

LAPLACE TRANSFORMATION AND APPLICATIONS

Laplace transformation – It's a transformation method used for solving differential equation.

Advantages

- The solution of differential equation using LT, progresses systematically.
- Initial conditions are automatically specified in transformed equation.
- The method gives complete solution in one operation. (Both complementary function and particular Integral in one operation)
- The Laplace Transform of a function, *f(t)*, is defined as

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

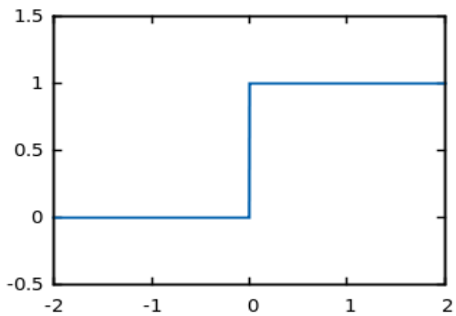
- Where *S* is the complex frequency
- Condition for Laplace transform to exist is

$$\int_{-\infty}^{\infty} f(t)\,e^{-st}dt \ < \ \infty$$

$$f(t) = L^{-1}F(S) = \frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} F(s)\,e^{st}ds$$

Unit step function

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$



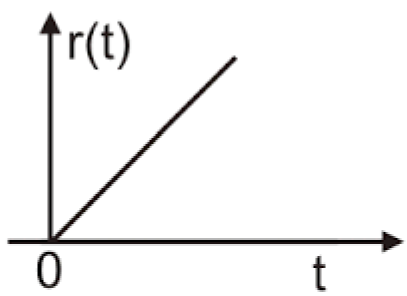
Delta function

$$\delta(t) = 0 \quad t \neq 0$$

$$\lim_{\epsilon \to 0} \int_0^{\epsilon} \delta(t) dt = 1$$

$$L\big(\delta(t)\big) = \int_0^{\infty} \delta(t) e^{-st} dt = e^{-0s} = 1$$

Ramp function



$$f(t) = t$$

$$F(s) = \int_0^{\infty} t e^{-st}\, dt$$

$$F(s) = \left[t \frac{e^{-st}}{s} \right]_0^{\infty} + \int_0^{\infty} \frac{1}{s} e^{-st} dt$$

$$F(s) = \frac{1}{s^2}$$

Laplace Transform of exponential function

$$f(t) = e^{-at}$$

$$L(e^{-at}\ ) = \int_0^{\infty} e^{-at} e^{-st} dt$$

$$= \int_0^\infty e^{-(s+a)t}\,dt$$

$$= \frac{1}{(s+a)}$$

$$f(t) = e^{at}$$

$$L(e^{at}) = \frac{1}{(s-a)}$$

$$f(t) = sin\omega t = \frac{1}{2j}(e^{j\omega t} - e^{-j\omega t})$$

$$f(t) = sin\omega t = \frac{1}{2j}(e^{j\omega t} - e^{-j\omega t})$$

$$L(sin\omega t) = L[\frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)]$$

$$= \frac{1}{2j}(L\left[e^{j\omega t}\right] - L\left[e^{-j\omega t}\right])$$

$$\frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right] = \frac{\omega}{s^2+\omega^2}$$

$$f(t) = cos\omega t = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$$

$$L(cos\omega t) = L[\frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right)]$$

$$= \frac{1}{2}(L\left[e^{j\omega t}\right] + L\left[e^{-j\omega t}\right])$$

$$\frac{1}{2}\left[\frac{1}{s-j\omega}+\frac{1}{s+j\omega}\right]=\frac{s}{s^2+\omega^2}$$

Laplace transform of derivative

Consider a function f(t)

W K T

$$L\big(f(t)\big) = F(s) = \int_0^{\infty} f(t)e^{-st}\,dt$$

Let $Let\;\; u = f(t),\;\; and\;\; dv = e^{-st}dt$

$$du = \left[\frac{df}{dt}\right]dt, \qquad v = -\frac{1}{s}e^{-st}$$

$$F(s) = \left[-\frac{f(t)}{s}e^{-st}\right]_0^{\infty} + \frac{1}{s}\int_0^{\infty}\frac{df}{dt}e^{-st}dt$$

$$F(s) = \frac{f(0)}{s} + \frac{1}{s}L\left[\frac{df}{dt}\right]$$

$L\left[\frac{df}{dt}\right] = s\,F(s) - \,f(0)$

In general

$$L\left[\frac{d^n f(t)}{dt^n}\right] = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0)\,\dots\dots - f^{n-1}(0)$$

Laplace Transform of Integration

$$L\left[\int_0^t f(t)\,dt\right] = \int_0^{\infty}\left[\int_0^t f(t)\,dt\right]e^{-st}dt$$

$$Let\quad u = \int_0^t f(t)\,dt\ , \qquad dv = e^{-st}dt$$

$$du = f(t)dt, \quad v = -\frac{1}{s}e^{-st}$$

$$L\left[\int_0^t f(t)\,dt\right] = \left[-\frac{e^{-st}}{s}\int_0^t f(t)\,dt\right]_0^\infty + \frac{1}{s}\int_0^\infty f(t)e^{-st}\,dt$$

$$L\left[\int_0^t f(t)\,dt\right] = +\left[\frac{1}{s}\int f(t)\,dt\right]_0 + \frac{F(s)}{s}$$

$$\left[\int f(t)\,dt\right]_0 \; is\ the\ value\ of\ integral\ f(t) as$$

$$t\ approches\ 0\ from + ve\ side$$

Laplace transform of some important functions

$$u(t) \rightarrow \frac{1}{s}$$

$$e^{-at} \rightarrow \frac{1}{(s+a)}$$

$$\sin \omega t \rightarrow \frac{\omega}{s^2+\omega^2}$$

$$\cos \omega t \rightarrow \frac{s}{s^2+\omega^2}$$

$$t \rightarrow \frac{1}{s^2}$$

$$t^2 \rightarrow \frac{2}{s^3}$$

$$t^n \rightarrow \frac{n!}{s^{n+1}}$$

$$e^{-at}t^n \rightarrow \frac{n!}{(s+a)^{n+1}}$$

$$e^{-at}\sin \omega t \rightarrow \frac{\omega}{(s+a)^2+\omega^2}$$

$$e^{-at}\cos \omega t \rightarrow \frac{s}{(s+a)^2+\omega^2}$$

$$\delta(t) \rightarrow 1$$

$$\sin h\ at \rightarrow \frac{a}{s^2 - a^2}$$

$$\cos h\ at \rightarrow \frac{s}{s^2 - a^2}$$

$$Shifting\ theorem$$

$$L[u(t-a)] = \int_a^\infty 1. e^{-st}\ dt$$

$$= e^{-as}\frac{1}{s}$$

$f(t-a)u(t-a)\ is\ function\ f(t)\ shifted\ to\ a$

$$L[f(t-a)u(t-a)] = e^{-as}F(s)$$

$$L^{-1}e^{-as}F(s) = f(t-a)u(t-a)$$

These equations tell us that transform of any function delayed to begin at time t=a, is e^{-as} times transform of the function when it begins at t=0. This is known as shifting theorem.

$$Given\ \ L[f(t)] = F(s)\ , \qquad L[f(t-a)u(t-a)] = e^{-as}F(s)$$

Initial value Theorem

It states that

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

Proof

$$\lim_{s\to\infty} \int_0^\infty \frac{df}{dt} e^{-st} dt = \lim_{s\to\infty} [sF(s) - f(0)] \quad \ldots\ldots\ 1$$

Substituting $s \rightarrow \infty$ in integration we have

$$0 = \lim_{s\to\infty} [sF(s) - f(0)]$$

$$f(0) = \lim_{s\to\infty}[sF(s)]$$
$$f(0) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

Final value theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Since s is not a function of t

$$\lim_{s\to 0}\int_0^\infty \frac{df}{dt}e^{-st}dt = \lim_{s\to 0}[sF(s) - f(0)]$$

Letting s→ 0 on LHS

$$\int_0^\infty \frac{df}{dt}dt = \lim_{t\to\infty}\int_0^t \frac{df}{dt}dt$$

$$\lim_{t\to\infty}[f(t) - f(0)] = \lim_{s\to 0}[sF(s) - f(0)]$$

Hence

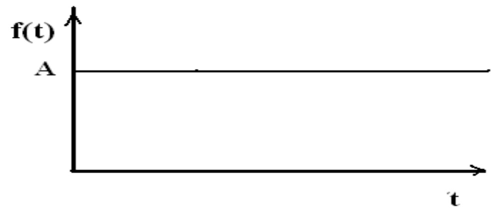
$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Wave form synthesis

Unit step function

$$f(t) = u(t) = 1 \quad t \geq 0$$
$$= 0 \quad t < 0$$
$$F(s) = \frac{1}{s}$$

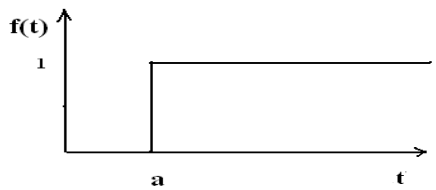
$$f(t) = A \quad t \geq 0$$
$$= 0 \quad t < 0$$
$$f(t) = Au(t)$$



$$F(s) = \frac{A}{S}$$

Delayed unit step

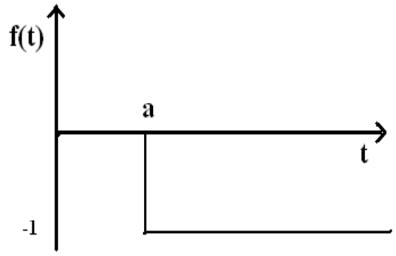
$$Consider\ a\ function\quad f(t) = 1 \quad t \geq a$$
$$= 0 \quad t < a$$
$$f(t) = u(t-a)$$



$$F(s) = e^{-as}\frac{1}{S}$$

Delayed –ve unit step

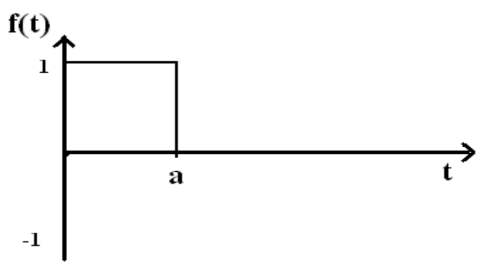
$$f(t) = 0 \quad t < a$$
$$= -1 \quad t \geq a$$
$$f(t) = -u(t-a)$$

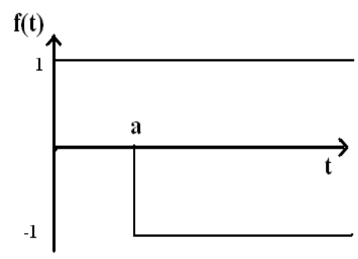


$$F(s) = -e^{-as}\frac{1}{S}$$

Waveform synthesis involving unit step function

$$f(t) = 1 \quad 0 \le t \le a$$
$$= 0 \quad for\ all\ other\ values\ of\ t$$
$$f(t) = u(t) - u(t-a)$$

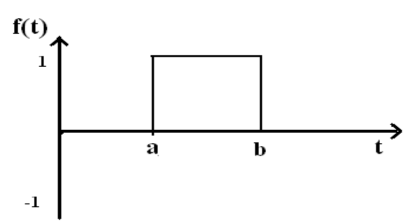




$$F(s) = \frac{1}{s} - e^{-as}\frac{1}{s}$$

Rectangular pulse

$$f(t) = 1 \quad a \le t \le b$$
$$= 0 \quad for\ all\ other\ values\ of\ t$$
$$f(t) = u(t-a) - u(t-b)$$



$$F(s) = e^{-as}\frac{1}{S} - e^{-bs}\frac{1}{S}$$

Laplace transform of periodic function

Let f(t) be a periodic function with period T.

Let f1(t),f2(t), f3(t) ….be the functions describing the first cycle, second cycle, third cycle ….

$$f(t) = f_1(t) + f_2(t) + f_3(t) \ldots.$$

$$f(t) = f_1(t) + f_1(t-T)u(t-T) + f_1(t-2T)u(t-2t) \ldots.$$

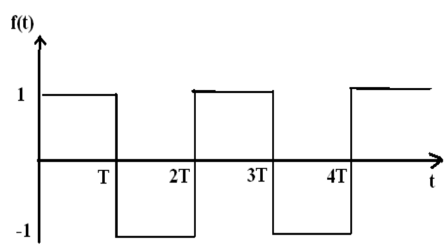
$$Let \quad L[f_1(t)] = F_1(s)$$

Therefore by shifting theorem

$$L[\,f(t)] = F_1(s)[1 + e^{-Ts} + e^{-2Ts} + \cdots]$$

$$L[\,f(t)] = \left[\frac{1}{1 - e^{-Ts}}\right] F_1(s)$$

Rectangular wave of time period 2T



$$Let\ f_1(t)\ be\ the\ first\ cycle\ of\ the\ waveform$$

$$f_1(t) = u(t) - 2u(t-T) + u(t-2T)$$

$$f(t) = f_1(t) + f_1(t-2T)u(t-2T) + f_1(t-4T)u(t-4T) \ldots.$$

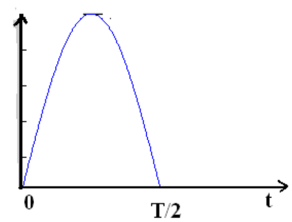
$$L[f_1(t)] = F_1(s) = \left[\frac{1}{s} - \frac{2e^{-Ts}}{s} + \frac{e^{-2Ts}}{s}\right]$$

$$L[\,f(t)] = \left[\frac{1}{1 - e^{-2Ts}}\right]\left[\frac{1}{s} - \frac{2e^{-Ts}}{s} + \frac{e^{-2Ts}}{s}\right]$$

$$L[f(t)] = \frac{1}{s}\left[\frac{(1-e^{-Ts})^2}{(1-e^{-2Ts})}\right]$$

$$= \frac{1}{s}\left[\frac{(1-e^{-Ts})^2}{(1-e^{-Ts})(1+e^{-Ts})}\right] = \frac{1}{s}\left[\frac{(1-e^{-Ts})}{(1+e^{-Ts})}\right]$$

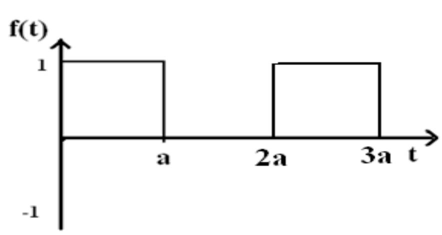
Half cycle of sine wave



$$f(t) = \sin\omega t \quad 0 < t < \frac{T}{2}$$
$$= 0 \; otherwise$$
$$f(t) = sin\omega t\, u(t) + \sin\left(\omega t - \frac{T}{2}\right) u\left(t - \frac{T}{2}\right)$$

$$F(s) = \frac{\omega}{s^2+\omega^2}\left[1 + e^{\frac{-Ts}{2}}\right]$$

Show that the transform of the square wave is $\frac{1}{s(1+e^{-as})}$



$$f(t) = f_1(t) + f_2(t) + f_3(t) \; ....$$

$$f(t) = f_1(t) + f_1(t-2a) + f_1(t-4a) \ldots.$$

$$f_1(t) = u(t) - u(t-a)$$

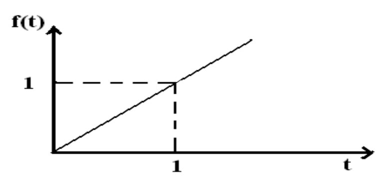
$$F_1(s) = \frac{1}{s}(1 - e^{-as})$$

$$F(s) = \frac{1}{(1 - e^{-2as})} F_1(s)$$

$$F(s) = \frac{(1 - e^{-as})}{s(1 - e^{-2as})} = \frac{(1 - e^{-as})}{s(1 - e^{-as})(1 + e^{-as})}$$

$$F(s) = \frac{1}{s(1 + e^{-as})}$$

Ramp Function

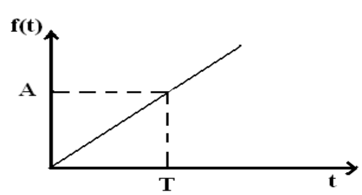


$$f(t) = t \quad t \geq 0$$
$$= 0 \quad t < 0$$

$$f(t) = tu(t)$$

$$F(s) = \frac{1}{s^2}$$

Ramp with slope A/T

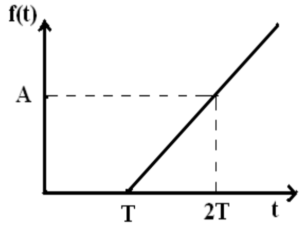


$$f(t) = \frac{A}{T} t \quad t \geq 0$$
$$= 0 \quad t < 0$$

$$f(t) = \frac{A}{T} tu(t)$$

$$F(s) = \frac{A}{T}\frac{1}{s^2}$$

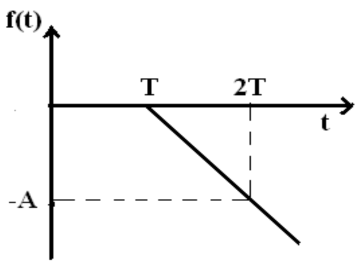
Shifted ramp



$$f(t) = \frac{A}{T}(t-T)u(t-T)$$

$$F(s) = e^{-Ts}\frac{A}{Ts^2}$$

Shifted ramp with negative slope



$$f(t) = -\frac{A}{T}(t-T)u(t-T)$$

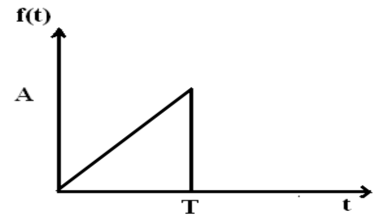
$$F(s) = -e^{-Ts}\frac{A}{Ts^2}$$

Addition of two ramp function

$$f(t) = \frac{A}{T}tu(t) - \frac{A}{T}(t-T)u(t-T)$$

$$F(s) = \frac{A}{Ts^2}[1 - e^{-Ts}]$$

Saw tooth waveform with slope A/T



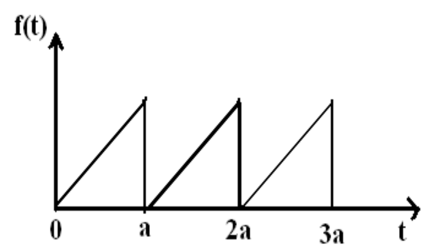
$$f(t) = \frac{A}{T} t u(t) - \frac{A}{T}(t-T)u(t-T) - Au(t-T)$$

$$F(s) = \frac{A}{T}\frac{1}{s^2} - e^{-Ts}\frac{A}{T}\frac{1}{s^2} - e^{-Ts}\frac{A}{s}$$

$$F(s) = \frac{A}{Ts^2}[1 - e^{-Ts}] - \frac{Ae^{-Ts}}{s}$$

For the waveform shown, show that the transform of this function is

$$F(s) = \frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$$



$$f(t) = f_1(t) + f_2(t) + f_3(t) \ldots.$$

$$f(t) = f_1(t) + f_1(t-a) + f_1(t-2a) \ldots.$$

$$f_1(t) = \frac{1}{a} t u(t) - \frac{1}{a}(t-a)u(t-a) - u(t-a)$$

$$F_1(s) = \frac{1}{as^2} - e^{-as}\frac{1}{as^2} - e^{-as}\frac{1}{s}$$

$$F_1(s) = \frac{1}{as^2}[1 - e^{-as}] - \frac{e^{-as}}{s}$$

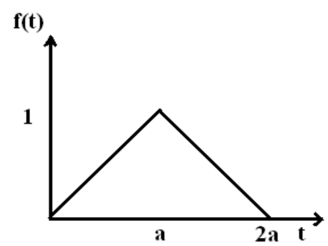
$$F(s) = \frac{1}{[1 - e^{-as}]} F_1(s)$$

$$F(s) = \frac{1}{[1 - e^{-as}]} \left[\frac{1}{as^2} [1 - e^{-as}] - \frac{e^{-as}}{s} \right]$$

$$F(s) = \frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$$

Hence proved

Triangular Waveform



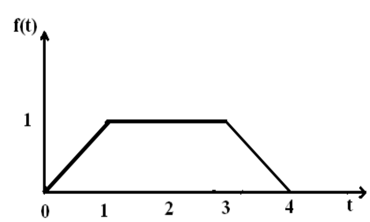
$$f(t) = tu(t) - 2(t-a)u(t-a) + (t-2a)u(t-2a)$$

$$F(s) = \frac{1}{s^2} - e^{-as}\frac{2}{s^2} + e^{-2as}\frac{1}{s^2}$$

$$F(s) = \frac{1}{s^2}[1 - 2e^{-as} + e^{-2as}]$$

$$F(s) = \frac{1}{s^2}[1 - e^{-as}]^2$$

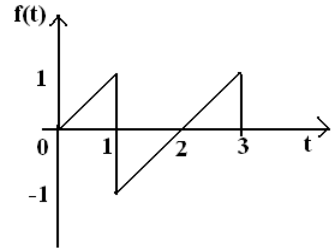
Trapezoidal wave



$$f(t) = tu(t) - (t-1)u(t-1) - (t-3)u(t-3) + (t-4)u(t-4)$$

$$F(s) = \frac{1}{s^2}[1 - e^{-s} - e^{-3s} + e^{-4s}]$$

Find the Laplace transform of the waveform shown in figure

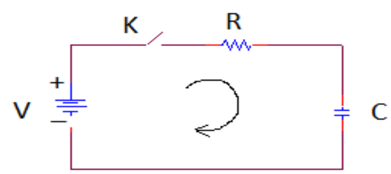


$$f(t) = tu(t) - 2u(t-1) - u(t-3) - (t-3)u(t-3)$$

$$F(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s} - \frac{e^{-3s}}{s} - \frac{e^{-3s}}{s^2}$$

Solution of networks using Laplace Transform

1) Consider a series RC network as shown in figure. It is assumed that the switch K is closed at t=0. Find the current flowing through the network.



Solution

Applying KVL, the equation for the circuit is

$$\frac{1}{C}\int_{-\infty}^{t} idt + Ri = Vu(t)$$

The transform of the equation is

$$\frac{1}{C}\left[\frac{I(s)}{s} + \frac{q(0)}{s}\right] + RI(s) = \frac{V}{s}$$

If the capacitor is initially uncharged, the above equation reduces to the form

$$I(s)\left[\frac{1}{Cs} + R\right] = \frac{V}{s}$$

Therefore

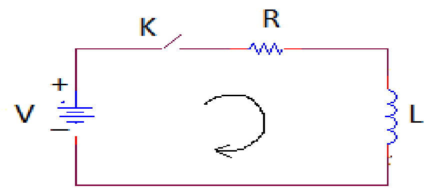
$$I(s) = \frac{V}{s\left[\frac{1}{Cs} + R\right]}$$

$$I(s) = \frac{V/R}{[s + 1/RC]}$$

$$i(t) = L^{-1}I(s) = L^{-1}\left[\frac{V/R}{s + 1/RC}\right]$$

$$i(t) = \frac{V}{R}e^{-t/RC}\ Amp$$

2) Consider a series RL network as shown in figure. It is assumed that the switch K is closed at t=0. Find the current flowing through the network.



Solution

Applying KVL, the equation for the circuit is

$$L\frac{di}{dt} + Ri = Vu(t)$$

The corresponding transformed equation is

$$L[sI(s) - i(0)] + RI(s) = \frac{V}{s}$$

Since i(0-)= 0, we have

$$[Ls + R]I(s) = \frac{V}{s}$$

$$I(s) = \frac{V}{s[Ls + R]}$$

$$I(s) = \frac{V/L}{s[s + R/L]}$$

To bring I(s) expression to the standard form, to take Laplace Inverse, let us apply partial fraction expansion for I(s)

$$I(s) = \frac{V/L}{s[s + R/L]} = \frac{A}{s} + \frac{B}{s + R/L}$$

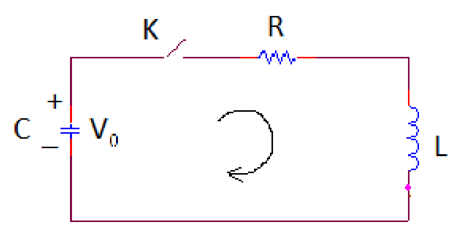
It can be found that A=V/L and B=$-V/L$

Therefore

$$I(s) = \frac{V}{L}\left[\frac{1}{s} - \frac{1}{s + R/L}\right]$$

$$i(t) = \frac{V}{L}\left(1 - e^{-\frac{Rt}{L}}\right)\ Amp$$

3) Consider a series RLC circuit with the capacitor initially charged to voltage V_0=1 volt



Solution

By applying KVL , the differential equation of the circuit can be written as

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{-\infty}^{t} idt = 0$$

The corresponding Transformation equation is

$$L[sI(s) - i(0-)] + RI(s) + \frac{1}{Cs}[I(s) + q(0-)] = 0$$

$$\frac{q(0-)}{Cs} = -\frac{V_0}{s}\ (for\ the\ polarities\ shown\ in\ figure)$$

$$i(0-) = 0\ A$$

Therefore

$$I(s)\left[Ls + R + \frac{1}{Cs}\right] = \frac{V_0}{s}$$

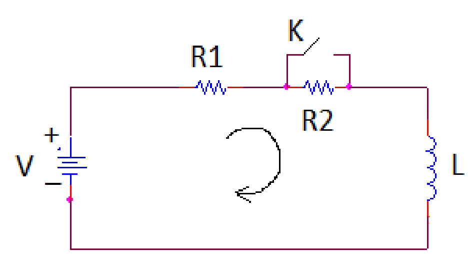
Substituting R=1ohm, L=1H, C=1/2 F and V_0=1 volt we have

$$I(s) = \frac{1}{s^2 + 2s + 2}$$

$$I(s) = \frac{1}{(s+1)^2 + 1}$$

$$i(t) = e^{-t} sint\ u(t) \quad Amp$$

4) In the circuit shown in figure, steady state is reached with switch K open. Obtain the expression for current when switch K is closed at t=0.
   Assume R1=1Ω ,R2=1 Ω, L=1H V=10V. Ω



Solution

Applying KVL, with the switch is closed

$$Vu(t) = R1i(t) + L\frac{di(t)}{dt}$$

Taking Laplace transform of the above equation yields

$$\frac{V}{s} = R1I(s) + L[sI(s) - i(0)]$$

$$i(0) = \frac{V}{R1+R2} = \frac{10}{3} = 3.333Amp$$

Substituting the values of R1, L and i(0-)

$$I(s) = \frac{10 + 3.333s}{s(s+1)} \ldots\ldots\ldots .1$$

Applying Partial fraction Expansion

$$I(s) = \frac{A}{s} + \frac{B}{(s+1)} \ldots\ldots .2$$

Solving for A and B

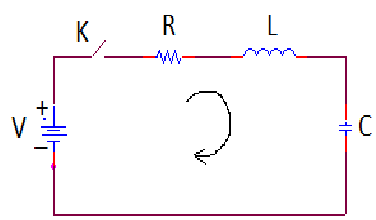
A=10 and B=−6.667

$$I(s) = \frac{10}{s} - \frac{6.667}{(s+1)}$$

Therefore

$$i(t) = [10 - 6.667e^{-t}]u(t) \ \ Amp$$

5) Derive the expression for current i(t) for the series RLC circuit shown. Assume zero initial conditions.



Solution

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t idt = V$$

The transformed equation is

$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = \frac{V}{s}$$

$$I(s) = \frac{V/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$I(s) = \frac{V}{L}\left[\frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}\right]$$

$$I(s) = \frac{V}{L}\left[\frac{1}{(s - S1)(s - S2)}\right]$$

Where $\quad S1, S2 = -\frac{R}{2L} \pm \sqrt{\left[\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}\right]}$

CASE 1: The roots are real and unequal $S1 \neq S2$

$$I(s) = \frac{V}{L(S1 - S2)}\left[\frac{1}{(s - S1)} - \frac{1}{(s - S2)}\right]$$

$$i(t) = \frac{V}{L(S1 - S2)}\left[e^{S1t} - e^{S2t}\right]$$

CASE2: S1=S2

$$I(s) = \frac{V}{L}\frac{1}{(s - S1)^2}$$

$$i(t) = \frac{V}{L}te^{S1t}\,u(t)\;Amp$$

CASE 3

$$S1, S2 = -\alpha \pm j\omega$$

Therefore

$$I(s) = \frac{V}{L}\left[\frac{1}{(s + \alpha + j\omega)(s + \alpha - j\omega)}\right]$$

$$I(s) = \frac{V}{L}\left[\frac{1}{(s + \alpha)^2 + \omega^2}\right]$$

$$I(s) = \frac{V}{L\omega}\left[\frac{\omega}{(s+\alpha)^2 + \omega^2}\right]$$

$$i(t) = \frac{V}{L\omega}[e^{-\alpha t} sin\omega t]$$

The Transformed Networks

The voltage – current relation of network elements can also be represented in frequency domain.

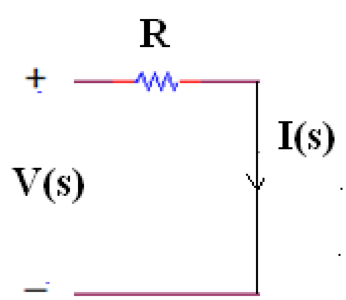
Resistor

For a resistor, the voltage-current relationship is

$$v(t) = Ri(t)$$

The Laplace transform of the above equation is

$$V(s) = RI(s)$$



Inductor

For an inductor, the voltage-current relationship is

$$v(t) = L\frac{di}{dt}$$

The Laplace transform of the above equation is

$$V(s) = L[sI(s) - i(0)] \ldots \ldots \ldots .. (1)$$

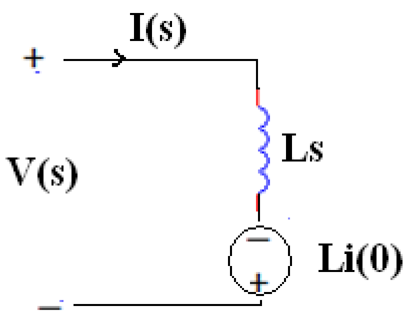
For an inductor, the voltage-current relationship can also be written as

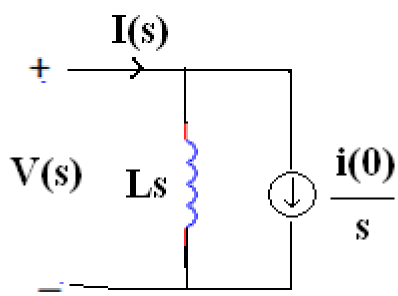
$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)dt$$

The Laplace transform of the above equation is

$$I(s) = \frac{1}{Ls}V(s) + \frac{i(0)}{s} \dots\dots\dots\dots.(2)$$

Equations (1) and (2) can be represented by the following circuits





Capacitor

For a capacitor the voltage-current relationship is

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

Laplace transform of above equation

$$V(s) = \frac{1}{Cs}I(s) + \frac{v(0)}{s} \dots\dots\dots(3)$$

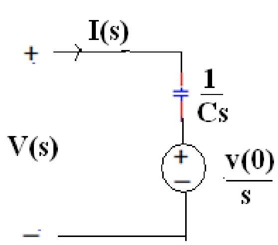
voltage-current relationship can also be written as

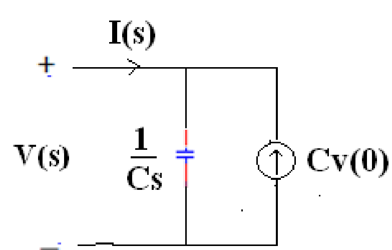
$$i(t) = C\frac{dv}{dt}$$

The Laplace transform of the above equation is

$$I(s) = C[sV(s) - v(0)] \dots\dots\dots..(4)$$

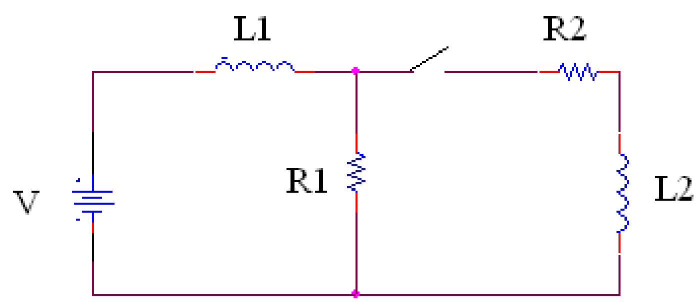
Equations (3) and (4) can be represented by the following circuits





Determine the current in the inductor L1and L2 for the circuit shown below.

The switch is closed at t=0 and the circuit has attained steady state before closing the switch. V1= 1 Volt, L1=2 H, L2=3 H, R1=R2=2Ω.



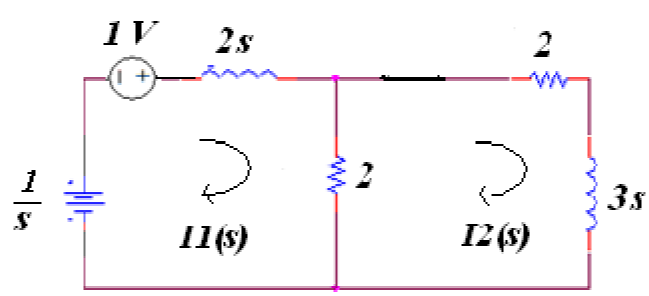
Solution

Before closing the switch the circuit has reached steady state. Hence the current through inductor L1 is

$$i_{L1}(0-) = i_{L1}(0+) = \frac{V}{R1} = \frac{1}{2} = 0.5\ A$$

$$i_{L2}(0+) = 0\ A$$

Hence the transformed network is shown below



Therefore the loop equations are

$$(2s+2)I_1(s) - 2I_2(s) = \frac{1}{s} + 1$$

$$-2I_1(s) + (3s+4)I_2(s) = 0$$

$$\begin{bmatrix} (2s+2) & -2 \\ -2 & (3s+4) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s}+1 \\ 0 \end{bmatrix}$$

By applying Cramer's rule

$$I_1(s) = \frac{\begin{vmatrix} \frac{1}{s}+1 & -2 \\ 0 & (3s+4) \end{vmatrix}}{(6s^2+14s+4)}$$

$$I_1(s) = \frac{(s+1)(3s+4)}{s(6s^2+14s+4)}$$

$$I_1(s) = \frac{(s+1)(3s+4)}{6s\left(s^2+\frac{14}{6}s+\frac{4}{6}\right)}$$

By applying partial fraction expansion

$$I_1(s) = \frac{(s+1)(3s+4)}{6s\left(s^2+\frac{14}{6}s+\frac{4}{6}\right)} = \frac{(s+1)(3s+4)}{6s\left(s+\frac{1}{3}\right)(s+2)}$$

$$I_1(s) = \frac{A}{s} + \frac{B}{\left(s+\frac{1}{3}\right)} + \frac{C}{(s+2)}$$

$$By\ solving\ A = 1 \qquad B = -\frac{3}{5} \qquad c = \frac{1}{10}$$

$$I_1(s) = \frac{1}{s} - \frac{3/5}{\left(s+\frac{1}{3}\right)} + \frac{1/10}{(s+2)}$$

$$i_1(t) = \left[1 - \frac{3}{5}e^{-\frac{t}{3}} + \frac{1}{10}e^{-2t}\right]u(t)$$

Similarly

$$I_2(s) = \frac{\begin{vmatrix} (2s+2) & \frac{1}{s}+1 \\ -2 & 0 \end{vmatrix}}{(6s^2+14s+4)}$$

$$I_2(s) = \frac{(s+1)}{s(3s^2+7s+2)}$$

$$I_2(s) = \frac{(s+1)}{3s(\ s+\frac{1}{3})(s+2)}$$

$$I_2(s) = \frac{A}{s} + \frac{B}{s+\frac{1}{3}} + \frac{C}{s+2}$$

By applying partial fraction expansion

$$A = \frac{1}{2} \qquad B = -\frac{2}{5} \qquad C = -\frac{1}{10}$$

Therefore

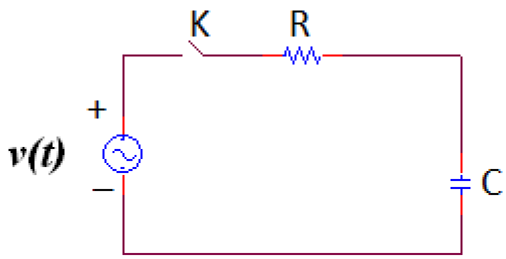
$$I_2(s) = \frac{1}{2}\left[\frac{1}{s}\right] - \frac{2}{5}\left[\frac{1}{s+\frac{1}{3}}\right] - \frac{1}{10}\left[\frac{1}{s+2}\right]$$

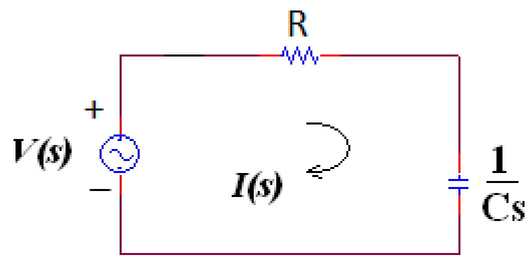
$$i_2(t) = \left[\frac{1}{2} - \frac{2}{5}e^{-\frac{t}{3}} - \frac{1}{10}e^{-2t}\right]u(t)\ Amp$$

Solution of networks with AC Excitation

For the network shown in figure find the voltage across the capacitor when the switch is closed at t=0.

Let R=2Ω, C=0.25 F and V(t)=0.5cost u(t)





The transformed network is shown below.

$$V(s) = \left(R + \frac{1}{Cs}\right) I(s)$$

Since $v(t) = 0.5\, cost$, $\quad V(s) = 0.5\, \frac{s}{s^2+1^2}$

$$V(s) = \frac{0.5s}{(s^2+1^2)} = \left(2 + \frac{4}{s}\right) I(s)$$

$$\frac{0.5s}{(s^2+1^2)} = \frac{(2s+4)}{s} I(s)$$

$$I(s) = \frac{0.5s^2}{(s^2+1^2)(2s+4)}$$

Voltage across the capacitor is given by

$$V_c(s) = \frac{1}{Cs} I(s)$$

$$V_c(s) = \frac{s}{(s^2+1)(s+2)}$$

$$V_c(s) = \frac{s}{(s^2+1)(s+2)} = \frac{As+B}{(s^2+1)} + \frac{C}{(s+2)}$$

$$\frac{s}{(s^2+1)(s+2)} = \frac{(As+B)(s+2)+C(s^2+1)}{(s^2+1)(s+2)}$$

Equating the numerators

$$s = (A+C)s^2 + (2A+B)s + (2B+C)$$

Equating the coefficients of s^2 $term, s\ term\ and\ constant\ term,$ we've

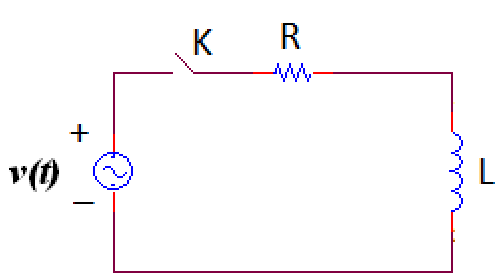
A=0.4 B=0.2 and C=-0.4

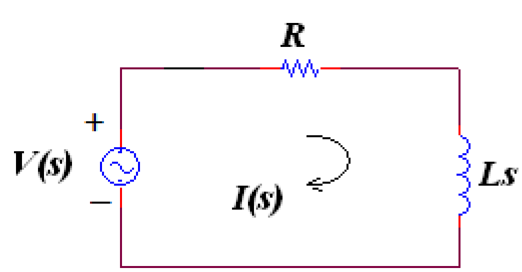
$$V_c(s) = \frac{0.4s}{(s^2+1)} + \frac{0.2}{(s^2+1)} - \frac{0.4}{(s+2)}$$

$$v_c(t) = [0.4\cos t\ + 0.2\sin t\ - 0.4e^{-2t}]\,u(t)\ \ Volts$$

2)Determine the current in the network when the switch is closed at t=0. Assume v(t)= 50 sin 25t, R=10 ohms, and L=5 H.

Solution





The transformed network is shown. Hence

$$V(s) = (R+Ls)I(s)$$

Since $v(t) = 50\sin 25\,t$, $V(s) = 50\ \frac{25}{s^2+25^2}$

$$50\left(\frac{25}{s^2+625}\right) = (10+5s)I(s)$$

$$I(s) = \frac{250}{(s^2+625)(s+2)}$$

By applying Partial fraction expansion

$$\frac{250}{(s^2+625)(s+2)} = \frac{As+B}{(s^2+625)} + \frac{C}{(s+2)}$$

$$\frac{250}{(s^2+625)(s+2)} = \frac{(As+B)(s+2)+C(s^2+625)}{(s^2+625)(s+2)}$$

$$250 = (A+C)s^2 + (2A+B)s + (2B+625)$$

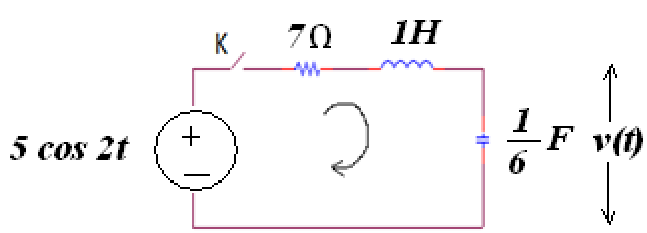
$$A+C=0 \qquad 2A+B=0 \qquad 2B+625=250$$

$$A=-0.397 \quad B=0.795 \quad C=0.397$$

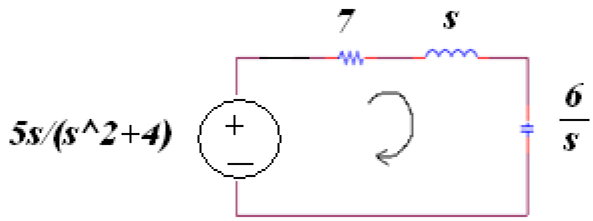
$$I(s) = \frac{-0.397s}{(s^2+625)} + \frac{0.795}{(s^2+625)} + \frac{0.397}{(s+2)}$$

$$i(t) = [-0.397\cos 25t + 0.032\sin 25\,t + 0.397e^{-2t}]u(t)\ Amp$$

3)For the network shown in figure find v(t) , if the switch is closed at t=0.



The transformed network is



$$\frac{5s}{s^2+4} = \left[7 + s + \frac{6}{s}\right] I(s)$$

$$I(s) = \frac{5s^2}{(s^2+4)(s^2+7s+6)}$$

$$V(s) = \frac{6}{s} I(s) = \frac{30s}{(s^2+4)(s^2+7s+6)}$$

$$V(s) = \frac{30s}{(s^2+4)(s+6)(s+1)}$$

By applying partial fraction expansion

$$\frac{30s}{(s^2+4)(s+6)(s+1)} = \frac{As+B}{(s^2+4)} + \frac{C}{(s+6)} + \frac{D}{(s+1)}$$

$$\frac{30s}{(s^2+4)(s+6)(s+1)} = \frac{(As+B)(s+6)(s+1) + C(s^2+4)(s+1) + D(s^2+4)(s+6)}{(s^2+4)(s+6)(s+1)}$$

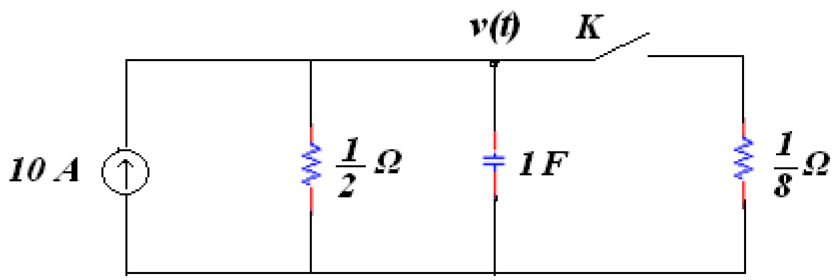
Evaluating the constants we have

$$A = \frac{3}{10} \quad B = \frac{42}{10} \quad C = \frac{9}{10} \quad and\ D = -\frac{6}{5}$$

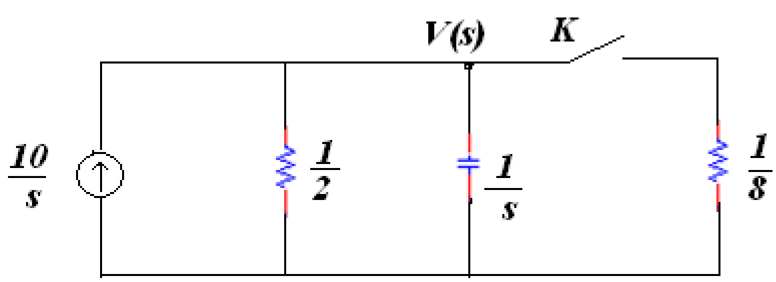
$$V(s) = \frac{3}{10}\left[\frac{s}{(s^2+4)}\right] + \frac{42}{10}\left[\frac{1}{(s^2+4)}\right] + \frac{9}{10}\left[\frac{1}{(s+6)}\right] - \frac{6}{5}\left[\frac{1}{(s+1)}\right]$$

$$v(t) = \left[\frac{3}{10} cos\ 2t + \frac{21}{10} \sin 2t + \frac{9}{10} e^{-6t} - \frac{6}{5} e^{-t}\right] u(t)$$

For the network shown the switch has been in open position for long time and it is closed at t=0. Find the voltage across the capacitor.



The transformed network at t=0- is



Let us find the solution of the circuit with switch K open.

$$\frac{10}{s} = 2V(s) + sV(s)$$

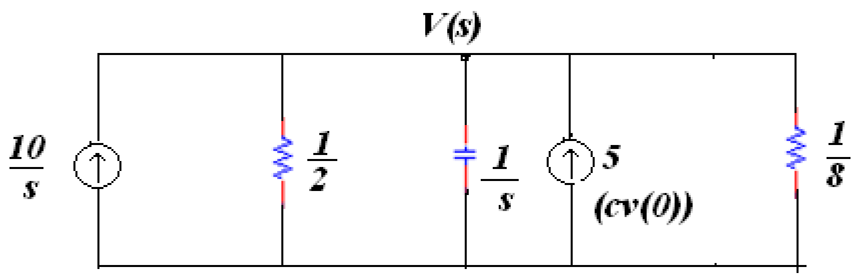
$$V(s) = \frac{10}{s(s+2)}$$

$Therefore\ V(t) at\ steady\ state\ is$

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} sV(s) = \left|\frac{10}{(s+2)}\right|_{s=0} = 5\ V$$

Therefore V(0+)=5 V

When the switch is closed at t=0 the transformed network is



By applying KCL we have

$$\frac{10}{s} + 5 = (2 + 8 + s)V(s)$$

$$\frac{(5s+10)}{s} = (s + 10)V(s)$$

$$V(s) = \frac{5s + 10}{s(s + 10)}$$

$By\ applying\ partial\ fraction\ expansion$

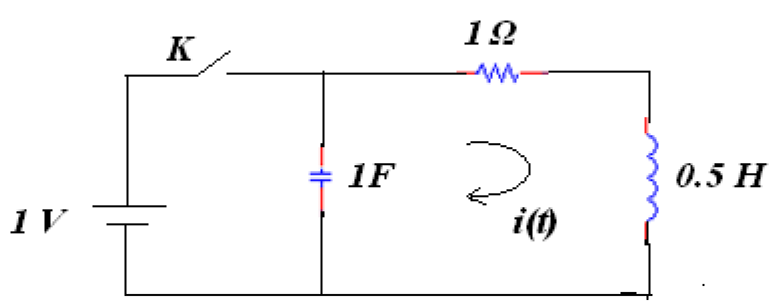
$$V(s) = \frac{5s+10}{s(s+10)} = \frac{A}{s} + \frac{B}{(s+10)}$$

We find $A = 1\ and\ B = 4$

$$V(s) = \frac{1}{s} + \frac{4}{(s + 10)}$$

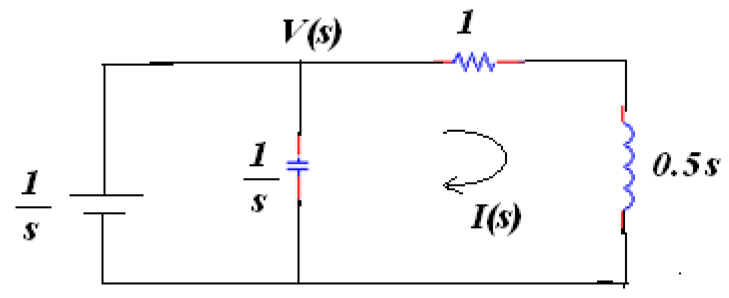
$$v(t) = [1 + 4e^{-10t}]u(t)$$

In the network shown the switch is opened at t=0. Steady state is reached before t=0. Find i(t)



Solution

At t=0- the transformed network is



$$V(s) = \frac{1}{s}$$

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} sV(s) = 1\ V$$

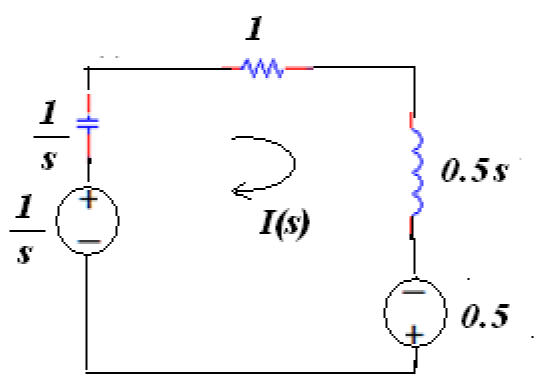
Applying KVL for outer loop

$$\frac{1}{s} = (1 + 0.5s)I(s)$$

$$I(s) = \frac{1}{s(1+0.5s)}$$

$$\lim_{t\to\infty} i(t) = \lim_{s\to 0} sI(s) = \lim_{s\to 0} \left|\frac{1}{(1+0.5s)}\right| = 1\ A$$

When the switch is opened the transformed network is



$$\frac{1}{s} + 0.5 = \left(\frac{1}{s} + 0.5s + 1\right) I(s)$$

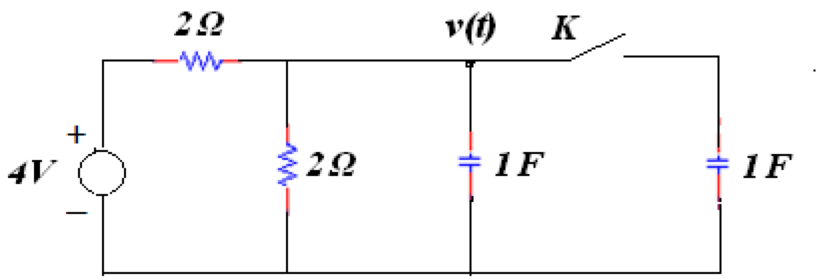
$$I(s) = \frac{(s+2)}{(s^2+2s+2)}$$

$$I(s) = \frac{(s+1)+1}{(s^2+2s+1)+1}$$

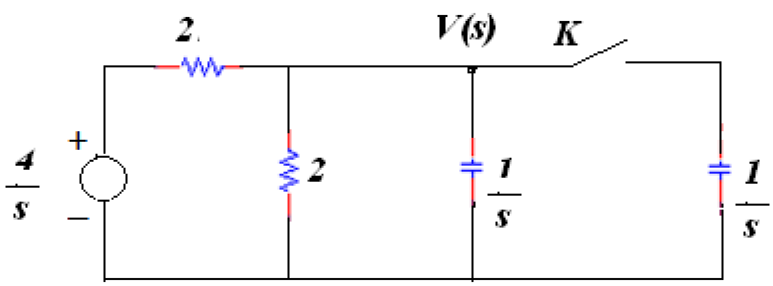
$$I(s) = \frac{(s+1)}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}$$

$$i(t) = e^{-t}\cos t + e^{-t}\sin t$$

The network shown in figure has attained steady state with switch K open. The switch is closed at t=0. Determine v(t)



The transformed network before closing the switch is



Writing the nodal equation for V(s)

$$\left[\frac{V(s)-\frac{4}{s}}{2}\right] + \frac{V(s)}{2} + sV(s) = 0$$

$$\frac{V(s)}{2} - \frac{2}{s} + \frac{V(s)}{2} + sV(s) = 0$$

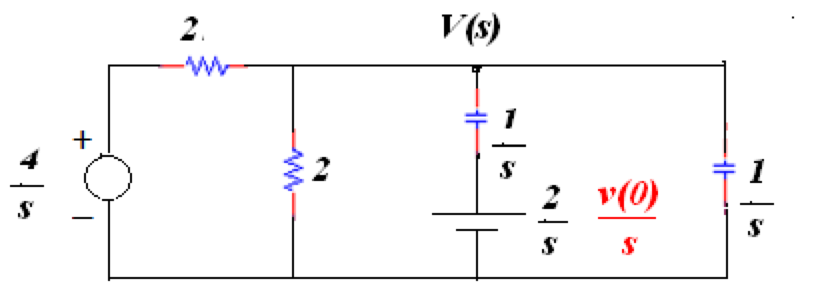
$$V(s)[1+s] = \frac{2}{s}$$

$$V(s) = \frac{2}{s(s+1)}$$

$$V \text{ at steady state is } \lim_{t\to\infty} v(t) = \lim_{s\to 0} sV(s) = 2\,V$$

$$\text{When the switch is closed } v(0+) = 2V$$

$$\text{The transformed}$$
$$\text{network at } t = 0 + \text{is}$$



Now writing the nodal equation for V(s)

$$\left[\frac{V(s) - \frac{4}{s}}{2}\right] + \frac{V(s)}{2} + s\left[V(s) - \frac{2}{s}\right] + sV(s) = 0$$

$$V(s) = \frac{2(s+1)}{2s(s+0.5)}$$

$$V(s) = \frac{A}{s} + \frac{B}{(s+0.5)}$$
$$A = 2 \quad and\ B = 1$$

$$V(s) = \frac{2}{s} + \frac{1}{(s+0.5)}$$
$$v(t) = [2 - e^{-0.5t}]u(t)\ volts$$

TWO POTR NETWORK

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Two Port Network

Overview

- The concept of a two-port network.
- The relationship between input and output current and voltages.
- Combinations of networks in series, parallel, and cascaded.

Two Port Network

- A pair of terminals through which a current may enter or leave a network is known as a port.
- Two terminal devices or elements (such as resistors, capacitors, and inductors) results in one – port network.
- Most of the circuits we have dealt with so far are two – terminal or one – port circuits. (Fig. 1(a))
- A two – port network is an electrical network with two separate ports for input and output.
- It has two terminal pairs acting as access points. The current entering one terminal of a pair leaves the other terminal in the pair. (Fig. 1(b))

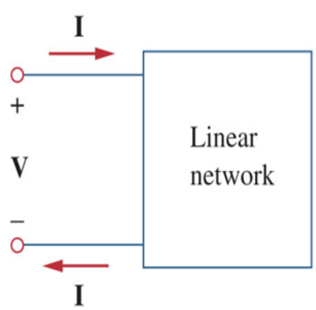


Fig. 1(a)

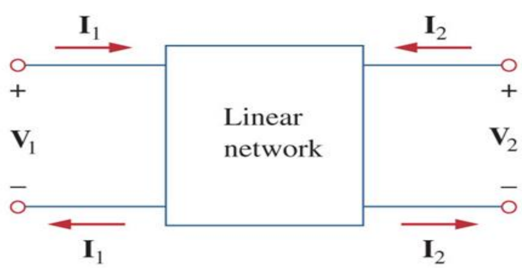


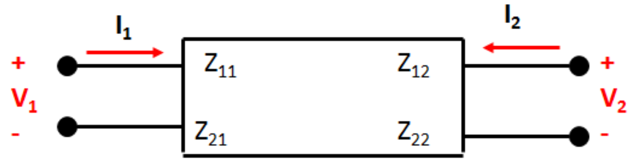
Fig. 1(b)

- To characterize a two-port network requires that we relate the terminal quantities V_1, V_2, I_1, and I_2.
- Out of these four, only two are independent.
- The terms that relate to these voltages and currents are called parameters.
- Impedance and admittance parameters are commonly used in the synthesis of filters.
- They are also important in the design and analysis of impedance-matching networks and power distribution networks.

Three types of two-port parameters are examined here: impedance, admittance & transmission.

Z – PARAMETER

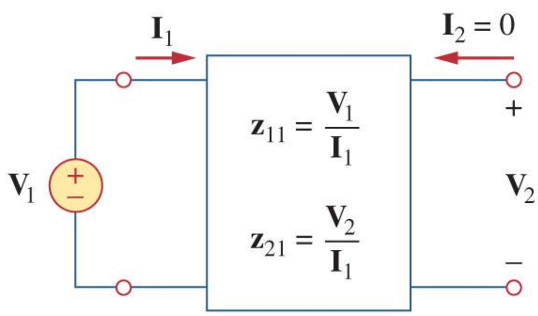
- Z – parameter is also called impedance parameter and the unit of Z – parameters is ohm (Ω)
- The “black box” replaced with Z-parameter is as shown below.



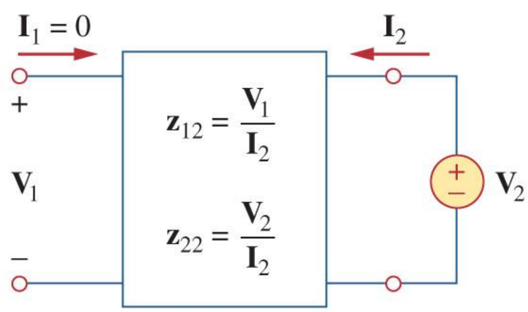
- A two-port network may be either voltage driven or current driven
- The terminal voltages can be related to the terminal currents as:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

The values of the parameters can be evaluated by setting the input or output port open circuits (i.e. set the current to zero).



(a)



(b)

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

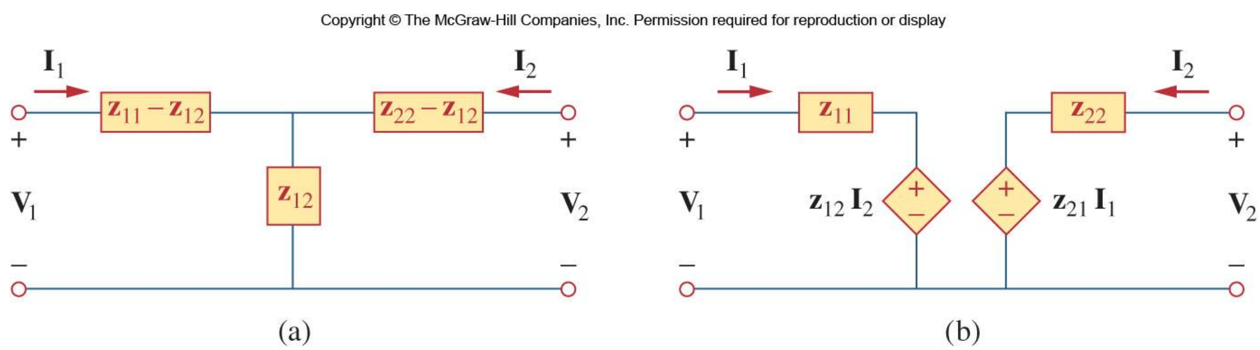
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

These are referred to as the open-circuit impedance parameters.

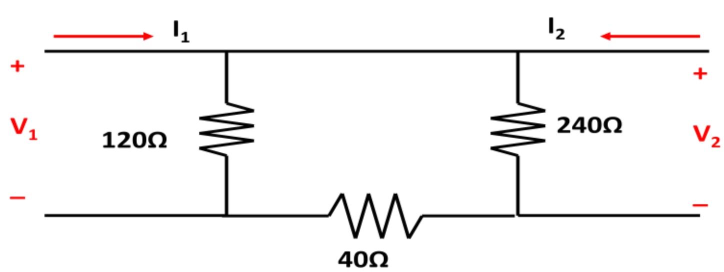
These parameters are as follows:

- z_{11} Open circuit input impedance
- z_{12} Open circuit transfer impedance from port 1 to port 2
- z_{21} Open circuit transfer impedance from port 2 to port 1
- z_{22} Open circuit output impedance
- When $z_{11}=z_{22}$, the network is said to be symmetrical.

It should be noted that an ideal transformer has no Z - parameters. The equivalent circuit for two port networks is shown below:

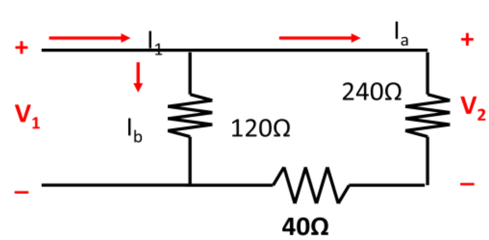


1. Find the Z – parameter of the circuit below.



Solution:

When I_2 = 0(open circuit port 2). Redraw the circuit.



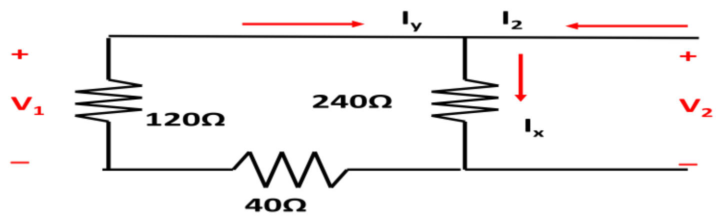
$$V_1 = 120 I_b ....(1) \qquad V_2 = 240 I_a ....(3)$$

$$I_b = \frac{280}{400} I_1 ......(2) \qquad I_a = \frac{120}{400} I_1 ....(4)$$

sub (1) $\rightarrow$ (2) sub (4) $\rightarrow$ (3)

$$\therefore Z_{11} = \frac{V_1}{I_1} = 84\Omega \qquad \therefore Z_{21} = \frac{V_2}{I_1} = 72\Omega$$

When $I_1 = 0$ (open circuit port 1). Redraw the circuit.



$$V_2 = 240 I_x ....(1) \qquad V_1 = 120 I_y ....(3)$$

$$I_x = \frac{160}{400} I_2 ....(2) \qquad I_y = \frac{240}{400} I_2 ....(4)$$

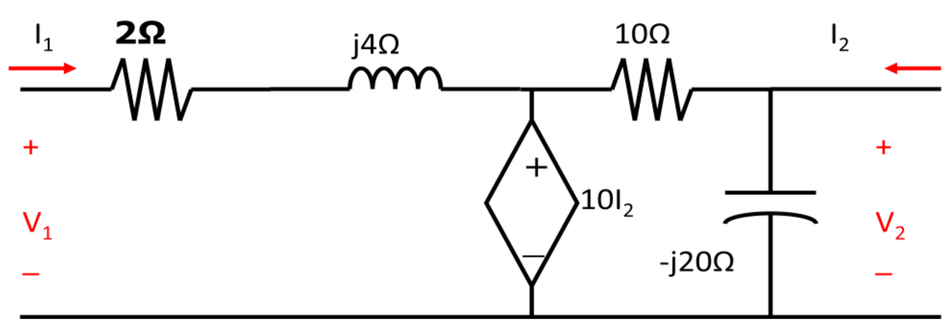
sub (1) $\rightarrow$ (2) sub (4) $\rightarrow$ (3)

$$\therefore \mathrm{Z}_{22} = \frac{V_2}{I_2} = 96\Omega \qquad \therefore Z_{12} = \frac{V_1}{I_2} = 72\Omega$$

In matrix form:

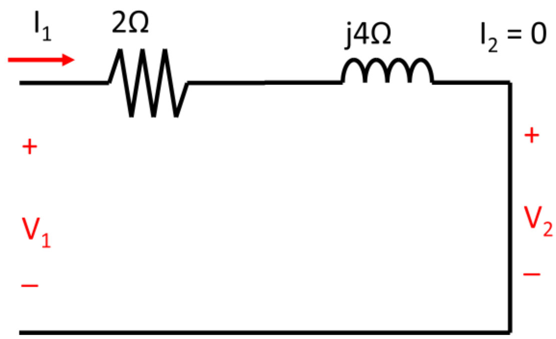
$$[Z] = \begin{bmatrix} 84 & 72 \\ 72 & 96 \end{bmatrix} \Omega$$

2. Find the Z – parameter of the circuit below



Solution:

i) $I_2 = 0$ (open circuit port 2). Redraw the circuit.



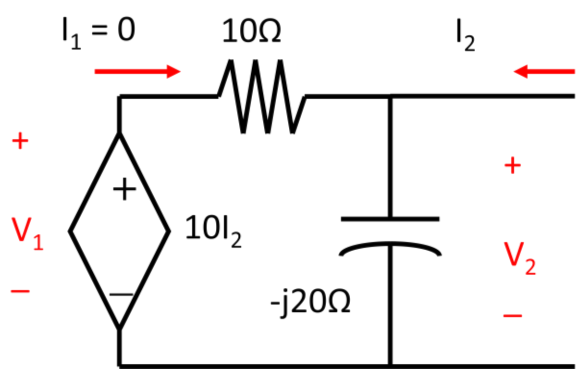
$$V_1 = I_1\,(2 + j4)$$

$$\therefore Z_{11} = \frac{V_1}{I_1} = (2 + j4)\Omega$$

$$V_2 = 0 \text{ (short circuit)}$$

$$\therefore Z_{21} = 0\Omega$$

ii) $I_1 = 0$ (open circuit port 1). Redraw the circuit.



$$V_1 = 10I_2$$

$$\therefore Z_{12} = \frac{V_1}{I_2} = 10\Omega$$

$$I_2 = \frac{V_2}{-j20} + \frac{V_2 - 10I_2}{10}$$

$$2I_2 = V_2\left(\frac{j}{20} + \frac{1}{10}\right)$$

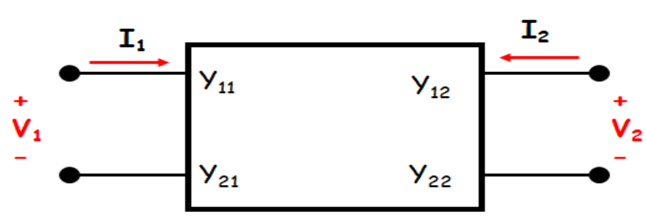
$$\therefore Z_{22} = \frac{V_2}{I_2} = (16\text{-}j8)\ \Omega$$

In matrix form;

$$[Z] = \begin{bmatrix} (2+j4) & 0 \\ 10 & (16 - j8) \end{bmatrix}\Omega$$

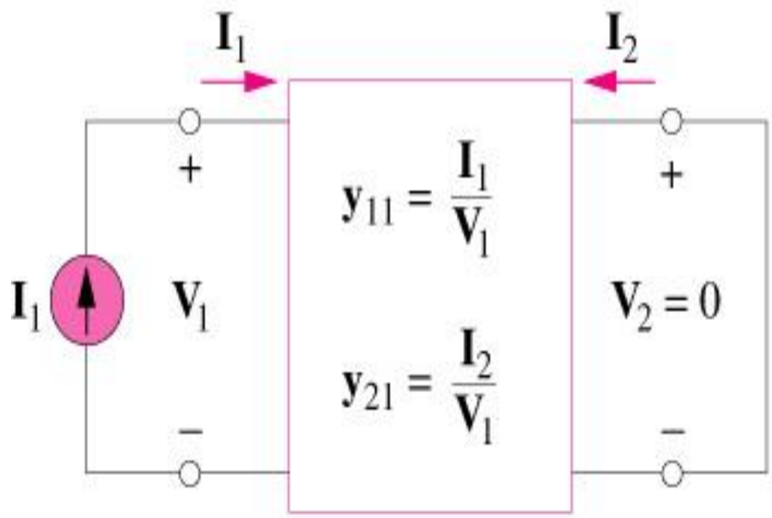
Y – PARAMETER

Y – Parameter also called admittance parameter and the unit is Siemens (S). The “black box” that we want to replace with the Y-parameter is shown below.

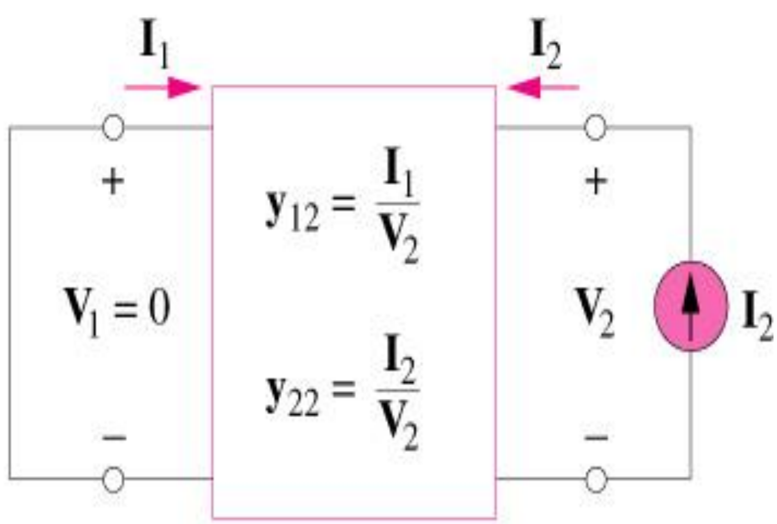


$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



(a)



(b)

$$y_{11} = \frac{I_1}{V_1}\Bigg|_{V_2=0} \quad \text{and} \quad y_{21} = \frac{I_2}{V_1}\Bigg|_{V_2=0}$$

y_{11} = Short-circuit input admittance

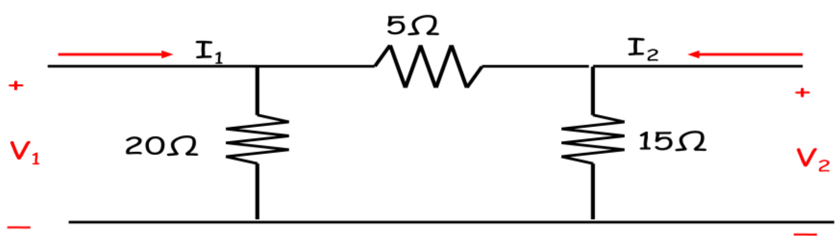
y_{21} = Short-circuit transfer admittance from port 1 to port 2

$$y_{12} = \frac{I_1}{V_2}\Bigg|_{V_1=0} \quad \text{and} \quad y_{22} = \frac{I_2}{V_2}\Bigg|_{V_1=0}$$

y_{12} = Short-circuit transfer admittance from port 2 to port 1

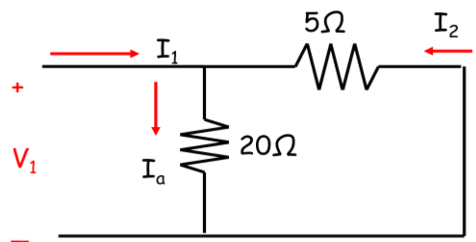
y_{22} = Short-circuit output admittance

1. Find the Y – parameter of the circuit shown below.



Solution:

i) $V_2 = 0$



$$V_1 = 20I_a ....(1)$$

$$I_a = \frac{5}{25} I_1 ....(2)$$

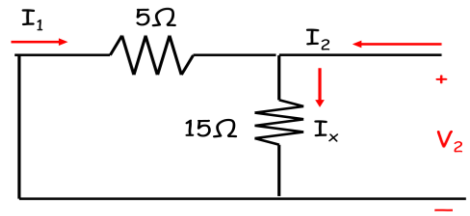
sub (1) $\rightarrow$ (2)

$$\therefore Y_{11} = \frac{I_1}{V_1} = \frac{1}{4} S$$

$$V_1 = -5I_2$$

$$\therefore Y_{21} = \frac{I_2}{V_1} = -\frac{1}{5} S$$

ii) $V_1 = 0$



$$V_2 = 15 I_x ....(3)$$

$$I_x = \frac{5}{25} I_2 ....(4)$$

sub (3) → (4)

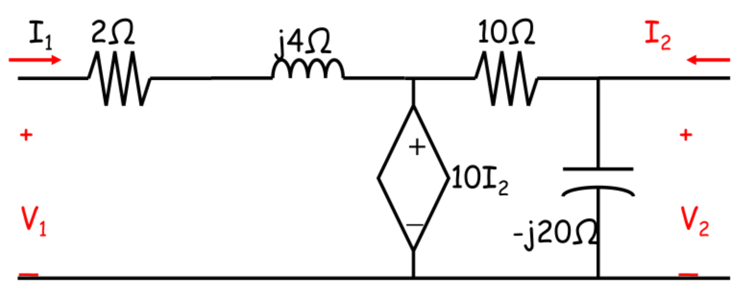
$$\therefore Y_{22} = \frac{I_2}{V_2} = \frac{4}{15} S$$

$$V_2 = -5 I_1$$

$$\therefore Y_{12} = \frac{I_1}{V_2} = -\frac{1}{5} S$$

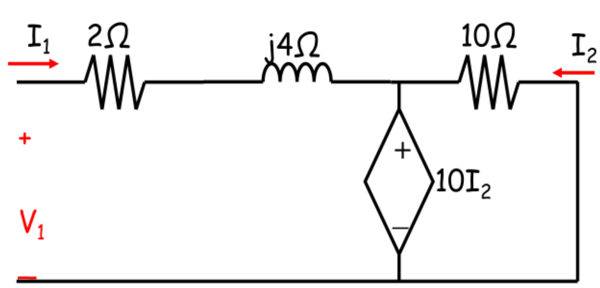
In matrix form, $[Y] = \begin{bmatrix} \frac{1}{4} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{4}{15} \end{bmatrix} S$

2. Find the Y – parameters of the circuit shown.



Solution:

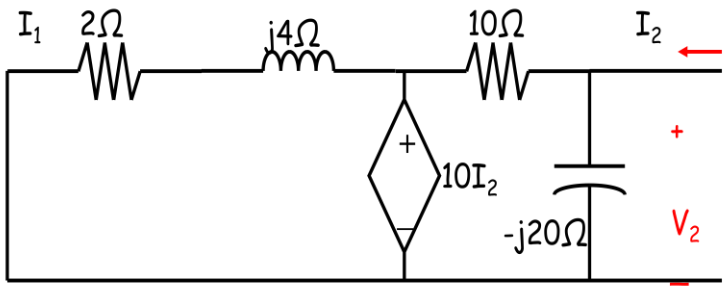
i) $V_2 = 0$ (short – circuit port 2). Redraw the circuit.



Applying KVL to the loop consisting of dependent source $10I_2$ and 10 Ω resistor we get,

$$10I_2 + 10I_2 = 0 \text{ or}$$
$$I_2 = 0$$
$$V_1 = (2 + j4)I_1$$
$$\therefore Y_{11} = \frac{I_1}{V_1} = \frac{1}{2 + j4} = (0.1 - j0.2)\ \text{S}$$
$$\therefore Y_{21} = \frac{I_2}{V_1} = 0\ \text{S}$$

ii) $V_1 = 0$ (short – circuit port 1). Redraw the circuit.



$$I_1 = \frac{-10I_2}{2 + j4} ....(1)$$
$$I_2 = \frac{V_2}{-j20} + \frac{V_2 - 10I_2}{10}$$
$$2I_2 = V_2\left(\frac{1}{10} + \frac{1}{-j20}\right) ....(2)$$
$$\therefore Y_{22} = \frac{I_2}{V_2} = (0.05 + j0.025)\ \text{S}$$

sub (2) $\to$ (1)

$$Y_{12} = \frac{I_1}{V_2} = (-0.1 + j0.075)\ \text{S}$$

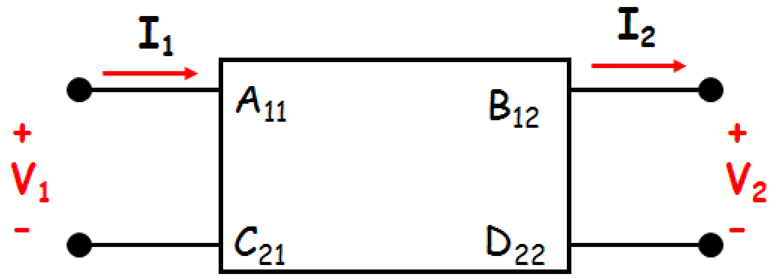
In matrix form;

$$\therefore [Y] = \begin{bmatrix} 0.1 - j0.2 & -0.1 + j0.075 \\ 0 & 0.05 + j0.025 \end{bmatrix} \text{S}$$

T (ABCD) PARAMETER

- T – parameter or also ABCD – parameter is a another set of parameters relates the variables at the input port to those at the output port.
- T – parameter also called *transmission parameters* because this parameter are useful in the analysis of transmission lines because they express sending – end variables (V_1 and I_1) in terms of the receiving – end variables (V_2 and $-I_2$).

- The “black box” replaced with T – parameter is as shown below.



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

T terms are called the transmission parameters or simply T or ABCD parameters, and each parameter has different units.

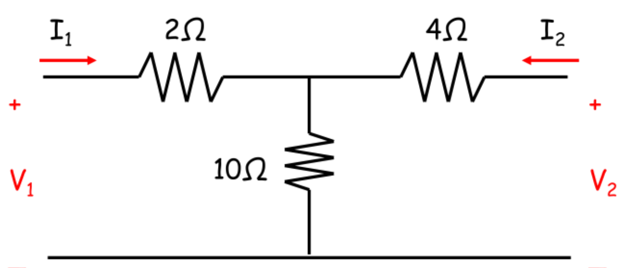
$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ A=open-circuit voltage ratio

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ C= open-circuit transfer admittance (S)

$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$ B= negative short-circuit transfer impedance (Ω)

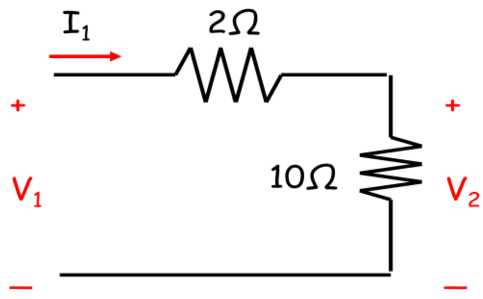
$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$ D=negative short-circuit current ratio

Find the ABCD – parameter of the circuit shown below.



Solution:

i) $I_2 = 0$,



$$V_2 = 10 I_1$$

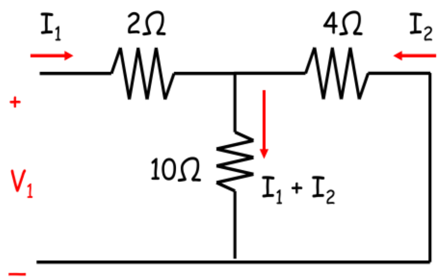
$$\therefore C = \frac{I_1}{V_2} = 0.1S$$

$$V_1 = 2I_1 + V_2$$

$$V_1 = 2\left(\frac{V_2}{10}\right) + V_2 = \frac{6}{5}V_2$$

$$\therefore A = \frac{V_1}{V_2} = 1.2$$

ii) $V_2 = 0$,



$$I_2 = -\frac{10}{14} I_1$$

$$\therefore D = -\frac{I_1}{I_2} = 1.4$$

$$V_1 = 2I_1 + 10(I_1 + I_2)$$

$$V_1 = 12 I_1 + 10 I_2$$

$$V_1 = 12\left(-\frac{14}{10} I_2\right) + 10 I_2$$

$$\therefore B = -\frac{V_1}{I_2} = 6.8\Omega$$

In matrix form; $[T] = \begin{bmatrix} 1.2 & 6.8\Omega \\ 0.1S & 1.4 \end{bmatrix}$

Conversion from Y to Z Parameters:

For the Y parameters we have, I = Y V …..(a)

For the Z parameters we have, V = Z I …..(b)

From (a), $V = Y^{-1} I$ …..(c)

Comparing (b) & (c) we have, $Z = Y^{-1}$

Therefore, $$Z = Y^{-1} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{22}}{\Delta Y} \end{bmatrix}$$

Where $\Delta Y = \det|Y|$

Conversion Table

$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Network Functions

Contents:

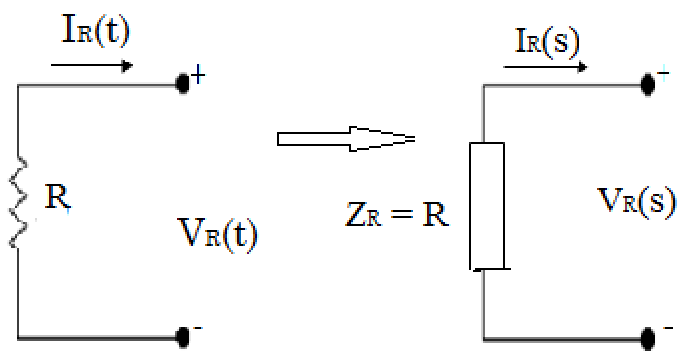
- Network functions of one port and two port networks
- Properties of poles and zeros of network functions.

V-I and I-V relations of basic elements:

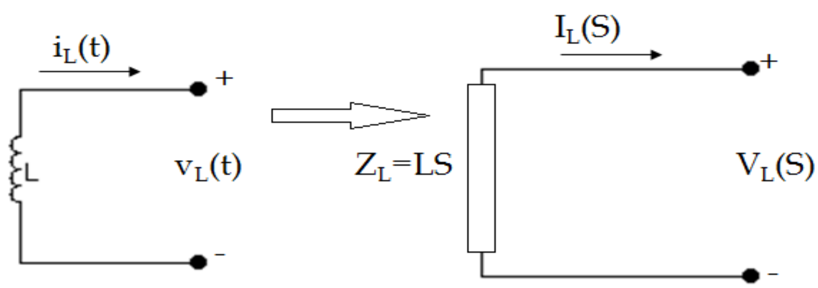
Component	Symbol	V-I relation	I-V relation
Resistor		$v_R(t) = i_R(t)R$	$i_R(t) = \frac{v_R(t)}{R}$
Capacitor		$v_c(t) = \frac{1}{C}\int i_c(t)dt$	$i_c(t) = C\frac{dv_c(t)}{dt}$
		$v_L(t) = L\frac{di_L(t)}{dt}$	$i_L(t) = \frac{1}{L}\int v_L(t)dt$

Inductor

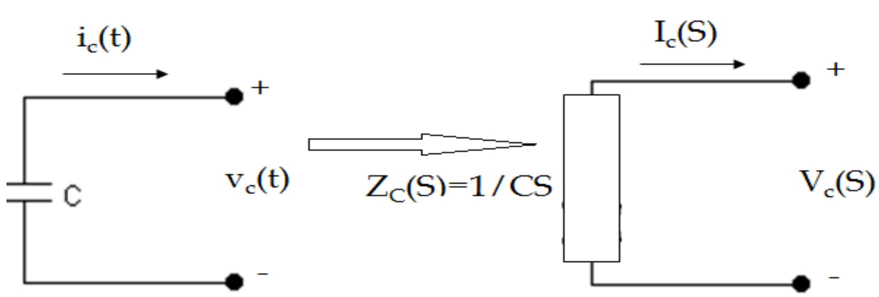
Transform Impedance (Resistor)



Transform Impedance (Inductor)



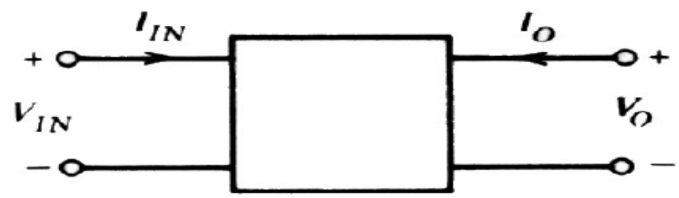
Transform Impedance (Capacitor)



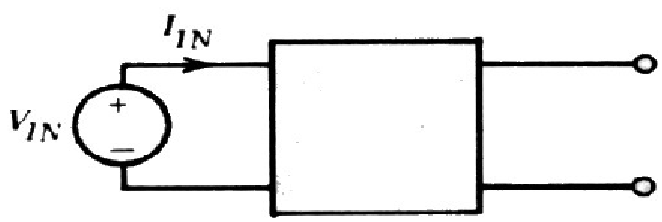
A network function is the Laplace transform of an impulse response. Its format is a ratio of two polynomials of the complex frequencies.

Consider the general two-port network shown in Figure (a). The terminal voltages and currents of the two-port can be related by two classes of network functions, namely, the driving point functions and the transfer functions.

Network Functions: Driving point impedance & admittance functions



(a) A two port network.



(b) Measuring input impedance.

I. The driving point functions relate the voltage at a port to the current at the same port.
II. These functions are a property of a single port.
III. For the input port the driving point impedance function $Z_{IN}(s)$ is defined as:

$$Z_{IN}(s) = V_{IN}(s)/I_{IN}(s)$$

Transfer function

This function can be measured by observing the current I_{IN} when the input port is driven by a voltage source V_{IN} (Figure (b)). The driving point admittance function $Y_{IN}(s)$ is the reciprocal of the impedance function, and is given by: $Y_{IN}(s) = I_{IN}(s)/V_{IN}(s)$

The output port driving point functions are defined in a similar way. The transfer functions of the two-port relate the voltage (or current) at one port to the voltage (or current) at the other port.

The possible forms of transfer functions are:

a) The voltage transfer function, which is a ratio of one voltage to another voltage. $\mathbf{G_{21}(s) = V_2(s)/V_1(s)}$

b) The current transfer function, which is a ratio of one current to another current. $\boldsymbol{\alpha_{21}(s) = I_2(s)/I_1(s)}$

c) The transfer impedance function, which is the ratio of a voltage to a current. $\mathbf{Z_{21}(s) = V_2(s)/I_1(s)}$

d) The transfer admittance function, which is the ratio of a current to a voltage. $\mathbf{Y_{21}(s) = I_2(s)/V_1(s)}$

The general form of a network function is:

$$H(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots + a_0}{b_m s^m + b_{m-1} s^{m-1} + b_{m-2} s^{m-2} + \cdots + b_0}$$

$$\text{where} \quad a_n \neq 0 \qquad b_m \neq 0$$

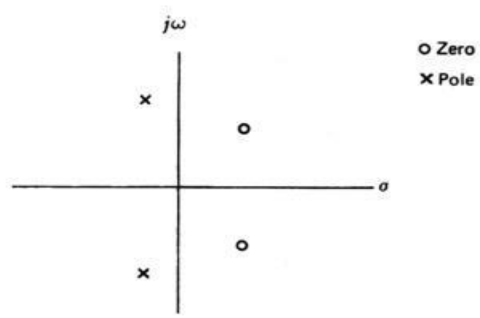
All the coefficients a_i and b_i are real

An alternate form of H(s)

$$H(s) = \frac{a_n(s - z_1)(s - z_2) \cdots (s - z_n)}{b_m(s - p_1)(s - p_2) \cdots (s - p_m)}$$

In the above expression z_1, z_2, ..., z_n are called the zeros of H(s), because H(s) = 0 when s = z_i. The roots of the denominator p_1, p_2, ..., p_m are called the poles of H(s). It can be seen that H(s) = ∞ at the poles, s = p_i.

The poles and zeros can be plotted on the complex s plane (s = σ + jω), which has the real part σ for the abscissa, and the imaginary part jω for the ordinate.



Poles and zeros plotted in the complex *s* plane.

Properties of all Network Functions:

1. Network functions are ratios of polynomials in s with real coefficients. A consequence of this property is that complex poles (and zeros) must occur in conjugate pairs.

Consider a complex root at (s = -a – jb) which leads to the factor (s + a + jb) in the network function. The jb term will make some of the coefficients complex in the polynomial, unless the conjugate of the complex root at (s = -a + jb) is also present in the polynomial. The product of a complex factor and its conjugate is

$$(s + a + jb)(s + a - jb) = s^2 + 2as + a^2 + b^2$$

which can be seen to have real coefficients.

2. The networks must be stable:

A bounded input excitation to the network must yield a bounded response. Or The output of a stable network cannot be made to increase indefinitely by the application of a bounded input excitation.

Stability of the general network function H(s)

1. If the network function has a simple pole on the real axis, the impulse response due to it (for t >= 0) will have the form:

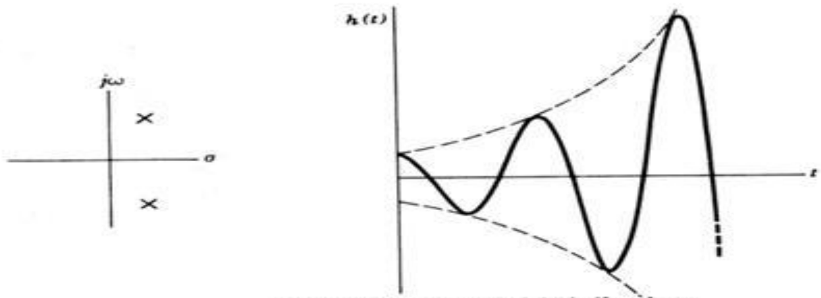
$$h(t) = \mathscr{L}^{-1} \frac{K_1}{s - p_1} = K_1 e^{p_1 t}$$

For p1 positive, the impulse response is seen to increase exponentially with time, representing an unstable circuit. Thus, ***H(s) cannot have poles on the positive real axis.***

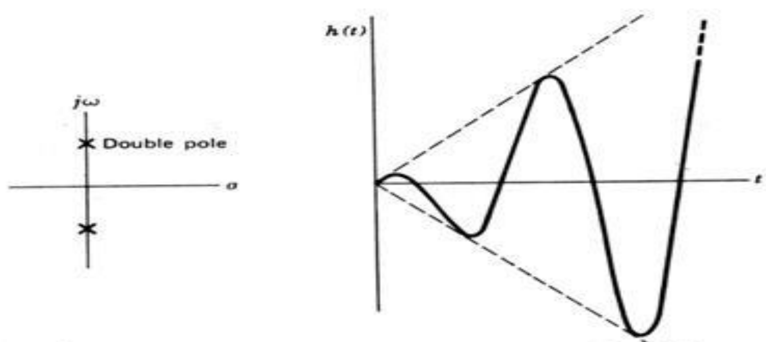
Suppose H(s) has a pair of complex conjugate poles at s = a +/- jb. The contribution to the impulse response due to this pair of poles is

$$\begin{aligned} h(t) &= \mathscr{L}^{-1}\left(\frac{K_1}{s - a - jb} + \frac{K_1}{s - a + jb}\right) = \mathscr{L}^{-1} \frac{2K_1(s - a)}{(s - a)^2 + b^2} \\ &= 2K_1 e^{at} \cos bt \end{aligned}$$

If ‘a’ is positive, corresponding to poles in the right half s plane, the response is seen to be an exponentially increasing sinusoid Therefore, ***H(s) cannot have poles in the right half s plane.*** An additional restriction on the poles of H(s) is that ***any poles on the imaginary axis must be simple.*** Higher order poles on the jω axis will also cause the network to be unstable.



Complex poles in the right half s plane.



Double poles on imaginary axis.

Summary:

- The network functions of all passive networks and all stable active network must be rational functions in ‘s’ with real coefficients.
- May not have poles in the right half s plane.
- May not have multiple poles on the jω axis.

Check to see whether the following are stable network functions:

$$\text{(a)}\ \frac{s}{s^2 - 3s + 4} \qquad \text{(b)}\ \frac{s - 1}{s^2 + 4}$$

The first function cannot be realized by a stable network because one of the coefficients in the denominator polynomial is negative. It can easily be verified that the poles are in the right half s plane.

The second function is stable. The poles are on the jω axis (at s = +/- 2j) and are simple. Note that the function has a zero in the right half s plane; however, this does not violate any of the requirements on network functions.

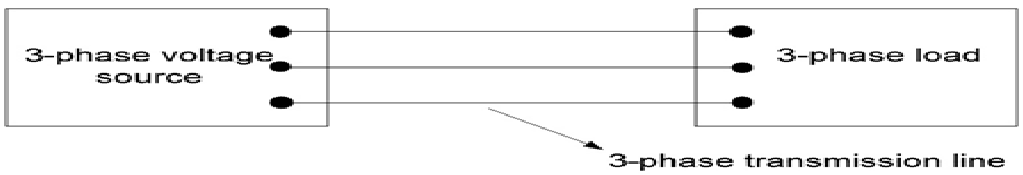
Subject: Electric Circuit Analysis

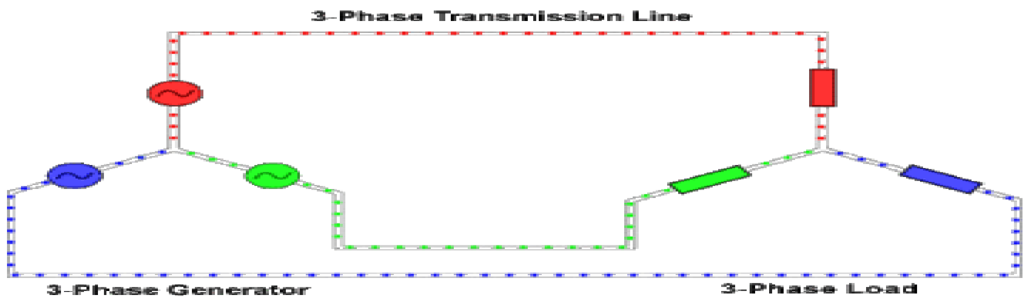
Unbalanced Three Phase Systems

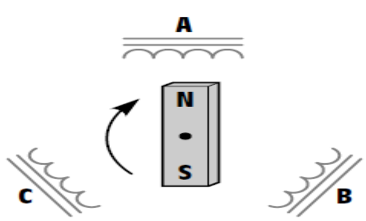
Review of A.C three phase system

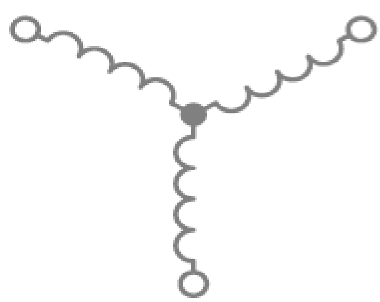
Three phase voltages are generated by the alternator(AC generator), when the rotating magnetic field sweeps across the stator conductors, hence emf's are induced in all the three phases, which are separated by 120 degrees.

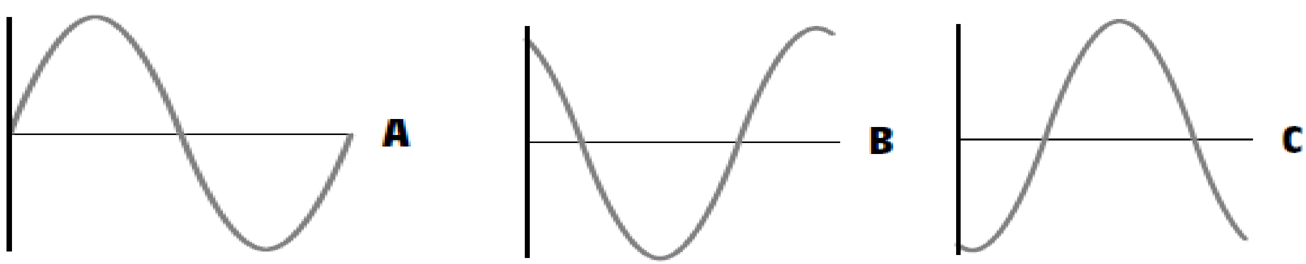
$E_A = E_m Sin\omega t$ $E_B = E_m Sin(\omega t-120^0)$ $E_C = E_m Sin(\omega t- 240^0)$ or $E_c = E_m Sin(\omega t+120^0)$

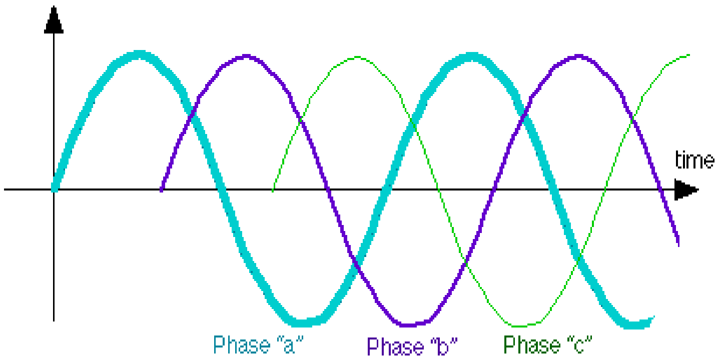


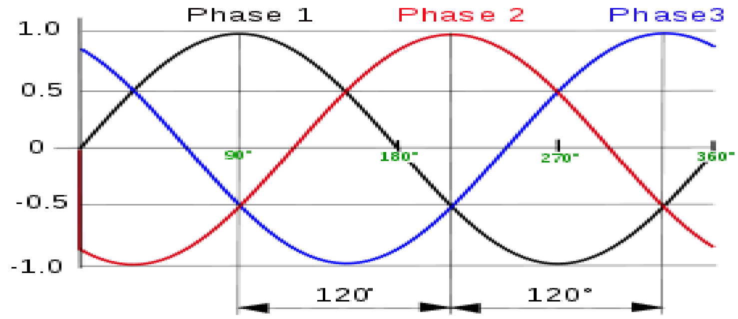






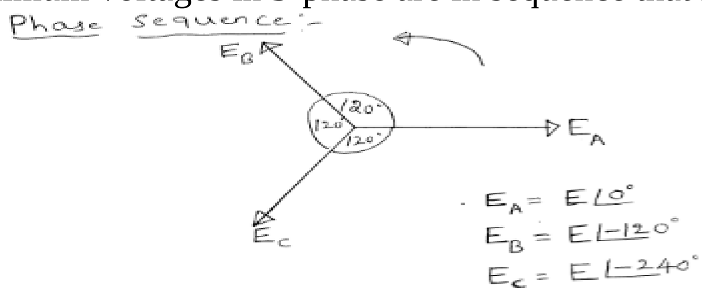






Phase Sequence

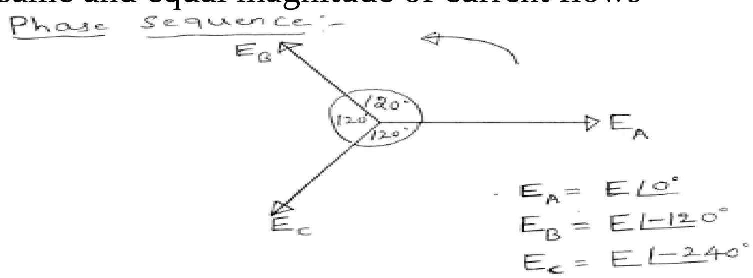
It is the order in which the maximum voltages in 3-phase are in sequence that is A, B, C.



Balanced 3-Phase Supply

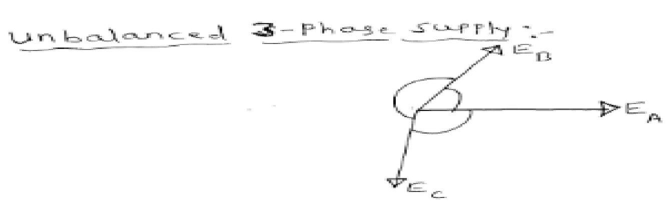
When all the three voltages in 3-phase supply having same magnitude but differs in phase by 120 degrees with respect to one another is called balanced 3-phase supply. OR

Also in all the three lines the same and equal magnitude of current flows



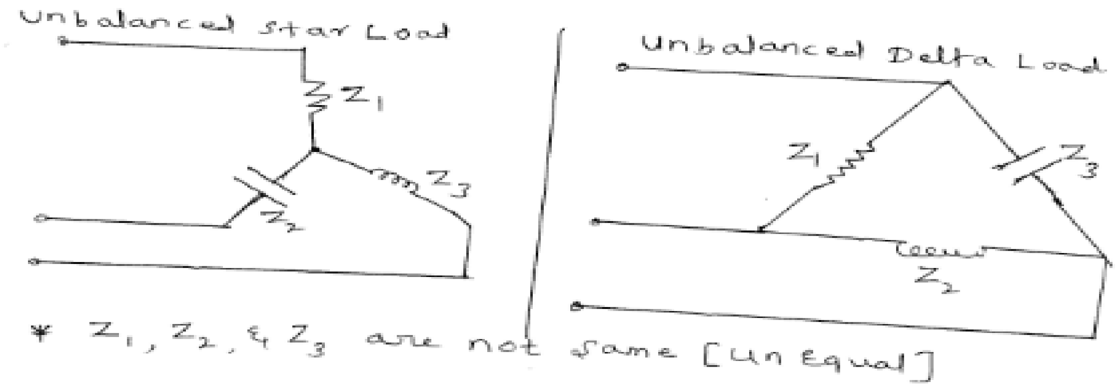
Un balanced 3-Phase Supply

The magnitude and phase angle in three phase supply are not similar that system is called un balanced 3-phase supply OR In all the three lines the magnitude of currents are different (un equal).



Unbalanced 3-Phase Load

Suppose among the three phase impedances, if any One of the phase impedance is different then it is called as unbalanced load.



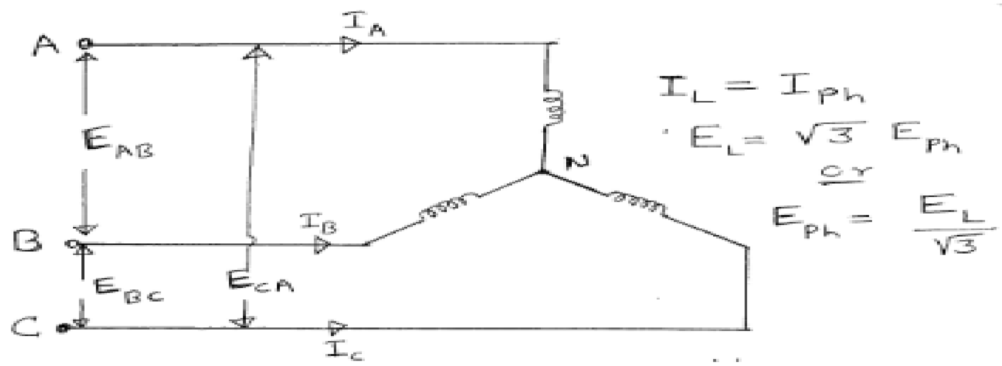
Star connected 3-phase system

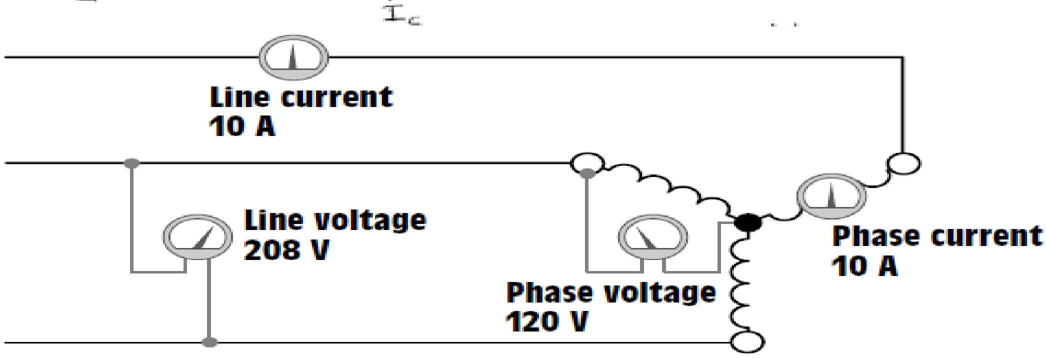
In star connected three phase system, all the three ends of coils are joined at one point N (Neutral) other 3 ends being free.

Consider E_{AB}, E_{BC}, E_{CA} are the line voltages called as E_L.

E_{AN} , E_{BN}, E_{CN} are the phase voltages called as E_{ph}.

In star connection Line current is equal to phase current $I_L = I_{Ph}$





$$E_{Line} = E_{Phase} \times 1.732$$

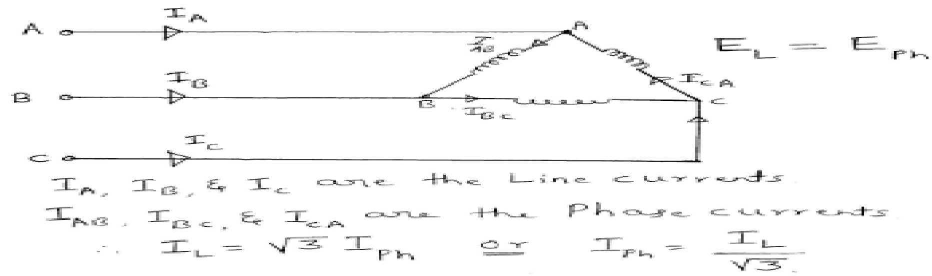
$$E_{Phase} = \frac{E_{Line}}{1.732}$$

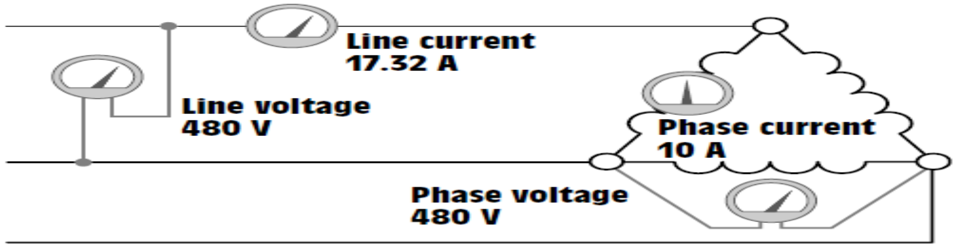
Delta connected 3-phase system

All the three coils are connected end to end to form delta connected three phase system

Consider I_A, I_B, I_C, are the line Currents called as I_L.

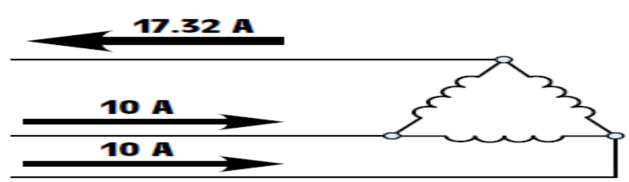
I_{AB} I_{BC} I_{CA} are the phase currents called as $I_{ph.}$
In Delta connection Line Voltage is equal to phase Voltage $E_L = E_{Ph}$





Voltage and current relationships in a delta connection.

In delta system line current is 17.32A and phase current is10A. the reason for this difference in current is that current flows through different windings at different times in a 3-phase circuit. During some periods of time current will flow between two lines only. At other times current will flow from two lines to the third. Delta connection is similar to parallel connection because there is always more than one path for current flow.



Division of currents in a delta connection.

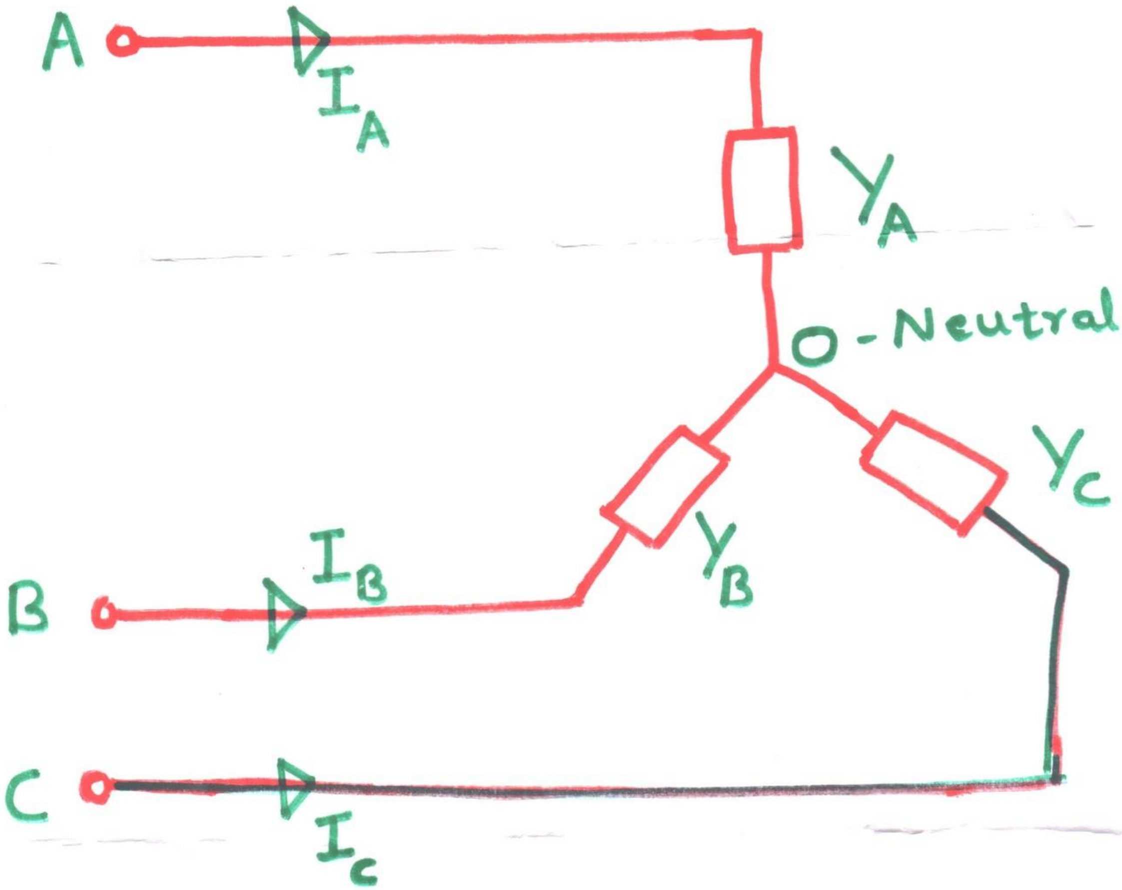
UnBalanced 3-Phase System :

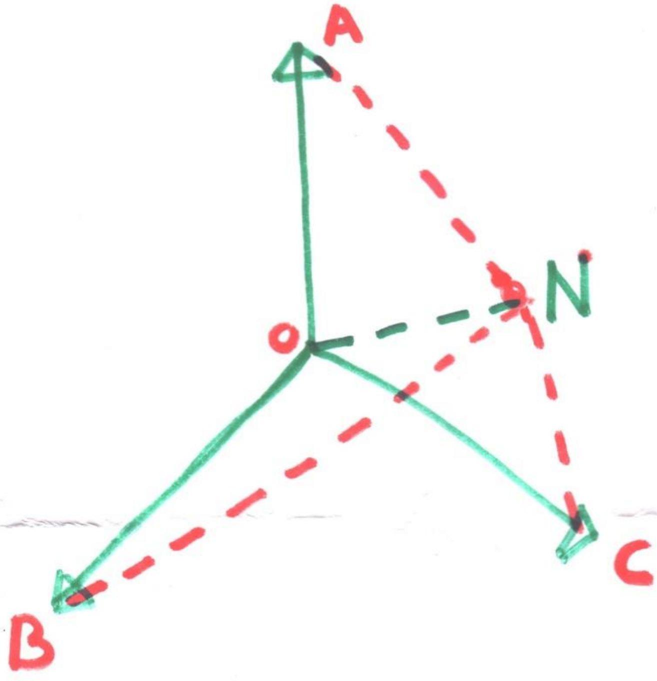
The loads in all three phases of either Star OR Delta connection are not identical to each other in all respects are called unbalanced load. In Unbalanced load the Line current in star and Phase current in Delta will be different.
“ The line or phase current giving rise to the flow of neutral current” in this context supply neutral and the star point is inter connected then it is called 3-phase 4-wire system. In unbalanced loading, due to the flow of unequal currents in each of the phase and voltage appears at the neutral.

Advantages of 3-phase system over 1-phase:

1.3-phase system is more efficient than 1-phase system
2.Cost of 3-phase equipment is less than 1-phase comparatively
3.3-phase system is 1.5 times more than 1-phase.
4.Harmonics in 3-phase system is minimum
5.Economy of 3-phase power transmission is cheaper than 1-phase
6. 3-phase system produces uniform torque but 1-phase gives pulsating torque
7.All 3-phase motors are self starting but 1-phase motors are not self starting
8.3-phase apparatus are compact in size, requires less material as compared to 1-phase.

To Find Voltage at Neutral point





$$I_A = V_{OA} \cdot Y_A \rightarrow (1)$$

$$I_B = V_{OB} \cdot Y_B \rightarrow (2)$$

$$I_C = V_{OC}\, Y_C \rightarrow (3)$$

Substitute Equations ① ② & ③ in ④

$$V_{OA} Y_A + V_{OB} Y_B + V_{OC} Y_C$$

$$[V_{AN} - V_{ON}] Y_A + [V_{BN} - V_{ON}] Y_B + [V_{CN} - V_{ON}] Y_C$$

Y_A, Y_B, and Y_C are the admittances.

$$[V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C] - V_{ON}[Y_A + Y_B + Y_C]$$

as per KCL $I_A + I_B + I_C = 0$

$$\therefore V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C - V_{ON}(Y_A + Y_B + Y_C) = 0$$

$$V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C = V_{ON}(Y_A + Y_B + Y_C)$$

$$\boxed{V_{ON} = \frac{V_{AN} Y_A + V_{BN} Y_B + V_{CN} Y_C}{Y_A + Y_B + Y_C}}$$

V_{ON} - represent neutral shift.

Example:1 Find the real power and neutral current for the given 3-phase star connected unbalanced load which is connected to 3-phase balanced supply voltage of 400V.

(1)

Given : $V_L = 400V$

$\therefore V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}$

$\boxed{V_{Ph} = 231V.}$

$Z_A = 25\angle 0^\circ,\ Z_B = 11\angle -20^\circ$

$Z_c = 15\angle 10^\circ$

$\therefore$ Line currents

$I_A = \frac{V_{Ph}}{Z_A} = \frac{231}{25\angle 0} = 9.24\angle 0^\circ A$

$I_B = \frac{V_{Ph}}{Z_B} = \frac{231\angle -120^\circ}{11\angle -20^\circ}$

$= 21\angle -100^\circ A.$

$I_c = \frac{V_{Ph}}{Z_c} = \frac{231\angle +120^\circ}{15\angle 10^\circ} = 15.4\angle 110^\circ A$

(2)

In star $I_L = I_{Ph}.$

$\therefore P_A = V_{Ph} \cdot I_A \cdot \cos[0^\circ]$

$= 231 \times 9.24 \times 1 = 2134W.$

$P_B = V_{Ph} \cdot I_B \cdot \cos[-120+100]$

$= 231 \times 21 \times \cos(-20^\circ)$

$P_B = 4558.45W.$

$P_c = V_{Ph} \cdot I_c \cos[120-110]$

$= 231 \times 15.4 \times \cos[10^\circ]$

$P_c = 3503.36W$

Total Power $= 10195.81 W$

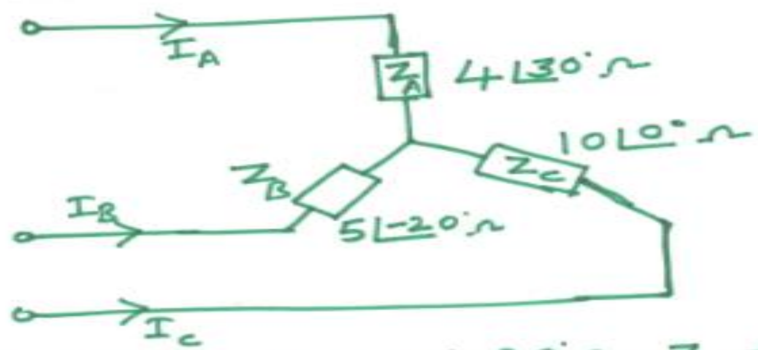
$\therefore$ Neutral current

$I_N = I_A + I_B + I_c$

$= 9.24\angle 0^\circ + 21\angle -100^\circ + 15.4\angle 110^\circ$

$I_N = (0.33 - j6.21) A.$

Example:2 Find the line currents and power drawn for the given 3-phase balanced star connected supply voltage of 50Volts and having unbalanced load impedances.



$Z_A = 4\angle 30\,\Omega \quad Z_B = 5\angle -20^\circ\,\Omega, \; Z_C = 10\angle 0^\circ\,\Omega$

$E_L = 50V.$

Phase Voltages

$$E_{AN} = \frac{E_L}{\sqrt{3}} = \frac{50}{\sqrt{3}} = 28.87\angle 0^\circ\,V$$

$$E_{BN} = 28.87\angle -120^\circ\,V$$

$$E_{CN} = 28.87\angle +120^\circ\,V$$

$$I_A = \frac{E_{AN}}{Z_A} = \frac{28.87\angle 0^\circ}{4\angle 30^\circ}$$

$$I_A = 7.217\angle -30^\circ\,A$$

$$I_B = \frac{E_{BN}}{Z_B} = \frac{28.87\angle -120^\circ}{5\angle -20^\circ}$$

$$= 5.772\angle -100^\circ\,A$$

$$I_C = \frac{E_{CN}}{Z_C} = \frac{28.87\angle +120^\circ}{10\angle 0^\circ}$$

$$I_C = 2.887\angle 120^\circ\,A.$$

: Power drawn by Z_A, Z_B & Z_C

$P_A = E_{AN} \cdot I_A \cos[0+30°]$

$= 28.87 \times 7.217 \times \cos 30°$

$P_A = 180.44 W$

$P_B = E_{BN} \times I_B \times \cos[-120+100°]$

$= 28.87 \times 5.772 \times \cos[-20°]$

$P_B = 156.59 W$

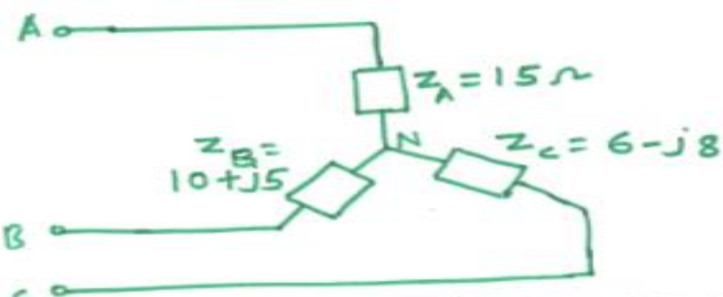
$P_C = E_{CN} \times I_C \times \cos[+120-120°]$

$= 28.87 \times 2.887 \times \cos[0°]$

$= 83.35 W.$

$P = P_A + P_B + P_C = \underline{420.38 W}.$

Ex.3-Find the neutral current which flows in the 3-phase unbalanced star connected load having balanced 3-phase supply voltage of 100Volts in ACB sequence, 50Hz.



Balanced supply Voltage = 100V

since sequence is A C B.

$E_{AN} = 100\angle 0°$

$E_{BN} = 100\angle 120°$ or $100\angle -240°$

$E_{CN} = 100\angle -120°$

$$\therefore \quad I_A = \frac{E_{AN}}{Z_A} = \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ \text{ A}$$

$$I_B = \frac{E_{BN}}{Z_B} = \frac{100\angle 120^\circ}{10+j5} = \frac{100\angle 120^\circ}{11.18\angle 26.56^\circ}$$

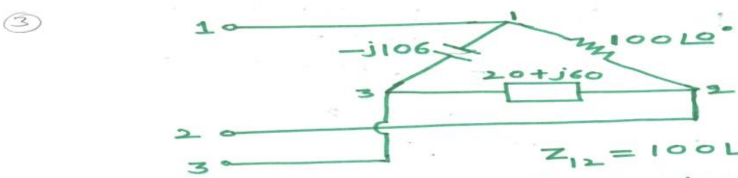
$$I_B = 8.95\angle 93.44 \text{ A}$$

$$Z_C = 6 - j8 = 10\angle -53.13\ \Omega$$

$$I_C = \frac{E_{CN}}{Z_C} = \frac{100\angle -120^\circ}{10\angle -53.13^\circ}$$

$$I_C = 10\angle -66.87^\circ \text{ A}$$

Example:4. Find the currents and power drawn in the given unbalanced Delta connected load supplied with balanced voltage of 100V.



$Z_{12} = 100\angle 0^\circ\ \Omega$

$Z_{13} = -j106\ \Omega = 106\angle -90^\circ\ \Omega$

$Z_{23} = [20 + j60]\ \Omega = 63.25\angle 71.6^\circ$

Supply Voltage = 100V

Let

$V_{12} = 100\angle 0^\circ$ V

$V_{23} = 100\angle -120^\circ$ V

$V_{31} = 100\angle +120^\circ$ V

With Phase shift of 120°

$$I_{12} = \frac{V_{12}}{Z_{12}} = \frac{100\angle 0^\circ}{100\angle 0^\circ} = 1 \text{ A.}$$

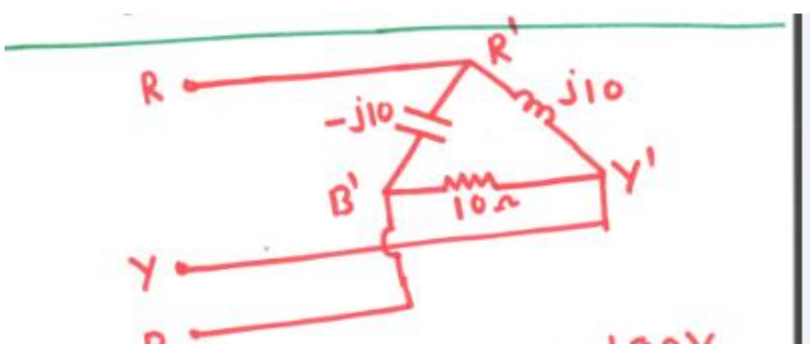
$$I_{23} = \frac{V_{23}}{Z_{23}} = \frac{100\angle -120^\circ}{63.25\angle 71.6} = 1.58\angle -191.6 \text{ A}$$

VTU e-Learning

④
$$I_{31} = \frac{V_{31}}{Z_{31}} = \frac{100\angle{+120^\circ}}{106\angle{-90^\circ}}$$
$$I_{31} = 0.943\angle{210^\circ}\ A$$
Apply KCL @ 1, 2, and 3.
$$I_1 + I_{31} - I_{12} = 0$$
$$\therefore I_1 = I_{12} - I_{31}$$
$$= 1\angle{0^\circ} - 0.943\angle{210^\circ}$$
$$= 1 - [-0.817 - j0.4715]$$
$$I_1 = 0.506\angle{68.79^\circ}\ A$$
then @ node 2. $I_2 + I_{12} - I_{23} = 0$
$$\therefore I_2 = I_{23} - I_{12}$$
$$= 1.58\angle{-191.6^\circ} - 1\angle{0^\circ}$$
$$I_2 = -2.55 + j0.318$$
$$= 2.57\angle{173^\circ}\ A$$

⑤ KCL @ node 3
$$I_3 + I_{23} - I_{31} = 0$$
$$I_3 = I_{31} - I_{23}$$
$$= 0.943\angle{210^\circ} - 1.58\angle{-191.6^\circ}$$
$$= 0.733 - j0.7895$$
$$I_3 = 1.08\angle{-47.13^\circ}\ A$$
Power calculation
$$P_1 = V_{12} \times I_1 \times \cos[0 - 68.79^\circ]$$
$$= 100 \times 0.506 \times \cos[-68.79^\circ]$$
$$= 18.31\ W.$$
$$P_2 = V_{23} \times I_2 \times \cos[-120^\circ - 173^\circ]$$
$$= 100 \times 2.57 \times \cos[-293^\circ]$$
$$= 100.42\ W.$$
$$P_3 = V_{31} \times I_3 \times \cos[+120^\circ + 47.13^\circ]$$
$$= 100 \times 1.08 \times \cos[167.13]$$
$$= -105.28\ W$$

Example:5. Find currents in the star connected unbalanced load.



$V_{RY} = 100\angle 0^\circ\ V$

$V_{YB} = 100\angle -120^\circ\ V$

$V_{BR} = 100\angle +120^\circ\ V$

$Z_{R'Y'} = j10\,\Omega = 10\angle 90^\circ$

$Z_{Y'B'} = 10\,\Omega$

$Z_{B'R'} = -j10 = 10\angle -90^\circ\,\Omega$

$$I_{R'Y'} = \frac{V_{RY}}{Z_{R'Y'}} = \frac{100\angle 0^\circ}{10\angle 90^\circ}$$

$$I_{R'Y'} = -j10\,A = 10\angle -90^\circ\,A$$

$$I_{Y'B'} = \frac{V_{YB}}{Z_{Y'B'}} = \frac{100\angle -120^\circ}{10} = 10\angle -120^\circ\,A$$

$$I_{B'R'} = \frac{V_{BR}}{Z_{B'R'}} = \frac{100\angle +120^\circ}{-j10} = \frac{100\angle +120^\circ}{10\angle -90^\circ} = 10\angle 210^\circ\,A$$

Apply KCL @ R', Y' & B'

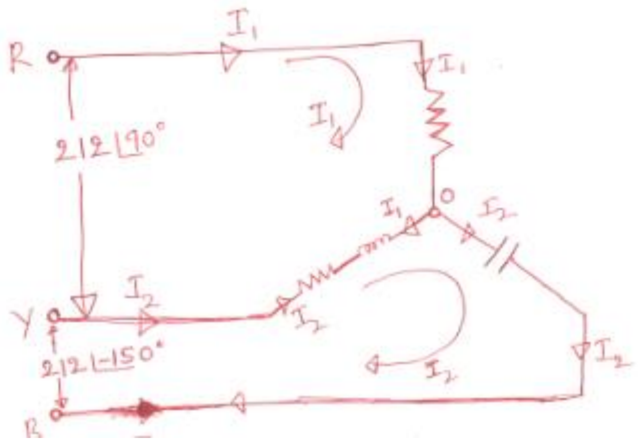
$$I_R = I_{R'Y'} - I_{B'R'} = 10\angle -90^\circ - 10\angle 210^\circ$$

$$= [8.66 - j5]\,A.$$

$$I_Y = I_{Y'B'} - I_{R'Y'}$$
$$= 10\angle{-120^\circ} - 10\angle{-90^\circ}$$
$$I_Y = [-5 + j1.34]\text{ A}$$

$$I_B = I_{B'R'} - I_{Y'B'}$$
$$= 10\angle{210^\circ} - 10\angle{-120^\circ}$$
$$I_B = [-3.66 + j3.6]\text{ A.}$$

Example:6. Find line currents using loop analysis.



consider the Loop R-O-Y:

$$-10 I_1 - (10+j10)(I_1 - I_2) + 212\angle 90^\circ = 0$$
$$I_1[-10-10-j10] + I_2[10+j10] + 212\angle 90^\circ = 0$$
$$\boxed{[-20-j10]\, I_1 + [10+j10]\, I_2 = -212\angle 90^\circ} \quad -(1)$$

consider the loop Y-O-B.

$$-[-j20]\, I_2 + 212\angle{-150^\circ} - (10+j10)(I_2 - I_1) = 0$$
$$\boxed{(10+j10)\, I_1 - I_2(10+j10) = -212\angle{-150^\circ}} \quad \rightarrow (2)$$

$$\Delta = \begin{vmatrix} -20-j10 & 10+j10 \\ 10+j10 & -10-j10 \end{vmatrix}$$

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Solving 1 & 2

$I_1 = 3.66\angle 15^\circ$ A $\qquad I_2 = 11.96\angle -114^\circ$ A

$I_R = I_1 = 3.66\angle 15^\circ$ A

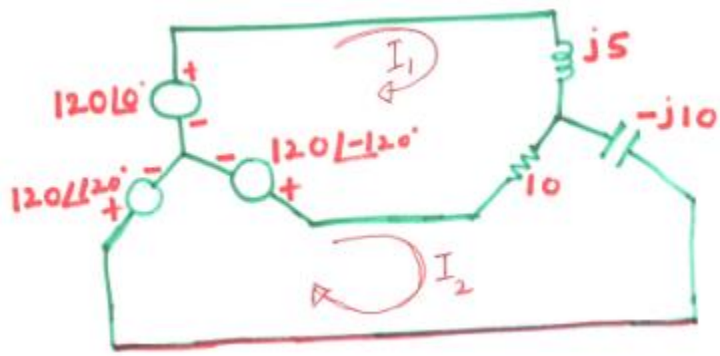
$I_B = -I_2 = 11.96\angle 66^\circ$

$I_Y = I_2 - I_1 = 11.96\angle -114^\circ - 3.66\angle 15^\circ$

$= -8.395 - j11.877$

$I_Y = 14.544\angle 125.75^\circ$ A

Example:7. Find currents using loop analysis.



Using Mesh analysis. Finding currents I_1 & I_2

$$-120\angle 120^\circ + 120\angle 0^\circ - (10+j5)I_1 + 10I_2 = 0$$

$$(10+j5)I_1 - 10I_2 = 120\sqrt{3}\angle 30^\circ \longrightarrow (1)$$

for mesh 2.

$$-120\angle 120^\circ + 120\angle -120^\circ - (10-j10)I_2 + 10I_1 = 0$$

$$-10I_1 + (10-j10)I_2 = 120\sqrt{3}\angle -90^\circ \longrightarrow (2)$$

matrix Equation

$$\begin{bmatrix} 10+j5 & -10 \\ -10 & 10-j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3}\angle 30^\circ \\ 120\sqrt{3}\angle -90^\circ \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 10+j5 & -10 \\ -10 & 10-j10 \end{bmatrix} = 50-j50 = 70.71\angle -45^\circ$$

$$\Delta_1 = \begin{bmatrix} 120\sqrt{3}\angle 30^\circ & -10 \\ 120\sqrt{3}\angle -90^\circ & 10-j10 \end{bmatrix} = 207.85(13.66-j13.66) = 4015\angle -45^\circ$$

$$\Delta_2 = \begin{bmatrix} 10+j5 & 120\sqrt{3}\angle 30^\circ \\ -10 & 120\sqrt{3}\angle -90^\circ \end{bmatrix}$$

$$= 207.85(13.66-j13.66) = \underline{3023.4\angle -20.1^\circ}$$

$$= \cancel{4015\angle -45^\circ}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4015.23\angle -45^\circ}{70.71\angle -45^\circ} = 56.78\,A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{3023.4\angle -20.1^\circ}{70.71\angle -45^\circ} = 42.75\angle 24.9^\circ \text{ A}$$

$I_A = I_1 = 56.78$ A

$I_C = -I_2 = 42.75$ A

$I_B = I_2 - I_1 = 38.78 + j18 - 56.78$

$= \underline{25.46\angle 135^\circ \text{ A}}$