Module-1

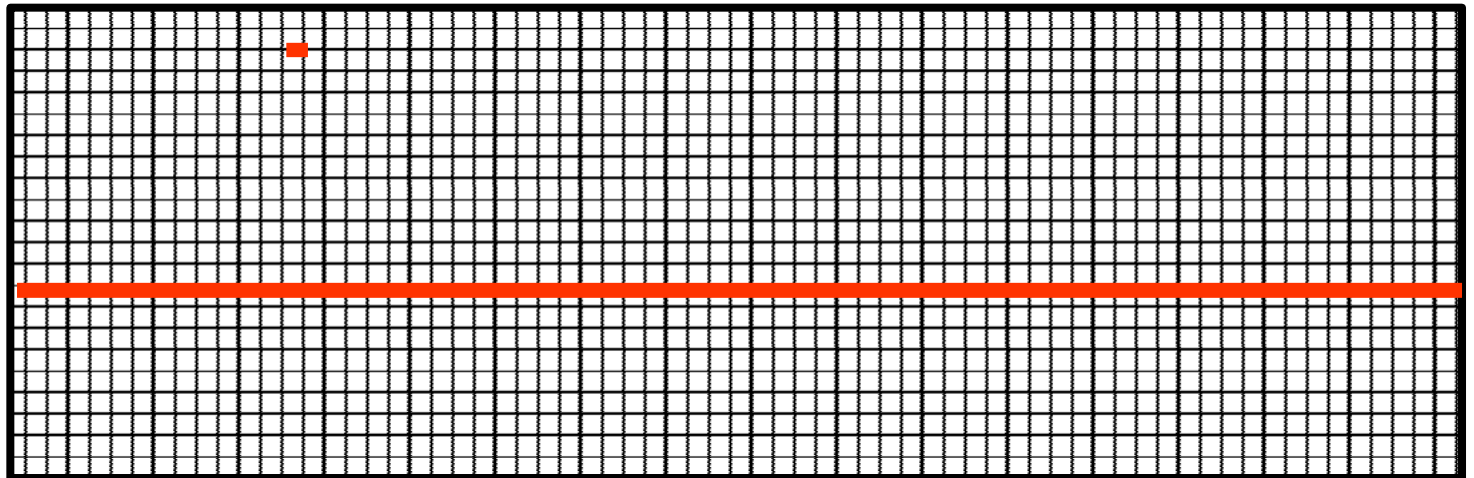# RANDOM SCAN DISPLAYS AND RASTER SCAN DISPLAYS

Prof. Rahul Palakar

RASTER SCAN DISPLAY

ಠ   **Raster:** A rectangular array of points or dot.

ಠ   An image is subdivided into a sequence of (usually horizontal) strips known as "scan lines" which can be further divided into discrete pixels for processing in a computer system.

A raster image is a collection of dots called pixels

RASTER IMAGE
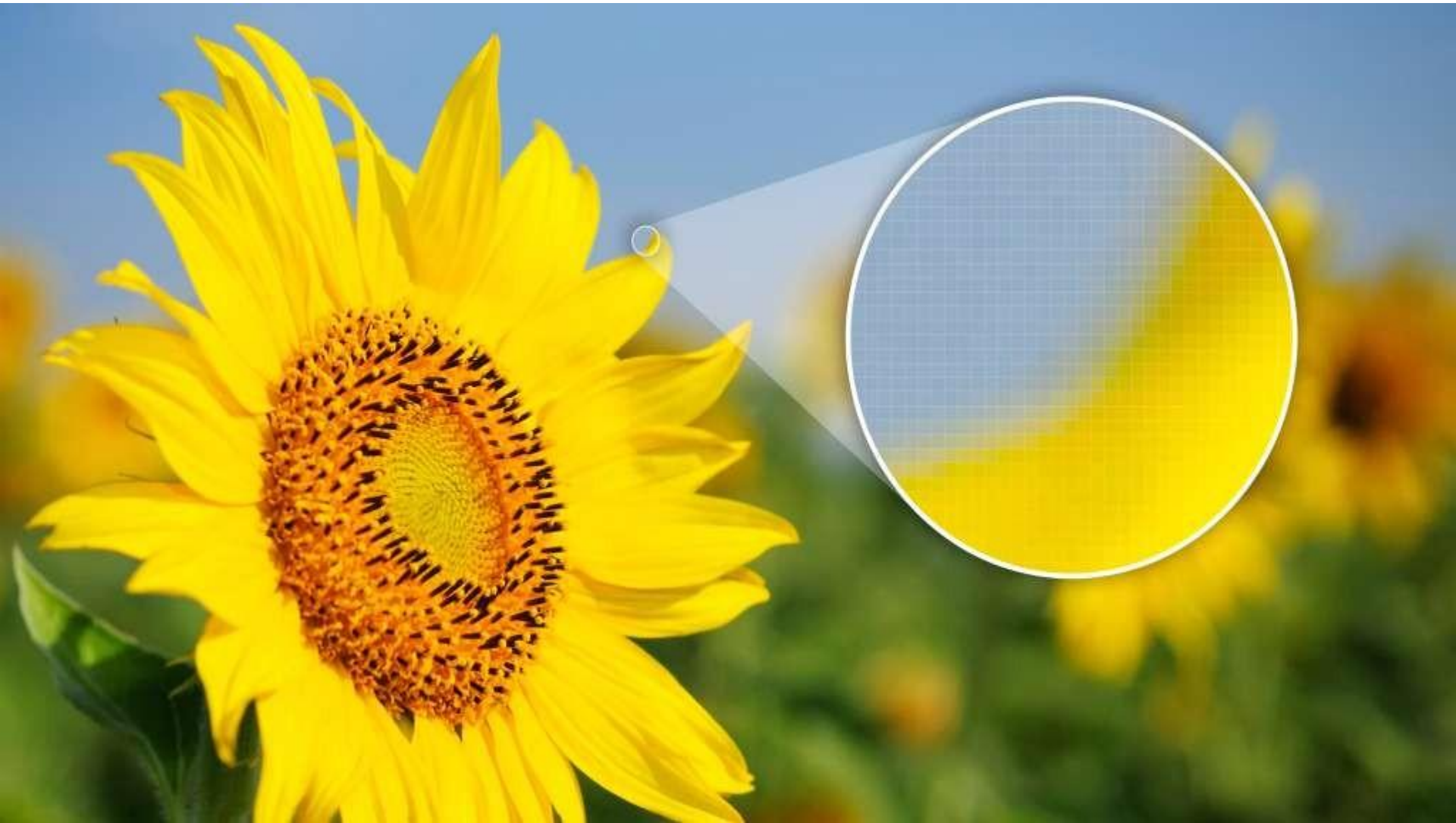
WORKING

- In a raster scan system, the electron beam is swept across the screen, one row at a time from top to bottom.

- As the electron beam moves across each row, the beam intensity is turned on and off to create a pattern of illuminated spots.

- The return to the left of the screen, after refreshing each scan line is called **Horizontal retrace**.

- At the end of each frame the electron beam returns to the top left corner of the screen to begin the next frame is called **Vertical retrace**:

Raster Scan Display
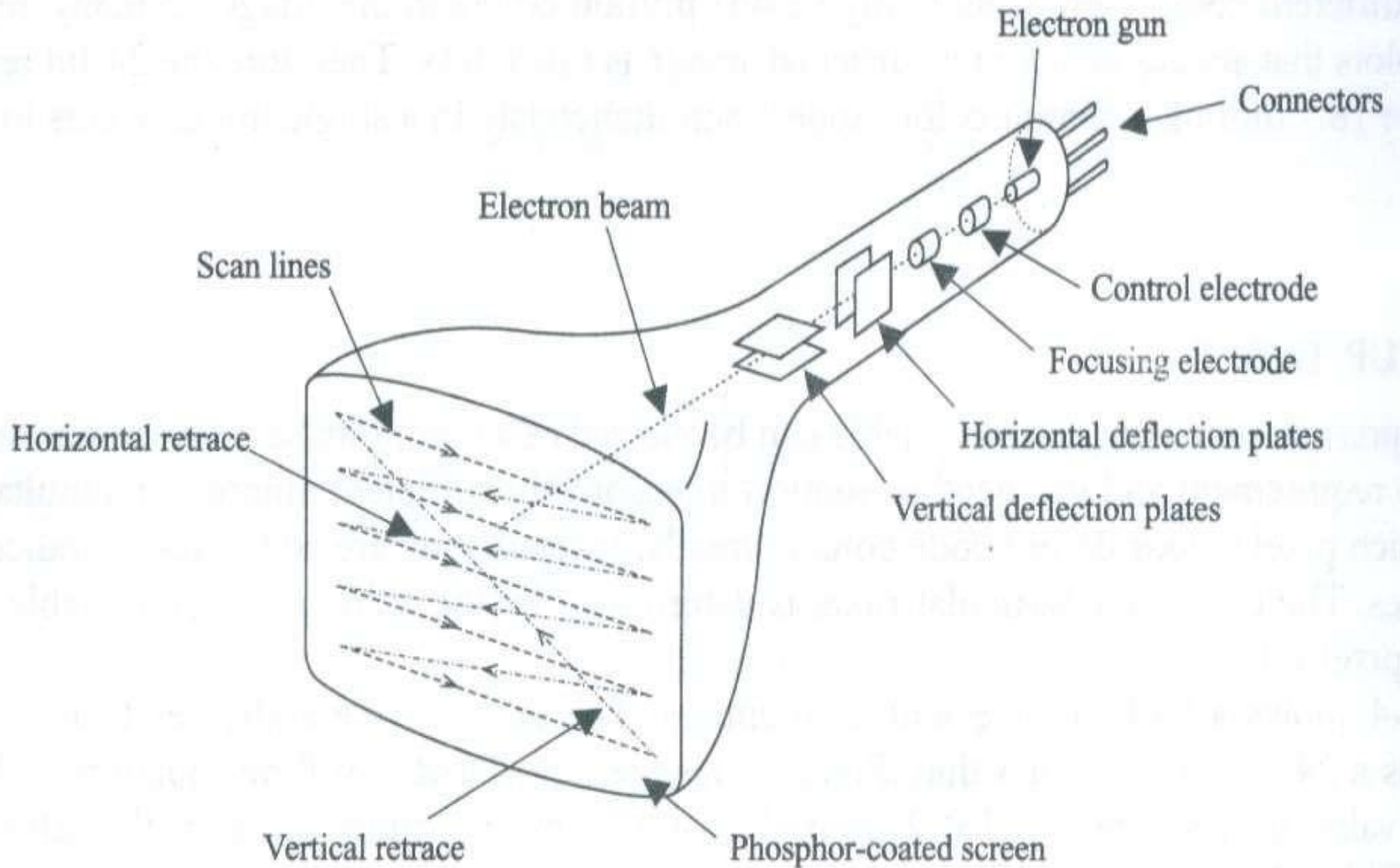
WORKING

- Picture definition is stored in a memory area called the **refresh buffer** or **frame buffer**.

- **Refresh buffer** or **frame buffer** is memory area that holds the set of intensity values for all the screen points.

- Stored intensity values then retrieved from refresh buffer and "**painted**" on the screen one row (**scan line**) at a time.
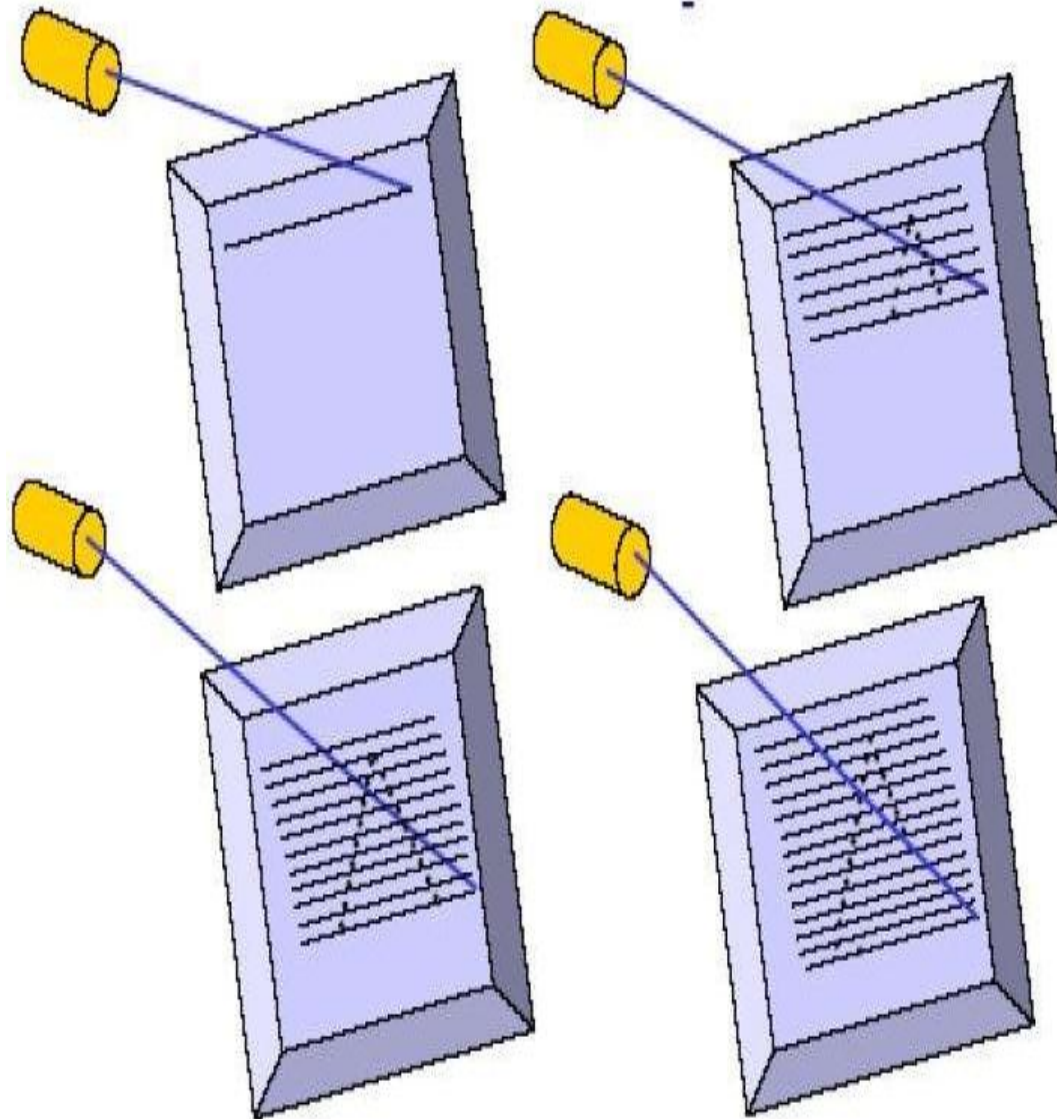
Object as set of discrete points across each scan line

- The quality of a raster image is determined by the total number of pixels (**resolution**), and the amount of information in each pixel (**color depth**)

- A black-and-white system: each screen point is either on or off, so only **one bit** per pixel is needed to control the intensity of screen positions. Such type of frame buffer is called Bit map

- High quality raster graphics system have **24 bits per pixel** in the frame buffer (a **full color** system or a **true color** system)

- Refreshing on raster scan displays is carriedout at the rate 60 to 80 frame per second.

INTERLACING

- On some raster systems (TV), each frame is displays in two passes using an interlaced refresh procedure.

- Interlacing is primarily used for slower refresh rates.

- An effective technique to avoid **Flicker**.(Flicker occurs on CRTs when they are driven at a low refresh rate, allowing the brightness to drop for time intervals sufficiently long to be noticed by a human eye)

INTERLACING
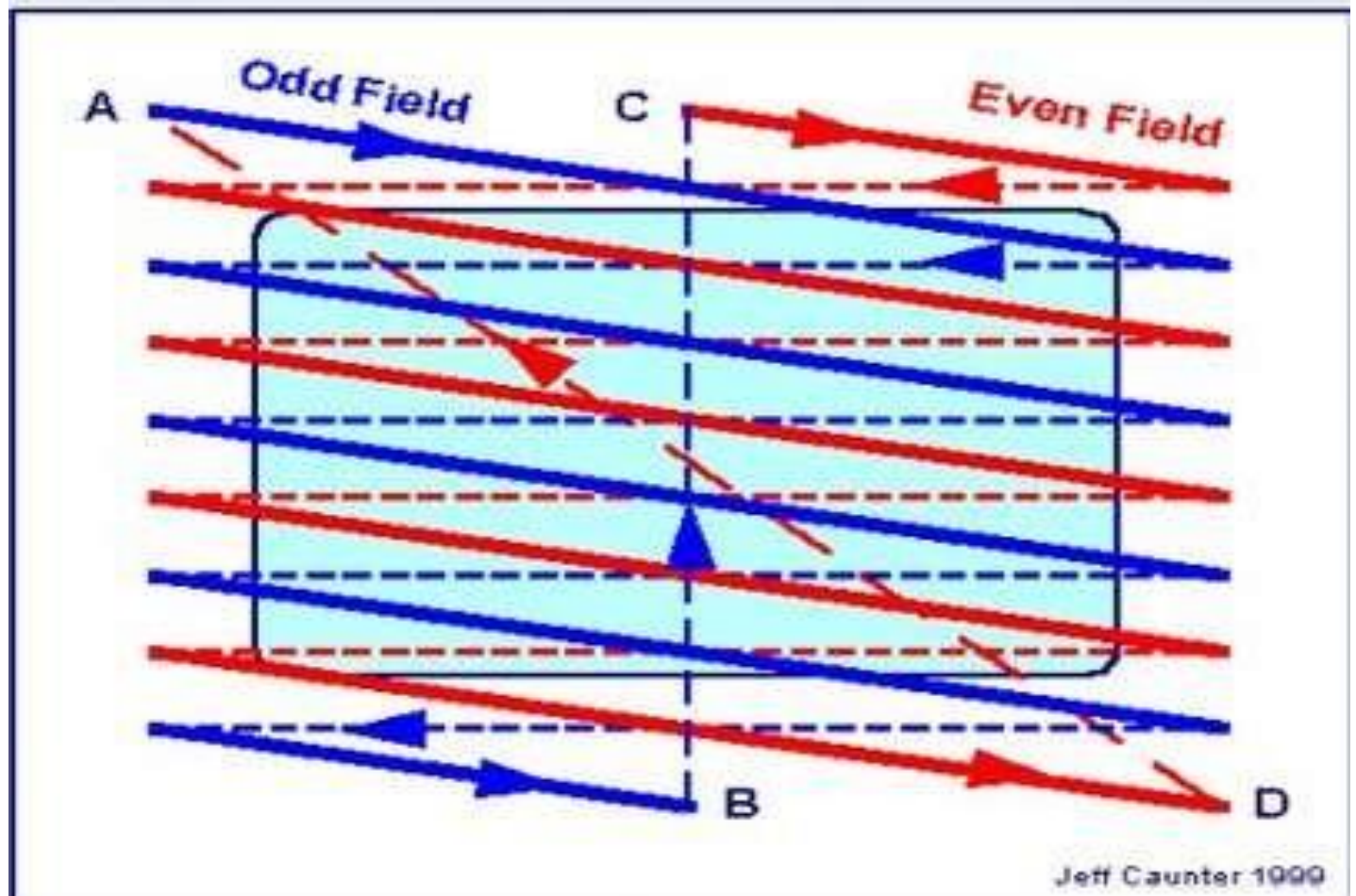
APPLICATIONS

- ❧ Suited for realistic display of screens

- ❧ Home television, computer printers create their images basically by raster scanning. Laser printers use a spinning polygonal mirror (or an optical equivalent) to scan across the photosensitive drum, and paper movement provides the other scan axis

- ➢ Common raster image formats include BMP (Windows Bitmap), JPEG (Joint Photographics Expert Group), GIF (Graphics Interchange Format) , PNG (Portable Network Graphic), PSD (Adobe PhotoShop)

DISADVANTAGE

- To increase size of a raster image the pixels defining the image are be increased in either number or size Spreading the pixels over a larger area causes the image to lose detail and clarity.

- Produces jagged lines that are plotted as discrete points.

Aliased

Anti-Aliased
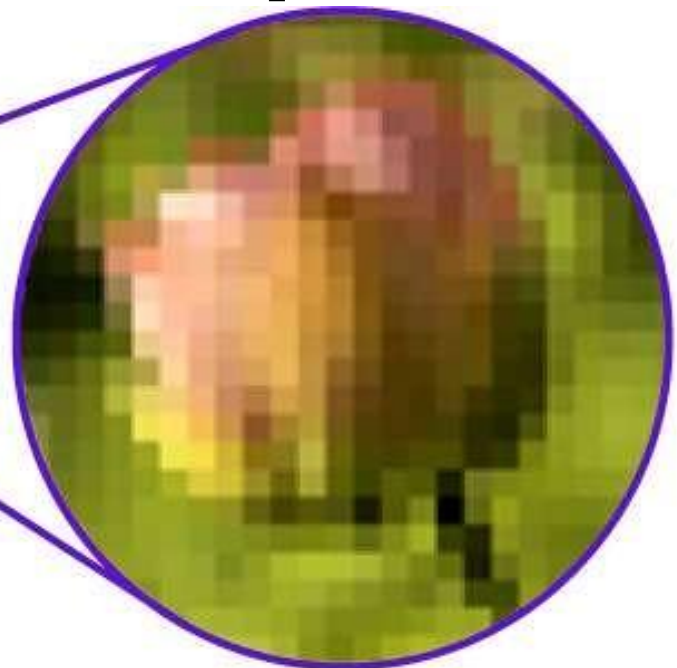
RANDOM SCAN DISPLAY

Random scan display is the use of geometrical primitives such as points, lines, curves, and polygons, which are all based upon **mathematical equation.**
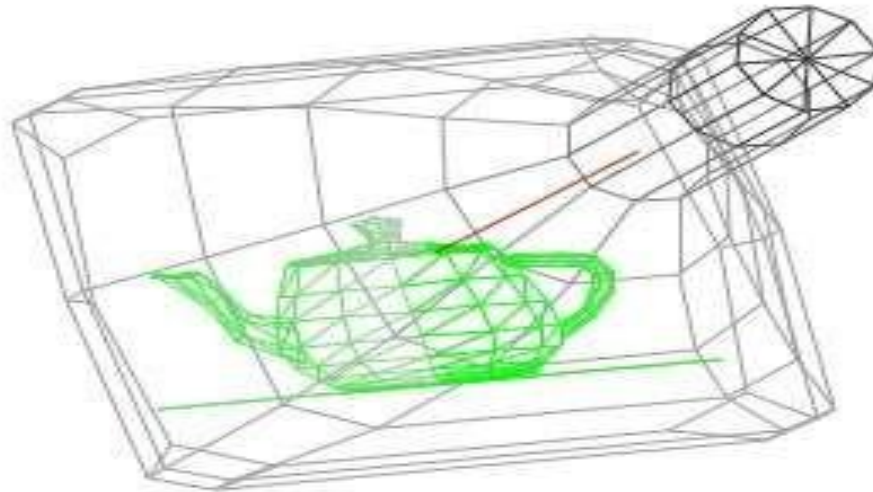
VECTOR IMAGE

WORKING

- When operated as a random-scan display unit, a CRT has the electron beam directed only to the parts of the screen where a picture is to be drawn.

- Random-scan monitors draw a picture one line at a time and for this reason they are also referred to as vector displays (or stroke-writing or calligraphic displays).

RASTER SCAN DISPLAY

(a)
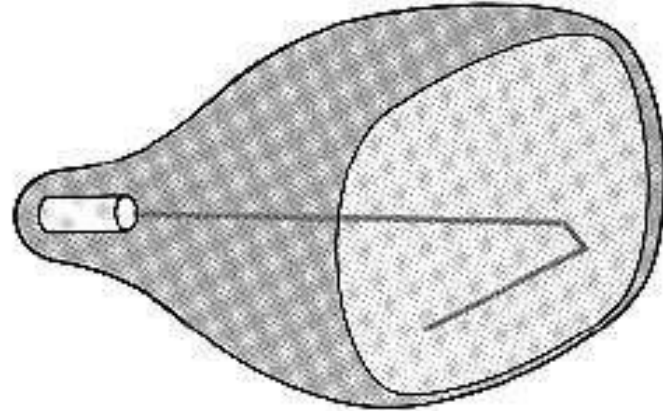
(b)
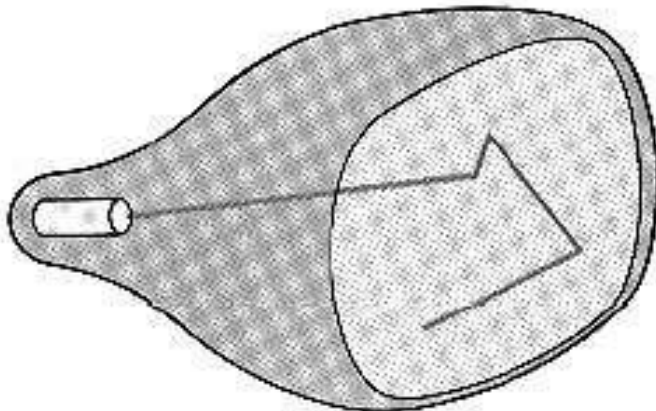
(c)
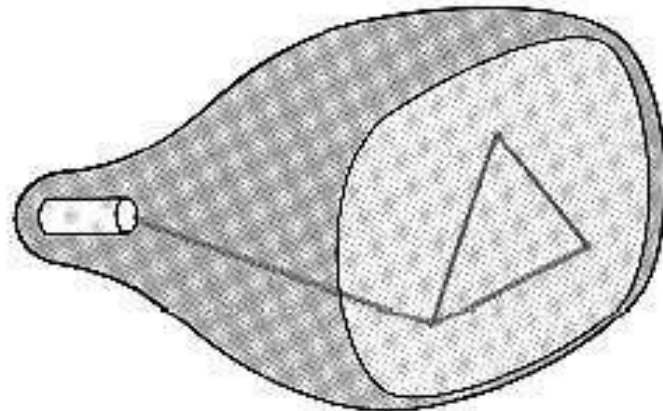
(d)

- **Refresh rate** depends on the number of lines to be displayed.
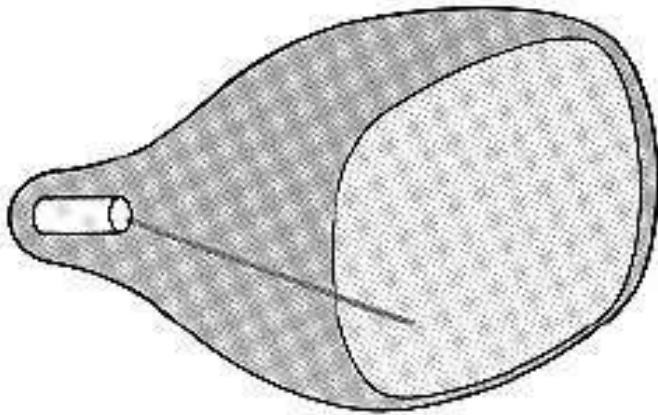
- Picture definition is now stored as a line-drawing commands an area of memory referred to as **refresh display file** (**display list**).

- To display a picture, the system cycle through the **set of commands** in the display file, drawing each component line in turn.

- Random scan displays are designed to draw all the component lines of a picture 30 to 60 times each second

➤A Raster system produces jagged lines that are plotted as discrete points sets.

Raster
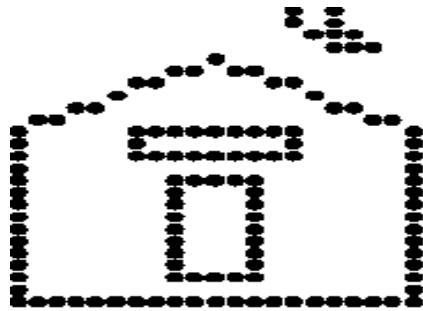
Outline primitives
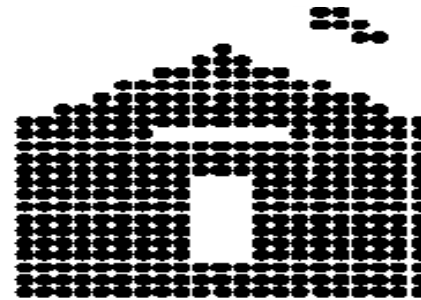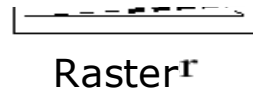
Filled primitives

➤Vector displays product smooth line drawing

Ideal Drawing
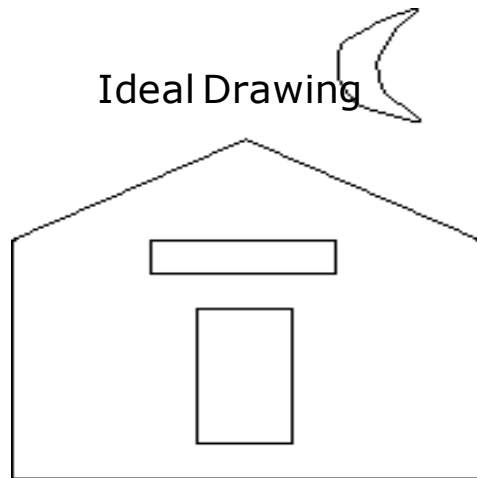
Vector Drawing

Ideal Drawing
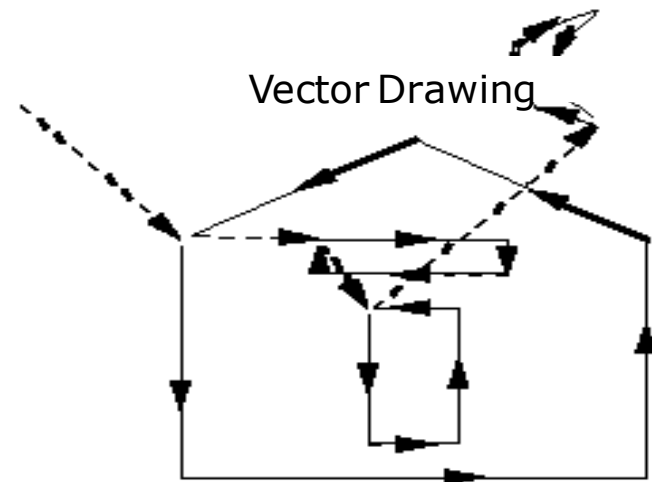
Vector Drawing

Random scan displays are designed for **line-drawing applications** and can not display realistic shaded scenes

Advantages

- Random scan displays have higher resolution than raster systems.

- Vector displays produce smooth line drawing.

- This minimal amount of information translates to a much smaller file size. (file size compared to large raster images)

- On zooming in, and it remains smooth

- The parameters of objects are stored and can be later modified.

Color CRT Monitors

■ A CRT monitor displays color pictures by using a combination of phosphors that emit different *color* lights.

Methods

1. Beam Penetration

2. Shadow Mask

Beam Penetration Method

- Two layers of *phosphor* (**red** and **green**) are coated onto the inside of the CRT screen.

- The display color depends on haw far the electron beam **penetrates** into the phosphor layers.

Beam Penetration Method

■ The speed of the electrons, and the screen color at any point, is controlled by the beam **acceleration voltage.**

Beam Penetration Method

The beam penetration method:

■Used with random scan monitors

■Only four colors are possible (**red**, **green**, **orange**, and **yellow**).

■Quality of pictures is not as good as with other methods.

Shadow Mask Method

The color CRT has:

■Three color *phosphor* dots (**red**, **green** and **blue**) at each point on the screen

■Three *electron guns*, each controlling the display of red, green and blue light.

Shadow Mask Method
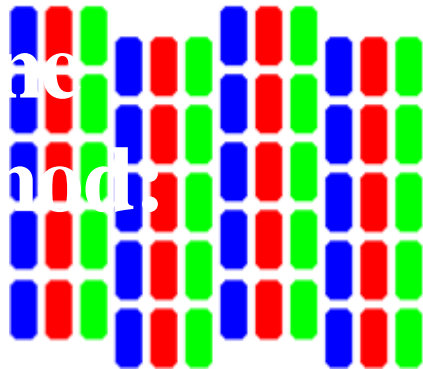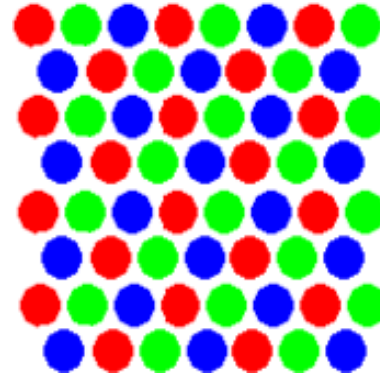
Shadow Mask Method
The delta-delta method:
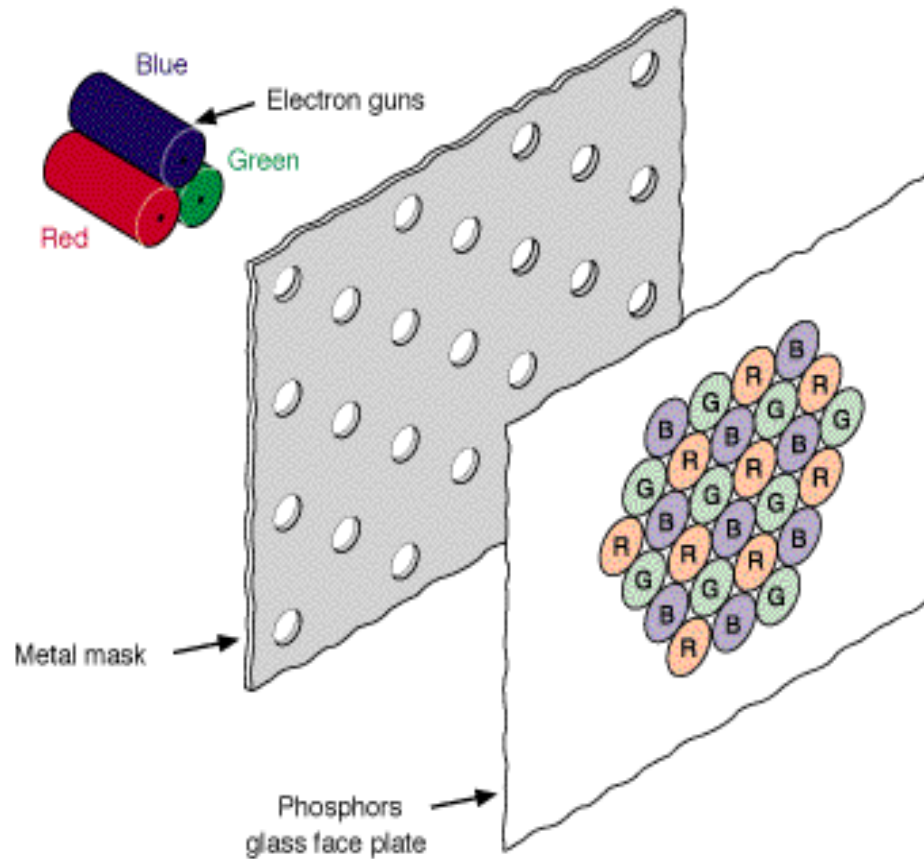
Shadow Mask Method
The in-line method:
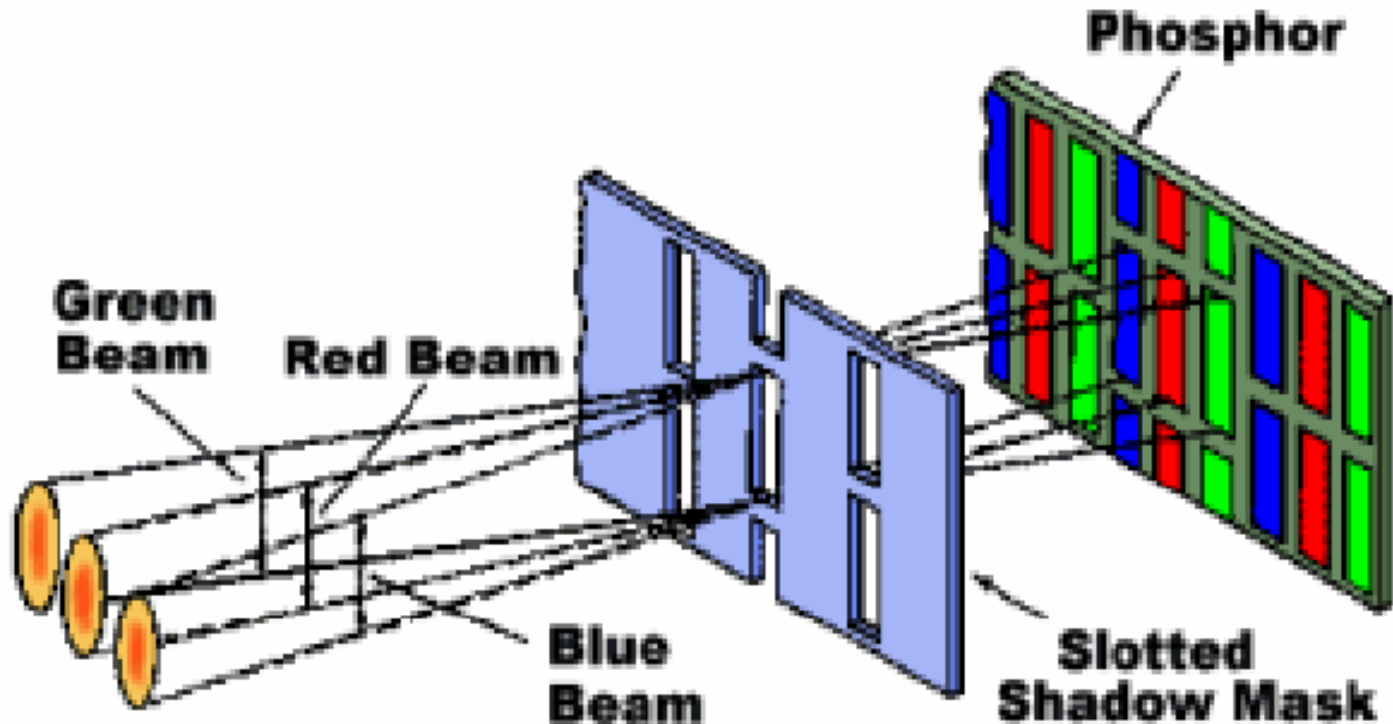
Shadow Mask Method

- We obtain color variations by varying the **intensity** levels of the three electron beam.

Shadow Mask Method

Shadow mask methods are:

- Used in raster scan system (including color TV)

- Designed as **_RGB_** monitors.

Shadow Mask Method

- High quality raster graphics system have **24** bits per pixel in the frame buffer (a **full color** system or a **true color** system)

Flat Panel Displays

- A class of video devices that have reduce volume and weight compared to a CRT.

- A significant feature of flat panel displays is that they are **thinner** than CRTs.

Flat Panel Displays

Current uses for flat panel displays:

- Small TV monitors

- Calculators

- Pocket video games

- Laptop computers

- Advertisement boars in

Flat Panel Displays

Flat panel displays:

- **Emissive** or **Emitters** Displays

- **Non-emissive** or **Non-emitters**

Emissive (or Emitters) Displays

- Emissive displays convert **electrical energy** into light.

- Examples: Plasma panel, thin- film electroluminescent displays, Light-Emitting Diodes (LED) and flat CRT.

Non-Emissive (or Non-Emitters) Displays

- Use **optical effects** to convert sunlight or light from some other source into graphics pattern.

- Example: **L**iquid-**C**rystal **D**evice (**LCD**)

Plasma Panel

- By applying high voltage to a pair of horizontal and vertical conductors, a small section of the gas (tiny neon bulb) at the intersection of the conductors break down into glowing plasma of electrons and ions.
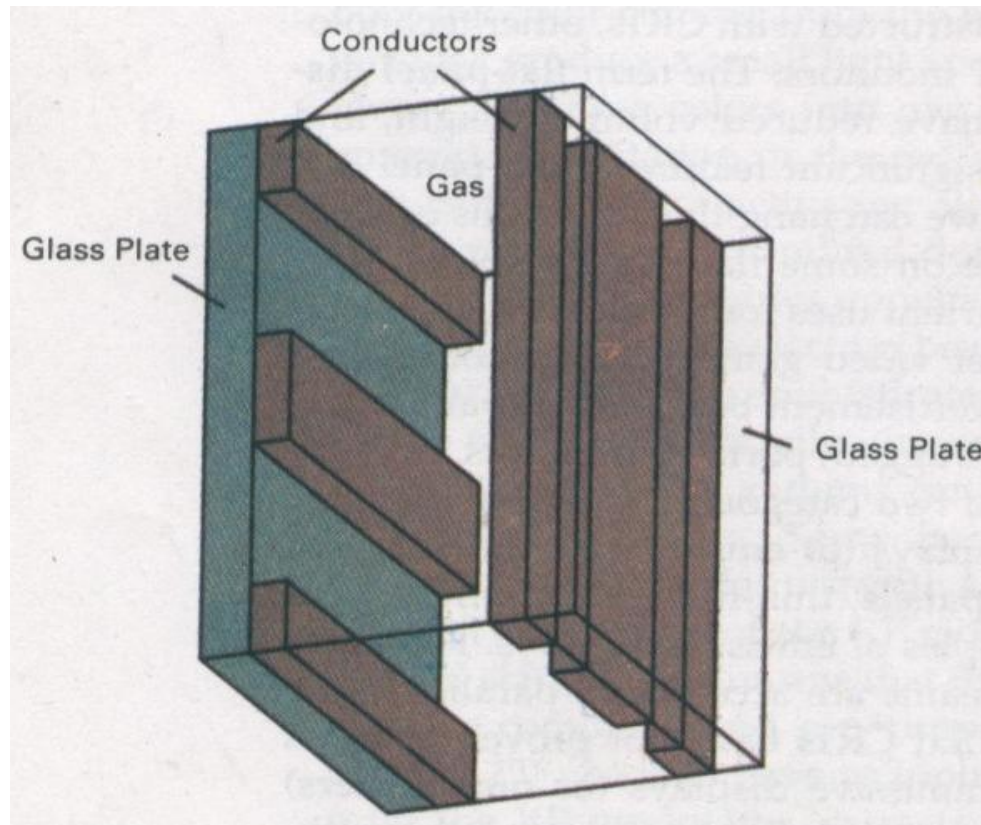
Thin Film Electroluminescent

- The region between the glass plates is filled with a phosphor, such as zinc sulfide doped with manganese.
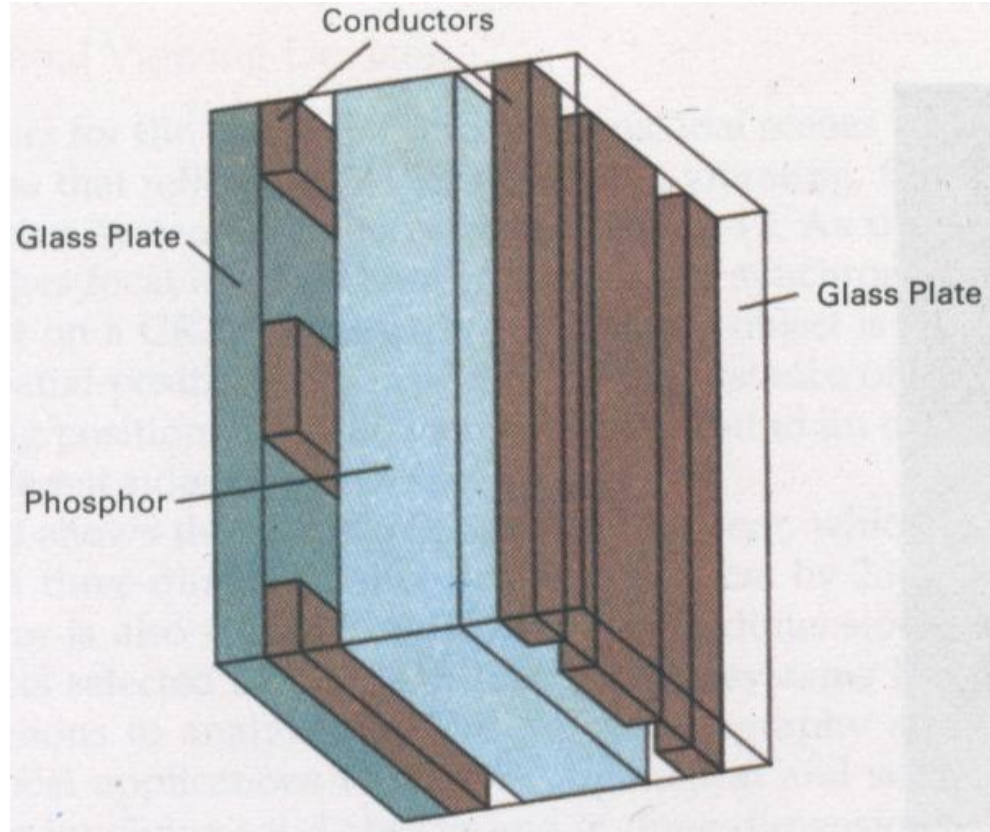
Liquid Crystal Displays (LCD)

- Used in small systems, such as calculators, laptop computers.

- Produce a picture by passing polarized light (from the surrounding or from an internal light source)   through a liquid-crystal material that can be aligned to either block or transmit the light.

Liquid Crystal Displays (LCD)

- **Liquid crystal:** These compounds have a crystalline arrangement of molecules, yet they flow like a liquid.

Liquid Crystal Displays (LCD)

- Two glass plates, each containing a light polarizer at right angles to the other plate, sandwich the liquid crystal materials.

- Rows of horizontal transparent conductor & columns of vertical conductors (put into glass plates)

On State
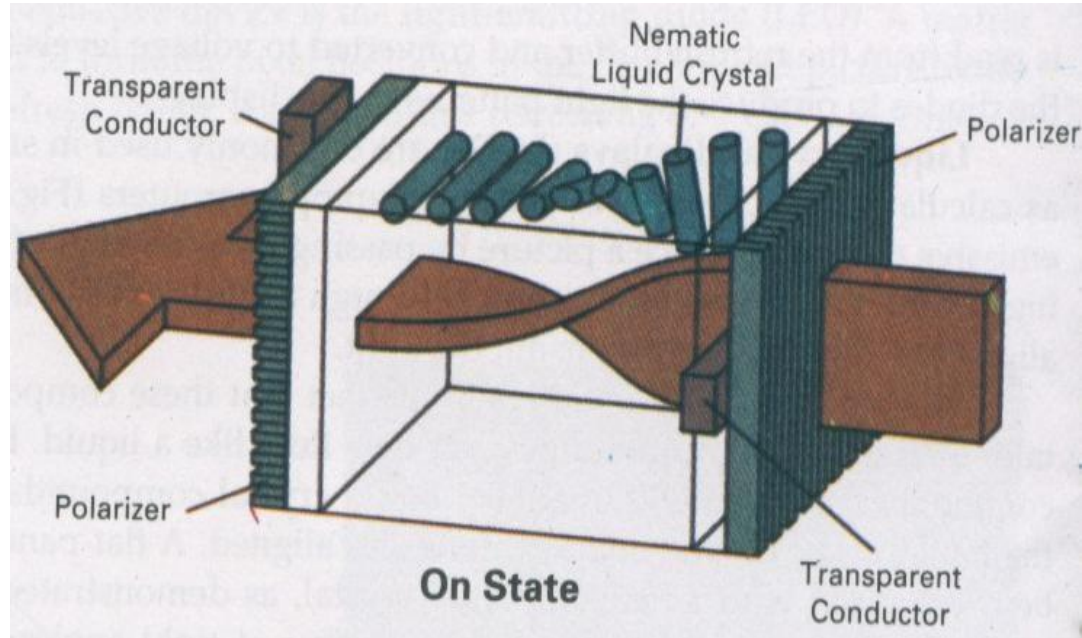
Liquid Crystal Displays (LCD)

- Polarized light passing through the material is ***twisted*** so that it will pass through the opposite polarizer.

- The light is then reflected back t the viewer.

Liquid Crystal Displays (LCD)
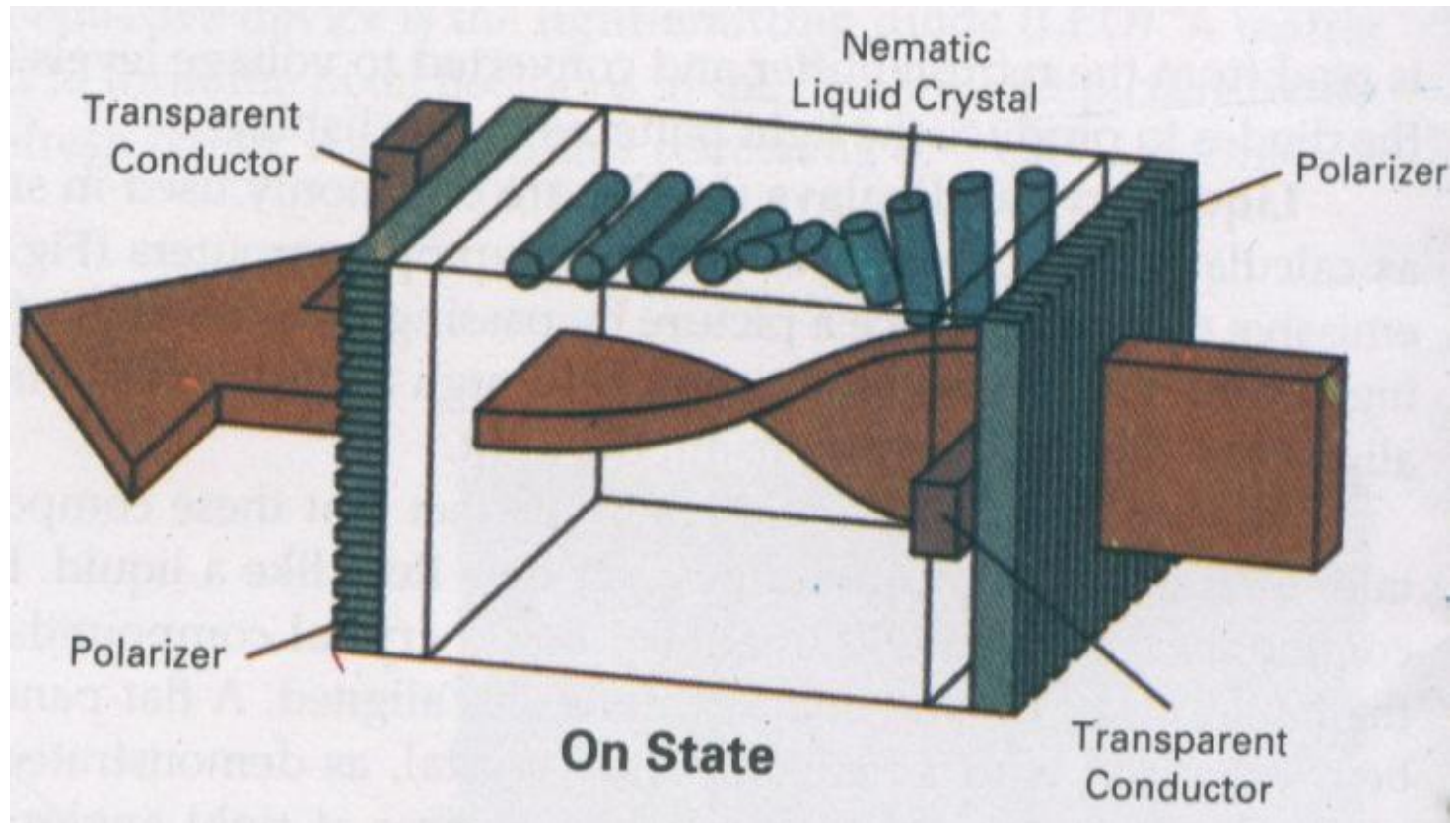
Liquid Crystal Displays (LCD)

- To turn off the pixel, we apply a voltage to the two intersecting conductor to align the molecules so that the light is not twisted.
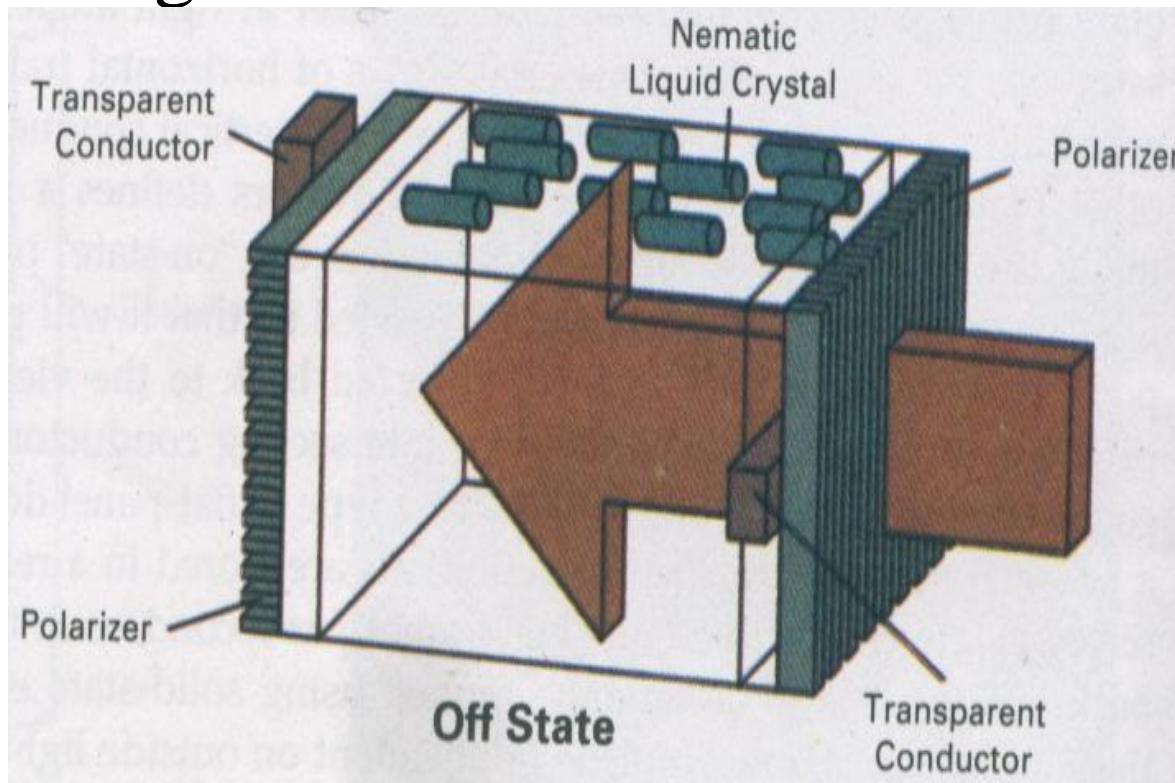
Liquid Crystal Displays (LCD)
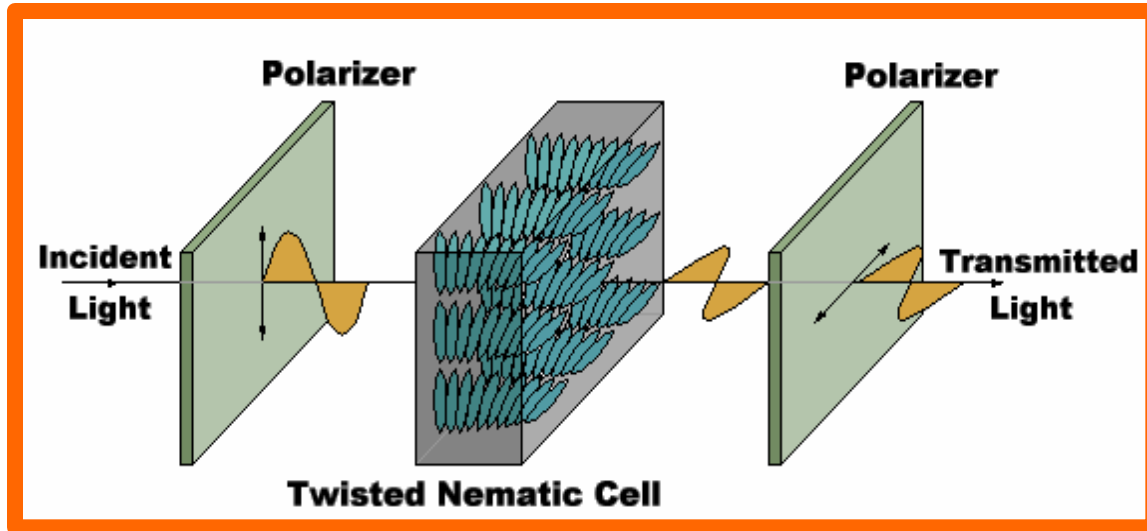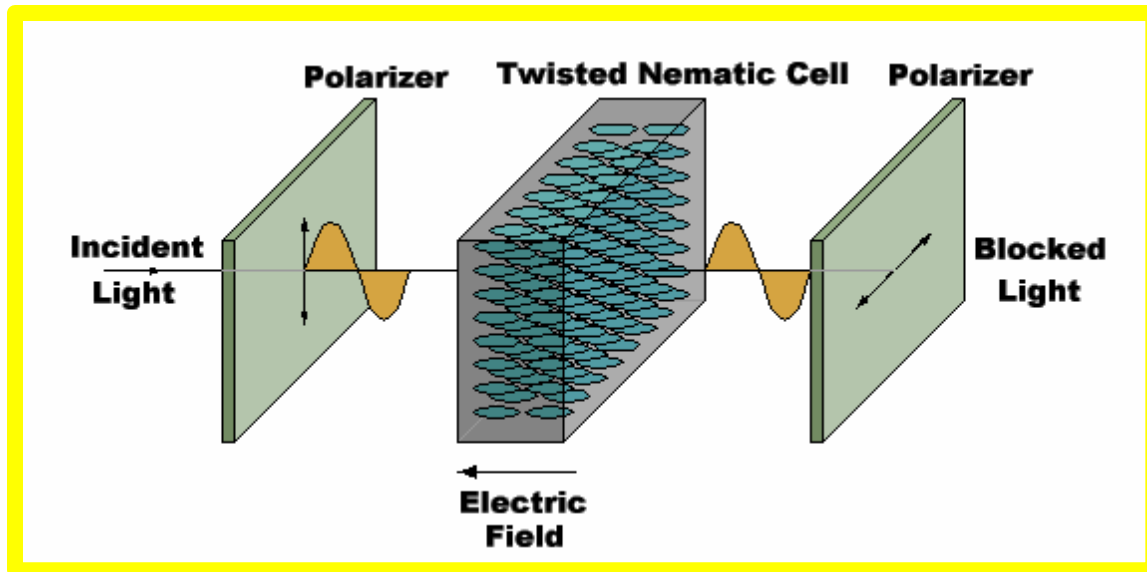
RASTER SCAN SYSTEM

ஐ **In addition to the central processing unit (CPU), a special processor, called the video controller or display controller, is used to control the operation of the display device.**

ஐ **A fixed area of the system memory is reserved for the frame buffer, and the video controller is given direct access to the frame buffer memory.**

Operation performed:

1. **Refreshing operation**

2. **Transformation (Areas of the screen can be enlarged, reduces, or moved during the refresh cycles)**

CPU

Peripheral devices

System bus

System memory | Frame buffer

Video controller

Monitor
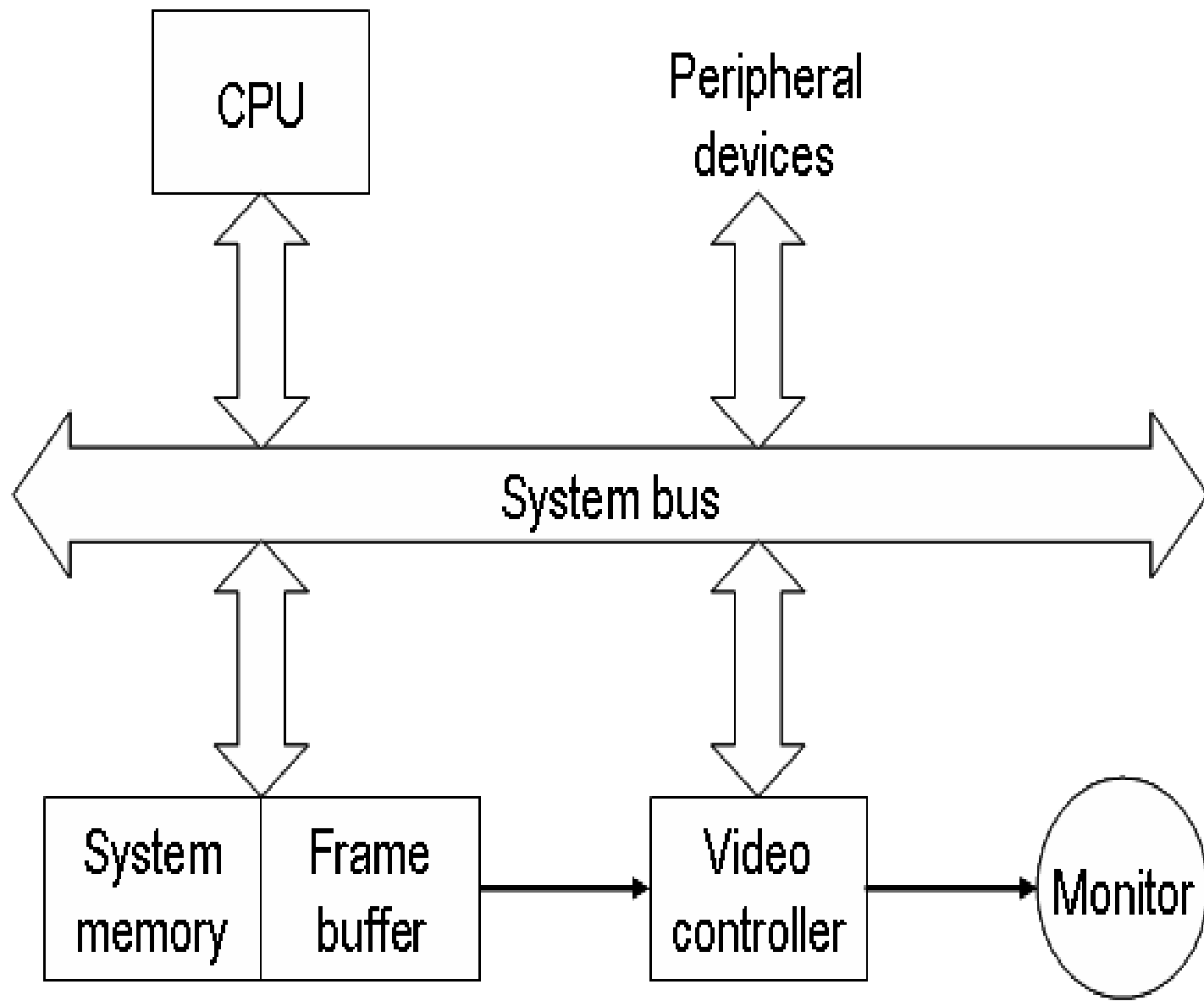
The Basic refresh operation of the video controller.

☙**Frame buffer** location, and the corresponding screen positions, are referenced in Cartesian coordinates

☙**Scan lines** are then labeled from y_{max} at the top of the screen to 0 at the bottom. Along each scan line, screen **pixel** positions are labeled from 0 to x_{max}

☙Two registers are used to store the coordinates of the screen pixels.

```
┌─────────────────────────────────┐
│                                 │
│     Raster Scan Generator       │
│                                 │
└─────────────────────────────────┘
        │                │
        ▼                ▼
┌──────────────┐   ┌──────────────┐
│              │   │              │
│  x Register  │   │  y Register  │
│              │   │              │
└──────────────┘   └──────────────┘
        │                │
        ▼                ▼
┌─────────────────────────────────┐
│                                 │
│        Memory Address           │
│                                 │
└─────────────────────────────────┘
              │
              ▼
┌─────────────────────────────────┐        ┌──────────────────┐
│                                 │        │                  │
│         Frame Buffer            │───────▶│    Intensity     │
│                                 │        │                  │
└─────────────────────────────────┘        └──────────────────┘
```
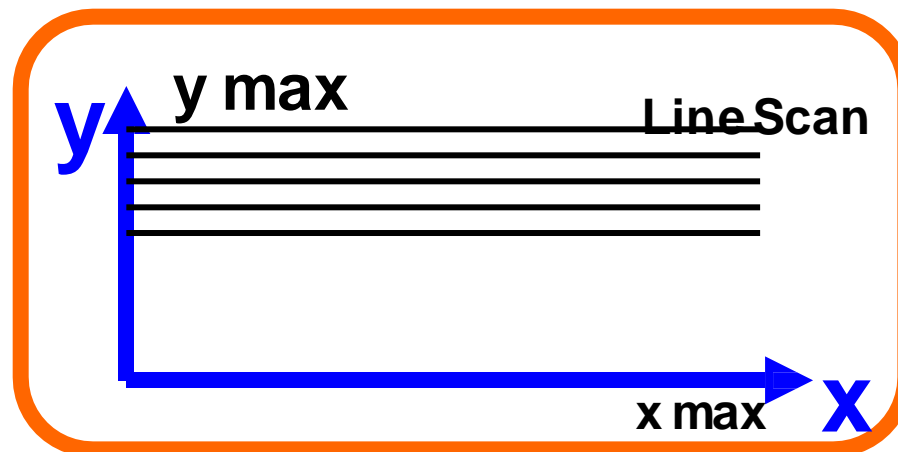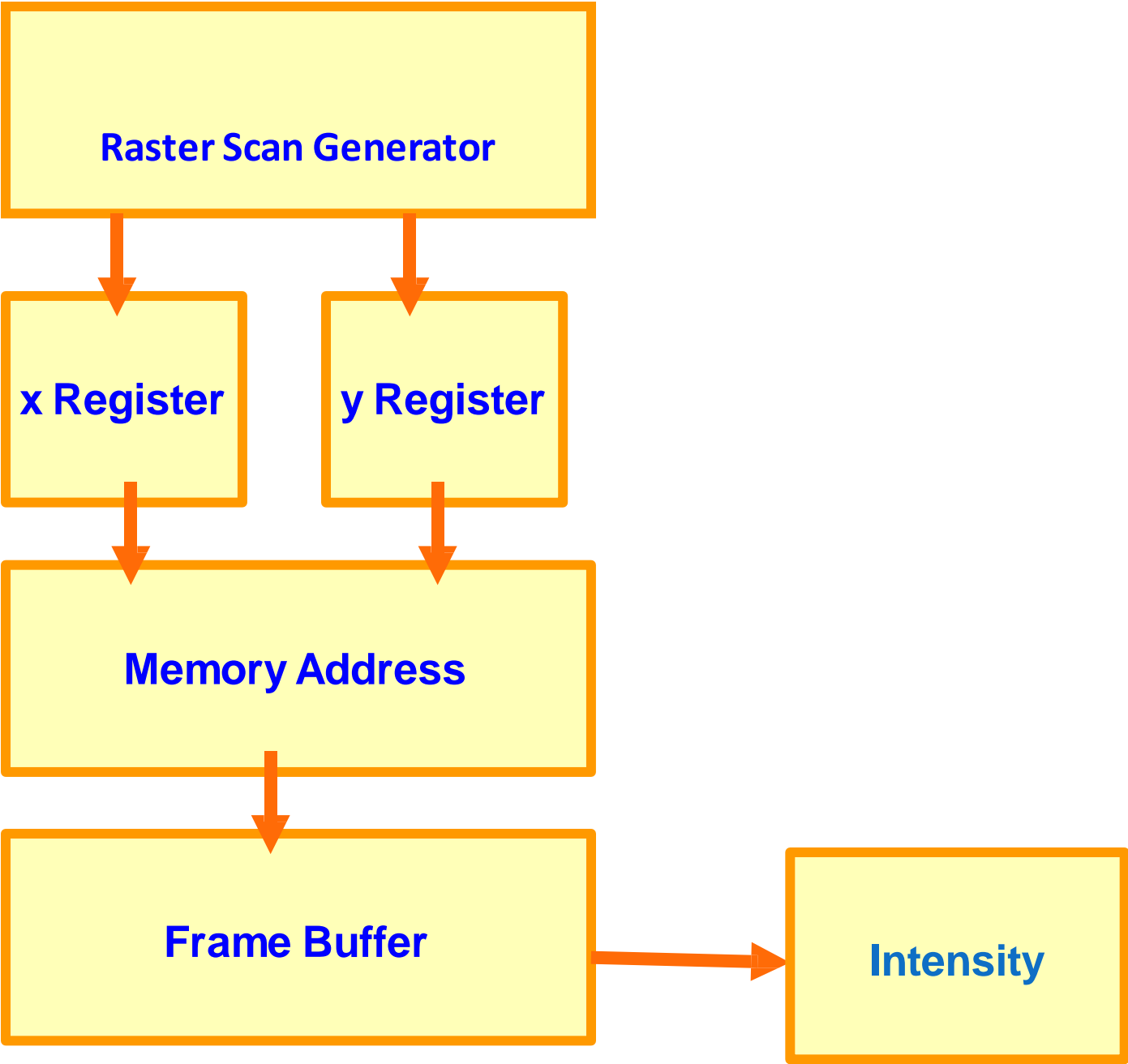
DISPLAY PROCESSOR

ɞThe purpose of the DP is to free the CPU   from the graphics chores.

ɞA major task of the display processor is

Scan Conversion.

ɞScan Conversion: is digitizing a picture definition given in an application program into a set of pixel intensity values for storage in the frame buffer.

I/O Devices

System Bus

Display Processor | CPU | System Memory

Frame Buffer → Video Controller → Monitor

Interface with host computer

(Display commands)   (Interaction data)

Display controller (DC) ← Keyboard
← Mouse

Video controller
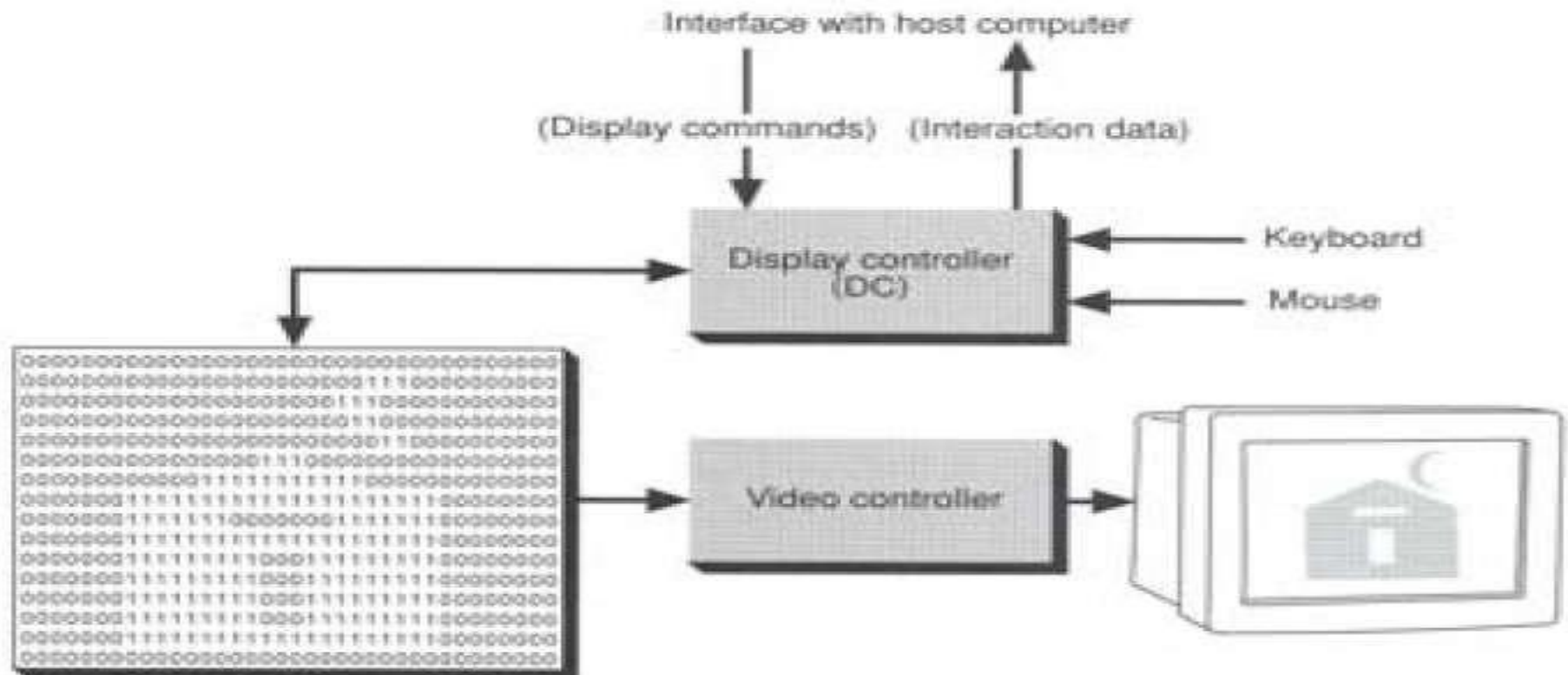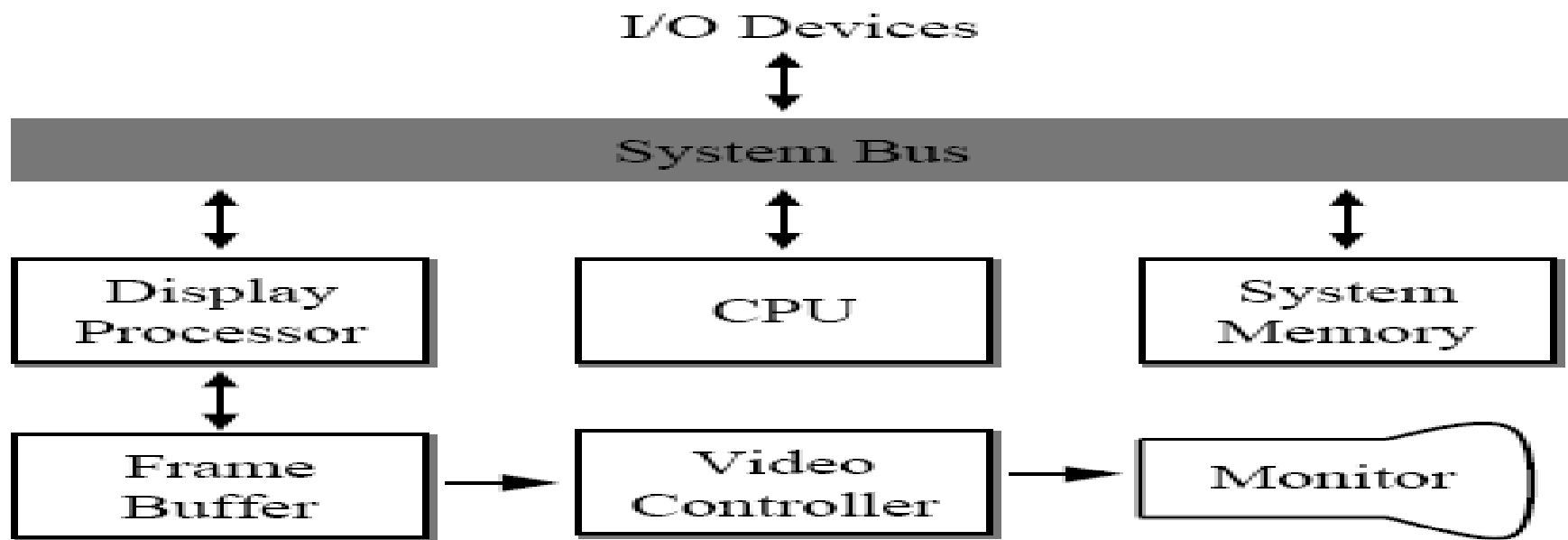
Random SCAN SYSTEM

ଛ Graphic commands are translated by the graphics package into a display file stored in the system memory.

ଛ This file is then accessed by the **display processor unit** (**DPU**)(graphic controller) to refresh the screen.

Random SCAN SYSTEM

HOST

Display List
Move 100, 100
Draw 200, 120
Text "cis681"
Move 30, 28
...

Draw 70, 20
Jump start

display commands

interaction

Display Controller (DC)

USER
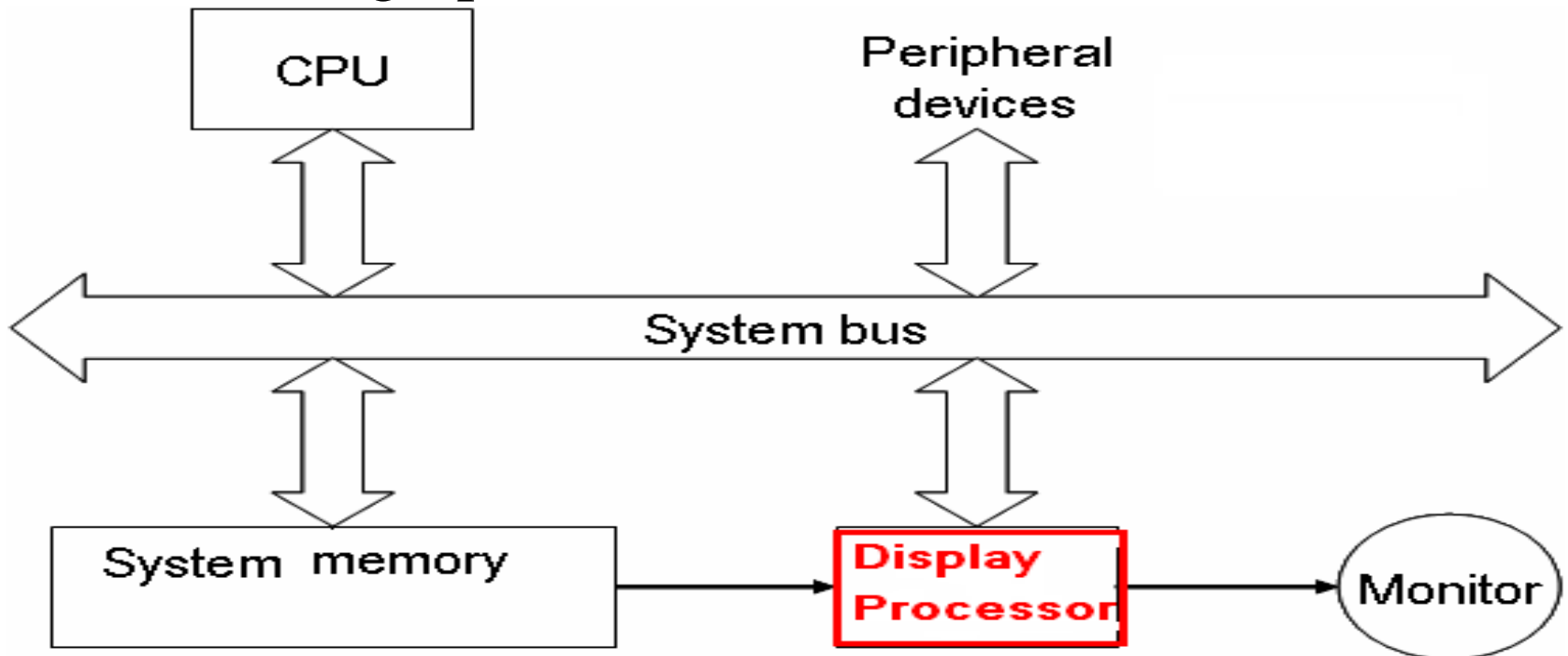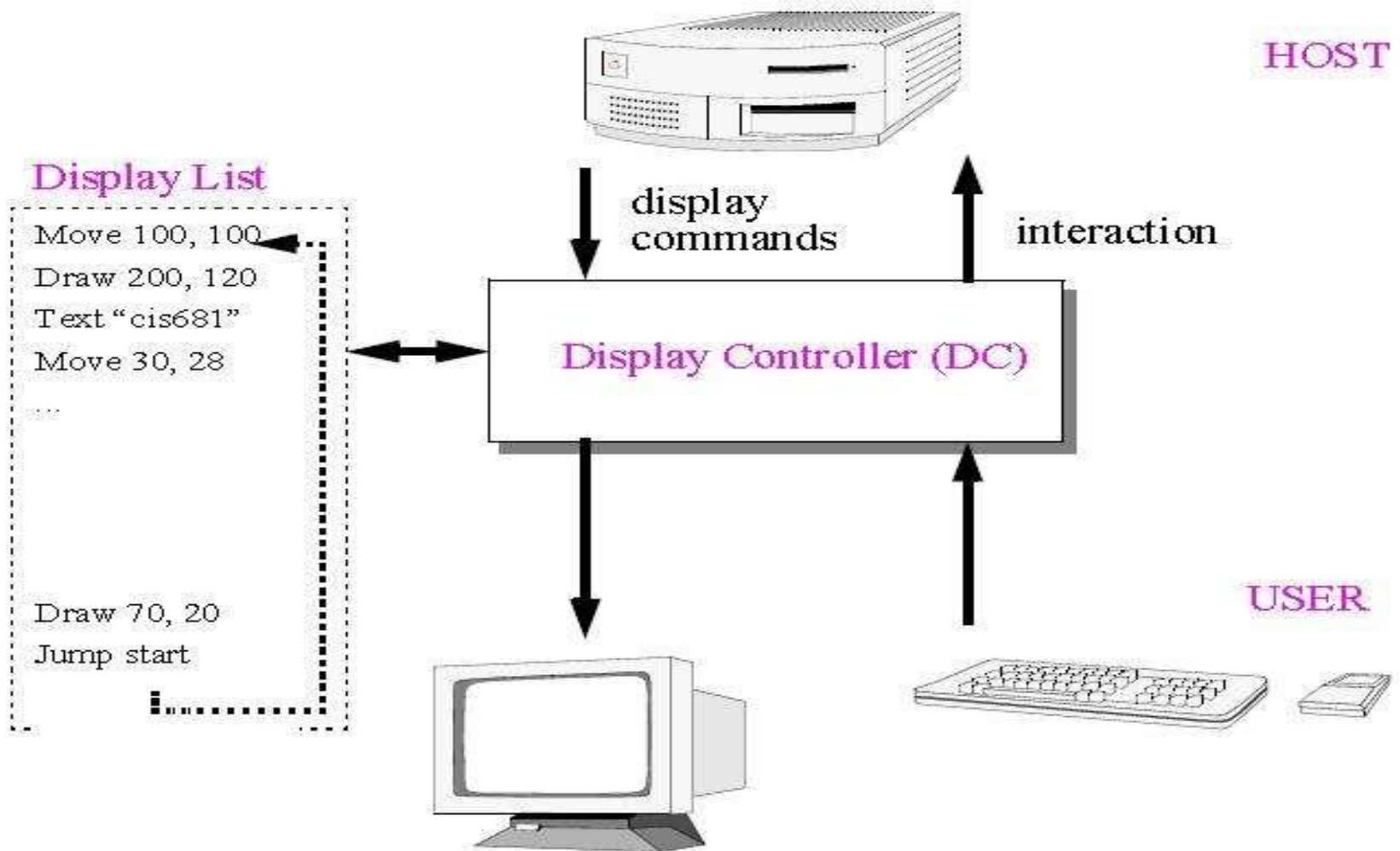
GRAPHICS WORKSTATIONS AND VIEWING SYSTEMS

- **Most graphics monitors today operate as raster-scan displays, and both CRT and flat-panel systems are in common use.**
- **Workstations range from small general-purpose computer systems to multi-monitor facilities, often with ultra-large viewing screens.**
- **For a personal computer, screen resolutions vary from about 640 by 480 to 1280 by 1024, and diagonal lengths measure from 12 inches to over 21 inches.**

An artist's workstation, featuring a monitor, a keyboard, a graphics tablet with a hand cursor, and a light table, in addition to data storage and telecommunications devices.

High-definition Graphics systems, with resolutions up to 2560 by 2048, are commonly used in medical imaging, air-traffic control, simulation, and CAD. A 2048 by 2048 flat-panel display

A high resolution (2048 by 2048) graphics monitor.

Many high-end graphics workstations also include large viewing screens, Often with specialized features.

The SGI Reality Center 2000D, featuring an ImmersaDesk R2 and displaying a large-screen stereoscopic view of pressure contours in a vascular blood-flow simulation superimposed on a volume-rendered anatomic data set.

A wide-screen view of a molecular system displayed on the three-channel SGI Reality Center 3300W.

- **Multi-panel display screens are used in a variety of applications that require "wall-sized" viewing areas.**
- **These systems are designed for presenting graphics displays at meetings, conferences, conventions, trade shows, retail stores, museums, and passenger terminals.**
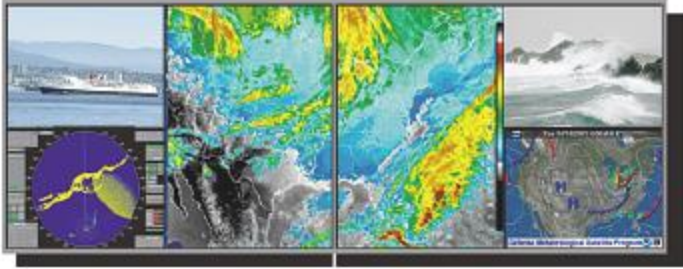
A multi-panel display system called the "Super Wall".

A large, curved-screen system can be useful for viewing by a group of people studying a particular graphics application

The 360^0 viewing screen in the NASA airport control-tower simulator, called the Future Flight Central Facility

GRAPHICAL INPUT DEVICES

WHAT IS AN INPUT DEVICE?

- An **input device** is any peripheral used to provide data and control signals to an information processing system.

LIST OF SOME INPUT DEVICES:

- **Mouse**
- **Keyboard**
- **Trackball**
- **Space ball**
- **Joystick**
- **Digitizer**
- **Dials**
- **Button boxes**

APPLICATION BASED INPUT DEVICES:

- **Data gloves**
- **Touch panels**
- **Image scanners**
- **Voice systems**

KEYBOARD

- **IT is a device primarily used to enter TEXT STRINGS.**

APLLICATIONS:

- **Used to enter Text Strings**

- **Short cuts to many Functions**

In Graphics:

- **Used to provide screen coordinates**

- **Menu selection**

- **Gaming controls**

- **And FOR entering many graphics function**

MOUSE

- **Hand-held BOX used to position the screen cursor**

- **Wheels or Rollers(now-a-days Laser lights) on the bottom are used to record the position of the screen**

- **Generally there are two or three buttons, used for operations like recording of the cursor positions or invoking of a function**

- **In order to increase the number of INPUT parameters, additional devices can be included**

- **The Z-MOUSE is an example of this**

Z-MOUSE

KEY FEATURES

▪**Has three buttons, a thumbwheel on the side, a trackball on the top and a standard mouse ball underneath**

▪**This provides SIX degrees of freedom to select the positions, rotations, etc.**

▪**Allows 3D viewing**

Applications:

•**Animation**

•**Auto CAD**

TRACKBALL AND SPACEBALL

TRACKBALL:
- It is a 2D positioning device.
- It consists a ball held by a socket containing sensors to detect the rotation of ball about TWO axis.
- User rolls the ball to move the cursor.
- They are often mounted on devices such as keyboards, Z-mouse, etc.

SPACEBALLS
- It provides SIX degrees of freedom.
- It is a fix device
- Movement detection is done using strain gauges.
- Cursor can move in any direction.
- It is more efficient then trackball.

APPLICATIONS

- **Used in CAD workstations.**
- **In animation.**
- **Sometimes on special Workstations such as the radar consoles in air-traffic control room.**
- **In Gaming consoles**
- **People with a mobility impairment use trackballs as an assistive technology input.**

JOYSTICK

- **Consists of a stick pivoted on a base.**
- **Used to steer the screen cursor.**
- **It also has one or two PUSH buttons as input switches to perform certain actions.**
- **Most joystick are 2D, but 3D do exist.**
- **Distance moved from the CENTER position corresponds to the screen cursor movement in that direction.**

APPLICATIONS

- **In Gaming consoles.**
- **In 3D animation.**
- **Used to drive machines like cranes, mining trucks, hydraulics, etc.**
- **Used as assistive technology pointing device such as in Electronic wheelchairs.**

DATA GLOVE

From Computer Desktop Encyclopedia
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Virtual Technologies, Inc.
Palo Alto, CA

DATA GLOVE

- **Used to grasp a "virtual" object.**
- **Uses sensors to detect the Hand and finger motion.**
- **Electromagnetic coupling between signals provides information about the position and orientation of the HAND.**
- **The system interfaces with the computer via a USB cable.**
- **Right- and left-handed models are available.**

APPLICATIONS

- **In 3D animation movies.**
- **Visual effects.**
- **Gestures can be categorized into useful information, such as to recognize Sign Language or other symbolic functions.**
- **3D Virtual environment Games.**

DIGITIZERS

- **Common device for drawing, painting, or interactively selecting coordinate positions on an object**

- is a computer input device that enables a user to hand-draw images, animations and graphics, with a special pen-like stylus,

- Software converts the analog touch input to lines or pressure-sensitive brush strokes in a document.

GRAPHICS TABLET

Wacom Cintiq Graphics Tablet

ComputerHope.com

• A **graphics tablet** is a computer input device that allows one to hand- draw images and graphics, similar to the way one draws images with a pencil and paper. These tablets may also be used to capture data or handwritten signatures.

• The common drawing TOOLS used to draw are **HAND CURSOR** and **STYLUS.**

• A **STYLUS** is a pencil -shaped pointing device.

- This Digitizing system uses electromagnetic resonance to detect the 3D positions of the STYLUS.

- This allows one to produce different shades of brush strokes with different pressure on its surface.

- Acoustic Tablets use sound waves to detect the position of the STYLUS.

APPLICATIONS

- **Used in generating Computer generated graphic images.**

- **Used in creating characters for Animation.**

- **In Technical drawings and CAD.**

- **Used for Handwriting recognition.**

IMAGE SCANNERS

- In computing, a **scanner** is a device that optically scans images, printed text, handwriting, or an object, and converts it to a digital image.

- When the scanning is performed, the gradation of gray scale or colors are recorded and stored in an array.

- Once scanned, any kind of transformations can be applied to the object image.

LIGHT PENS

- A **light pen** is a computer input device in the form of a light-sensitive wand used in conjunction with a computer's CRT TV set or monitor

- Allows the users to point to displayed objects and to draw objects on screen

- The position points are highly accurate and sensitive

- It generates electrical pulse which records the position of the electron beam

- Not very much popular

GRAPHICS NETWORKS

- Multiuser environments and computer networks are now common elements in many graphics applications.

- Various resources, such as processors, printers, plotters, and data files, can be distributed on a network and shared by multiple users.

- A graphics monitor on a network is generally referred to as a **graphics server**, or simply a **server.**

- The monitor includes standard input devices such as a keyboard and a mouse or trackball.

- In that case, the system can provide input, as well as being an output server.

- The computer on the network that is executing a graphics application program is called the *client*, and the output of the program is displayed on a **server.**

- When operating on a network, a client computer transmits the instructions for displaying a picture to the monitor (server).

- This is accomplished by collecting the instructions into packets before transmission, instead of sending the individual graphics instructions one at a time over the network.

GRAPHICS ON THE INTERNET

- A great deal of graphics development is now done on the **Internet**, which is a global network of computer networks.

- **World Wide Web** provides a hypertext system that allows users to locate and view documents that can contain text, graphics, and audio.

- Resources, such as graphics files, are identified by a **uniform resource locator** (URL).

Each URL Contains 2 parts:

1. the protocol for transferring the document.
2. the server that contains the document and, optionally, the location (directory) on the server.

- For example, the URL http://www.siggraph.org indicates a document that is to be transferred with the **h**yper**t**ext **t**ransfer **p**rotocol (http) and that the server is www.siggraph.org, which is the home page of the Special Interest Group in Graphics (SIGGRAPH) of the Association for Computing Machinery.

- Documents on the Internet can be constructed with the **H**ypertext **M**arkup **L**anguage (HTML).

- HTML provides a simple method for describing a document containing text, graphics, and references (hyperlinks) to other documents.

GRAPHICS SOFTWARE

There are two broad classifications for computer-graphics software:

1. Special purpose packages.

2. general programming packages.

Special-purpose packages are designed for nonprogrammers who want to generate pictures, graphs, or charts in some application area without worrying about the graphics procedures that might be needed to produce such displays.

- By contrast, a general programming package provides a library of graphics functions that can be used in a programming language such as C, C++, Java, or Fortran.

- A set of graphics functions is often called a computer-graphics application programming interface (CG API), because the library provides a software interface between a programming language (such as C++) and the hardware.

Coordinate Representations

- To generate a picture using a programming package, we first need to give the geometric descriptions of the objects that are to be displayed.

- These descriptions determine the locations and shapes of the objects.

Ex: a box is specified by the positions of its corners (vertices), and a sphere is defined by its center position and radius.

- With few exceptions, general graphics packages require geometric descriptions to be specified in a standard, right-handed, Cartesian-coordinate reference frame.

- If coordinate values for a picture are given in some other reference frame they must be converted to Cartesian coordinates before they can be input to the graphics package.

- In general, several different Cartesian reference frames are used in the process of constructing and displaying a scene.

1. First, we can define the shapes of individual objects, such as trees or furniture, within a separate coordinate reference frame for each object - *are called* ***modeling coordinates***, *or sometimes* ***local coordinates*** *or* ***master coordinates.***

2. Once the individual object shapes have been specified, we can construct ("model") a scene by placing the objects into appropriate locations within a scene reference frame called ***world coordinates***- *involves the transformation of the individual modeling-coordinate frames to specified positions and orientations within the world-coordinate frame.*

Geometric descriptions in modeling coordinates and world coordinates can be given in any convenient floating-point or integer values, without regard for the constraints of a particular output device.
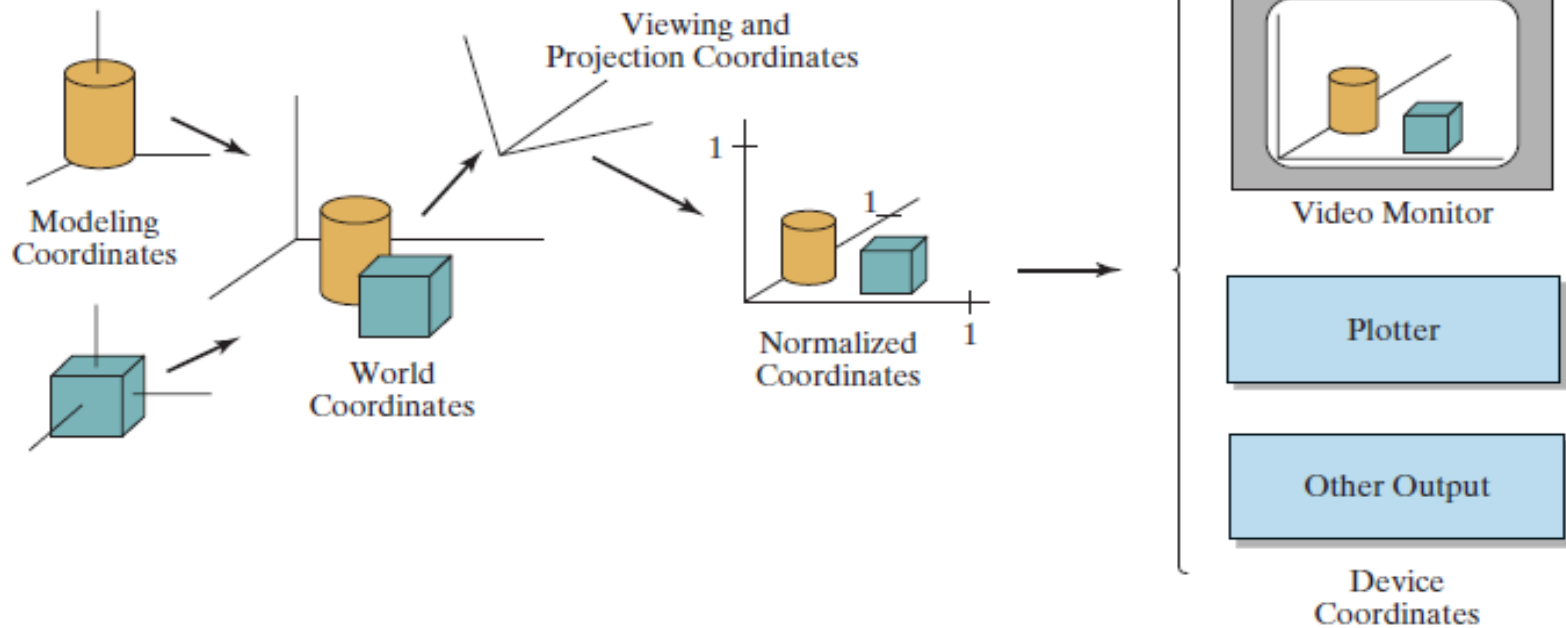
- After all parts of a scene have been specified, the overall world-coordinate description is processed through various routines onto one or more output-device reference frames for *display - is called the* **viewing pipeline***.*

3. World coordinate positions are first converted to viewing coordinates corresponding to the view we want of a scene, based on the position and orientation of a hypothetical camera.

Then object locations are transformed to a two-dimensional projection of the scene, which corresponds to what we will see on the output device.

4. The scene is then stored in normalized coordinates, where each coordinate value is in the range from −1 to 1 or in the range from 0 to 1, depending on the system- *are also referred to as* *normalized device coordinates*.

5. Finally, the picture is scan converted into the refresh buffer of a raster system for display. The coordinate systems for display devices are generally called device coordinates, or screen coordinates in the case of a video monitor.

Modeling Coordinates

World Coordinates

Viewing and Projection Coordinates

Normalized Coordinates

Video Monitor

Plotter

Other Output

Device Coordinates

$$(x_{mc}, y_{mc}, z_{mc}) \rightarrow (x_{wc}, y_{wc}, z_{wc}) \rightarrow (x_{vc}, y_{vc}, z_{vc}) \rightarrow (x_{pc}, y_{pc}, z_{pc})$$

$$\rightarrow (x_{nc}, y_{nc}, z_{nc}) \rightarrow (x_{dc}, y_{dc})$$

Bresenham Circle Drawing Algorithm

Explanation :

- In this algorithm, we sample at unit interval and determine the nearest pixel on circle path each step.

- In this first quadrant of circle from X = 0 to X = Y, the slope of the curve varies from 0 to -1.

- So, we take unit step in positive X-direction over this octant and using a decision parameter we can determine which of the two possible Y – position is closer to the circle path at each step.

- And as above step we find the Y – position in other seven octant.

Bresenham's circle algorithm
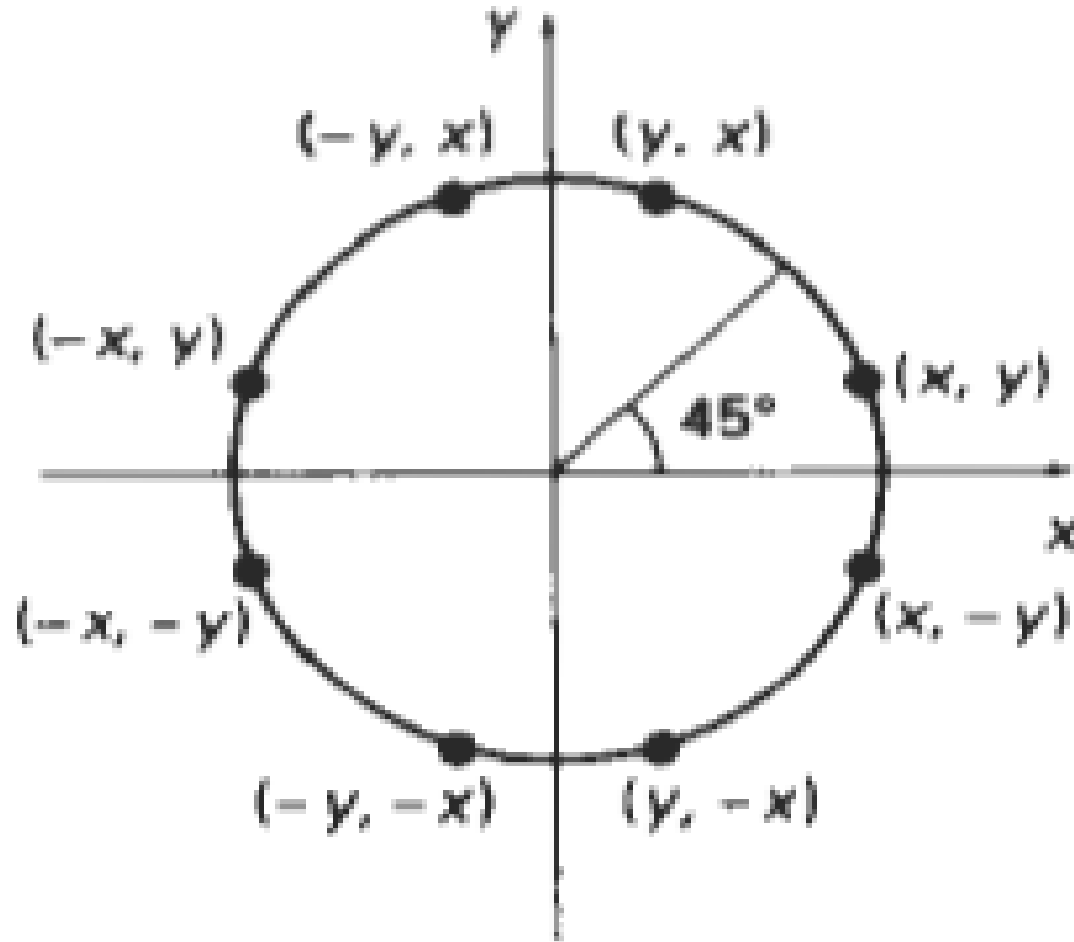
Bresenham Circle Drawing Algorithm

- Circle function is,

$$f_{circle}(X, Y) = X^2 + Y^2 - r^2$$

- Any point (X, Y) on the boundary of the circle with radius r satisfies the equation $f_{circle}(X, Y) = 0$

- In general the relative position of any point (X, Y) can be determined by sign of the circle function

$$f_{circle}(X, Y) \begin{cases} < 0, & (x, y) \text{ inside circle} \\ = 0, & (x, y) \text{ on the circle} \\ > 0. & (x, y) \text{ outside circle} \end{cases}$$

$M_1 (X_k, Y_k)$ $M_3 (X_k + 1, Y_k)$

d_1

d_2

$M_2 (X_k + 1, Y_k - 1)$

Y_k

$Y_k - 1$

r

X_k $X_k + 1$
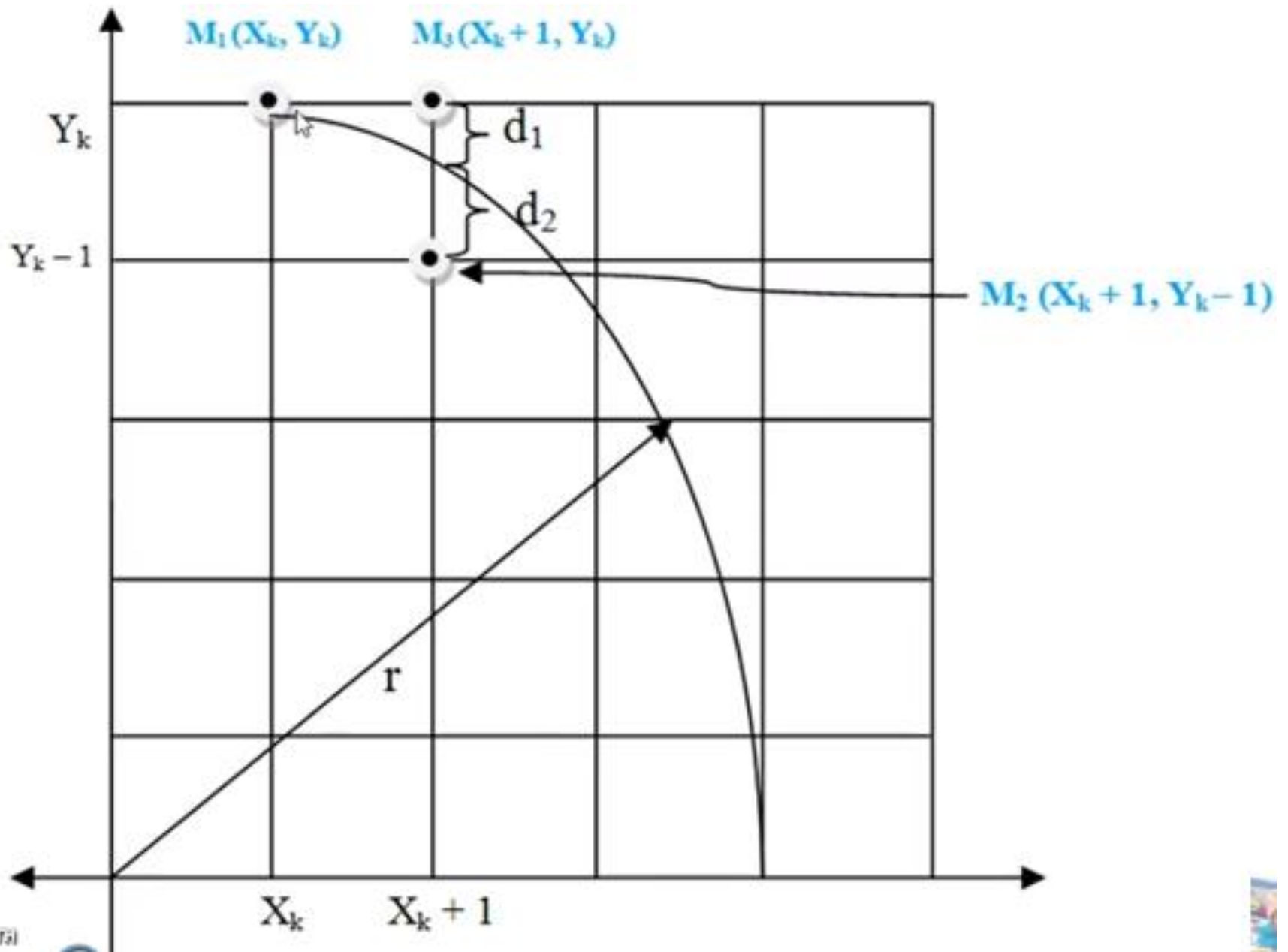
- After point M_1, do we choose M_2 or M_3 ?

    d_1 = distance of M_3 from circle

    d_2 = distance of M_2 from circle

$$\therefore d_1 = (X_k + 1)^2 + Y_k^2 - r^2 \qquad \text{[always +ve]}$$

$$\therefore d_2 = (X_k + 1)^2 + (Y_k - 1)^2 - r^2 \qquad \text{[always -ve]}$$

- Decision parameter $P_k = d_1 + d_2$

$$P_k = (X_k + 1)^2 + Y_k^2 - r^2 + (X_k + 1)^2 + (Y_k - 1)^2 - r^2$$

$$= 2(X_k + 1)^2 + Y_k^2 + (Y_k - 1)^2 - 2r^2 \quad \text{---------(1)}$$

- So, if $P_k < 0$ then circle is closer to M_3(above point),

    $(X_{k+1}, Y_k) = (X_k + 1, Y_k)$

- Otherwise, $P_k \geq 0$ then circle is closer to M_2(below point),

    $(X_{k+1}, Y_k) = (X_k + 1, Y_k - 1)$

- To find next decision parameter, P_{k+1}

$$P_{k+1} = 2(X_{k+1} + 1)^2 + Y_{k+1}^2 + (Y_{k+1} - 1)^2 - 2r^2 \quad \text{-----(2)}$$

- Difference between equation (2) – (1)

$$\begin{aligned}
P_{k+1} - P_k &= 2(X_{k+1} + 1)^2 + Y_{k+1}^2 + (Y_{k+1} - 1)^2 - 2r^2 \\
&\quad - 2(X_k + 1)^2 - Y_k^2 - (Y_k - 1)^2 + 2r^2
\end{aligned}$$

$$\begin{aligned}
&= 2(X_k + 1 + 1)^2 + Y_{k+1}^2 + (Y_{k+1} - 1)^2 \\
&\quad - 2(X_k + 1)^2 - Y_k^2 - (Y_k - 1)^2 \qquad \text{(where } X_{k+1} = X_k + 1\text{)}
\end{aligned}$$

$$\begin{aligned}
&= 2(X_k + 2)^2 + Y_{k+1}^2 + (Y_{k+1} - 1)^2 \\
&\quad - 2(X_k + 1)^2 - Y_k^2 - (Y_k - 1)^2
\end{aligned}$$

$$\begin{aligned}
&= 2(X_k^2 + 4X_k + 4) + Y_{k+1}^2 + Y_{k+1}^2 - 2Y_{k+1} + 1 \\
&\quad - 2(X_k^2 + 2X_k + 1) - Y_k^2 - Y_k^2 + 2Y_k - 1
\end{aligned}$$

$$\begin{aligned}
&= 2X_k^2 + 8X_k + 8 + 2Y_{k+1}^2 - 2Y_{k+1} \\
&\quad - 2X_k^2 - 4X_k - 2 - 2Y_k^2 + 2Y_k
\end{aligned}$$

$$= 4X_k + 6 + 2Y_{k+1}^2 - 2Y_{k+1} - 2Y_k^2 + 2Y_k$$

$$P_{k+1} = P_k + 4X_k + 2(Y_{k+1}^2 - Y_k^2) - 2(Y_{k+1} - Y_k) + 6$$

- If $P_k < 0$ (M_3 is select) then $Y_{k+1} = Y_k$

$$P_{k+1} = P_k + 4X_k + 2(Y_k^2 - Y_k^2) - 2(Y_k - Y_k) + 6$$

$$P_{k+1} = P_k + 4X_k + 6$$

- If $P_k \geq 0$ (M_2 is select) then $Y_{k+1} = Y_k - 1$

$$P_{k+1} = P_k + 4X_k + 2[(Y_k - 1)^2 - Y_k^2)] - 2[(Y_k - 1) - Y_k)] + 6$$
$$= P_k + 4X_k + 2[Y_k^2 - 2Y_k + 1 - Y_k^2] - 2[Y_k - 1 - Y_k] + 6$$
$$= P_k + 4X_k - 4Y_k + 2 + 2 + 6$$
$$= P_k + 4X_k - 4Y_k + 10$$

$$P_{k+1} = P_k + 4(X_k - Y_k) + 10$$

- Here initial decision parameter P_0,

$$P_0 = d_1 + d_2$$

$$= [1^2 + r^2 - r^2] + [1^2 + (r - 1)^2 - r^2]$$

$$= 1 + 1 + r^2 - 2r + 1 - r^2$$

$$P_0 = 3 - 2r$$

Algorithm:

Step-1: Input radius r and circle centre (X_c, Y_c) obtained the first point $(X_0, Y_0) = (0, r)$

Step-2: Calculate the initial value of decision parameter as

$$P_0 = 3 - 2r$$

Step-3: At each X_k position, starting at k = 0, perform, the following test:

If $P_k < 0$, the next point is $(X_k + 1, Y_k)$

$$P_{k+1} = P_k + 4X_k + 6$$

otherwise the next point is $(X_k + 1, Y_k - 1)$

$$P_{k+1} = P_k + 4(X_k - Y_k) + 10$$

Step-4: Determine the symmetry points in other seven octant

Step-5: Move each pixel position (X, Y) into circular path

$$X = X + X_c \quad \text{and} \quad Y = Y + Y_c$$

Step-6: Repeat step 3 to 5 until $X \geq Y$.