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MATDIP301

Third Semester B.E. Degree Examination, June 2012

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Express $z = \dfrac{2-\sqrt{3}i}{1+i}$ in the form a + ib. **(06 Marks)**

b. Find modulus and amplitude of $z = \dfrac{3+i}{2+i}$. **(07 Marks)**

c. Find all the values of $z = \left(\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right)^{3/4}$. **(07 Marks)**

2 a. Find the nth derivative of $y = e^{ax}\cos(bx+c)$. **(06 Marks)**

b. If $y = \sin(m\sin^{-1}x)$ prove that $(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} + (m^2-n^2)y_n = 0$. **(07 Marks)**

c. Expand $y = \log(1+x)$ in Maclaurins series upto 5th term. **(07 Marks)**

3 a. If $u = \dfrac{x^2y^2}{x+y}$, find the value of $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$. **(06 Marks)**

b. If $u = 3x^2 + y^2$ and $x^2 - y^2 = 1$, find $\dfrac{du}{dx}$. **(07 Marks)**

c. If $x = r\cos\phi$, $y = r\sin\phi$, z = z, find $\dfrac{\partial(x, y, z)}{\partial(r, \phi, z)}$. **(07 Marks)**

4 a. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$ and hence obtain $\int_0^{\pi/2} \sin^4 x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^1 x^2(1-x^2)^{7/2}dx$. **(07 Marks)**

c. Evaluate $\int_0^1\int_0^3 x^3y^3 dx\,dy$. **(07 Marks)**

5 a. Evaluate $\int_0^1\int_0^2\int_0^3 (x+y+z)dz\,dy\,dx$. **(06 Marks)**

b. Evaluate $\int_0^\infty x^2e^{-4x}\,dx$ using gamma function. **(07 Marks)**

c. Find $\beta\left(\dfrac{5}{2}, \dfrac{3}{2}\right)$ in terms of gamma function.. **(07 Marks)**

6 a. Solve the equation $\sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = 0$. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{x-y}{x+y}$. **(07 Marks)**

c. Solve $\frac{dy}{dx} = (x+y)^2$. **(07 Marks)**

7 a. Solve $\frac{dy}{dx} = \frac{\sin 2x - \tan y}{x \sec^2 y}$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + x^2 y = x^2$. **(07 Marks)**

c. Solve $\frac{dy}{dx} + \sin xy = \sin x \cos x$. **(07 Marks)**

8 a. Solve $(D^2 + a^2)y = x^2$. **(06 Marks)**

b. Solve $(D^3 + D^2 - D - 1)y = e^{2x}$. **(07 Marks)**

c. Solve $(D^4 - 1)y = \sin x + 2$. **(07 Marks)**

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