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**Third Semester B.E. Degree Examination, June 2012**  
**Engineering Mathematics – III**

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting  
at least TWO questions from each part.**

**PART – A**

- 1 a. Find the Fourier series of the function,  

$$f(x) = \begin{cases} 2-x, & 0 \leq x \leq 4 \\ x-6, & 4 \leq x \leq 8 \end{cases}$$
. Hence deduce that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$  (07 Marks)
- b. Find the half range cosine series for,  $f(x) = x(\pi - x)$  in  $0 < x < \pi$ . (06 Marks)
- c. Analyse harmonically the data given below and express 'y' in Fourier series upto the second harmonics: (07 Marks)

$x^\circ$	0	60	120	180	240	300	360
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

- 2 a. Find the complex Fourier transform of  $f(x)$  where  

$$f(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$$
. Hence find the value of  $\int_0^\infty \frac{x \cos x - \sin x}{x^3} dx$ . (07 Marks)
- b. Find the Fourier cosine transform of the function,  

$$f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4-x, & 1 < x < 4 \\ 0, & x > 4 \end{cases}$$
 (06 Marks)
- c. Find the complex Fourier transform of  $e^{-a^2 x^2}$ ,  $a > 0$ . Hence deduce that  $e^{-x^2/2}$  is self reciprocal under the complex Fourier transform. (07 Marks)
- 3 a. Find the general solution of,  $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ . (07 Marks)
- b. Form the partial differential equation by eliminating the arbitrary functions from,  
 $z = f(x+ct) + g(x-ct)$ . (06 Marks)
- c. Solve  $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$  by the method of separation of variables. (07 Marks)
- 4 a. Derive one-dimensional heat equation in standard form. (07 Marks)
- b. Solve the Laplace equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  for its various possible solutions, by the method of separation of variables. (06 Marks)
- c. A string is stretched tightly between two points at a distance 'l' apart. The motion of the string is started by displacing the string into the form  $u = u_0 \sin\left(\frac{\pi x}{l}\right)$ , from which it is released from rest. Find the displacement  $u(x, t)$  of any point at a distance x from one end at any time t. (07 Marks)

**PART – B**

- 5 a. Using Regula-Falsi method, find a root of  $x^6 - x^4 - x^3 - 1 = 0$  in  $(1, 2)$ , correct to four decimal places. Carryout three iterations. (07 Marks)
- b. Apply Gauss-Seidel iterative method, to solve  $x + 2y + 5z = 20$ ;  $5x + 2y + z = 12$ ;  $x + 4y + 2z = 15$ . (06 Marks)
- c. Using power method, find the largest eigen value and the corresponding eigen vector of the matrix  $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ , starting with the initial eigen vector  $(1 \ 0 \ 0)^T$ . (07 Marks)

- 6 a. Find the values of  $f(38)$  and  $f(85)$  using suitable interpolation formulae, given (07 Marks)

x:	40	50	60	70	80	90
y=f(x):	184	204	226	250	276	304

- b. Evaluate  $\int_0^1 \sqrt{\sin x + \cos x} dx$  correct to two decimal places using Simpson's  $\left(\frac{1}{3}\right)^{\text{rd}}$  rule taking seven equidistant ordinates. (06 Marks)
- c. Fit an interpolating polynomial for the data:

x:	0	1	4	8	10
y = f(x):	-5	-14	-125	-21	355

using Newton's general interpolation formula. Hence find  $f(2)$ . (07 Marks)

- 7 a. Obtain Euler's equation for the variational problem in the form:

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0.$$

Modify this equation when  $f$  is independent of  $y$ . (07 Marks)

- b. Define a geodesic on a surface. Prove that the geodesics on a plane are straight lines. (06 Marks)

- c. Solve the variational problem  $\delta \int_0^{\frac{\pi}{2}} (y^2 - y'^2) dx$  under the conditions  $y(0)=0$ ,  $y\left(\frac{\pi}{2}\right) = 2$ . (07 Marks)

- 8 a. Find the z-transforms of, i)  $\sin n\theta$  ii)  $\cos n\theta$ . (07 Marks)

- b. Find the inverse z-transform of,  $\frac{2z^2 + 3z}{(z+2)(z-4)}$ . (06 Marks)

- c. Using z-transforms, solve  $y_{n+2} - 5y_{n+1} + 6y_n = 1$  with  $y_0 = 0$  and  $y_1 = 1$ . (07 Marks)

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