USN

10CS34

Third Semester B.E. Degree Examination, June 2012

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Let S = {21, 22, 23, ….., 39, 40}. Determine the number of subsets A of S such that :
  - i) $|A| = 5$
  - ii) $|A| = 5$ and the largest element in A is 30
  - iii) $|A| = 5$ and the largest element is at least 30
  - iv) $|A| = 5$ and the largest element is at most 30
  - v) $|A| = 5$ and A consists only of odd integers. **(10 Marks)**

b. Prove or disprove: For non-empty sets A and B, $P(A \cup B) = P(A) \cup P(B)$ where P denotes power set. **(05 Marks)**

c. In a group of 30 people, it was found that 15 people like Rasagulla, 17 like Mysorepak, 15 like Champakali, 8 like Rasagulla and Mysorepak, 11 like Mysorepak and Champakali, 8 like Champakali and Rasagulla and 5 like all three. If a person is chosen from this group, what is the probability that the person will like exactly 2 sweets? **(05 Marks)**

2 a. Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology. **(05 Marks)**

b. Write dual, negation, converse, inverse and contrapositive of the statement given below :
  If Kabir wears brown pant, then he will wear white shirt. **(05 Marks)**

c. Define $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$. Represent $p \vee q$ and $p \to q$ using only $\uparrow$. **(05 Marks)**

d. Establish the validity or provide a counter example to show the invalidity of the following arguments : **(05 Marks)**

i)
$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \neg r \\ \hline \therefore q \end{array}$$

ii)
$$\begin{array}{l} p \\ p \to r \\ p \to (q \vee \neg r) \\ \neg q \vee \neg s \\ \hline \therefore s \end{array}$$

3 a. For the universe of all polygons with three or four sides, define the following open statements:

  i(x): all the interior angles of x are equal
  h(x): all sides of x are equal
  s(x): x is a square
  t(x): x is a triangle

Translate each of the following statements into an English sentence and determine its truth value:

  - i) $\forall x\, [s(x) \leftrightarrow (i(x) \wedge h(x))]$
  - ii) $\exists x\, [t(x) \to (i(x) \leftrightarrow h(x))]$

Write the following statements symbolically and determine their truth values.

  - iii) Any polygon with three or four sides is either a triangle or a square
  - iv) For any triangle if all the interior angles are not equal, then all its sides are not equal. **(08 Marks)**

3 b. Let p(x, y) denote the open statement x divides where the universe consists of all integers. Determine the truth values of the following statements. Justify your answers.
i) $\forall x\ \forall y\ [p(x, y) \wedge p(y, x) \rightarrow (x = y)]$ ii) $\forall x\ \forall y\ [p(x, y) \vee p(y, x)]$ **(06 Marks)**

c. Prove that for every integer n, n^2 is even if and only if n is even. **(06 Marks)**

4 a. Prove $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \quad \forall n \in z^+$. **(06 Marks)**

b. Prove $2^n < n!\ \ \forall\ n > 3$ and $n \in z^+$. **(06 Marks)**

c. Define an integer sequence recursively by

$a_0 = a_1 = a_2 = 1$

$a_n = a_{n-1} + a_{n-3}\ \forall\ n \geq 3$.

Prove that $a_{n+2} \geq (\sqrt{2})^n \quad \forall\ n \geq 0$. **(08 Marks)**

PART – B

5 Let A = {α, β, γ}, B = {θ, η}, C = {λ, μ, ν}.

a. Find $(A\cup B)\times C$, $A\cup(B\times C)$, $(A\times B)\cup C$ and $A\times(B\cup C)$. **(08 Marks)**

b. Give an example of a relation from A to B × B which is not a function. **(04 Marks)**

c. How many onto functions are there from (i) A to B, (ii) B to A? **(02 Marks)**

d. i) Write a function $f : A\rightarrow C$ and a function $g : C\rightarrow A$. Find $g \circ f : A\rightarrow A$.
ii) Write an invertible function $f : A\rightarrow C$ and find its inverse. **(06 Marks)**

6 a. Let A = {1, 2, 3, 4}, B = {w, x, y, z} and C = {p, q, r, s}. Consider R_1 = {(1, x), (2, w), (3, z)} a relation from A to B, R_2 = {(w, p), (z, q), (y, s), (x, p)} a relation from B to C.
i) What is the composite relation $R_1 \circ R_2$ form A to C?
ii) Write relation matrices $M(R_1)$, $M(R_2)$ and $M(R1 \circ R2)$
iii) Verify $M(R_1)\cdot M(R_2) = M(R_1 \circ R_2)$ **(06 Marks)**

b. Let A = {1, 2, 3, 6, 9, 12, 18} and define a relation R on A as xRy iff x|y. Draw the Hasse diagram for the poset (A, R). **(06 Marks)**

c. Let A = {1, 2, 3, 4, 5} × {1, 2, 3, 4, 5} and define R as $(x_1, y_1)R(x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$.
i) Verify that R is an equivalence relation on A.
ii) Determine the equivalence class [(1, 3)].
iii) Determine the partition induced by R. **(08 Marks)**

7 a. Define a binary operation ∗ on Z as $x * y = x + y - 1$. Verify that (Z, ∗) is an abelian group. **(07 Marks)**

b. Let $f : G \rightarrow H$ be a group homomorphism onto H. If G is an abelian group, prove that H is also abelian. **(07 Marks)**

c. The encoding function $E : Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$.
i) Determine all the code words.
ii) Find the associated parity-check matrix H. **(06 Marks)**

8 a. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R)$, prove that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a unit of this ring if and only if $ad - bc \neq 0$. **(08 Marks)**

b. Let R be a ring with unity and a, b be units in R. Prove that ab is a unit of R and that $(ab)^{-1} = b^{-1}a^{-1}$. **(06 Marks)**

c. Find multiplicative inverse of each (non-zero) element of Z_7. **(06 Marks)**

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