USN | | | | | | | | | | **06MAT21**

Second Semester B.E. Degree Examination, June 2012
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose your answers for the following : **(04 Marks)**

i) The radius of curvature at a point (r, θ) of $r = ae^{\theta\cot\alpha}$ is
A) $r\mathrm{cosec}\alpha$ B) $\mathrm{cosec}\alpha$ C) $\cot\alpha$ D) none of these

ii) The radius of the circle of curvature is
A) 1 B) $\frac{1}{\rho}$ C) ρ D) ρ^2

iii) The value of C of the Lagrange's mean value theorem for $f(x) = \tan^{-1}x$ in [0, 1] is
A) 0.125 B) 0.523 C) $\pi/4$ D) $\pi/2$

iv) Maclaurin's series expansion of sinx is
A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} ....$ B) $1 - \frac{x^2}{2!} + \frac{x^3}{3!} + ...$
C) $x + \frac{x^3}{3!} + ...$ D) $1 + x + \frac{x^2}{2} + ...$

b. Find the radius of curvature for the curve $y = a \log \sec (x/a)$ at any point (x, y). **(04 Marks)**

c. State and prove Lagrange's mean value theorem. **(06 Marks)**

d. Expand $e^{\sin x}$, using Maclaurin's series upto the term containing x^4. **(06 Marks)**

2 a. Choose your answers for the following : **(04 Marks)**

i) Lim $\sec\left(\frac{\pi}{2x}\right)$ logx is equal to
A) $\pi/2$ B) $2/\pi$ C) π D) $\pi/3$

ii) The basic fundamental indeterminate forms are
A) $\frac{0}{0}$ B) $\frac{\infty}{\infty}$ C) both A and B D) none of these

iii) Find the critical point of the function $f(x, y) = \sin x + \sin y + \sin(x + y)$ is
A) (1, 1) B) $(\pi/3, \pi/3)$ C) $(\pi/2, \pi/2)$ D) none of these

iv) In a plane triangle ABC, the maximum value of COSA · COSB · COSC is
A) 3/4 B) 3/8 C) 1/8 D) 5/8

b. Evaluate $\lim\limits_{x \to \pi/2} (2x\tan x - \pi\sec x)$. **(04 Marks)**

c. Expand $e^{ax} \sin by$ in powers of x and y as far as terms of 3rd degree. **(06 Marks)**

d. Show that the maximum value of $xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$ is $3a^2$. **(06 Marks)**

3 a. Choose your answers for the following : **(04 Marks)**

i) $\int_0^2\int_0^x (x+y)\,dxdy$ is equal to

A) 3 B) 4 C) 5 D) none of these

ii) The volume of the cylinder with base radius 'a' and height 'h' is

A) r^2h B) $\frac{2}{3}rh$ C) πr^2h D) none of these

iii) The value of $\beta(m, n)$ is

A) $\int_0^\infty x^{m-1}(1-x)^{n-1}dx$ B) $\int_0^1 x^{m-1}(1-x)^x dx$

C) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ D) none of these

iv) If n is a positive integer, then $\overline{\big|n+1}$ is equal to

A) $n\overline{\big|n}$ B) $(n-1)\ \overline{\big|n-1}$ C) $n\ \overline{\big|n+1}$ D) $n!$

b. Calculate by double integration the volume generated by the revolution of the cardioid $r = a(1 - \cos\theta)$ about its axis. **(04 Marks)**

c. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\ dxdydz$. **(06 Marks)**

d. Show that $\int_0^{\pi/2}\sqrt{\sin\theta}\ d\theta\ \times \int_0^{\pi/2}\frac{1}{\sqrt{\sin\theta}}d\theta = \pi$. **(06 Marks)**

4 a. Choose your answers for the following : **(04 Marks)**

i) $\vec{F}$ is said to be solenoidal, if

A) $\int_c \vec{F}.d\vec{r} = 0$ B) $\int_c \vec{F}\times d\vec{r} = 0$ C) $\vec{F}\times\vec{r} = 0$ D) none of these

ii) If $\vec{F} = 3xyi + y^2j$ and C is the curve, in the xy-plane, $y = x^2$ from (0, 0) to (1, 1), then $\int F \times dr$ is :

A) Constant B) Variable C) zero D) none of these

iii) Green's theorem in the plane is a special case of

A) Gauss theorem B) Euler's theorem

C) Baye's theorem D) Stoke's theorem.

iv) Stoke's theorem is a relation between

A) a line integral and a surface integral B) a surface and volume integral

C) two volume integrals D) a line and volume integral.

b. If $\vec{F} = 3xyi - y^2j$, evaluate $\int_C \vec{F}.d\vec{r}$ along the curve $y = 2x^2$ in the xy-plane from (0, 0) to (1, 2). **(04 Marks)**

c. Evaluate, by Green's theorem, $\int_C (xy + y^2)\ dx + x^2dy$, where C is bounded by $y = x$ and $y = x^2$. **(06 Marks)**

d. Prove that the cylindrical co-ordinates system is orthogonal. **(06 Marks)**

PART – B

5 a. Choose your answers for the following : **(04 Marks)**

i) Solution of the differential equation $(D^2 + a^2)\,y = 0$ is
A) $C_1e^{ax} + C_2e^{-ax}$ B) C_1 Cosax + C_2 Sinax
C) $(C_1 + C_2x)$ Cos ax D) None of these

ii) P.I. of the differential equation $(D^2 + 3D + 2)\,y = e^x$ is
A) $\frac{1}{6}e^x$ B) $\frac{1}{3}e^x$ C) $\frac{e^x}{2}$ D) e^x

iii) The roots of the A.E with differential equation $(D^3 + 2D^2 - D - 2)\,y = 0$ are
A) (1, 1, -2) B) (-1, 1, -2) C) (1, 1, 2) D) (-1, -1, 2)

iv) C.F of $(D^2 + 1)\,y = x^3$ is
A) $(c_1 + c_2\,x)\,e^x$ B) $(c_1x + c_2)\,e^{-x}$
C) $(c_1\,\mathrm{Cos}x + c_2\,\mathrm{Sin}x)\,e^x$ D) $(c_1\,\mathrm{Cos}x + c_2\,\mathrm{Sin}x)$

b. Solve $(D^3 + 1)\,y = e^x$. **(04 Marks)**

c. Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$. **(06 Marks)**

d. Solve by the method of undetermined co-efficients the equation $y'' + 4y = x^2 + e^{-x}$. **(06 Marks)**

6 a. Choose your answers for the following : **(04 Marks)**

i) The Wronskin of e^x and e^{-x} is
A) 2 B) -1 C) 0 D) -2

ii) To transform $(ax + 1)^2\,y'' + (ax + 1)\,y' + y = \phi(x)$ into a L.D.E with constant coefficients, put t =
A) logx B) log (ax + 1) C) e^t D) x

iii) Solve the initial value problem $x'' + 4x' + 29\,x = 0$ satisfying the conditions $x(0) = 0$, $x'(0) = 15$ is
A) e^{-2t} (3 Sin5t) B) $3e^{-2t}$ C) 3 Sin5t D) none of these

iv) $(C_1 + C_2x)\,e^x$ is the general solution of
A) $(D + 1)^2\,y = 0$ B) $(D - 1)^2\,y = 0$
C) $(D^2 - 1)\,y = 0$ D) $(D^2 + 1)\,y = 0$

b. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. **(04 Marks)**

c. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$, by the method of variation of parameter. **(06 Marks)**

d. Solve the initial value problem $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y + 2\,\mathrm{Cosh}\,x = 0$, given $y = 0$, $\frac{dy}{dx} = 1$ at $x = 0$. **(06 Marks)**

7 a. Choose your answers for the following : **(04 Marks)**

i) Laplace transform of te^{-t} Sint is
A) $\frac{2(s-1)}{(s^2+s+1)^2}$ B) $\frac{s+1}{(s^2-s+1)^2}$ C) $\frac{s+1}{(s^2-s+2)^2}$ D) $\frac{2(s+1)}{(s^2+2s+2)^2}$

ii) Laplace transform of Sin 3t is
A) $\frac{s}{s^2+9}$ B) $\frac{3}{s^2+9}$ C) $\frac{2}{s^2-9}$ D) $\frac{1}{s^2-9}$

iii) Laplace transform of $f'(t)$ is
A) $s\,f(s) - f(0)$ B) $s\,f(s) + f(0)$ C) $s\,f(0) - f'(0)$ D) $s\,f'(0) - f(0)$

iv) Laplace transform of t^3 is equal to
A) $\frac{3!}{s^3}$ B) $\frac{6}{s^2}$ C) $\frac{6}{s^4}$ D) $\frac{5}{s^4}$

b. Find L.T of $e^{2t}\,Cos^2 t$. **(04 Marks)**

c. If f(t) is a periodic function of period 'T', then show that $L\{f(t)\} = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t)\,dt$. **(06 Marks)**

d. Find $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$. **(06 Marks)**

8 a. Choose your answers for the following : **(04 Marks)**

i) Inverse Laplace transform of $\frac{1}{s^2 - a^2}$ is

A) $\frac{Cosat}{a}$ B) Sinat C) Coshat D) $\frac{Sinhat}{a}$

ii) Inverse Laplace transform of $\frac{s+2}{s^2 - 4s + 13}$ is

A) e^{-2t} Cos3t + $\frac{4}{3}e^{2t}$ Sin 3t B) e^{2t} Sin3t + $\frac{3}{4}$ e^{-2t} Cos3t

C) e^{2t} Sin3t $-\frac{4}{3}$ e^{-2t} Cos3t D) e^{2t} Cos3t $+\frac{4}{3}e^{2t}$ Sin3t

iii) Inverse Laplace transform of $\frac{s}{(s^2 + a^2)^2}$ is

A) $\frac{1}{2a}$t Cosat B) $\frac{1}{2a}$t Sinat C) t Cos^2at D) $\frac{t}{2}$Sinat

iv) $L^{-1}\left\{\frac{1}{s^n}\right\}$ is possible only when n is

A) n > 1 B) n ≥ - 1 C) n = 1, 2, ….. D) n < 1.

b. Find the $L^{-1}\left\{\frac{s^2 - 2s + 1}{s^3}\right\}$. **(04 Marks)**

c. Find $L^{-1}\left\{\frac{3s+7}{s^2 - 2s - 3}\right\}$. **(06 Marks)**

d. Applying L.T method, solve x″ - 2x′ + x = e^{2t} given that x(0) = 0 and x′(0) = -1. **(06 Marks)**

* * * * *